Reduced-Order Models: Fundamentals and Applications

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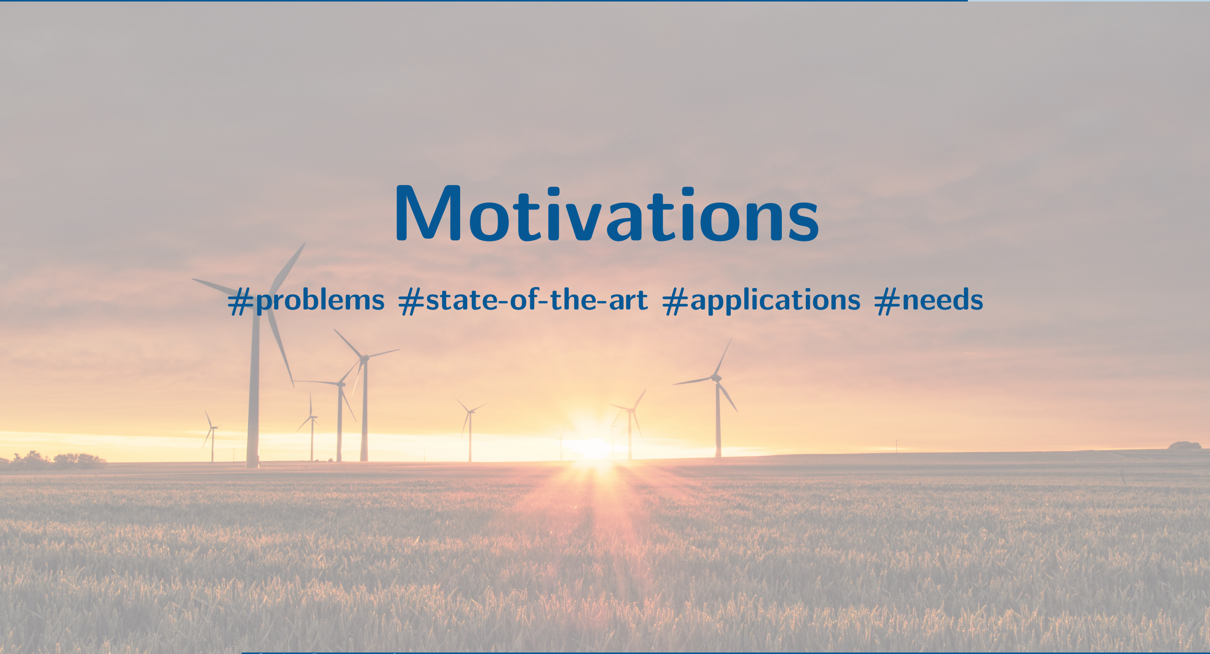
SISSA mathLab: our current efforts, aims and perspectives

A team developing **Advanced Reduced Order Methods** for parametric PDEs with a special focus on **Computational Fluid Dynamics**.

Acknowledgements for the presentation setting: Gianluigi Rozza, Moaad Khamlich, Anna Ivagnes, Francesco Romanò, Pierfrancesco Siena, Marco Tezzele, Caterina Balzotti, Michele Girfoglio, Federico Pichi, Maria Strazzullo, Martina Cracco



Motivations

#problems #state-of-the-art #applications #needs

Leading Motivation: computational science and engineering challenges

Numerical simulations are a booming field for mathematics and computational modelling.

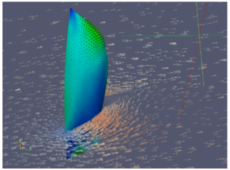
They are able to deal with **multiphysics** problems, as well as systems characterized by **multiple spatial and temporal scales**.

In fact, in the last years, by both **industrial and clinical** research partners there is a growing demand of:

* **efficient computational tools** for
* **many query** and **real time** computations,
* **parametrized formulations**,
* simulations of increasingly **complex systems** with uncertain scenarios

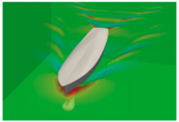
Strategies to speed up computations:

* **High Performance Computing** (HPC)
* **Model Order Reduction** (MOR)

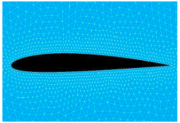


Overview of the physical parametric problems

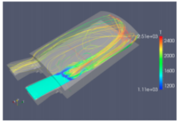
Industrial Flows



Naval Eng.

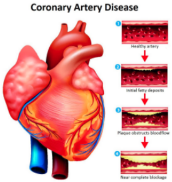


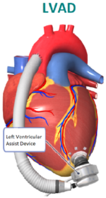
Aeronautics

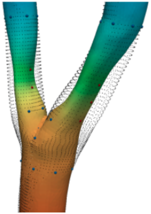


Industrial App.

Biomedical Applications







The fields of application are many: **naval**, **nautical, mechanical, civil**, **aeronautical**, **industrial** as well as **biomedical engineering**.

More in general, the response any application is going to deal with a wide range of parameters, both **dimensionless numbers** and **geometrical parameters**.

SISSA mathLab: our current efforts, aims and perspectives

Goals of our research group:

* to face and overcome **several limitations** of the **state of the art** for parametric **ROM** in CFD;
* to improve capabilities of reduced order methodologies for **more demanding applications** in **industrial**, **medical** and **applied sciences settings**;
* to carry out important methodological developments in **Numerical Analysis**, with special emphasis on mathematical modelling and a more extensive exploitation of **Computational Science and Engineering**
* focus on **Computational Fluid Dynamics** as a central topic to enhance broader applications in **multiphysics** and **coupled settings**, as well as more realistic models (e.g. aeronautical, mechanical, naval, off-shore, wind, sport, biomedical engineering and also cardiovascular surgery planning).



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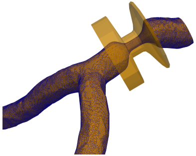
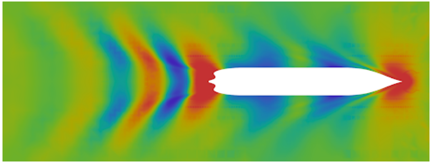
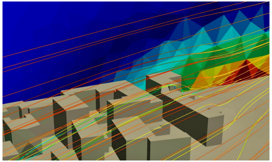
#ROM #Naval #Cardio #Environment

Applications of Reduced Order Models to Naval, Biomedical and Environmental fields







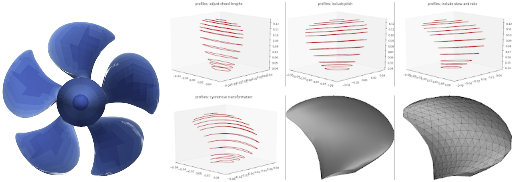
Motivations: Naval Projects

SOPHYA

Seakeeping Of Pleaning Hull YAchts

(in cooperation with MONTECARLOYACHTS and MICAD)

The main goal is to obtain a **hull of optimal shape** to improve the **seakindness and comfort** of the planing hulls in non calm sea conditions.



Python package developed: BladeX

prelica

Reduction of noise generated by propellers

(in cooperation with EFLUIDS and CETENA)

The aim of the project was to predict and reduce the **hydro-acoustic noise** generated by **propeller blades** and obtain a reliable parametrization and deformation of the blade.

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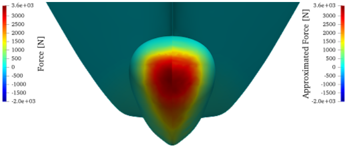
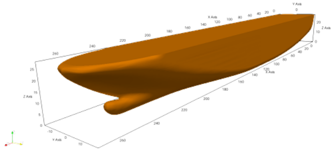
Motivations: Naval Projects

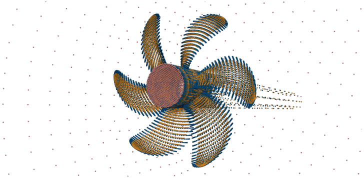
Data-driven Hull Shape Optimization System

(in cooperation with FINCANTIERI)

The aim was to develop an efficient shape optimization framework to **reduce the hydrodynamic ship resistance**. The aim is to achieve a remarkable reduction of the fuel consumption in the new cruise ships by optimized **hull shapes**.







Python packages maintained: BladeX PyGeM

Data-driven ROMs for blade shape optimization

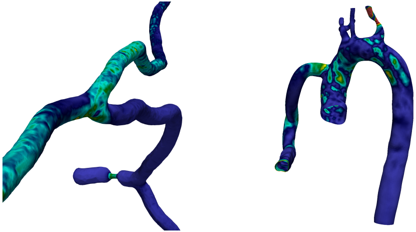
(in cooperation with FINCANTIERI)

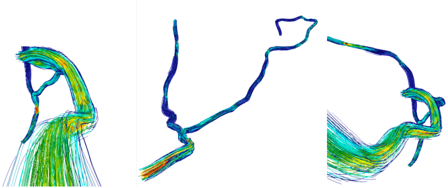
The aim of the project is to develop the shape that maximizes the **efficiency** of the **propeller** in cruise ships, avoiding the erosion and the cavitation phenomenon (formation of vapor-filled cavities).

Motivations: Biomedical Projects

Data-driven ROMs for patient-specific data.
(in cooperation with Ospedale Luigi Sacco)

The aim was to develop an efficient ROM framework to reduce the computational time, in order to achieve a **real time evaluation** to choose the best **surgical plan**.





Data-driven ROMs for optimal control problem.
(in cooperation with University of HOUSTON)

The aim of the project is to develop a framework with the control on the **outlet boundary conditions**.

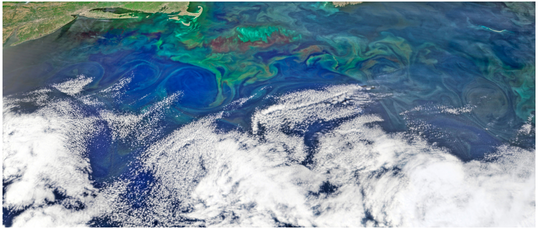
P. Siena, M. Girfoglio, F. Ballarin, G. Rozza, "Data-driven reduced order modelling for patient-specific hemodynamics of coronary artery bypass grafts with physical and geometrical parameters", 2022, arXiv preprint arXiv:2203.13682.

C. Balzotti, P. Siena, M. Girfoglio, A. Quaini, G. Rozza, "A data-driven Reduced Order Method for parametric optimal blood flow control: application to coronary bypass graft", 2022, arXiv preprint arXiv:2206.15384

Motivations: Environmental Projects

ROMs for atmospheric and ocean flows
(in cooperation with University of HOUSTON)

The interest is in **geophysical flows** for **ocean** and **weather forecast**. We propose a framework which combines **ROM** and **LES**, and leads to a **Speed-up** ≈ 100 for **long time predictions**.



Pollutant Control in the Gulf of Trieste

The goal is to monitor, manage and predict dangerous marine phenomena in a **fast way**. In particular we want to keep the pollutant loss under a safeguard **threshold**.

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Overview

#technology #software #webcomputing #offline-online #fast-computing

Intrusive Reduced Order Methods: a brief overview

- $()_h$: **Full Order Methods** (FEM, FV, FD, SEM) are **high fidelity solutions** - to be **accelerated**;
- $()^{ROM}$: **Reduced Order Methods** (ROM) - the **accelerator**.

- Input parameters:

$$\boldsymbol{\mu} \text{ (geometry, physical properties, etc.)}$$

- Parametrized PDE:

$$\mathcal{A}(u(\boldsymbol{\mu}); \boldsymbol{\mu}) = r(\boldsymbol{\mu}) \quad \rightsquigarrow \quad \underbrace{\boldsymbol{A}_h(\boldsymbol{\mu})\boldsymbol{u}_h(\boldsymbol{\mu}) = r_h(\boldsymbol{\mu})}_{\text{full order}} \quad \rightsquigarrow \quad \underbrace{\boldsymbol{A}^{ROM}(\boldsymbol{\mu})\boldsymbol{u}^{ROM}(\boldsymbol{\mu}) = r^{ROM}(\boldsymbol{\mu})}_{\text{reduced order}}$$

- Output:

$$u(\boldsymbol{\mu}) \quad \approx \quad \underbrace{\mathbf{u}_h(\boldsymbol{\mu})}_{\text{full order}} \quad \approx \quad \underbrace{\mathbf{u}^{ROM}(\boldsymbol{\mu})}_{\text{reduced order}}$$

- Input-Output evaluation: (black-box)

$$\boldsymbol{\mu} \quad \rightarrow \quad \mathbf{u}_h(\boldsymbol{\mu}) \quad \rightarrow \quad \mathbf{u}^{ROM}(\boldsymbol{\mu})$$

J. S. Hesthaven, G. Rozza, B. Stamm., "Certified Reduced Basis Methods for Parametrized Partial Differential Equations", SpringerBriefs in Mathematics, Springer, 2015

P. Benner, W. Schilders, S. Grivet-Talocia, A. Quarteroni, G. Rozza, L. M. Silveira , "Model Order Reduction, Volume 1,2,3", De Gruyter, 2020

J. S. Hesthaven, C. Pagliantini, G. Rozza, "Reduced basis methods for time-dependent problems", Acta Numerica, 2022

G. Rozza, G. Stabile, F. Ballarin, "Advanced Reduced Order Methods and Applications in Computational Fluid Dynamics", SIAM Press, 2022

Intrusive Reduced Order Methods: a brief overview

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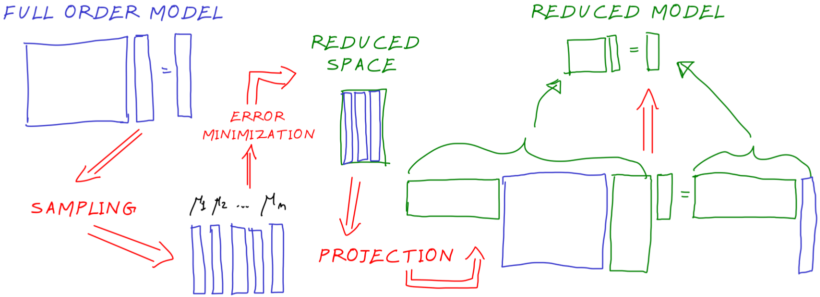

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ROMs: libraries developed at SISSA mathLab (software)

* Development of new high-quality **open source software libraries** [1], that can be freely employed and extended by anyone around the world, and are now the basis of teaching activities, collaborations and projects with several academic and industrial partners.
* High-level integration of newly developed reduced order methods with an already existing computational pipeline, in order to propose to use all the scientific developments carried out in the **ERC AROMA-CFD project** as an add-on onto the existing computational pipeline, rather than a completely different one.

[1] mathlab.sissa.it/cse-software

Historical Perspective

#POD #RB #continuation
#timeline #wind-engineering #pioneers

Acknowledgements: Luca Bruno

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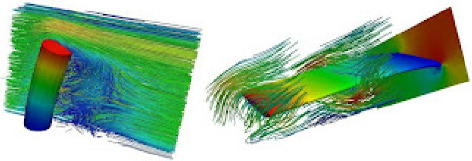
Model Order Reduction: Historical Perspectives

Two **historical approaches**:

- **Proper Orthogonal Decomposition** (POD) to simulate time dependent flows and as efficient way to model turbulent flows [1] [2]
- **Reduced Basis Methods** as continuation methods in non-linear structural mechanics [3] [4] [5].

At the state of the art these two approaches are also merged.

- Other methods: **Principal Generalized Decomposition** (PGD), **Hierarchical Model Reduction** (HMR). [6] [7]



[1] **Ravindran, S. S.** (2000). A reduced-order approach for optimal control of fluids using proper orthogonal decomposition. International journal for numerical methods in fluids, 34(5), 425-448.

[2] **Sirovich, L.** (1987). Turbulence and the dynamics of coherent structures. I. Coherent structures. Quarterly of applied mathematics, 45(3), 561-571.

[3] **Noor, A. K., & Peters, J. M.** (1980). Reduced basis technique for nonlinear analysis of structures. AIAA Journal, 18(4), 455-462.

[4] **Noor, A. K.**, (1982). On making large nonlinear problems small, Computer Methods in Applied Mechanics and Engineering 34, 955 – 985.

[5] **Fox, R.L. and Miura, H.**, (1971). An approximate analysis technique for design calculations, AIAA Journal 9, 174–179.

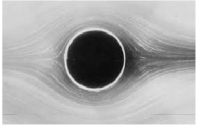
[6] **Benner, P., Schilders, W., Grivet-Talocia, S., Quarteroni, A., Rozza, G., & Miguel Silveira, L.** (2020-2021). Model Order Reduction: Volumes 1,2,3, De Gruyter.

[7] **Stein, E., De Borst, R., Hughes, T. J.** (2004). Encyclopedia of computational mechanics, *Model Order Reduction*, Wiley.

[8] **Chinesta, Huerta, Rozza, Willcox**, Encyclopedia Comp. Mechanics, 2015

Historical contributions and challenges

- After few attempts in the 80's, only after 2000's, when **computational resources** were available, there was a **new era for Reduced Order Methods**[a].
- **Several research groups** developed in Europe and North America in aeronautics/astronautics/mechanical engineering department/applied mathematics focused on ROMs.



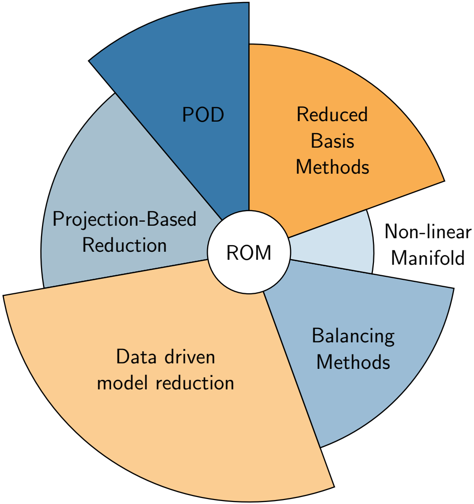

- Nowadays several initiatives at the European level contributed in the consolidation of the **community (COST EU-MORNET)**.
- In ECCOMAS/SIAM congresses several sessions are now dedicated to ROMs and applications.

[a]There was **lack of computational power** for offline steps because Numerical Analysis was still far to provide efficient tools.

ROM in Math and Numerical Analysis

1971	Fox & Miura
1978	Almroth, Stern & **Brogan**
1980s	Noor
1983	Fink & **Rheinbold**
1987	Porshing & **Lee**
1989	Peterson (CFD)
1996	Balmes
1998	Ito & **Ravindran**
1999	Christensen, Brons, **Sorensen**
2000	Patera, Maday et al
2005	Kunish, Volkwein et al **Willcox, Farhat et al**

ROM developments



Snapshots - based approaches (off-line)

(On-line)

P. Benner, W. Schilders, S. Grivet-Talocia, A. Quarteroni, G. Rozza, L. M. Silveira , "Model Order Reduction, Volume 1,2,3", De Gruyter, 2020

Building the Reduced Space: POD vs RB

Full Order Model: $\boldsymbol{A}_h(\boldsymbol{\mu})\boldsymbol{u}_h(\boldsymbol{\mu}) = r_h(\boldsymbol{\mu})$ with $\mathbf{u}_h \in \mathbb{V}_h$ and $dim(\mathbb{V}_h) = N_h$

Parameters in a training set: $\boldsymbol{\mu}_i \in [\boldsymbol{\mu}_1, \dots, \boldsymbol{\mu}_{N_t}]$

Proper Orthogonal Decomposition:

- **Solve FOM** for all training set $\boldsymbol{u}_h^i = \boldsymbol{u}_h(\boldsymbol{\mu}_i)$
- **Eigenvalue problem**:

$$\boldsymbol{C}_{ij} = (\boldsymbol{u}_h^i)^T \boldsymbol{M} \boldsymbol{u}_h^j \quad \rightarrow \quad \boldsymbol{C}\boldsymbol{V} = \boldsymbol{V}\boldsymbol{\lambda}$$

- Keep the most energetic N_r **POD modes**:

$$\xi_i(\boldsymbol{x}) = \frac{1}{N_t}\sum_{j=1}^{N_t} \boldsymbol{V}_{ji}\boldsymbol{u}_h^j(\boldsymbol{x}) \text{ for } i = 1, \dots, N_r$$

$$\Xi = [\xi_1, \cdots, \xi_{N_r}] \in \mathbb{R}^{N_h \times N_r}$$

Optimal in $||\cdot||_2$ sense

Reduced basis:

- We provide an **error estimator** $\eta(\boldsymbol{\mu}, \Xi)$
- $\Xi := \{\boldsymbol{u}_h(\boldsymbol{\mu}_0)\}$
- Iteratively
  - Find largest error

$$\eta(\boldsymbol{\mu}^*, \Xi) = \max_i \eta(\boldsymbol{\mu}^i, \Xi)$$

  - Expand

$$\Xi =: \{\Xi, \boldsymbol{u}_h(\boldsymbol{\mu}^*)\}$$

Optimal in $||\cdot||_\infty$ sense

- **Reduced solution**: $\boldsymbol{u}^{ROM}(\mu) = \sum_{i=0}^{N_r} u_i^R(\mu)\xi_i(\boldsymbol{x})$, $\boldsymbol{u}^{ROM}(\mu) = \Xi\boldsymbol{u}^R(\mu)$ for $\boldsymbol{u}^R \in \mathbb{R}^{N_r}$

J. S. Hesthaven, G. Rozza, B. Stamm., "Certified Reduced Basis Methods for Parametrized Partial Differential Equations", SpringerBriefs in Mathematics, Springer, 2015

Reducing the model

Full Order Model: $\boldsymbol{A}_h(\boldsymbol{\mu})\boldsymbol{u}_h(\boldsymbol{\mu}) = r_h(\boldsymbol{\mu})$ with $\mathbf{u}_h \in \mathbb{V}_h$ and $dim(\mathbb{V}_h) = N_h$
Reduced space: Ξ

Galerkin projection:

- **Reduced solution**:

$$\boldsymbol{u}^{ROM}(\boldsymbol{\mu}) = \sum_{i=0}^{N_r} u_i^R(\boldsymbol{\mu})\xi_i(\boldsymbol{x}) = \Xi\boldsymbol{u}^R$$

- **Project on the left**:

$$\underbrace{\Xi^T \boldsymbol{A}_h(\boldsymbol{\mu})\Xi}_{\boldsymbol{A}^{ROM}(\boldsymbol{\mu})\in\mathbb{R}^{N_r\times N_r}} \ \boldsymbol{u}^R(\boldsymbol{\mu}) = \underbrace{\Xi^T r_h(\boldsymbol{\mu})}_{r^{ROM}(\boldsymbol{\mu})}$$

- **ROM** $\boldsymbol{A}^{ROM}(\boldsymbol{\mu})\boldsymbol{u}^R(\boldsymbol{\mu}) = r^{ROM}(\boldsymbol{\mu})$

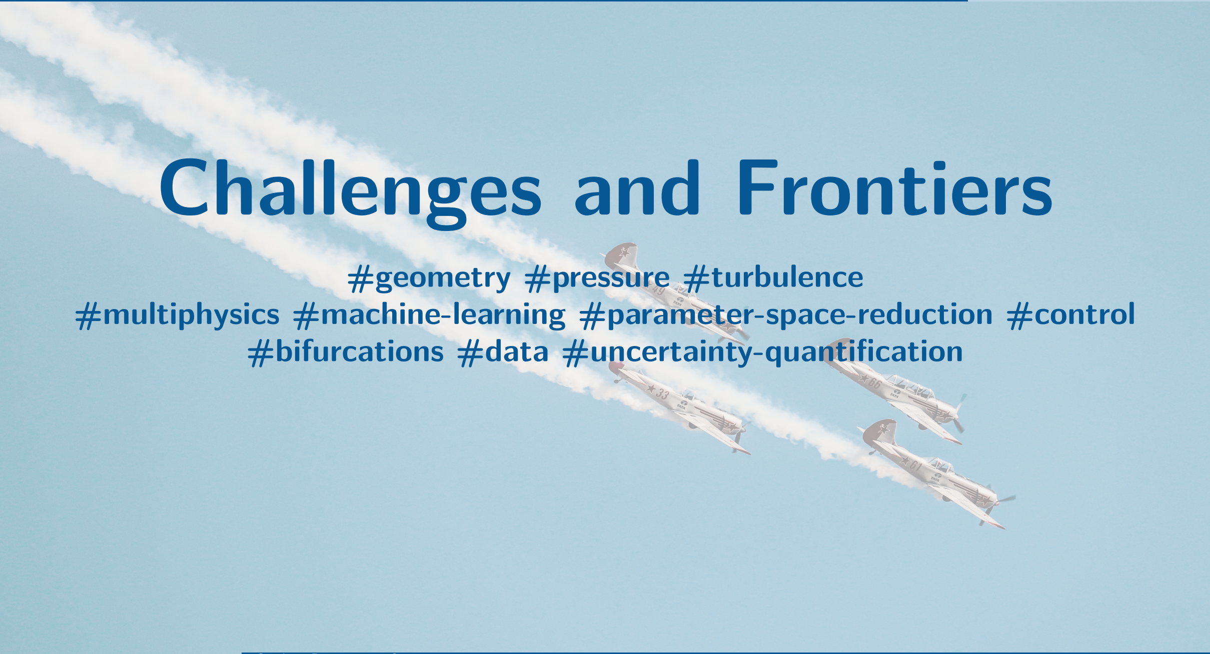
Regression technique:

- Find some values for the **map**

$$\boldsymbol{\mu} \rightarrow \boldsymbol{u}^R(\boldsymbol{\mu})$$

  for example on training set $\boldsymbol{u}^R(\boldsymbol{\mu}^i)$
- **Approximate** the map $\boldsymbol{u}^R(\boldsymbol{\mu})$ with $\hat{\boldsymbol{u}}^R(\boldsymbol{\mu})$
  - Neural Networks
  - Interpolation functions
  - Least square approximations
  - ...
- **ROM** is the evaluation of $\hat{\boldsymbol{u}}^R(\boldsymbol{\mu})$

J. S. Hesthaven, G. Rozza, B. Stamm., "Certified Reduced Basis Methods for Parametrized Partial Differential Equations", SpringerBriefs in Mathematics, Springer, 2015



Challenges and Frontiers

#geometry #pressure #turbulence
#multiphysics #machine-learning #parameter-space-reduction #control
#bifurcations #data #uncertainty-quantification

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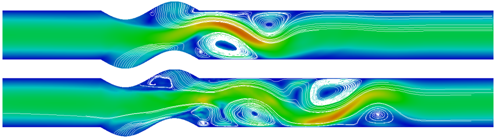
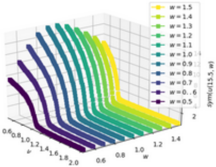
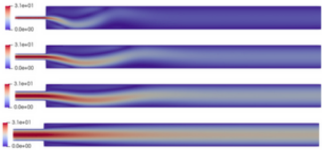
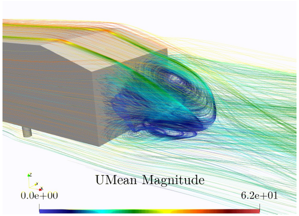
Strategic Fields of development for ROMs

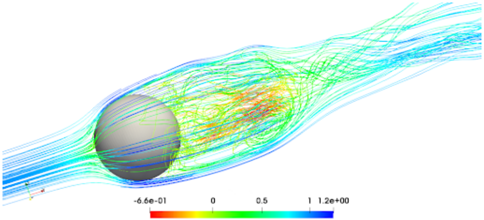
- **Geometric Parametrization** (shape design)
- Increasing Reynolds number
- Improve **Pressure recovery** (accuracy)
- **Coupling** multi-physics
- **Automatic-learning** developments
- **Turbulence** (ROMs for RANS and LES)
- **Parameter space reduction**
- Flow **control** and data assimilation
- **Bifurcations** and flow stability









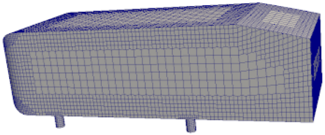


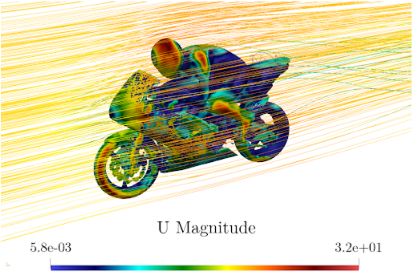
ROM challenges in CFD

Classical Discretization techniques:

FEM (Finite Element Method)	**FVM** (Finite Volume Method)

increasing Reynolds number →





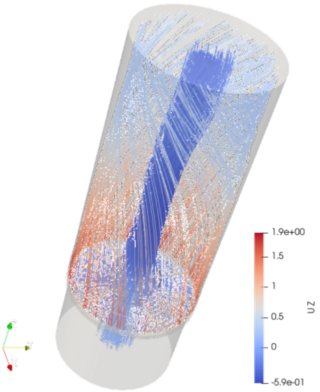
Key problems:

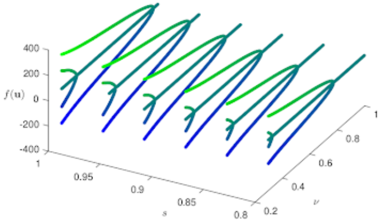
- **Stabilization issues** and fulfillment of *inf-sup* condition
  - stabilization approaches (SUPG, variational multi-scale approach)
  - SUP-ROM: addition of supremizer modes to enrich the velocity space
  - PPE-ROM: replacement of the continuity equation with the pressure Poisson equation
- **Turbulence treatment**
- Reliability of classical and reduced discretization methods in **real world applications**

ROM challenges in CFD

Other options:

SEM (Spectral Element Method)	**DG** (Discontinuous Galerkin)





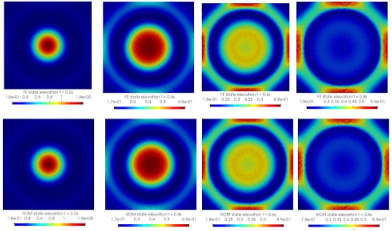
Key problems:

* Applications to **compressible flows**
* **Bifurcations** and study of flow stability
* Study of **multiphysics problems** (Fluid-Structure Interaction, heat transfer, ...)

Stabilization techniques for FEM-based ROM

FEM (Finite Element Method)

- appropriate selection of velocity and pressure spaces to fulfill the **inf-sup condition** $(\mathbb{P}_k - \mathbb{P}_{k-1})$ [2] : supremizers
- spurious oscillation terms in **advection-dominated** problems
- introduction of stabilization methods, as **SUPG** (Streamline Upwind Petrov–Galerkin) for $\mathbb{P}_k - \mathbb{P}_k$ spaces, $k \geq 0$. OFFLINE-ONLINE stabilization.



[1]**What is the *inf-sup* condition?**

Consider the saddle-point problem:

Find u in V, p in Q such that, $\forall$v in V and $\forall$ q in Q:

$$\begin{cases} a(u,v) + b(v,p) = \langle f, v \rangle \\ b(u,q) = \langle g, q \rangle . \end{cases}$$

The inf-sup condition is the sufficient condition for the **uniqueness** of the solution (u,p). It is defined as:

$$\inf_{q \in Q, q \neq 0} \sup_{v \in V, v \neq 0} \frac{b(v,q)}{\|v\|_V \|q\|_Q} \geq \beta,$$

$\beta \geq 0$.

Rozza, G., & Veroy, K. (2007). On the stability of the reduced basis method for Stokes equations in parametrized domains. Computer methods in applied mechanics and engineering, 196(7), 1244-1260.

Stabilization techniques for FV-based ROM

FVM (Finite Volume Method)

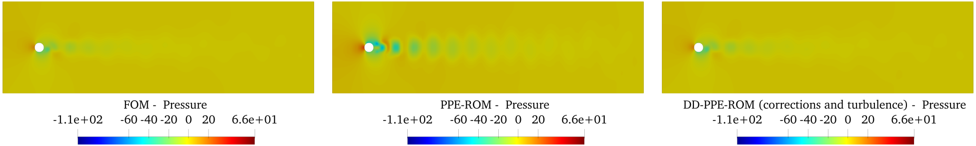
- lack of accuracy in pressure recovery with standard POD-Galerkin approaches
- stabilization techniques:
  - **SUP-ROM**: introduction of supremizer modes $\mathbf{s}_i = \mathbf{s}(\chi_i)$ (where χ_i are the pressure modes) such that:

$$\begin{cases} \Delta \mathbf{s}_i = -\nabla \chi_i \text{ in } \Omega \\ \mathbf{s}_i = \mathbf{0} \text{ on } \partial\Omega \end{cases}$$

  - **PPE-ROM**: replacement of the continuity equation with the pressure Poisson equation

$$\nabla \cdot \mathbf{u} = 0 \rightarrow \Delta p = -\nabla \cdot (\nabla \cdot (\mathbf{u} \otimes \mathbf{u}))$$

- **Variational Multi-Scale** ROM approach: introduction of data-driven *correction* terms in the reduced system.



Stabilization techniques: the Variational Multi-Scale (VMS) ROM approach

When building a Reduced Order Model, different modal regimes can be chosen:

- **under-resolved** regime: fewer ROM modes than the number required to accurately approximate the dynamics of the flow;
- **marginally-resolved** regime: the number of ROM basis functions is enough to represent the main features of the system, but **the standard G-ROM yields inaccurate approximations**;
- **fully-resolved** regime: the number of modes is enough to represent the underlying dynamics of the system.

How to improve the accuracy in the marginally-resolved regime?

Include the contribution of the neglected modes in the ROM by adding extra **correction/closure** terms

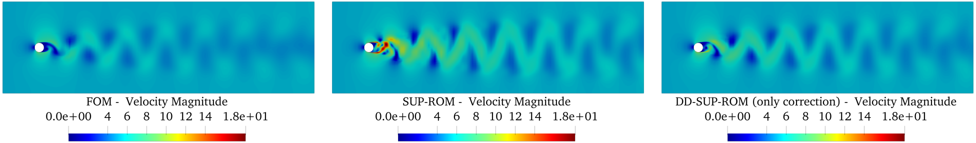
Standard ROM

$$\begin{cases} \dot{\mathbf{a}} = \boldsymbol{f}(\mathbf{a}, \mathbf{b}) \,, \\ \boldsymbol{h}(\mathbf{a}, \mathbf{b}) = \mathbf{0} \,. \end{cases}$$

DD-VMS-ROM

$$\begin{cases} \dot{\mathbf{a}} = \boldsymbol{f}(\mathbf{a}, \mathbf{b}) + \tau_u \,, \\ \boldsymbol{h}(\mathbf{a}, \mathbf{b}) = \tau_p \,, \end{cases}$$

where **a** and **b** are the reduced coefficient vectors for velocity and pressure.



Ivagnes, A., Stabile, G., Mola, A., Iliescu, T., & Rozza, G. (2022). Pressure Data-Driven Variational Multiscale Reduced Order Models. arXiv preprint arXiv:2205.15118.

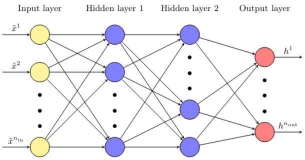
Artificial Intelligence: CSE applications with neural networks

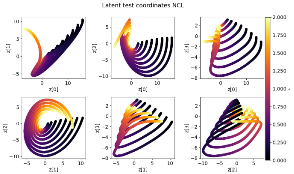
Artificial Intelligence can be applied in several fields of **Computational Fluid Dynamics** and **Model Order Reduction**.

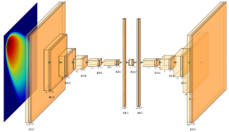
Feedforward Neural Networks

Used as **regression technique** for:

- approximation in reduced order models (POD-NN)
- inclusion of **turbulence modelling** in non-intrusive reduced order models







Autoencoders

- dimensionality reduction
- features extraction

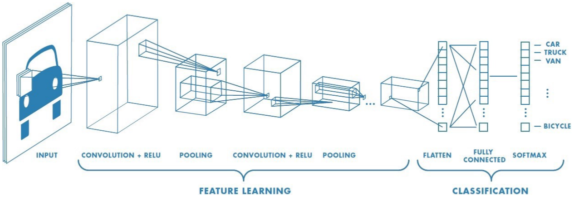
Meneghetti, L., Demo, N., & Rozza, G. (2021). A Dimensionality Reduction Approach for Convolutional Neural Networks. arXiv preprint arXiv:2110.09163.

Romor, F., Stabile, G., & Rozza, G. (2022). Non-linear manifold ROM with Convolutional Autoencoders and Reduced Over-Collocation method. arXiv preprint arXiv:2203.00360.

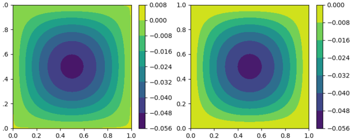
Artificial Intelligence: physics informed CSE applications

Convolutional Neural Networks

- object detection
- image recognition



PINA: a Python package for the approximated resolution of PDEs and ODEs with physics-informed neural networks.
https://github.com/mathLab/PINA



Analytical solution (left), PINN solution (right) to a Poisson problem.

Physics-Informed Neural Networks

- inclusion of the physical laws and governing equations in the loss function
- strategy to approximately solve PDEs

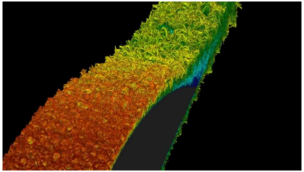
Meneghetti, L., Demo, N., Rozza, G., & Sonego, M. Convolutional Neural Networks for object detection in professional appliances.

Demo, N., Strazzullo, M., & Rozza, G. (2021). An extended physics informed neural network for preliminary analysis of parametric optimal control problems. arXiv preprint arXiv:2110.13530.

ROM for turbulent flows (Zancanaro et al.)

Why not a Direct Numerical Simulation?

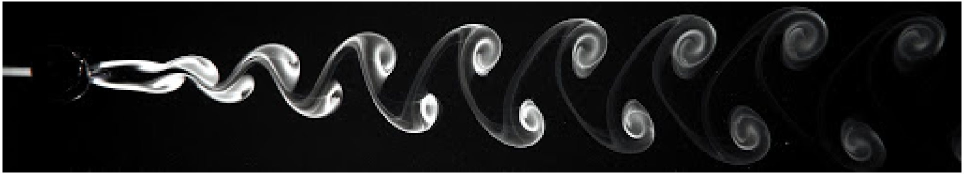
- **huge dimension** of the problem is required;
- **computational time** gets unaffordable;
- for many applications such an accuracy is **not needed**.



Involved conservation laws: **Reynolds averaged Navier-Stokes equations**

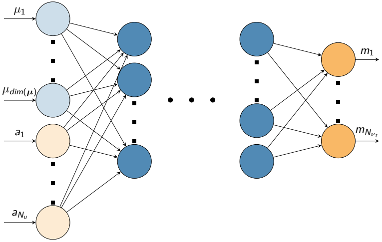
$$\begin{cases} \nabla \cdot (\overline{\boldsymbol{u}} \otimes \overline{\boldsymbol{u}}) - (\nu + \nu_t)\, \nabla^2 \overline{\boldsymbol{u}} + \nabla \overline{p} + \dfrac{2}{3} \nabla k = 0 \\ \nabla \cdot \overline{\boldsymbol{u}} = 0 \end{cases} \tag{1}$$

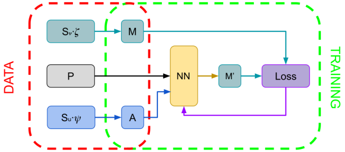
where $\nu = \mu/\rho$ is the kinematic viscosity, ν_t stands for the eddy viscosity while k represents the turbulent kinetic energy, being $\overline{\boldsymbol{u}} \in \mathbb{V}_h$ and $\overline{p} \in \mathbb{Q}_h$.



Turbulent flows - Turbulence treatment

The reconstruction of the **eddy viscosity field** can be done with a fully-connected **neural network** [2].





Reduced eddy viscosity field:

$\nu_{t\,r} = \sum_{i=1}^{N_{\nu_t}} m_i(\boldsymbol{x})\zeta_i(\boldsymbol{\mu})$, where:

- $\mathbf{m} = (m_i)_{i=1}^{N_{\nu_t}}$ is the eddy viscosity coefficient vector;
- $(\zeta_i)_{i=1}^{N_{\nu_t}}$ are the eddy viscosity modes.

Mapping from known modes:

$$\mathcal{F} : \mathbb{R}^{dim(\boldsymbol{\mu})+N_u} \to \mathbb{R}^{N_{\nu_t}}, \quad \mathcal{F}(\boldsymbol{\mu}, \overline{\boldsymbol{a}}(\boldsymbol{\mu})) = \boldsymbol{m}$$

Neural network mapping:

$\tilde{\boldsymbol{m}} = \mathcal{F}_{network}(\boldsymbol{\mu}, \overline{\boldsymbol{a}}(\boldsymbol{\mu}))$, with $\mathcal{F}_{network}$ regression map

Loss function of the neural network:

$$\mathcal{L} = \sum_{i=1}^{N_{\nu_t}} ||\boldsymbol{m}_i - \tilde{\boldsymbol{m}}_i||_{L^2}$$

[2] **Zancanaro M., Mrosek M., Stabile G., Othmer C., Rozza G.**, (2021), *Hybrid neural network reduced order modelling for turbulent flows with geometric parameters*, MDPI-Fluids

Conclusions

- It is time to better integrate **Data, Modelling, Analysis, Numerics, Control, Optimization and Uncertainty Quantification** in a new **parametrized, reduced and coupled paradigm**;
- we need to draw the attention to the fact that **"Science and Engineering could advance with Mathematics (CSE)"**;
- **Applied Mathematics as propeller for methodological innovation and technology transfer** by a new generation of talented computational scientists.

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- COST, European Union Cooperation in Science and Technology, TD 1307 EU-MORNET Action (http://www.eu-mor.net)
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- SMACT;
- FSE FVG;