TULSF 9th edition

SISSA Trieste, 7 November 2024

Schedule

09:30 - 10:00	**Registration**
10:00 - 10:45	**Pietro De Poi** (Università di Udine) Geproci sets on skew lines in $\mathbb{P}^3$ with two transversals
11:30 - 12:15	**Giulia Menara** (Università di Trieste) Eulerian Magnitude Homology in Erdős-Rényi Random Graphs: Substructures and Torsion
12:15 - 14:00	**Lunch time**
14:00 - 14:45	**Muhammad Sohaib Khalid** (SISSA) Destabilising subvarieties and Nakai–Moishezon type criteria
15:15 - 16:00	**Alex Casarotti** (Università di Ferrara) On the unirationality of conic bundles with eight singular fibers
16:00 - 16:45	**Coffee break**
16:45 - 17:30	**Matej Filip** (University of Ljubljana) Smoothing Gorenstein toric singularities and mirror symmetry

Book of abstracts

1 Alex Casarotti (Università di Ferrara)

On the unirationality of conic bundles with eight singular fibers

Abstract: We investigate the problems of unirationality and rationality for conic bundles $S \to \mathbb{P}^1$ over a C_1 field k, which can be described as the zero locus of a hypersurface in the projectivization of a rank-3 vector bundle over $\mathbb{P}^1$. Conic bundles can be classified by the degree d of the discriminant, i.e. the number of points on the base where the corresponding fiber is not a smooth conic. Unirationality for $d < 8$ was already established by Kollár and Mella in 2014, while the case for general d remains open. In this work we focus on the next case $d = 8$, and explicitly show that for all four possible types of such conic bundles S, the set of unirational ones forms an open subset of the parameter space. We also examine algebraic constraints, depending on the base field k, under which a general S is not rational over k.

2 Pietro De Poi (Università di Udine)

Geproci sets on skew lines in $\mathbb{P}^3$ with two transversals

Abstract: A set of points Z in $\mathbb{P}^3$ is an (a, b)-geproci set (for GEneral PROjection is a Complete Intersection) if its projection from a general point to a plane is a complete intersection of curves of degrees a and b. we will report on some results in order to pursue classification of geproci sets. Specifically, we will show how to classify (m, n)-geproci sets Z which consist of m points on each of n skew lines, assuming the skew lines have two transversals in common. We will show in this case that $n < 7$. Moreover we will show that all geproci sets of this type and with no points on the transversals are contained in the F_4 configuration. We conjecture that a similar result is true for an arbitrary number m of points on each skew line, replacing containment in F_4 by containment in a half grid obtained by the so-called standard construction.

3 Matej Filip (University of Ljubljana)

Smoothing Gorenstein toric singularities and mirror symmetry

Abstract: We establish a correspondence between one-parameter deformations of an affine Gorenstein toric variety X, defined by a polytope P, and mutations of a Laurent polynomial f, whose Newton polytope is equal to P. If the Newton polytope P of f is two dimensional and there exists a set of mutations of f that mutate P to a smooth polygon, then, under certain assumptions, we show that the Gorenstein toric variety, defined by P, admits a smoothing. This smoothing is obtained by proving that the corresponding one-parameter deformation families are unobstructed and that the general fiber of this deformation family is smooth. Our assumptions hold for all polygons that are affine equivalent to a facet of a reflexive three-dimensional polytope Q, and thus we are able to provide applications to mirror symmetry and deformation theory of the Fano toric variety corresponding to Q.

4 Muhammad Sohaib Khalid (SISSA)

Destabilising subvarieties and Nakai–Moishezon type criteria

Abstract: The Nakai–Moishezon criterion in algebraic geometry and Yau's solution of the Calabi conjecture, when taken together, can be viewed as establishing a correspondence between the solvability of the complex Monge–Ampère equation and the positivity of certain intersection numbers involving proper subvarieties. In complex geometry, many other examples of such correspondences have been discovered recently. In this talk, we will review these discoveries and consider the set of "destabilising subvarieties" for geometric PDEs, that is, those subvarieties which violate the associated positivity criteria, and present some resutls about their geometry and cardinality. This is joint work with Sjöström Dyrefelt.

5 Giulia Menara (Università di Trieste)

Eulerian Magnitude Homology in Erdős-Rényi Random Graphs: Substructures and Torsion

Abstract: Magnitude was first introduced by Leinster in 2008 [1]. It is a notion analogous to the Euler characteristic of a category, and it captures the structure and complexity of a metric space. Magnitude homology was defined in 2014 by Hepworth and Willerton [2] as a categorification of magnitude in the context of simple undirected graphs, and although the construction of the boundary map suggests that magnitude homology groups are strongly influenced by the graph substructures, it is not straightforward to detect such subgraphs. In this talk, I introduce eulerian magnitude homology [3]. I will do this by defining the eulerian magnitude chain complex, a subcomplex of the magnitude chain complex exhibiting a more explicit connection to the combinatorics of the graph. I will illustrate how eulerian magnitude homology enables a more accurate analysis of graph substructures and then apply these results to Erdős-Rényi random graphs and obtain an asymptotic estimate for the Betti numbers of the eulerian magnitude homology groups on the diagonal. Finally, I will discuss the regimes where an Erdős-Rényi random graph has torsion-free eulerian magnitude homology groups [4].

References:

[1] T. Leinster,The Euler characteristic of a category, Documenta Mathematica (13) (2008) 21–49.

[2] R. Hepworth, S. Willerton, Categorifying the magnitude of a graph. Homo- topy, Homology and Applications 16(2) (2014) 1–30.

[3] C. Giusti, G. Menara, Eulerian magnitude homology: subgraph structure and random graphs. arXiv: 2403.09248 (2024).

[4] G. Menara, On torsion in eulerian magnitude homology of Erdos-Renyi ran- dom graphs. arXiv: 2409.03472 (2024).