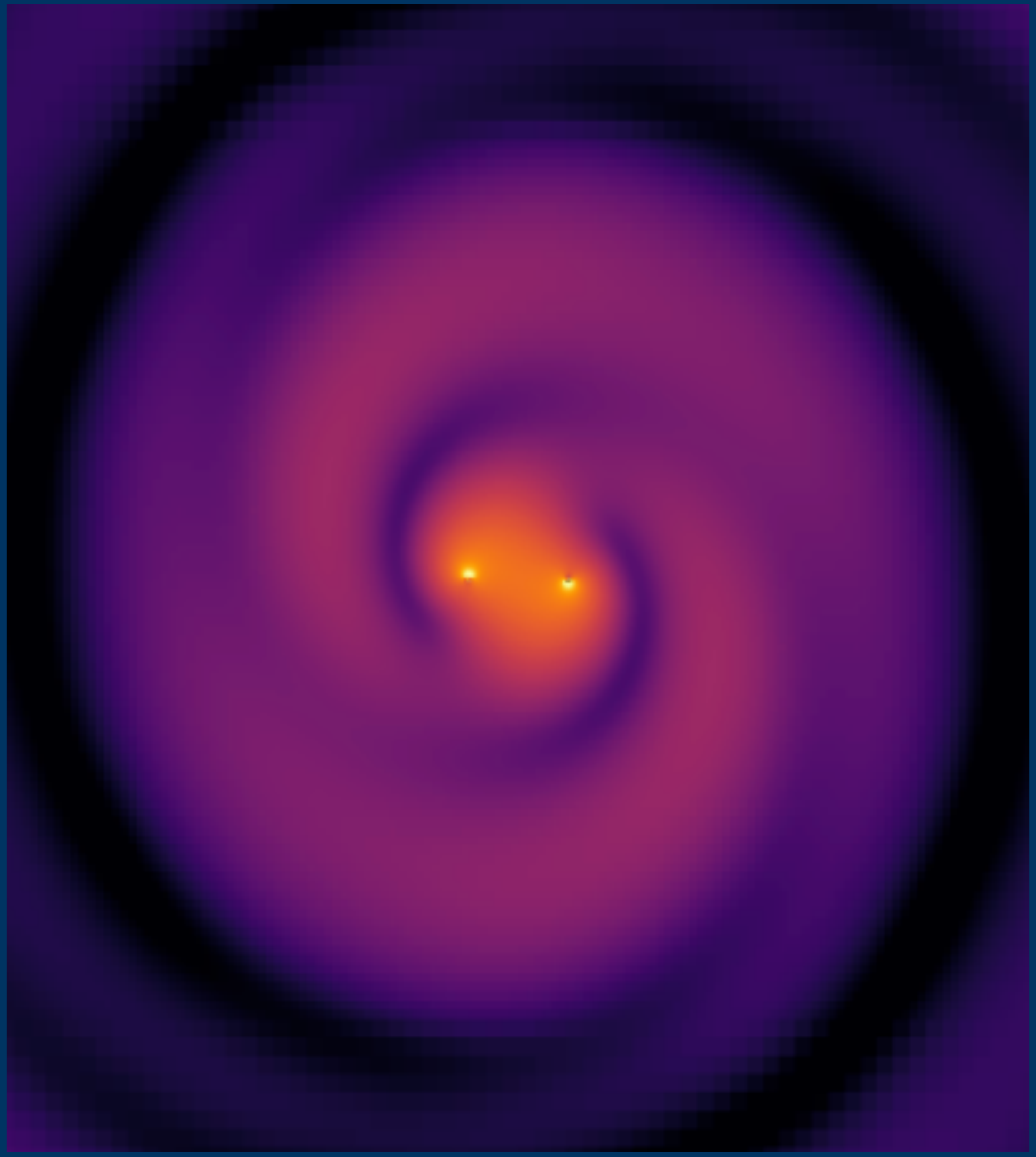
NR: What is it good for?

KATY CLOUGH



Overview

- **Light/wave dark matter**

  NR work has shown:

  - Dark matter can cluster around binaries.

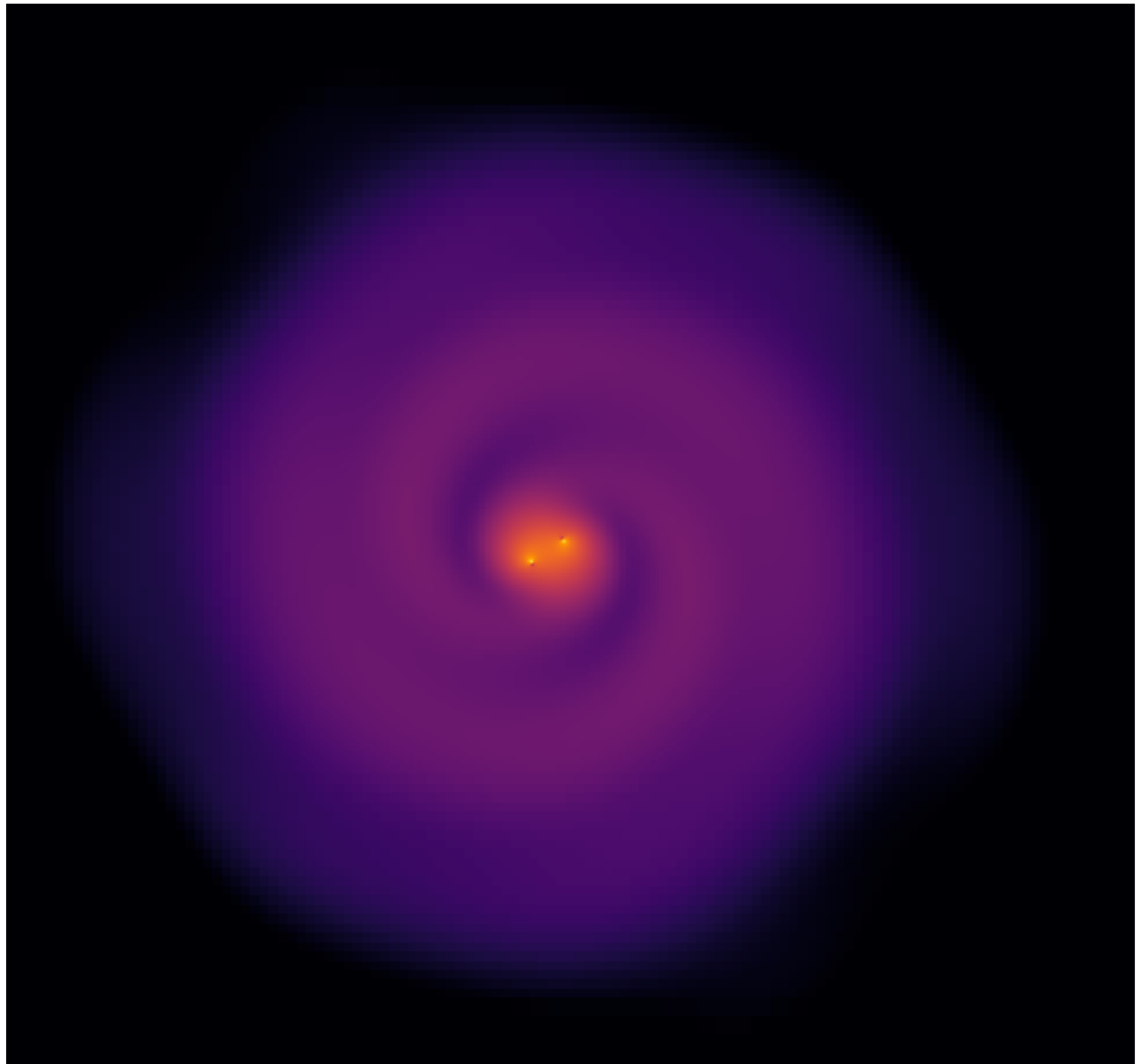
  - We can infer its density through GW observations.

- **Heavy/particle dark matter**

  NR work is needed (?):

  - Are there (similar BH mass) scenarios where particle DM can cluster? (e.g. Kozai Lidov)

  - What is the impact of third bodies / (P)BH clusters in the strong field regime?



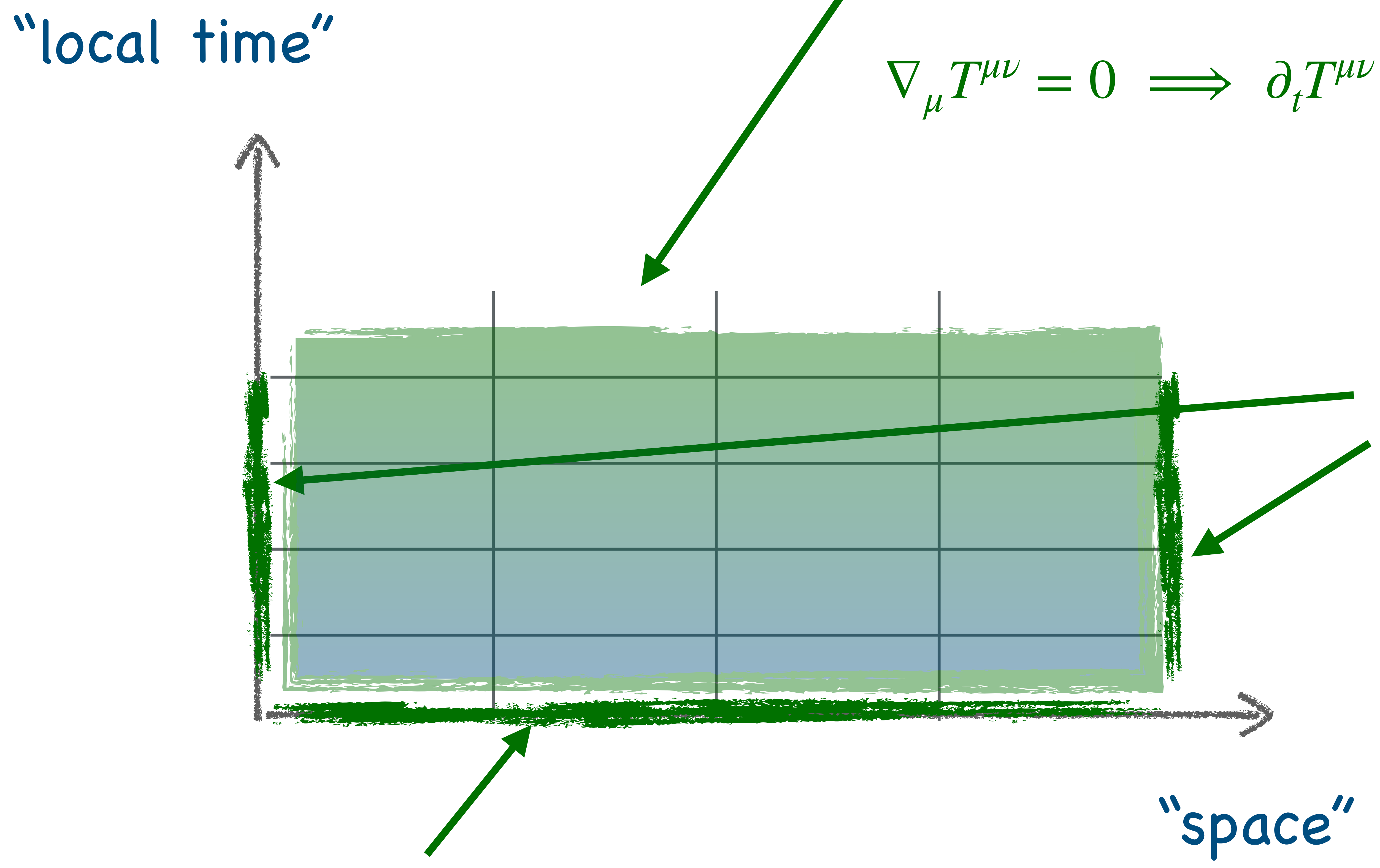
AURREKOETXEA ET AL, *PHYS.REV.D* 110 (2024) 8, 083011

Some general observations

NR: the lazy way to solve the Einstein Equations

Fill using Einstein equation $\partial_{tt} g_{\mu\nu} = f(\partial_t g_{\mu\nu}, g_{\mu\nu}, T_{\mu\nu})$

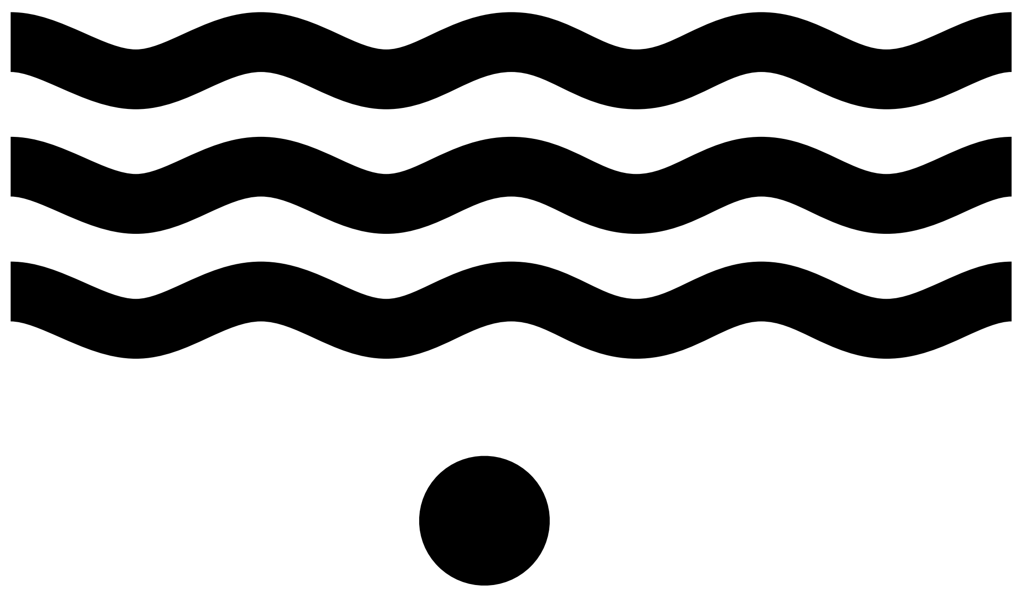
$$\nabla_\mu T^{\mu\nu} = 0 \implies \partial_t T^{\mu\nu} = f(T_{\mu\nu}, g_{\mu\nu})$$



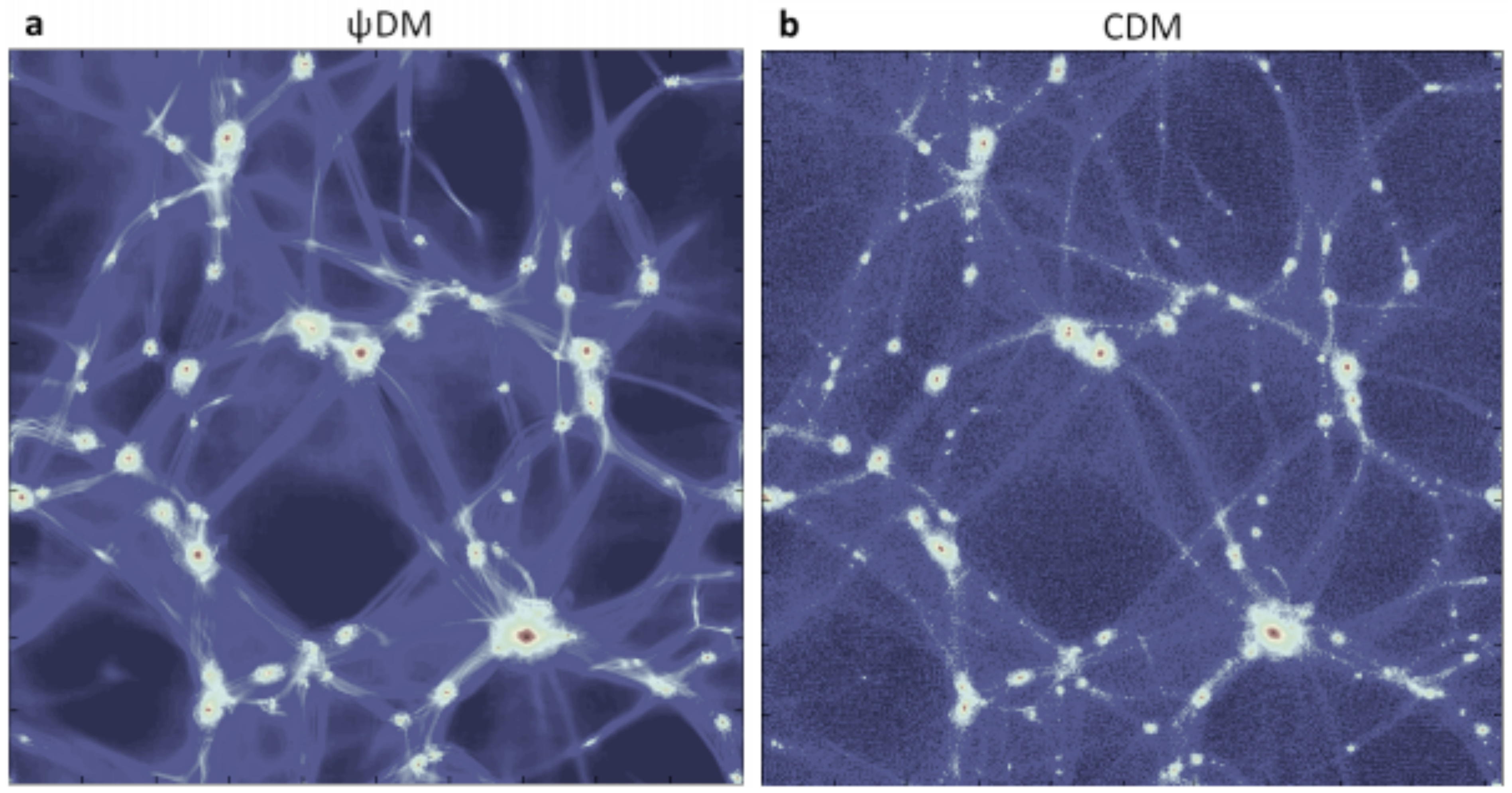
boundary conditions $(\partial_{xx} g_{\mu\nu}, \partial_x g_{\mu\nu},\ g_{\mu\nu},\ T_{\mu\nu})$

initial data $(\partial_t g_{\mu\nu},\ g_{\mu\nu},\ T_{\mu\nu})$

Wave versus particle: the NR perspective

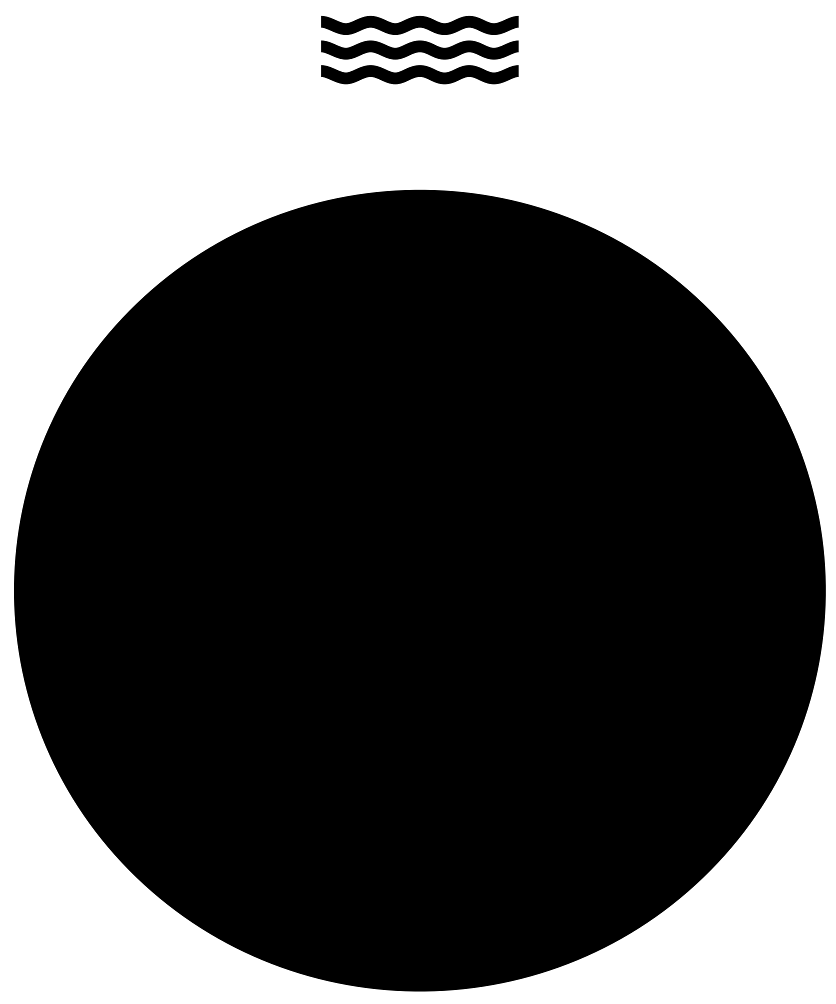


Wave



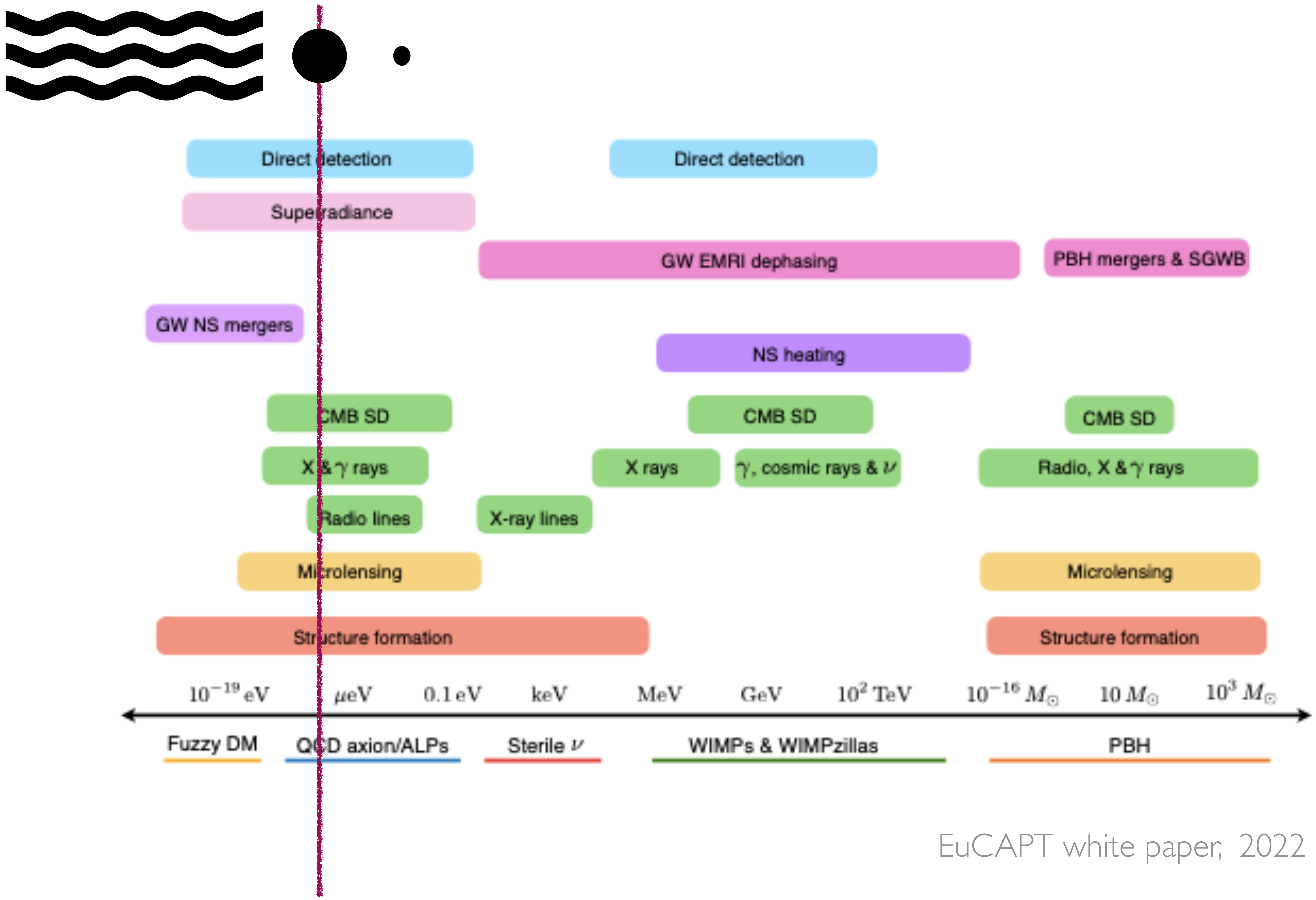
Schive et al. 2014
Cosmic structure as the quantum interference of a coherent dark wave

See also Wave Dark Matter review by Lam Hui
Ann.Rev.Astron.Astrophys. 59 (2021) 247-289



Particle

Wave versus particle: the NR perspective

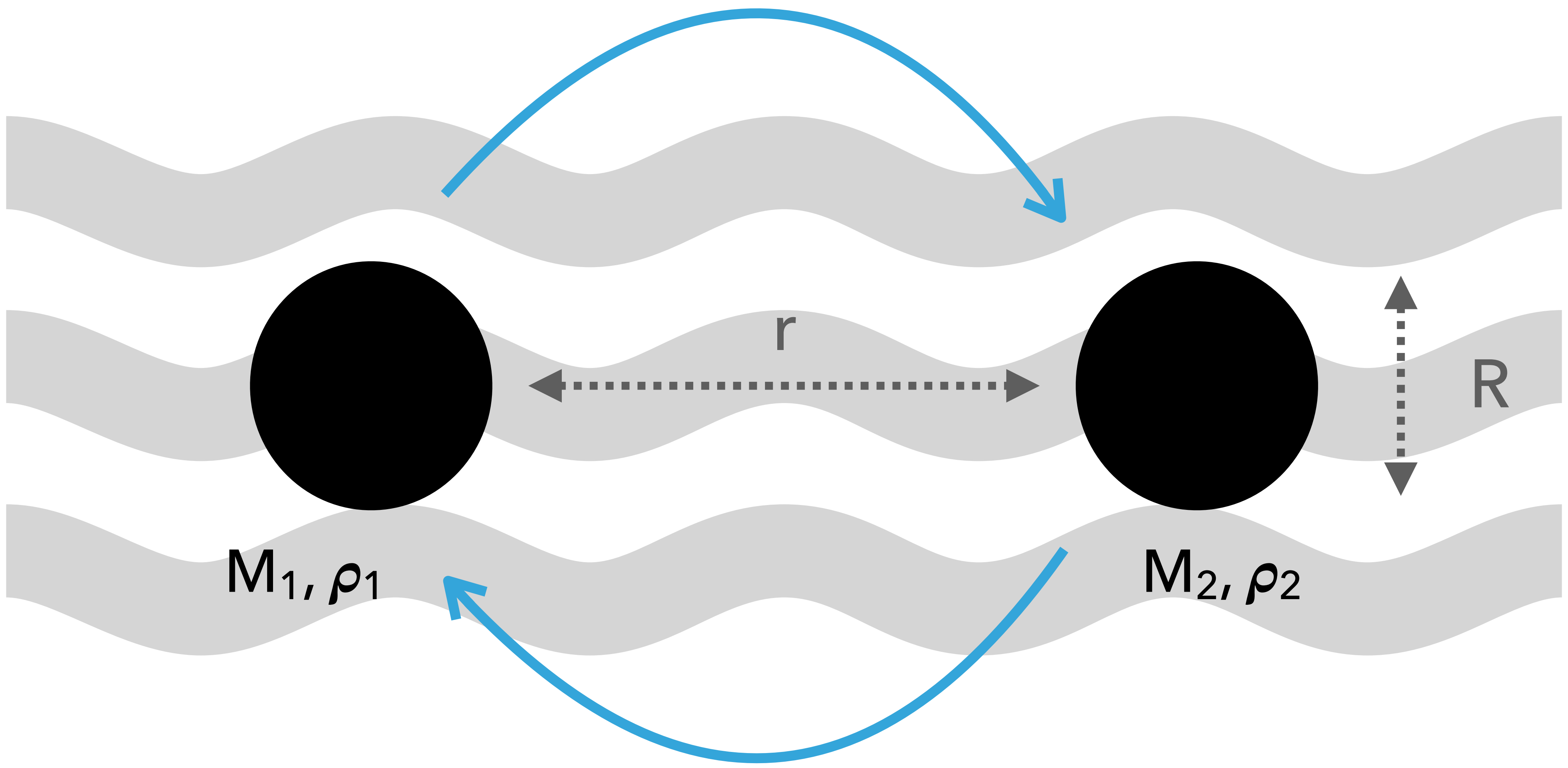


EuCAPT white paper, 2022

Similarity of scales is a feature and a bug

DYNAMICAL SPACETIME

NO PERTURBATIVE EXPANSION



GRAVITATIONAL BACKREACTION (STRONG GRAVITY)

Dark matter density - the NR perspective

Tiny effect at average galactic densities

$$\rho \sim 1\ \mathrm{GeV/cm^3}\ \textbf{or}\ 1\ \mathrm{M_\odot/pc^3}$$

(Particle physicist) (Astrophysicist)

Dark matter density - the NR perspective

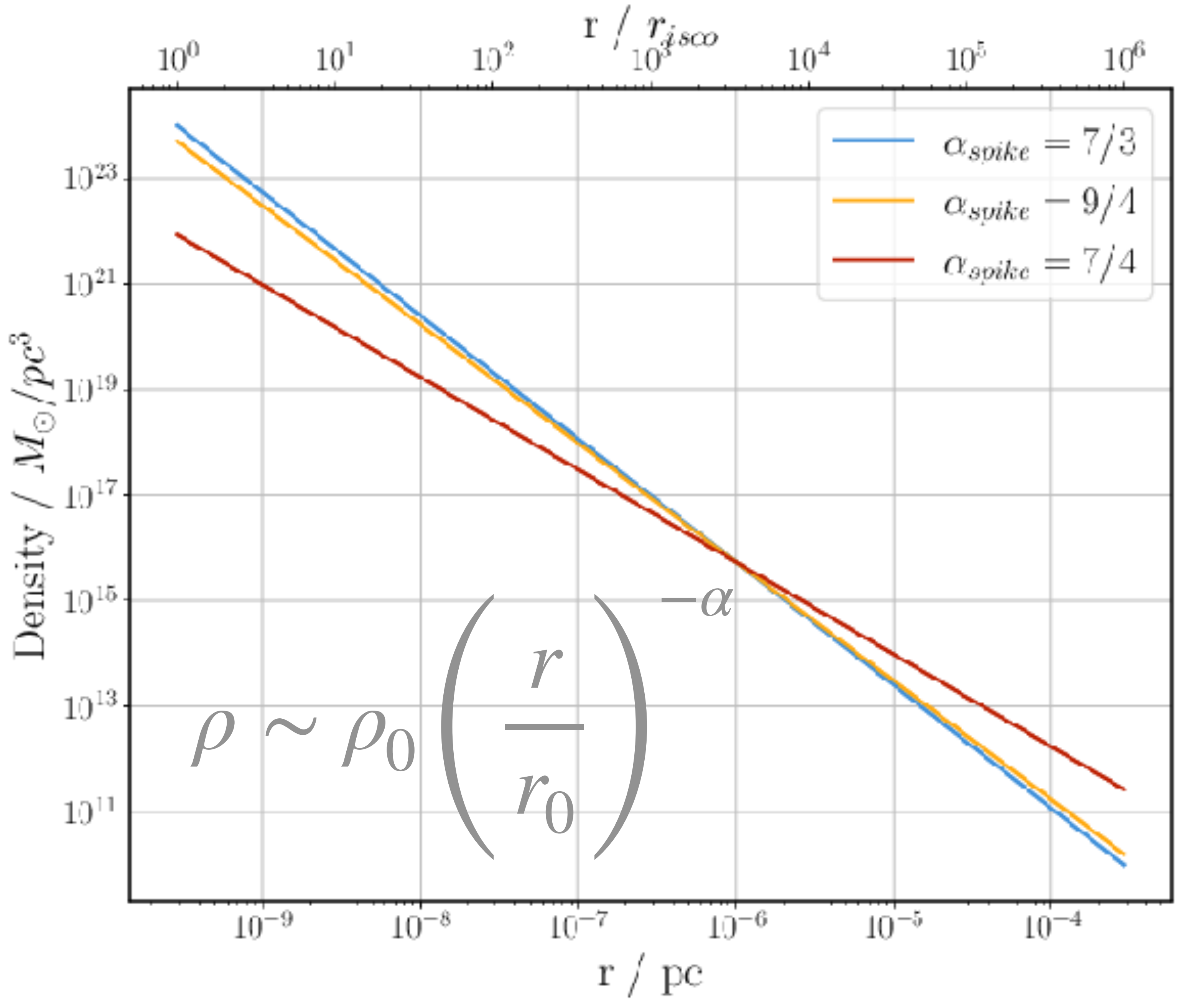
Tiny effect at average galactic densities

$$\frac{\rho}{1/R_s^2} \sim 10^{-30} \left(\frac{M_{BH}}{10^6 M_\odot} \right)^2$$

(Numerical relativist)

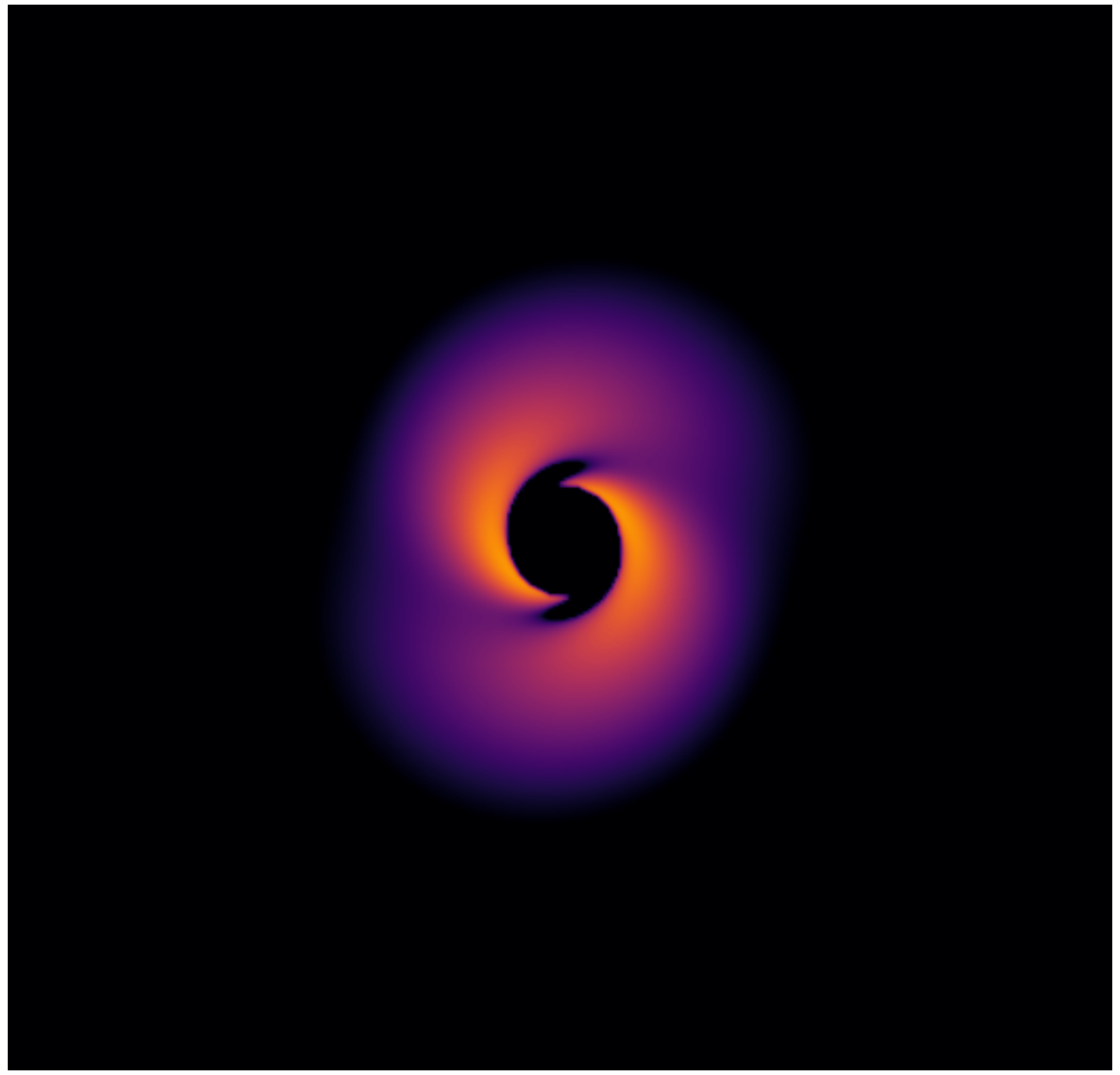
Dark matter density - the NR perspective

However, potential for significant enhancements around BHs



Accretion

Becker et.al. 2021
Circularization vs. Eccentrification in Intermediate Mass Ratio Inspirals inside Dark Matter Spikes



Superradiance

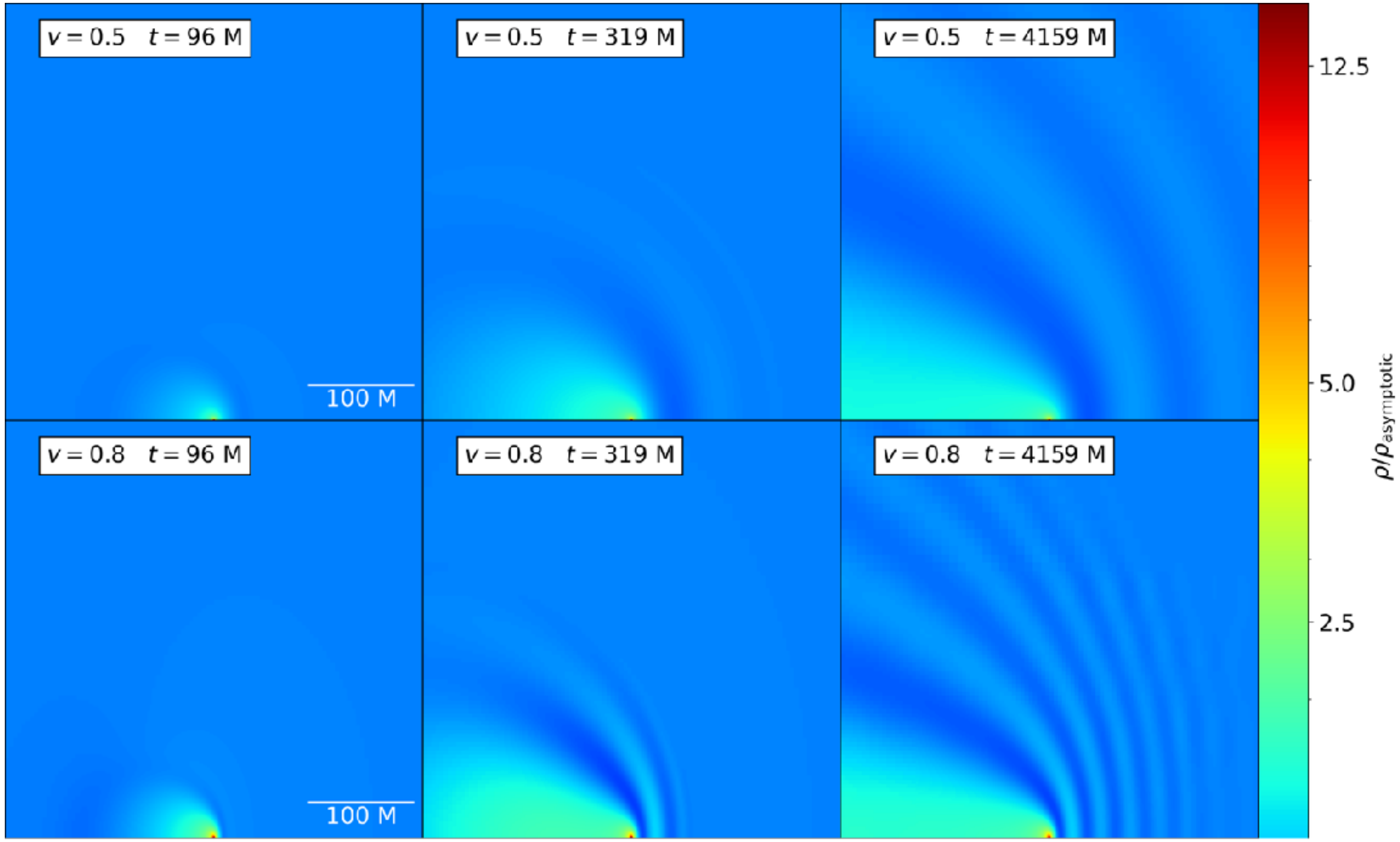
Review by Brito et. al. (updated 2020)
Superradiance: New Frontiers in Black Hole Physics

Dark matter density - the NR perspective

Best constraint on BH scales from S2 star motion around Sag A* (1% of mass within $10^4 R_s$) A. Foschi et al. (GRAVITY), Mon. Not. Roy. Astron. Soc. 524, 1075 (2023)

$$\frac{\rho}{1/R_s^2} \sim 10^{-15} \left(\frac{M_{BH}}{10^6 M_\odot} \right)^2$$

Conclusion: may be able to neglect back reaction beyond first order

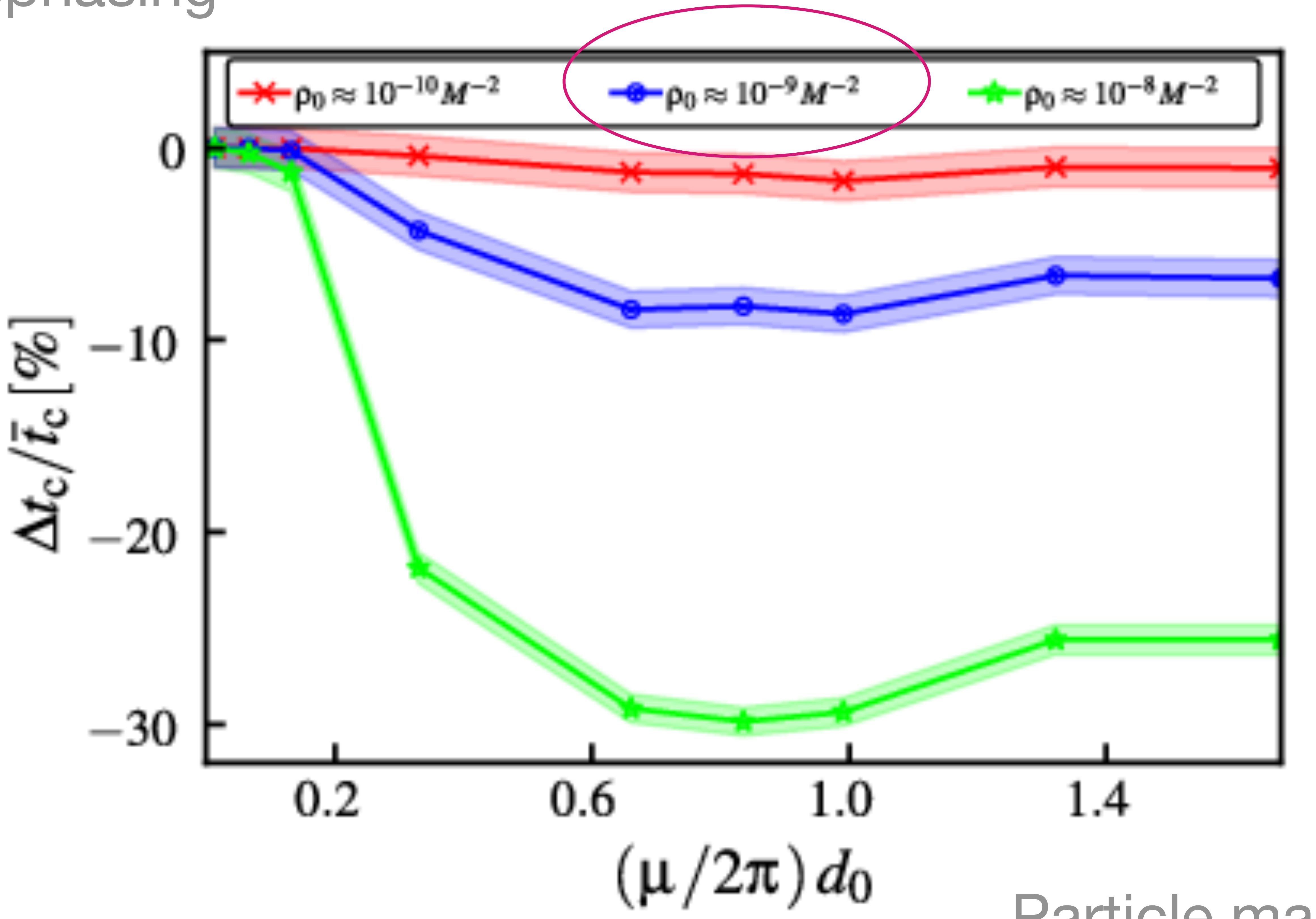


Energy density of DM

D Traykova, T. Helfer, KC et. al.
Phys.Rev.D 104 (2021) 10, 103014,

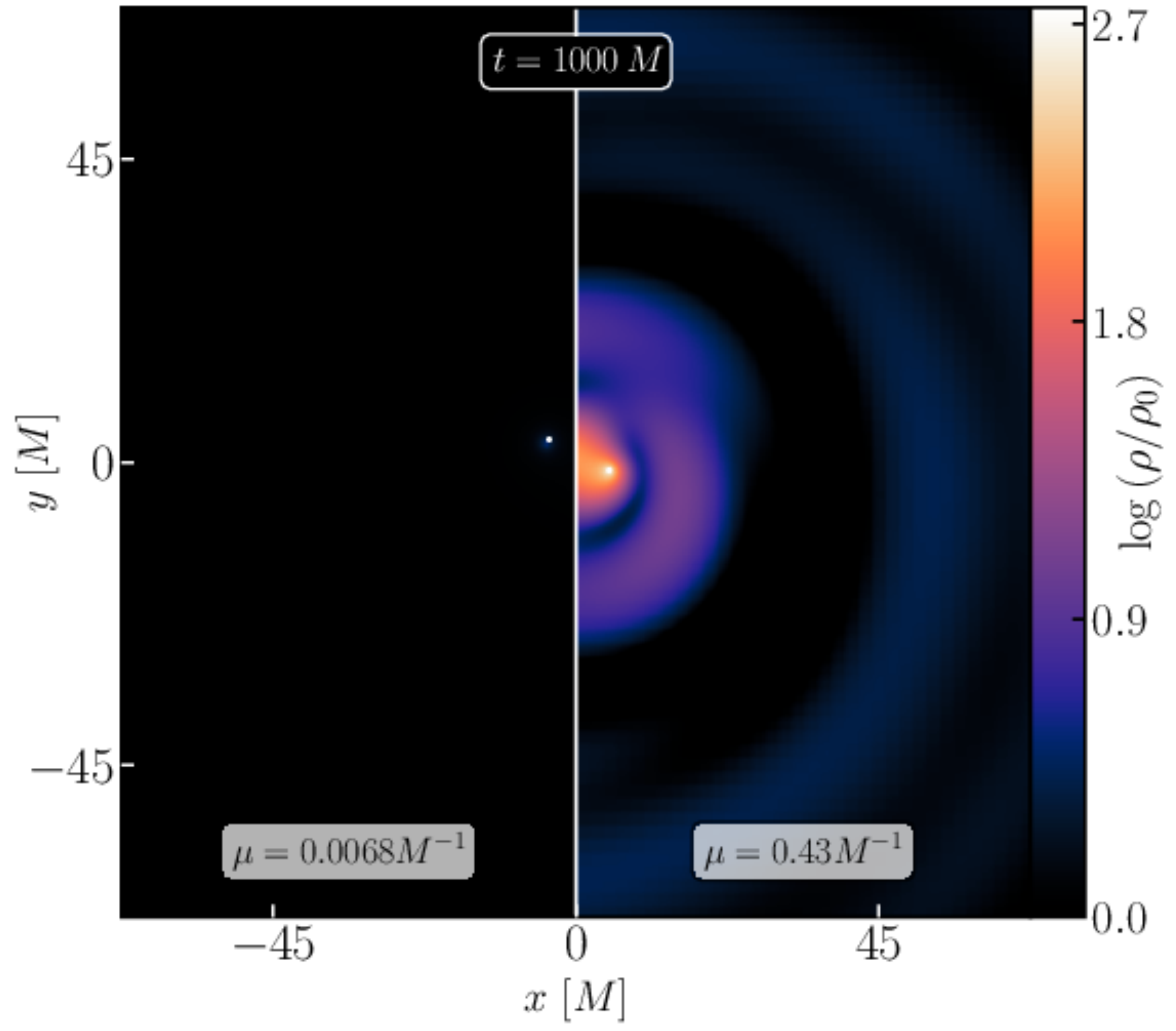
OR: Include backreaction at potentially unrealistic densities

Dephasing



Particle mass

Energy density of DM



J. Aurrekoetxea, KC, J Bamber, P Ferreira 2023
Phys.Rev.Lett. 132 (2024) 21, 211401

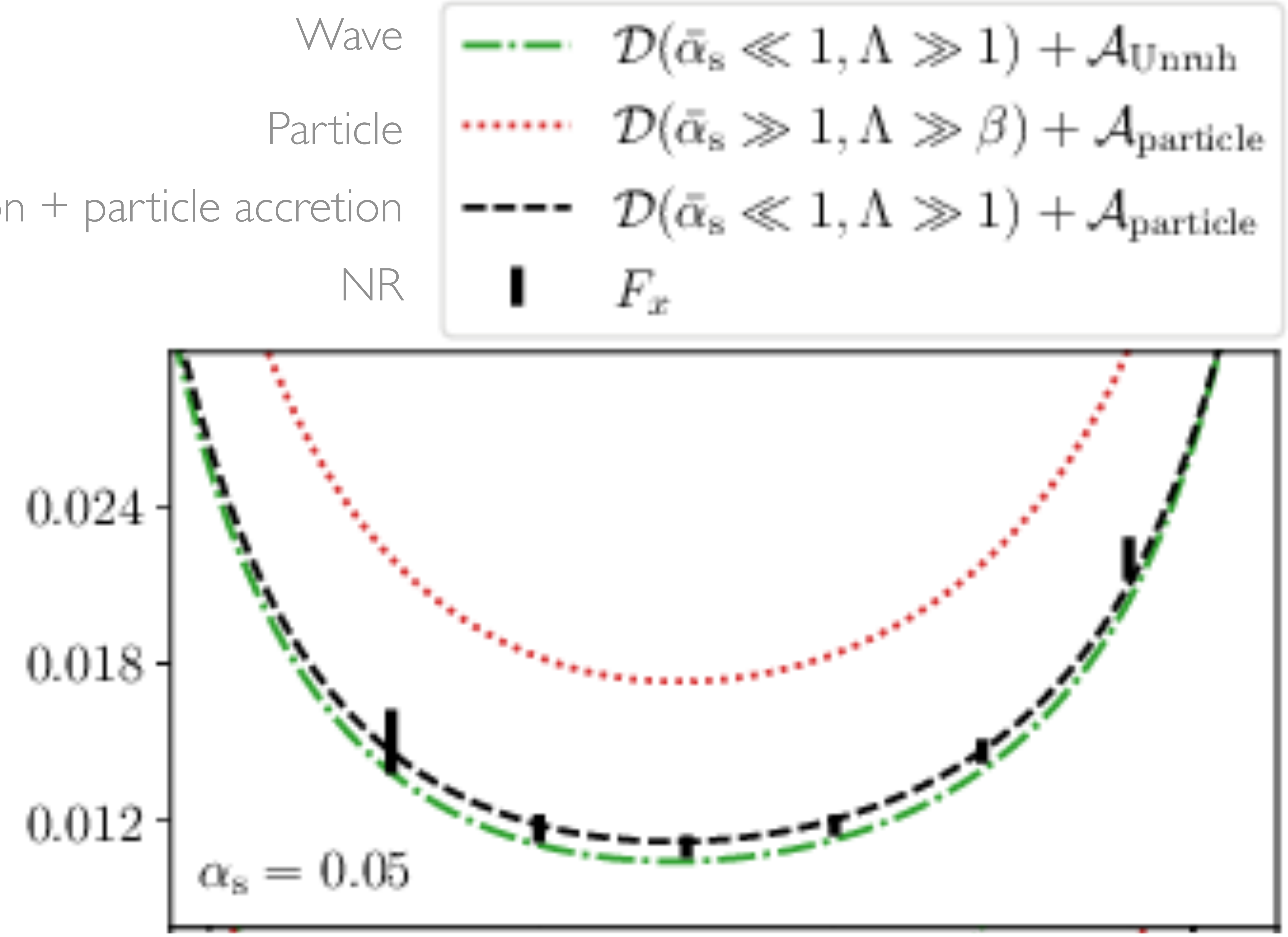
Best used to highlight new effects, and validate and extend analytics

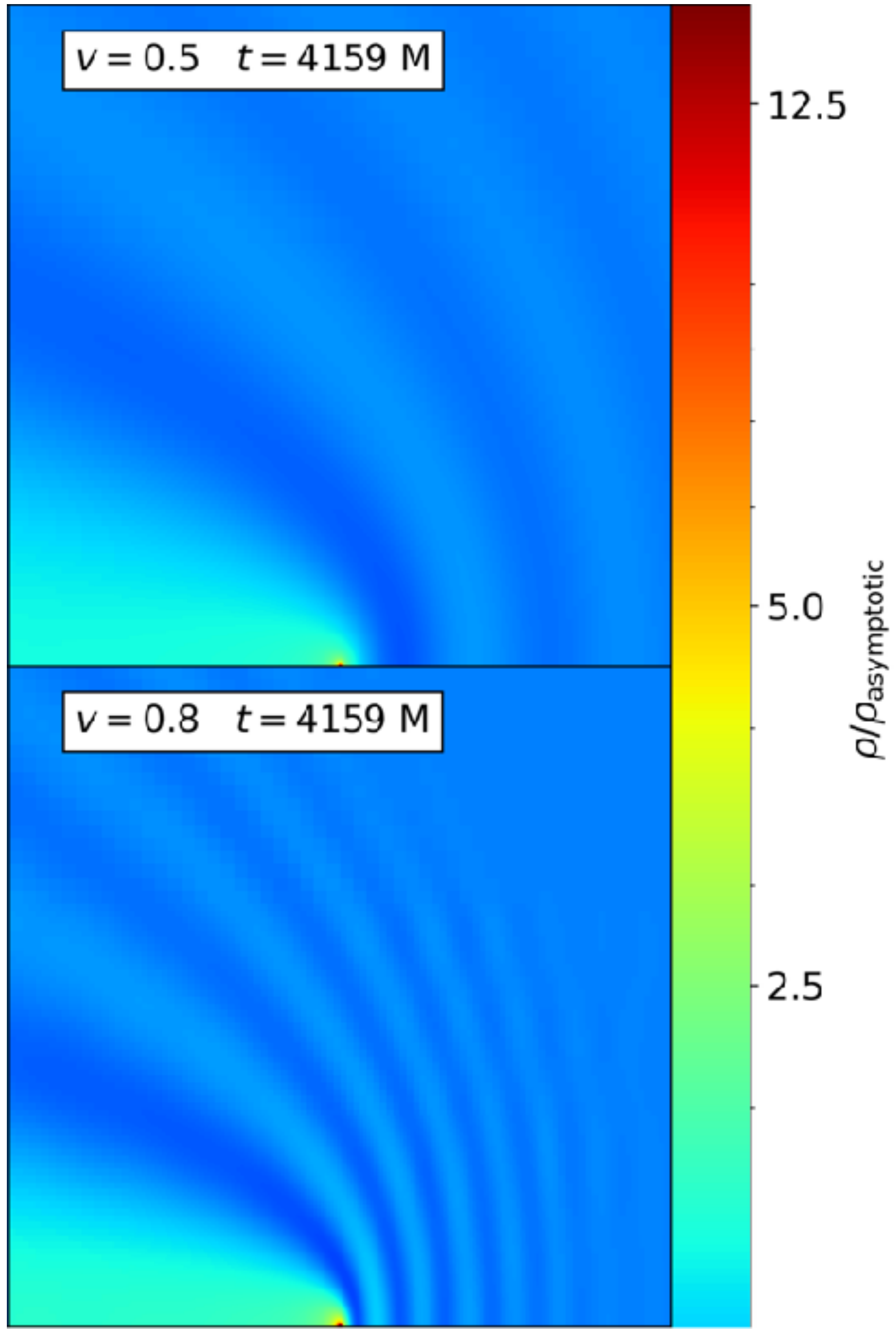
Wave

Particle

Wave dynamical friction + particle accretion

NR



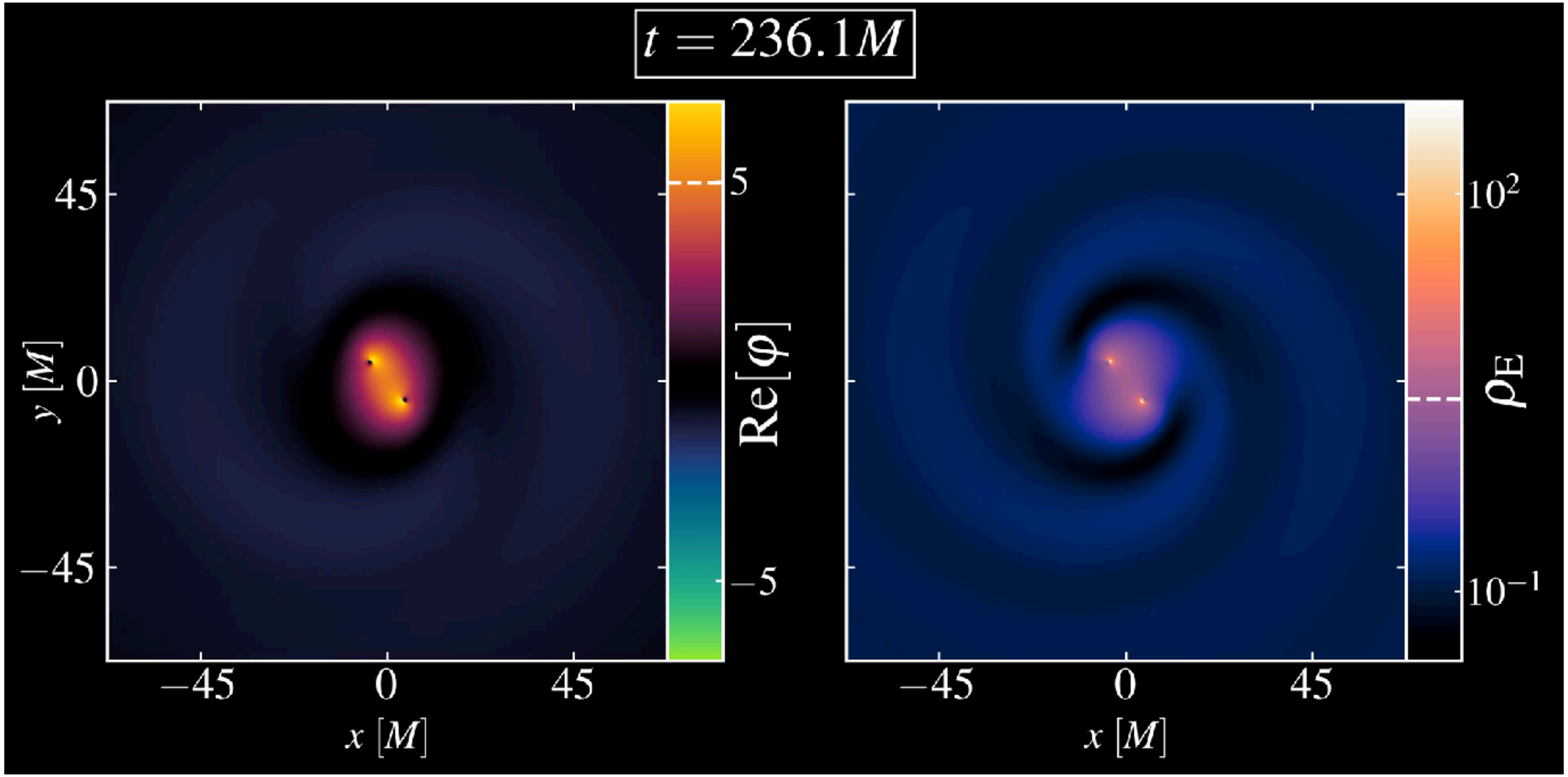


D Traykova, R Vicente, KC et. al.
Phys.Rev.D 108 (2023) 12, L121502

Light / wave dark matter

Accretion: wave like case seems to resist dispersal, and forms a central overdensity

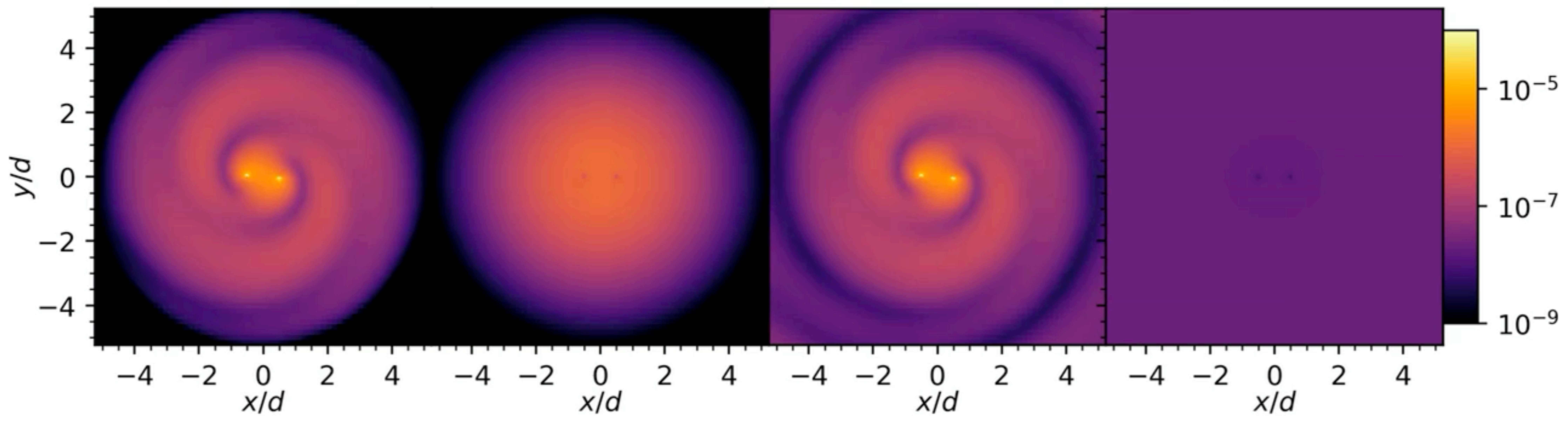
Field



energy density

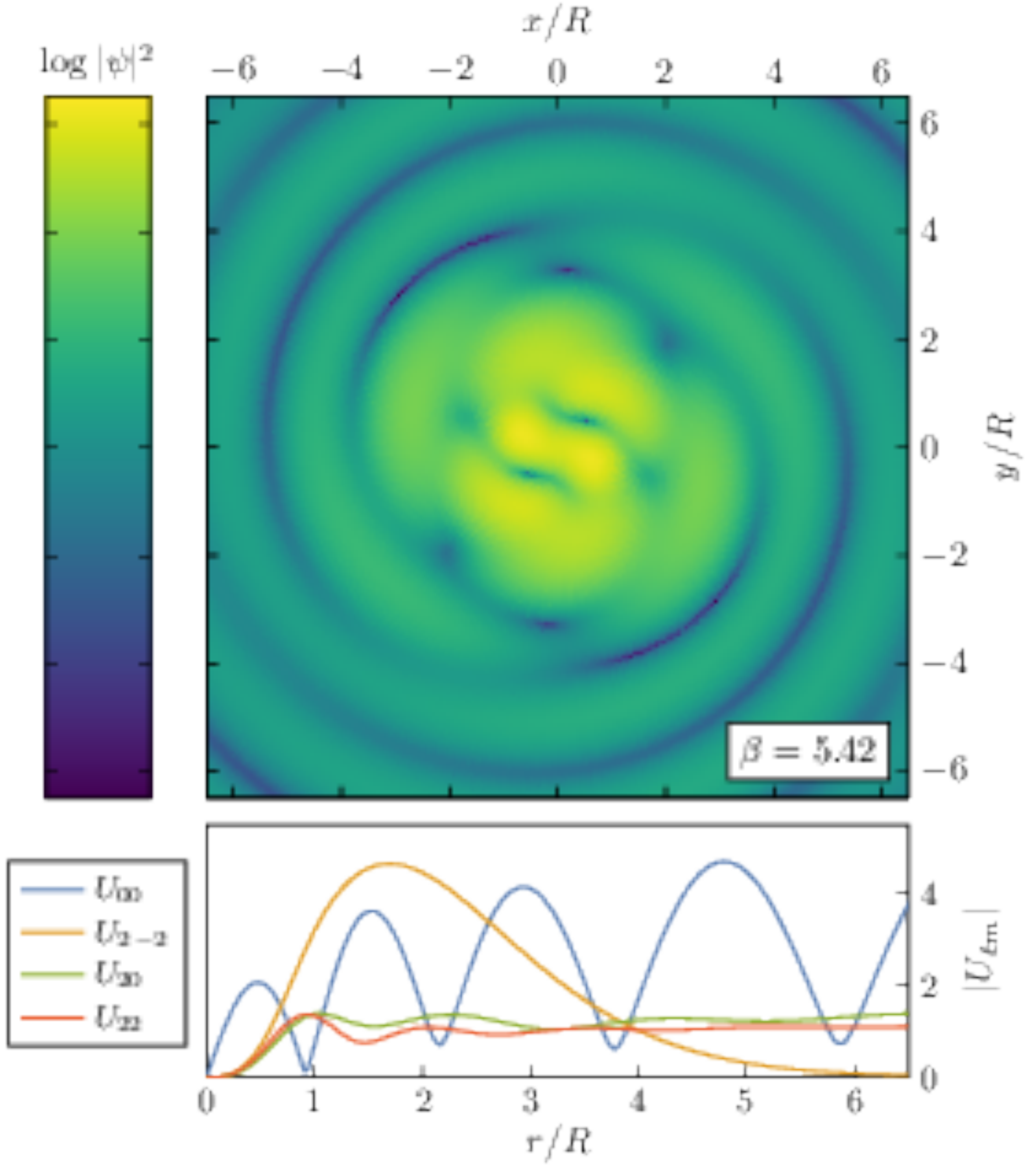
J. Bamber, J. Aurrekoetxea, KC, P. Ferreira 2023
Phys Rev D 107 2, 024035

Different initial profiles converge over time to the same one

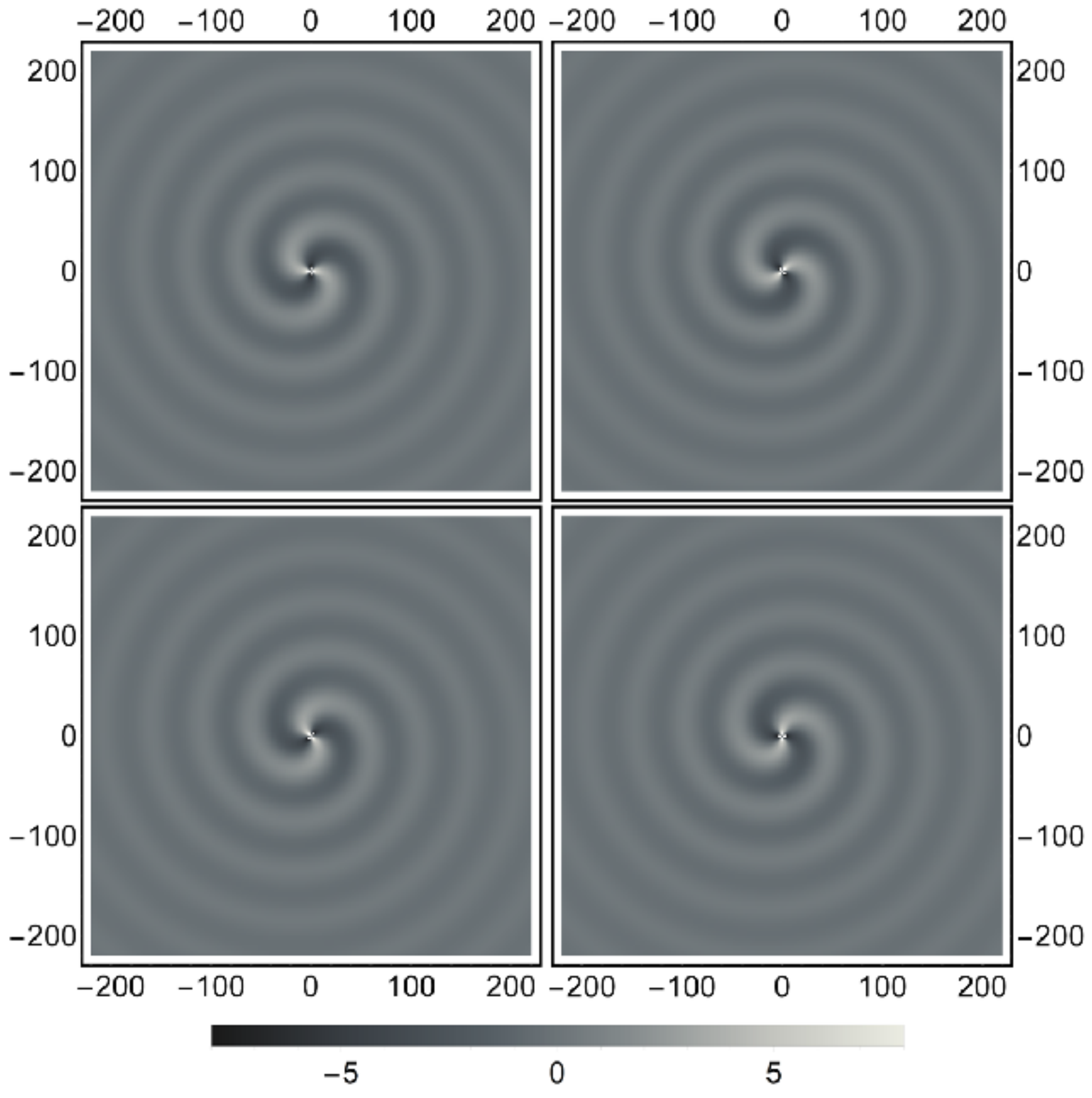


Energy density of DM

Semi-analytic description for this scattering in the non relativistic regime

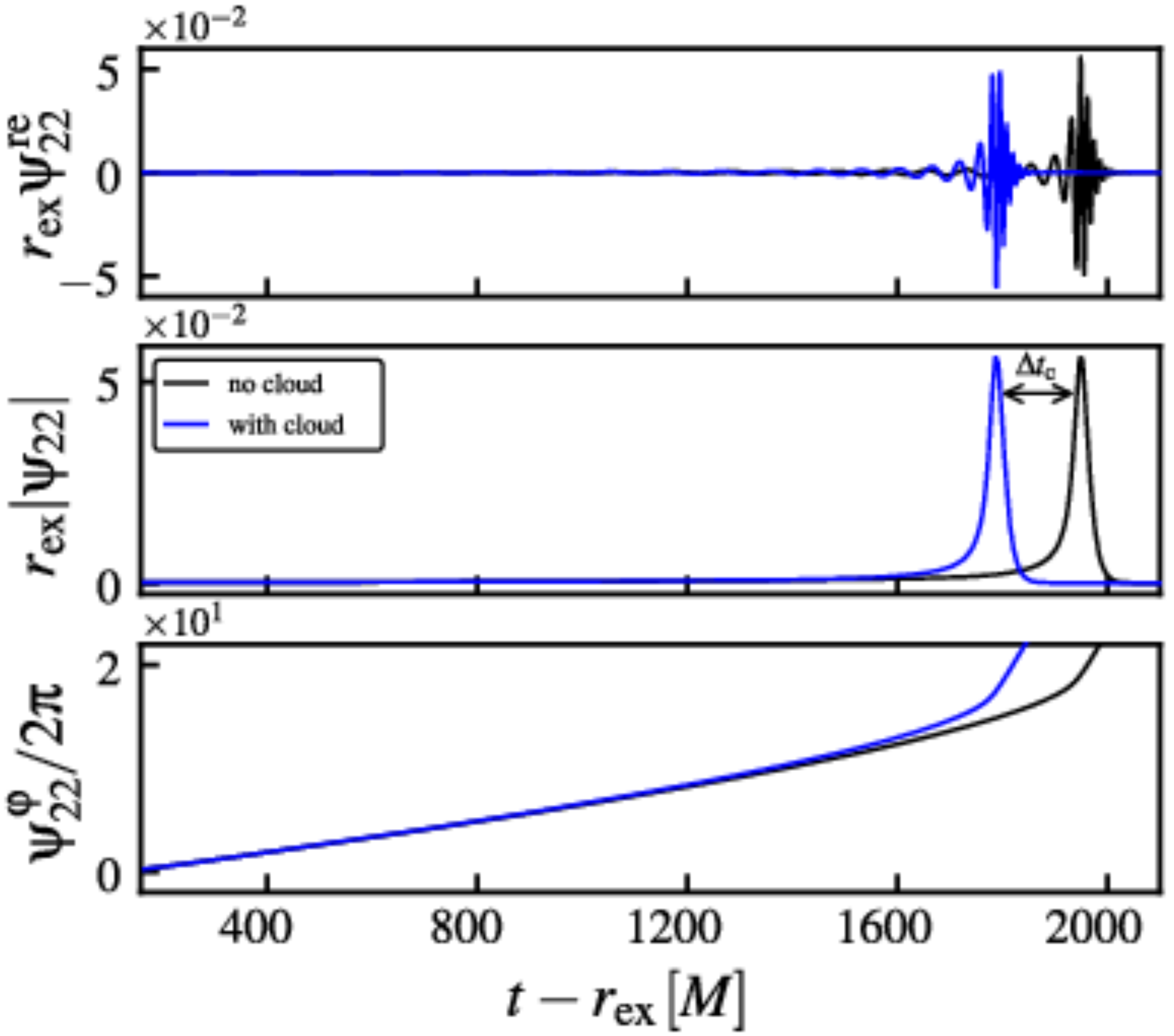


GM Tomaselli
Phys.Rev.D 111 (2025) 6, 063075

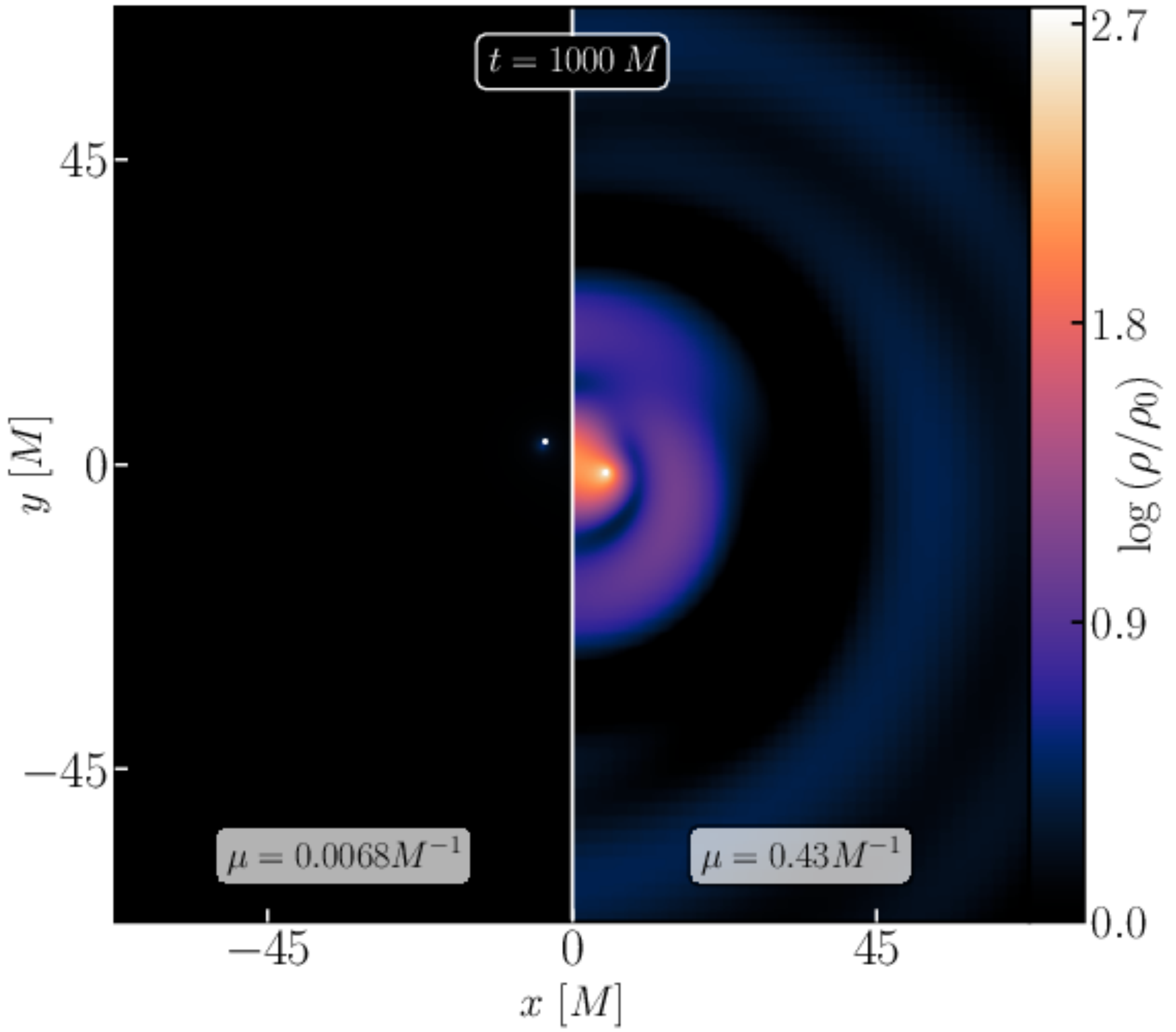


L. Annulli, V Cardoso, R Vicente 2020
Phys.Rev.D 102 (2020) 6, 063022

Such an overdensity leads to a dephasing

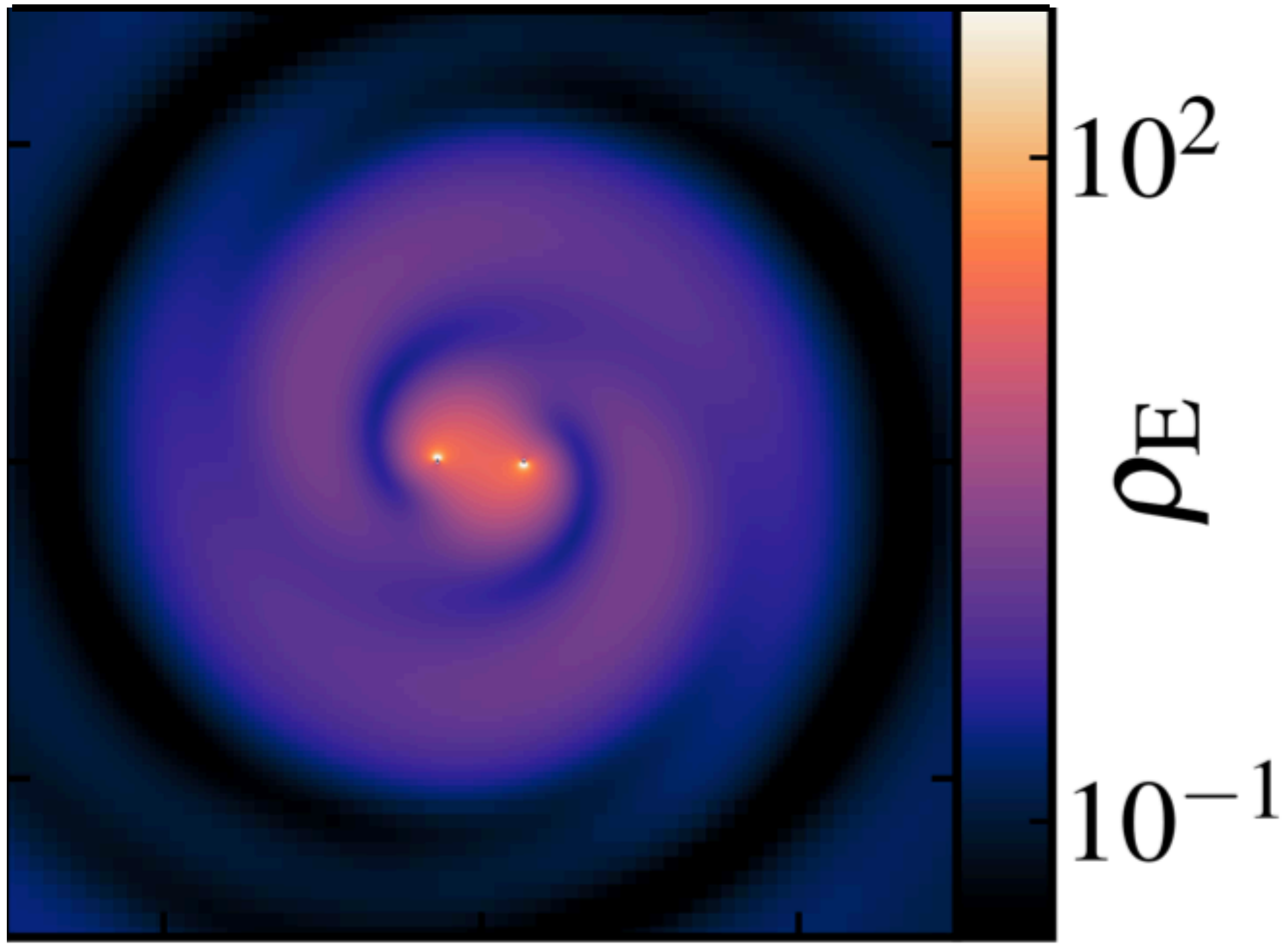


Energy density of DM

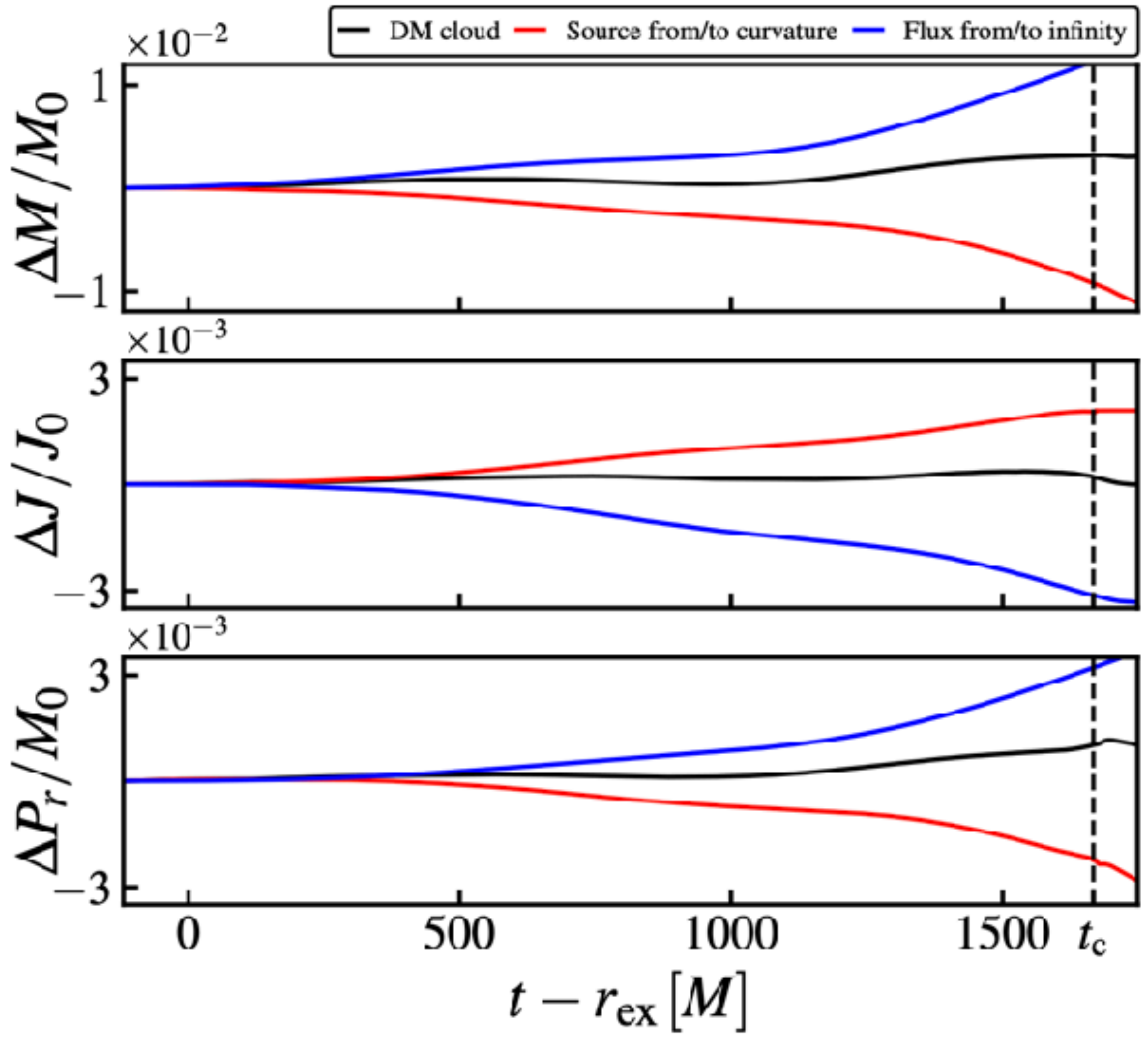


J. Aurrekoetxea, KC, J Bamber, P Ferreira 2023
Phys.Rev.Lett. 132 (2024) 21, 211401

Can measure gravitational “forces” on the binary

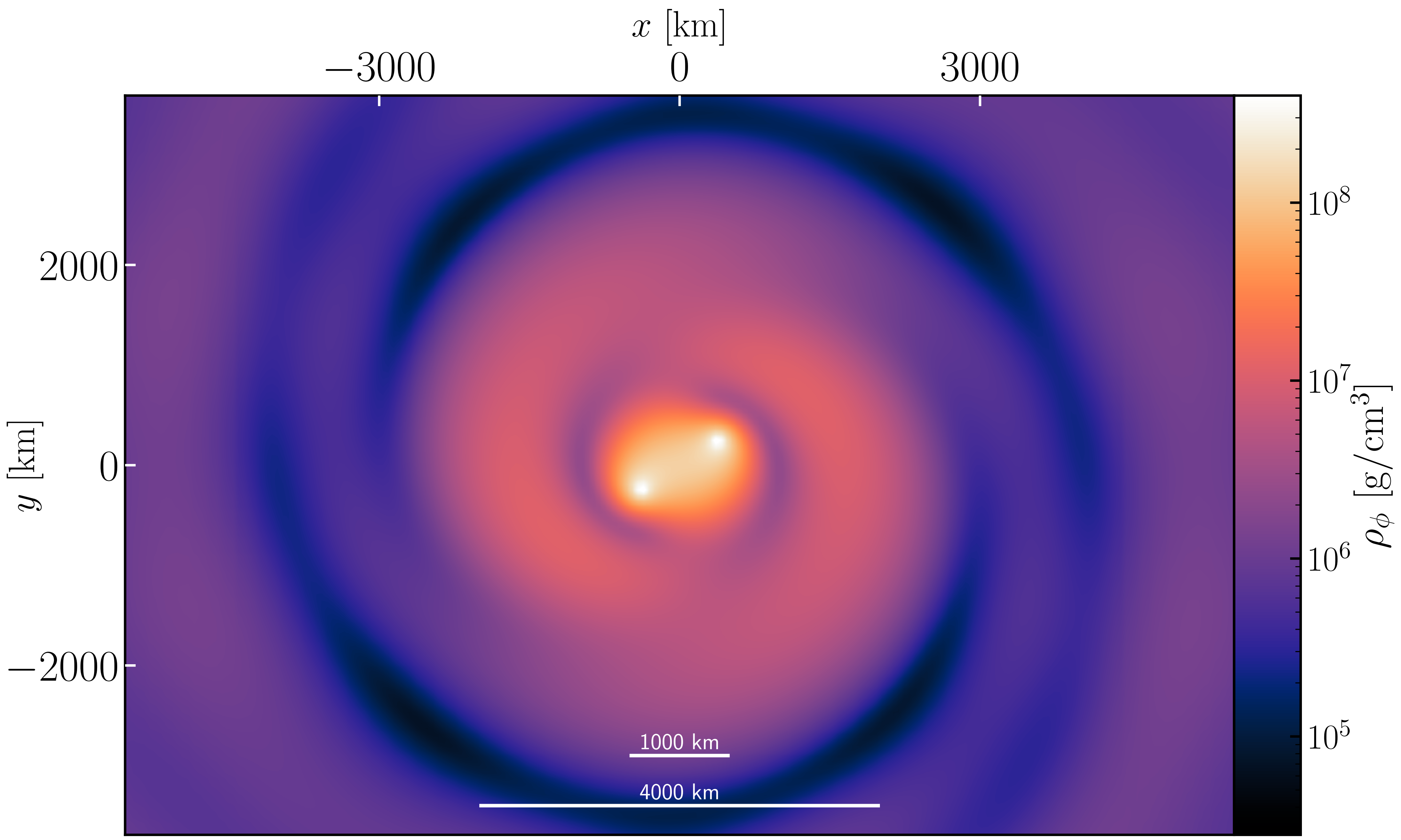


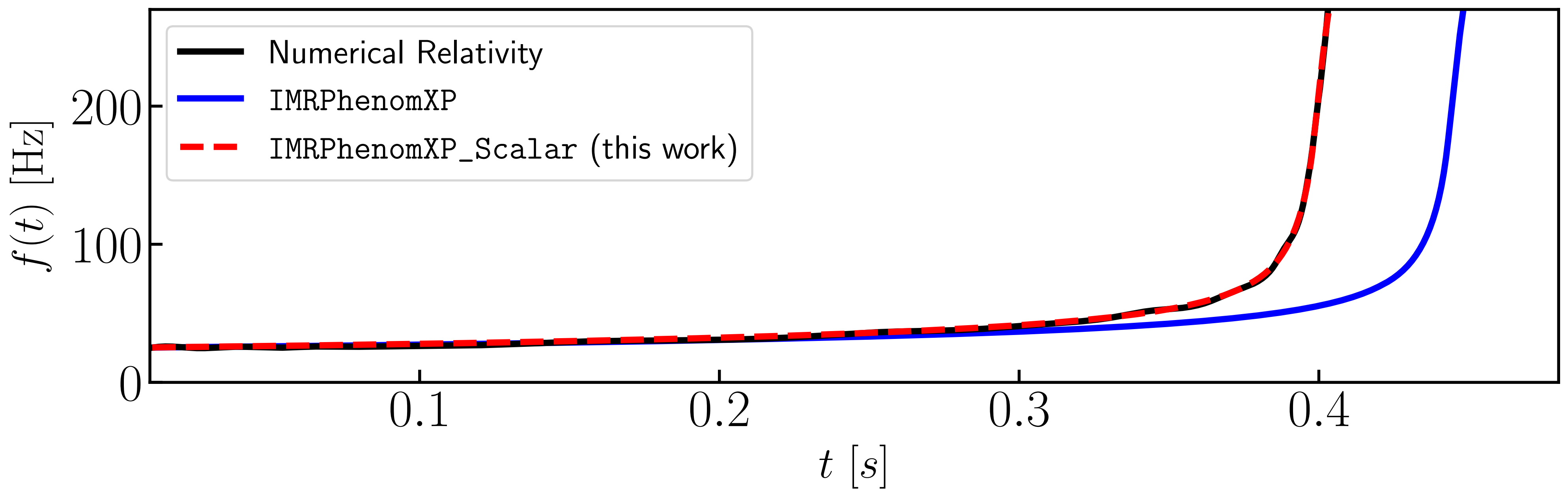
Energy density of DM

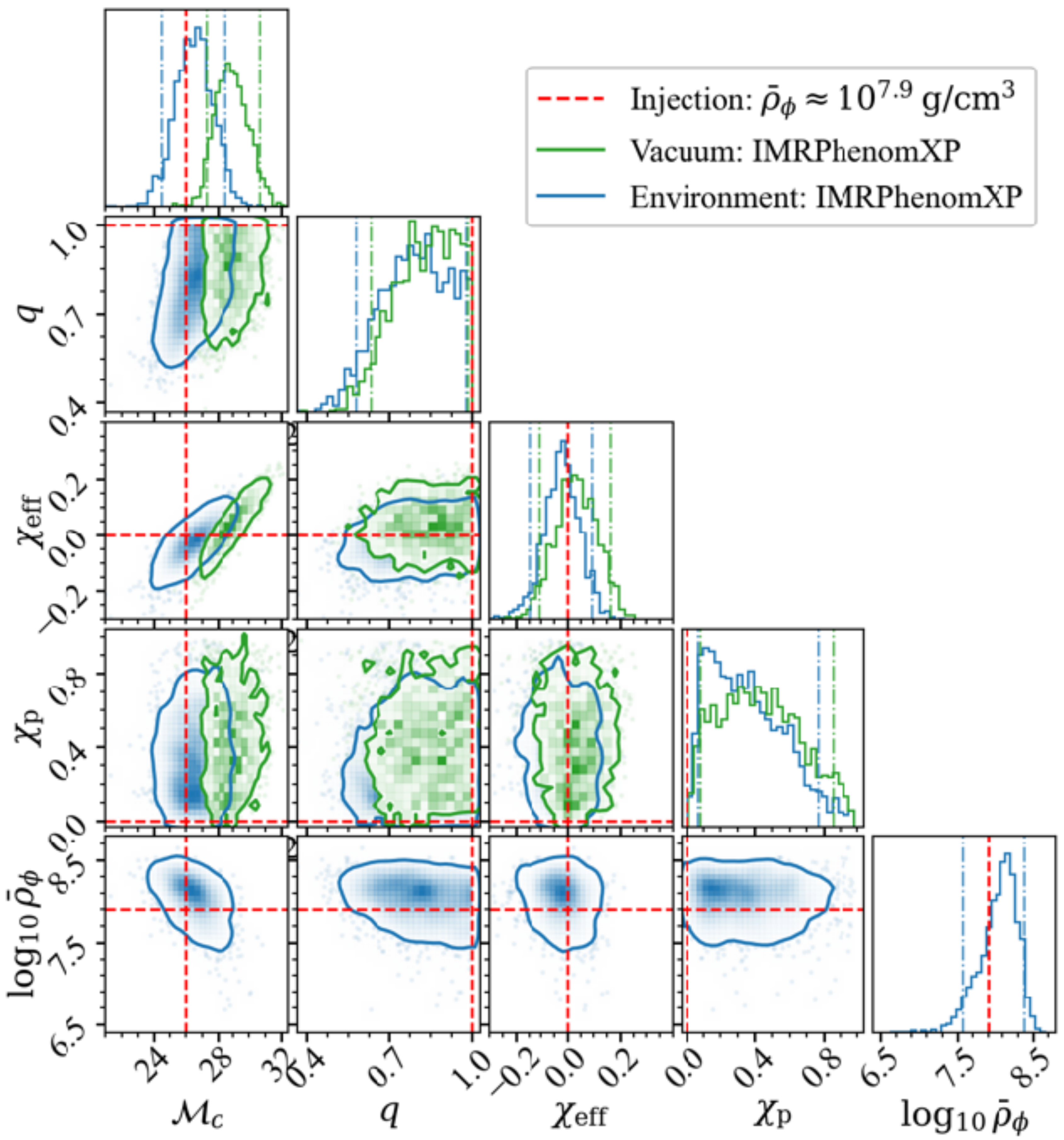


J. Aurrekoetxea, KC, J Bamber, P Ferreira 2023
Phys.Rev.Lett. 132 (2024) 21, 211401

Can construct a model and use it to infer DM density from observations







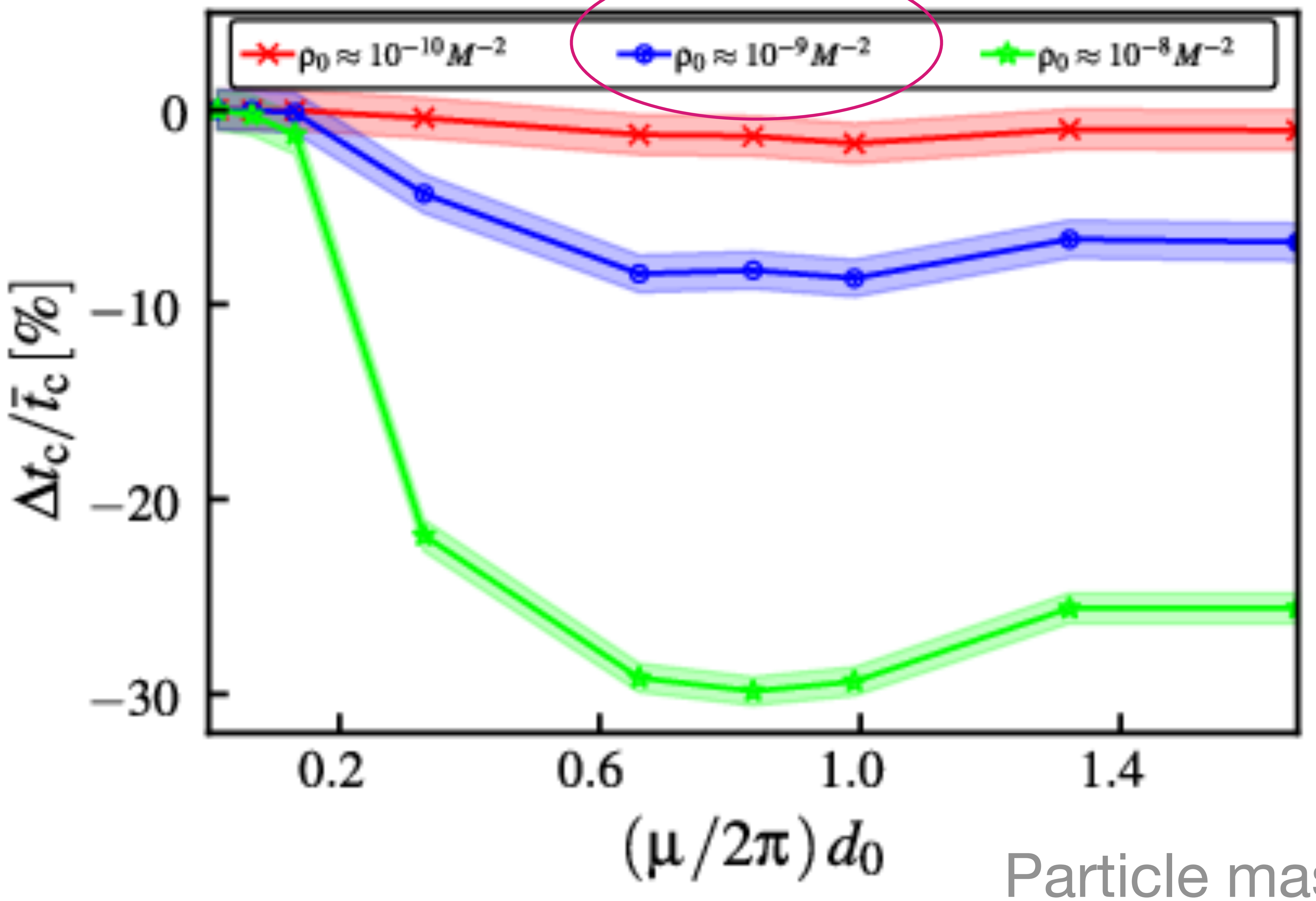
More from Rodrigo next!

J. Aurrekoetxea, , S Roy, R Vicente KC, P Ferreira 2025
In prep

Surprisingly large and persistent effect at higher masses

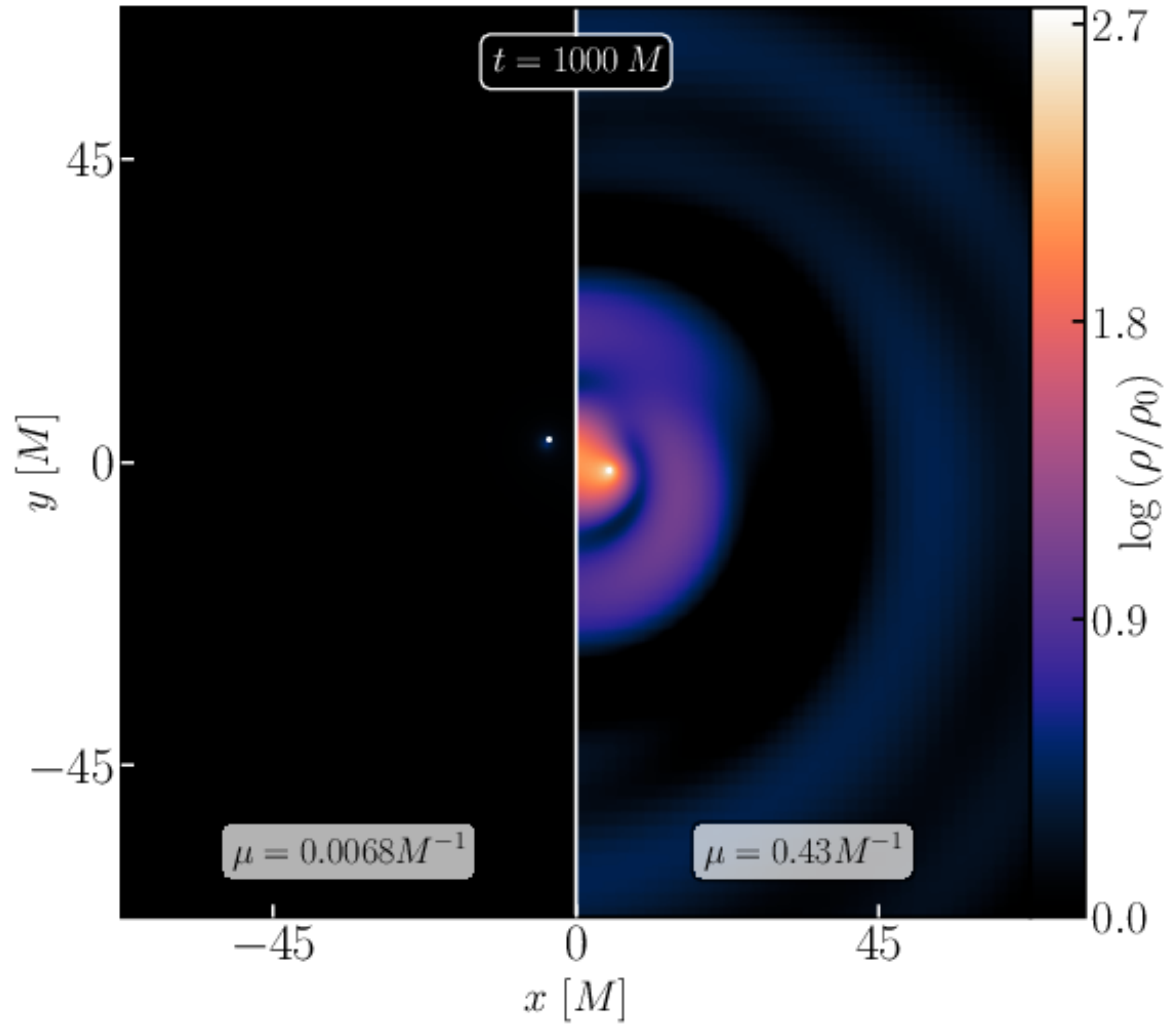
dephasing

Recall that S2 star constraints are $\rho \sim 10^{-15} M^{-2}$ ***so this is high***



Particle mass

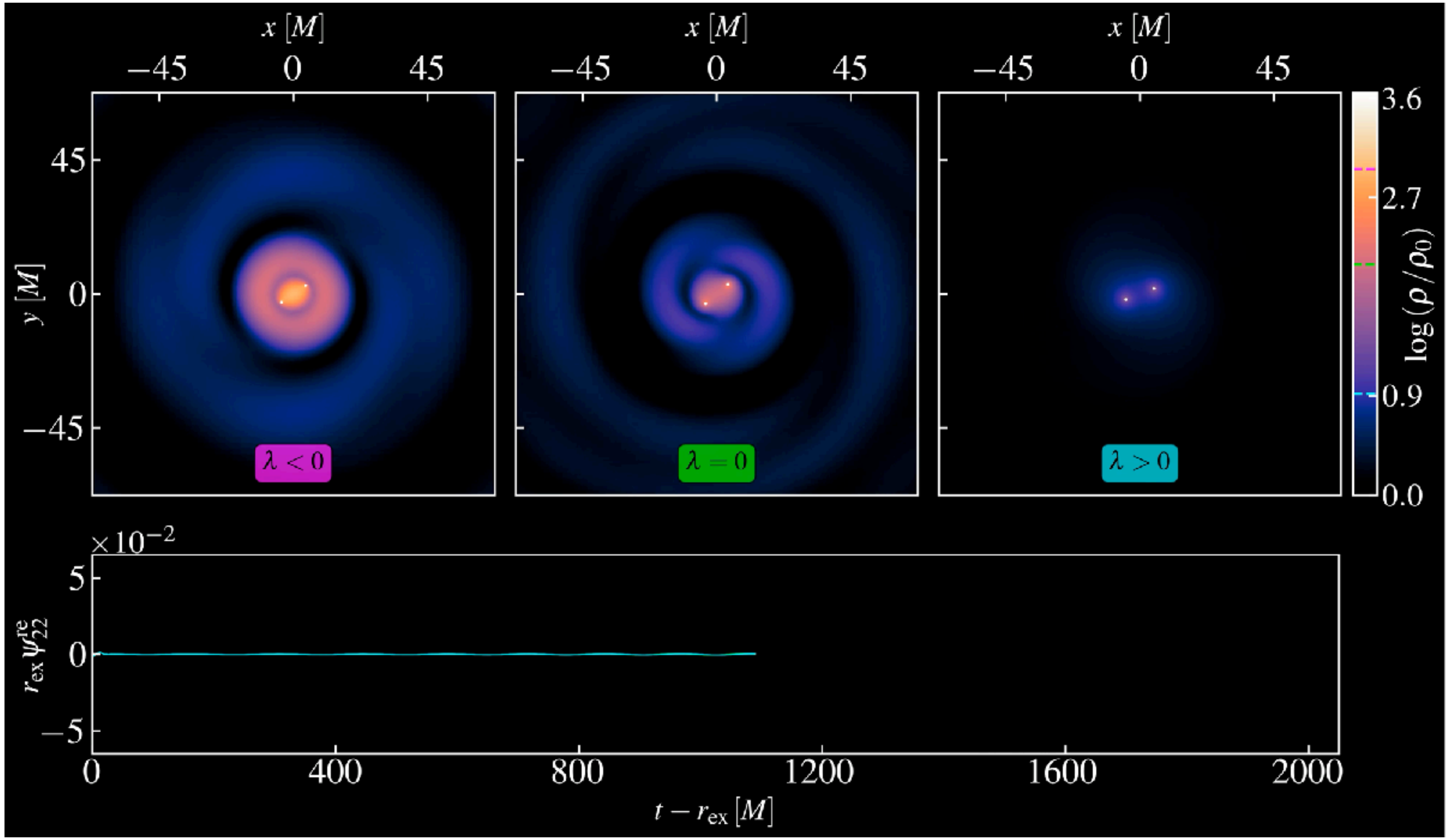
Energy density of DM



J. Aurrekoetxea, KC, J Bamber, P Ferreira 2023
Phys.Rev.Lett. 132 (2024) 21, 211401

What about self interactions?

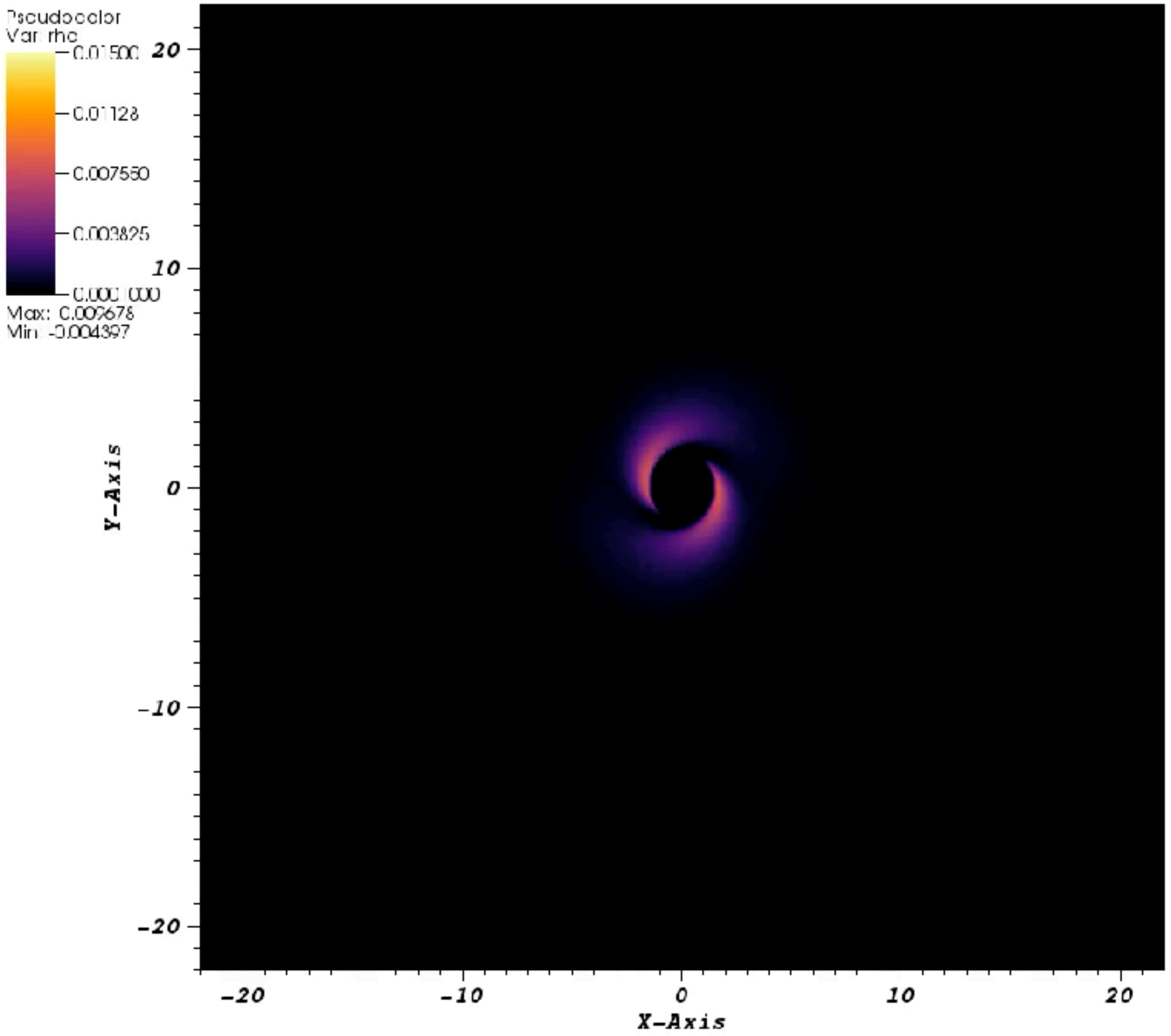
Attractive



Repulsive

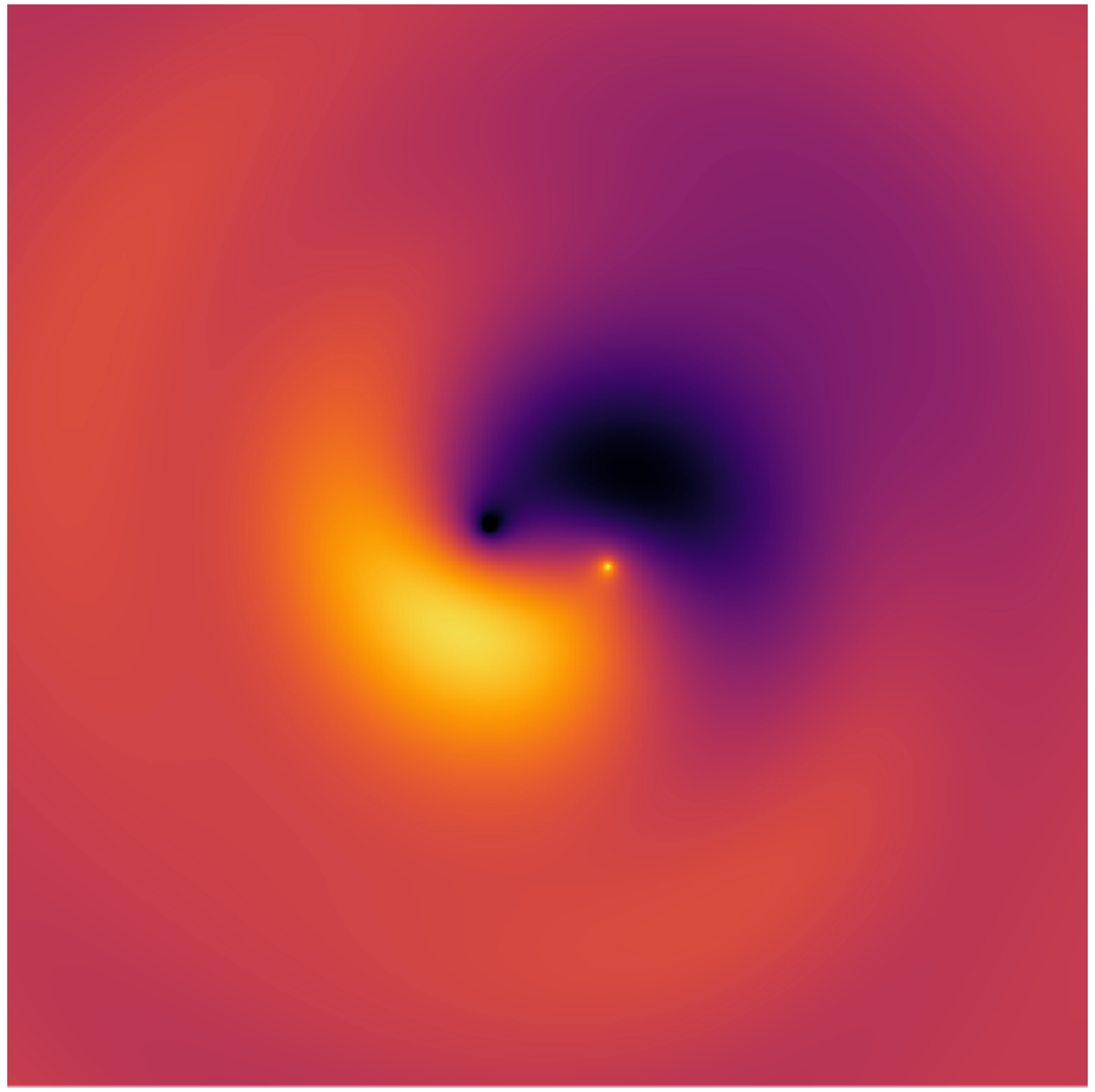
J. Aurrekoetxea, J. Marsden, KC, P Ferreira 2024
Phys.Rev.D 110 (2024) 8, 083011

What about superradiance?

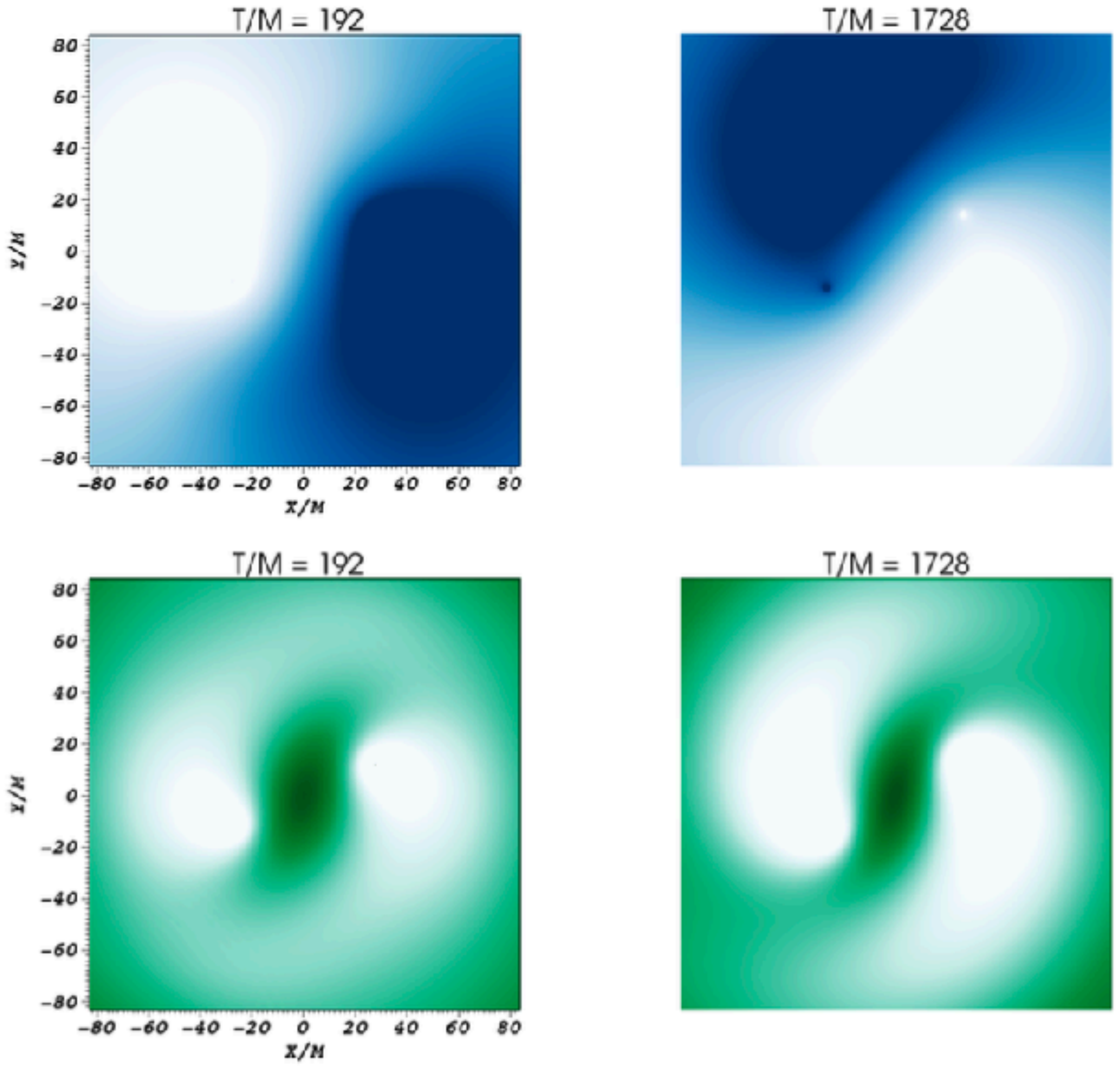


Energy density of a vector field during a superradiant phase.

Starting with a "pre-grown" cloud around one object, there is evidence from NR that this persists in a binary

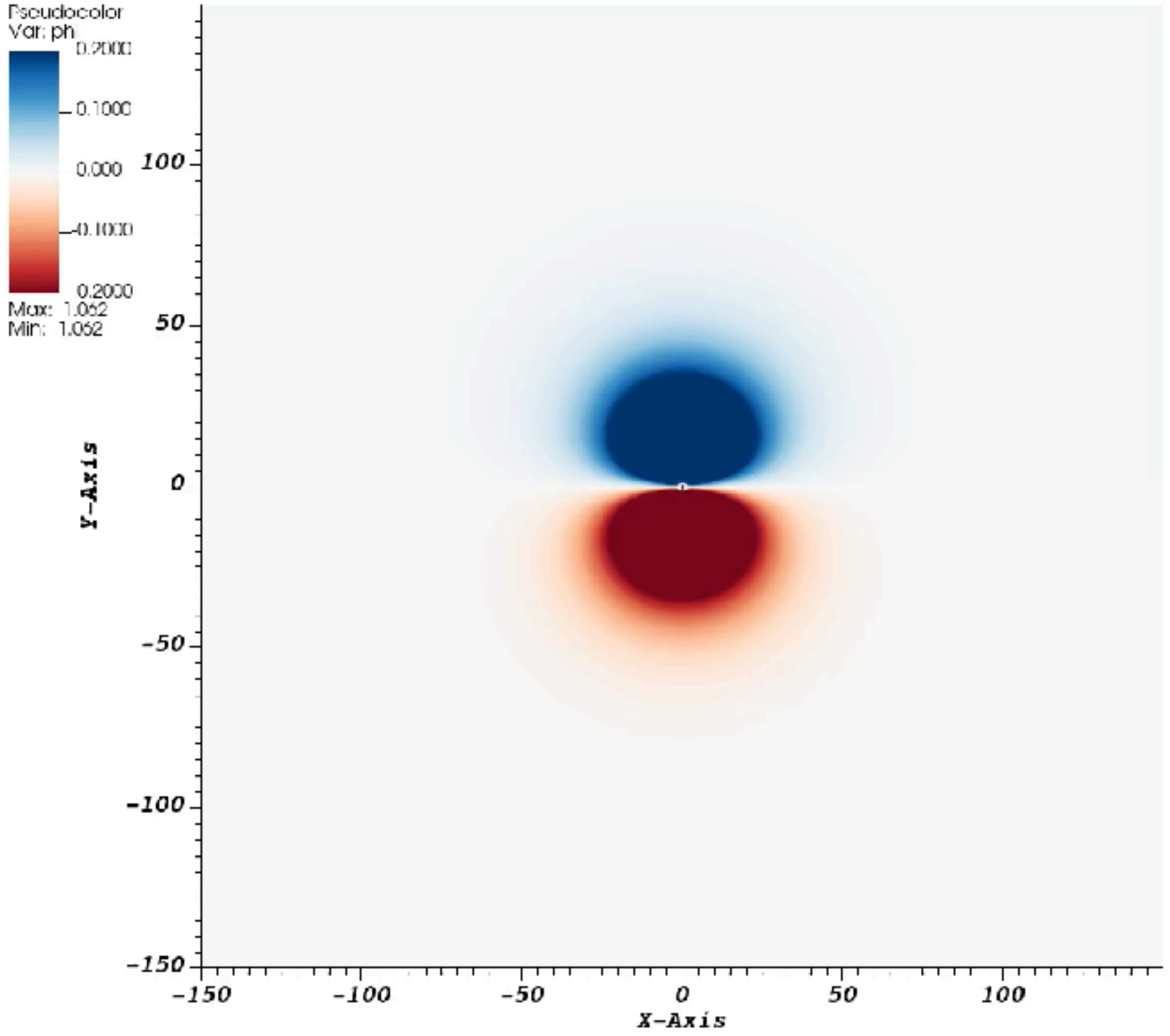


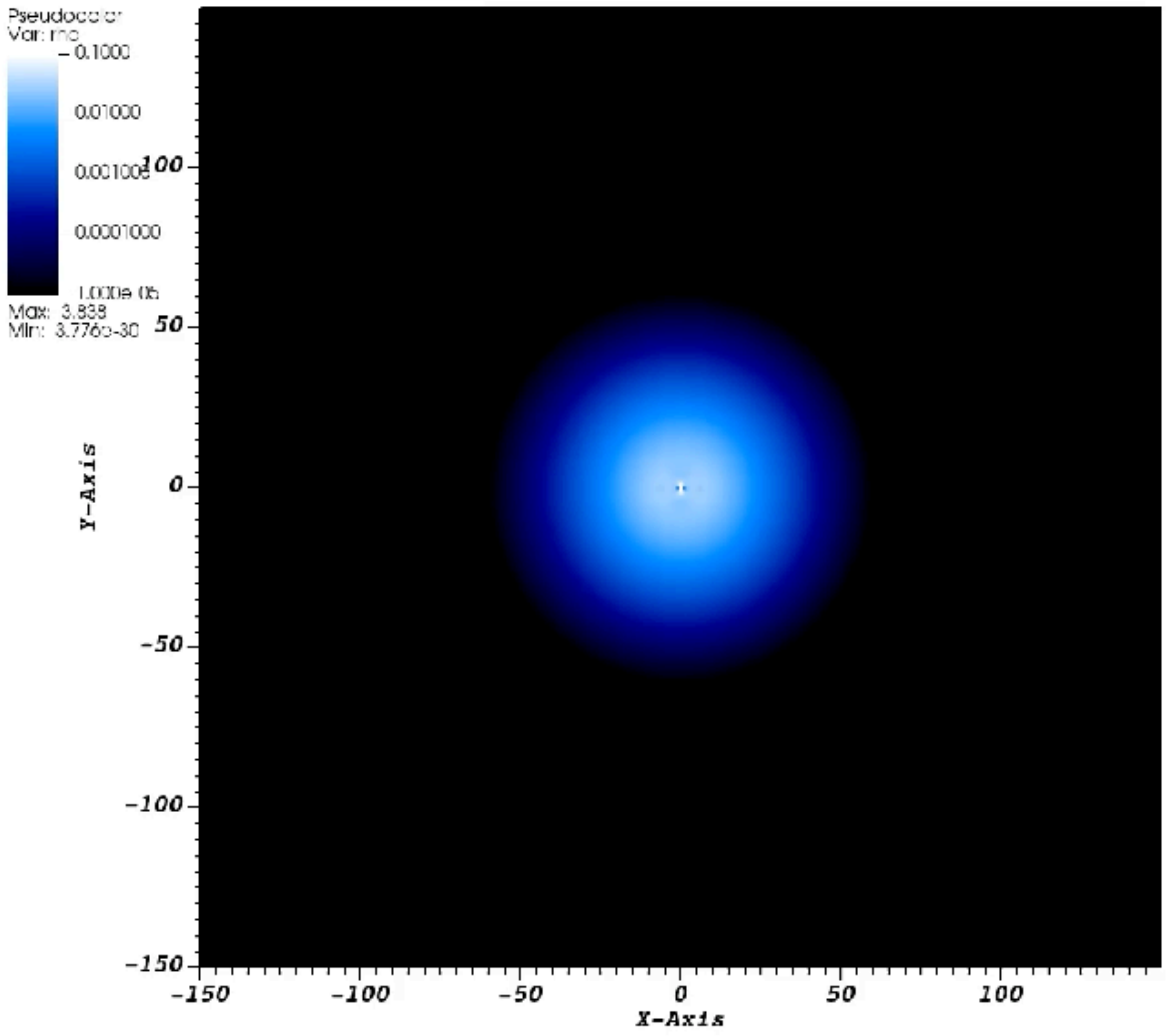
GRChombo simulation



T Ikeda, L. Bernard, V Cardoso, M Zilihao 2021
Phys.Rev.D 103 (2021) 2, 024020

Although not always…



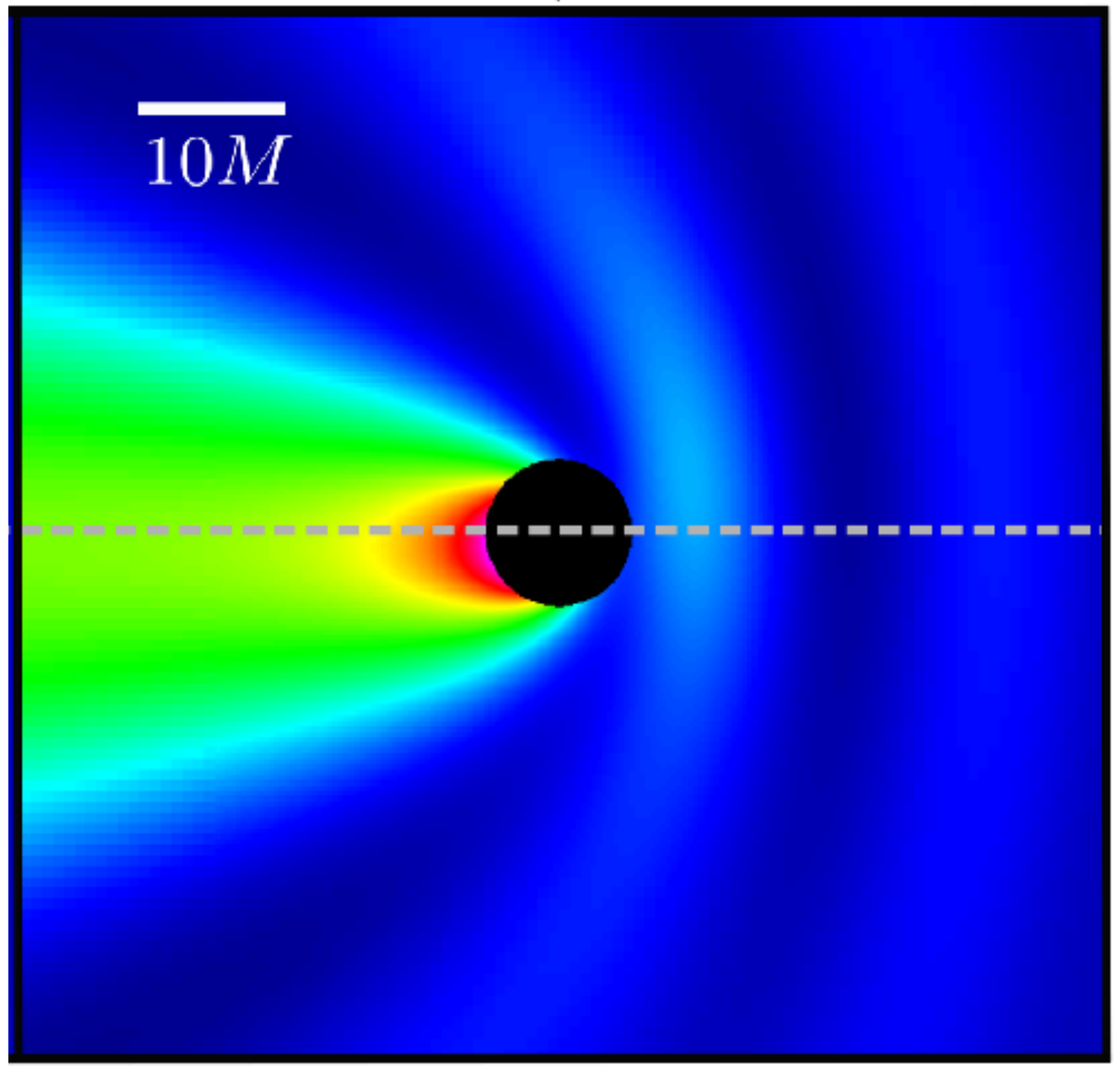


Scalar field and density starting from initial superradiant state around single BH

What next?

- Better characterisation of the quasi stationary states
- Extend to cases with scalar angular or linear momentum (arising from superradiance or halo environment)
- Effect of unequal mass - potential for kicks and centre of mass drift
- Effect of spin - gravitational Magnus effects
- Can self interactions be measured?
- …

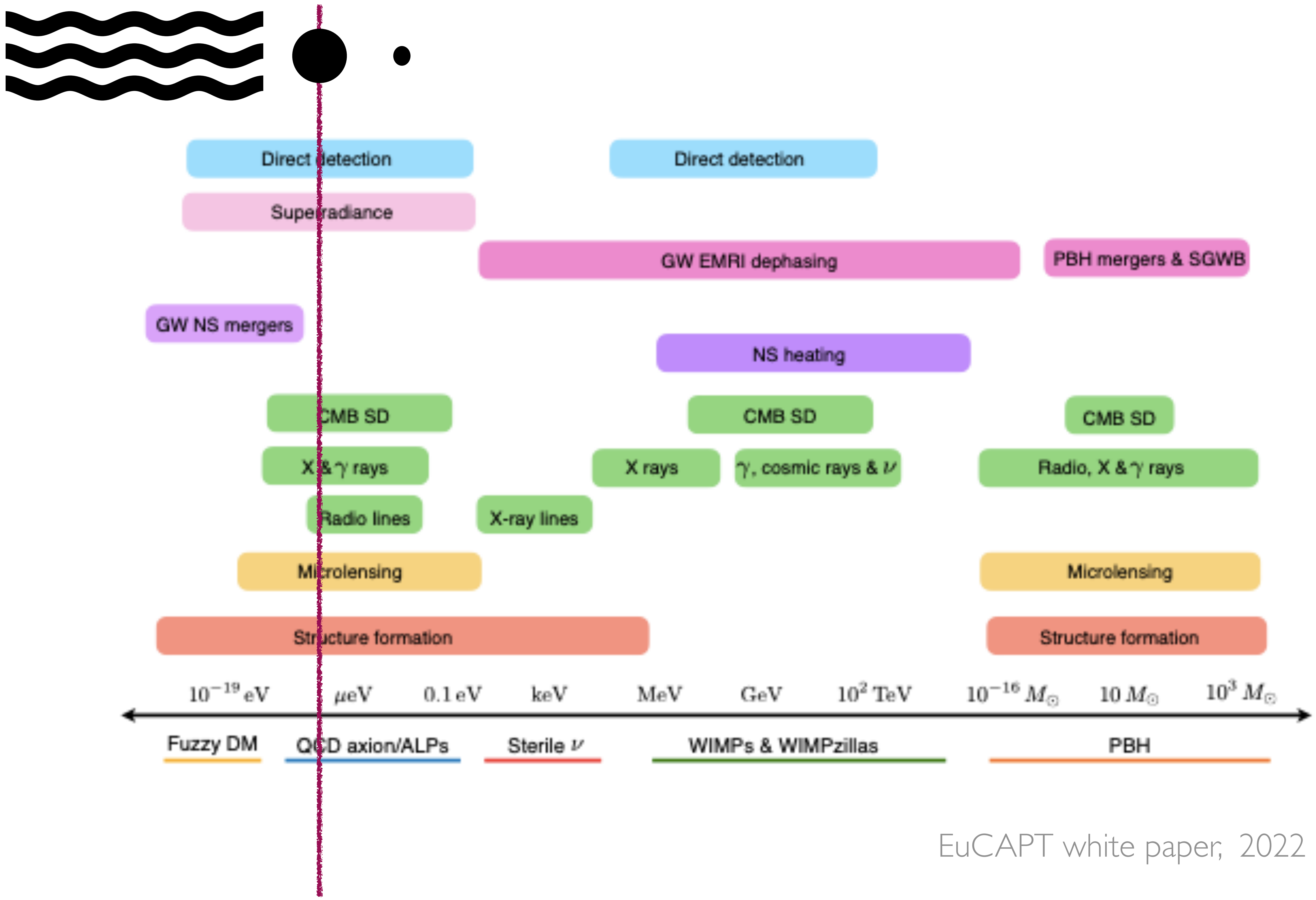
$v = 0.5, a/M = 0.99$



Z Wang, T Helfer, D Traykova, KC, E Berti 2024
Phys.Rev.D 110 (2024) 2, 024009

Heavy / particle dark matter

Wave versus particle: the NR perspective

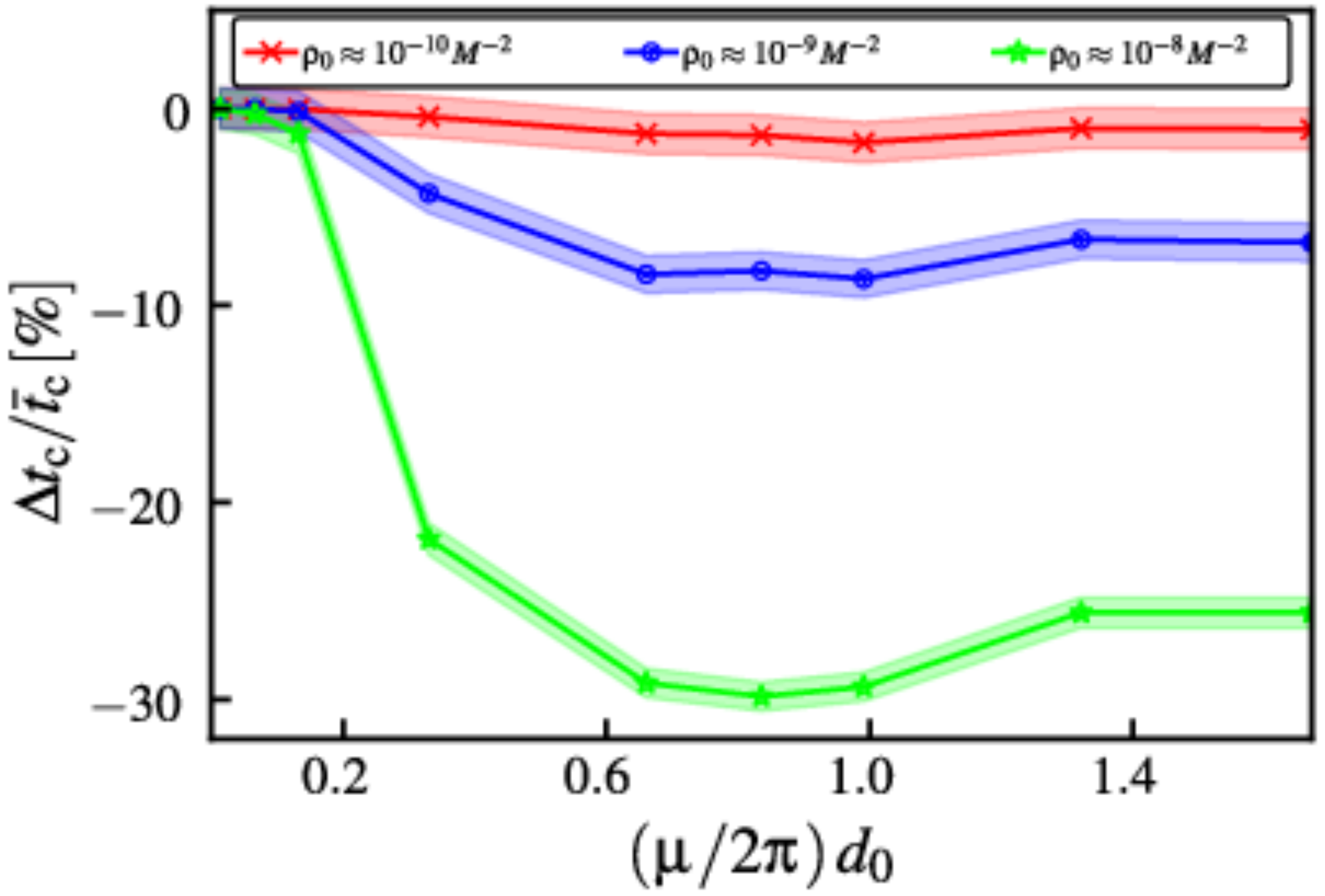


EuCAPT white paper, 2022

What next?

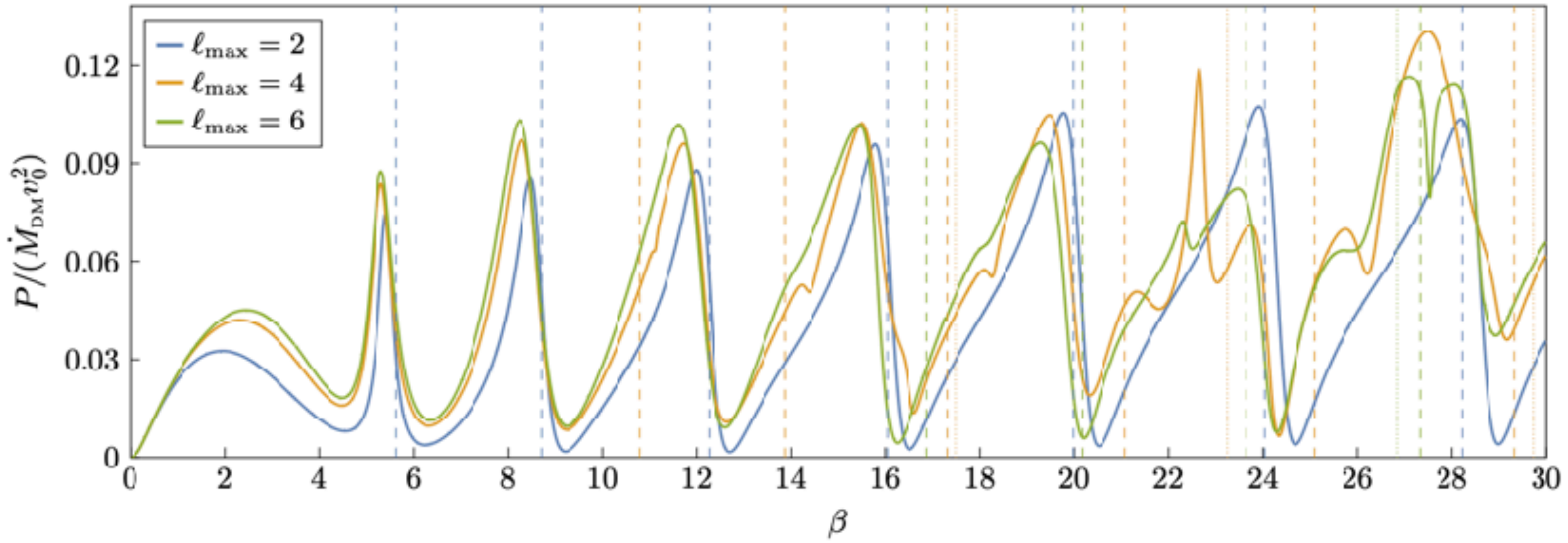
- Are there (similar BH mass) scenarios where particle DM can cluster? (e.g. Kozai Lidov)
- What is the impact of third bodies / (P)BH clusters in the strong field regime?
- How can we take advantage of improvements in performance in NR (e.g. using GPUs)?
- …

Some evidence that quasi stationary states may extend to higher masses



J. Aurrekoetxea, KC, J Bamber, P Ferreira 2023
Phys.Rev.Lett. 132 (2024) 21, 211401

Higher masses



GM Tomaselli
Phys.Rev.D 111 (2025) 6, 063075

Higher masses

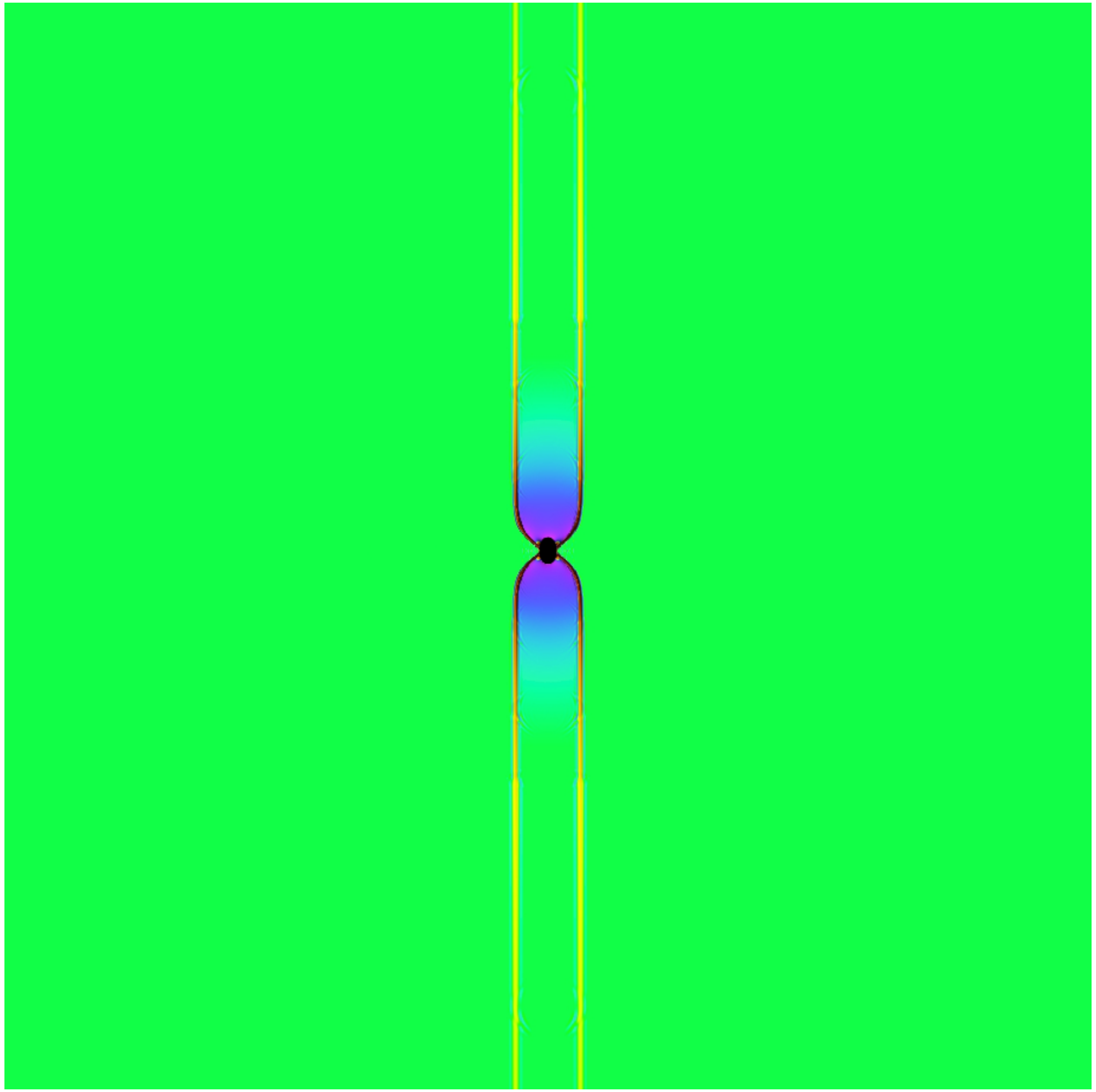
Will need Einstein-Vlasov

Using particles to study BH formation from the collision of wave-fronts

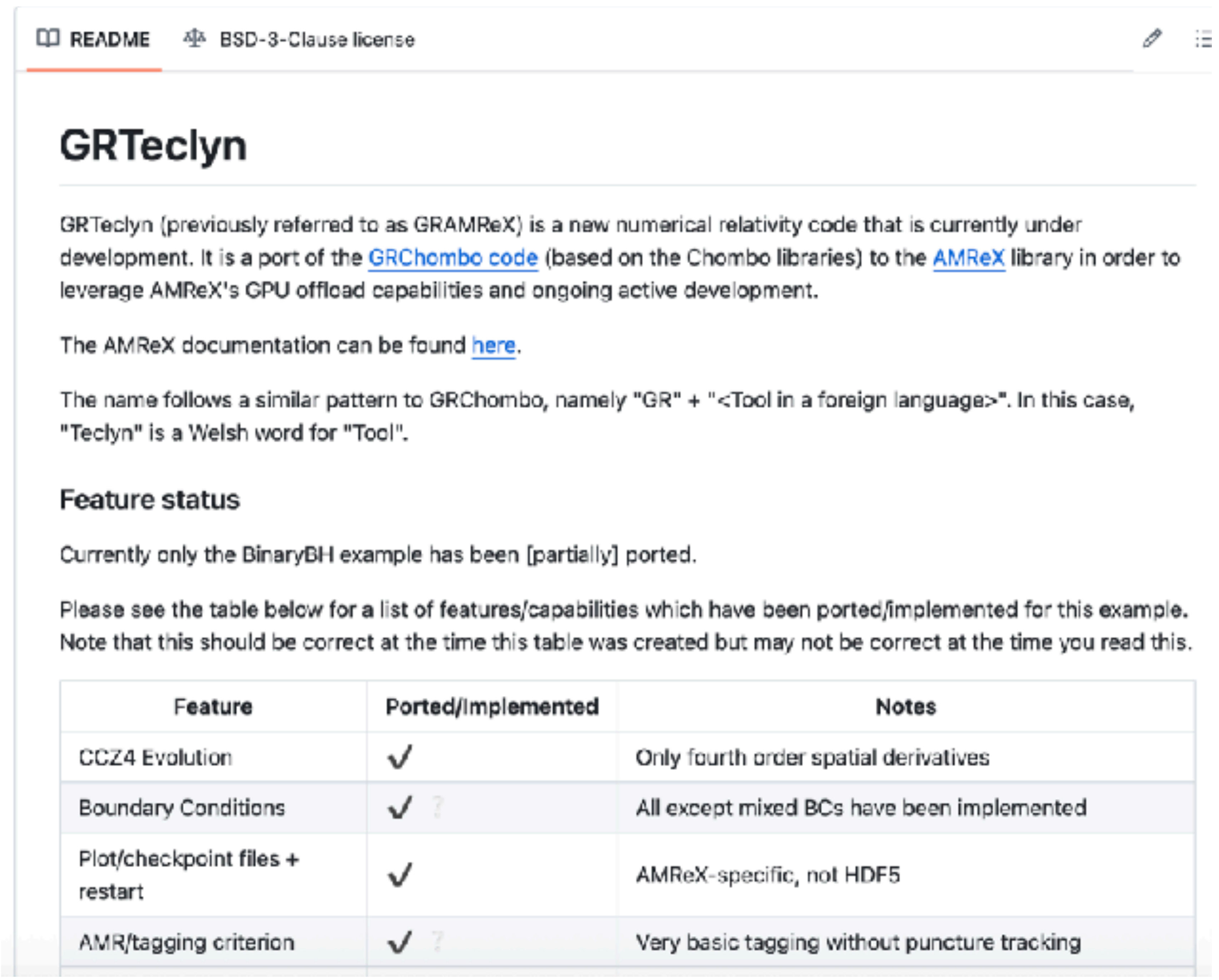
Particles move on geodesics

Particles backreact (locally) on the spacetime - interpolation of particles onto a stress-energy field

F Pretorius and W.E. East 2018
Phys.Rev.D 98 (2018) 8, 084053



GRTeclyn - a new NR code with particles (and GPUs)



README   BSD-3-Clause license

GRTeclyn

GRTeclyn (previously referred to as GRAMReX) is a new numerical relativity code that is currently under development. It is a port of the GRChombo code (based on the Chombo libraries) to the AMReX library in order to leverage AMReX's GPU offload capabilities and ongoing active development.

The AMReX documentation can be found here.

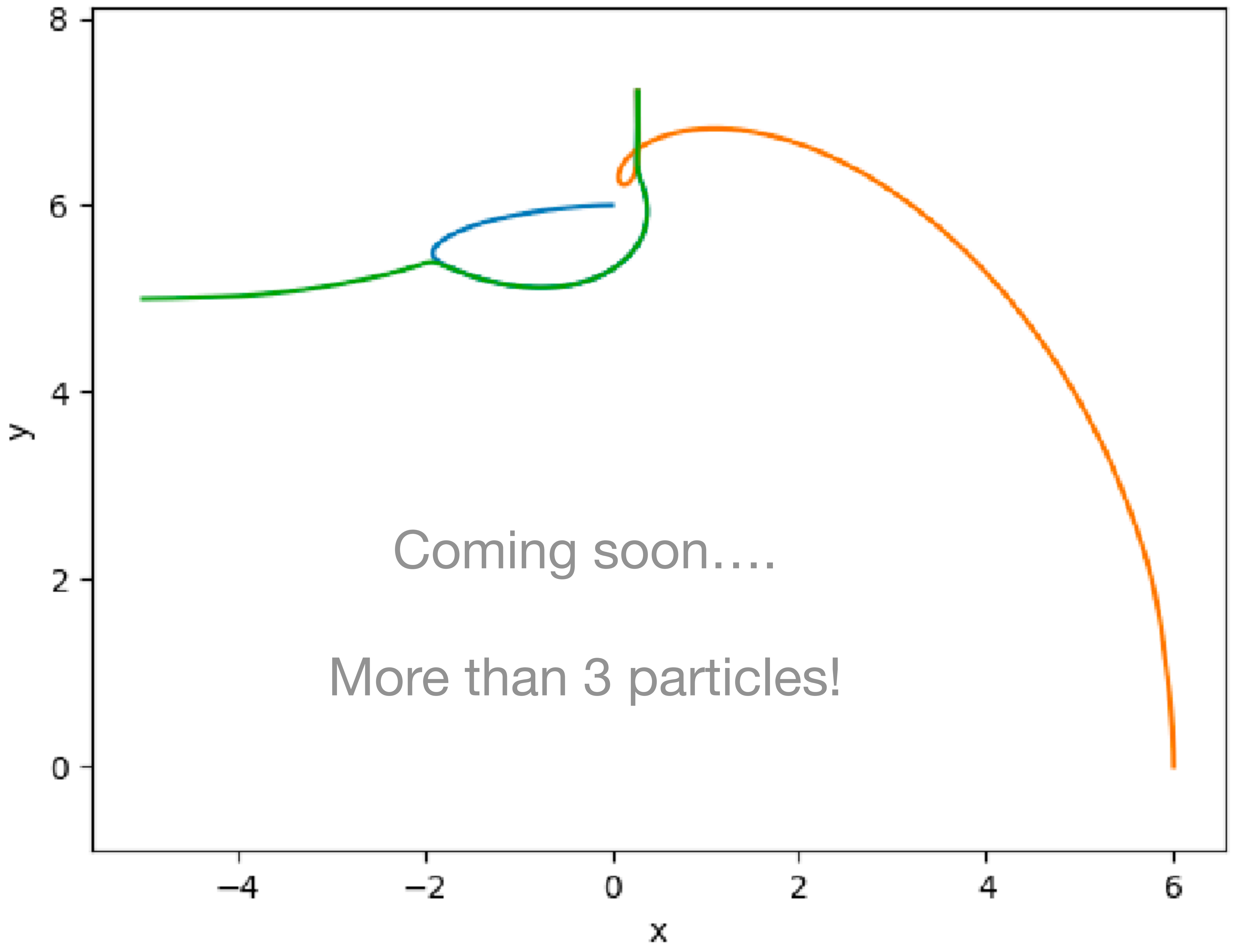
The name follows a similar pattern to GRChombo, namely "GR" + "<Tool in a foreign language>". In this case, "Teclyn" is a Welsh word for "Tool".

Feature status

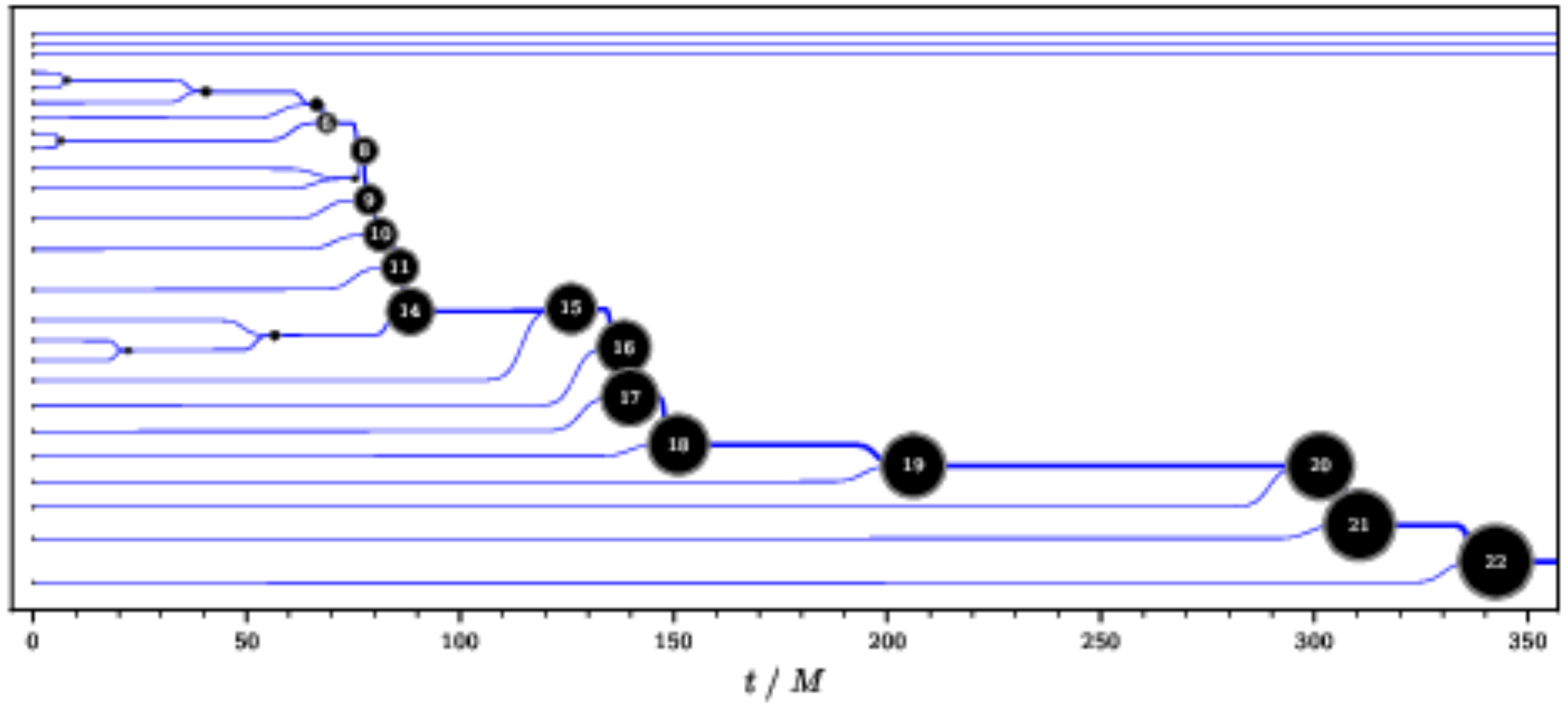
Currently only the BinaryBH example has been [partially] ported.

Please see the table below for a list of features/capabilities which have been ported/implemented for this example. Note that this should be correct at the time this table was created but may not be correct at the time you read this.

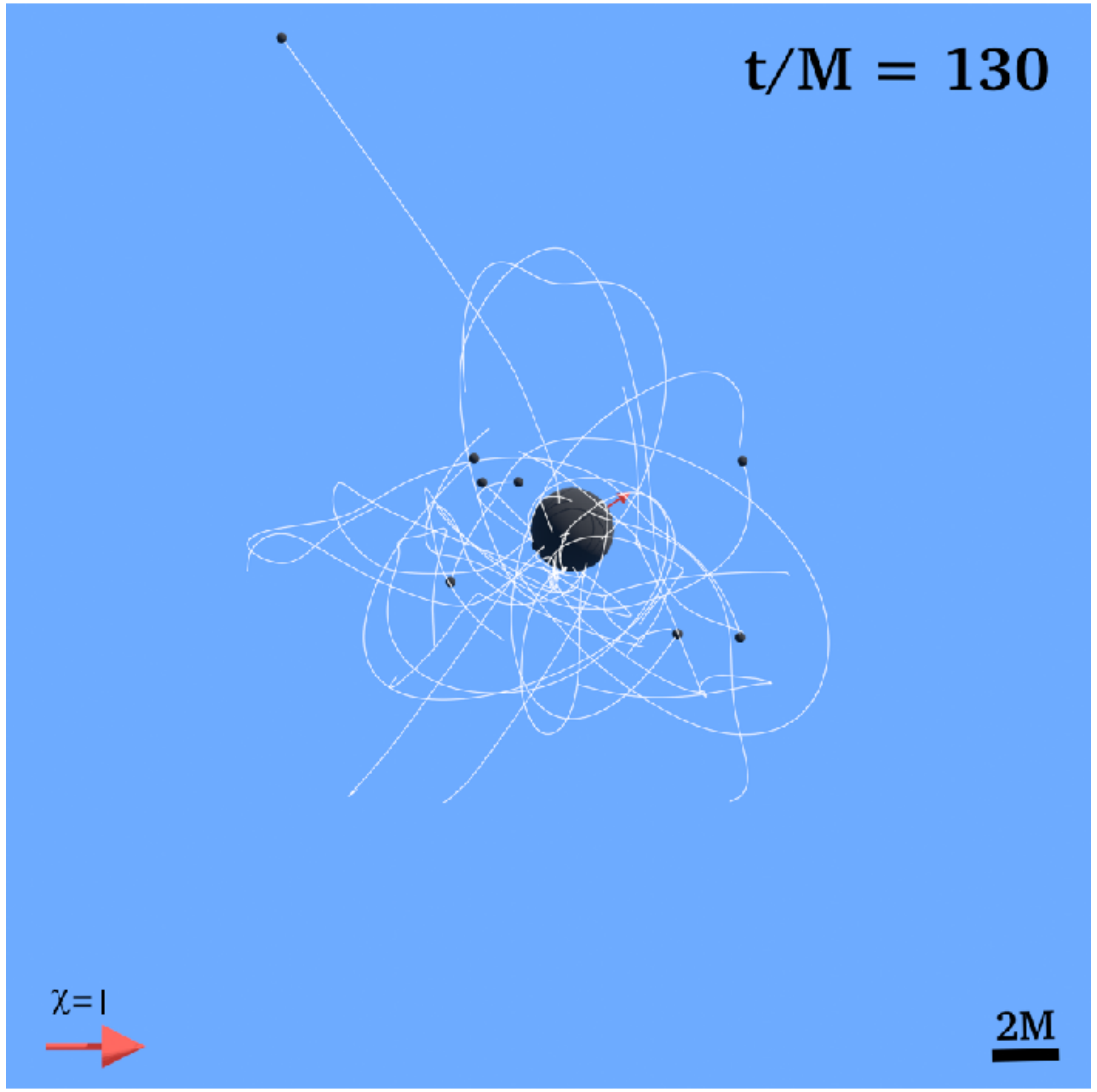
Feature	Ported/Implemented	Notes
CCZ4 Evolution	✓	Only fourth order spatial derivatives
Boundary Conditions	✓	All except mixed BCs have been implemented
Plot/checkpoint files + restart	✓	AMReX-specific, not HDF5
AMR/tagging criterion	✓	Very basic tagging without puncture tracking



Full NR simulations of clusters of BHs



25 BHs with initially small separation merge hierarchically



Jamie Bamber, Stuart L. Shapiro, Milton Ruiz, Antonios Tsokaros 2025
e-Print: 2505.01495 [gr-qc]

Questions?