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Dynamics of a soliton in an external potential

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Joint work with A. Fusé, A. Maspero, S. Sansottera

Consider the nonlinear Schrödinger equation

$$-i\psi_t = -\Delta\psi - \beta(|\psi|^2)\psi + \epsilon V\psi\,, \quad \beta \in C^\omega\,, \quad V \in \mathcal{S} \cap C^\omega\,;$$

it is well known that, when $\epsilon = 0$, under suitable conditions on β, the NLS admitts traveling wave solutions (soliton for short). When $\epsilon \neq 0$, heuristic considerations suggest that the soliton should move as a particle subject to a mechanical force due to an effective potential computed from V. The problem is to understand if this is true or not.

In this talk I will show that the soliton does not exchange neither energy nor angular momentum with the rest of the field for times of the order ϵ^{-r} for any r. This allows to deduce some informations on the trajectory of the soliton.

The proof is composed by two steps: first one shows that the Hamiltonian of the NLS can be rewritten in suitable coordinates as follows

$$H = \epsilon^{1/2} H_{mech}(p,q) + \tfrac{1}{2}\langle EL_0\phi, \phi\rangle + h.o.t.$$

where $H_{mech}(p,q)$ is the hamiltonian of a mechanical particle in a central potential and describes the motion of the soliton's barycenter, while ϕ is a function representing the "free" field.

The second step consistes in applying a Nekhoroshev type theorem; in turn this requires to verify a nondegeneracy hypothesis on $H_{mech}(p,q)$ (quasiconvexity). This is obtained by applying a result that we recently got ensuring that such an assumption is always satisfied in the central motion problem except for Harmonic end Keplerian potentials.

In the talk I will also try to give the ideas of the proof of this second result.

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