## Dynamics of a soliton in an external potential

Thursday, June 8, 2017 9:30 AM (40 minutes)

Joint work with A. Fusé, A. Maspero, S. Sansottera

Consider the nonlinear Schrödinger equation
$-i \psi_{t}=-\Delta \psi-\beta\left(|\psi|^{2}\right) \psi+\epsilon V \psi, \quad \beta \in C^{\omega}, \quad V \in \mathcal{S} \cap C^{\omega} ;$
it is well known that, when $\epsilon=0$, under suitable conditions on $\beta$, the NLS admitts traveling wave solutions (soliton for short). When $\epsilon \neq 0$, heuristic considerations suggest that the soliton should move as a particle subject to a mechanical force due to an effective potential computed from $V$. The problem is to understand if this is true or not.

In this talk I will show that the soliton does not exchange neither energy nor angular momentum with the rest of the field for times of the order $\epsilon^{-r}$ for any $r$. This allows to deduce some informations on the trajectory of the soliton.

The proof is composed by two steps: first one shows that the Hamiltonian of the NLS can be rewritten in suitable coordinates as follows
$H=\epsilon^{1 / 2} H_{\text {mech }}(p, q)+\frac{1}{2}\left\langle E L_{0} \phi, \phi\right\rangle+$ h.o.t.
where $H_{\text {mech }}(p, q)$ is the hamiltonian of a mechanical particle in a central potential and describes the motion of the soliton's
barycenter, while $\phi$ is a function representing the "free"
field.
The second step consistes in applying a Nekhoroshev type theorem; in turn this requires to verify a nondegeneracy hypothesis on $H_{m e c h}(p, q)$ (quasiconvexity). This is obtained by applying a result that we recently got ensuring that such an assumption is always satisfied in the central motion problem except for Harmonic end Keplerian potentials.

In the talk I will also try to give the ideas of the proof of this second result.

Primary author: Prof. BAMBUSI, Dario (Dipartimento di Matematica, Università degli studi di Milano)

Presenter: Prof. BAMBUSI, Dario (Dipartimento di Matematica, Università degli studi di Milano)

