

Parabolic Hecke eigensheaves

Abstract

In this series of lectures I will discuss the tamely ramified geometric Langlands correspondence over the complex numbers. I will use the case of $GL(2)$ local systems on the projective line with tame ramification at five points to illustrate a general method for constructing Hecke eigensheaves by combining Fourier-Mukai duality with non-abelian Hodge theory. I will explain how the program converts the construction problem into a purely algebraic geometric question which can be solved explicitly by a higher dimensional version of the spectral cover construction. The focus will be on the projective geometry of the moduli spaces involved, and on the singularities and geometric subtleties needed for the correct formulation of the correspondence.

Tentatively the four lectures will cover the following topics:

- 1) Moduli spaces of $GL(2)$ parabolic bundles and parabolic Higgs bundles on \mathbb{P}^1 with tame ramification at five points. Symplectic leaves and Hitchin fibers.
- 2) Wobbly bundles and the modular spectral cover. Geometry of the parabolic Hecke correspondence. Formulation of the parabolic Hecke eigensheaf problem for Higgs bundles.
- 3) Abelianization and the geometry of the abelianized Hecke correspondence. Standard divisors and the parabolic Picard groups of relevant moduli spaces.
- 4) Pullbacks, pushforwards, and tensor products of tame parabolic Higgs bundles. Parabolic Hecke eigensheaf construction and verification of all non-abelian Hodge theory and Hecke eigensheaf constraints.

Some references:

Ron Donagi, Tony Pantev “Langlands duality for Hitchin systems”, *Invent. math.* (2012) 189: 653.
<https://arxiv.org/abs/math/0604617>

Ron Donagi, Tony Pantev “Geometric Langlands and non-Abelian Hodge theory”, *Surveys in Differential Geometry* Volume 13 (2008), 85 –116.

R. Donagi, T. Pantev, C. Simpson “Direct Images in Non Abelian Hodge Theory”.
<https://arxiv.org/abs/1612.06388>.

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