Geometry of Integrable systems

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SISSA

Book of Abstracts

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Methods of tangent and cotangent coverings for Dubrovin-Novikov integrability operators

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The well-established method of tangent and cotangent covering for searching integrability operators, like Hamiltonian, symplectic and recursion operators, was introduced by Kersten, Krasil'shchik and Verbovetsky in 2003. The method consists in describing integrability operators of a given PDE as linear functions of some odd variables which are in the kernel of linearization or adjoint linearization of the PDE. We apply the method to the search of Dubrovin-Novikov integrability operators for hydrodynamic-type PDEs. We recover known results, like: Tsarev's compatibility conditions between a hydrodynamic-type system and a first-order local Dubrovin-Novikov Hamiltonian operator; a geometric interpretation of nonlocalities in Ferapontov's nonlocal homogeneous operators. We obtain new results, like a new system of PDEs that expresses the compatibility of third-order Dubrovin-Novikov and a hydrodynamic-type system, as well as new (integrable?) systems of that type. We will discuss several interesting problems and conjectures that are emerging from the interaction between the two theories.

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Some analytical aspects of the Kontsevich matrix model

Author: Mattia Cafasso[1]

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In this seminar I will show that the Kontsevich integral on $n \times n$ matrices ($n < \infty$) is the isomonodromic tau function
associated to a 2×2 Riemann–Hilbert problem. This approach allows us to gain control of the analysis
of the convergence as $n \to \infty$. By an appropriate choice of the external source matrix in Kontsevich's
integral, I'll show that the limit produces the isomonodromic tau function of a special tronquée solution
of the first Painlevé hierarchy, and I will identify the solution in terms of the Stokes'data of the associated
linear problem . Time permitting I will adress the problem of universality for the Kontsevich matrix model. This is a joint work with M. Bertola.

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Topological recursion of Eynard-Orantin and the Harmonic Oscillator

Author: Vasilisa Shramchenko[1]

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We apply the Chekhov-Eynard-Orantin topological recursion to the curve corresponding to the quantum harmonic oscillator and demonstrate that the result is equivalent to the WKB wave function. We also show that using the multi-differentials obtained by the topological recursion from the harmonic oscillator curve, one generates naturally the so-called Poincaré polynomials associated with the orbifolds of the metric ribbon graphs.

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Discrete solitons for the focusing Ablowitz-Ladik equation with non-zero boundary conditions via inverse scattering transform

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Soliton solutions of the focusing Ablowitz-Ladik (AL) equation with nonzero boundary conditions at infinity are derived within the framework of the inverse scattering transform (IST). After reviewing the relevant aspects of the direct and inverse problems, explicit soliton solutions will be discussed which are the discrete analog of the Tajiri-Watanabe and Kutznetsov-Ma solutions to the focusing NLS equation on a finite background. Then, by performing suitable limits of the above solutions, discrete analog of the celebrated Akhmediev and Peregrine solutions will also be presented. These solutions, which had been recently derived by direct methods, are obtained for the first time within the framework of the IST, thus providing a spectral characterization of the solutions and a description of the singular limit process.

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Complex reflection groups, logarithmic connections and bi-flat F-manifolds

Authors: Alessandro Arsie[1]; Paolo Lorenzoni[2]

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We show that bi-flat F-manifolds can be interpreted as natural geometrical structures encoding the almost duality for Frobenius manifolds without metric. Using this framework, we extend Dubrovin's duality between orbit spaces of Coxeter groups and Veselov's $\vee$-systems, to the orbit spaces of exceptional well-generated complex reflection groups of rank 2 and 3. On the Veselov's $\vee$-systems side, we provide a generalization of the notion of $\vee$-systems that gives rise to a dual connection which coincides with a Dunkl-Kohno-type connection associated with such groups. In particular, this allows us to treat on the same ground several different examples including Coxeter and Shephard groups. Remarkably, as a byproduct of our results, we prove that in some examples basic flat invariants are not uniquely defined. As far as we know, such a phenomenon has never been pointed out before. Based on joint works with Alessandro Arsie.

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Interface motion features for Euler fluids models

Author: Giovanni Ortenzi[1]

Co-authors: Colin Thomson [2]; Gregorio Falqui [3]; Marco Pedroni [4]; Roberto Camassa [5]

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In Euler fluids, the interaction between interfaces and boundaries can lead to interface singularities in finite times.
We give an example of this behavior in some long wave models for two-fluid systems.

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”Integrable” gap probabilities for the Generalized Bessel process

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We consider the gap probability for the Generalized Bessel process, a determinantal point process which arises as critical limiting kernel near the hard edge of the spectrum of a certain random matrix ensemble. We prove that such probability can be expressed in terms of the Fredholm determinant of a suitable Its-Izergin-Korepin-Slavnov integrable operator and linked in a canonical way to Riemann-Hilbert (RH) problem. Starting from the RH problem, we can construct a Lax pair and link the gap probability to the Painlevè III hierarchy. Moreover, we are able to construct a system of two coupled Hamiltonians which can be hopefully identified with the 2-dimensional Garnier system LH(2+3).
The talk is based on some previous results and an on-going project with Dr. Mattia Cafasso (Université Angers, France).

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Dressing networks: towards an integrability approach for understanding network structures

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A large variety of real world systems can be naturally modelled by networks, i.e. graphs whose nodes represent the components of a system linked (interacting) according to specific statistical rules. A network is realised by a graph typically constituted by a large number of nodes/links. Fluid and magnetic models in Physics are just two among many classical examples of systems which can

be modelled by simple or complex networks. In particular "extreme" conditions (thermodynamic regime), networks, just like fluids and magnets, exhibit a critical collective behaviour intended as a drastic change of state due to a continuous change of the model parameters.
Using an approach to thermodynamics, recently introduced to describe a general class of van der Waals type models and magnetic systems in mean field approximation, we analyse the integrable structure of corresponding networks and use the theory of integrable conservation laws combined with a suitable "dressing" procedure to calculate order parameters outside and inside the critical region.

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Methods of tangent and cotangent coverings for Dubrovin-Novikov integrability operators

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The well-established method of tangent and cotangent covering for searching integrability operators, like Hamiltonian, symplectic and recursion operators, was introduced by Kersten, Krasil'shchik and Verbovetsky in 2003. The method consists in describing integrability operators of a given PDE as linear functions of some odd variables which are in the kernel of linearization or adjoint linearization of the PDE. We apply the method to the search of Dubrovin-Novikov integrability operators for hydrodynamic-type PDEs. We recover known results, like: Tsarev's compatibility conditions between a hydrodynamic-type system and a first-order local Dubrovin-Novikov Hamiltonian operator; a geometric interpretation of nonlocalities in Ferapontov's nonlocal homogeneous operators. We obtain new results, like a new system of PDEs that expresses the compatibility of third-order Dubrovin-Novikov and a hydrodynamic-type system, as well as new (integrable?) systems of that type. We will discuss several interesting problems and conjectures that are emerging from the interaction between the two theories.

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KP theory, total positivity and rational degenerations of M-curves

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We establish connections between two objects, naturally arising in the theory of the Kadomtsev-Petviashvily equation: totally nonnegative Grassmannians and rational degenerations of the M-curves (Riemann surfaces with an antiholomorphic involution and the maximal possible number of real ovals) with a collection of marked points.

More precisely, we show that a KP divisor satisfying the reality conditions on a degenerate M-curve is canonically associated to any point in the totally non–negative Grassmannian.

In the case of a certain rational degeneration of hyperelliptic M-curves, we also solve the inverse problem and explain the connection to the finite Toda system.

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From matrix resolvents to tau functions

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We introduce a simple method of computing tau functions for a wide class of integrable systems, including the Drinfeld–Sokolov hierarchy and the Toda lattice hierarchy. This method expresses the generating series of logarithmic derivatives of a tau function in terms of matrix resolvents. We also discuss possible extension of this method to other integrable systems. The talk is based on a series of joint works with Marco Bertola and Boris Dubrovin.

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Painlevé functions, Fredholm determinants and combinatorics

Author: Oleg Lisovyi[1]

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We will derive Fredholm determinant representation for isomonodromic tau functions of Fuchsian systems with n regular singular points on the Riemann sphere and generic monodromy in GL(N,\mathbb C). The corresponding operator acts in the direct sum of N(n-3) copies of L^2(S^1). Its kernel will be expressed in terms of fundamental solutions of n-2 elementary 3-point Fuchsian systems whose monodromy is determined by monodromy of the relevant n-point Fuchsian system via a decomposition of the punctured sphere into pairs of pants. For N=2 these building blocks have hypergeometric representations, the kernel becomes completely explicit and has Cauchy type. In this case principal minor expansion of the Fredholm determinant yields multivariate series representation for the tau function of the Garnier system obtained earlier via its identification with Fourier transform of Liouville conformal block (or a dual Nekrasov-Okounkov partition function). Further specialization to n=4 will provide an explicit series representation of the general solution to Painlevé VI equation.

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Dynamics of a soliton in an external potential

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Joint work with A. Fusé, A. Maspero, S. Sansottera

Consider the nonlinear Schrödinger equation

$$-i\psi_t = -\Delta\psi - \beta(|\psi|^2)\psi + \epsilon V\psi \ , \quad \beta \in C^\omega \ , \quad V \in \mathcal{S} \cap C^\omega \ ;$$

it is well known that, when $\epsilon = 0$, under suitable conditions on β, the NLS admitts traveling wave solutions (soliton for short). When $\epsilon \neq 0$, heuristic considerations suggest that the soliton should move as a particle subject to a mechanical force due to an effective potential computed from V. The problem is to understand if this is true or not.

In this talk I will show that the soliton does not exchange neither energy nor angular momentum with the rest of the field for times of the order ϵ^{-r} for any r. This allows to deduce some informations on the trajectory of the soliton.

The proof is composed by two steps: first one shows that the Hamiltonian of the NLS can be rewritten in suitable coordinates as follows

$$H = \epsilon^{1/2} H_{mech}(p,q) + \tfrac{1}{2}\langle EL_0\phi, \phi\rangle + h.o.t.$$

where $H_{mech}(p,q)$ is the hamiltonian of a mechanical particle in a central potential and describes the motion of the soliton's barycenter, while ϕ is a function representing the "free" field.

The second step consistes in applying a Nekhoroshev type theorem; in turn this requires to verify a nondegeneracy hypothesis on $H_{mech}(p,q)$ (quasiconvexity). This is obtained by applying a result that we recently got ensuring that such an assumption is always satisfied in the central motion problem except for Harmonic end Keplerian potentials.

In the talk I will also try to give the ideas of the proof of this second result.

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The many faces of Ricci curvature

Author: Mauro Carfora[1]

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Powered by Ricci flow theory, this classical subject has undergone remarkable developments in recent years. We have seen the conjunction of a variety of ideas from topology, geometric analysis, optimal transport, and renormalization group theory. In this talk we touch upon some of these themes as well as on some unconventional aspects that Ricci curvature still holds in store.

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The quantum FPU problem

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The quantum version of the Fermi-Pasta-Ulam problem is presented.
More precisely, a chain of many quantum particles pairwise interacting
through any potential, with fixed ends, is considered.
It is shown that, close to thermal equilibrium, the dynamics of the system is described, to the first perturbative order, by a simple, effective quantum field theory, namely the quantum Korteweg-de Vries equation.

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Polynomial Tau Functions and Bilinearization of the Drinfeld–Sokolov Hierarchies

Authors: Clément du Crest de Villeneuve[1]; Di Yang[2]; Mattia Cafasso[1]

[1] *Université d'Angers*

[2] *Max Planck Institute*

The Drinfeld–Sokolov hierarchies are sequences of partial differential equations introduced in 1985. Each sequence of PDE's is called an integrable hierarchy and possesses a bihamiltonian structure. Moreover, one can express all the components of any solution as logarithmic derivatives of a single function called the tau function. In this talk I aim to show how to compute polynomial tau functions (the simplest ones) of the Drinfled–Sokolov hierarchies in terms of Toeplitz determinants. Furthermore, this allows us to search for Hirota equations that could be satisfied by the computed tau functions. For instance, in the Drinfeld–Sokolov hierarchy of type B2, we showed that there is no Hirota equation of degree 2 nor 4 satisfied by the computed tau functions and only one for degree 6 an 8.

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Algebraic structures for the double ramification hierarchy

Author: Alexandr Buryak[1]

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The double ramification (DR) hierarchy is a system of evolutionary PDEs with one spatial variable associated to an arbitrary cohomological field theory. The class of DR hierarchies includes a lot of important hierarchies such as the KdV, Gelfand-Dickey, ILW, Toda hierarchies. Remarkably, the DR hierarchies are endowed with a lot of algebraic structures that can be described very explicitly. I will discuss the recursion operators, quantization, tau-structure and, in the case of conformal cohomological field theories, conjectural bi-Hamiltonian structure.

The talk is closely related to the subsequent talk of Paolo Rossi who will, in particular, speak about the conjectural equivalence between the DR hierarchy and the hierarchy of topological type.

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On the DR/DZ equivalence conjecture

Author: Paolo Rossi[1]

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Building on the material introduced in A. Buryak's talk I will report on the progress towards the proof of the conjecture, first proposed by Buryak and then refined and studied in a series of joint papers with Dubrovin, Guéré and myself, that the double ramification and Dubrovin-Zhang hierarchies are in fact equivalent through a Miura transformation that preserves the tau-structure (a normal Miura).

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TBA

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Almost periodicity of Stokes matrices of Quantum Cohomology of Grassmannians

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Work in progress, joint with B. Dubrovin and D. Guzzetti. Quantum cohomology is a fundamental tool for the description of the enumerative geometry of smooth projective varieties, and more general symplectic manifolds. An intriguing conjecture relates Quantum Cohomology of a Fano manifold X of Hodge-Tate type with the geometry of the derived category of coherent sheaves $\mathcal{D}^b(X)$. In this seminar I will present a property of almost periodicity of Stokes matrices associated to the points of small Quantum Cohomology of complex Grassmannians, and I will discuss the "mirror counterpart" in terms of exceptional objects in their derived categories.

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Time evolution of a solvable quantum many body system

Author: Vieri Mastropietro[1]

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The Luttinger model is a solvable model describing interacting fermions in one dimension; the solution was found in 1965 by Mattis and Lieb but only equilibrium properties were explored. We present some recent results, obtained in collaboration with Langmann,
Lebowitz and Moosavi (CMP 2016; PRB 2017) on the time evolution of such model starting from a non-equilibrium situation,
that is a domain wall initial state, i.e., a state with different densities on its left and right sides, or a non uniform temperature profile state. The system approaches a non equilibrium steady state carrying a (density or heat) current

with remarkable universality properties, and explicit formulas for the time evolution of several physical properties can be obtained.
Physically the system describes a one dimensional metals as realized in quantum wires.
The analysis is rigorous in the first case, and exact in the second, and
depends crucially on the solvability of the model; how much the behavior changes if solvability is broken is an important
open question.
E. Langmann, J. Lebowitz, V.Mastropietro, P Moosavi (CMP 2016; PRB 2017)

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Poisson cohomology of multidimensional Dubrovin-Novikov Poisson structures and their normal forms.

Author: Guido Carlet[1]

[1] *Amsterdam University*

After a review of the theory of classification of dispersive Poisson and bi-Hamiltonian structures under Miura type transformations, we will present our recent results on the cohomology of scalar multidimensional Poisson structures of DN type and the complete classification of their dispersive deformations in the case of two independent variables.

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Kontsevich-Penner model and open intersection numbers

Author: Giulio Ruzza[1]

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I will report on a joint work in progress with Prof. M. Bertola. We identify the Kontsevich-Penner matrix integral with a suitable isomonodromic tau function. This allows us to derive explicit formulae for multipoint functions of this model, which are conjectured to describe intersection numbers on the moduli space of open Riemann surfaces.