Integrable Systems: Geometrical and Analytical Approaches

Monday, June 3, 2024 - Friday, June 7, 2024

SISSA

Book of Abstracts

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Asymptotic analysis of integrable dispersive PDEs: random multisoliton solutions

Author: Kenneth McLaughlin[1]

[1] *Tulane University*

Corresponding Author: kmclaughlin@tulane.edu

I will present a series of examples of soliton gasses, in which the interplay of Riemann-Hilbert analysis and randomness provides a detailed description of interesting phenomena. Time will be reserved to discuss future research challenges. Our research group includes Manuela Girotti (Emory), Aikaterini Gkogkou (Tulane), Tamara Grava (SISSA), Robert Jenkins (University of Central Florida), Guido Mazzuca (Tulane), Alexander Minakov (Prague), and Maxim Yattselev (IUPUI), and I will be sampling from work with various subsets of the team.

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Bargmann gas solution to the mKdV equation

Author: Manuela Girotti[1]

[1] *Emory University*

Corresponding Author: manuela.girotti@emory.edu

We construct a new class of (gas) solutions of the mKdV equation as a limit of $2N$ solitons, eventually in the presence of an additional dispersive coefficient. We also analyze the long-time behaviour of the profile: the solution is asymptotically an elliptic solution at both $x \to \pm\infty$, with same parameters, but different shift $x_0^{\pm}$.
This is a joint work with Ken McLaughlin (Tulane U.) and Robert Jenkins (UCF).

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Exactly solvable interacting particle systems

Author: Chiara Franceschini[1]

[1] *UniMoRe*

Corresponding Author: chiara.franceschini@unimore.it

In statistical physics systems of interacting particles can be used as toy models to study non equilibrium phenomena. A way to do that is to place the system in contact with an external environment that generates a current through it, place the system out of equilibrium and so reversibility is lost. In these boundary driven models the first natural question one would like to answer regards the stationary measure: stationary states are considered the simplest beyond equilibrium but are, in general, not known explicitly. I will present two models recently introduced where the stationary measure can be obtained.
Joint work with: Gioia Carinci, Rouven Frassek, Davide Gabrielli, Cristian Giardinà, Frank Redig, Dimitrios Tsagkarogiannis.

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On some tautological relations between decorated trees and the DR/DZ conjecture

Author: Danilo Lewanski[None]

Corresponding Author: danilo.lewanski@units.it

We study some relations between trees decorated by the double ramification cycles times \lambda_g and trees decorated by the Omega class, via virtual localisation. We then employ virtual localisation to deduce some polynomiality properties of the Omega class. If time permits, some connections with the DR/DZ conjecture will be discussed.

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Bi-flat F-manifolds, Frolicher Nijenhuis Bicomplexes and Integrable Systems

Author: Paolo Lorenzoni[1]

[1] *Università di Milano-Bicocca*

Corresponding Author: paolo.lorenzoni@unimib.it

We introduce some geometric structures related to the notion of bi-flat F-manifolds and we show their applications to the theory of Integrable Systems.
Based on joint works with Alessandro Arsie, Franco Magri and Sara Perletti.

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Gradient blowups in multidimensional Burgers-Hopf equation

Author: Giovanni Ortenzi[1]

[1] *Università di Milano Bicocca*

Corresponding Author: giovanni.ortenzi@unimib.it

The dust equation, also known as homogeneous or pressureless Euler equation (Zel'dovich, 1970), is a toy-model for the fluid dynamic Euler equation in which pressure is neglected. This PDE is the multidimensional analogue of the Burgers-Hopf equation and is a good training ground for the analysis of the catastrophe structures in more than one spatial dimension. In the talk, the gradient blowups for this model are characterized with particular attention to the behavior of the vorticity and the effects of external fields.

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Maxwell–Bloch equations without spectral broadening: the long-time asymptotics of an input pulse in a long two-level laser amplifier

Authors: Oleksandr Minakov[1]; Volodymyr Kotlyarov[None]

[1] *Charles University*

Corresponding Author: minakov.ilt@gmail.com

We study the problem of propagation of an input electromagnetic pulse through a long two-level laser amplifier under trivial initial conditions. we consider an unstable model described by the Maxwell–Bloch equations without spectral broadening. We obtain rigorous asymptotic results at large times, in two regions: in a region near the light cone and in the tail region. The region near the light cone is described by pulses of narrowing width and growing amplitude, while in the tail region, a new type of solitons is found against an oscillating background.

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Generalized Hydrodynamics for the Volterra lattice

Author: Guido Mazzuca[1]

[1] *Tulane University*

Corresponding Author: gmazzuca@tulane.edu

Despite needing a complete mathematical foundation, the theory of Generalized Hydrodynamics has been used to obtain precise approximations of the correlation functions for several integrable models. For example, H. Spohn used this theory to compute the correlation function of the Toda lattice.

We consider another integrable model, namely the Volterra lattice. We introduce the Generalized Gibbs Ensemble for this lattice and relate it to the Anti-symmetric β ensemble, a classical random matrix model for anti-symmetric matrices. This connection allows us to explicitly compute the density of states for the Volterra lattice in terms of the random matrix ensemble one. Given this density, we can apply the theory of GHD to obtain a linear approximation of the correlation function for our model.

This talk is mainly based on the following paper:

- G. M., \textit{Generalized Hydrodynamics for the Volterra lattice: ballistic and nonballistic behavior of correlation functions.} arXiv preprint: 2404.08499

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Biorthogonal measures, polymer partition functions, and random matrices.

Author: Mattia Cafasso[None]

Corresponding Author: mattia.cafasso@univ-angers.fr

In this talk, I will describe a particular class of biorthogonal measures related to discrete and semi-discrete polymers (Log-Gamma, O'Connell-Yor, and mixed). More precisely, I will show that the Laplace transform of the partition function of the mentioned polymers coincides with the multiplicative statistics of these biorthogonal measures. This result can be seen as a finite N variant of the connection between the narrow wedge solution of the KPZ equation and the Airy point process. It generalizes previous results of Imamura and Sasamoto for the (homogeneous) O'Connell-Yor polymer. Time permitting, I will show some applications to the small-temperature limit of these

polymers and their relation with matrix models. These results have been obtained jointly with Tom Claeys.

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Quantum intersection numbers on the moduli space of curves and the Gromov-Witten invariants of the projective line

Author: Alexandr Buryak[1]

[1] *HSE Moscow & Skoltech*

Corresponding Author: buryaksh@gmail.com

I will talk about our recent joint work with Xavier Blot, where we related the quantum intersection numbers on the moduli space of curves to the stationary relative Gromov-Witten invariants of the projective line with an insertion of a Hodge class.
As a corollary, this gives a new geometric interpretation of the standard intersection numbers of psi-classes on the moduli space of curves. We also give a new, much shorter proof, of an explicit formula for the "purely quantum" intersection numbers, which was obtained before by Xavier Blot, and which relates these numbers of the one-part double Hurwitz numbers.

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The extended Ablowitz-Ladik hierarchy and a generalized Frobenius manifold

Author: Youjin Zhang[1]

[1] *Tsinghua University*

Corresponding Author: youjin@tsinghua.edu.cn

We define a certain extension of the Ablowitz-Ladik hierarchy, and show that it possesses a tau structure and Virasoro symmetries. We prove that this integrable hierarchy coincides with the topological deformation of the Principal Hierarchy of a generalized Frobenius manifold with non-flat unity.

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Affine Gaudin Bethe Ansatz and the ODE/IM correspondence

Author: Davide Masoero[1]

[1] *Lisbon University*

Corresponding Author: dmasoero@gmail.com

We consider the monodromy-free opers corresponding to solutions of the Affine Gaudin Bethe Ansatz equations.
We define and study the spectral determinants (called Q functions) for these opers. We conjecture that the Q-functions obtained from the Affine Gaudin Bethe Ansatz coincide with the Q-functions of the Bazhanov-Lukyanov-Zamolodchikov opers with the monster potential, which are related to

the quantum KdV flows according to the ODE/IM correspondence. We give supporting evidence for this conjecture.

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Discrete Painlevé equations in random partitions and planar maps

Author: Sofia Tarricone[1]

[1] *IPhT, CEA Paris-Saclay*

Corresponding Author: sofia.tarricone@ipht.fr

The aim of this talk is to explain the connections between the discrete Painlevé I-II equations and certain random and combinatoric models. In the first part of the talk, we will start from a classical result by Borodin, which allows us to calculate the probability distributions of the first parts of random partitions with Poissonized Plancherel measure via a recurrence relation using solutions of the discrete Painlevé II equation. Then, we will see a generalization of this result, that we proved with T. Chouteau, in the case of the so-called "multicritical" Schur measures. In the second part, we will discuss instead the appearance of the discrete Painlevé I equation in the context of counting problems for planar quadrangulation with given geodesic distance and how to explain it from an orthogonal polynomials perspective (work in progress with J. Bouttier).

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On Dubrovin-Novikov brackets

Author: Guido Carlet[1]

[1] *Université de Bourgogne*

Corresponding Author: gcarlet@gmail.com

Dubrovin and Novikov initiated the study of local homogeneous differential-geometric Poisson brackets of arbitrary degree k in their seminal 1984 paper. Despite several results in low degree, very little is known about their structure for arbitrary k. After an introduction to the topic we report on our recent results on the structure of DN brackets of degree k. We show that certain linear combinations of the coefficients of a DN bracket define k connections which are all flat and that the Poisson cohomology of a DN bracket is related with the Chevalley-Eilenberg cohomology of a Lie algebra which is naturally associated with the bracket. Joint work with M. Casati.

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Quasimodular functions from Donaldson-Thomas invariants

In this talk, I will explain how to construct quasimodular forms from Betti numbers of moduli spaces of one-dimensional coherent sheaves on the projective plane. This gives a proof of some predictions from theoretical physics about the refined topological string theory in the Nekrasov-Shatashvili limit. The proof of this result combines various tools of modern enumerative algebraic geometry, such as derived categories, Gromov-Witten invariants, and tropical geometry. This work is in part joint with Honglu Fan, Shuai Guo, and Longting Wu.

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Borel multi-transforms, and quantum differential equations of P^1-bundles

Author: Giordano Cotti[1]

[1] *Universitade de Lisboa, Portugal*

Corresponding Author: gcotti@sissa.it

The qDEs define a class of ordinary differential equations in the complex domain, whose study represents a challenging and active area in both contemporary geometry and mathematical physics. The qDEs define rich invariants attached to smooth projective varieties. These equations encapsulate information not only about the enumerative geometry of varieties but also, conjecturally, about their topology and complex geometry. The way to disclose such a huge amount of data is through the study of the asymptotics and monodromy of their solutions.

In this talk, the speaker will address the problem of explicitly integrating the quantum differential equations of varieties and will report on his progress in a long-term project on this topic. Focusing on the case of projectivizations of vector bundles, he will first introduce a family of integral transforms and special functions (the integral kernels). Then, he will show how to use these tools to find explicit integral representations of solutions. Based on arXiv:2005.08262 (Memoirs of the EMS, 2022) and arXiv:2210.05445 (Journal Math. Pures et Appl. 2024).

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Quantum KdV, quasimodular forms, and shifted symmetric functions

Author: Giulio Ruzza[1]

[1] *Universidade de Lisboa*

Corresponding Author: gruzza@fc.ul.pt

I will review recent progress in the study of the spectrum of the quantum double ramification hierarchy associated with the trivial cohomological field theory, i.e., of the quantum KdV hierarchy in its first Poisson structure. A remarkable relation to quasimodular forms naturally suggests a conjecture for the expansion to all orders of the spectrum, and we verify the conjecture explicitly at first order. Based on joint work with Jan-Willem van Ittersum and work in progress with Jan-Willem van Ittersum and Don Zagier.

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The extended Ablowitz-Ladik hierarchy and a generalized Frobenius manifold

We define a certain extension of the Ablowitz-Ladik hierarchy, and show that it possesses a tau structure and Virasoro symmetries. We prove that this integrable hierarchy coincides with the topological deformation of the Principal Hierarchy of a generalized Frobenius manifold with non-flat unity.

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Resurgent large genus asymptotics of intersection numbers

Author: Elba Garcia-Failde[1]

[1] *Universitaire de Jussieu*

I will present a new approach to the computation of the large genus asymptotics of intersection numbers, in particular of Witten—Kontsevich, Theta and r-spin intersection numbers. Our technique is based on a resurgent analysis of the generating series of such intersection numbers, and relies on the presence of a quantum curve and the determinant formulae. With this approach, we are able to extend the recent results of Aggarwal with the computation of subleading corrections, solving a conjecture of Guo–Yang and initiating the development of a universal technique to explore large order asymptotics of enumerative problems that appear in the context of topological recursion. Based on joint work with B. Eynard, A. Giacchetto, P. Gregori and D. Lewański.

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Large deviations for the log-Gamma polymer

Author: Tom Claeys[1]

[1] *UCLouvain*

Corresponding Author: tom.claeys@uclouvain.be

I will present a conjecture about large deviations of the partition function of the log-Gamma polymer. We can rigorously prove our result, except for one step for which we only have heuristic evidence. I will show that the conjectured large deviation rate function matches with that of last passage percolation with exponential weights in the zero-temperature limit, and with the lower tail of the Tracy-Widom distribution for moderate deviations.
The talk will be based on joint work with Julian Mauersberger.

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Confinement effects in fluid-boundary interactions

Author: Roberto Camassa[1]

[1] *Chapel Hill*

Hydrodynamics has motivated many of the advances that lie at the foundations of the mathematics of completely integrable models; the interplay between a fluid and its boundaries adds to the richness, both from a mathematical and a physical perspective, of phenomena that can arise in this area. This talk will present examples of how boundary effects lead to notable outcomes. These effects can be analytically predicted by simple mathematical models (and their integrability), and observed in "simple"experimental setups.

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Classical W-algebras and generalized Drinfeld-Sokolov hierarchies

Author: Daniele Valeri[1]

[1] *Roma III*

Classical affine W-algebras W(g,O) are algebraic structures associated to a simple Lie algebra g and a nilpotent orbit O. In this talk we want to review their definition, the construction (for all but seven nilpotent orbits for exceptional Lie algebras) of an integrable hierarchy of PDEs known as generalized Drinfeld-Sokolov hierarchy and how to derive the tau-structure of the hierarchy.