Time evolution of a quantum solvable many body system (the Luttinger model)

Vieri Mastropietro

University of Milan

Collaboration with E.Langmann J.Lebowitz,P.Moosavi;
Comm Math Phys 349, 551 (2017); Phys Rev B 2017

Introduction

- How do closed quantum many-body systems out of equilibrium eventually equilibrate? A classical question whose interest has been renewed by recent experiments on ultracold atomic gases (see e.g. Eisertt et al. Nature (2015)).

Introduction

- How do closed quantum many-body systems out of equilibrium eventually equilibrate? A classical question whose interest has been renewed by recent experiments on ultracold atomic gases (see e.g. Eisertt et al. Nature (2015)).
- Simplest situation (partitioned protocol): one prepares an isolated system of quantum particles (bosons or fermions) in an initial state at time $t = 0$ with different density or temperature profiles to the left and right and then lets it evolve according to its internal translation invariant Hamiltonian (Classical case; Lebowitz Spohn (1977)).

Introduction

- How do closed quantum many-body systems out of equilibrium eventually equilibrate? A classical question whose interest has been renewed by recent experiments on ultracold atomic gases (see e.g. Eisertt et al. Nature (2015)).
- Simplest situation (partitioned protocol): one prepares an isolated system of quantum particles (bosons or fermions) in an initial state at time $t = 0$ with different density or temperature profiles to the left and right and then lets it evolve according to its internal translation invariant Hamiltonian (Classical case; Lebowitz Spohn (1977)).
- One can consider for instance a $d = 1$ system on the interval $[-L/2, L/2]$ for $L > 0$ prepared in an initial state which is different to the left and right of the origin, evolve the system in time t, and consider an interval $[-\ell, \ell]$ for $L > \ell > 0$, followed by first letting $L \to \infty$ and then $t \to \infty$ while keeping ℓ fixed but arbitrary.

Introduction

- After a long time, one could get *thermal equilibrium* or an approach to some form of steady state.
- Many body Interaction, integrabillity, disorder influence the evolution. In particular for integrable or MBL systems thermalization can not happen.

Introduction

- After a long time, one could get *thermal equilibrium* or an approach to some form of steady state.
- Many body Interaction, integrabillity, disorder influence the evolution. In particular for integrable or MBL systems thermalization can not happen.
- In general to study transport one needs reservoir, which can stochastic or Hamiltonian; the partioned protocol is can be seen an Hamiltonian reservoir.

Introduction

- After a long time, one could get *thermal equilibrium* or an approach to some form of steady state.
- Many body Interaction, integrabillity, disorder influence the evolution. In particular for integrable or MBL systems thermalization can not happen.
- In general to study transport one needs reservoir, which can stochastic or Hamiltonian; the partioned protocol is can be seen an Hamiltonian reservoir.
- On expect to reach a final stationary state with density or heat curremt I. One defines

$$\sigma = \frac{I}{(\mu_L - \mu_R)/L} \qquad \sigma = \frac{I}{(T_L - T_R)/L}$$

Introduction

- After a long time, one could get *thermal equilibrium* or an approach to some form of steady state.
- Many body Interaction, integrabillity, disorder influence the evolution. In particular for integrable or MBL systems thermalization can not happen.
- In general to study transport one needs reservoir, which can stochastic or Hamiltonian; the partioned protocol is can be seen an Hamiltonian reservoir.
- On expect to reach a final stationary state with density or heat curremt I. One defines

$$\sigma = \frac{I}{(\mu_L - \mu_R)/L} \qquad \sigma = \frac{I}{(T_L - T_R)/L}$$

- In general $\sigma \sim L^\alpha$, $\alpha = 0$ correspond to a normal conductor and $\alpha = 1$ to a perfect conductor. When $\alpha = 0$ the current is proportional to the gradient temperature(Fourier law).

Solvable models

- In the non interacting case there is no hope to get Fourier law.

Solvable models

- In the non interacting case there is no hope to get Fourier law.
- Taking into account the interaction is a very non trivial problem; one needs the dynamical properties of an N-particle Schroedinger equation.

Solvable models

- In the non interacting case there is no hope to get Fourier law.
- Taking into account the interaction is a very non trivial problem; one needs the dynamical properties of an N-particle Schroedinger equation.
- One can study certain solvable models. One very natural class of them are spin chains like the XX or the XXZ spin chain, solvable by Berthe ansatz or Baxter methods. XX: Araki-Ho (2000), Ogata (2002), Aschbacher Pillet (2003); 2 chemical potential Antal, Racz, Rakos, Schutz (1999); XXZ: Bernard and Doyon (2012); Karrasch, Ilan, Moore,(2013); Lancaster Mitra (2010), Goldstein, Andrei (2013); Bertini, Collura, De Nardis Fagotti r 2016; Bernard et al (2016).

Solvable models

- In the non interacting case there is no hope to get Fourier law.
- Taking into account the interaction is a very non trivial problem; one needs the dynamical properties of an N-particle Schroedinger equation.
- One can study certain solvable models. One very natural class of them are spin chains like the XX or the XXZ spin chain, solvable by Berthe ansatz or Baxter methods. XX: Araki-Ho (2000), Ogata (2002), Aschbacher Pillet (2003); 2 chemical potential Antal, Racz, Rakos, Schutz (1999); XXZ: Bernard and Doyon (2012); Karrasch, Ilan, Moore,(2013); Lancaster Mitra (2010), Goldstein, Andrei (2013); Bertini, Collura, De Nardis Fagotti r 2016; Bernard et al (2016).
- Solvable models are believed to miiss thermalization and Fourier law. Even in such solvable interacting models exact results are very few for complexity due to Bethe ansatz; usal analysis done assuming conformal symmetry, which is broken in realistic models.

Solvable models

- In the non interacting case there is no hope to get Fourier law.
- Taking into account the interaction is a very non trivial problem; one needs the dynamical properties of an N-particle Schroedinger equation.
- One can study certain solvable models. One very natural class of them are spin chains like the XX or the XXZ spin chain, solvable by Berthe ansatz or Baxter methods. XX: Araki-Ho (2000), Ogata (2002), Aschbacher Pillet (2003); 2 chemical potential Antal, Racz, Rakos, Schutz (1999); XXZ: Bernard and Doyon (2012); Karrasch, Ilan, Moore,(2013); Lancaster Mitra (2010), Goldstein, Andrei (2013); Bertini, Collura, De Nardis Fagotti r 2016; Bernard et al (2016).
- Solvable models are believed to miiss thermalization and Fourier law. Even in such solvable interacting models exact results are very few for complexity due to Bethe ansatz; usal analysis done assuming conformal symmetry, which is broken in realistic models.
- Here I will expose some result on a solvable model for which exact results can be derived. Continuum but with intrinsic length.

The Luttinger model

- The Luttinger model is the "Ising model" for condensed matter. It describes left ad right moving fermions with linear dispersion relation. Luttinger (1963); Mattis Lieb (1965); Haldane (1981) Recent review Mattis Mastropietro, Luttinger model, World Scientific (2013).

The Luttinger model

- The Luttinger model is the "Ising model" for condensed matter. It describes left ad right moving fermions with linear dispersion relation. Luttinger (1963); Mattis Lieb (1965); Haldane (1981) Recent review Mattis Mastropietro, Luttinger model, World Scientific (2013).
- The exact solution was used to investigate equilibrium properties; remarkably we show that it can be used also to investigate non equilibrium many body dynamics. The case of two densities is rigorously analyzed; the two temperature is at the moment only exact.

The Luttinger model

- The Luttinger model is the "Ising model" for condensed matter. It describes left ad right moving fermions with linear dispersion relation. Luttinger (1963); Mattis Lieb (1965); Haldane (1981) Recent review Mattis Mastropietro, Luttinger model, World Scientific (2013).
- The exact solution was used to investigate equilibrium properties; remarkably we show that it can be used also to investigate non equilibrium many body dynamics. The case of two densities is rigorously analyzed; the two temperature is at the moment only exact.
- $$H_\lambda = \sum_{r=\pm} \int_{-L/2}^{L/2} dx : \tilde{\psi}^+_r(x)(-irv_F\partial_x - \mu)\tilde{\psi}^-_r(x) :$$
$$+\lambda \int_{-L/2}^{L/2} dxdy\lambda v(x-y) \sum_{r,r'} : \tilde{\psi}^+_r(x)\tilde{\psi}^-_r(x) :: \tilde{\psi}^+_{r'}(y)\tilde{\psi}^-_{r'}(y) :$$
$\{\tilde{\psi}^+_r(x), \tilde{\psi}^+_{r'}(x)\} = \delta_{r,r'}\delta(x-y)$, :: denotes Wick ordering, $v(x-y)$ is exponentially decaying.

The Luttinger model

- In terms of $\tilde\psi^\pm_{r'}(x)=e^{\pm i r p_F x}\psi^\pm_{r'}(x)$, $v_F p_F=\mu_0$, $:\tilde\psi^+_r(x)\tilde\psi^-_r(x):=:\psi^+_r(x)\psi^-_r(x):+\frac{\mu_0}{2\pi v_F}$

$$H_\lambda=\sum_{r=\pm}\int_{-L/2}^{L/2}dx:\psi^+_r(x)(-irv_F\partial_x)\psi^-_r(x)$$

$$+\lambda\sum_{r,r'}\int_{-L/2}^{L/2}dxdy\lambda v(x-y):\psi^+_r(x)\psi^-_r(x)::\psi^+_{r'}(y)\psi^-_{r'}(y):$$

The Luttinger model

- In terms of $\tilde{\psi}^{\pm}_{r'}(x) = e^{\pm i r p_F x}\psi^{\pm}_{r'}(x)$, $v_F p_F = \mu_0$,
  $:\tilde{\psi}^{+}_{r}(x)\tilde{\psi}^{-}_{r}(x) :=: \psi^{+}_{r}(x)\psi^{-}_{r}(x) : + \frac{\mu_0}{2\pi v_F}$

$$H_\lambda = \sum_{r=\pm} \int_{-L/2}^{L/2} dx : \psi^{+}_{r}(x)(-i r v_F \partial_x)\psi^{-}_{r}(x)$$

$$+\lambda \sum_{r,r'} \int_{-L/2}^{L/2} dx dy \lambda v(x-y) : \psi^{+}_{r}(x)\psi^{-}_{r}(x) :: \psi^{+}_{r'}(y)\psi^{-}_{r'}(y) :$$

- If $\rho_r(x) =: \psi^{+}_{+}(x)\psi^{-}_{+}(x)$: anomalous commutation reations (ML1966)
  $[\rho_r(p), \rho_{r'}(-p')] = r\delta_{r,r'} \frac{Lp}{2\pi} \delta_{p,p'}$

The Luttinger model

- In terms of $\tilde{\psi}^{\pm}_{r'}(x) = e^{\pm i r p_F x}\psi^{\pm}_{r'}(x)$, $v_F p_F = \mu_0$,
  $: \tilde{\psi}^{+}_{r}(x)\tilde{\psi}^{-}_{r}(x) :=: \psi^{+}_{r}(x)\psi^{-}_{r}(x) : + \frac{\mu_0}{2\pi v_F}$

$$H_\lambda = \sum_{r=\pm} \int_{-L/2}^{L/2} dx : \psi^{+}_{r}(x)(-irv_F\partial_x)\psi^{-}_{r}(x)$$

$$+\lambda \sum_{r,r'} \int_{-L/2}^{L/2} dxdy\lambda v(x-y) : \psi^{+}_{r}(x)\psi^{-}_{r}(x) :: \psi^{+}_{r'}(y)\psi^{-}_{r'}(y) :$$

- If $\rho_r(x) =: \psi^{+}_{+}(x)\psi^{-}_{+}(x)$: anomalous commutation reations (ML1966)
  $[\rho_r(p), \rho_{r'}(-p')] = r\delta_{r,r'}\frac{Lp}{2\pi}\delta_{p,p'}$
- GS of H_0 $|\Psi_0\rangle$ $a^{-}_{r,k}|\Psi_0\rangle = a^{+}_{r,-k}|\Psi_0\rangle = 0$ for all $rk > 0$;
  $: a^{+}_{r,k}a^{-}_{r',k'} := a^{+}_{r,k}a^{-}_{r',k'} - \langle\Psi_0|a^{+}_{r,k}a^{-}_{r',k'}|\Psi_0\rangle$

The Luttinger model

- In terms of $\tilde{\psi}^{\pm}_{r'}(x) = e^{\pm irp_F x}\psi^{\pm}_{r'}(x)$, $v_F p_F = \mu_0$,
  $:\tilde{\psi}^{+}_{r}(x)\tilde{\psi}^{-}_{r}(x) :=: \psi^{+}_{r}(x)\psi^{-}_{r}(x) : + \frac{\mu_0}{2\pi v_F}$

$$H_\lambda = \sum_{r=\pm}\int_{-L/2}^{L/2} dx : \psi^{+}_{r}(x)(-irv_F\partial_x)\psi^{-}_{r}(x)$$

$$+\lambda\sum_{r,r'}\int_{-L/2}^{L/2} dxdy\lambda v(x-y) : \psi^{+}_{r}(x)\psi^{-}_{r}(x) :: \psi^{+}_{r'}(y)\psi^{-}_{r'}(y) :$$

- If $\rho_r(x) =: \psi^{+}_{+}(x)\psi^{-}_{+}(x) :$ anomalous commutation reations (ML1966)
  $[\rho_r(p), \rho_{r'}(-p')] = r\delta_{r,r'}\frac{Lp}{2\pi}\delta_{p,p'}$
- GS of H_0 $|\Psi_0\rangle$ $a^{-}_{r,k}|\Psi_0\rangle = a^{+}_{r,-k}|\Psi_0\rangle = 0$ for all $rk > 0$;
  $: a^{+}_{r,k}a^{-}_{r',k'} := a^{+}_{r,k}a^{-}_{r',k'} - \langle\Psi_0|a^{+}_{r,k}a^{-}_{r',k'}|\Psi_0\rangle$
- $|\Psi_\lambda\rangle$ is the interacting ground state.

Equilibrium properties

- 2-point function $\langle \Psi_\lambda | \tilde{\psi}^+_r(x) \tilde{\psi}^-_r(y) | \Psi_\lambda \rangle$

$$= \frac{i e^{-irp_F(x-y)}}{2\pi r(x-y) + i0^+} \exp\left(\int_0^\infty dp \frac{\eta_\lambda(p)}{p} (\cos p(x-y) - 1) \right)$$

with $\eta_\lambda(p) = (1 - [\lambda \hat{v}(p)/(\pi + \lambda \hat{v}(p))]^2)^{-1/2} - 1.$

Equilibrium properties

- 2-point function $\langle \Psi_\lambda | \tilde\psi_r^+(x) \tilde\psi_r^-(y) | \Psi_\lambda \rangle$

$$= \frac{ie^{-irp_F(x-y)}}{2\pi r(x-y) + i0^+} \exp\left(\int_0^\infty dp \frac{\eta_\lambda(p)}{p} (\cos p(x-y) - 1) \right)$$

with $\eta_\lambda(p) = (1 - [\lambda \hat v(p)/(\pi + \lambda \hat v(p))]^2)^{-1/2} - 1$.

- The Fermi momentum appear as the oscillating factor.

Equilibrium properties

- 2-point function $\langle \Psi_\lambda | \tilde{\psi}^+_r(x) \tilde{\psi}^-_r(y) | \Psi_\lambda \rangle$

$$= \frac{i e^{-irp_F(x-y)}}{2\pi r(x-y) + i0^+} \exp\left(\int_0^\infty dp \frac{\eta_\lambda(p)}{p} (\cos p(x-y) - 1) \right)$$

with $\eta_\lambda(p) = (1 - [\lambda \hat{v}(p)/(\pi + \lambda \hat{v}(p))]^2)^{-1/2} - 1$.

- The Fermi momentum appear as the oscillating factor.
- The interaction modifies the long distance decay; Critical anomalous exponents $O(|x-y|^{-1-\eta(0)})$; The occupation number is discontinuous! On the other hand for small $x-y$ $O(|x-y|^{-1})$

Equilibrium properties

- 2-point function $\langle \Psi_\lambda | \tilde{\psi}_r^+(x) \tilde{\psi}_r^-(y) | \Psi_\lambda \rangle$

$$= \frac{ie^{-irp_F(x-y)}}{2\pi r(x-y) + i0^+} \exp\left(\int_0^\infty dp \frac{\eta_\lambda(p)}{p} (\cos p(x-y) - 1) \right)$$

with $\eta_\lambda(p) = (1 - [\lambda \hat{v}(p)/(\pi + \lambda \hat{v}(p))]^2)^{-1/2} - 1$.

- The Fermi momentum appear as the oscillating factor.
- The interaction modifies the long distance decay; Critical anomalous exponents $O(|x-y|^{-1-\eta(0)})$; The occupation number is discontinuous! On the other hand for small $x-y$ $O(|x-y|^{-1})$
- The integral is diverging for local iteraction; one can introduce an infinite wave function renormalization but the for small distances $O(|x-y|^{-1-\eta(0)})$ (the theory is scale invariant).

Equilibrium properties

- 2-point function $\langle \Psi_\lambda | \tilde{\psi}^+_r(x) \tilde{\psi}^-_r(y) | \Psi_\lambda \rangle$

$$= \frac{i e^{-irp_F(x-y)}}{2\pi r(x-y) + i0^+} \exp\left(\int_0^\infty dp \frac{\eta_\lambda(p)}{p} (\cos p(x-y) - 1) \right)$$

with $\eta_\lambda(p) = (1 - [\lambda \hat{v}(p)/(\pi + \lambda \hat{v}(p))]^2)^{-1/2} - 1$.

- The Fermi momentum appear as the oscillating factor.
- The interaction modifies the long distance decay; Critical anomalous exponents $O(|x-y|^{-1-\eta(0)})$; The occupation number is discontinuous! On the other hand for small $x-y$ $O(|x-y|^{-1})$
- The integral is diverging for local iteraction; one can introduce an infinite wave function renormalization but the for small distances $O(|x-y|^{-1-\eta(0)})$ (the theory is scale invariant).
- The ldensity is $\rho(x) = \rho_+(x) + \rho_-(x)$ and the current $j(x) = v_F(\rho_+(x) - \rho_-(x))$ (from continuity equation).

Non Equilibrium properties

In order to describe the evolution of a domain wall state we consider the ground state of an Hamiltonian with different chemical potentials in the left and right sides ($\mu_L = \mu_0 + \mu$, $\mu_L = \mu_0 - \mu$)

$$H_{\lambda,\mu} = H_\lambda - \mu \int_{-L/2}^{L/2} dx W(x)(\rho_+(x) + \rho_-(x))$$

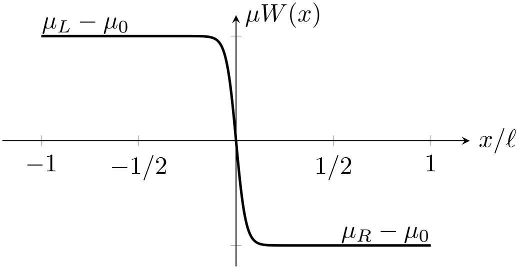


Figure: The chemical potential

Evolution

- We consider the ground state of $H_{\lambda,\mu}$, which of course have different densities in the L and R side, and we evolve it with the Luttinger Hamiltonian $H_{\lambda'}$

$$|\Psi^{\lambda'}_{\lambda,\mu}(t)\rangle = e^{-iH_{\lambda'}t}|\Psi_{\lambda,\mu}\rangle. \quad (1)$$

Evolution

- We consider the ground state of $H_{\lambda,\mu}$, which of course have different densities in the L and R side, and we evolve it with the Luttinger Hamiltonian $H_{\lambda'}$

$$|\Psi^{\lambda'}_{\lambda,\mu}(t)\rangle = e^{-iH_{\lambda'}t}|\Psi_{\lambda,\mu}\rangle. \quad (1)$$

- If $\lambda = \lambda'$ it corresponds to an experiment in which one switches off an external field producing an excess of density on one side of the system compared to the other at time t $=$ 0 and considers the evolution of the system under the translation invariant Hamiltonian for time t ¿ 0.

Evolution

- We consider the ground state of $H_{\lambda,\mu}$, which of course have different densities in the L and R side, and we evolve it with the Luttinger Hamiltonian $H_{\lambda'}$

$$|\Psi_{\lambda,\mu}^{\lambda'}(t)\rangle = e^{-iH_{\lambda'}t}|\Psi_{\lambda,\mu}\rangle. \tag{1}$$

- If $\lambda = \lambda'$ it corresponds to an experiment in which one switches off an external field producing an excess of density on one side of the system compared to the other at time t = 0 and considers the evolution of the system under the translation invariant Hamiltonian for time t ¿ 0.
- $\lambda \neq \lambda'$ there is in addition an interaction quench, i.e., at t = 0 there is also a change from coupling λ to λ'

Evolution

- We consider the ground state of $H_{\lambda,\mu}$, which of course have different densities in the L and R side, and we evolve it with the Luttinger Hamiltonian $H_{\lambda'}$

$$|\Psi^{\lambda'}_{\lambda,\mu}(t)\rangle = e^{-iH_{\lambda'}t}|\Psi_{\lambda,\mu}\rangle. \tag{1}$$

- If $\lambda = \lambda'$ it corresponds to an experiment in which one switches off an external field producing an excess of density on one side of the system compared to the other at time t = 0 and considers the evolution of the system under the translation invariant Hamiltonian for time t ¿ 0.
- $\lambda \neq \lambda'$ there is in addition an interaction quench, i.e., at t = 0 there is also a change from coupling λ to λ'
- If $\mu = 0$ there only a quench (Cazalilla $\lambda = 0$)

The non interacting case

- $$\langle \Psi^0_{0,\mu}(t)|\rho(x)|\Psi^0_{0,\mu}(t)\rangle = \frac{\mu}{2\pi v_F}\left(W(x - v_F t) + (x + v_F t)\right),$$
  $$\langle \Psi^0_{0,\mu}(t)|j(x)|\Psi^0_{0,\mu}(t)\rangle = \frac{\mu}{2\pi}\left(W(x - v_F t) - W(x + v_F t)\right).$$

  A central region $(-v_F t, v_F t)$ around $x = 0$ with zero total density, relative to the large constant ground state density, bounded by two fronts moving with constant velocity.

The non interacting case

- $$\langle \Psi^0_{0,\mu}(t)|\rho(x)|\Psi^0_{0,\mu}(t)\rangle = \frac{\mu}{2\pi v_F}\left(W(x - v_F t) + (x + v_F t)\right),$$
  $$\langle \Psi^0_{0,\mu}(t)|j(x)|\Psi^0_{0,\mu}(t)\rangle = \frac{\mu}{2\pi}\left(W(x - v_F t) - W(x + v_F t)\right).$$
  A central region $(-v_F t, v_F t)$ around $x = 0$ with zero total density, relative to the large constant ground state density, bounded by two fronts moving with constant velocity.
- The shape of the fronts does not change with time; as $t \to \infty$, the system reaches a state with vanishing total density everywhere.

The non interacting case

- $$\langle \Psi^0_{0,\mu}(t)|\rho(x)|\Psi^0_{0,\mu}(t)\rangle = \frac{\mu}{2\pi v_F}\left(W(x-v_Ft)+(x+v_Ft)\right),$$
$$\langle \Psi^0_{0,\mu}(t)|j(x)|\Psi^0_{0,\mu}(t)\rangle = \frac{\mu}{2\pi}\left(W(x-v_Ft)-W(x+v_Ft)\right).$$
  A central region $(-v_F t, v_F t)$ around $x=0$ with zero total density, relative to the large constant ground state density, bounded by two fronts moving with constant velocity.
- The shape of the fronts does not change with time; as $t\to\infty$, the system reaches a state with vanishing total density everywhere.
- Similarly, the current is non-zero in the same region, and, as $t\to\infty$, it is tends to the non-vanishing value $\mu/2\pi = \frac{e^2}{h}(\mu_L-\mu_R)$ everywhere.

The non interacting case

- The two-point correlation function without interaction is given by

$$\langle \Psi^0_{0,\mu}(t)|\psi^+_r(x)\psi^-_r(y)|\Psi^0_{0,\mu}(t)\rangle =$$

$$\frac{i}{2\pi r(x-y)+i0^+}\exp\left(-irv_F^{-1}\mu\int_{y-rv_F t}^{x-rv_F t}dzW(z)\right).$$

For finite t, the two-point correlation function is not translation invariant.

The non interacting case

- The two-point correlation function without interaction is given by

$$\langle \Psi^0_{0,\mu}(t)|\psi^+_r(x)\psi^-_r(y)|\Psi^0_{0,\mu}(t)\rangle =$$

$$\frac{i}{2\pi r(x-y)+i0^+}\exp\left(-irv_F^{-1}\mu\int_{y-rv_Ft}^{x-rv_Ft}dzW(z)\right).$$

For finite t, the two-point correlation function is not translation invariant.

- $$\lim_{t\to\infty}\langle \Psi^0_{0,\mu}(t)|\tilde\psi^+_r(x)\tilde\psi^-_r(y)|\Psi^0_{0,\mu}(t)\rangle$$

$$= \frac{ie^{-irv_F^{-1}(\mu_0+r\mu/2)(x-y)}}{2\pi r(x-y)+i0^+}$$

The non interacting case

- The two-point correlation function without interaction is given by
$$\langle\Psi^0_{0,\mu}(t)|\psi^+_r(x)\psi^-_r(y)|\Psi^0_{0,\mu}(t)\rangle =$$
$$\frac{i}{2\pi r(x-y)+i0^+}\exp\left(-irv_F^{-1}\mu\int_{y-rv_Ft}^{x-rv_Ft}dzW(z)\right).$$
For finite t, the two-point correlation function is not translation invariant.
- $$\lim_{t\to\infty}\langle\Psi^0_{0,\mu}(t)|\tilde\psi^+_r(x)\tilde\psi^-_r(y)|\Psi^0_{0,\mu}(t)\rangle$$
$$=\frac{ie^{-irv_F^{-1}(\mu_0+r\mu/2)(x-y)}}{2\pi r(x-y)+i0^+}$$
- The final steady state is the ground state of free fermions with different chemical potentials $\mu_\pm=\mu_0\pm\mu/2$ for right- and left-moving fermions, obtained from the two-point correlation function. This is what gets if an external potential is applied $\mu_+-\mu_-=eV$.

The current satisfies the following relation in the non-interacting case:

$$I = \frac{e^2}{h}(\mu_L - \mu_R) = \frac{e^2}{h}(\mu_+ - \mu_-), \tag{2}$$

Landauer conductance $I/(\mu_+ - \mu_-) = \frac{e^2}{h}$.

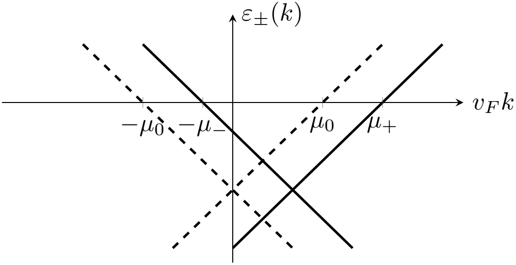


Figure: Fermi sea at infinity

The current satisfies the following relation in the non-interacting case:

$$I = \frac{e^2}{h}(\mu_L - \mu_R) = \frac{e^2}{h}(\mu_+ - \mu_-), \quad (2)$$

Landauer conductance $I/(\mu_+ - \mu_-) = \frac{e^2}{h}$.

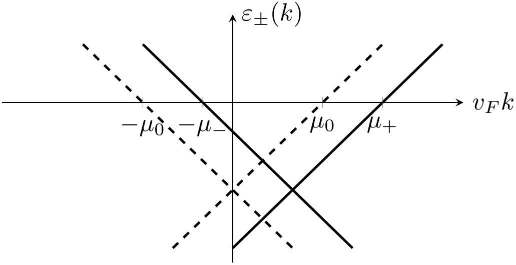


Figure: Fermi sea at infinity

How the interaction modifies the above picture?

The interacting case

- $$\langle\Psi^{\lambda'}_{\lambda,\mu}(t)|\rho(x)|\Psi^{\lambda'}_{\lambda,\mu}(t)\rangle = \frac{\mu}{2\pi}\int_{-\infty}^{\infty}\frac{dp}{2\pi}\frac{K_\lambda(p)}{v_\lambda(p)}\hat{W}(p)2\cos(pv_{\lambda'}(p)t)e^{ipx}$$

$$\langle\Psi^{\lambda'}_{\lambda,\mu}(t)|j(x)|\Psi^{\lambda'}_{\lambda,\mu}(t)\rangle = \frac{\mu}{2\pi}\int_{-\infty}^{\infty}\frac{dp}{2\pi}\frac{K_\lambda(p)}{v_\lambda(p)}\hat{W}(p)v_{\lambda'}(p)(-2i\sin(pv_{\lambda'}(p)t))e^{ipx}$$

with the *renormalized Fermi velocity*

$$v_\lambda(p) = \sqrt{1+2\hat{\lambda}(p)/\pi}$$

and the *Luttinger parameter*

$$K_\lambda(p) = \frac{1}{\sqrt{1+2\hat{\lambda}(p)/\pi}}.$$

The interacting case

- $$\langle \Psi^{\lambda'}_{\lambda,\mu}(t)|\rho(x)|\Psi^{\lambda'}_{\lambda,\mu}(t)\rangle = \frac{\mu}{2\pi}\int_{-\infty}^{\infty}\frac{dp}{2\pi}\frac{K_\lambda(p)}{v_\lambda(p)}\hat{W}(p)2\cos(pv_{\lambda'}(p)t)e^{ipx}$$

  $$\langle \Psi^{\lambda'}_{\lambda,\mu}(t)|j(x)|\Psi^{\lambda'}_{\lambda,\mu}(t)\rangle = \frac{\mu}{2\pi}\int_{-\infty}^{\infty}\frac{dp}{2\pi}\frac{K_\lambda(p)}{v_\lambda(p)}\hat{W}(p)v_{\lambda'}(p)(-2i\sin(pv_{\lambda'}(p)t))e^{ip}$$

  with the *renormalized Fermi velocity*

  $$v_\lambda(p) = \sqrt{1+2\hat{\lambda}(p)/\pi}$$

  and the *Luttinger parameter*

  $$K_\lambda(p) = \frac{1}{\sqrt{1+2\hat{\lambda}(p)/\pi}}.$$

- The system evolves ballistically but non-local interaction produces dispersion effects.

Plots

Evolution of the density from the non interacting domain wall GS with $\lambda' = -0.96$, range $0, 00025l$, $t = 0, 2l, 4l, 6l$

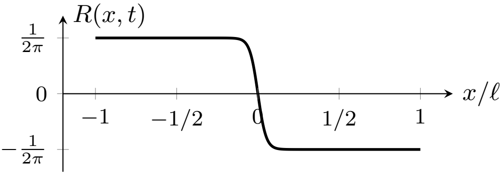
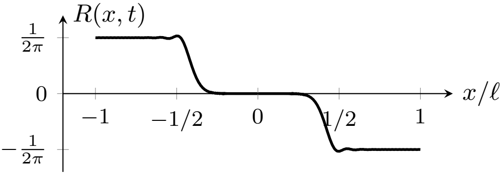


Figure: $t = 0$



Plots

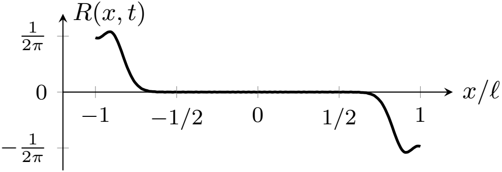


Figure: $t = 4l$

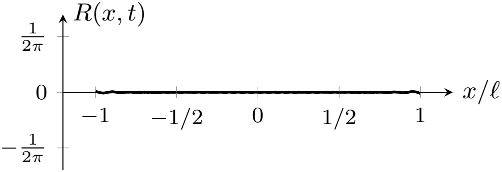


Figure: $t = 6l$

Plots

Evolution of the current from the non interacting domain wall GS with $\lambda' = -0.96$, range $0, 00025l$, $t = 0, 2l, 4l, 6l$

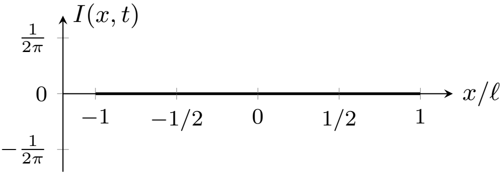
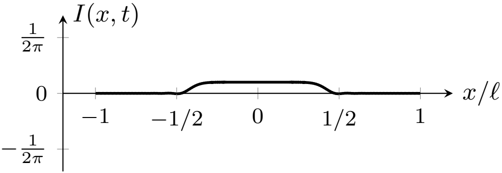


Figure: $t = 0$



Plots

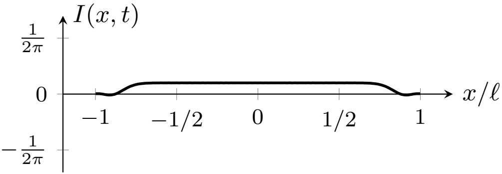


Figure: $t = 4l$

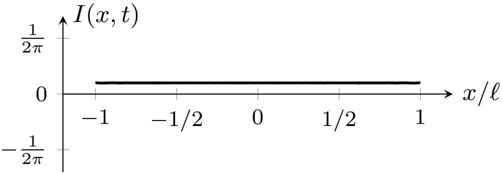


Figure: $t = 6l$

Plots

Evolution of the density from the non interacting domain wall GS with $\lambda' = -0.96$, range $0, 00025l$, $t = 0, 2l, 4l, 6l$

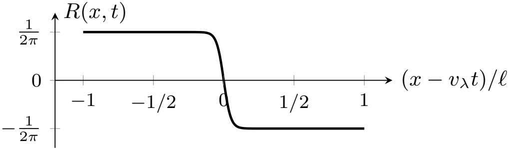


Figure: $t = 0$

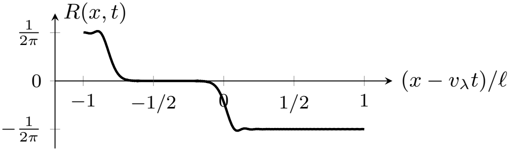


Figure: $t = 2l$

Plots

Evolution of the density from the non interacting domain wall GS with $\lambda' = -0.96$, range $0, 00025l$, $t = 2l, 4l$

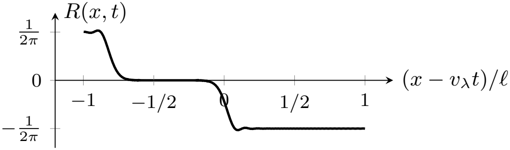


Figure: $t = 2l$

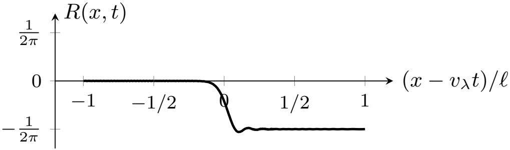


Figure: $t = 2l$

The 2-point function

- $$\langle\Psi^{\lambda'}_{\lambda,\mu}(t)|\psi^+_r(x)\psi^-_r(y)|\Psi^{\lambda'}_{\lambda,\mu}(t)\rangle = e^{-ir^{-1}A_r(x,y,t)(x-y)}S_r(x,y,t)$$

  with $A_r(x,y,t)$

  $$= \mu\int_{-\infty}^{\infty}\frac{dp}{2\pi}\frac{K_\lambda(p)}{v_\lambda(p)}\hat W(p)\left(\cos(pv_{\lambda'}(p)t) - irv_{\lambda'}(p)\sin(pv_{\lambda'}(p)t)\right)\frac{e^{ipx}-e^{ipy}}{ip(x-y)}$$

  and

  $$S_r(x,y,t) = \langle\Psi_\lambda|e^{iH_{\lambda'}t}\psi^+_r(x)\psi^-_r(y)e^{-iH_{\lambda'}t}|\Psi_\lambda\rangle$$

The 2-point function

- $$\langle\Psi^{\lambda'}_{\lambda,\mu}(t)|\psi^+_r(x)\psi^-_r(y)|\Psi^{\lambda'}_{\lambda,\mu}(t)\rangle = e^{-ir^{-1}A_r(x,y,t)(x-y)}S_r(x,y,t)$$
  with $A_r(x,y,t)$
  $$= \mu\int_{-\infty}^{\infty}\frac{dp}{2\pi}\frac{K_\lambda(p)}{v_\lambda(p)}\hat{W}(p)\left(\cos(pv_{\lambda'}(p)t) - irv_{\lambda'}(p)\sin(pv_{\lambda'}(p)t)\right)\frac{e^{ipx}-e^{ipy}}{ip(x-y)}$$
  and
  $$S_r(x,y,t) = \langle\Psi_\lambda|e^{iH_{\lambda'}t}\psi^+_r(x)\psi^-_r(y)e^{-iH_{\lambda'}t}|\Psi_\lambda\rangle$$

- $S_r(x,y,t) = \frac{i}{2\pi r(x-y)+i0^+}\times$
  $$\exp\left(\int_0^\infty dp\frac{\eta_{\lambda,\lambda'}(p) - \gamma_{\lambda,\lambda'}(p)\cos(2pv_{\lambda'}(p)t)}{p}(\cos p(x-y)-1)\right)$$
  If $\lambda=\lambda'$ then $\eta_{\lambda,\lambda}=\eta_\lambda$ and $\gamma_\lambda=0$ (it reduces to the quilibrium correlation). $\eta_{\lambda,\lambda'} = \frac{K_\lambda(K^{-2}_{\lambda'}+1)+K^{-1}_\lambda(K^2_{\lambda'}+1)}{4}-1$, $\gamma_{\lambda,\lambda'} = \frac{K_\lambda(K^{-2}_{\lambda'}-1)+K^{-1}_\lambda(K^2_{\lambda'}-1)}{4}$. Quench: is the evolution of the GS of H_λ

Large time behavior

- In the limit $t \to \infty$ a stationary state is reached
- zero density

$$\lim_{t\to\infty} \langle \Psi^{\lambda'}_{\lambda,\mu}(t)|\rho(x)|\Psi^{\lambda'}_{\lambda,\mu}(t)\rangle = 0$$

stationary current

$$\lim_{t\to\infty} \langle \Psi^{\lambda'}_{\lambda,\mu}(t)|j(x)|\Psi^{\lambda'}_{\lambda,\mu}(t)\rangle = \frac{(\mu_L - \mu_R)}{2\pi} \frac{K_\lambda v_{\lambda'}}{v_\lambda}$$

- The limiting current depends on the interaction; $\lambda = \lambda'$ equilibrium result by Kubo. If $\lambda \neq \lambda'$ memory of the initial state.

The 2-point function

- In the limit translation invariance is recovered
$\lim_{t\to\infty}\langle\Psi^{\lambda'}_{\lambda,\mu}(t)|\tilde\psi^+_r(x)\tilde\psi^-_r(y)|\Psi^{\lambda'}_{\lambda,\mu}(t)\rangle$

$$= \frac{ie^{-ir^{-1}(\mu_0+r\mu K_\lambda v_{\lambda'}/2v_\lambda)(x-y)}}{2\pi r(x-y)+i0^+}\exp\left(\int_0^\infty dp\frac{\eta_{\lambda,\lambda'}(p)}{p}\left(\cos p(x-y)-1\right)\right)$$

The 2-point function

- In the limit translation invariance is recovered
$\lim_{t\to\infty}\langle\Psi^{\lambda'}_{\lambda,\mu}(t)|\tilde\psi^+_r(x)\tilde\psi^-_r(y)|\Psi^{\lambda'}_{\lambda,\mu}(t)\rangle$

$$= \frac{i e^{-i r^{-1}(\mu_0 + r\mu K_\lambda v_{\lambda'}/2v_\lambda)(x-y)}}{2\pi r(x-y)+i0^+}\exp\left(\int_0^\infty dp\frac{\eta_{\lambda,\lambda'}(p)}{p}(\cos p(x-y)-1)\right)$$

- If $\lambda=\lambda'$ (no quench) is the ground state correlation of luttinger with different chemical potential for left and right going fermion. generalized gibbs ensembles

The 2-point function

- In the limit translation invariance is recovered $\lim_{t\to\infty}\langle\Psi^{\lambda'}_{\lambda,\mu}(t)|\tilde\psi^+_r(x)\tilde\psi^-_r(y)|\Psi^{\lambda'}_{\lambda,\mu}(t)\rangle$

$$= \frac{ie^{-ir^{-1}(\mu_0+r\mu K_\lambda v_{\lambda'}/2v_\lambda)(x-y)}}{2\pi r(x-y)+i0^+}\exp\left(\int_0^\infty dp\frac{\eta_{\lambda,\lambda'}(p)}{p}(\cos p(x-y)-1)\right)$$

- If $\lambda=\lambda'$ (no quench) is the ground state correlation of luttinger with different chemical potential for left and right going fermion. generalized gibbs ensembles
- If $\lambda\neq\lambda'$ we do not know if is the gibbs expectation of some hamiltonian; still suggests that there are two chemical potentials for left and right going particles

Universality

- Fermions with different chemical potentials for right- and left-moving particles,

$$\mu_\pm = \mu_0 \pm \frac{\mu}{2} \frac{K_\lambda v_{\lambda'}}{v_\lambda}, \tag{3}$$

- the final state depends on the details of the time evolution and the initial state but the Landauer conductance is universal:

$$G = \frac{I}{\mu_+ - \mu_-} = \frac{\mu K_\lambda v_{\lambda'}}{2\pi v_\lambda} \frac{v_\lambda}{\mu K_\lambda v_{\lambda'}} = \frac{1}{2\pi}$$

Universality

- There has been an extensive debate on the universality or not of transport in luttinger liquis.

Universality

- There has been an extensive debate on the universality or not of transport in luttinger liquis.
- Alekseev, Cheianov, Froehlich (1998) considered a system with different chemical potentials for particles with positive or negative velocity and get universality.

Universality

- There has been an extensive debate on the universality or not of transport in luttinger liquis.
- Alekseev, Cheianov, Froehlich (1998) considered a system with different chemical potentials for particles with positive or negative velocity and get universality.
- If we start from a partitioned system with different chemical potentials in left and right side we recover dynamically the same model without quench. Universality is recovered in a non equilibrium setting.

Heat

- We consider now as initial state a (smooth) temperature profile $T(x)$.

Heat

- We consider now as initial state a (smooth) temperature profile $T(x)$.
- if $H(x)$ is the energy density operator defining the Hamiltonian, $\mathbf{H} = \int dx\, H(x)$, then the initial state is given by $\rho_{neq} = e^{-G}/\mathrm{Tr}e^{-G}$ with

$$G = \int dx\, \beta(x) H(x), \tag{4}$$

where $\beta(x) \equiv T(x)^{-1} = \beta[1 + \varepsilon W(x)]$ for some function $W(x)$ with β the average inverse temperature and the deviation from equilibrium.

Heat

- We consider now as initial state a (smooth) temperature profile $T(x)$.
- if $H(x)$ is the energy density operator defining the Hamiltonian, $\mathbf{H} = \int dx\, H(x)$, then the initial state is given by $\rho_{neq} = e^{-G}/\mathrm{Tr} e^{-G}$ with

$$G = \int dx\, \beta(x) H(x), \tag{4}$$

where $\beta(x) \equiv T(x)^{-1} = \beta[1 + \varepsilon W(x)]$ for some function $W(x)$ with β the average inverse temperature and the deviation from equilibrium.

- We will mainly be concerned with the case of a step-like profile $T(x)$ equal to T_L (T_R) far to the left (right), e.g., $W(x) = -(1/2)\tanh(x/)$ with > 0, where β and are determined by $\beta(\mp\infty) = T_{L,R}^{-1}$. The evolution of the system is given by $\mathbf{H}$ and

$$< O(t) >_{\text{neq}} = Tr \rho_{neq}(t) O = Tr \rho_{neq} O(t), \tag{5}$$

where $O(t) = e^{i\mathbf{H}t} O e^{-i\mathbf{H}t}$, $\rho_{neq}(t) = e^{i\mathbf{H}t} \rho_{neq} e^{-i\mathbf{H}t}$. If $= 0$ is an equilibrium expectation value with temperature $T = \beta^{-1}$

Local interaction

- In the non interaction, or with local interaction

$$E(x,t) = \frac{1}{2}\left[G(x - vt) + G(x + vt)\right],$$
$$J(x,t) = \frac{}{2}\left[G(x - vlt) - G(x - vt)\right]$$

with

$$G(x) = \frac{\pi}{6v}\frac{1}{\beta(x)^2} + \frac{v}{12\pi}\left(\frac{\beta''(x)}{\beta(x)} - \frac{1}{2}\left(\frac{\beta'(x)}{\beta(x)}\right)^2\right)$$
$$= \frac{\pi}{6v}T(x)^2 - \frac{v}{12\pi}(Sg)(x)$$

with $g = \int_0^x dx' T(x')/T$.

Local interaction

- In the non interaction, or with local interaction

$$E(x,t) = \frac{1}{2}\left[G(x-vt)+G(x+vt)\right],$$
$$J(x,t) = \frac{}{2}\left[G(x-vlt)-G(x-vt)\right]$$

with

$$G(x) = \frac{\pi}{6v}\frac{1}{\beta(x)^2} + \frac{v}{12\pi}\left(\frac{\beta''(x)}{\beta(x)} - \frac{1}{2}\left(\frac{\beta'(x)}{\beta(x)}\right)^2\right)$$
$$= \frac{\pi}{6v}T(x)^2 - \frac{v}{12\pi}(Sg)(x)$$

with $g = \int_0^x dx' T(x')/T$.

- Exact resummation of power series in ε;S_g is the Schwartzian derivative (natural object in CFT).

Local interaction

- In the non interaction, or with local interaction

$$E(x,t) = \frac{1}{2}\left[G(x-vt)+G(x+vt)\right],$$
$$J(x,t) = \frac{}{2}\left[G(x-vlt)-G(x-vt)\right]$$

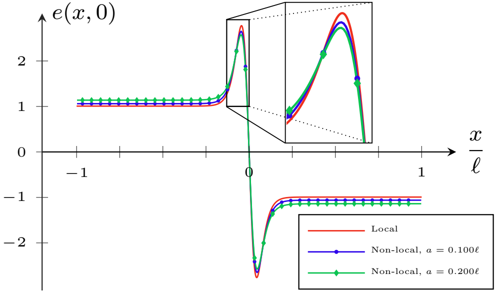
with

$$G(x) = \frac{\pi}{6v}\frac{1}{\beta(x)^2} + \frac{v}{12\pi}\left(\frac{\beta''(x)}{\beta(x)} - \frac{1}{2}\left(\frac{\beta'(x)}{\beta(x)}\right)^2\right)$$
$$= \frac{\pi}{6v}T(x)^2 - \frac{v}{12\pi}(Sg)(x)$$

with $g = \int_0^x dx' T(x')/T$.

- Exact resummation of power series in ε;S$_g$ is the Schwartzian derivative (natural object in CFT).
- Ballistic motion of the fronts; the S_g term, proportional to derivative, generates a peak. One could imagine that the energy is proportional to the temperature profile; instead an extra term appear. Absent in previous analysis (Bernard, Doyon)

Heat



$e(x,0)$
2
1
0
−1
−2
−1
0
1
$\frac{x}{\ell}$
Local
Non-local, $a = 0.100\ell$
Non-local, $a = 0.200\ell$

Figure:

Non Local interaction

In the case of non local interaction there is an extra length; results only at first order

$$E(x,t) = E_0 + \varepsilon E_1(x,t) + O(\varepsilon^2)$$
$$J(x,t) = \varepsilon J_1(x,t) + O(\varepsilon^2),$$

where E_0 is equal to $\lim_{t\to\infty} E(x,t)$ and

$$E_1(x,t) = -\sum_{r,r'} \frac{dp}{2\pi} \int \frac{dq}{4\pi} \hat{W}(p) A(p-q,q)$$

$$J_1(x,t) = -\sum_{r,r'} \frac{dp}{2\pi} \int \frac{dq}{4\pi} \hat{W}(p) \frac{i}{p} \frac{\partial}{\partial t} A(p-q,q)$$

with

$$A(p,p') = e^{i(p+p')x - i[r(p)+r'(p')]t} \times \frac{[r(p)+r'(p')]^2}{4(p)(p')} \frac{[re^{2\varphi(p)} + r'e^{2\varphi(p')}]^2}{4e^{2[\varphi(p)+\varphi(p')]}}$$
$$\times \frac{e^{\beta[r(p)+r'(p')]}-1}{r(p)+r'(p')} \frac{r(p)}{e^{\beta r(p)}-1} \frac{r'(p')}{e^{\beta r'(p')}-1}.$$

heat

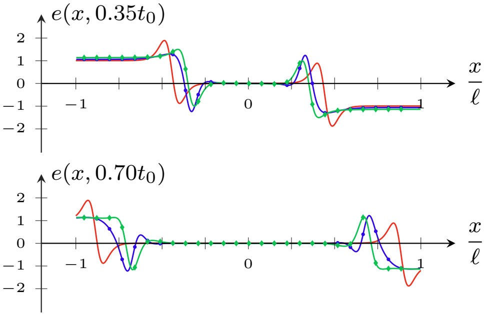


Figure: Evolution of the energy; the fronts move ballistically, a NESS is reachef, in the non local case dispersion effects are visible

heat

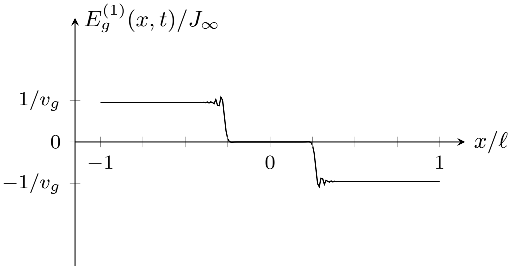


Figure: $t = 2l$

heat

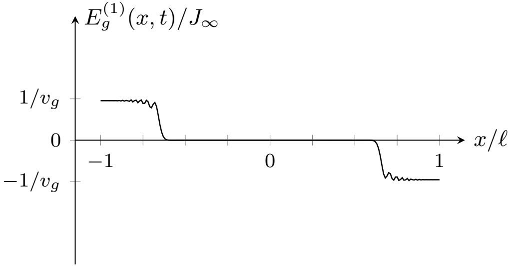


Figure: $t = 2l$

heat

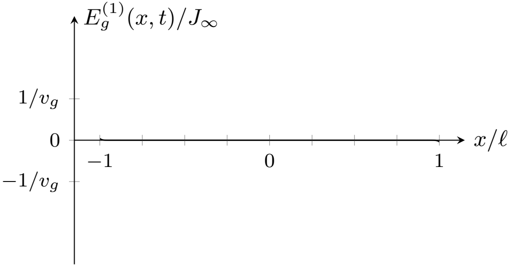


Figure: $t = 2l$

heat

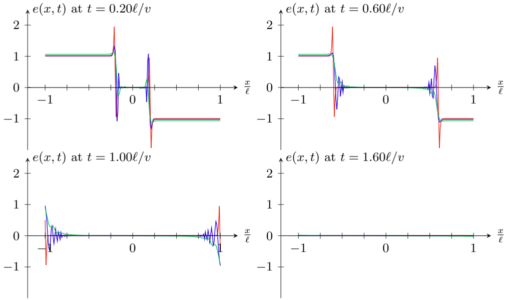


Figure: $t = 2l$

NESS

- A stationary state is reached carrying a current.

$$\lim_{t\to\infty} Tr\rho_{neq}(t)O = \frac{Tre^{-\beta_+ H_+ - \beta_- H_-}O}{\mathrm{Tr}e^{-\beta_+ H_+ - \beta_- H_-}}$$

with $\beta_\pm = T_{L,R}^{-1}$. This NESS describes a translation invariant state factorized into right- and left-moving plasmons (the bosonic modes diagonalizing the Hamiltonian)at equilibrium with temperatures $T_\pm = 1/\beta_\pm$.

NESS

- A stationary state is reached carrying a current.

$$\lim_{t\to\infty} Tr\rho_{neq}(t)O = \frac{Tre^{-\beta_+ H_+ - \beta_- H_-} O}{\mathrm{Tr}e^{-\beta_+ H_+ - \beta_- H_-}}$$

with $\beta_\pm = T_{L,R}^{-1}$. This NESS describes a translation invariant state factorized into right- and left-moving plasmons (the bosonic modes diagonalizing the Hamiltonian)at equilibrium with temperatures $T_\pm = 1/\beta_\pm$.

- As $\int dx\, H(x) = \sum_r H_r$ with H_r using the continuity equation to show that $\int dx\, J(x) = \frac{1}{2}\sum_r r \int dq\, \frac{d\omega(q)}{dq}(q)\tilde\rho_r(-q)\tilde\rho_r(q)$, we obtain

$$\lim_{t\to\infty} E(x,t) = w_\lambda + \sum_r \int_+ \frac{dq}{2\pi} \frac{\omega(q)}{e^{\beta_r \omega(q)} + 1},$$

$$\lim_{t\to\infty} J(x,t) = \sum_r r \int_+ \frac{dq}{2\pi} \frac{d\omega(q)}{dq} \frac{\omega(q)}{e^{\beta_r \omega(q)} + 1}$$

NESS

- By the change of variables $u=\beta_r\omega(q)$ we obtain

$$\lim_{t\to\infty} J(x,t)=\sum_r r\frac{\pi T_r^2}{12}=\frac{\pi}{12}(T_L^2-T_R^2)\equiv J \tag{6}$$

The final heat current only depends on $T_{L,R}$ and is independent of microscopic details. Such universal behavior, previously observed in CFT, , thus remains true even when scale invariance is broken.

NESS

- By the change of variables $u = \beta_r \omega(q)$ we obtain

$$\lim_{t\to\infty} J(x,t) = \sum_r r \frac{\pi T_r^2}{12} = \frac{\pi}{12}(T_L^2 - T_R^2) \equiv J \tag{6}$$

  The final heat current only depends on $T_{L,R}$ and is independent of microscopic details. Such universal behavior, previously observed in CFT, , thus remains true even when scale invariance is broken.

- Thhe energy density in the NESS as a sum of energy densities at equilibrium with temperatures $T_{L,R}$ and is non-universal. Indeed, it depends on the interaction, and only in the local case, when $(p) =$ and $\varphi(p) = \varphi$ are constant, does it simplify to

$$\lim_{t\to\infty} E(x,t) = \sum_r \frac{\pi}{12} T_r^2 = \frac{\pi}{12}(T_L^2 + T_R^2)$$

Conclusions

- We consider the non rquilibrium evolution of a solvable model of interacting fermions
- The system reaches a steady state (NESS)which is not a themal state due to conserved quantities.

Conclusions

- We consider the non rquilibrium evolution of a solvable model of interacting fermions
- The system reaches a steady state (NESS)which is not a themal state due to conserved quantities.
- Remarkable universality properties
- NESS carries a current; no Fourier law.
- What happens breaking integrability? A thermal state is reached? analogy with classical dynamics ?