School and Workshop on Algebraic Geometry and Physics 2017

Report of Contributions

https://indico.sissa.it/e/swagp2017

Contribution ID: **1** Type: **not specified**

Decorated character varieties, Painlevé equations and quantization

Abstract

In this lecture course I will introduce some notions in (quantum)Teichmuller theory for orientable non compact Riemann surfaces. We will then discuss colliding boundary components and discuss the Painlevé equations as an example. Finally we will link with the theory of Cherednik algebra.

Tentative schedule:

Lecture 1: Teichmuller theory for orientable non compact Riemann surfaces: Thurston shear coordinates, Fock decomposition, Goldman bracket and quantisation. Complexification and SL_2–character varieties.

Lecture 2: Colliding holes in orientable non compact Riemann surfaces. Bordered cusps, Teichmuller theory for Riemann surfaces with bordered cusps. Example: confluence of the Painlevé equations as hole collision. Quantisation.

Lecture 3: Cherednik algebra associated to the root system C_1. Basic representation. Spherical sub-algebra and Askey–Wilson algebra. Askey–Wilson polynomials and q-Askey scheme.

Lecture 4. C_1 Cherednik algebra and sixth Painlevé equation. Confluence of the Painlevé equation as Whittaker degeneration of the C_1 Cherednik algebra. Open problems.

Presenter: MAZZOCCO, Marta (University of Loughborough, UK)

Contribution ID: **2** Type: **not specified**

Parabolic Hecke eigensheaves

Abstract

In this series of lectures I will discuss the tamely ramified geometric Langlands correspondence over the complex numbers. I will use the case of $GL(2)$ local systems on the projective line with tame ramification at five points to illustrate a general method for constructing Hecke eigensheaves by combining Fourier-Mukai duality with non-abelian Hodge theory. I will explain how the program converts the construction problem into a purely algebraic geometric question which can be solved explicitly by a higher dimensional version of the spectral cover construction. The focus will be on the projective geometry of the moduli spaces involved, and on the singularities and geometric subtleties needed for the correct formulation of the correspondence.

Tentatively the four lectures will cover the following topics:

1) Moduli spaces of $GL(2)$ parabolic bundles and parabolic Higgs bundles on $\mathbb{P}^1$ with tame ramification at five points. Symplectic leaves and Hitchin fibers.

2) Wobbly bundles and the modular spectral cover. Geometry of the parabolic Hecke correspondence. Formulation of the parabolic Hecke eigensheaf problem for Higgs bundles.

3) Abelianization and the geometry of the abelianized Hecke correspondence. Standard divisors and the parabolic Picard groups of relevant moduli spaces.

4) Pullbacks, pushforwards, and tensor products of tame parabolic Higgs bundles. Parabolic Hecke eigensheaf construction and verification of all non-abelian Hodge theory and Hecke eigensheaf constraints.

Some references:

Ron Donagi, Tony Pantev "Langlands duality for Hitchin systems", Invent. math. (2012) 189: 653.
https://arxiv.org/abs/math/0604617

Ron Donagi, Tony Pantev "Geometric Langlands and non-Abelian Hodge theory", Surveys in Differential Geometry Volume 13 (2008), 85 –116.

R. Donagi, T. Pantev, C. Simpson "Direct Images in Non Abelian Hodge Theory".
https://arxiv.org/abs/1612.06388.

Presenter: PANTEV, Tony (University of Pennsylvania, USA)

Contribution ID: **3** Type: **not specified**

Geometry of character varieties via arithmetic techniques

Abstract

1) General setup: varieties parametrizing n-dimensional representations of the fundamental group of a Riemann surface, moduli space of Higgs bundles on a curve. Weil conjectures, application of arithmetic techniques to obtain geometric information.

2) Tools: Frobenius formula for counting representations, bookkeeping using symmetric functions, generating functions, Macdonald polynomials.

3) Derivation of the main formula for the number of points of the character varieties over finite fields. Connection to quiver varieties. Conjecture for the mixed polynomial.

4) Special cases of the conjecture, geometry and combinatorics. Connection to the Hilbert scheme of points on the affine plane.

Presenter: RODRÍGUEZ VILLEGAS, Fernando (ICTP, Italy)

Contribution ID: **4** Type: **not specified**

Higgs stacks and Lie-like structures

Abstract

1) Moduli stacks of coherent sheaves, quiver sheaves (including Higgs sheaves) on smooth projective curves

2) Global nilpotent cone, its irreducible components, moduli space of chains, their Jordan decompositions

3) Hall algebras techniques, including Eisenstein series, Harder residue theorem, volume computation; determination of the volume and Poincaré polynomial of the above moduli spaces

4) some more recent research (cohomological Hall algebras of Higgs sheaves, …)

Presenter: SCHIFFMANN, Olivier (Université de Paris-Sud Orsay and CNRS, France)

Contribution ID: **5** Type: **not specified**

Character varieties and cohomological DT invariants

Tuesday, June 20, 2017 4:10 PM (1 hour)

Abstract

I will pick up where Sven left off, and explain how the cohomology of smooth character varieties for a Riemann surface S may (conjecturally) be considered as the cohomological DT invariants for the category of representations of the fundamental group of S. That these cohomological DT invariants exist at all is a consequence of the cohomological integrality theorem, and that they should be given by the above conjecture is strongly suggested by work of Hausel and Villegas. I'll try to explain all this, along with some more recent developments.

Presenter: DAVISON, Ben (Glasgow University, UK)

Contribution ID: **6** Type: **not specified**

Introducing Donaldson-Thomas theory with an eye towards character varieties

Monday, June 19, 2017 4:10 PM (1 hour)

Abstract

Despite the fact that Donaldson-Thomas theory has been developed to study moduli spaces in a 3-dimensional context, it can (conjecturally) also be used to analyse moduli in 2-dimensional situations as for example sheaves on K3 surfaces, Higgs bundles and character varieties. We will focus on character varieties as annoying technical difficulties disappear and everything can be proven rigorously by a technique called dimension reduction. In the first of two talks on this topic I am going to introduce Donaldson-Thomas theory in order to set the scene for the second talk provided by my collaborator Ben Davison.

Presenter: MEINHARDT, Sven (University of Sheffield, UK)

Contribution ID: **7** Type: **not specified**

Cohomology of character varieties

Friday, June 23, 2017 11:20 AM (1 hour)

Abstract

A conjecture of Hausel, Letellier and Rodriguez-Villegas gives an explicit formula for the mixed Hodge polynomials of character varieties, which hints on a connection with Hilbert schemes. In another development Gorsky, Oblomkov, Rasmussen and Shende conjecture a connection between Hilbert schemes and homological invariants of torus knots and links. The purpose of this talk is to show that certain cell decompositions of character varieties produce an explicit connection between the two conjectures, allows us to calculate the cohomologies in some examples, and implies the so-called curious hard Lefschetz property.

Presenter: MELLIT, Anton (IST Austria)

Contribution ID: **9** Type: **not specified**

Symplectic resolutions for Higgs moduli spaces

Thursday, June 22, 2017 12:10 PM (30 minutes)

Abstract

In this talk I will present some recent work on the algebraic symplectic geometry of the singular moduli spaces of Higgs bundles of degree 0 and rank n on a compact Riemann surface X of genus g. In particular, I will show how to to prove that such moduli spaces are symplectic singularities, in the sense of Beauville, and admit a projective symplectic resolution if and only if $g = 1$ or $(g, n) = (2, 2)$. These results are an application of a recent paper by Bellamy and Schedler [BS16] via the so-called Isosingularity Theorem.

Presenter: TIRELLI, Andrea (Imperial College London)

Contribution ID: **10** Type: **not specified**

Topological mirror symmetry via p-adic integration

Monday, June 19, 2017 2:30 PM (30 minutes)

Abstract

The topological mirror symmetry conjecture of Hausel-Thaddeus predicts an equality of Hodge numbers of certain SL_n and PGL_n -Higgs moduli spaces which are known to be mirror partners in the sense of Strominger, Yau and Zaslow. In my talk I will explain how to prove this conjecture my means of p-adic integration along the Hitchin fibration. This is joint work Michael Groechening and Paul Ziegler.

Presenter: WYSS, Dimitri (EPF Lausanne/IST Austria)

Contribution ID: **11** Type: **not specified**

DT invariants from holomorphic curves and a geometric interpretation of Block- Göttsche invariants

Tuesday, June 20, 2017 2:30 PM (30 minutes)

Abstract

I will first describe a general analogy between DT invariants of 3-Calabi-Yau categories and holomorphic disks in holomorphic symplectic varieties. In some specific case, I will then explain an extension of this result relating refined DT invariants and holomorphic curves of higher genus. Along the way, I will give a new geometric interpretation of Block-Göttshe refined tropical curve counting.

Presenter: BOUSSEAU, Pierrick (Imperial College London)

Contribution ID: **12** Type: **not specified**

On the E-polynomial of parabolic Sp_{2n}-character varieties

Wednesday, June 21, 2017 11:40 AM (30 minutes)

Abstract

In this talk, we find the E-polynomials of a family of parabolic Sp_{2n}-character varieties via arithmetic methods, namely counting points over finite fields. We deduce relevant topological information like Euler characteristic and number of the connected components.

Presenter: CAMBÒ, Vincenzo (SISSA, Trieste)

Contribution ID: **13** Type: **not specified**

Higgs stacks and Lie-like structures - First Lecture

Monday, June 19, 2017 10:00 AM (50 minutes)

Abstract

Moduli stacks of coherent sheaves, quiver sheaves (including Higgs sheaves) on smooth projective curves

Presenter: SCHIFFMANN, Olivier (Université de Paris-Sud Orsay and CNRS, France)

Contribution ID: **14** Type: **not specified**

Higgs stacks and Lie-like structures - Second Lecture

Monday, June 19, 2017 3:00 PM (50 minutes)

Abstract

Global nilpotent cone, its irreducible components, moduli space of chains, their Jordan decompositions

Presenter: SCHIFFMANN, Olivier (Université de Paris-Sud Orsay and CNRS, France)

Contribution ID: **15** Type: **not specified**

Higgs stacks and Lie-like structures - Third Lecture

Tuesday, June 20, 2017 10:00 AM (50 minutes)

Abstract

Hall algebras techniques, including Eisenstein series, Harder residue theorem, volume computation; determination of the volume and Poincaré polynomial of the above moduli spaces

Presenter: SCHIFFMANN, Olivier (Université de Paris-Sud Orsay and CNRS, France)

Contribution ID: **16** Type: **not specified**

Higgs stacks and Lie-like structures - Fourth Lecture

Wednesday, June 21, 2017 9:30 AM (50 minutes)

Abstract

Some more recent research (cohomological Hall algebras of Higgs sheaves, …)

Presenter: SCHIFFMANN, Olivier (Université de Paris-Sud Orsay and CNRS, France)

Contribution ID: **17** Type: **not specified**

Decorated character varieties, Painlevé equations and quantization - First Lecture

Monday, June 19, 2017 12:10 PM (50 minutes)

Abstract

Teichmuller theory for orientable non compact Riemann surfaces: Thurston shear coordinates, Fock decomposition, Goldman bracket and quantisation. Complexification and SL_2–character varieties.

Presenter: MAZZOCCO, Marta (University of Loughborough, UK)

Contribution ID: **18** Type: **not specified**

Decorated character varieties, Painlevé equations and quantization - Second Lecture

Tuesday, June 20, 2017 3:00 PM (50 minutes)

Abstract

Colliding holes in orientable non compact Riemann surfaces. Bordered cusps, Teichmuller theory for Riemann surfaces with bordered cusps. Example: confluence of the Painlevé equations as hole collision. Quantisation.

Presenter: MAZZOCCO, Marta (University of Loughborough, UK)

Contribution ID: **19** Type: **not specified**

Decorated character varieties, Painlevé equations and quantization - Third Lecture

Wednesday, June 21, 2017 10:50 AM (50 minutes)

Abstract

Cherednik algebra associated to the root system C_1. Basic representation. Spherical sub-algebra and Askey–Wilson algebra. Askey–Wilson polynomials and q-Askey scheme.

Presenter: MAZZOCCO, Marta (University of Loughborough, UK)

Contribution ID: **20** Type: **not specified**

Decorated character varieties, Painlevé equations and quantization - Fourth Lecture

Thursday, June 22, 2017 3:40 PM (50 minutes)

Abstract

C_1 Cherednik algebra and sixth Painlevé equation. Confluence of the Painlevé equation as Whittaker degeneration of the C_1 Cherednik algebra. Open problems.

Presenter: MAZZOCCO, Marta (University of Loughborough, UK)

Contribution ID: **21** Type: **not specified**

Parabolic Hecke eigensheaves - First Lecture

Thursday, June 22, 2017 11:20 AM (50 minutes)

Abstract

Moduli spaces of $GL(2)$ parabolic bundles and parabolic Higgs bundles on $\mathbb{P}^1$ with tame ramification at five points. Symplectic leaves and Hitchin fibers.

Presenter: PANTEV, Tony (University of Pennsylvania, USA)

Contribution ID: **22** Type: **not specified**

Parabolic Hecke eigensheaves - Second Lecture

Thursday, June 22, 2017 2:30 PM (50 minutes)

Abstract

Wobbly bundles and the modular spectral cover. Geometry of the parabolic Hecke correspondence. Formulation of the parabolic Hecke eigensheaf problem for Higgs bundles.

Presenter: PANTEV, Tony (University of Pennsylvania, USA)

Contribution ID: **23** Type: **not specified**

Parabolic Hecke eigensheaves - Third Lecture

Friday, June 23, 2017 10:00 AM (50 minutes)

Abstract

Abelianization and the geometry of the abelianized Hecke correspondence. Standard divisors and the parabolic Picard groups of relevant moduli spaces.

Presenter: PANTEV, Tony (University of Pennsylvania, USA)

Contribution ID: **24** Type: **not specified**

Parabolic Hecke eigensheaves - Fourth Lecture

Friday, June 23, 2017 2:30 PM (50 minutes)

Abstract

Pullbacks, pushforwards, and tensor products of tame parabolic Higgs bundles. Parabolic Hecke eigensheaf construction and verification of all non-abelian Hodge theory and Hecke eigensheaf constraints.

Presenter: PANTEV, Tony (University of Pennsylvania, USA)

Contribution ID: **25** Type: **not specified**

Relating stability conditions and cluster varieties

Tuesday, June 20, 2017 12:10 PM (1 hour)

Abstract

I will describe the relationship between two spaces associated to a quiver with potential. The first is a complex manifold parametrizing Bridgeland stability conditions on a certain triangulated category, and the second is a cluster variety with a natural Poisson structure. For quivers of type A_n, I will describe a local biholomorphism from the space of stability conditions to the cluster variety. This result is a special case of work in progress with Tom Bridgeland.

Presenter: ALLEGRETTI, Dylan (University of Sheffield, UK)

Contribution ID: **26** Type: **not specified**

Geometry of character varieties via arithmetic techniques - First Lecture

Monday, June 19, 2017 11:20 AM (50 minutes)

Abstract

General setup: varieties parametrizing n-dimensional representations of the fundamental group of a Riemann surface, moduli space of Higgs bundles on a curve. Weil conjectures, application of arithmetic techniques to obtain geometric information.

Presenter: RODRÍGUEZ VILLEGAS, Fernando (ICTP, Italy)

Contribution ID: **27** Type: **not specified**

Geometry of character varieties via arithmetic techniques - Second Lecture

Tuesday, June 20, 2017 11:20 AM (50 minutes)

Abstract

Tools: Frobenius formula for counting representations, bookkeeping using symmetric functions, generating functions, Macdonald polynomials.

Presenter: RODRÍGUEZ VILLEGAS, Fernando (ICTP, Italy)

Contribution ID: **28** Type: **not specified**

Geometry of character varieties via arithmetic techniques - Third Lecture

Thursday, June 22, 2017 10:00 AM (50 minutes)

Abstract

Derivation of the main formula for the number of points of the character varieties over finite fields. Connection to quiver varieties. Conjecture for the mixed polynomial.

Presenter: RODRÍGUEZ VILLEGAS, Fernando (ICTP, Italy)

Contribution ID: **29** Type: **not specified**

Geometry of character varieties via arithmetic techniques - Fourth Lecture

Friday, June 23, 2017 3:20 PM (50 minutes)

Abstract

Special cases of the conjecture, geometry and combinatorics. Connection to the Hilbert scheme of points on the affine plane.

Presenter: RODRÍGUEZ VILLEGAS, Fernando (ICTP, Italy)