

PRIMORDIAL BLACK HOLES IN THE MULTI-MESSENGER ERA
TRIESTE 2025

SINGLE FIELD INFLATION

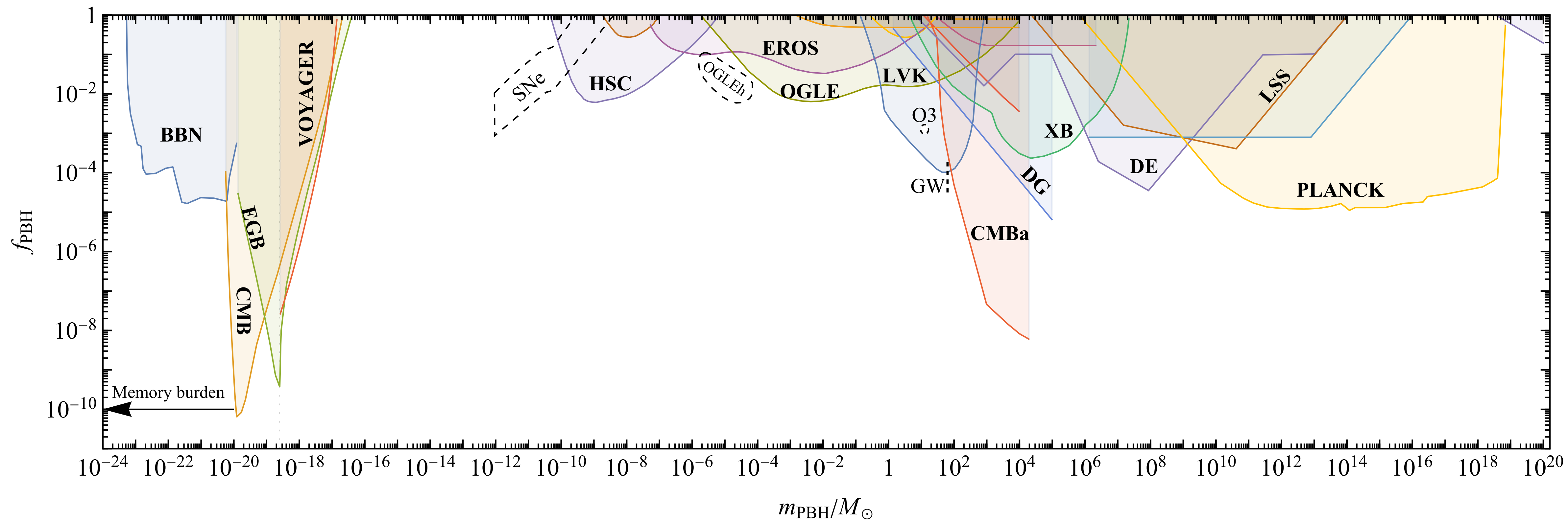
FOR PRIMORDIAL BLACK HOLES

Hardi Veermäe
KBFI, Tallinn

November 4, 2025

 **Keemilise ja
Bioloogilise Füüsika Instituut**
National Institute of Chemical Physics and Biophysics

FOUNDATIONS OF THE UNIVERSE
CoE TK202



CONSTRAINTS ON THE PBH PARAMETER SPACE

[github.com/vianvask/PBHconstraints]

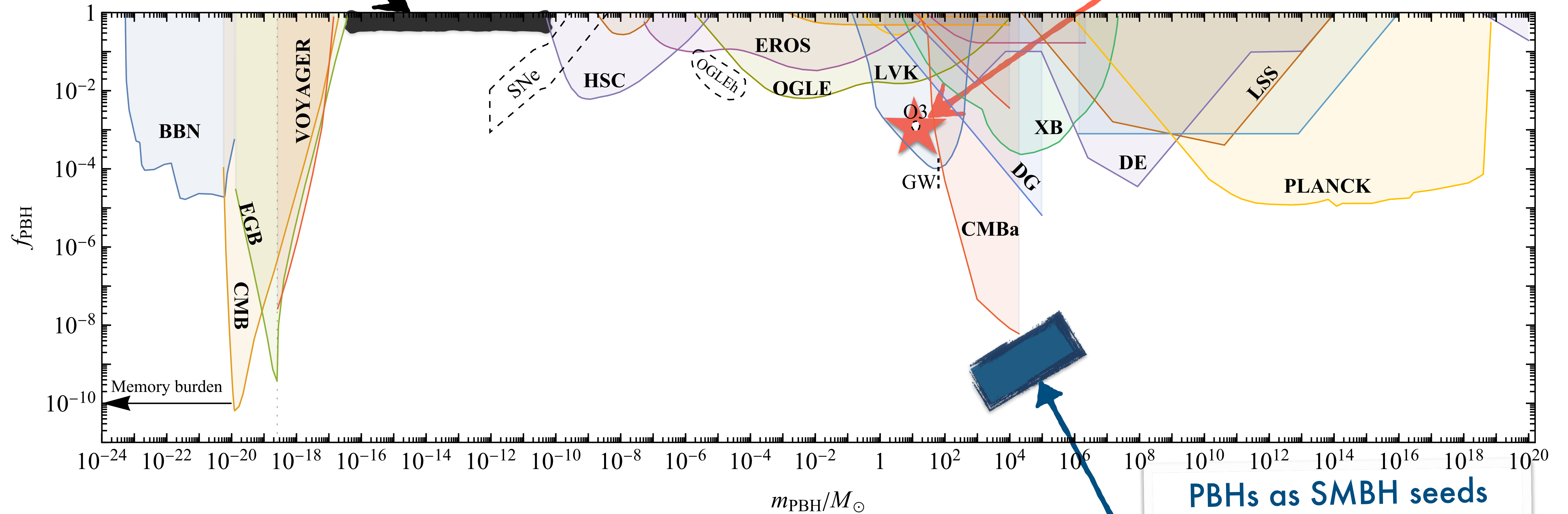
PBHs as all dark matter

$$M_{\text{PBH}} \approx 10^{17} - 10^{22} \text{ g}$$

observed BH binaries

$$M_{\text{PBH}} \approx 1 - 100 M_{\odot}$$

$$f_{\text{PBH}} \approx 10^{-3}$$



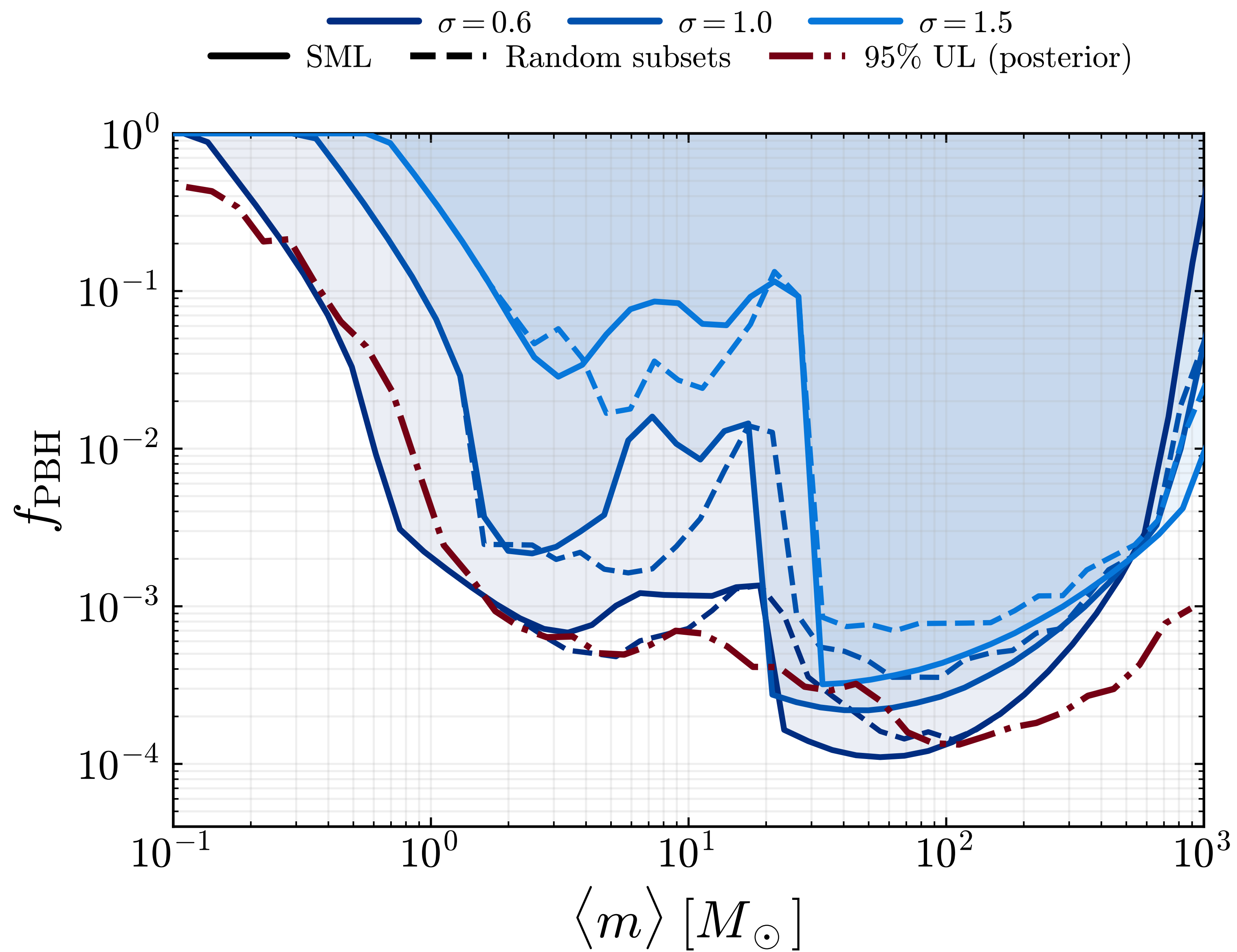
CONSTRAINTS ON THE PBH PARAMETER SPACE

[github.com/vianvask/PBHconstraints]

PBHs as SMBH seeds

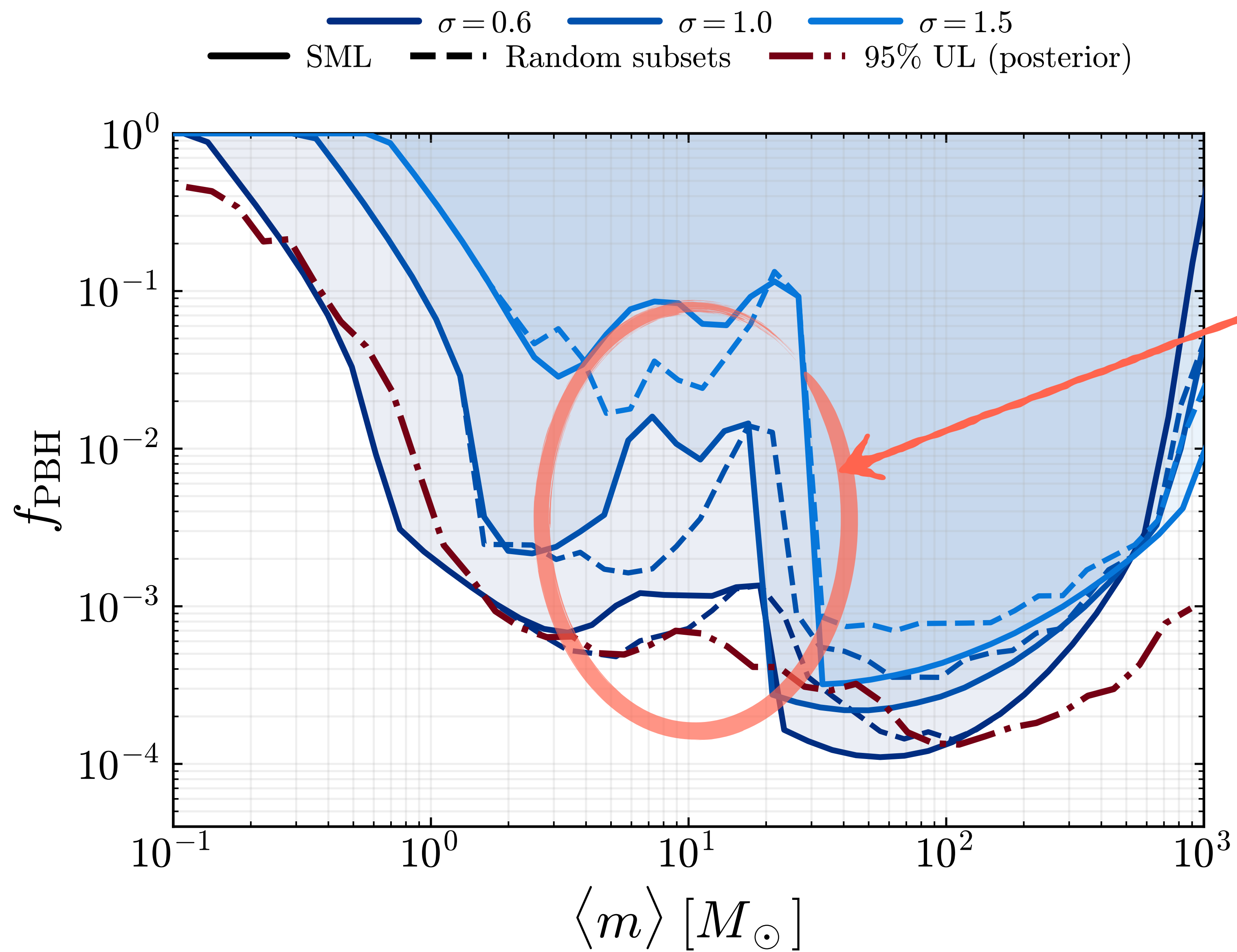
$$M_{\text{PBH}} \approx 10^3 - 10^6 M_{\odot}$$

$$f_{\text{PBH}} \approx 10^{-10} \frac{M_{\text{PBH}}}{10^3 M_{\odot}}$$



MASS FUNCTION

$$\psi \propto \exp \left[-\frac{\ln^2(m/m_c)}{2\sigma^2} \right]$$



*here, some LVK
 binaries could be
 primordial*

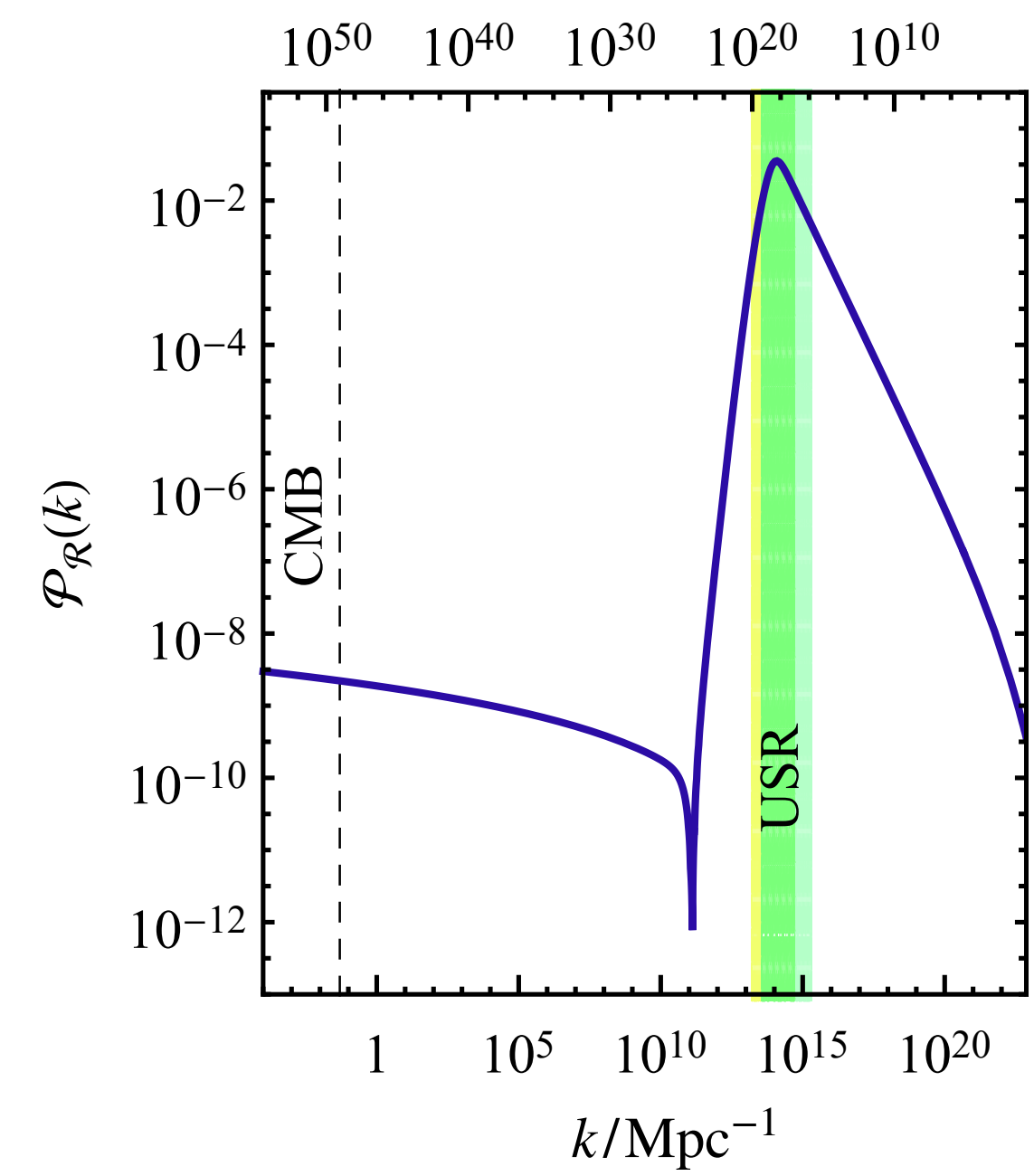
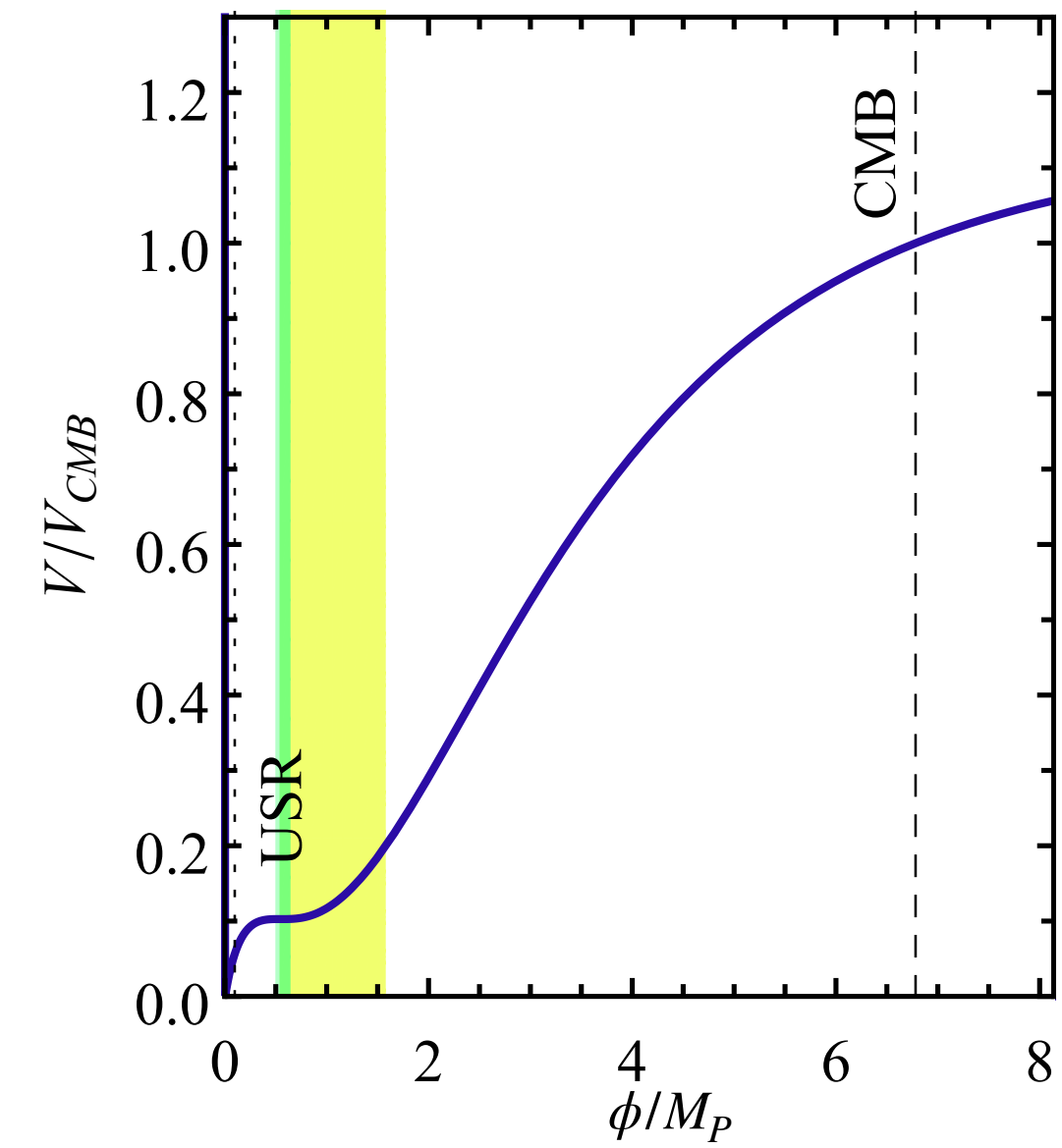
MASS FUNCTION

$$\psi \propto \exp \left[-\frac{\ln^2(m/m_c)}{2\sigma^2} \right]$$

SPECTRAL SHAPES

IN SINGLE FIELD INFLATION

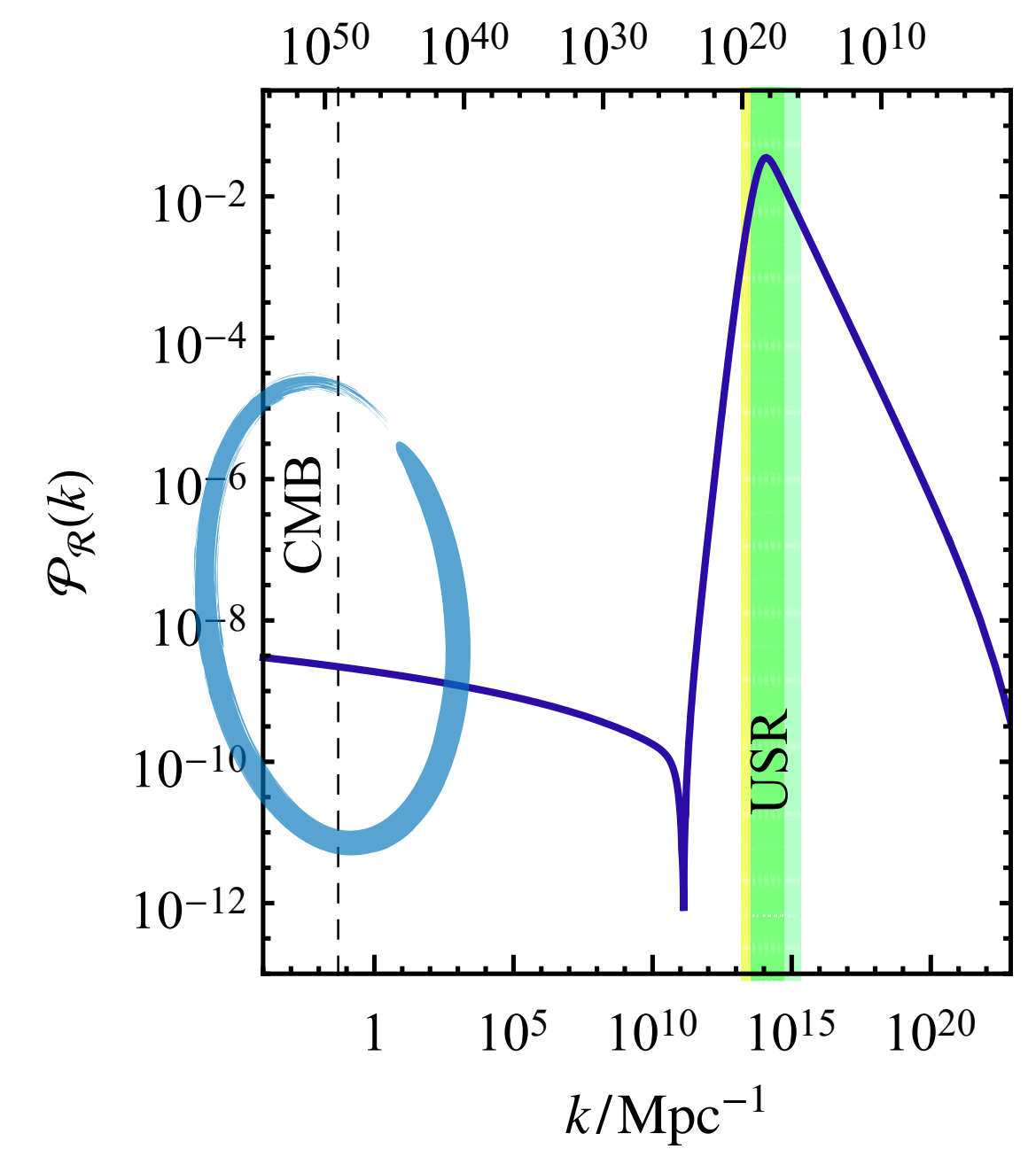
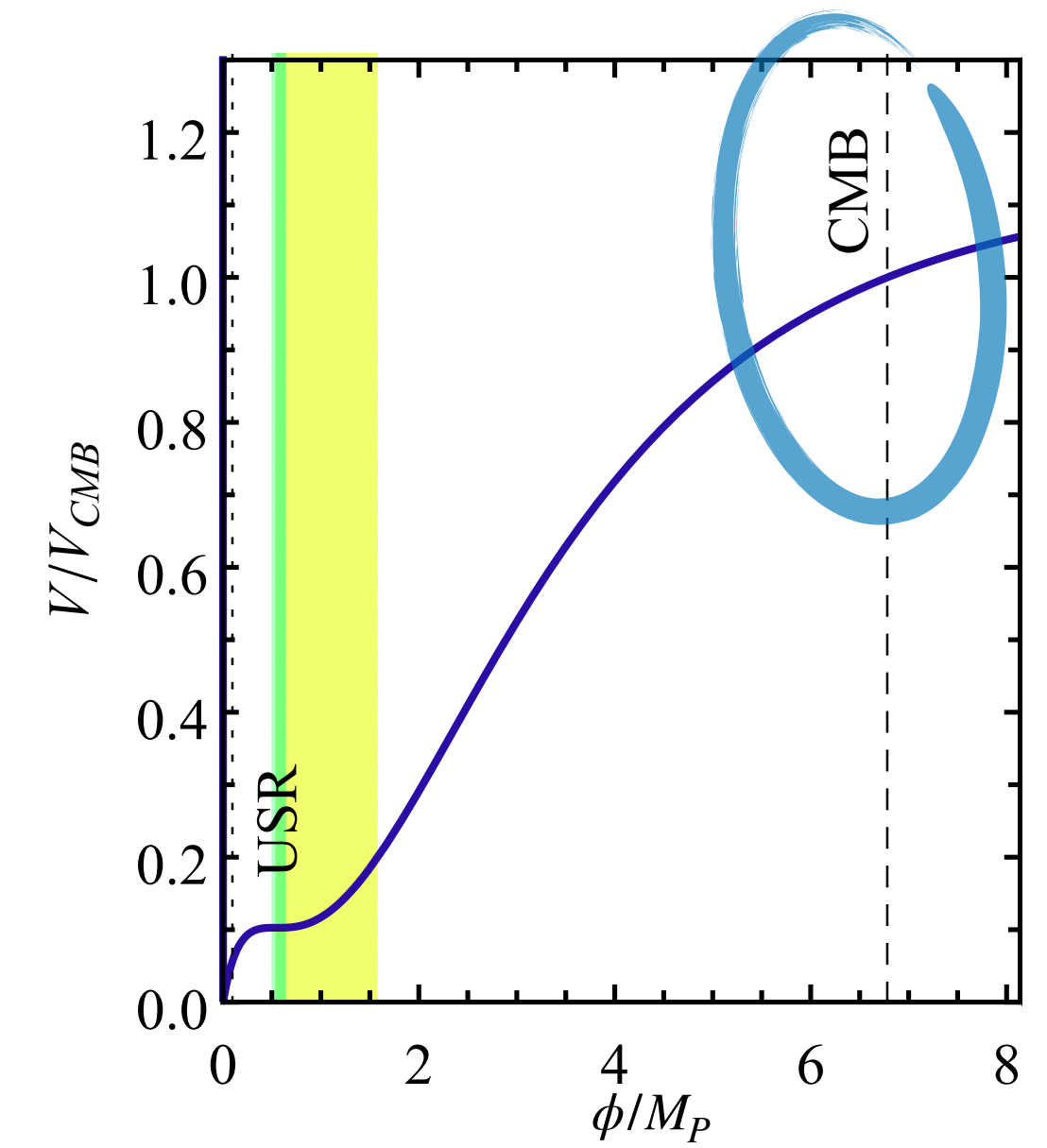
INFLATIONARY TIMELINE



INFLATIONARY TIMELINE

SR:

$$P_{\zeta}(k_{\text{CMB}}) \propto \left. \frac{H^4}{\dot{\phi}^2} \right|_{\text{CMB}} \approx 10^{-9}$$



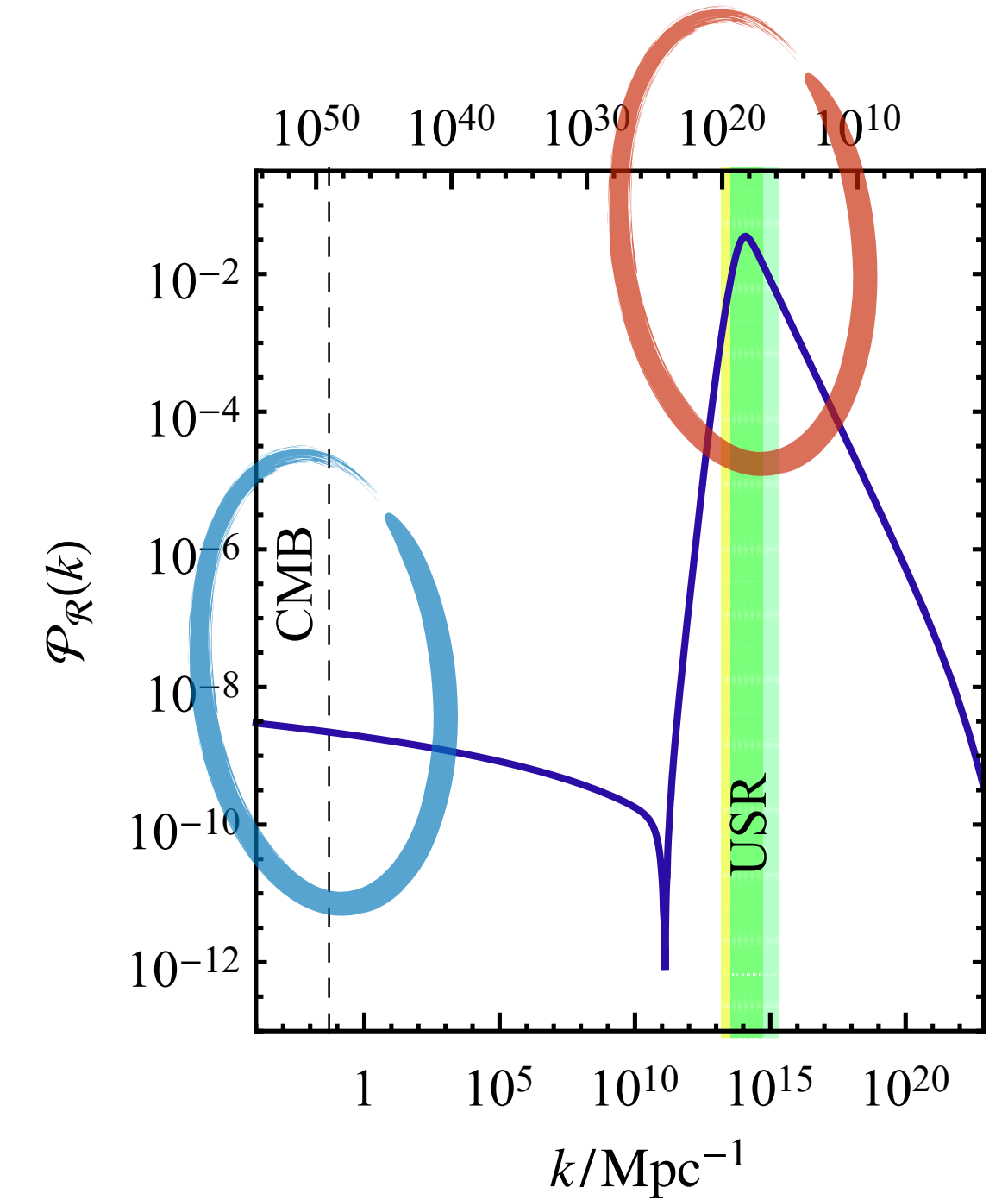
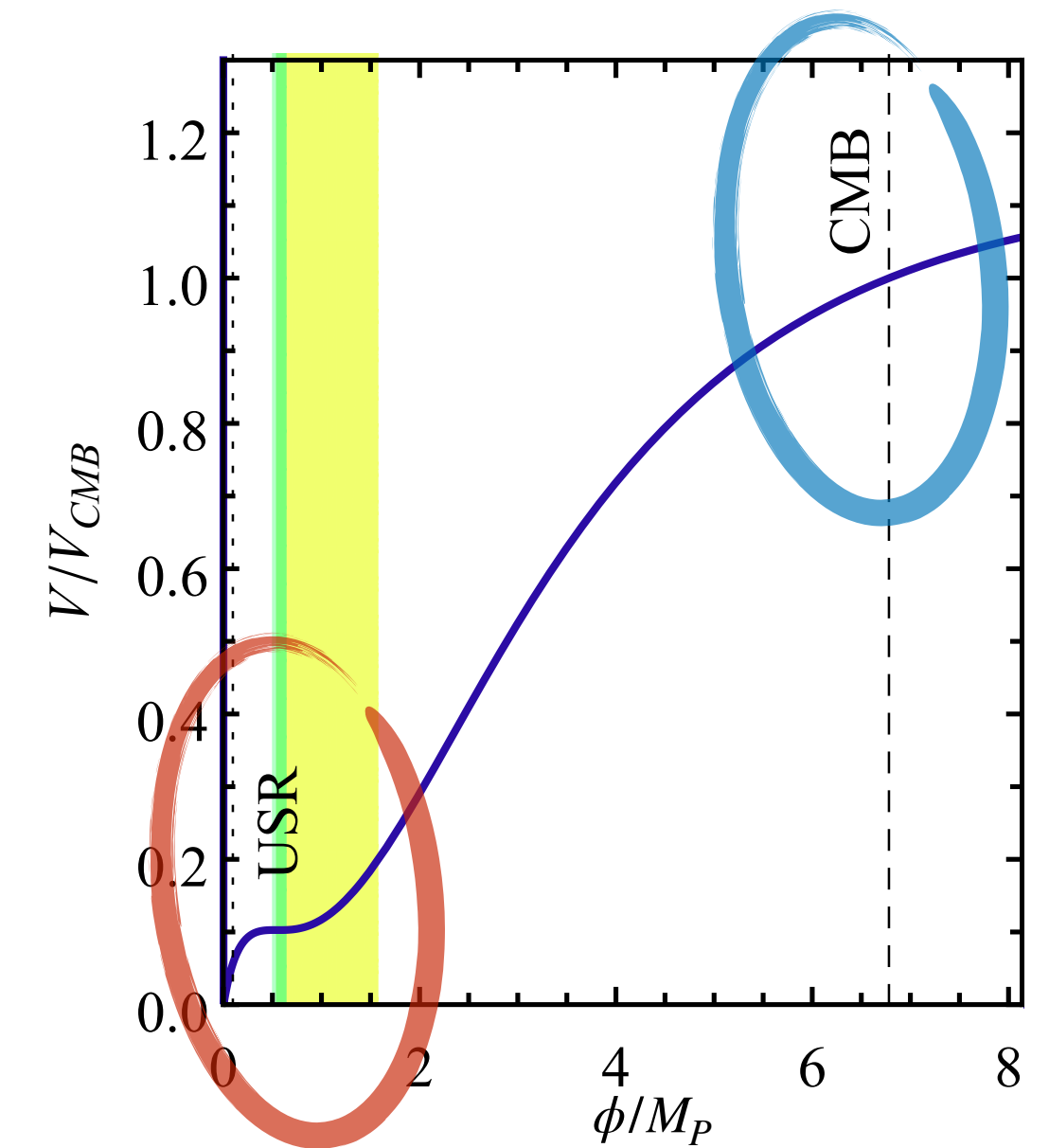
INFLATIONARY TIMELINE

SR:

$$P_{\zeta}(k_{\text{CMB}}) \propto \left. \frac{H^4}{\dot{\phi}^2} \right|_{\text{CMB}} \approx 10^{-9}$$

CR:

$$P_{\zeta}(k_{\text{CMB}}) \propto \left. \frac{H^4}{\dot{\phi}^2} \right|_{\text{CR}} \approx 10^{-3} - 10^{-2}$$

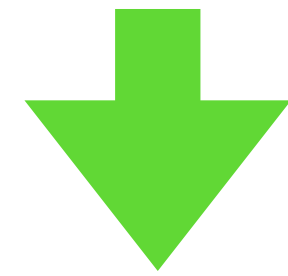


INFLATIONARY TIMELINE

SR:

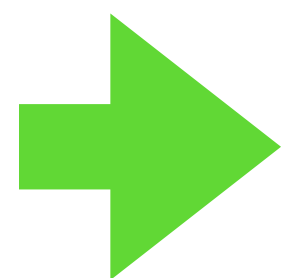
$$P_{\zeta}(k_{\text{CMB}}) \propto \left. \frac{H^4}{\dot{\phi}^2} \right|_{\text{CMB}} \approx 10^{-9}$$

USR

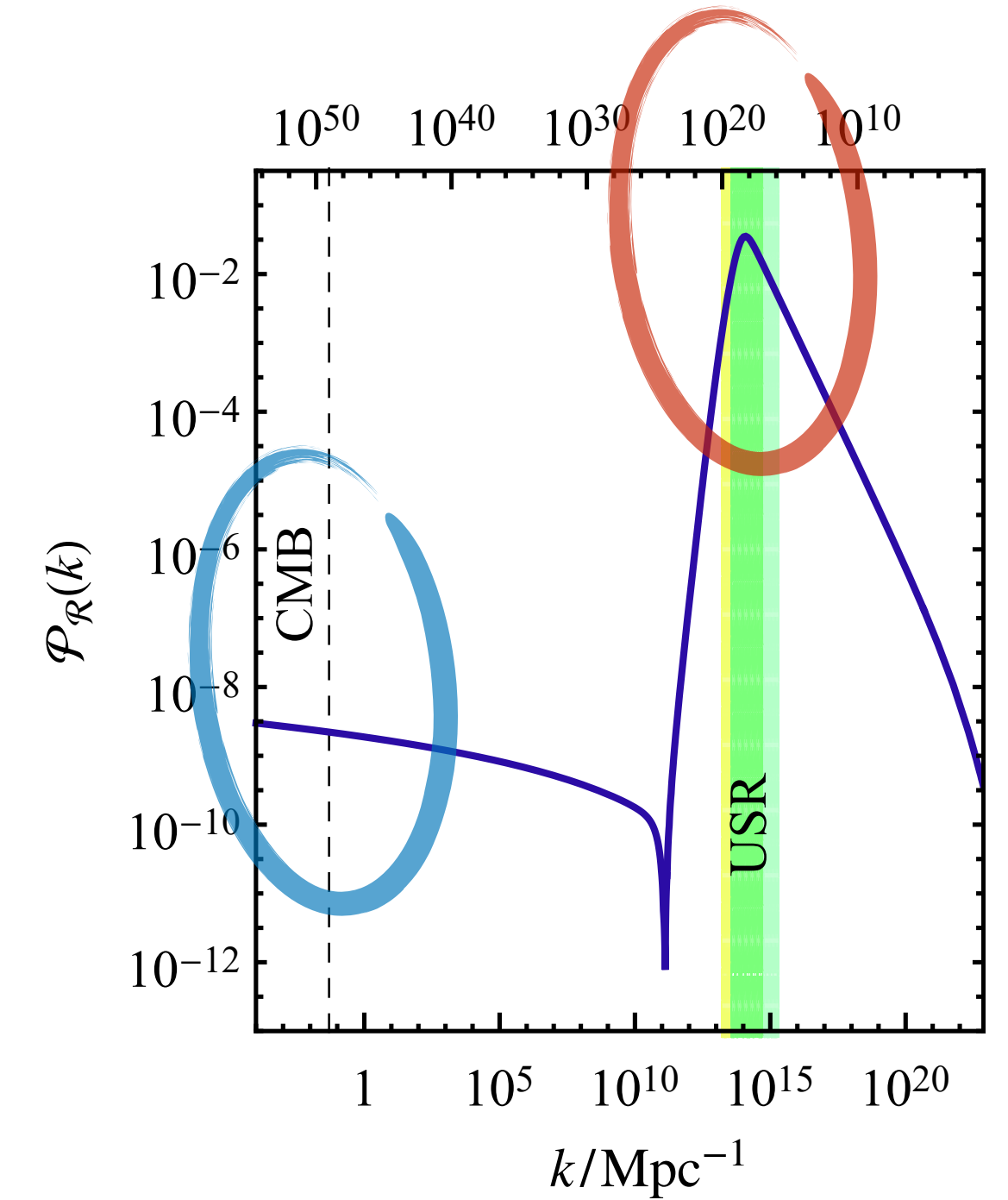
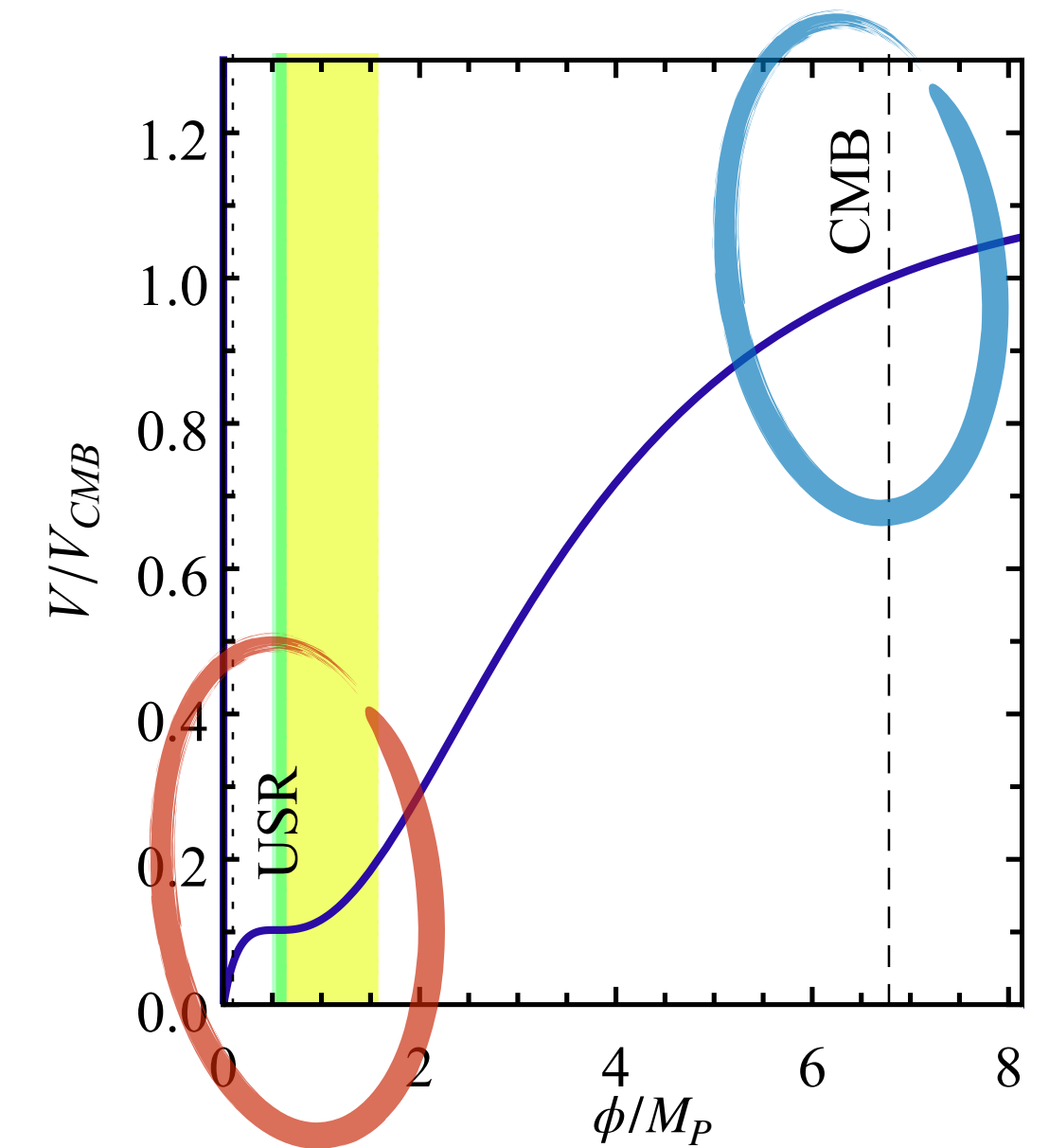


CR:

$$P_{\zeta}(k_{\text{CMB}}) \propto \left. \frac{H^4}{\dot{\phi}^2} \right|_{\text{CR}} \approx 10^{-3} - 10^{-2}$$

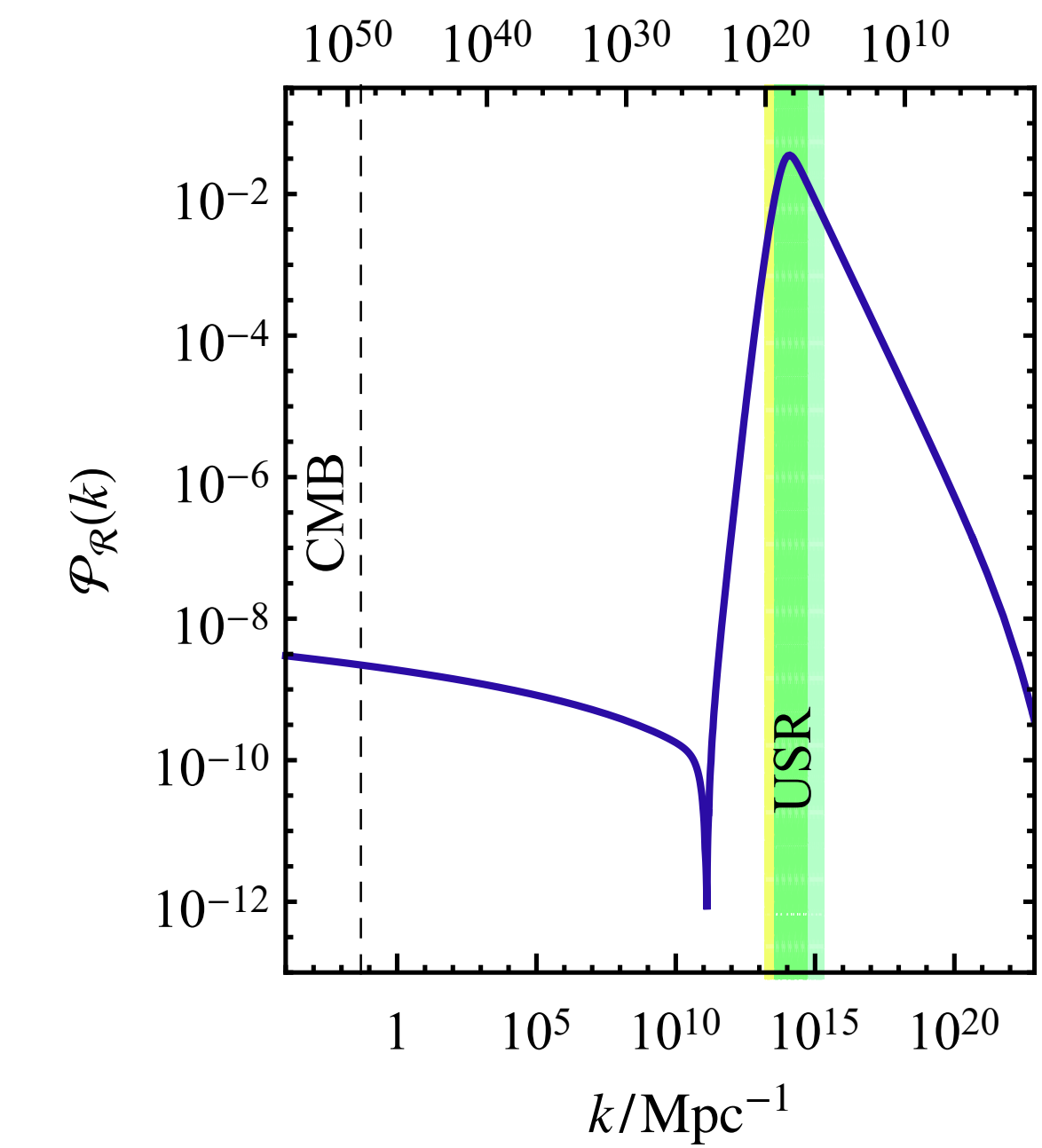
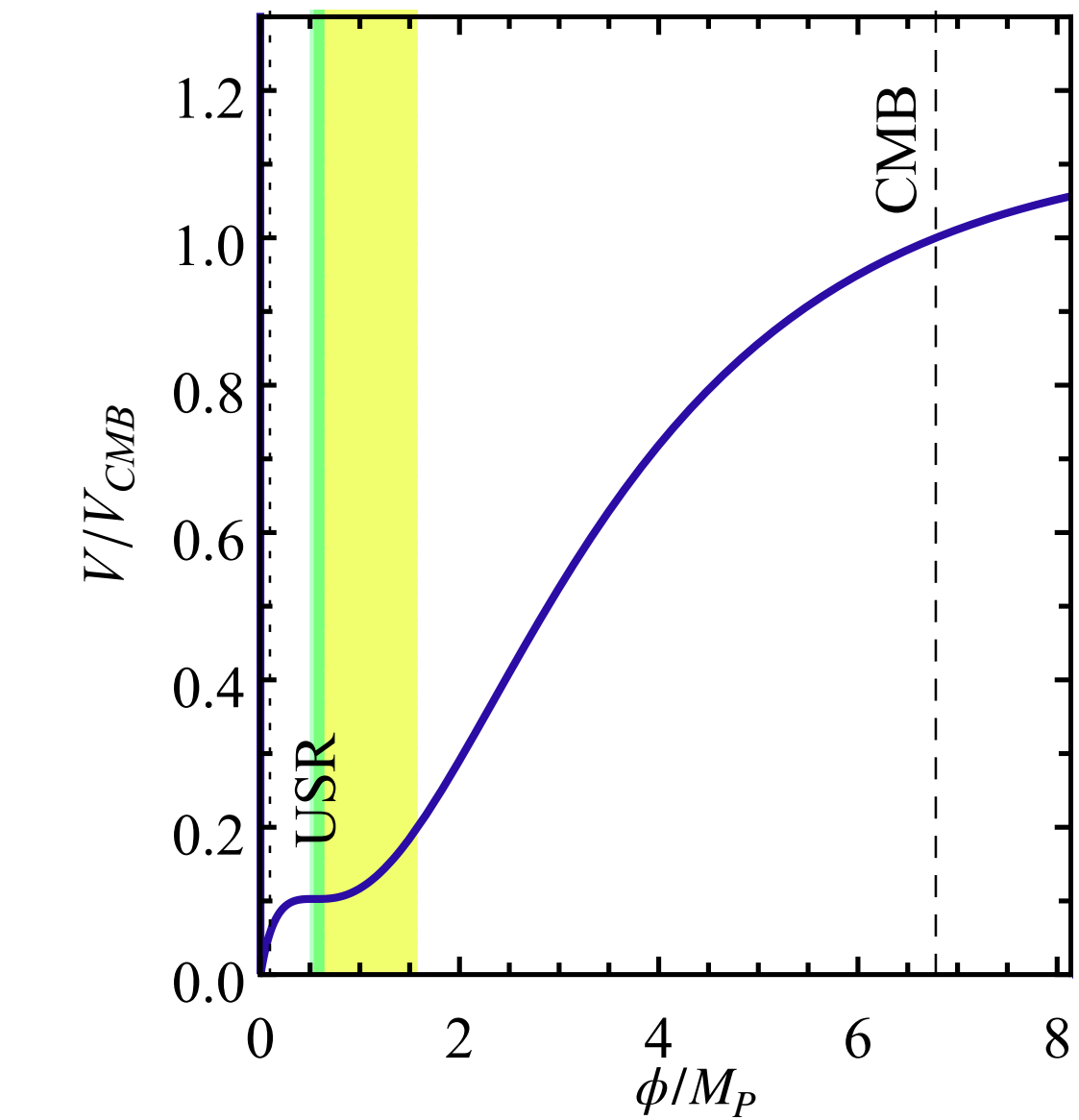


$$\dot{\phi}_{\text{CR}} \lesssim 10^{-3} \dot{\phi}_{\text{CMB}}$$



INFLATIONARY TIMELINE

[2205.13540 Karam et al]

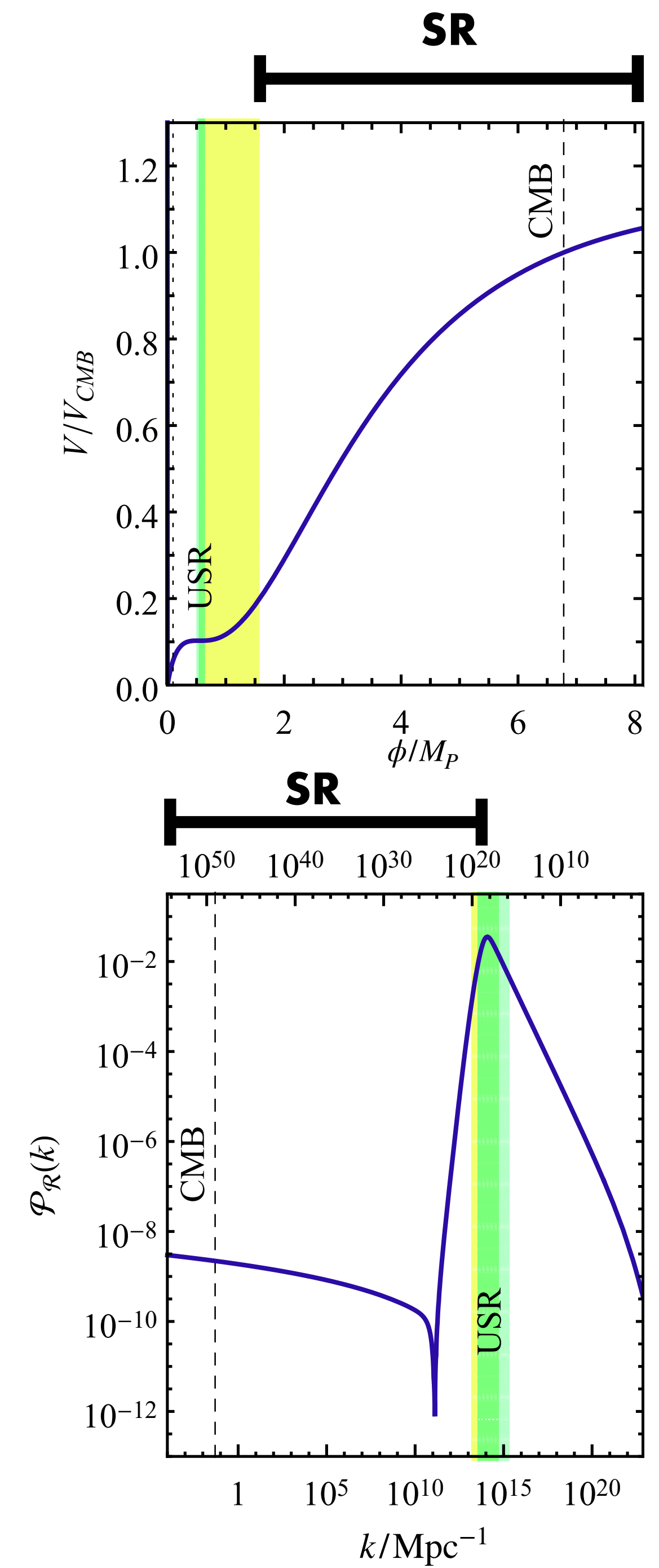


INFLATIONARY TIMELINE

[2205.13540 Karam et al]

1. SLOW-ROLL (SR)

lasts $\mathcal{O}(30)$ e-folds



INFLATIONARY TIMELINE

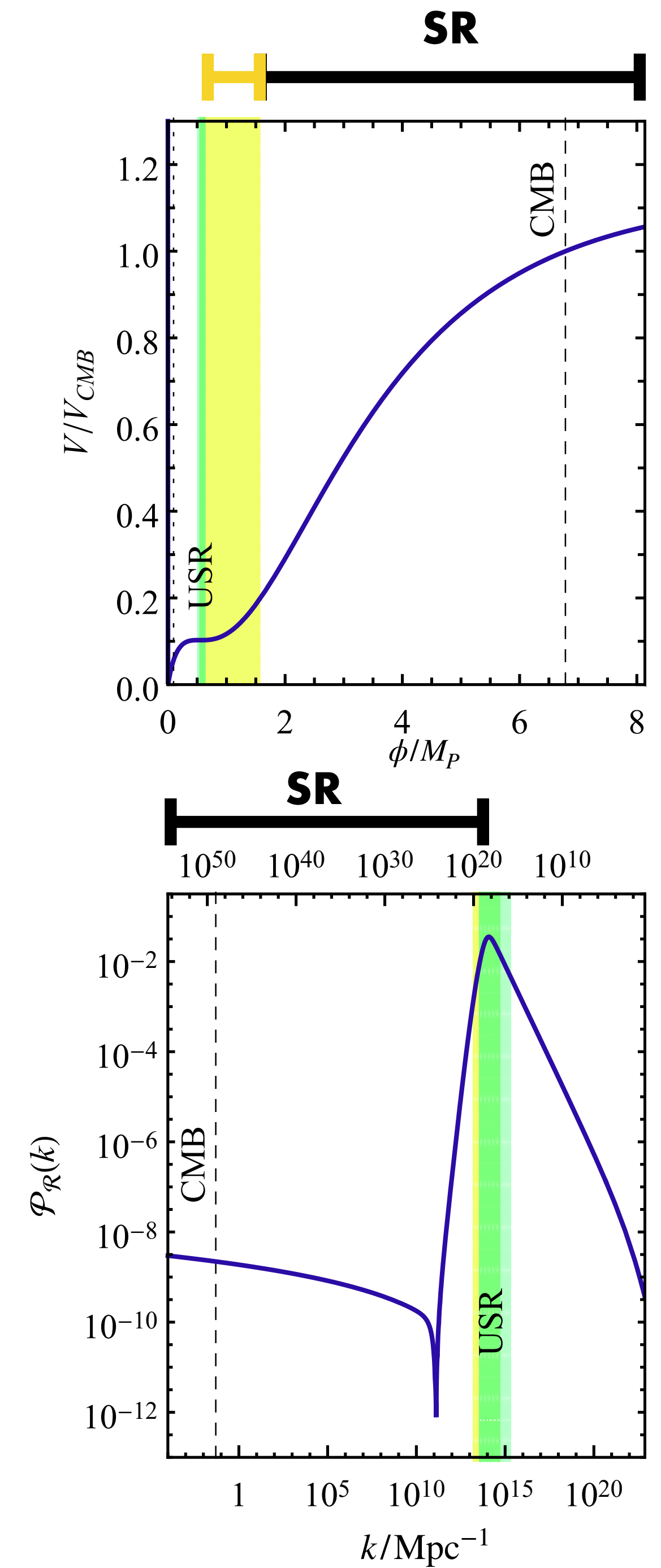
[2205.13540 Karam et al]

1. SLOW-ROLL (SR)

lasts $\mathcal{O}(30)$ e-folds

2. TRANSITION from SR to USR

lasts $\lesssim 1$ e-folds



INFLATIONARY TIMELINE

[2205.13540 Karam et al]

1. SLOW-ROLL (SR)

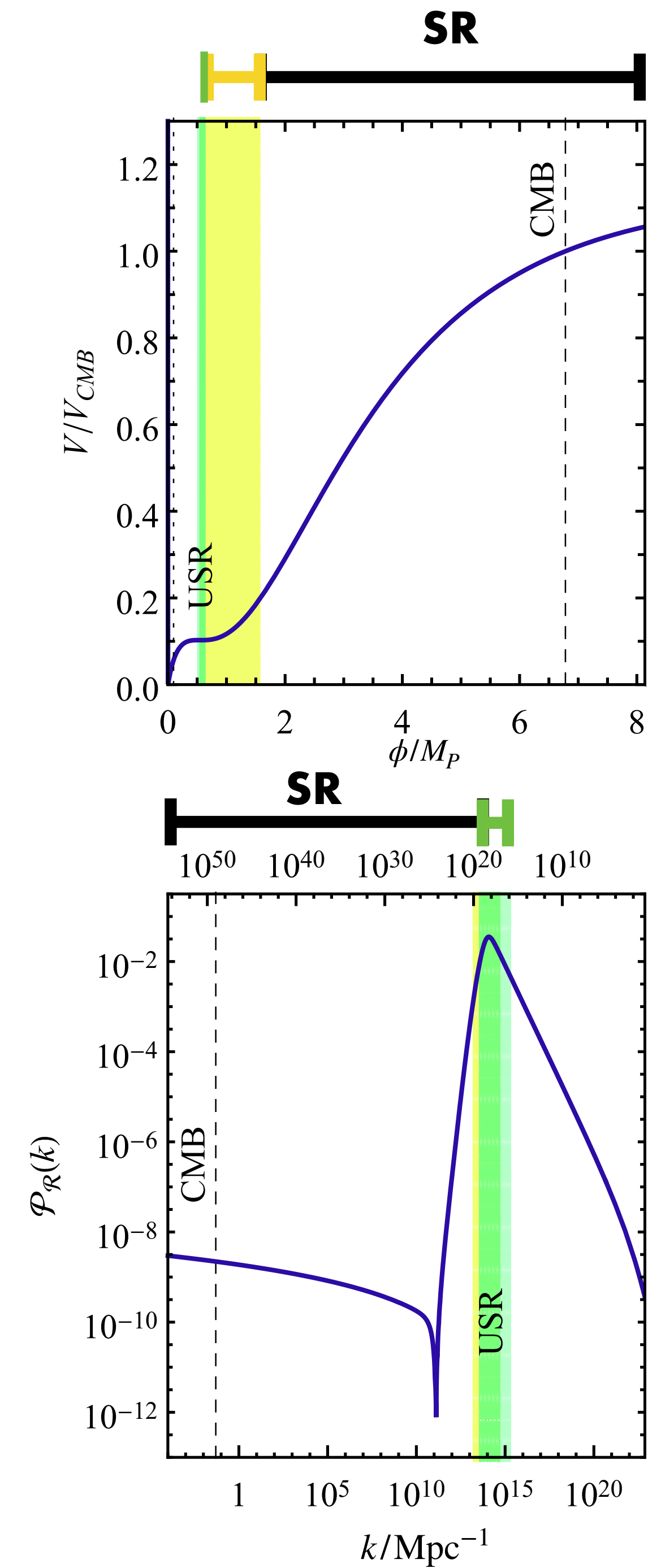
lasts $\mathcal{O}(30)$ e-folds

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3. ULTRA-SLOW-ROLL (USR)

lasts $\mathcal{O}(3)$ e-folds



INFLATIONARY TIMELINE

[2205.13540 Karam et al]

1. SLOW-ROLL (SR)

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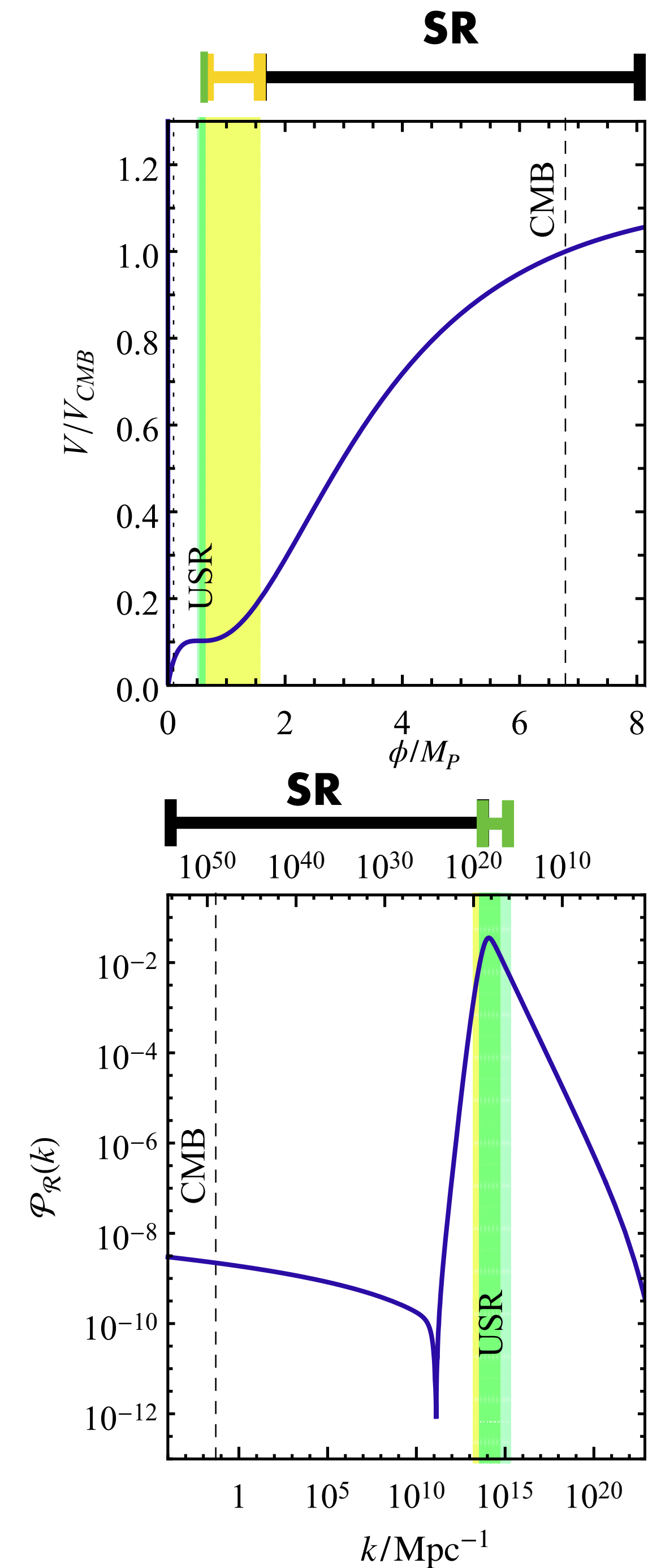
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3. ULTRA-SLOW-ROLL (USR)

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4. TRANSITION from USR to CR



INFLATIONARY TIMELINE

[2205.13540 Karam et al]

1. SLOW-ROLL (SR)

lasts $\mathcal{O}(30)$ e-folds

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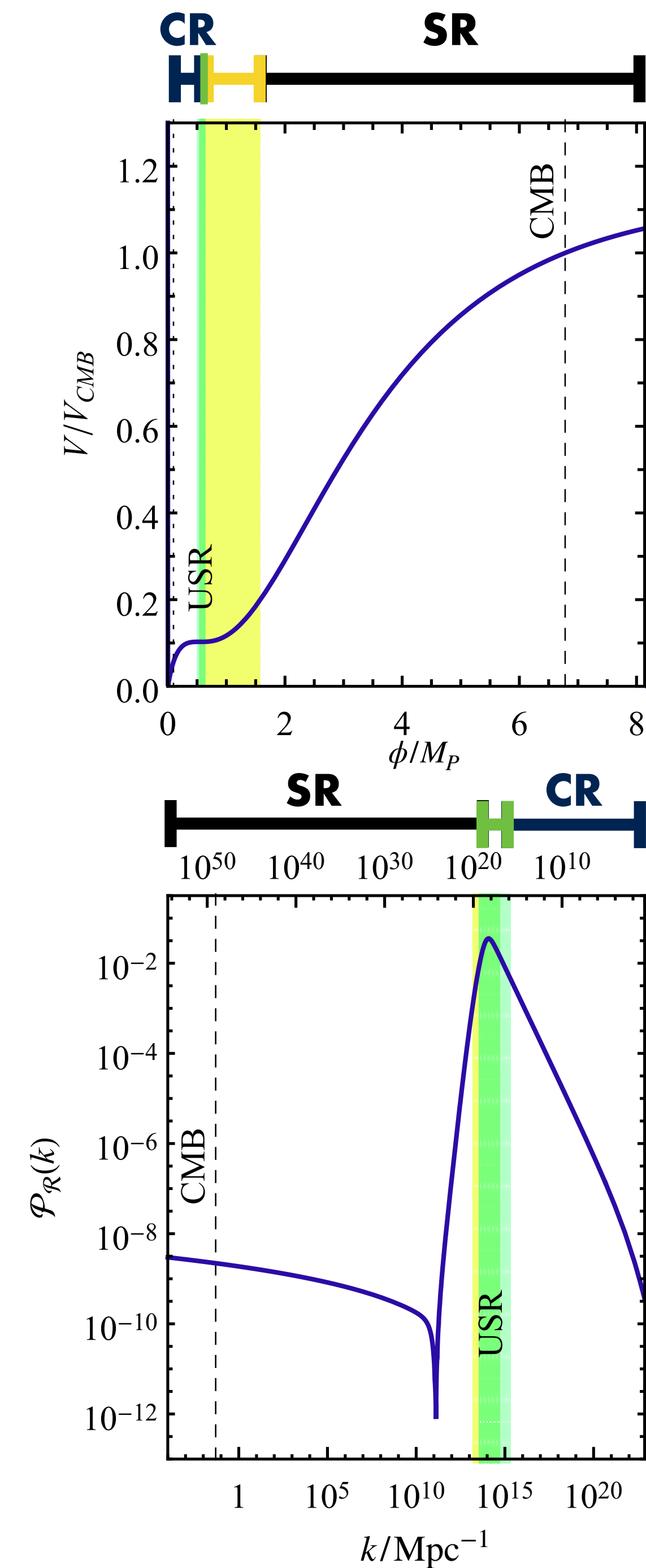
3. ULTRA-SLOW-ROLL (USR)

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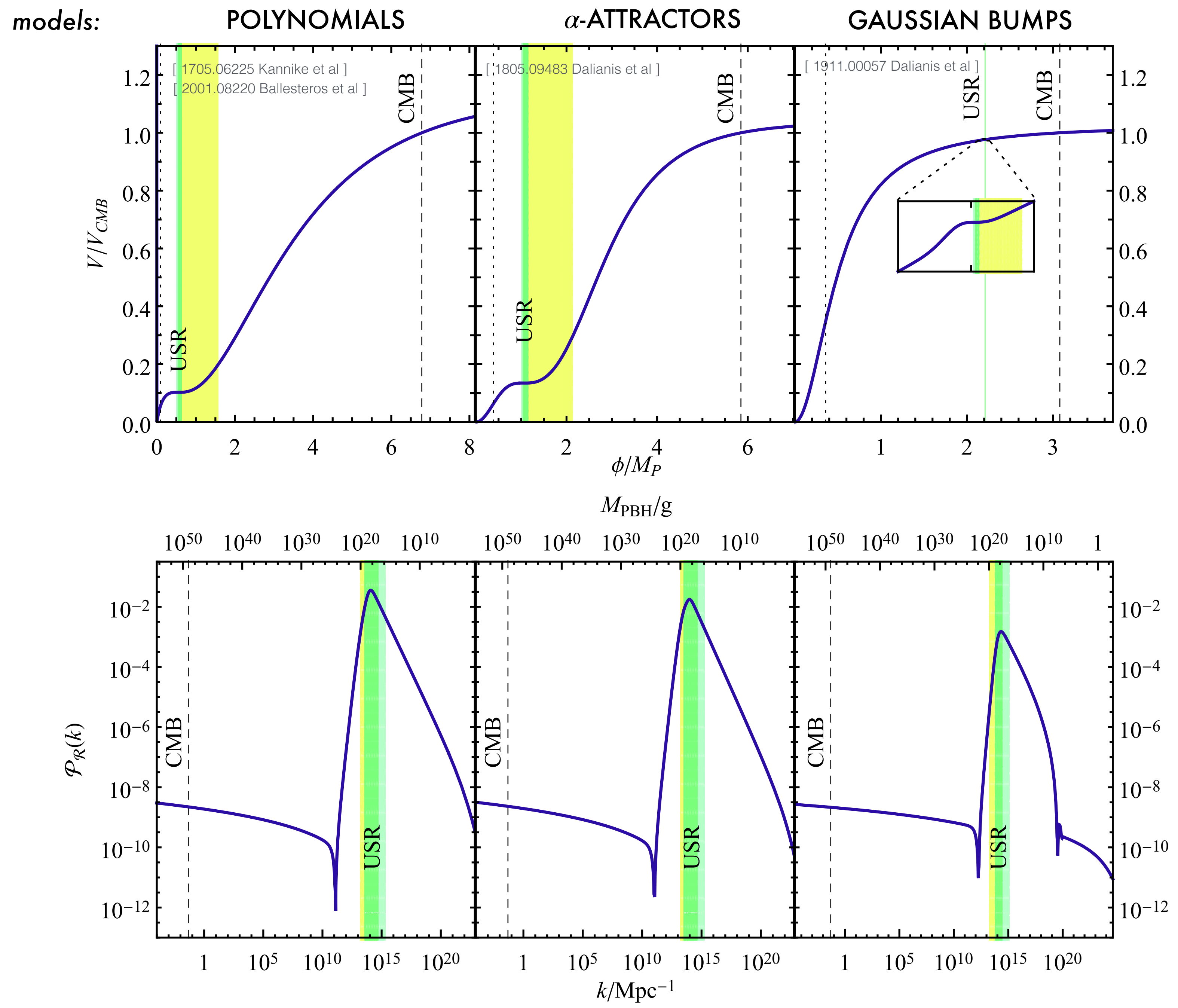
4. TRANSITION from USR to CR

5. CONSTANT-ROLL (CR)

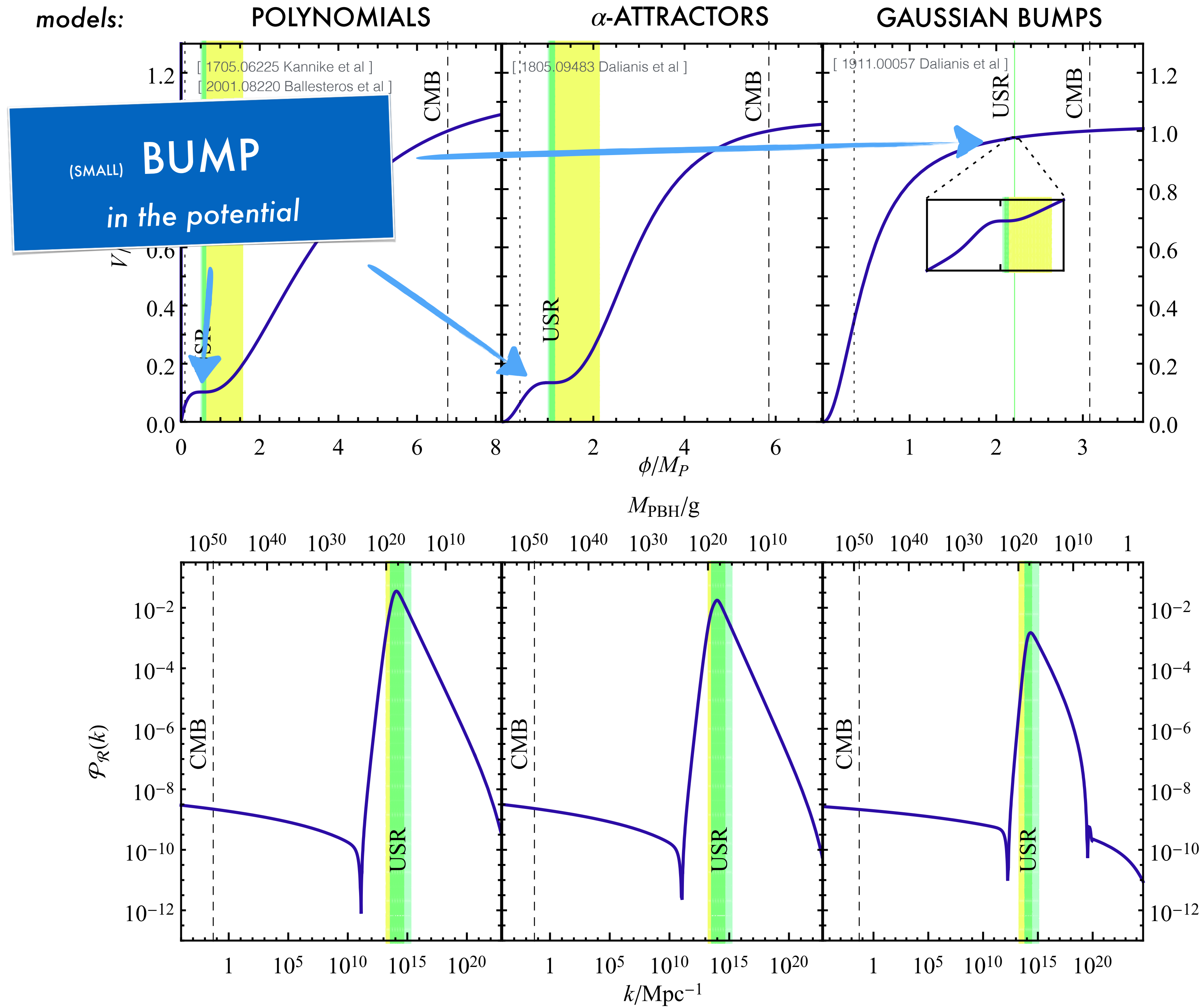
...or SR



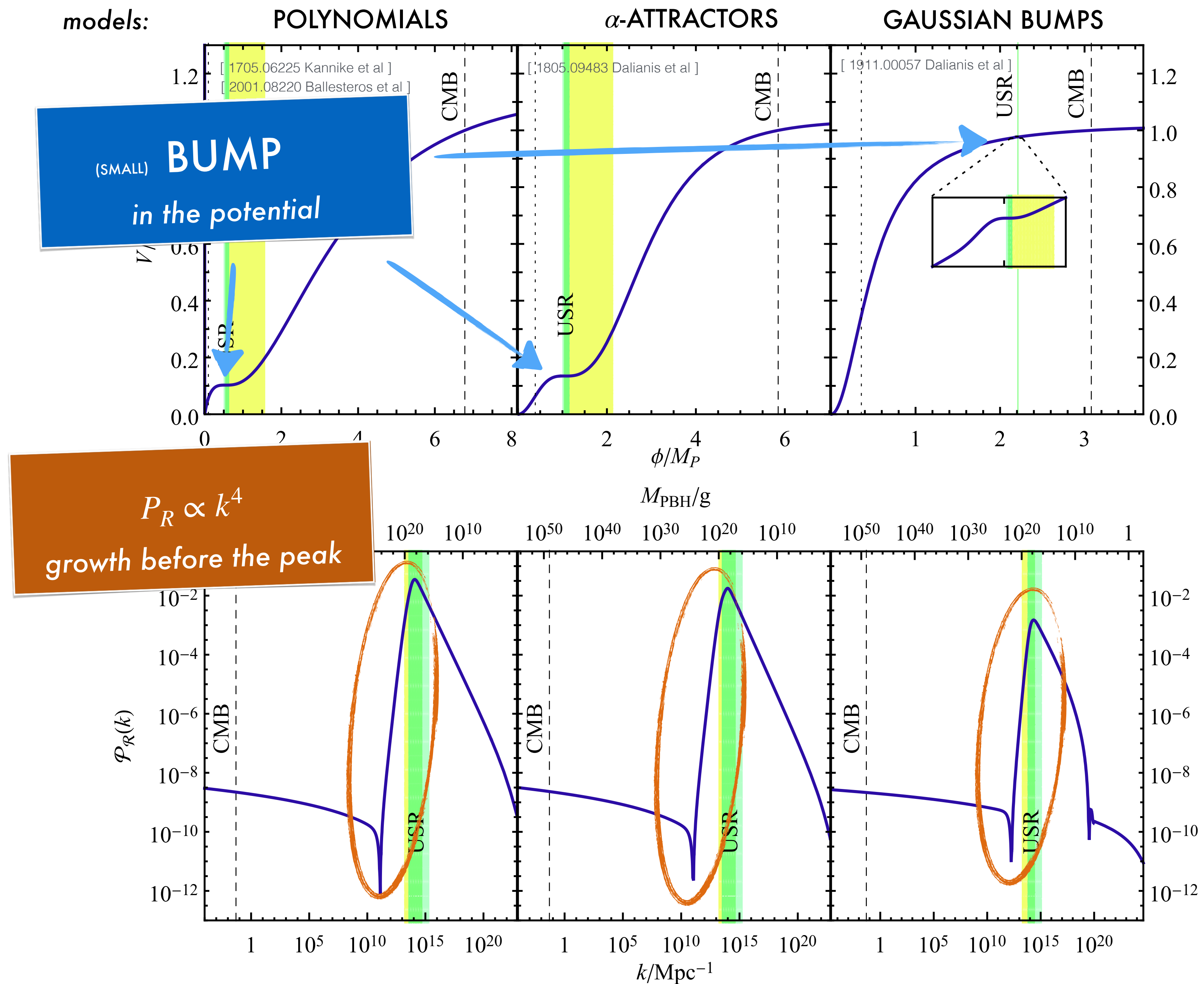
TYPICAL SPECTRA



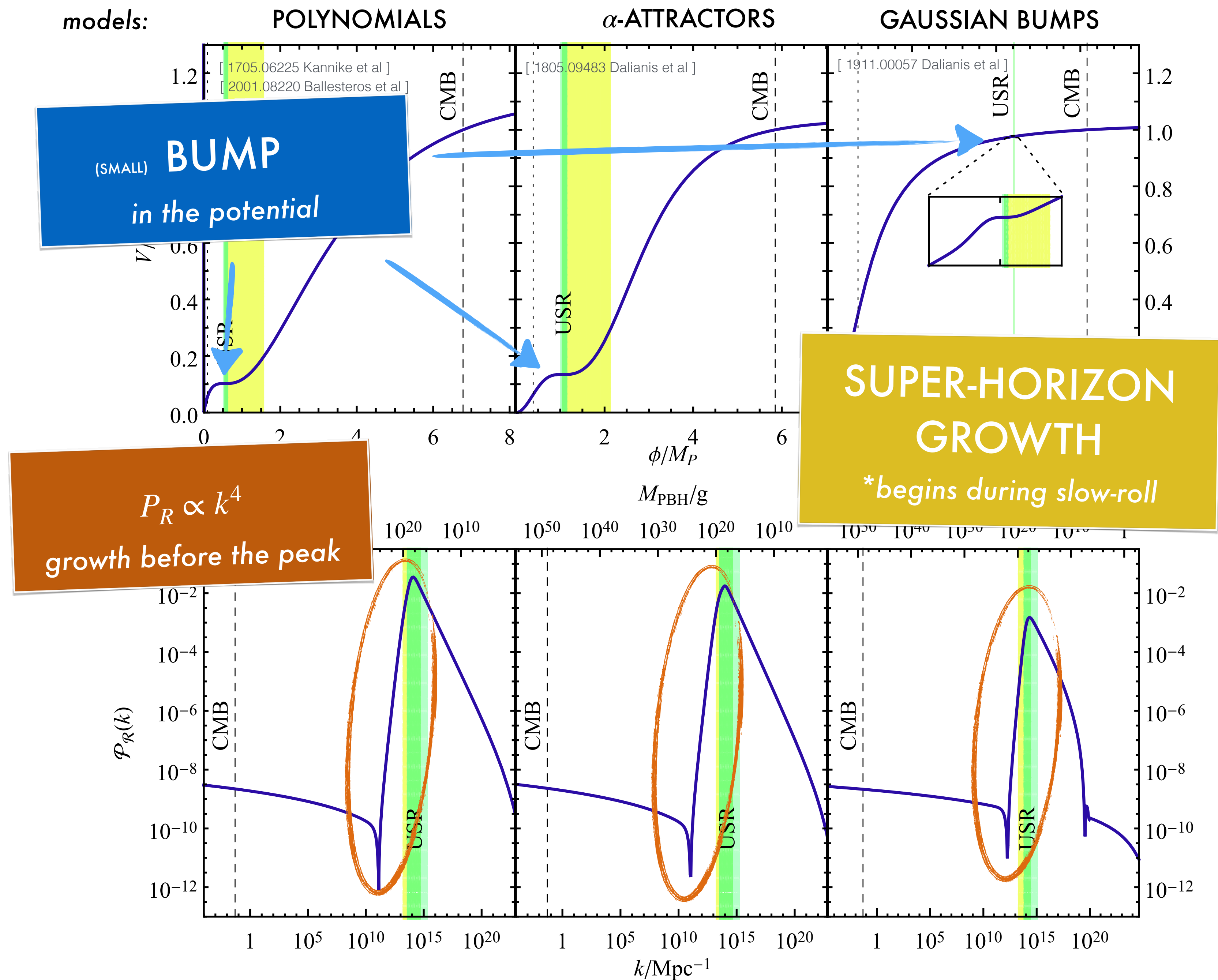
TYPICAL SPECTRA



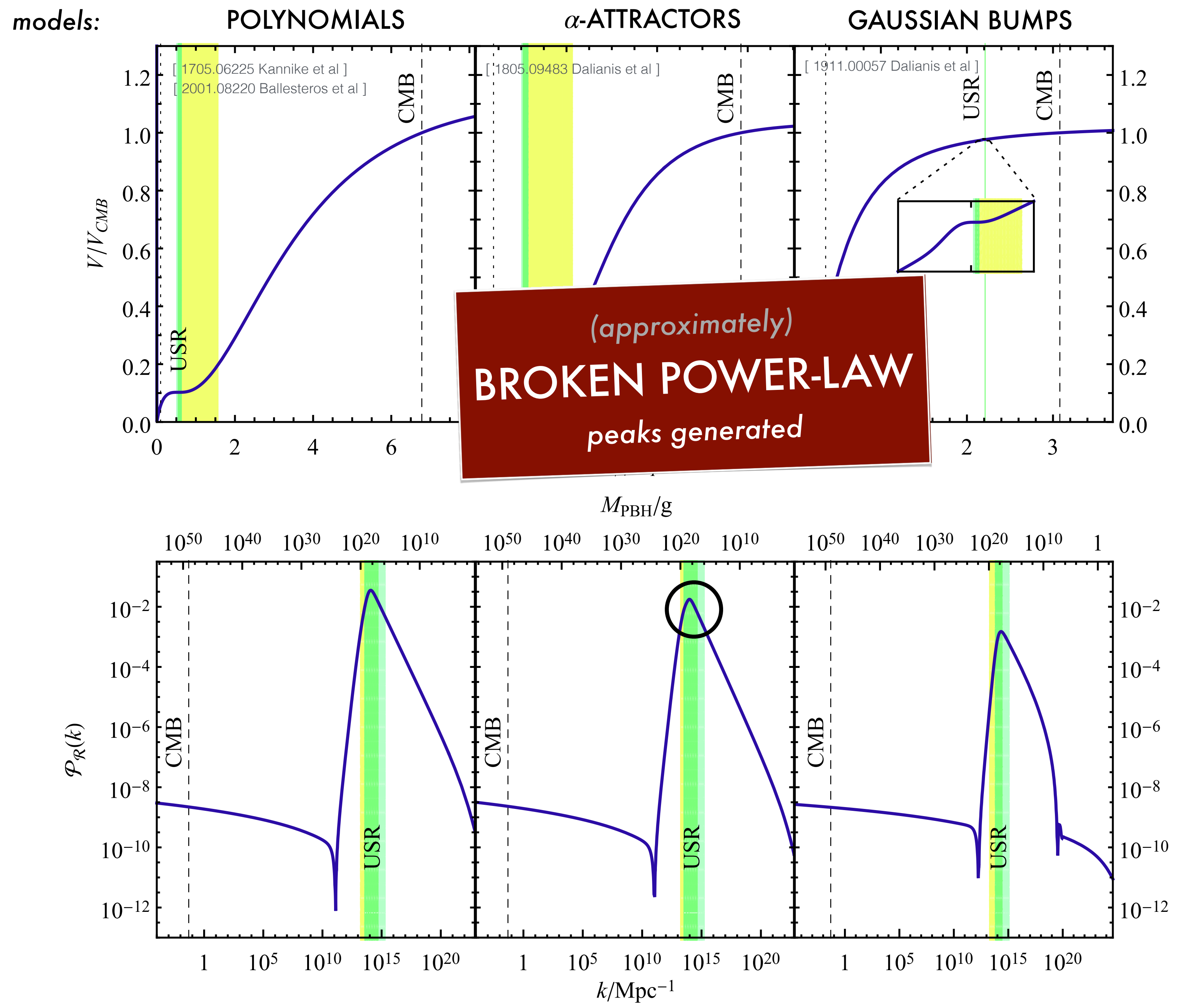
TYPICAL SPECTRA



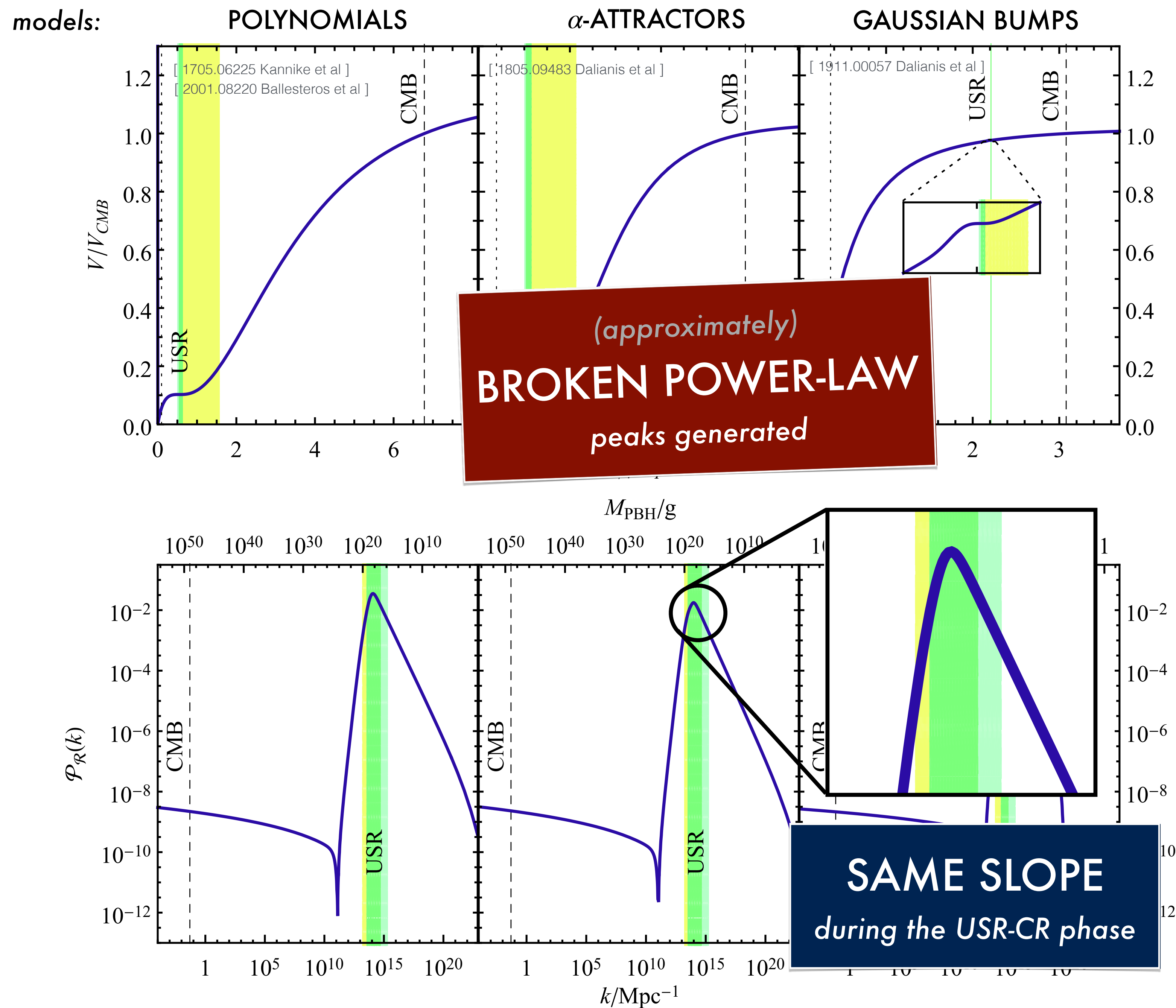
TYPICAL SPECTRA



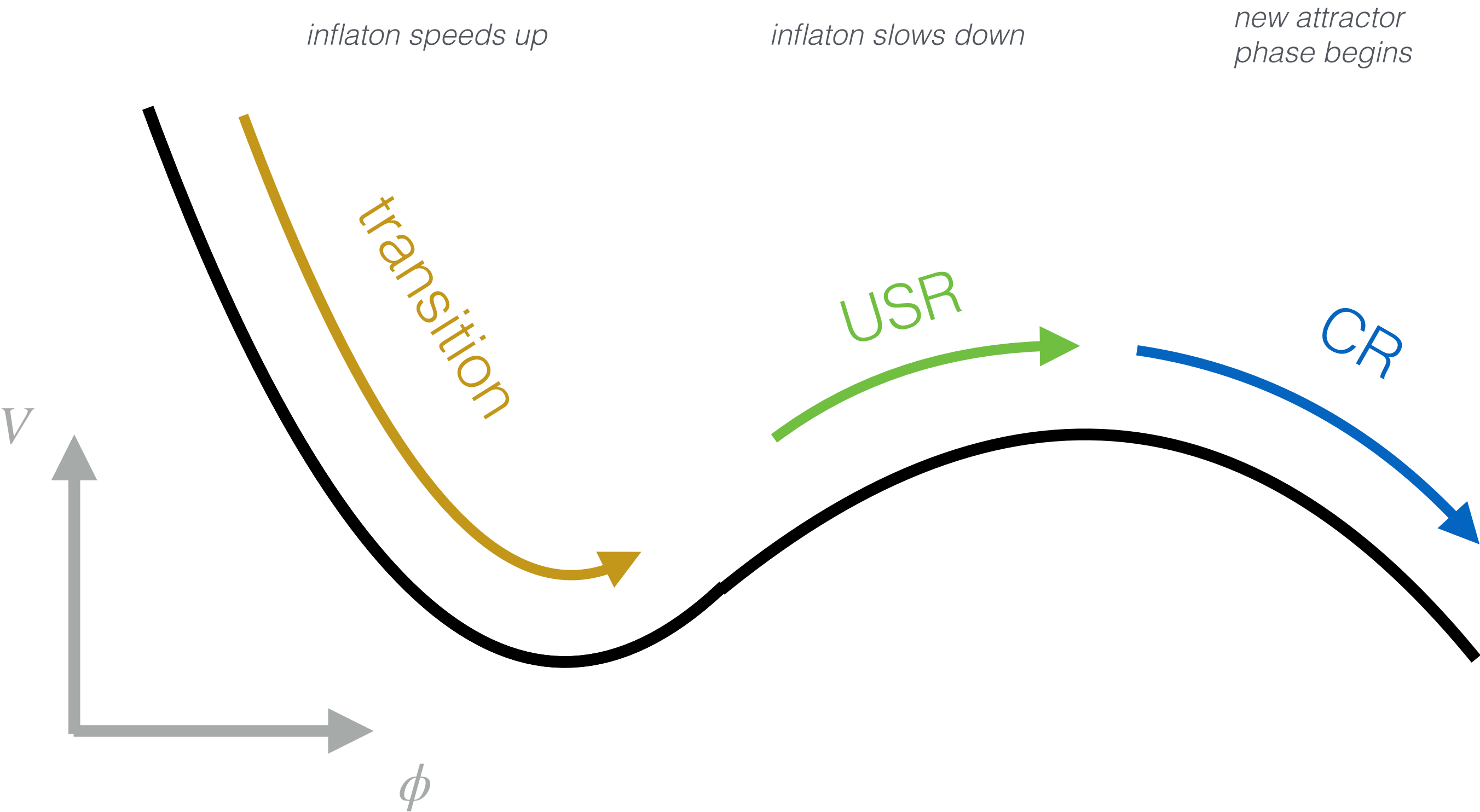
TYPICAL SPECTRA



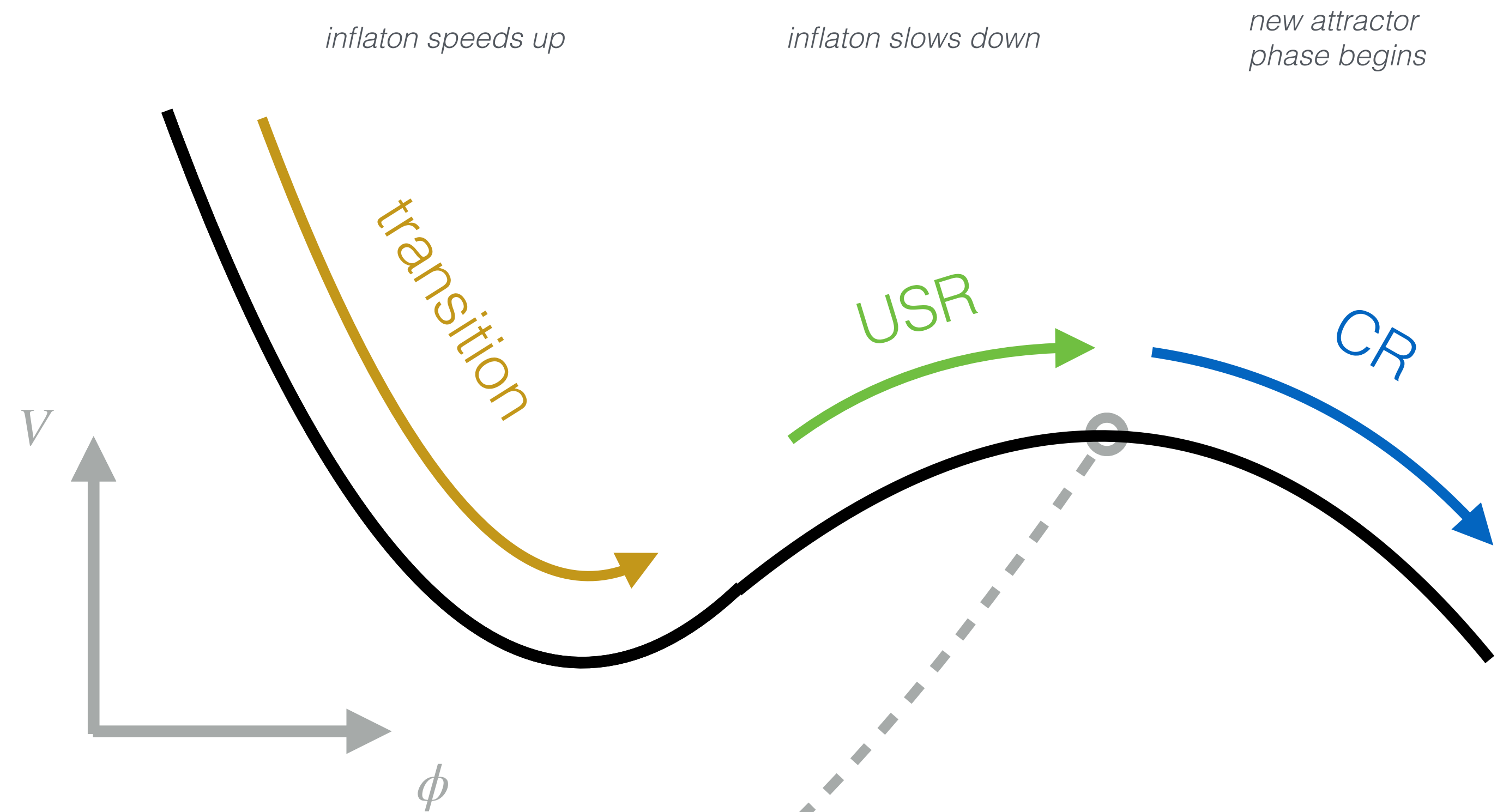
TYPICAL SPECTRA



CONSTANT- ν INFLATION AND WANDS DUALITY



CONSTANT- $\mathcal{V}$ INFLATION AND WANDS DUALITY

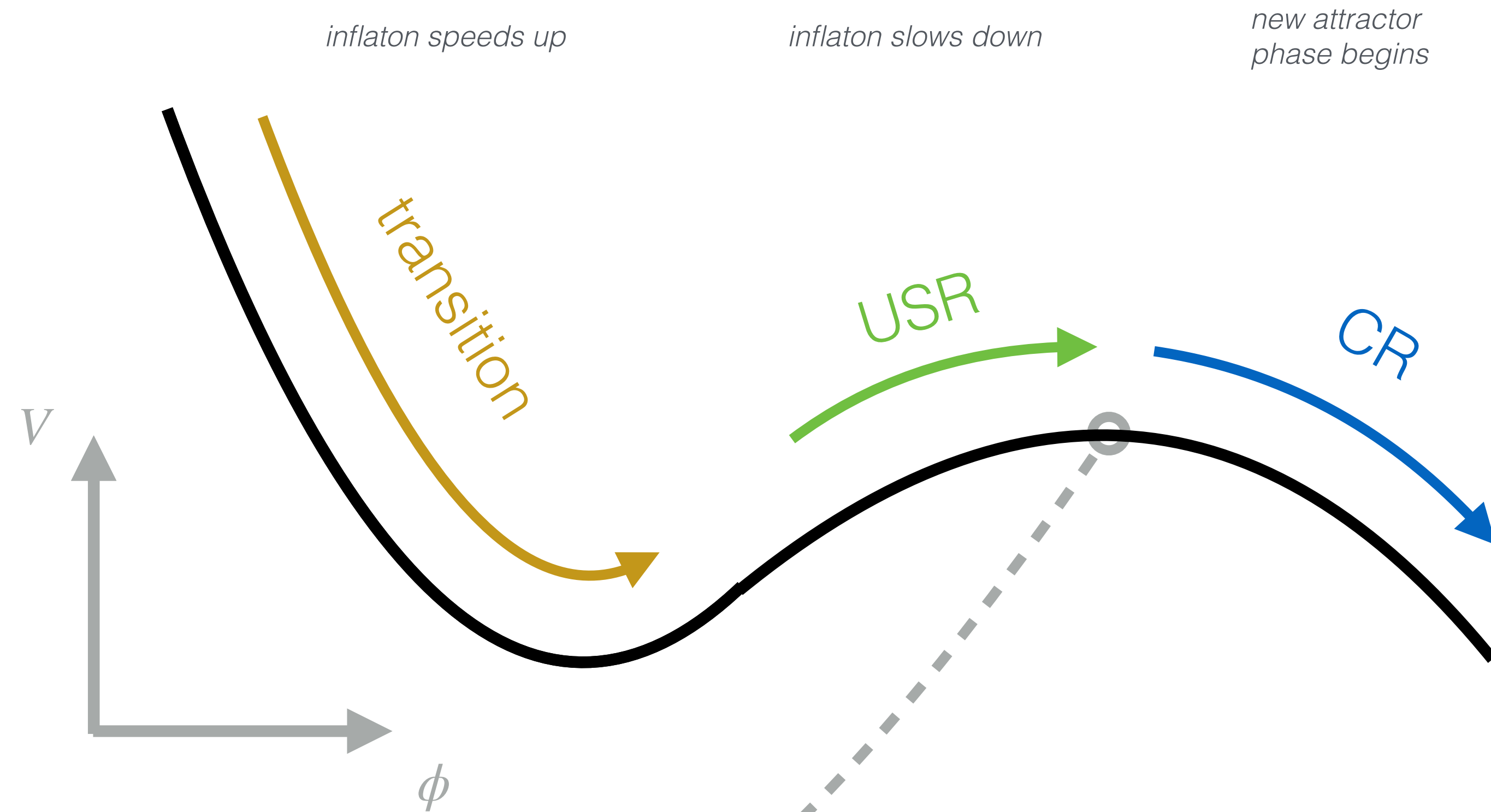


expansion around the local maximum:

$$\partial_N^2 \phi + 3\partial_N \phi \approx -\eta_{V,c} (\phi - \phi_c)$$

$$\eta_{V,c} \equiv V''(\phi_c)/V(\phi_c)$$

CONSTANT- ν INFLATION AND WANDS DUALITY



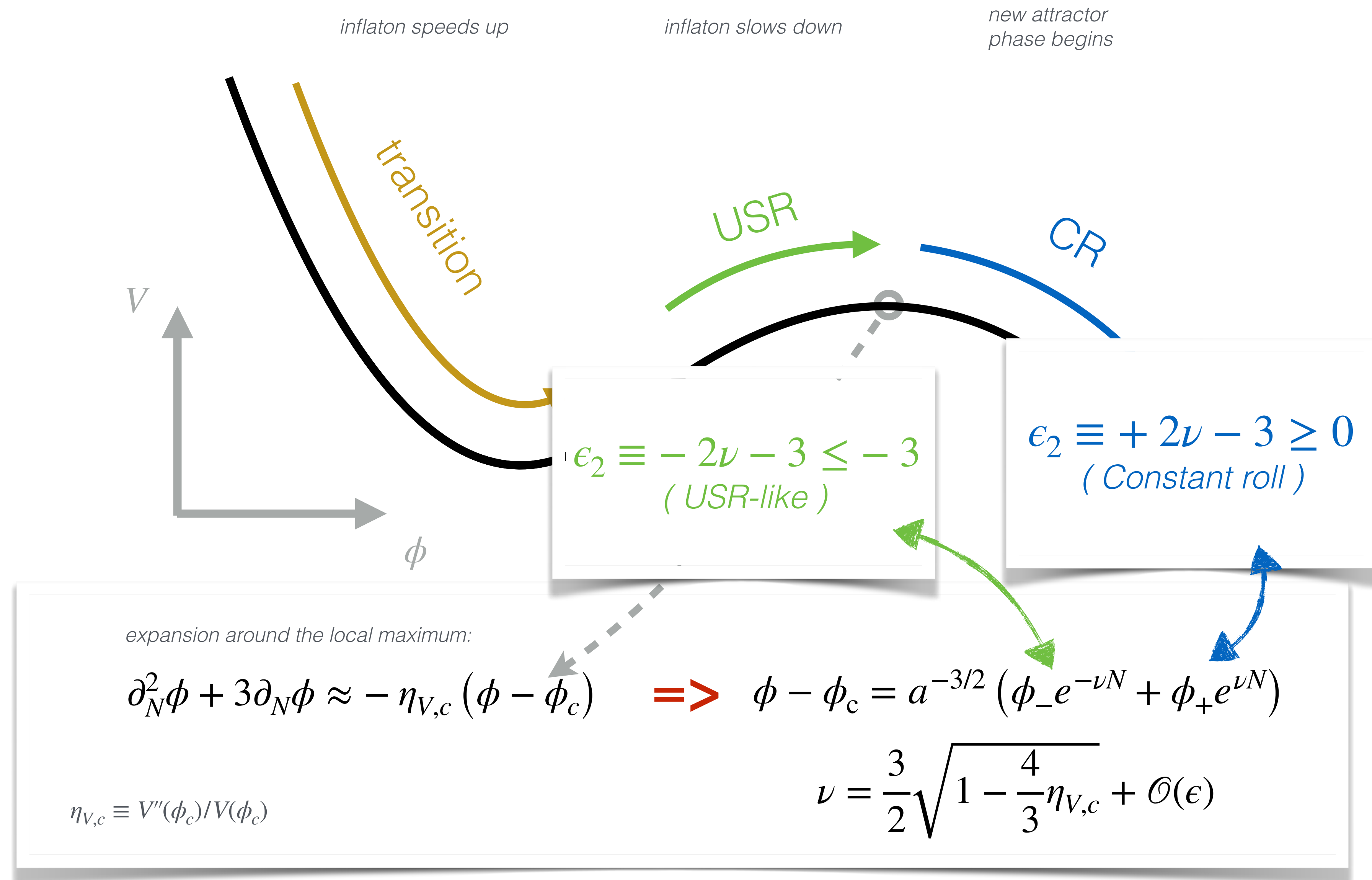
expansion around the local maximum:

$$\partial_N^2 \phi + 3\partial_N \phi \approx -\eta_{V,c} (\phi - \phi_c) \quad \Rightarrow \quad \phi - \phi_c = a^{-3/2} (\phi_- e^{-\nu N} + \phi_+ e^{\nu N})$$

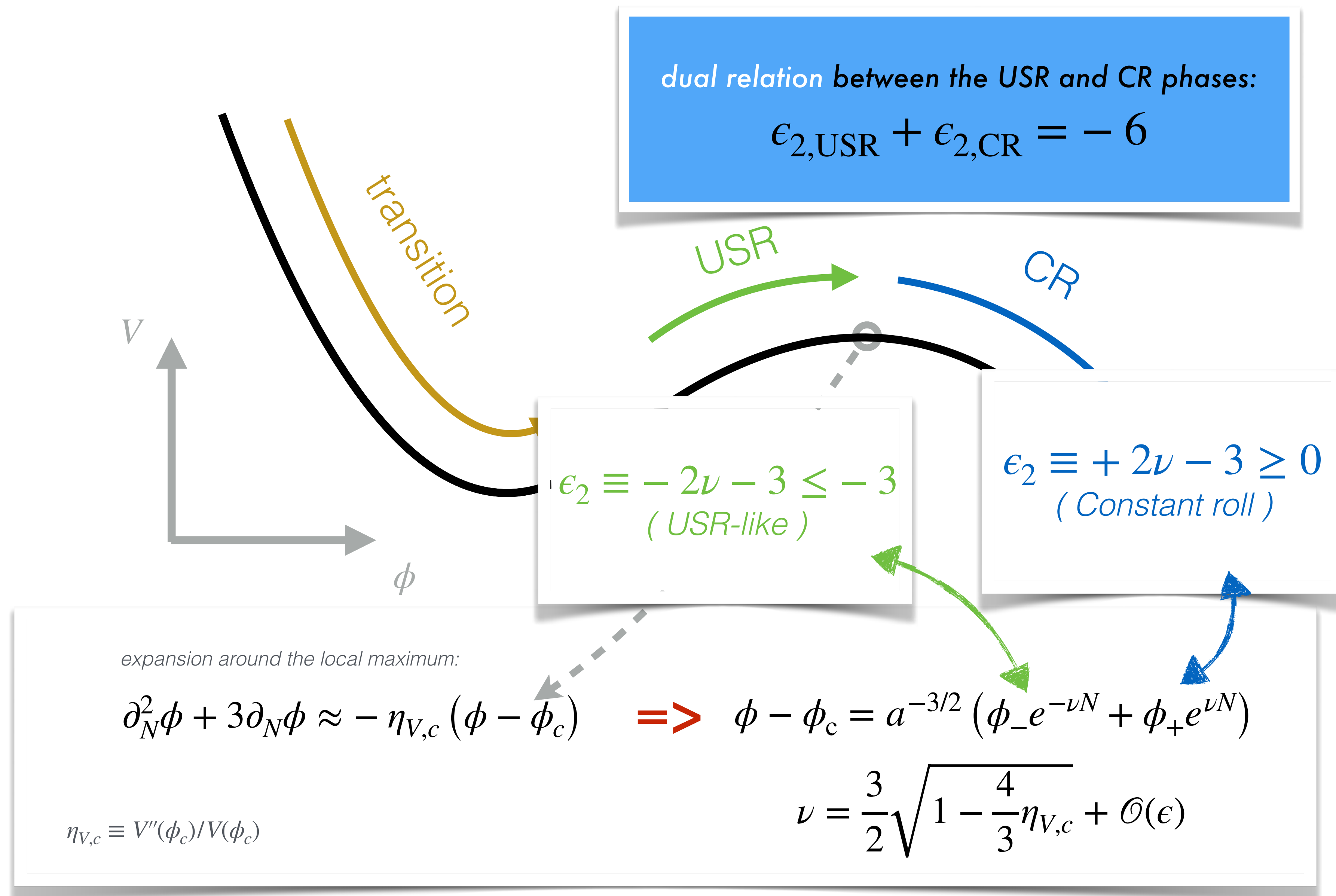
$$\eta_{V,c} \equiv V'''(\phi_c)/V(\phi_c)$$

$$\nu = \frac{3}{2} \sqrt{1 - \frac{4}{3} \eta_{V,c}} + \mathcal{O}(\epsilon)$$

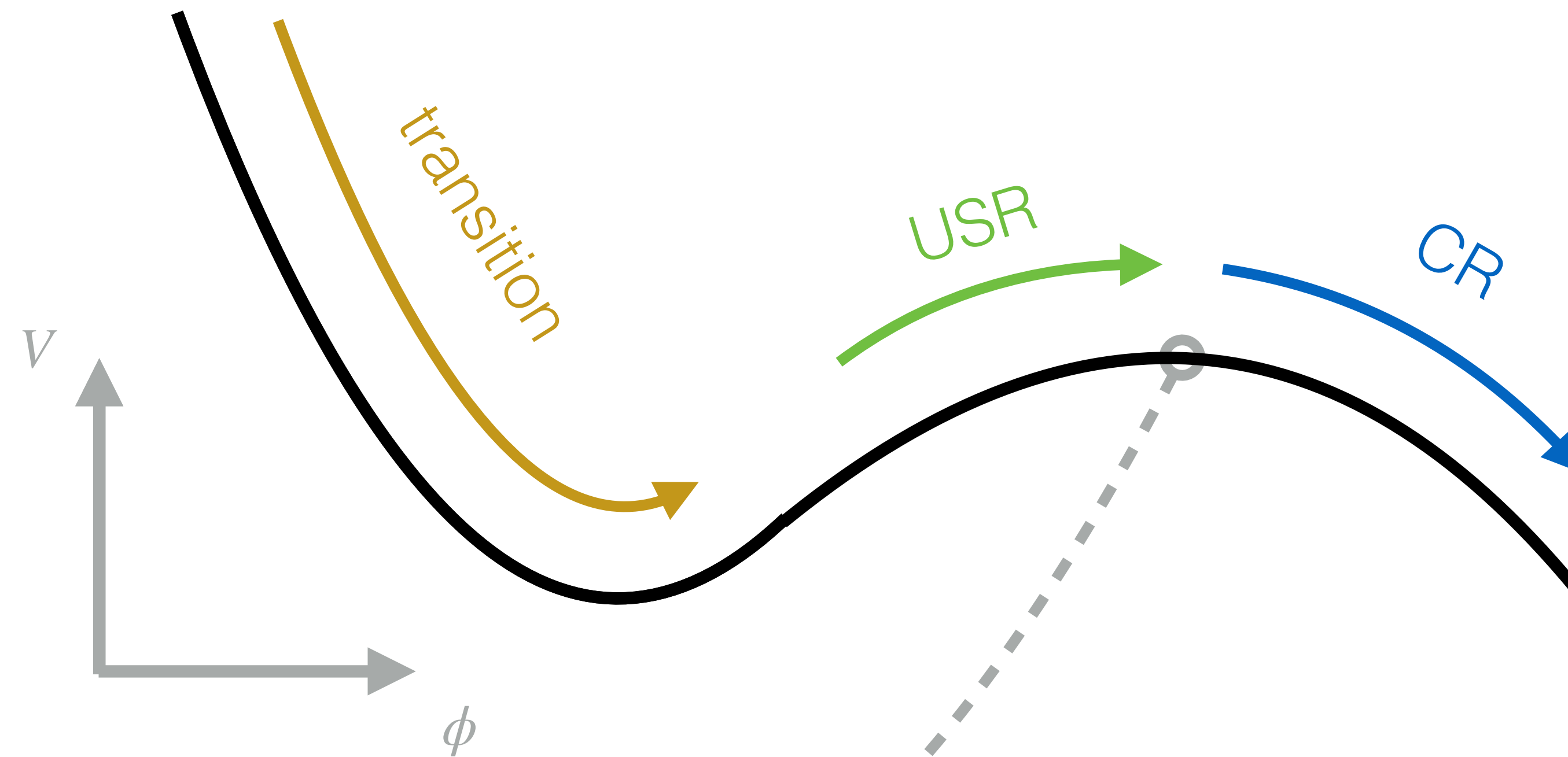
CONSTANT- ν INFLATION AND WANDS DUALITY



CONSTANT- ν INFLATION AND WANDS DUALITY



CONSTANT- ν INFLATION



expansion around the local maximum:

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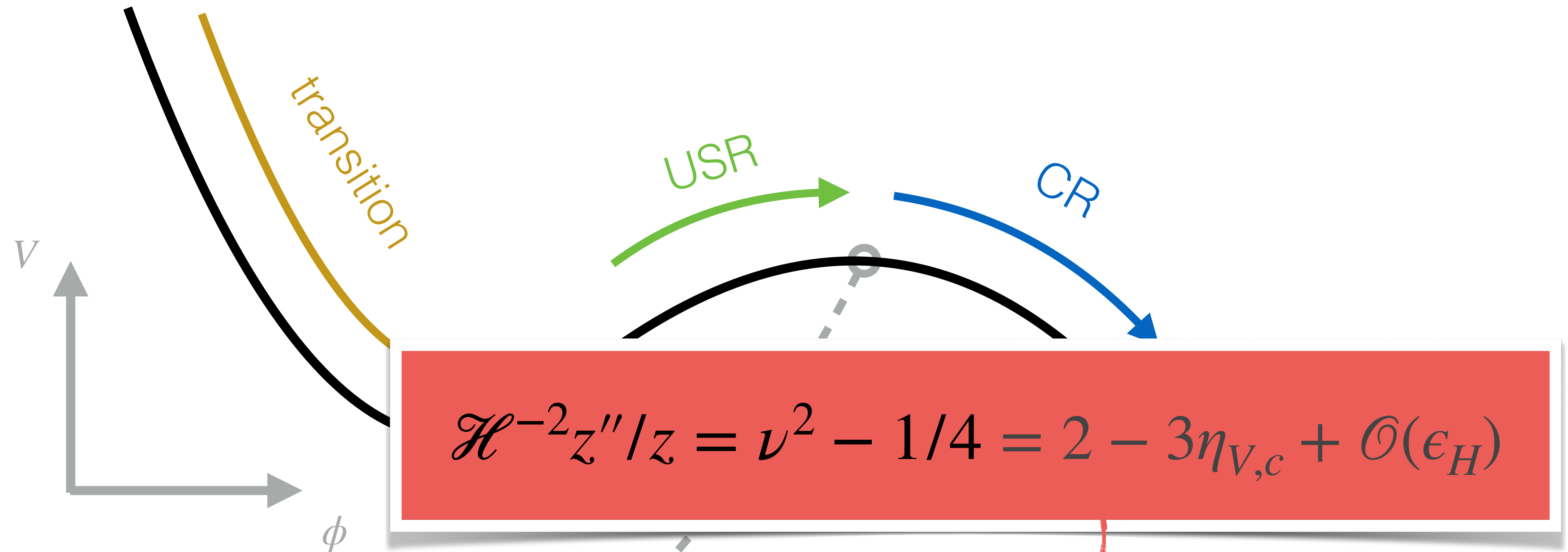
$\Rightarrow$

mode evolution:

$$u_k'' + \left(\frac{z''}{z} - k^2 \right) u_k = 0$$

$$z \equiv a\partial_N \phi = a\dot{\phi}/H$$

CONSTANT- ν INFLATION



expansion around the local maximum:

$$\partial_N^2 \phi + 3\partial_N \phi \approx -\eta_{V,c} (\phi - \phi_c)$$

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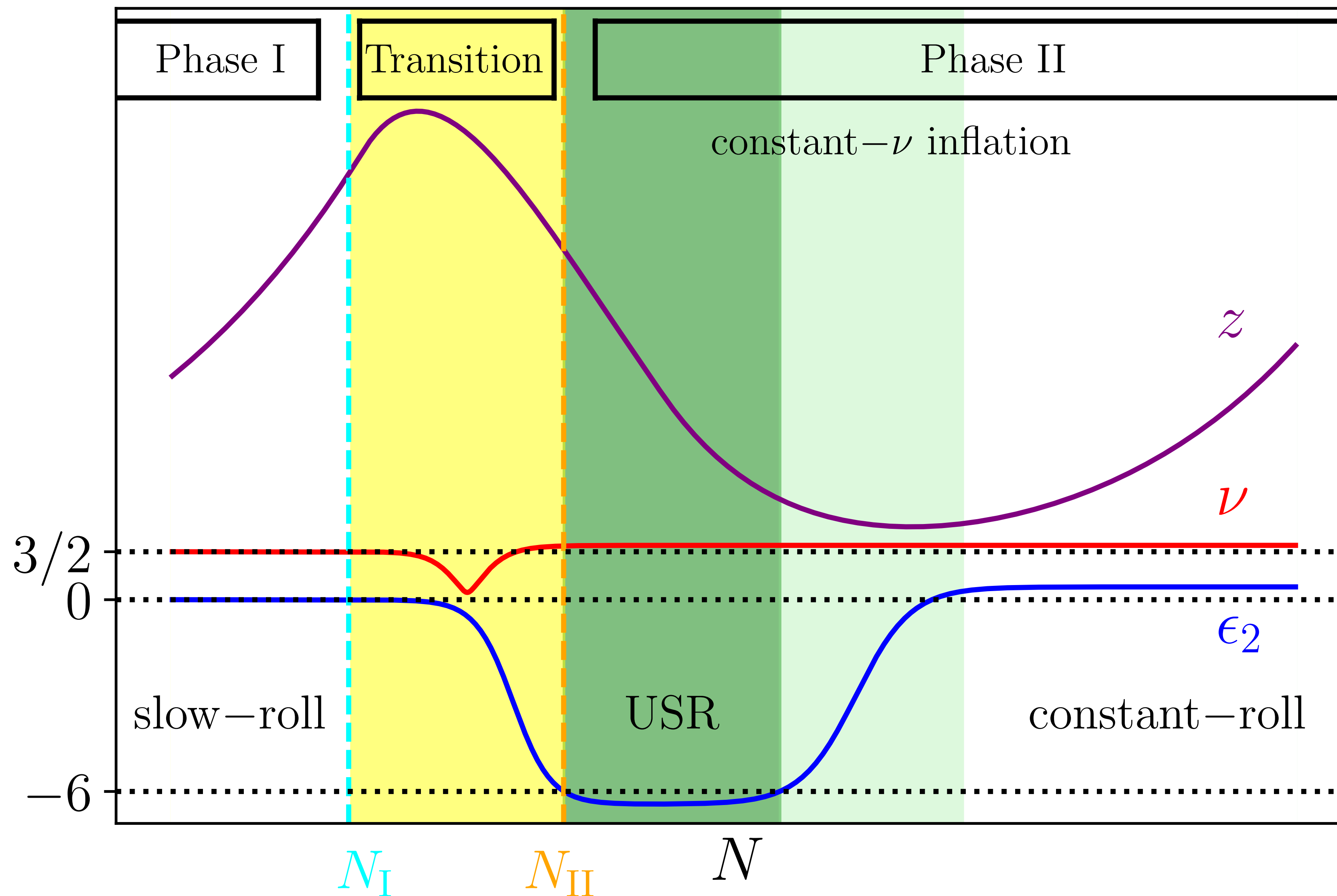
$\Rightarrow$

mode evolution:

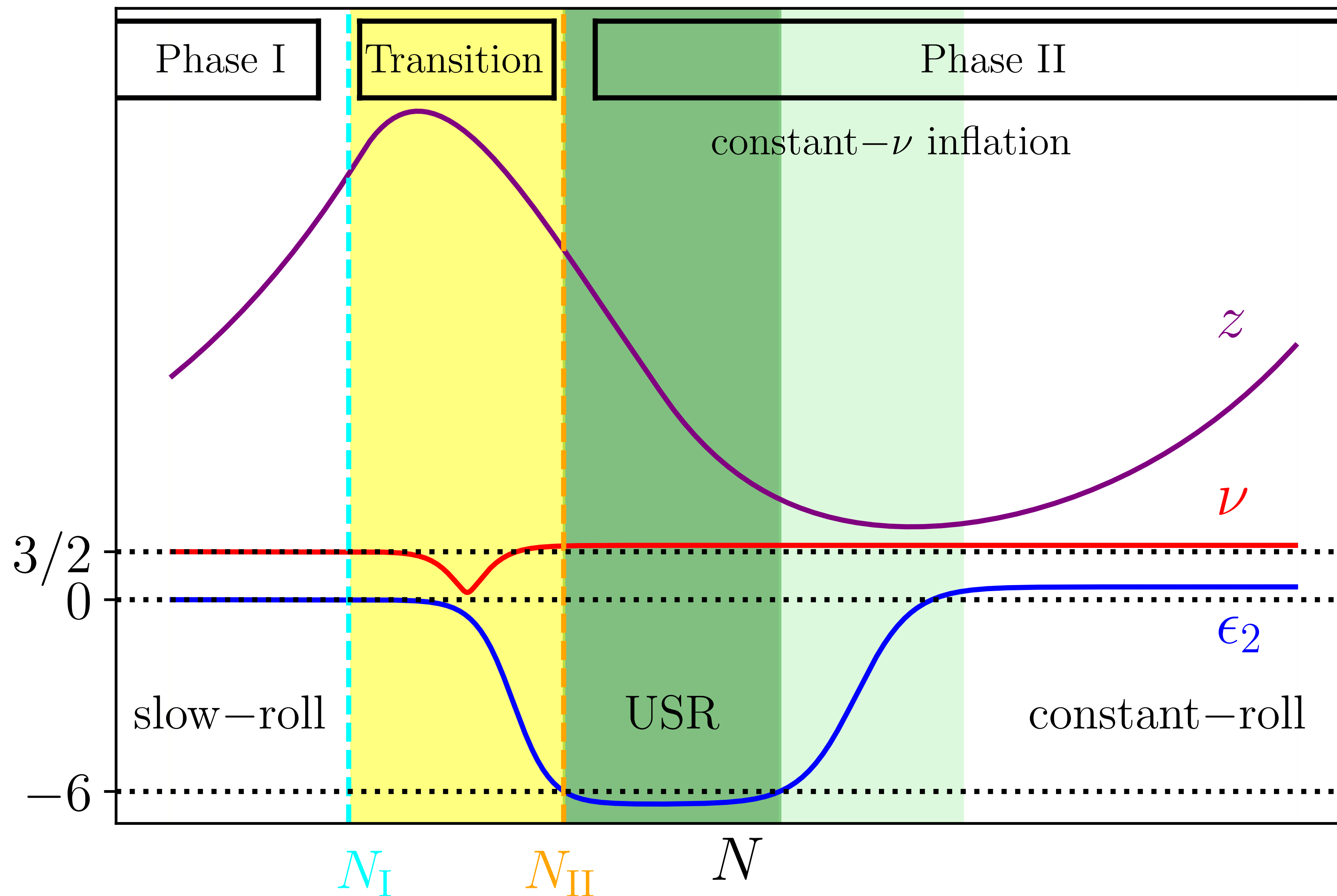
$$u_k'' + \left(\frac{z''}{z} - k^2 \right) u_k = 0$$

$z \equiv a\partial_N \phi = a\dot{\phi}/H$

mode evolution the same during USR and CR



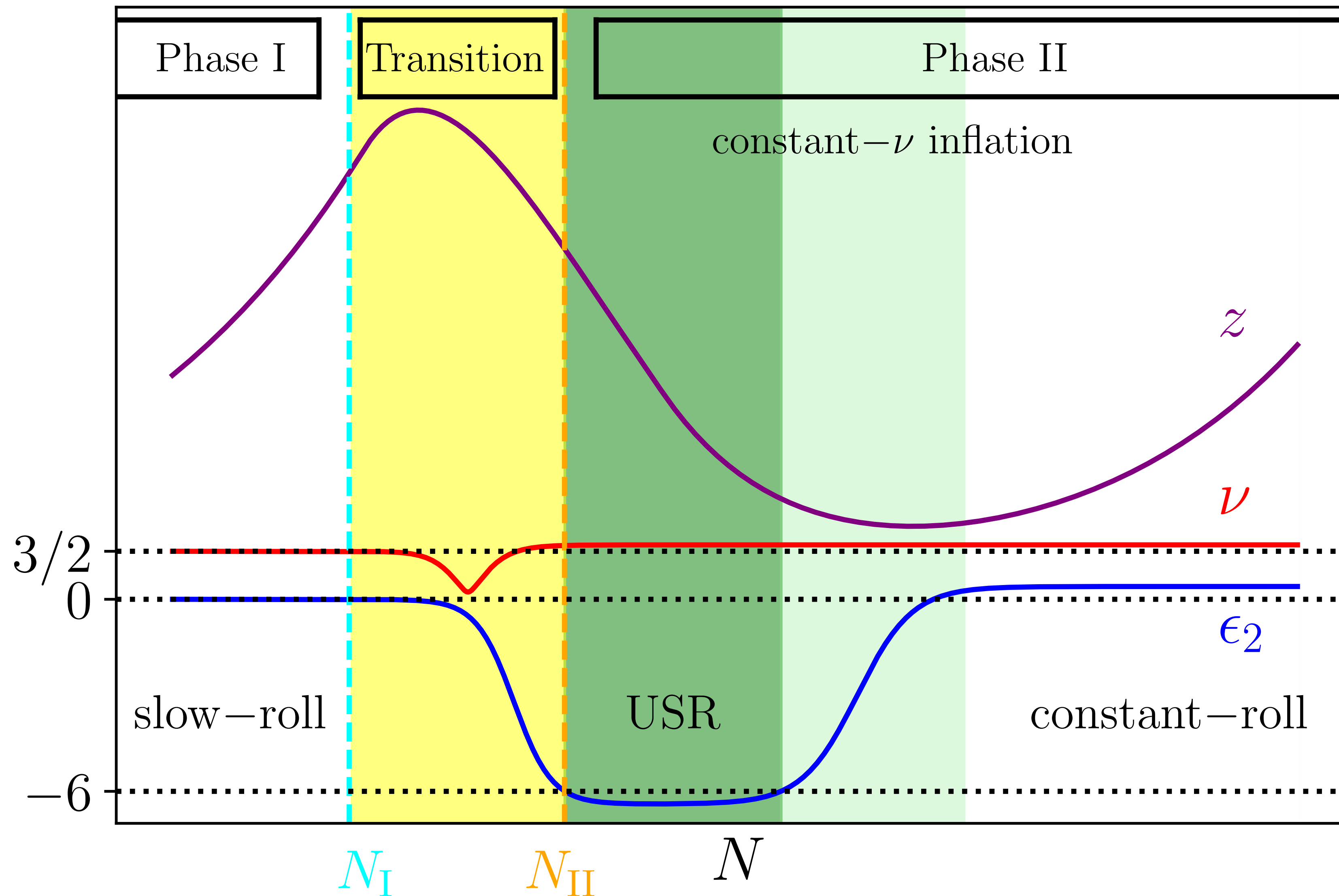
$$\epsilon_1 \equiv \partial_N \phi^2 / 2, \quad \epsilon_2 \equiv \partial_N \epsilon_1 / \epsilon_1$$



*dual relation between the USR
and CR phases:*

$$\epsilon_{2,\text{USR}} + \epsilon_{2,\text{CR}} = -6$$

$$\epsilon_1 \equiv \partial_N \phi^2 / 2, \quad \epsilon_2 \equiv \partial_N \epsilon_1 / \epsilon_1$$



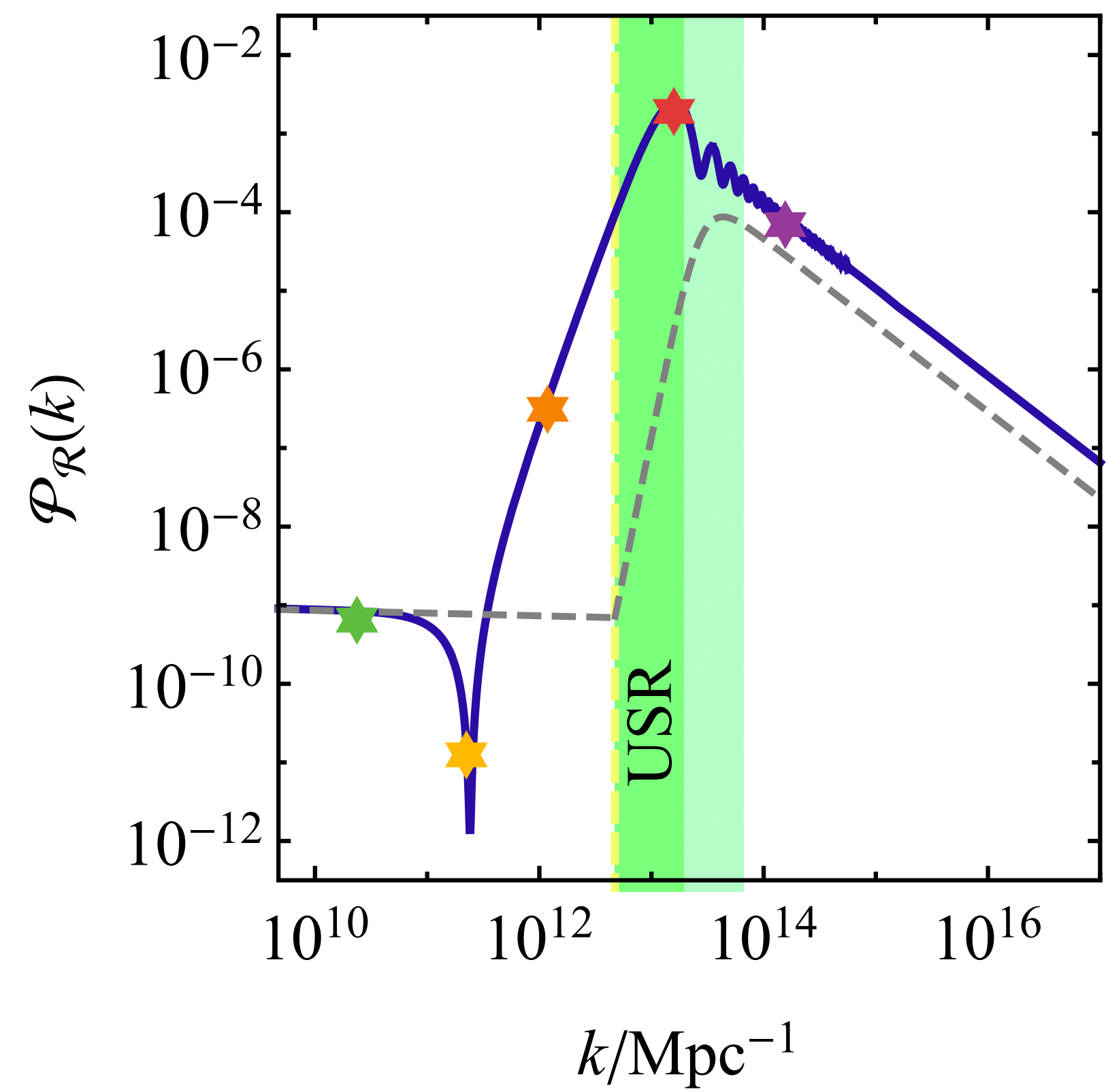
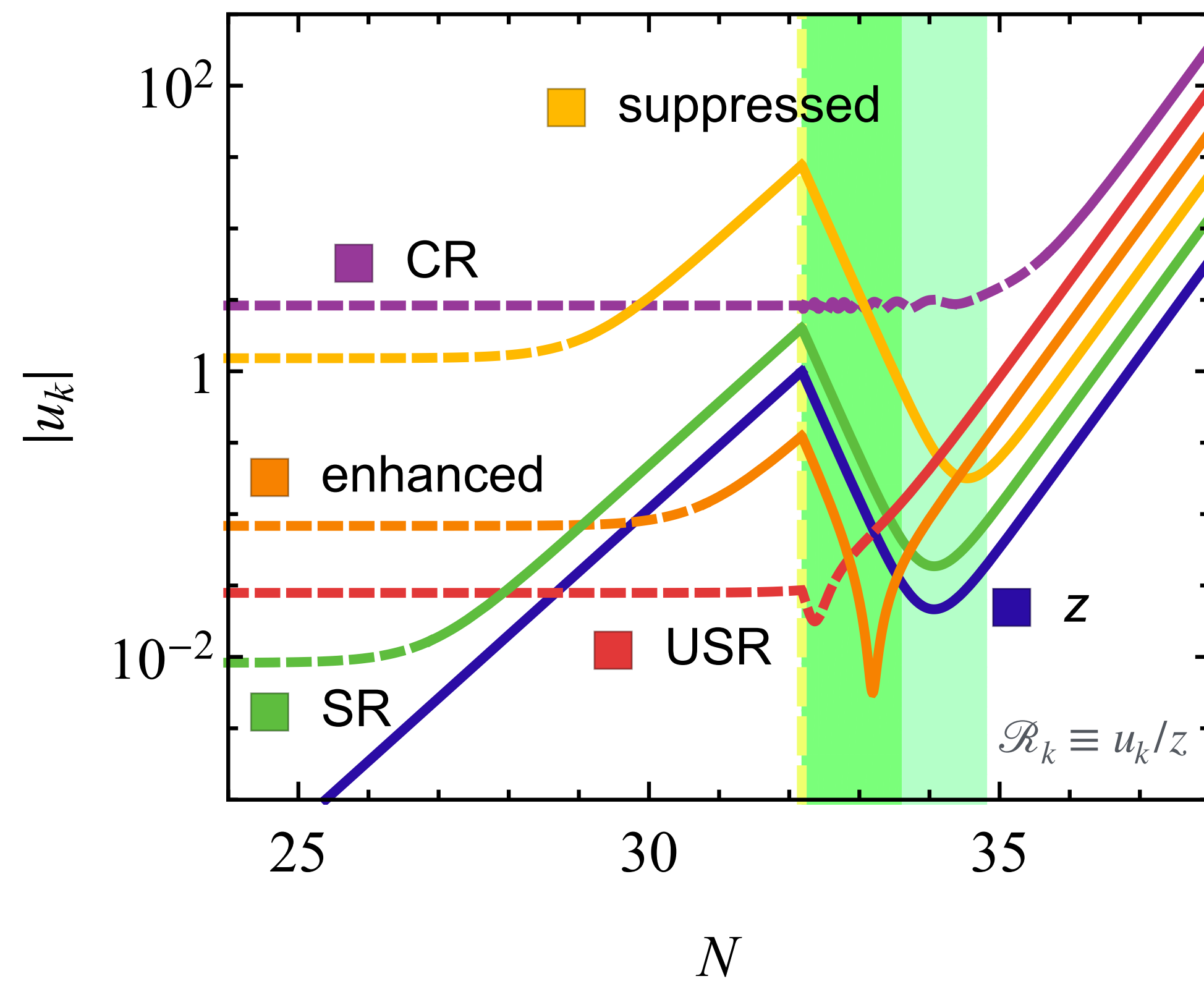
$$\nu^2 = \frac{9}{4} - \epsilon_1 + \frac{3}{2}\epsilon_2 - \frac{1}{2}\epsilon_1\epsilon_2 + \frac{1}{4}\epsilon_2^2 + \frac{1}{2}\epsilon_2\epsilon_3 = \text{const.}$$

dual relation between the USR and CR phases:

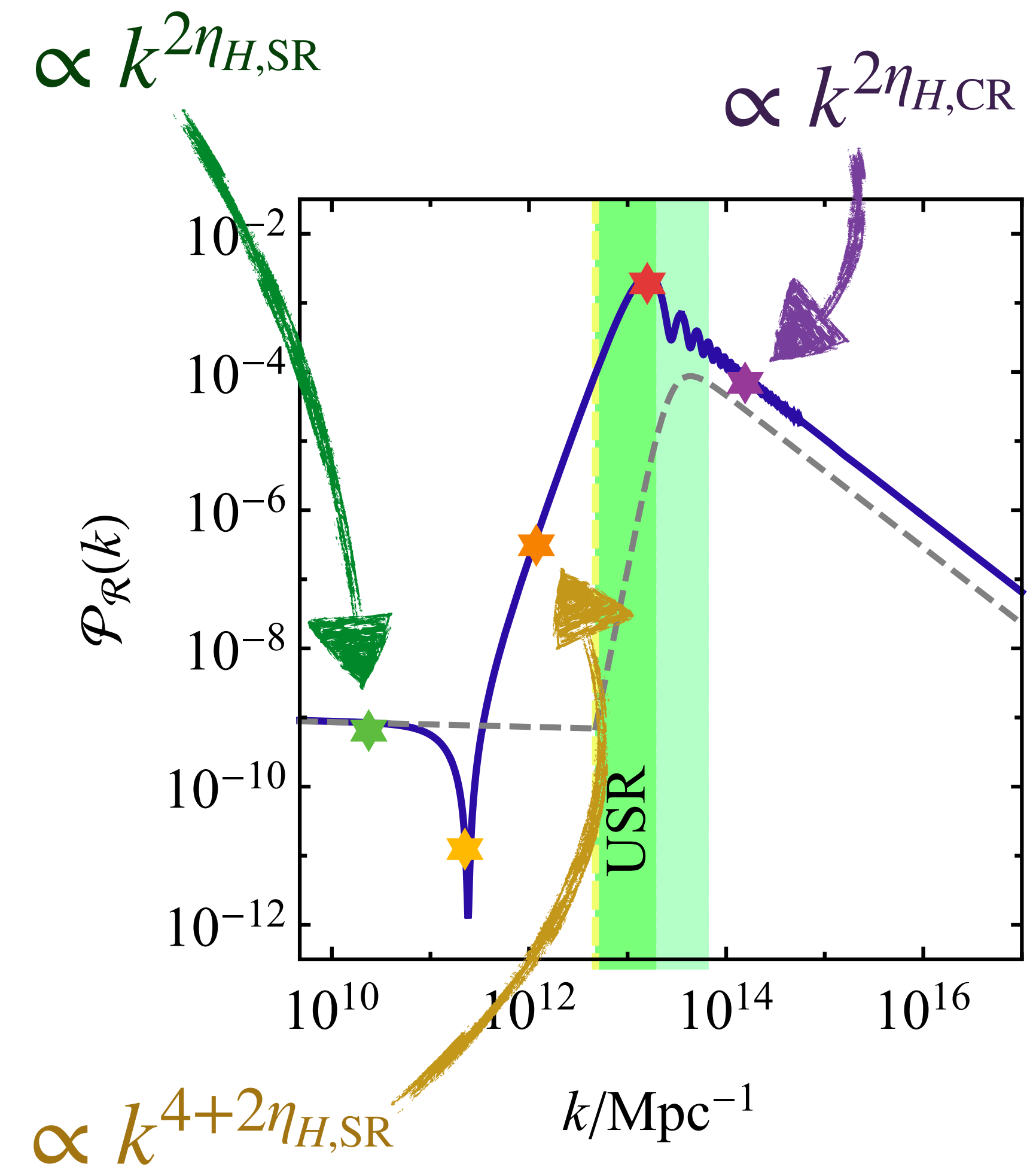
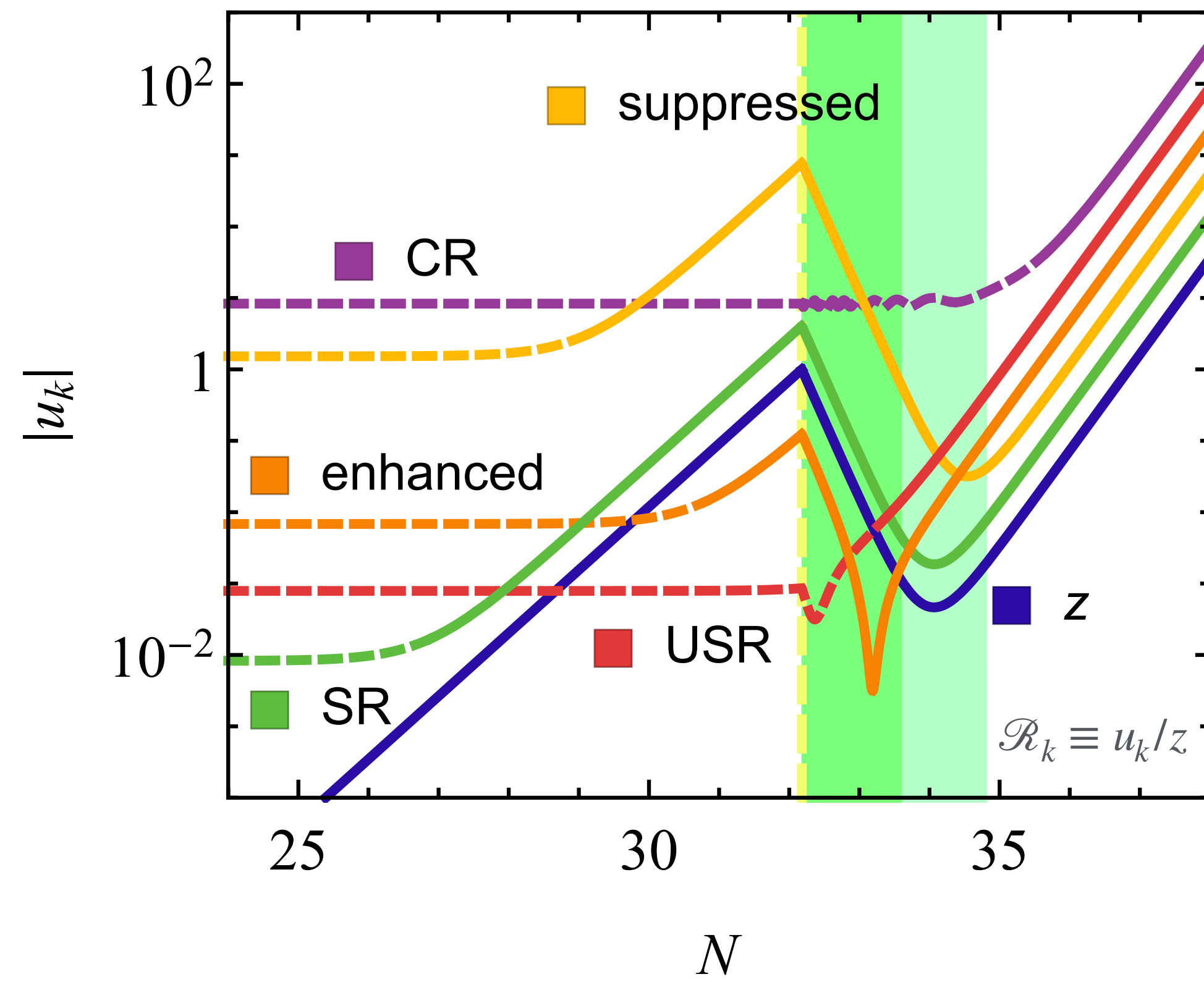
$$\epsilon_{2,\text{USR}} + \epsilon_{2,\text{CR}} = -6$$

$$\epsilon_1 \equiv \partial_N \phi^2 / 2, \quad \epsilon_2 \equiv \partial_N \epsilon_1 / \epsilon_1$$

MODE EVOLUTION



MODE EVOLUTION



INFLATIONARY TIMELINE

1. SLOW-ROLL (SR)

CMB

2. TRANSITION from SR to USR

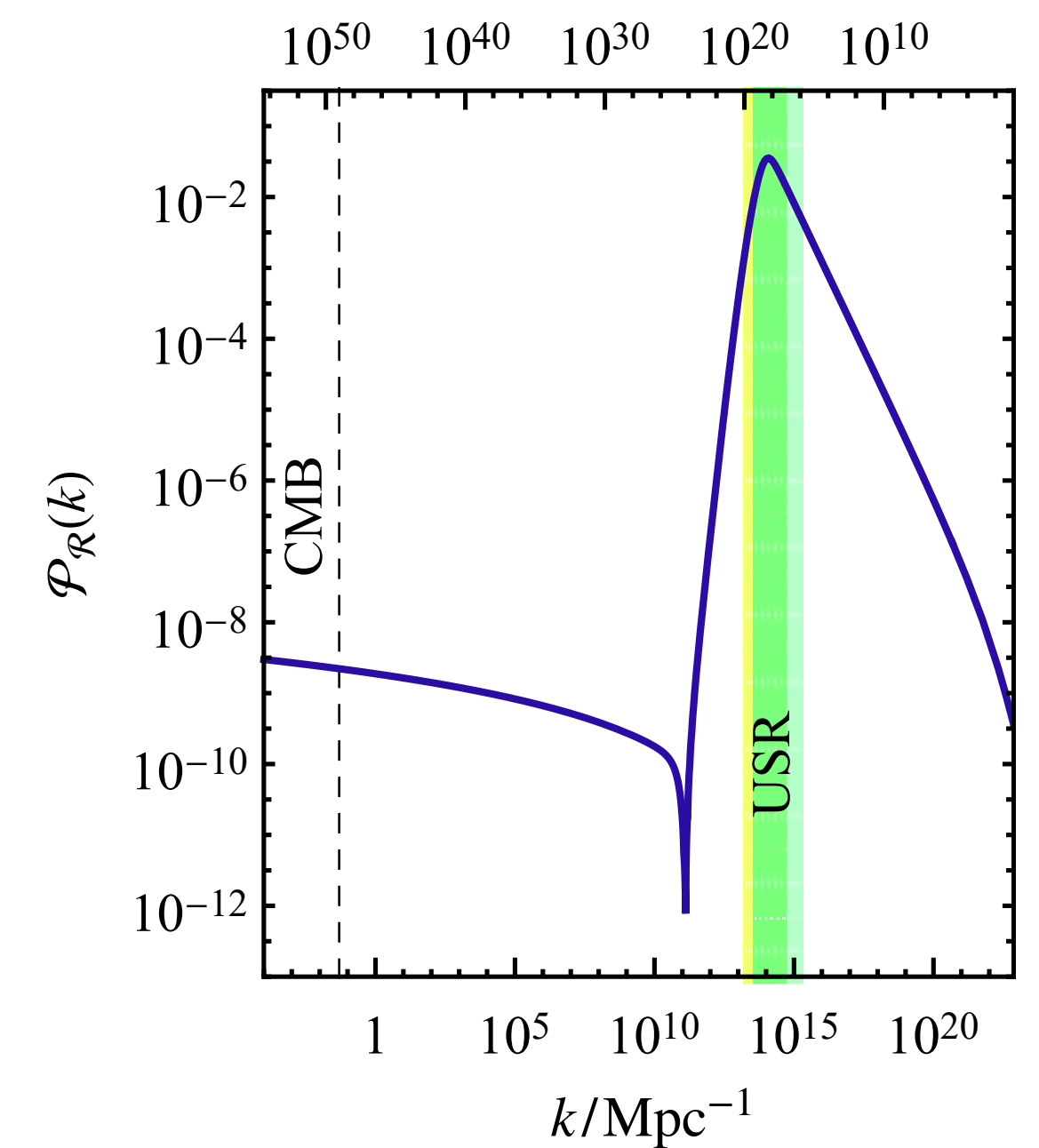
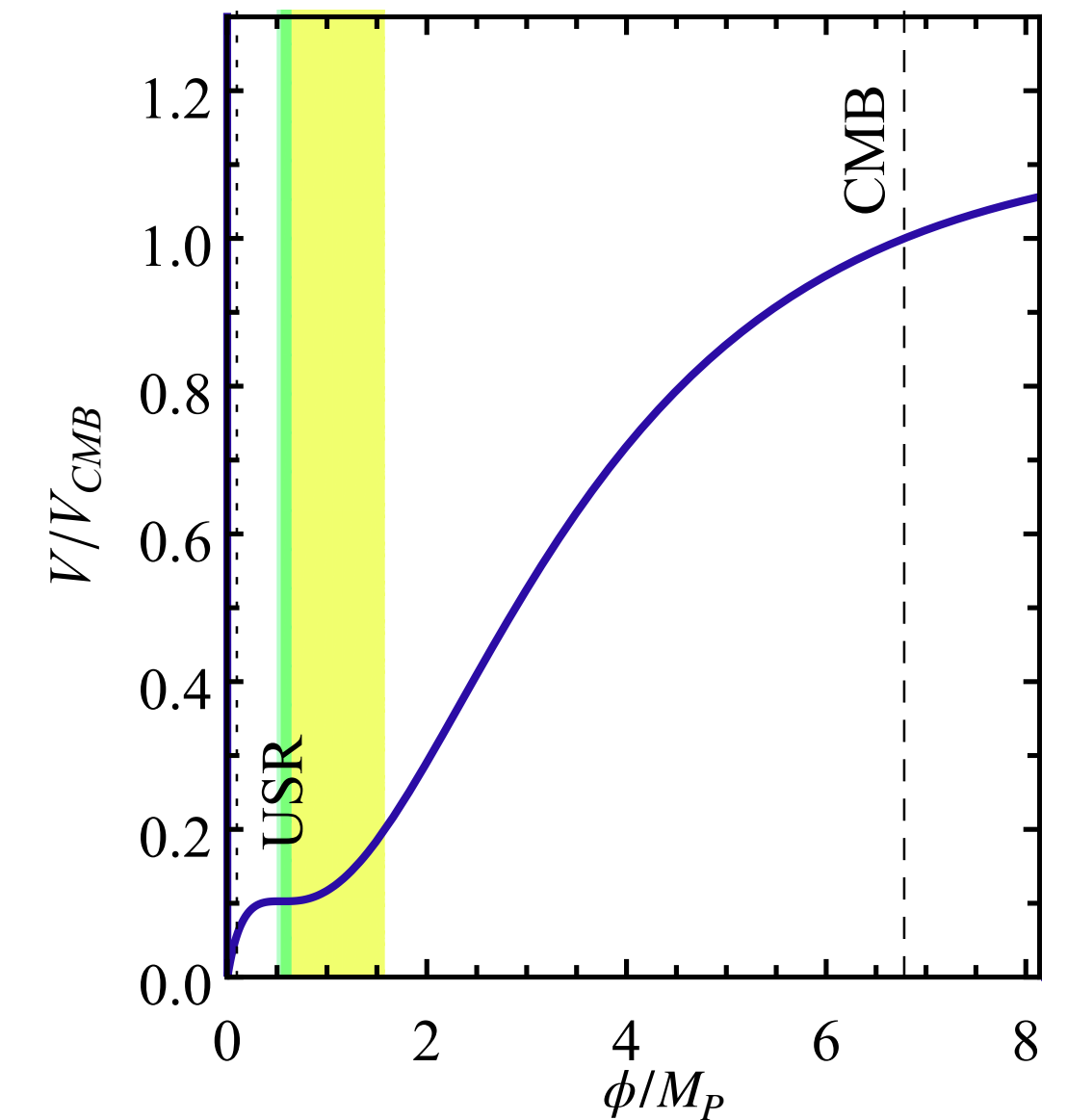
PEAK SHAPE

3. ULTRA-SLOW-ROLL (USR)

4. TRANSITION from USR to CR

5. CONSTANT-ROLL (CR)

CONSTANT - ν



INFLATIONARY TIMELINE

1. SLOW-ROLL (SR)

CMB

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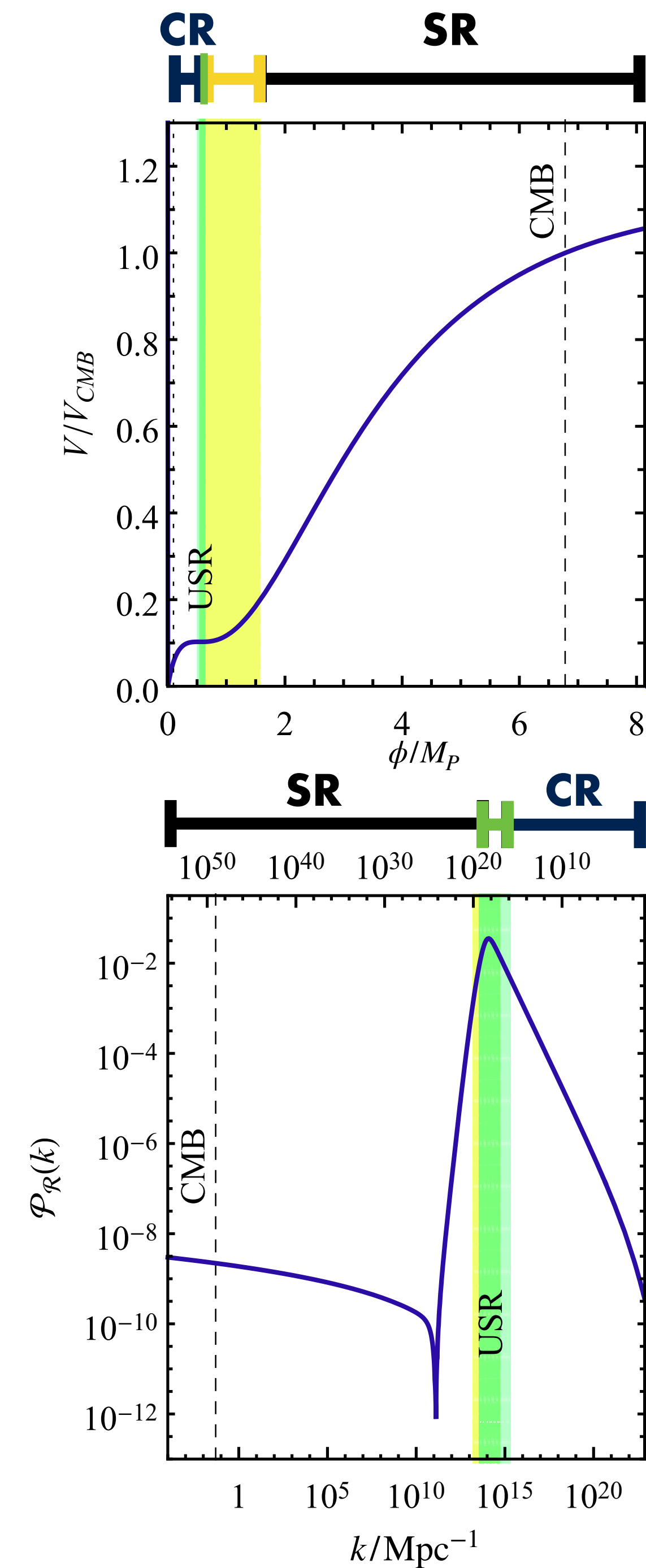
PEAK SHAPE

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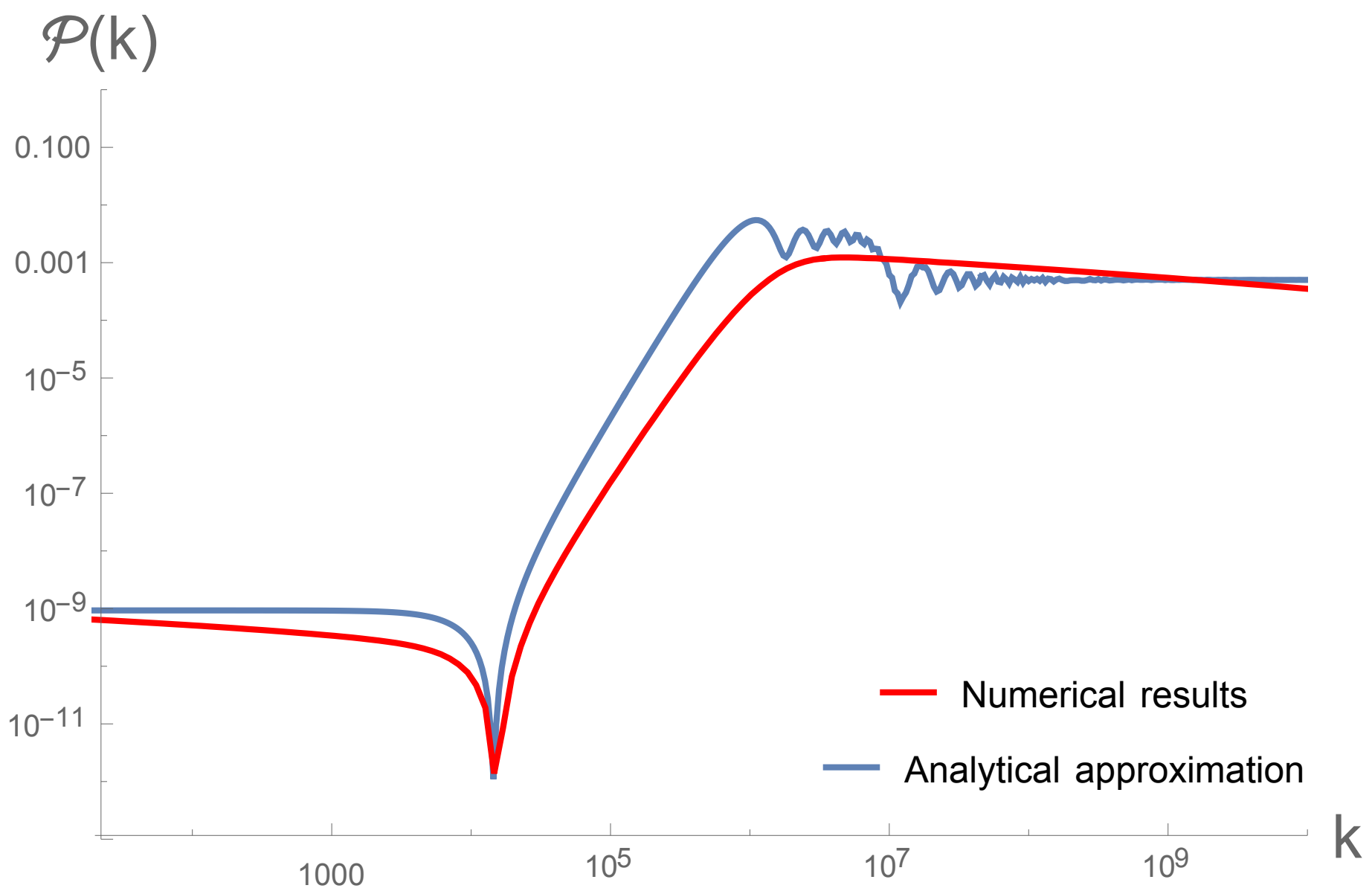
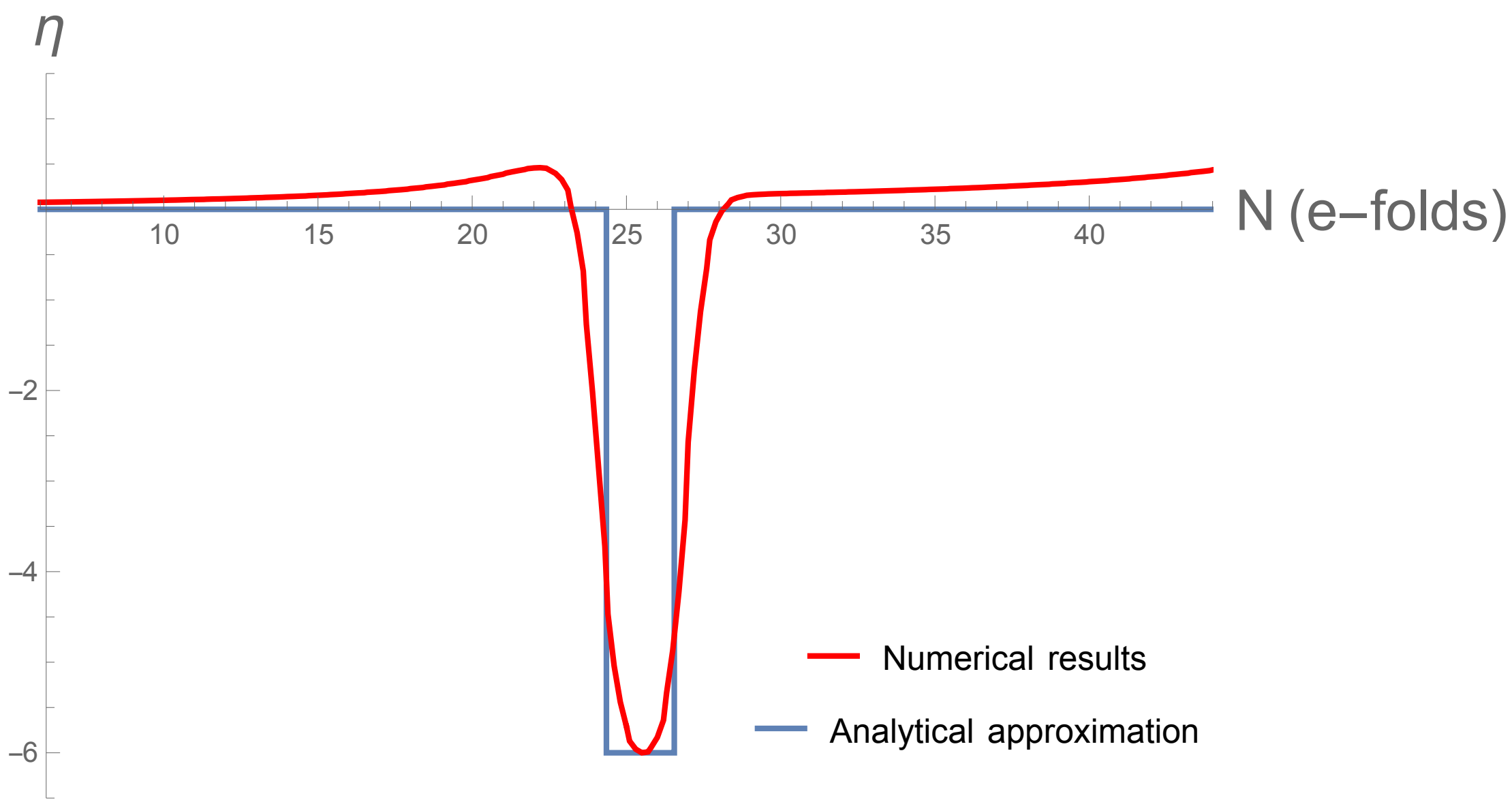
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CONSTANT - ν

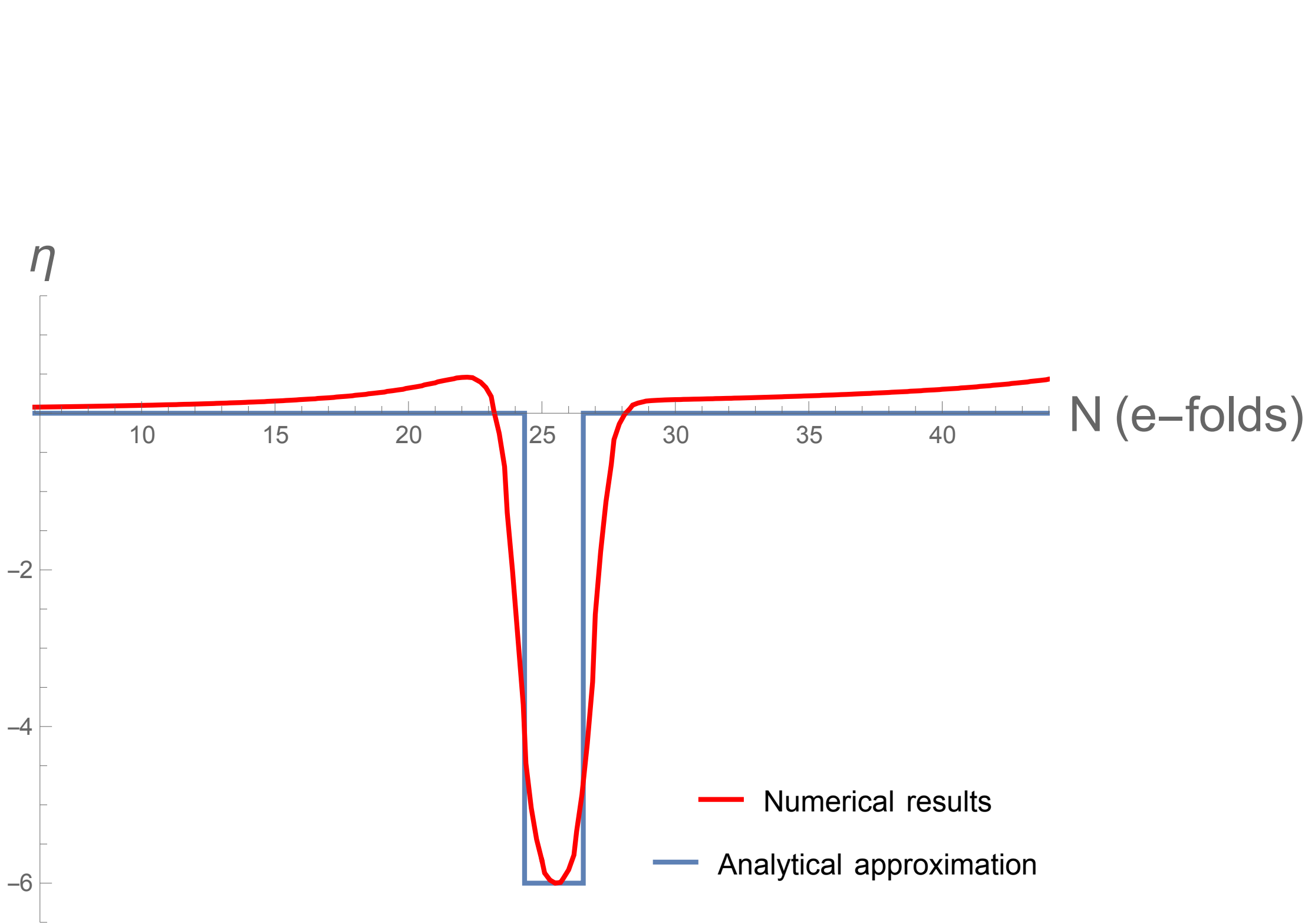


A SIMPLIFIED MODEL: *INSTANTANEOUS TRANSITIONS* in η

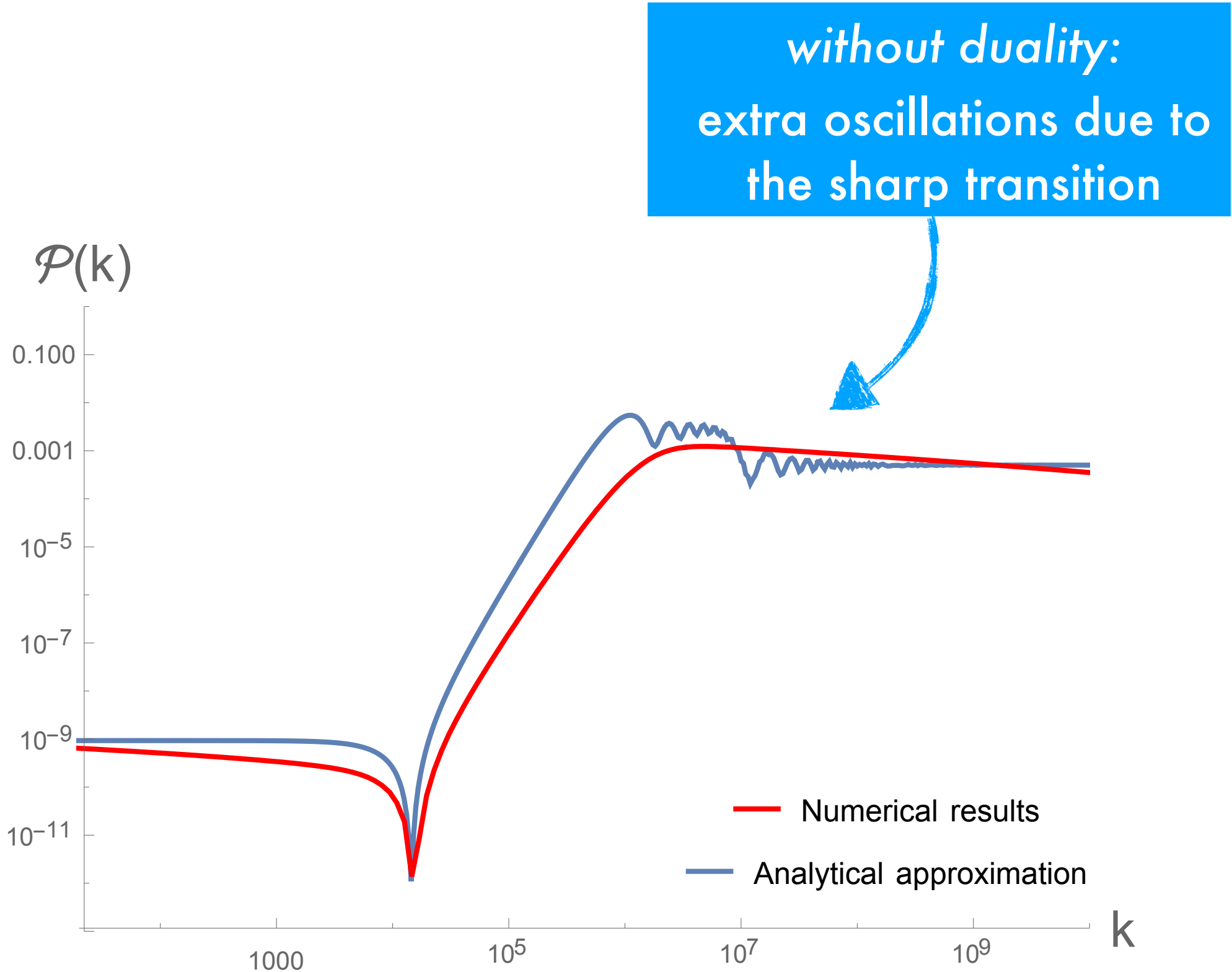


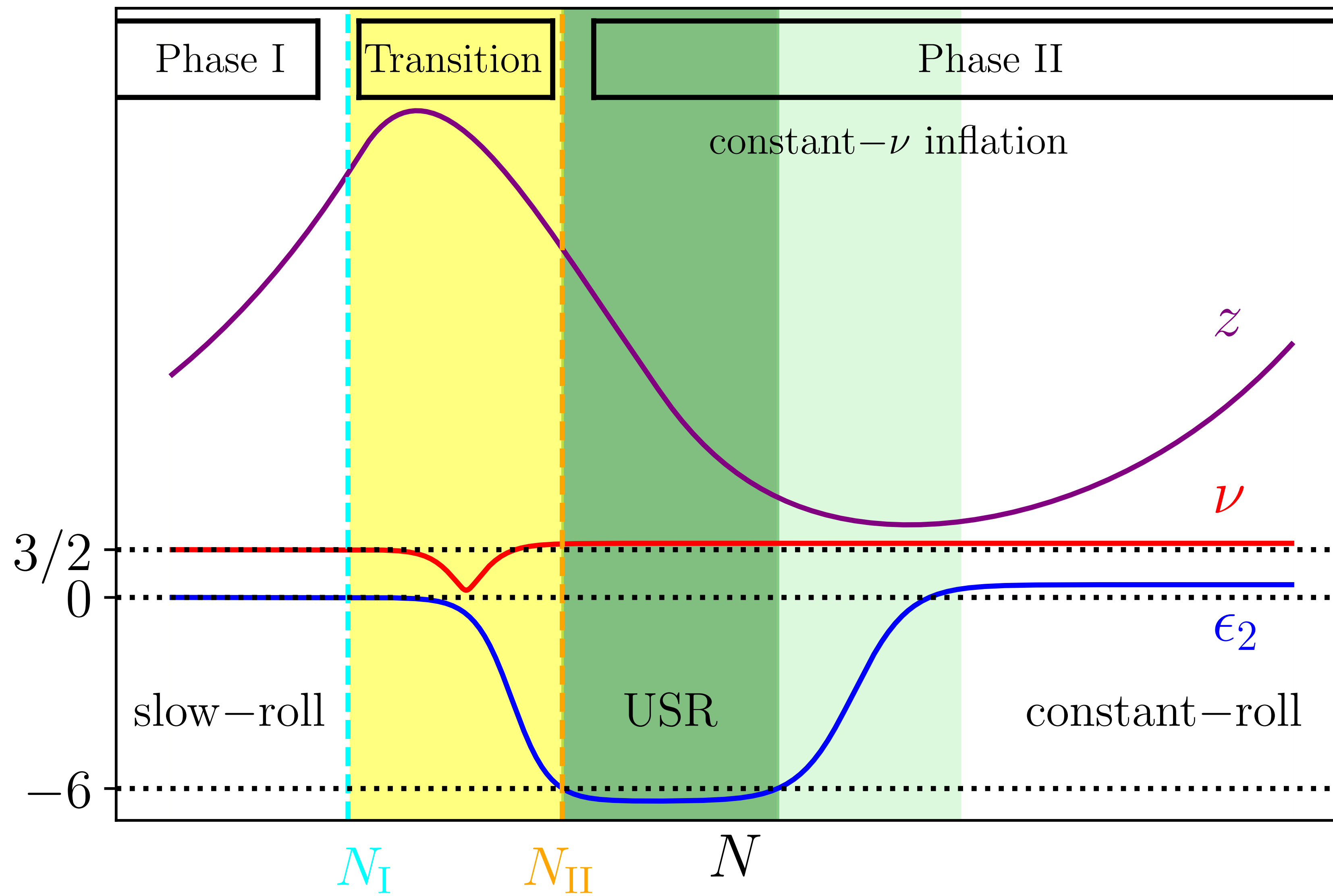
ansatz for η (or ϵ_2)

A SIMPLIFIED MODEL: INSTANTANEOUS TRANSITIONS in η



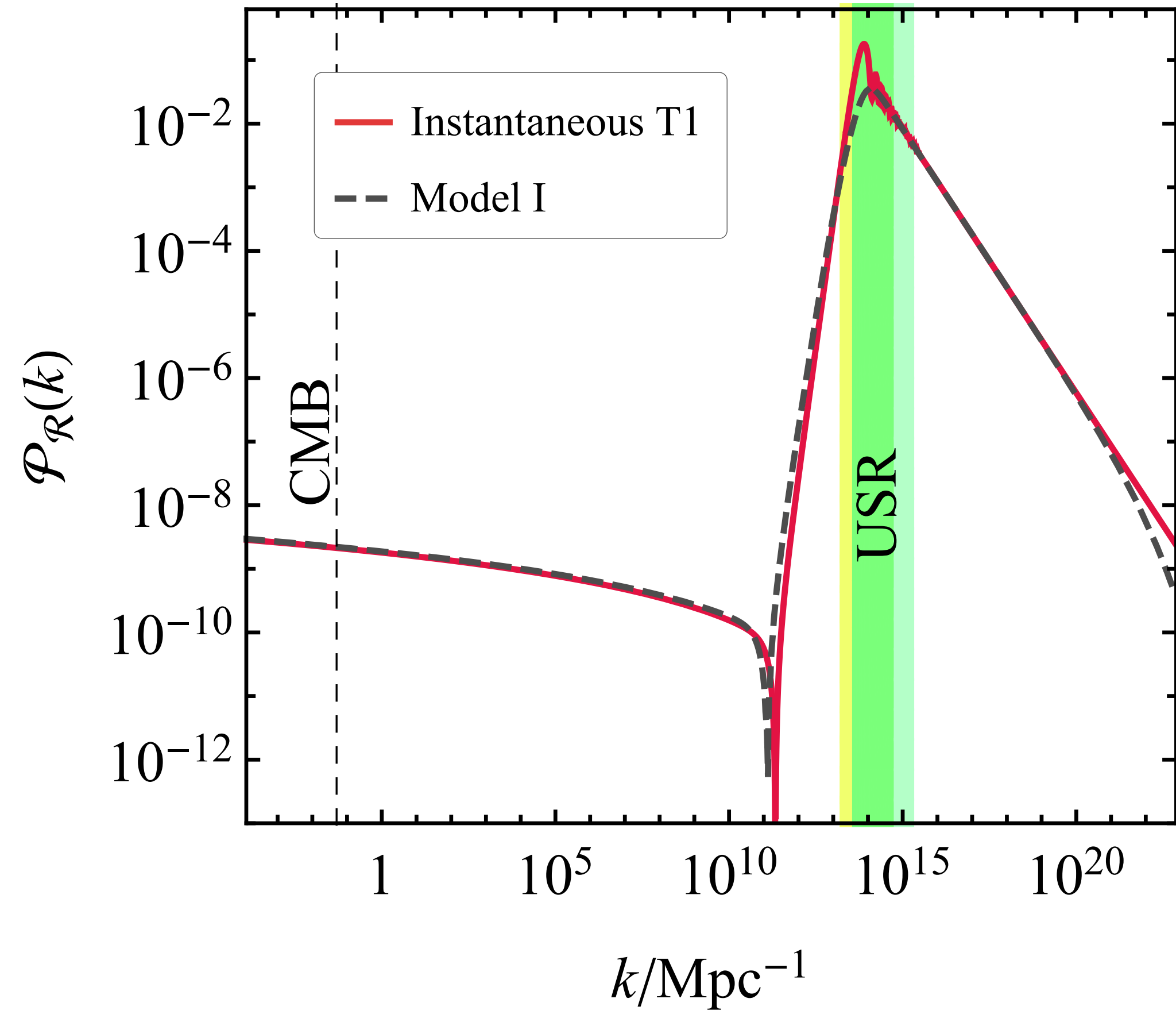
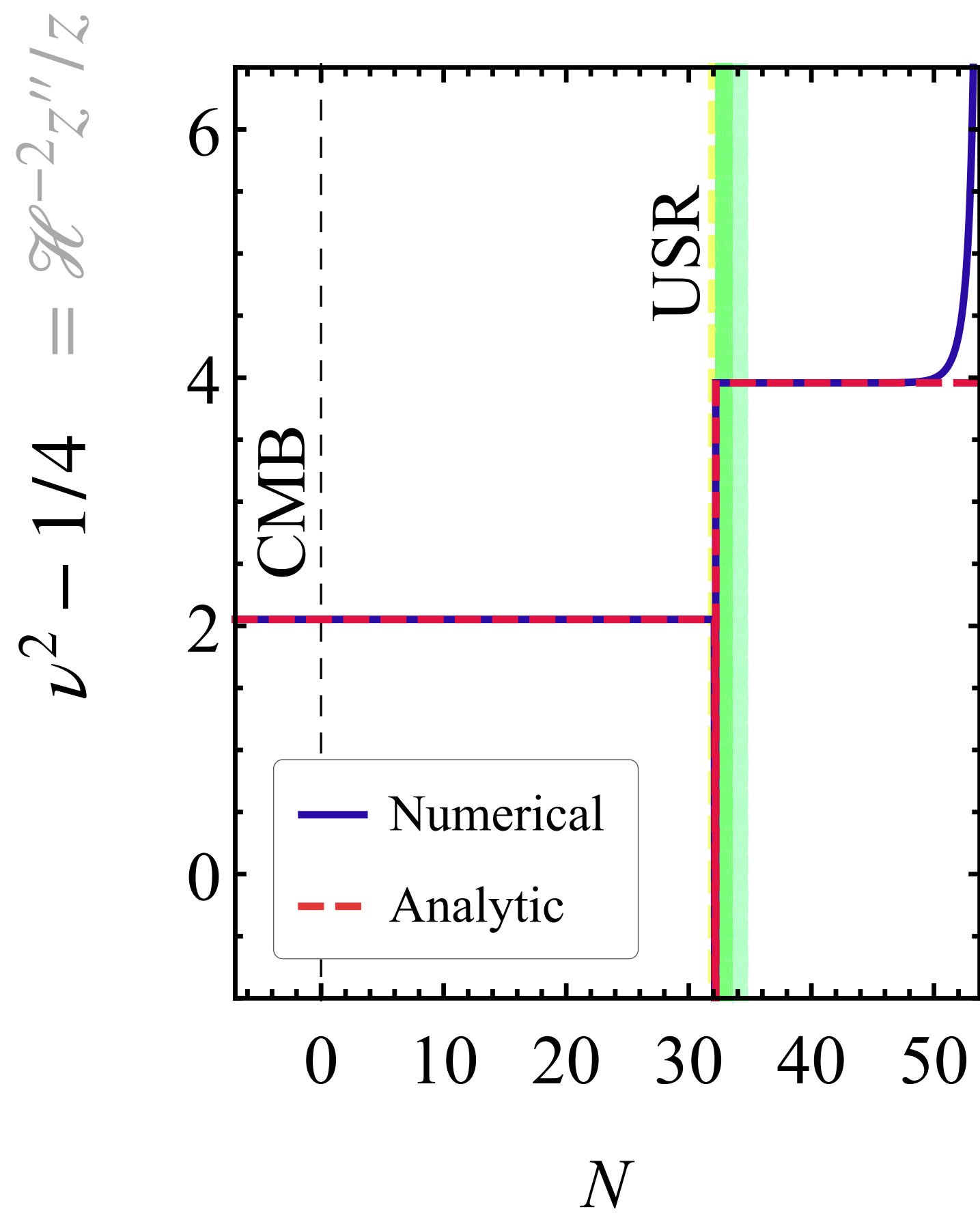
ansatz for η (or ϵ_2)





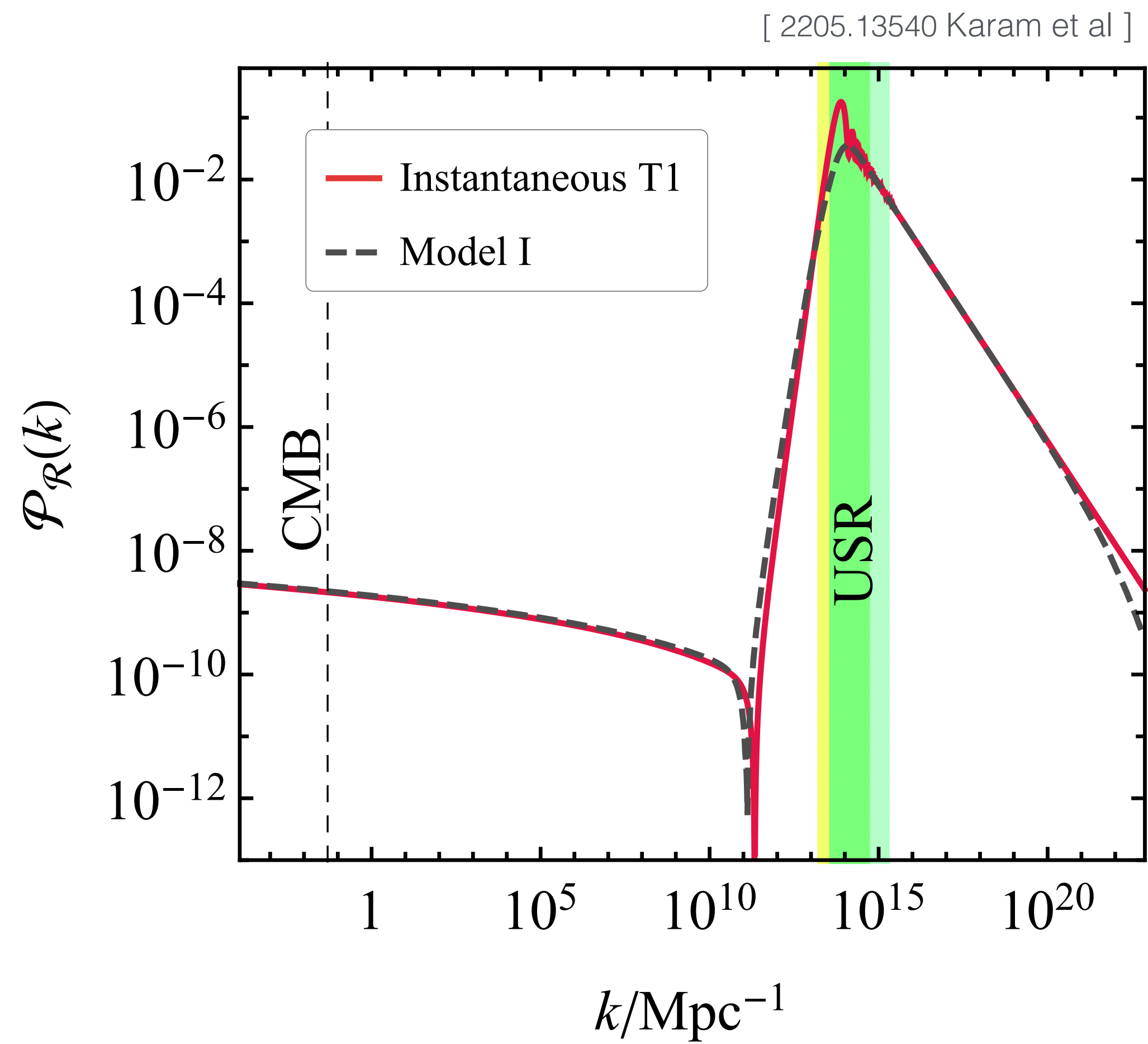
A SIMPLIFIED MODEL: INSTANTANEOUS TRANSITIONS

[2205.13540 Karam et al]



A SIMPLIFIED MODEL: *INSTANTANEOUS TRANSITIONS*

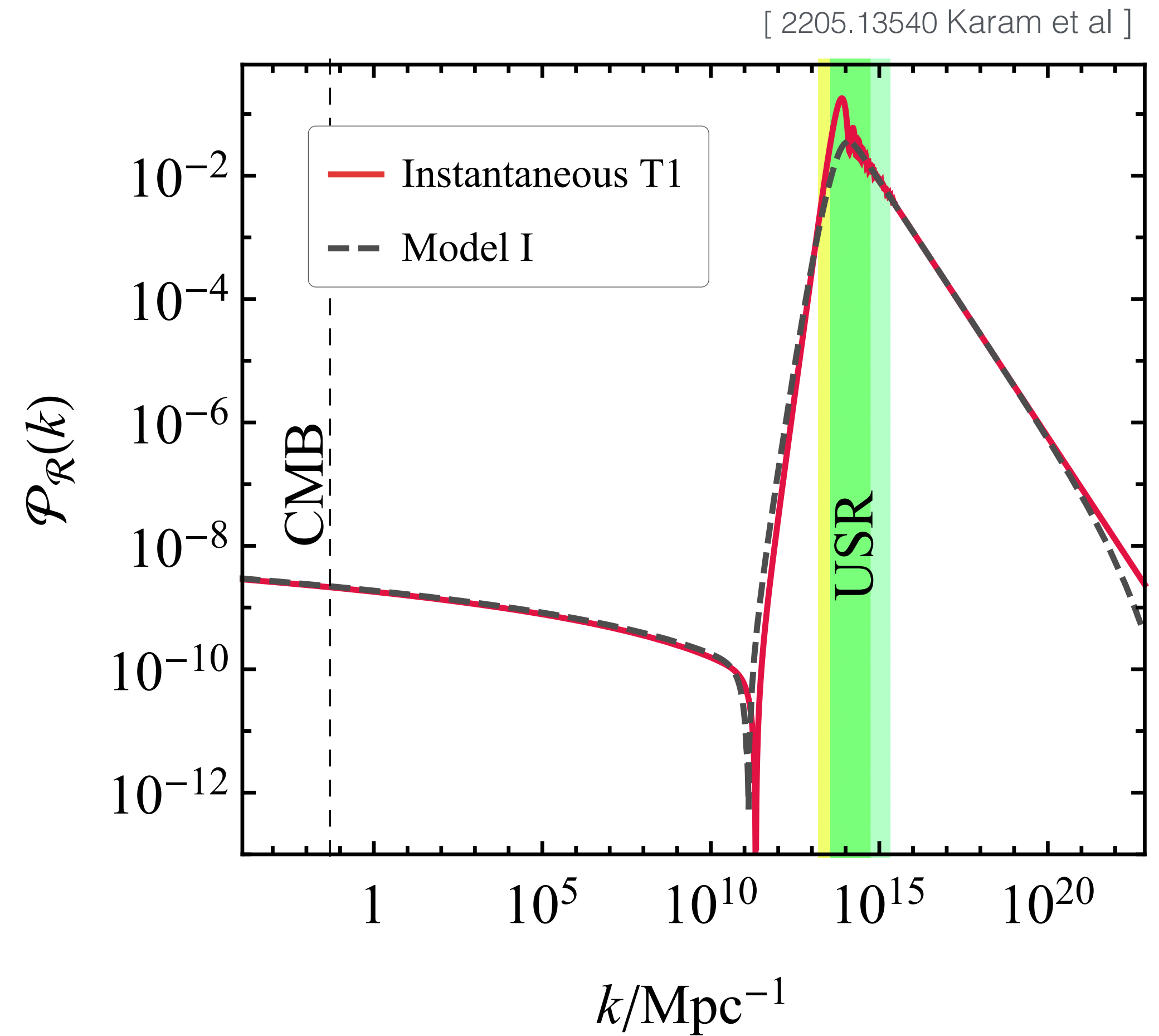
CURVATURE POWER SPECTRUM ...



A SIMPLIFIED MODEL: INSTANTANEOUS TRANSITIONS

CURVATURE POWER SPECTRUM ...

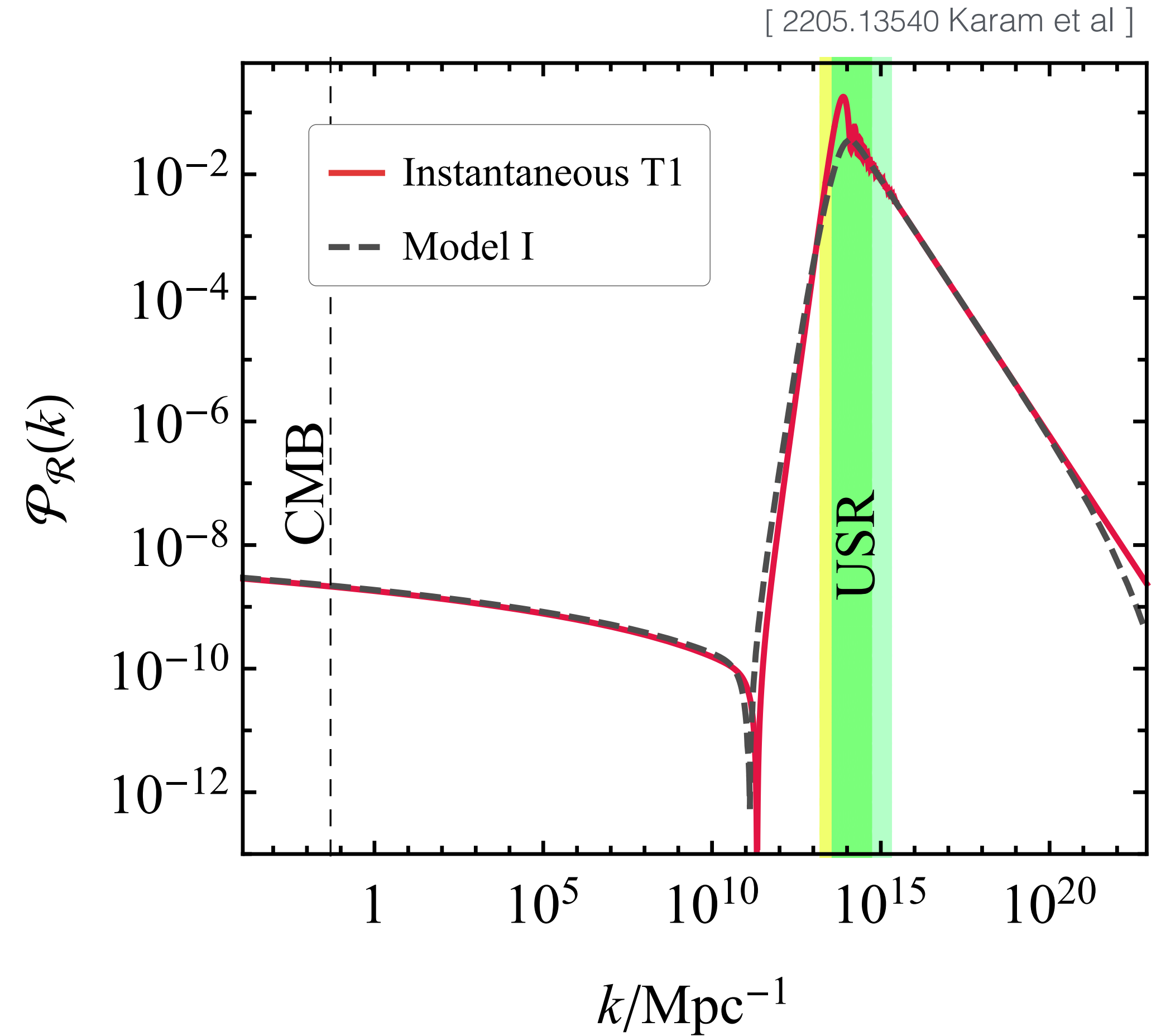
1. ...is exactly solvable



A SIMPLIFIED MODEL: INSTANTANEOUS TRANSITIONS

CURVATURE POWER SPECTRUM ...

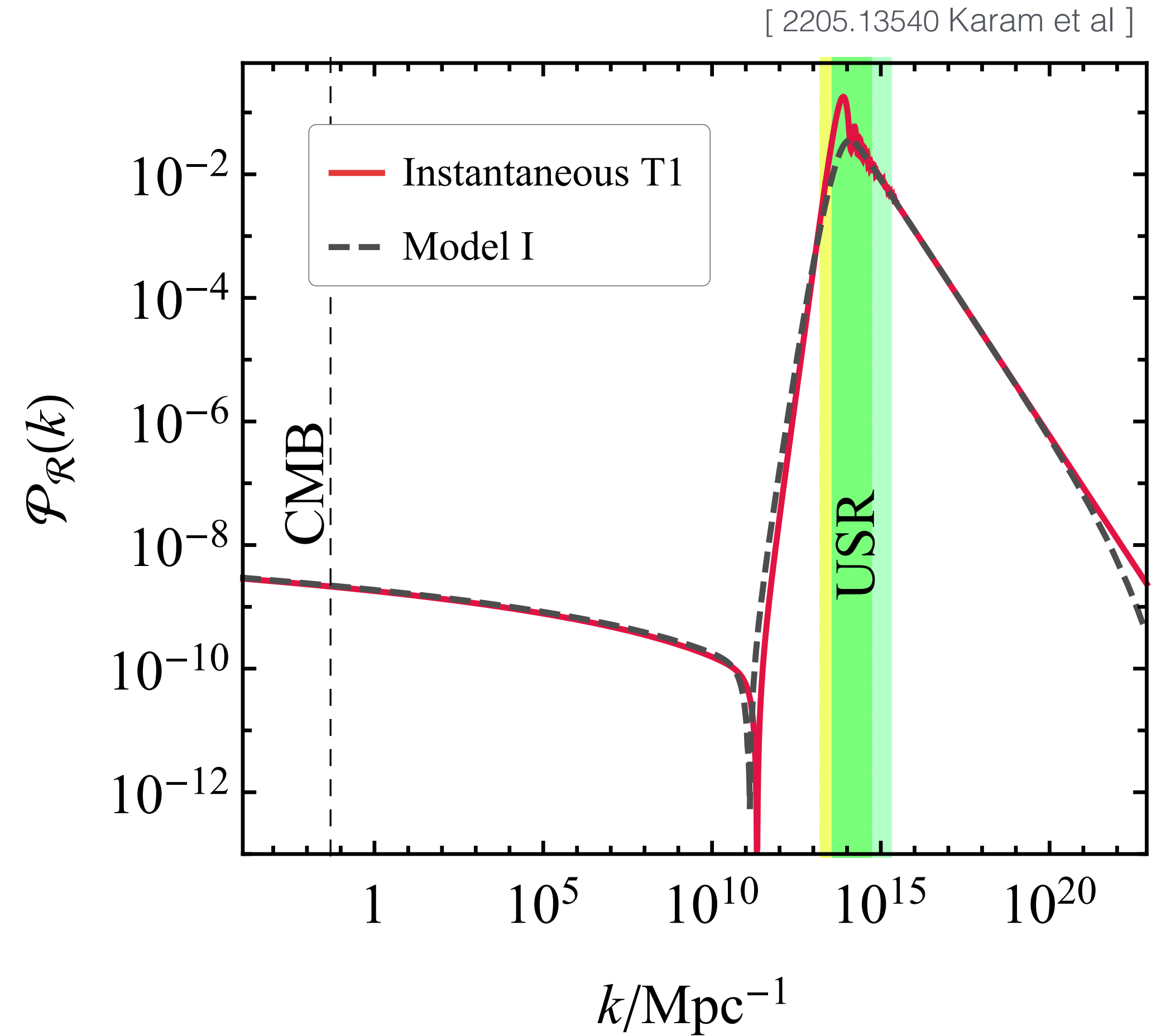
1. ...is exactly solvable
2. ...is viable for both PBHs and CMB



A SIMPLIFIED MODEL: INSTANTANEOUS TRANSITIONS

CURVATURE POWER SPECTRUM ...

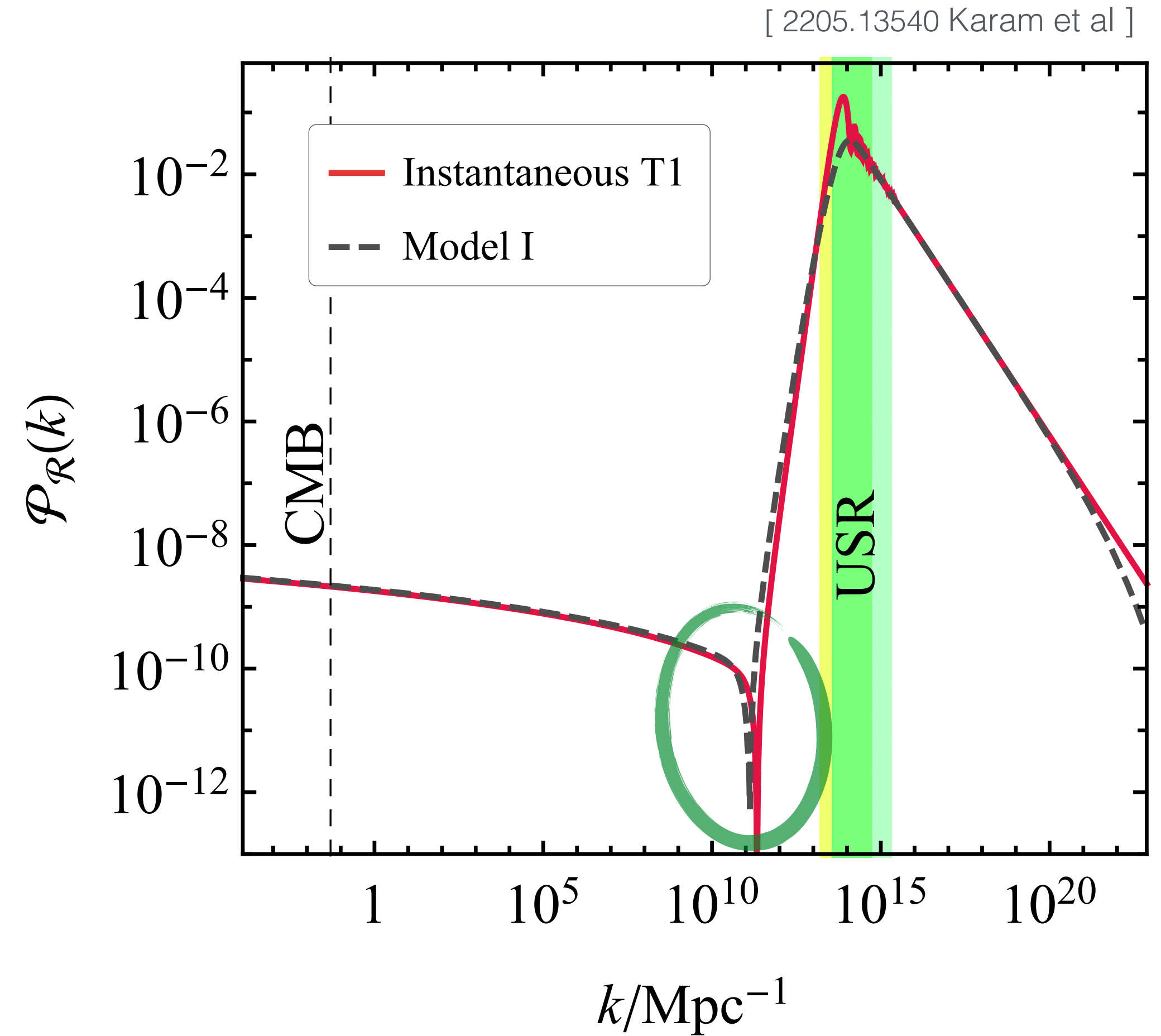
1. ...is exactly solvable
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A SIMPLIFIED MODEL: INSTANTANEOUS TRANSITIONS

CURVATURE POWER SPECTRUM ...

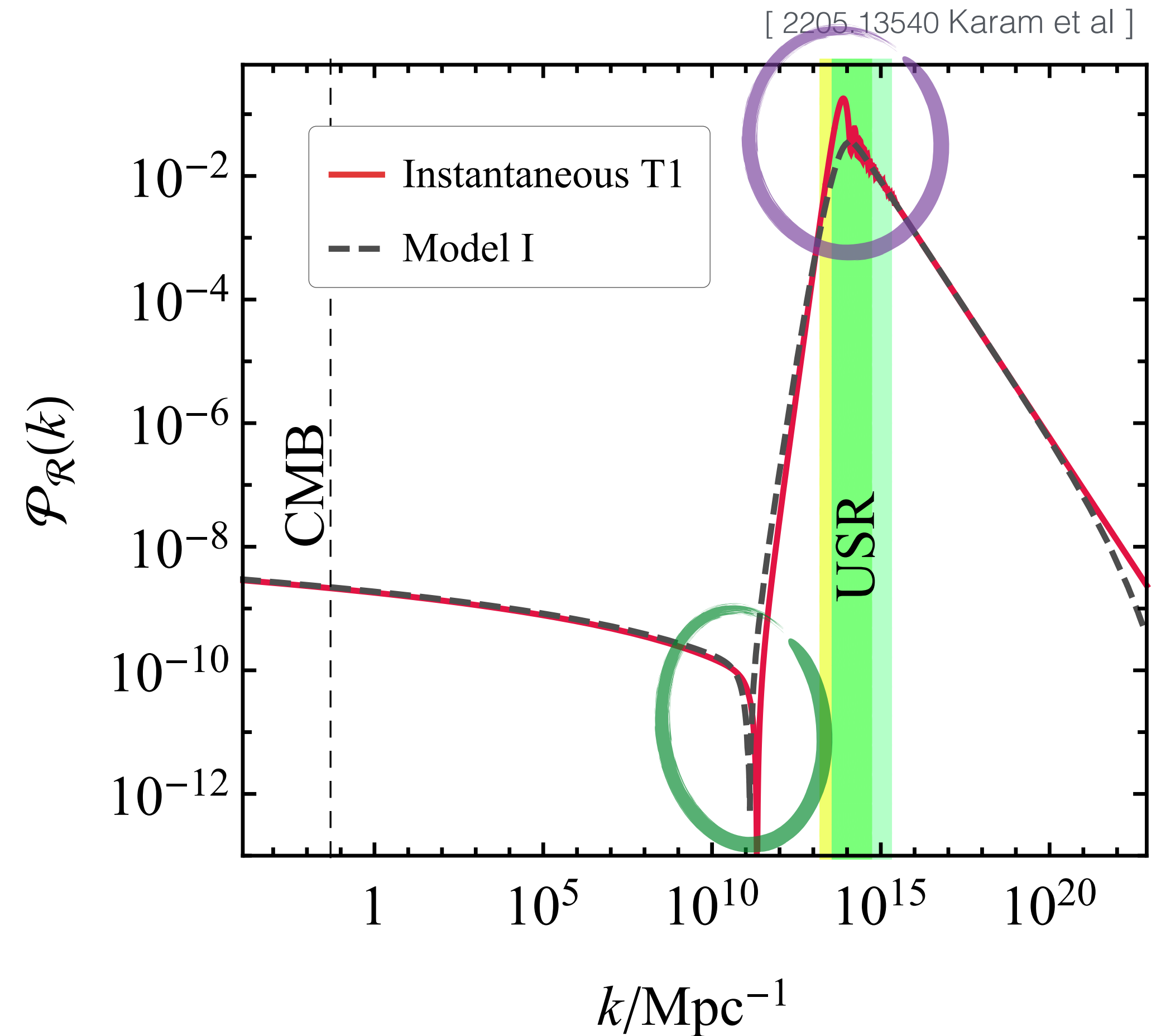
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2. ...is viable for both PBHs and CMB
3. ...approximates USR models well
4. ...contains a dip



A SIMPLIFIED MODEL: INSTANTANEOUS TRANSITIONS

CURVATURE POWER SPECTRUM ...

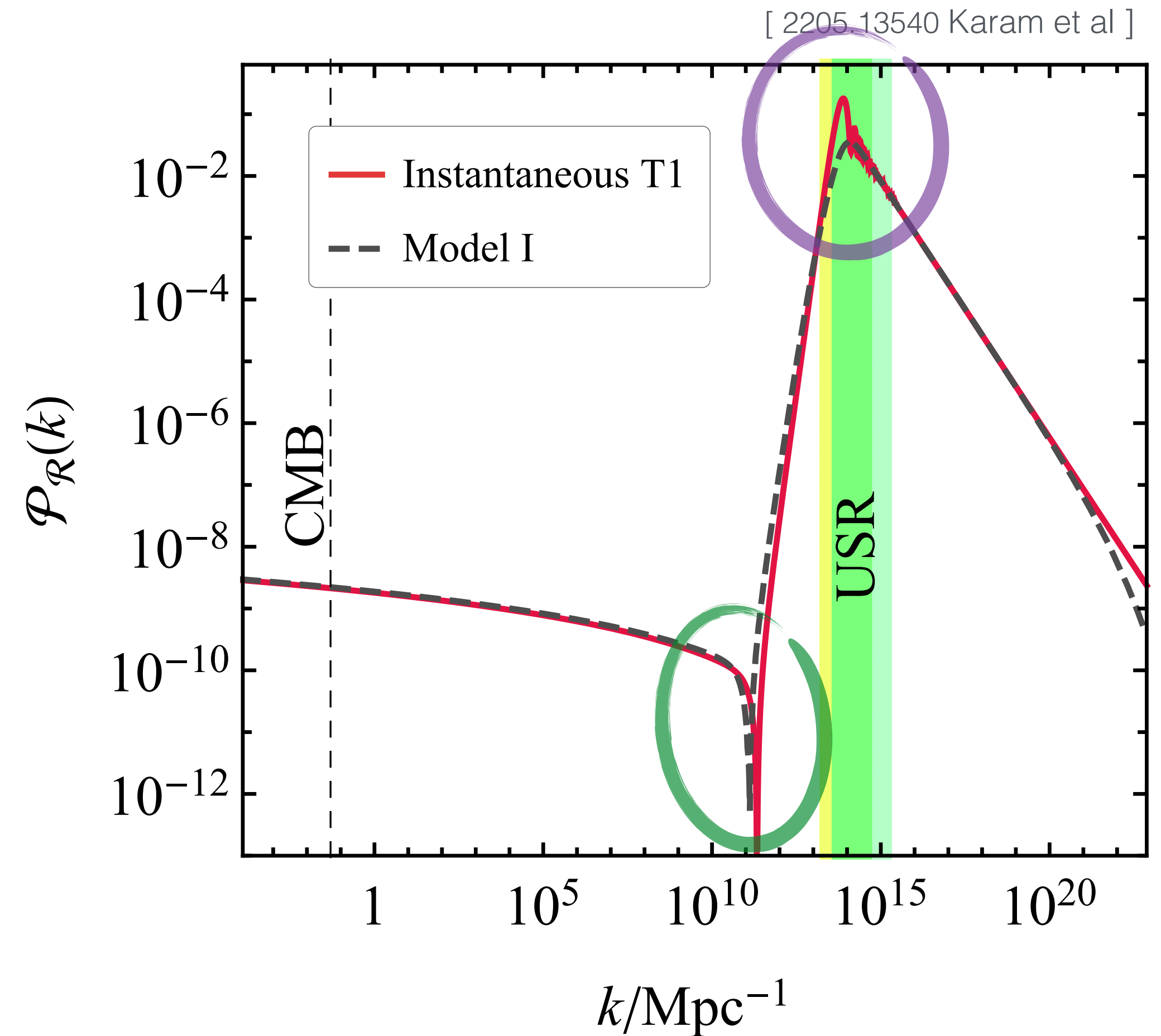
1. ...is exactly solvable
2. ...is viable for both PBHs and CMB
3. ...approximates USR models well
4. ...contains a dip
5. ...has oscillatory features



A SIMPLIFIED MODEL: INSTANTANEOUS TRANSITIONS

CURVATURE POWER SPECTRUM ...

1. ...is exactly solvable
2. ...is viable for both PBHs and CMB
3. ...approximates USR models well
4. ...contains a dip
5. ...has oscillatory features



=> PEAK SHAPE DEPENDS ON THE SR to USR TRANSITION

BEYOND INSTANTANEOUS TRANSITIONS

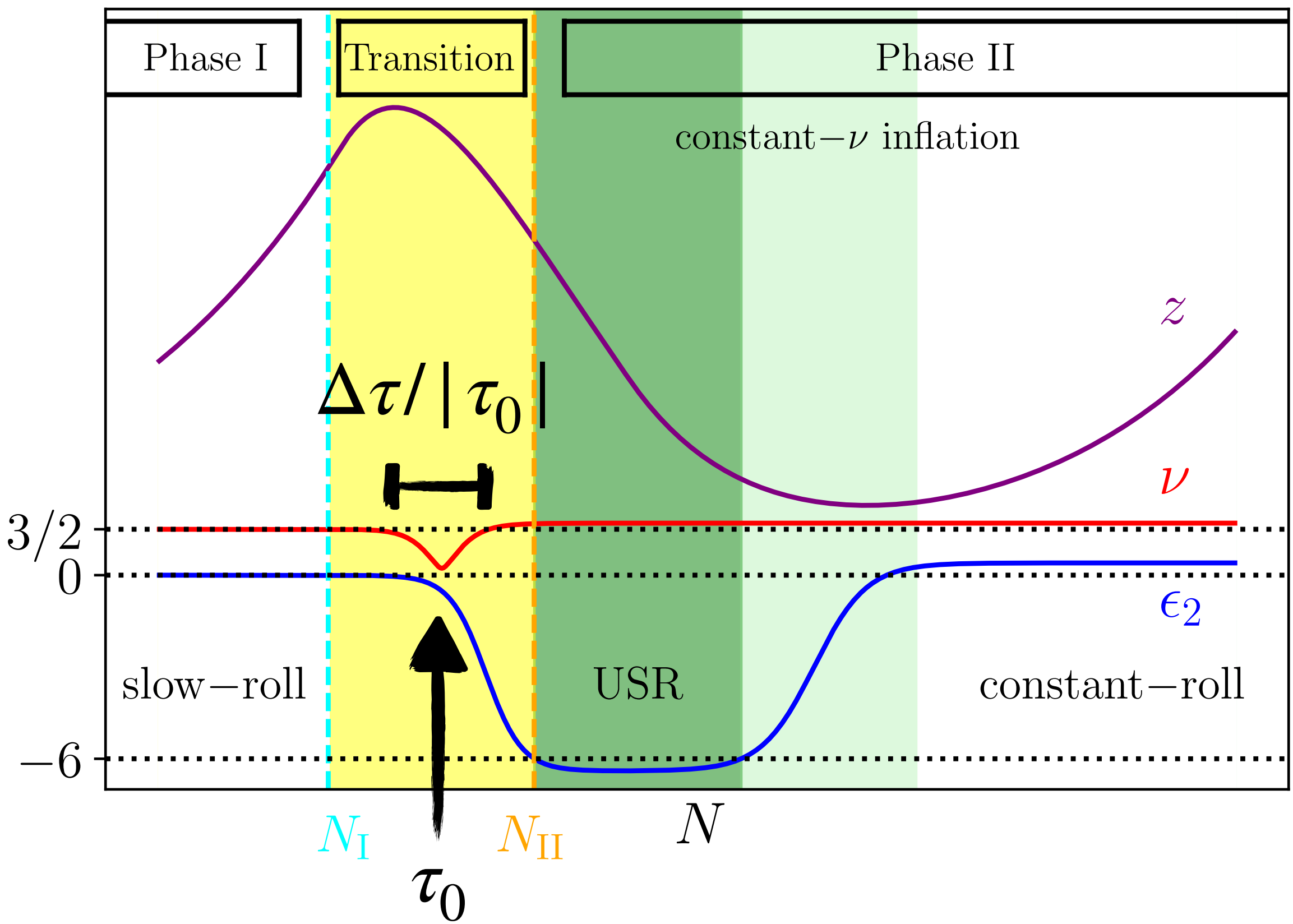
$$u_k'' + (z''/z - k^2) u_k = 0$$

PERTURBATIVE IN THE UV

$$k|\tau_0| \gg 1$$

PERTURBATIVE IN THE IR

$$k|\tau_0| \ll 1$$



MODE EVOLUTION

PHASE I (slow-roll)

$$u_{\text{I},k}(\tau) = a_{\text{I},1}(k)u_{\text{I},k}^{(1)}(\tau) + a_{\text{I},2}(k)u_{\text{I},k}^{(2)}(\tau)$$

DAMPED GROWING

TRANSITION

$$\mathbf{a}_{\text{II}}(k) = T(k) \mathbf{a}_{\text{I}}(k)$$

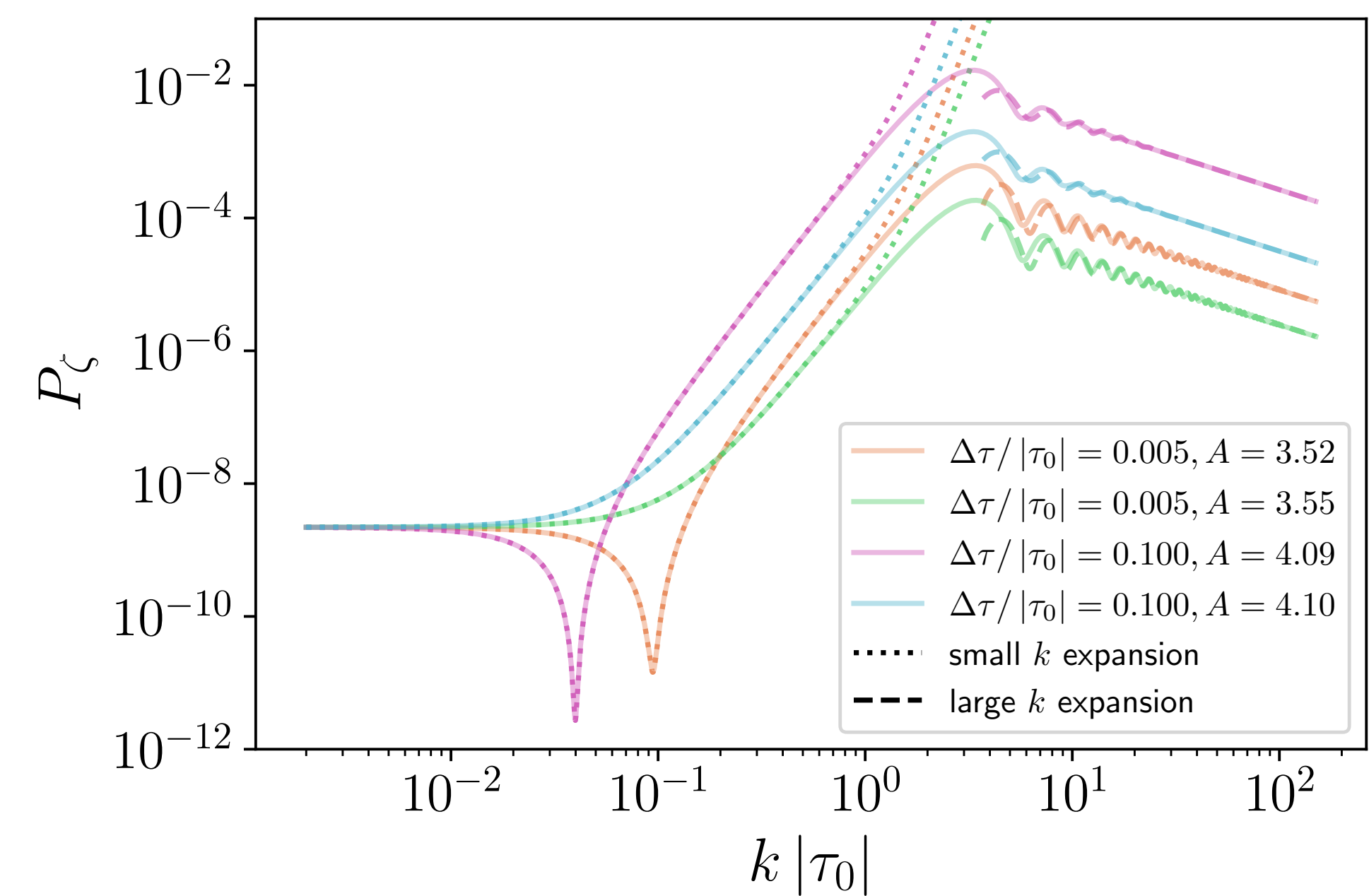
transfer matrix

PHASE II (constant ν)

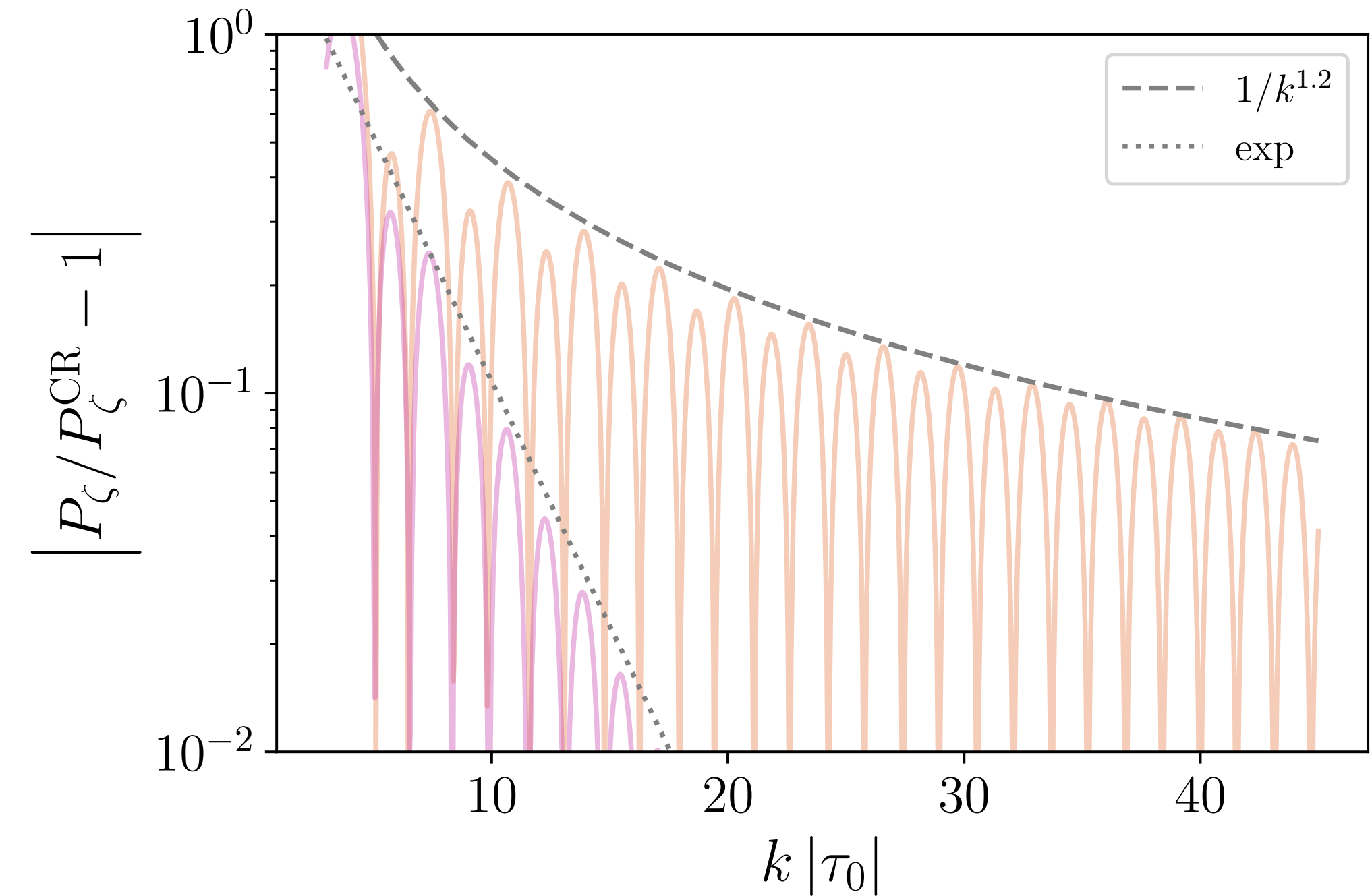
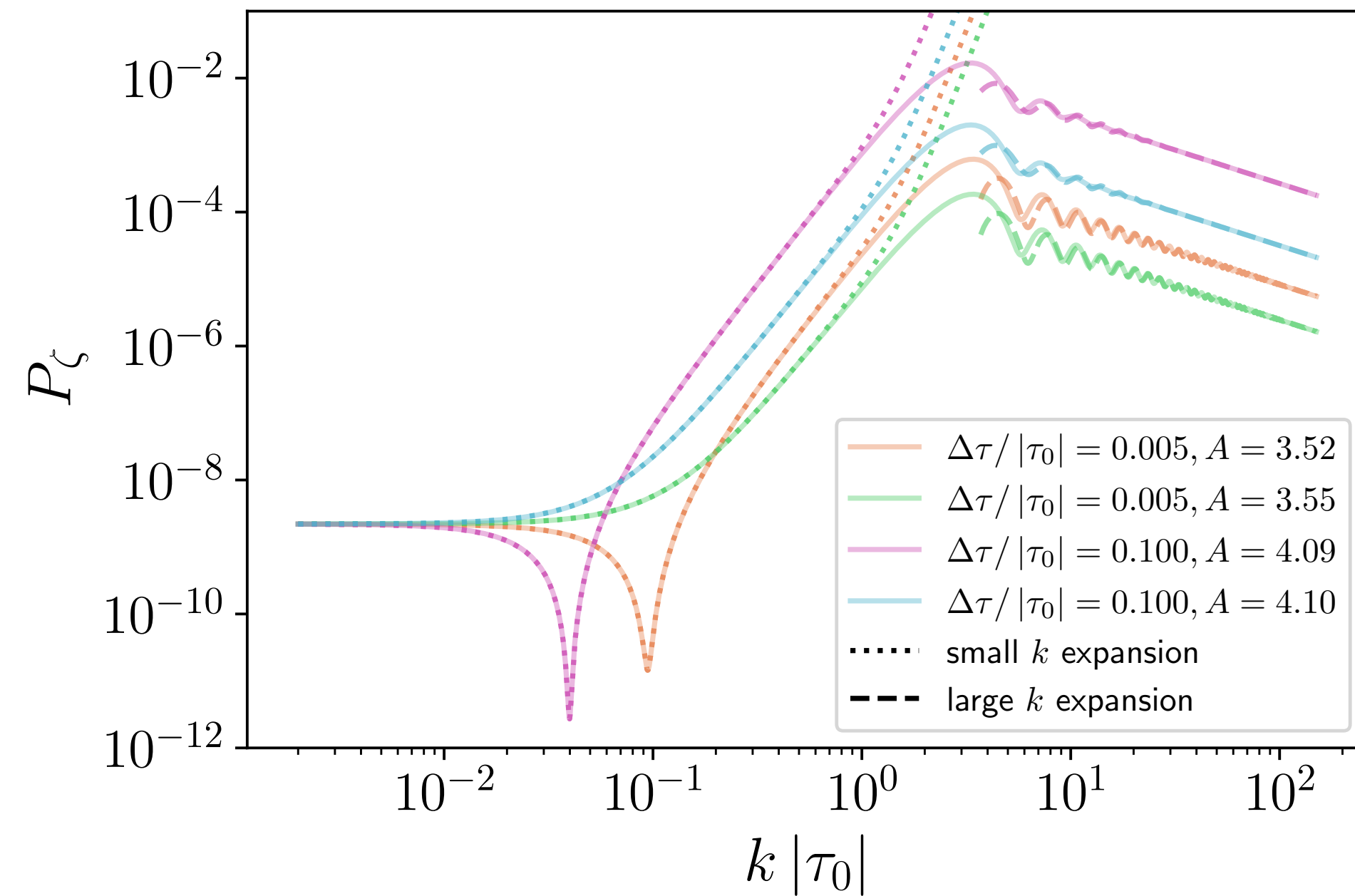
$$u_{\text{II},k}(\tau) = a_{\text{II},1}(k)u_{\text{II},k}^{(1)}(\tau) + a_{\text{II},2}(k)u_{\text{II},k}^{(2)}(\tau)$$

DAMPED GROWING

1. LONGER TRANSITIONS ERASE SPECTRAL OSCILLATIONS



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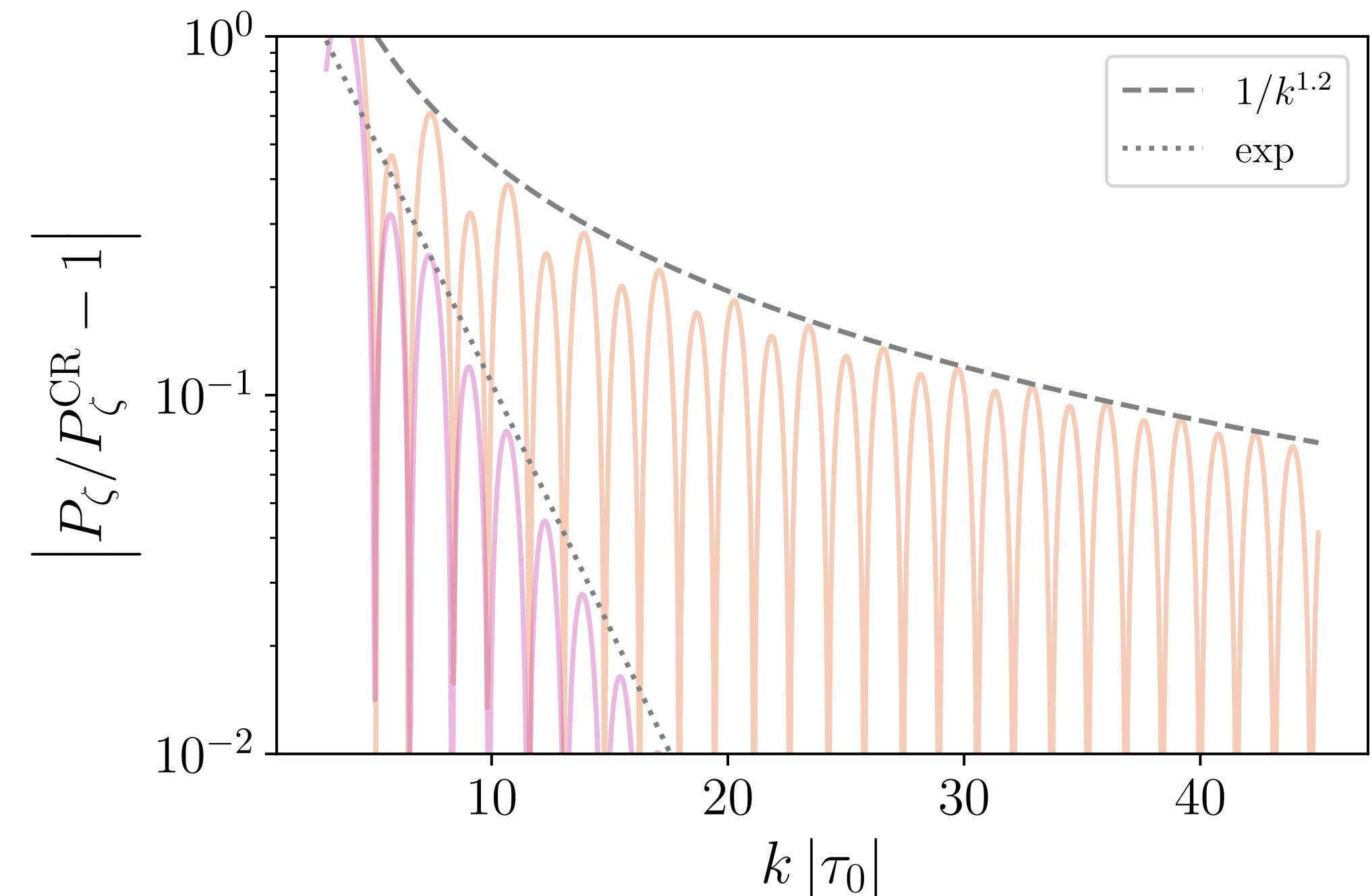
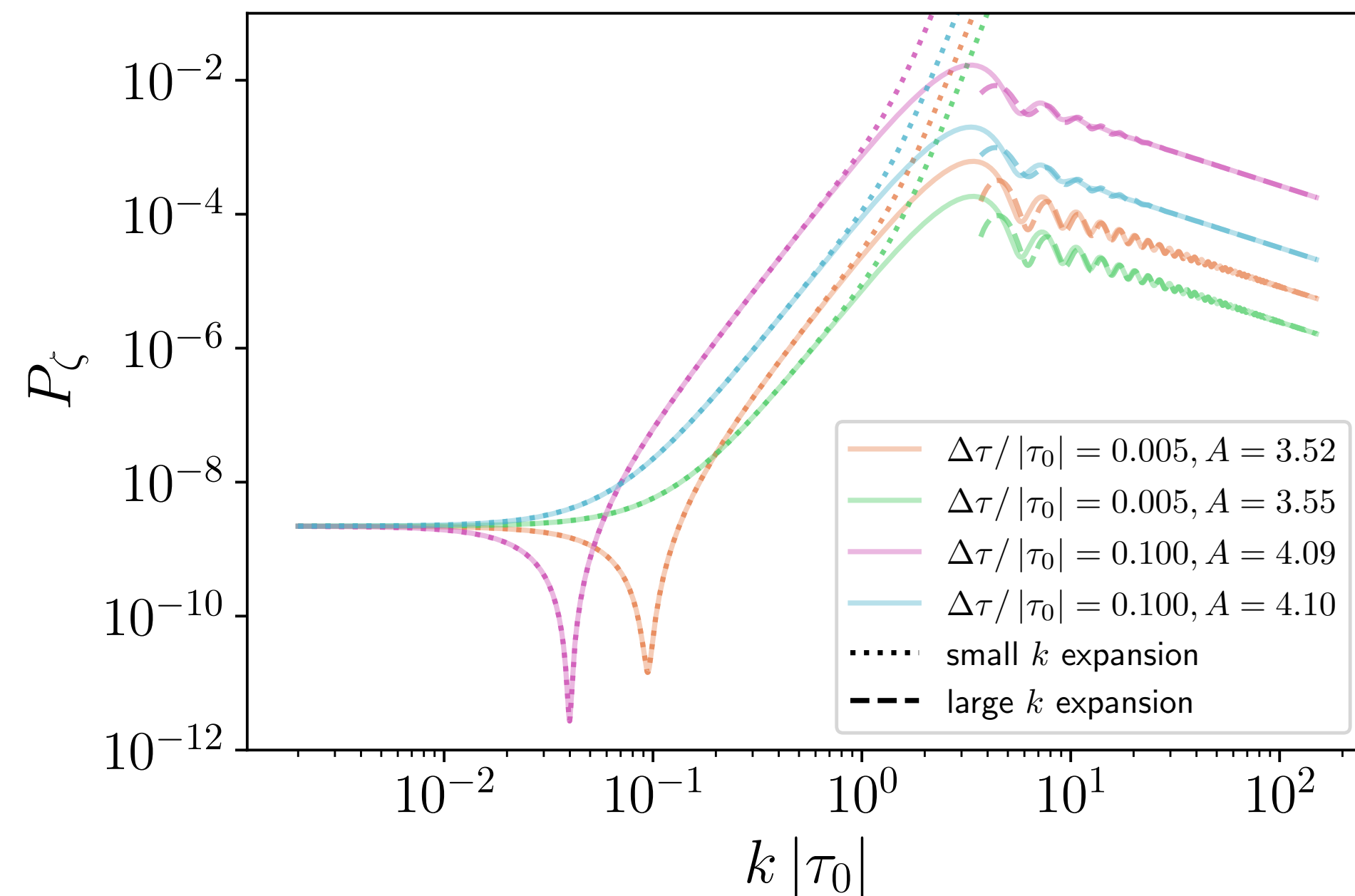
perturbative estimate of spectral oscillations:

$$\frac{P_\zeta(k)}{P_\zeta^{\text{(CR)}}(k; \nu_{\text{II}})} = 1 + \frac{1}{k} \text{Re} \left\{ e^{i\pi\nu_{\text{II}}} \left[\tilde{A}(2k) + i \frac{\tilde{B}_{\text{II}}(2k)}{2k\tau_{\text{II}}} - i \frac{\tilde{B}_{\text{I}}(2k)}{2k\tau_{\text{I}}} \right] \right\}$$

$$\tilde{A}(k) \equiv \int_{\tau_{\text{I}}}^{\tau_{\text{II}}} d\tau' e^{ik\tau'} \frac{z''}{z}(\tau')$$

$$\tilde{B}_i(k) \equiv \frac{e^{ik\tau_i}}{\tau_i} \left(\nu_i^2 - \frac{1}{4} \right)$$

1. LONGER TRANSITIONS ERASE SPECTRAL OSCILLATIONS



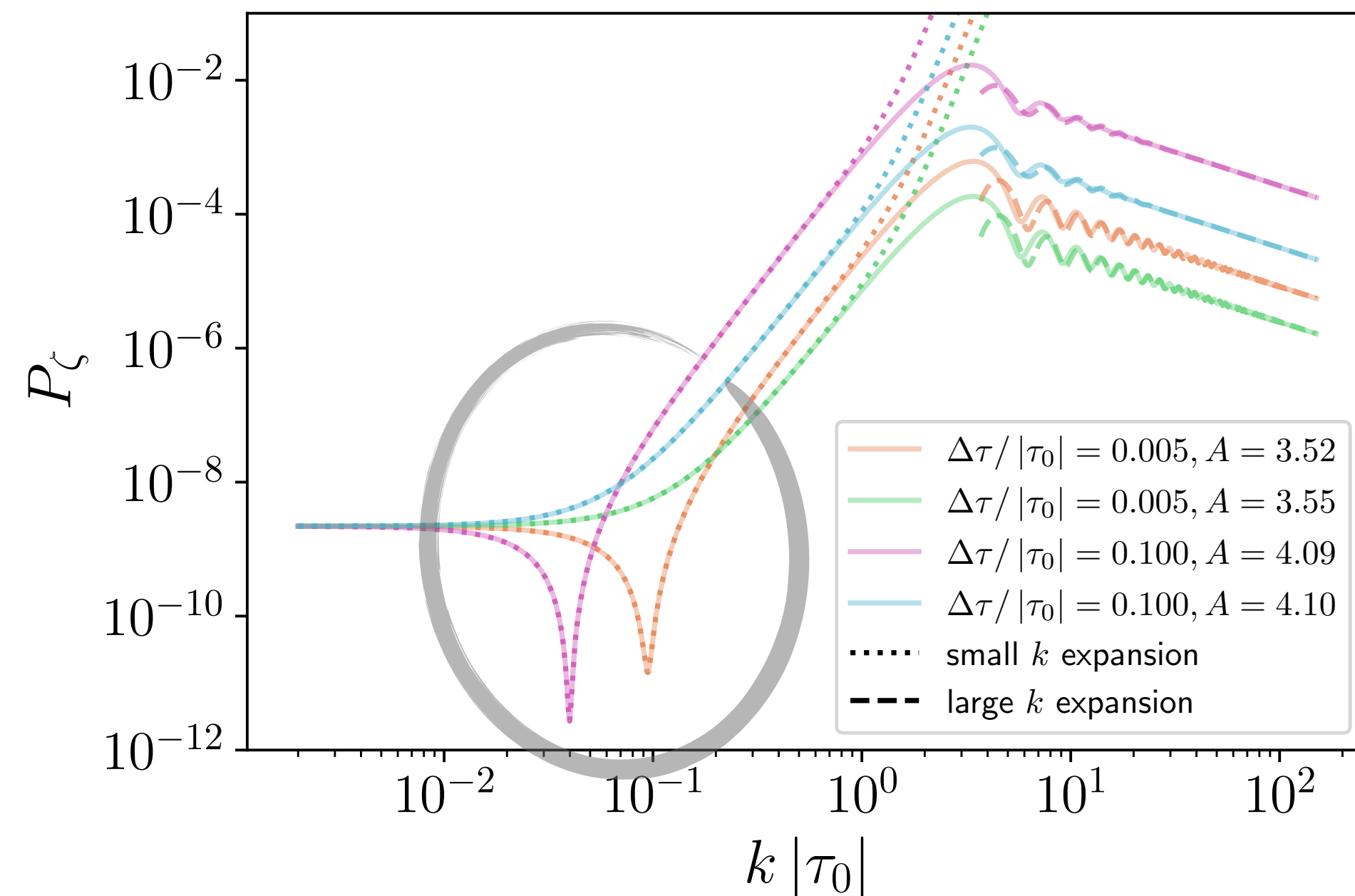
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2. INFLATON VELOCITY REVERSAL CREATES DIPS

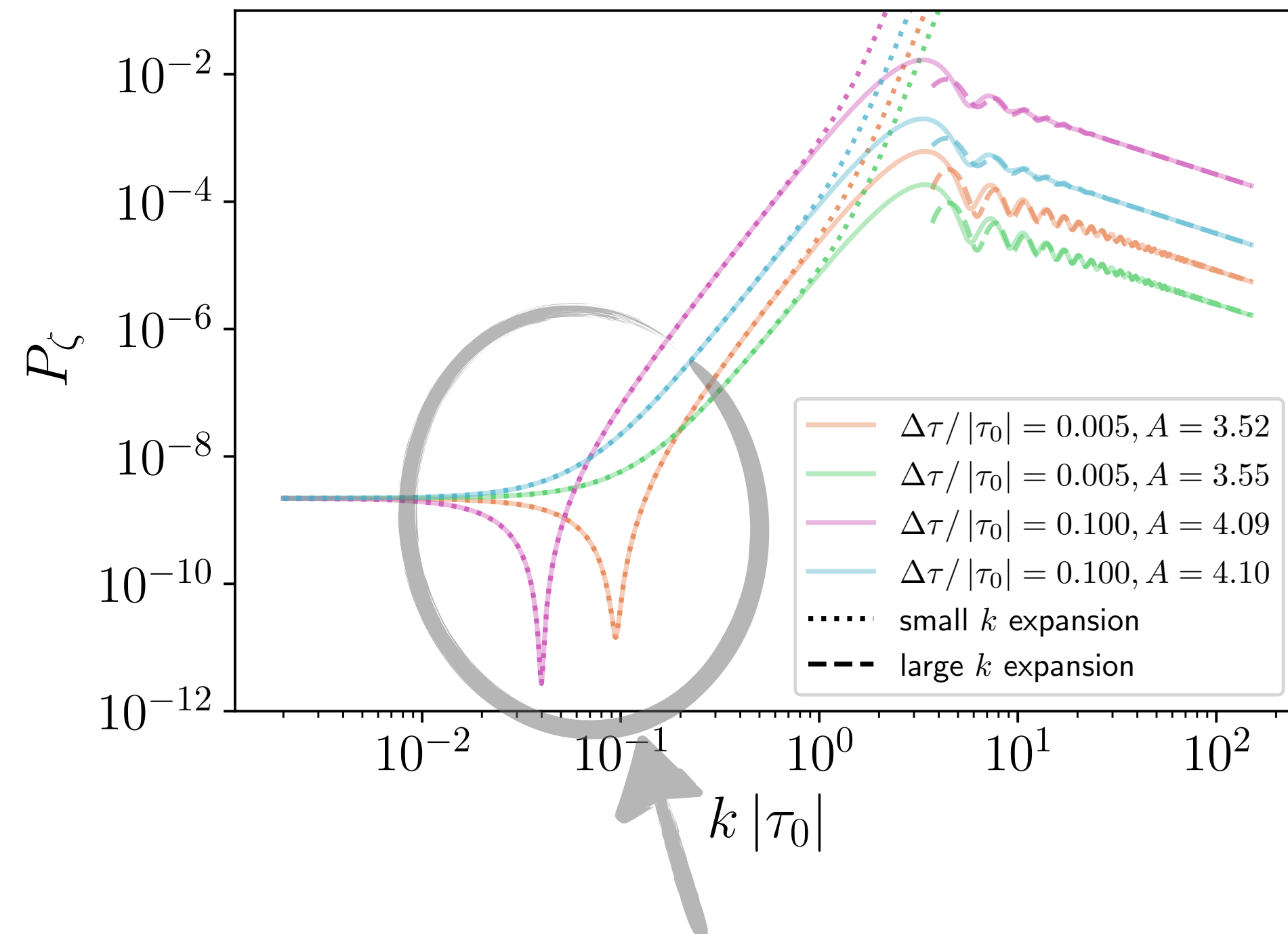


$$\mathcal{P}_\zeta(k) \stackrel{k \ll k_*}{\equiv} \mathcal{P}_\zeta^{\text{CMB}} \left\{ \left[1 + A_2 \left(\frac{k}{k_*} \right)^2 + A_4 \left(\frac{k}{k_*} \right)^4 \right]^2 + B_0^2 \left(\frac{k}{k_*} \right)^6 \right\}$$

$$\frac{A_2}{k_*^2} \simeq - \left[\int_{-\infty}^{\tau^*} d\tau' z(\tau')^2 \right] \mathcal{H} \int_{\tau_0}^0 \frac{d\tau'}{z(\tau')^2}, \quad A_2 \gtrsim A_4^2$$

$A_2 < 0$ if $z = a\dot{\phi}/H$ does not cross 0

2. INFLATON VELOCITY REVERSAL CREATES DIPS



GREEN and **BLUE** spectra for models in which $\dot{\phi}$ changes sign

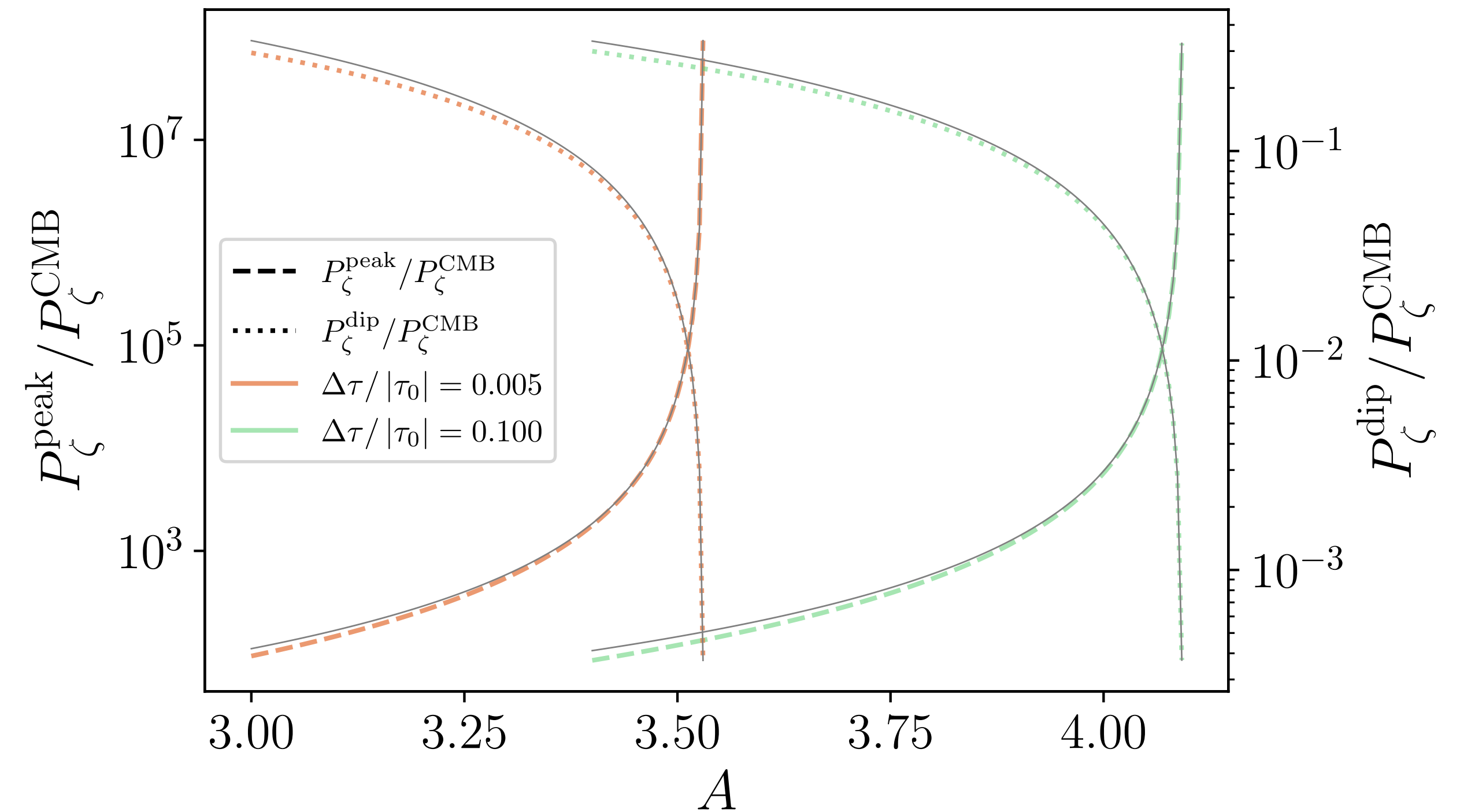
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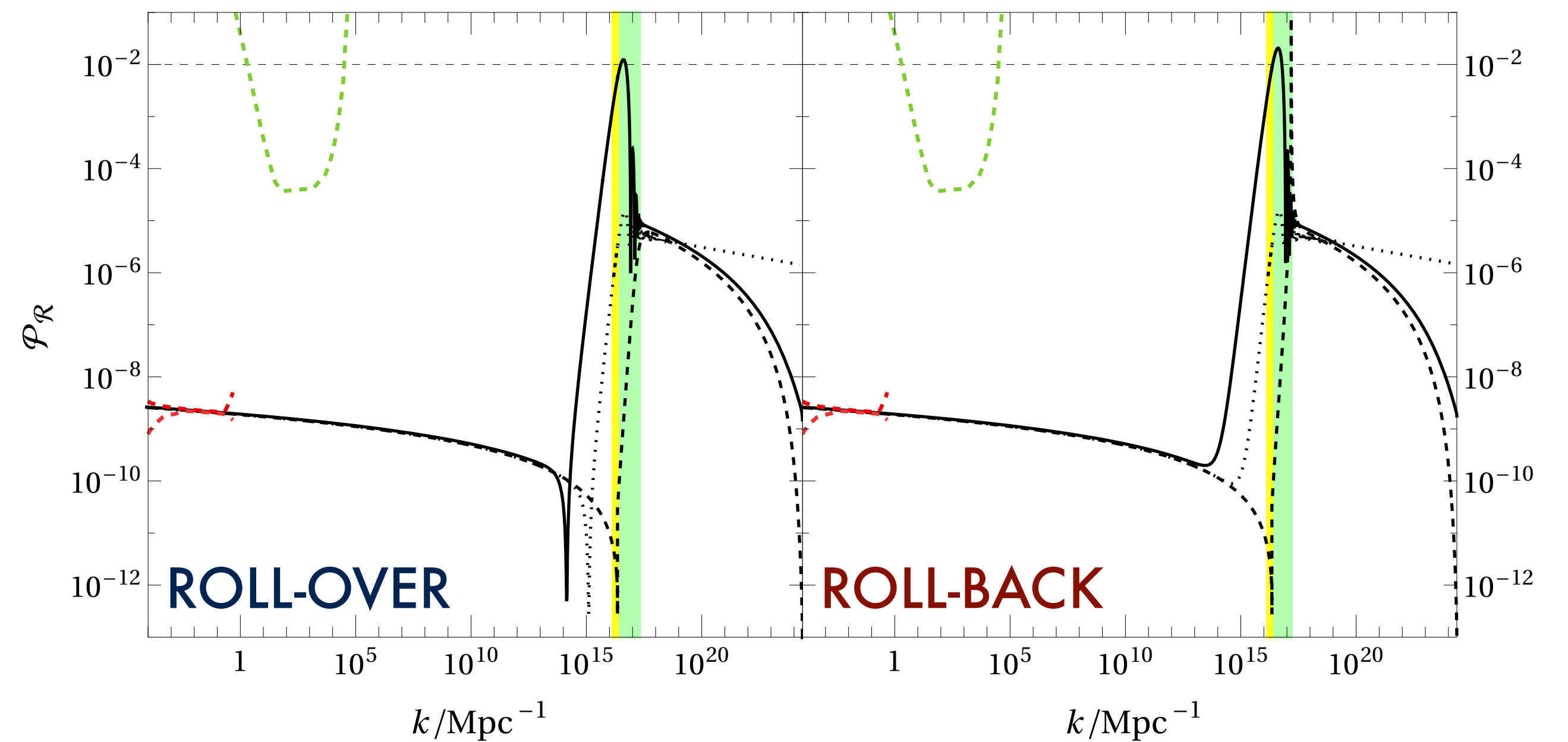
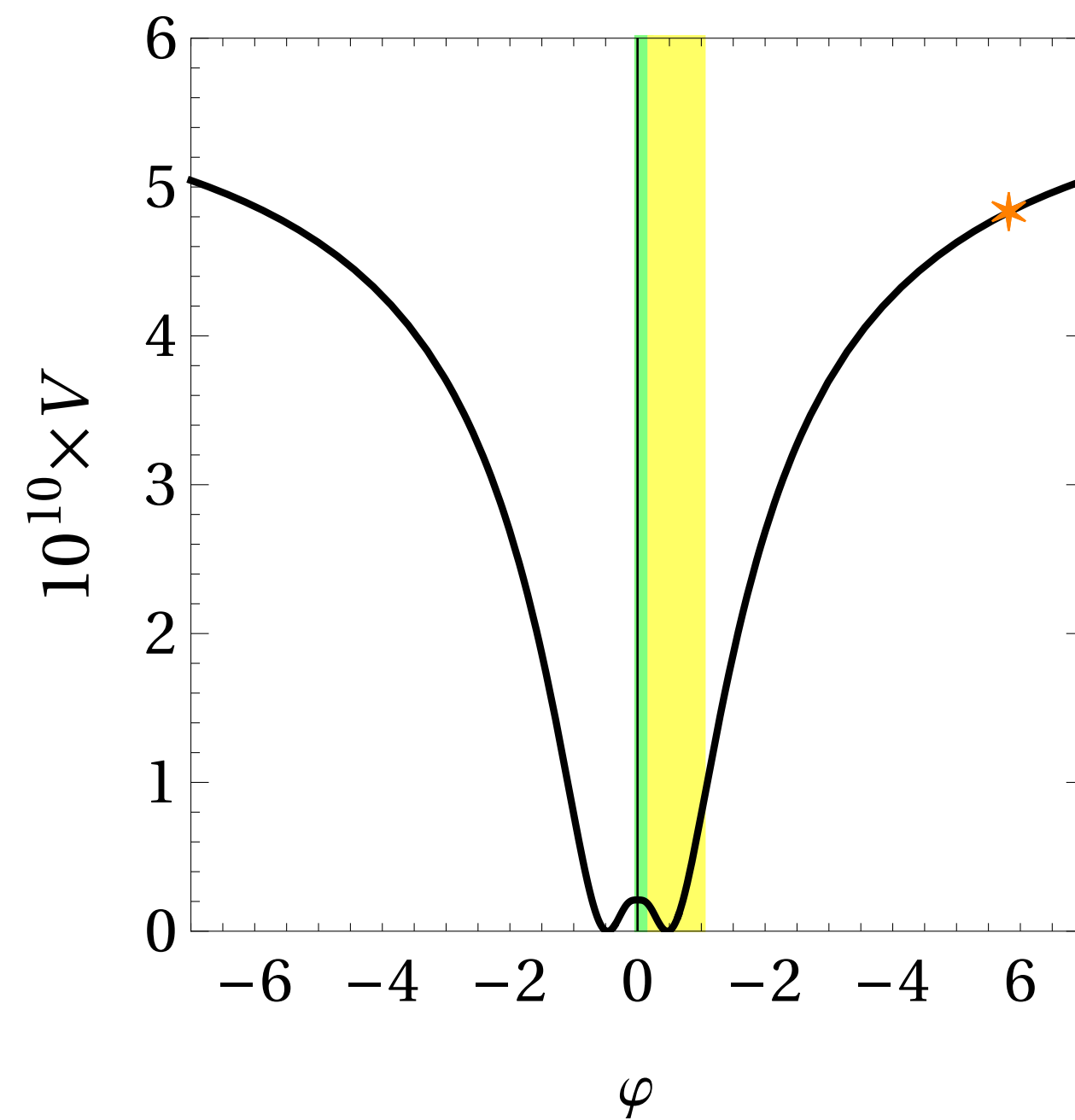
3. TALLER PEAKS IMPLY DEEPER DIPS

$$\frac{P_{\zeta}^{\text{peak}}}{P_{\zeta}^{\text{CMB}}} \propto \left(\frac{P_{\zeta}^{\text{dip}}}{P_{\zeta}^{\text{CMB}}} \right)^{-2}$$



4. FASTER ROLL => TALLER WIGGLES

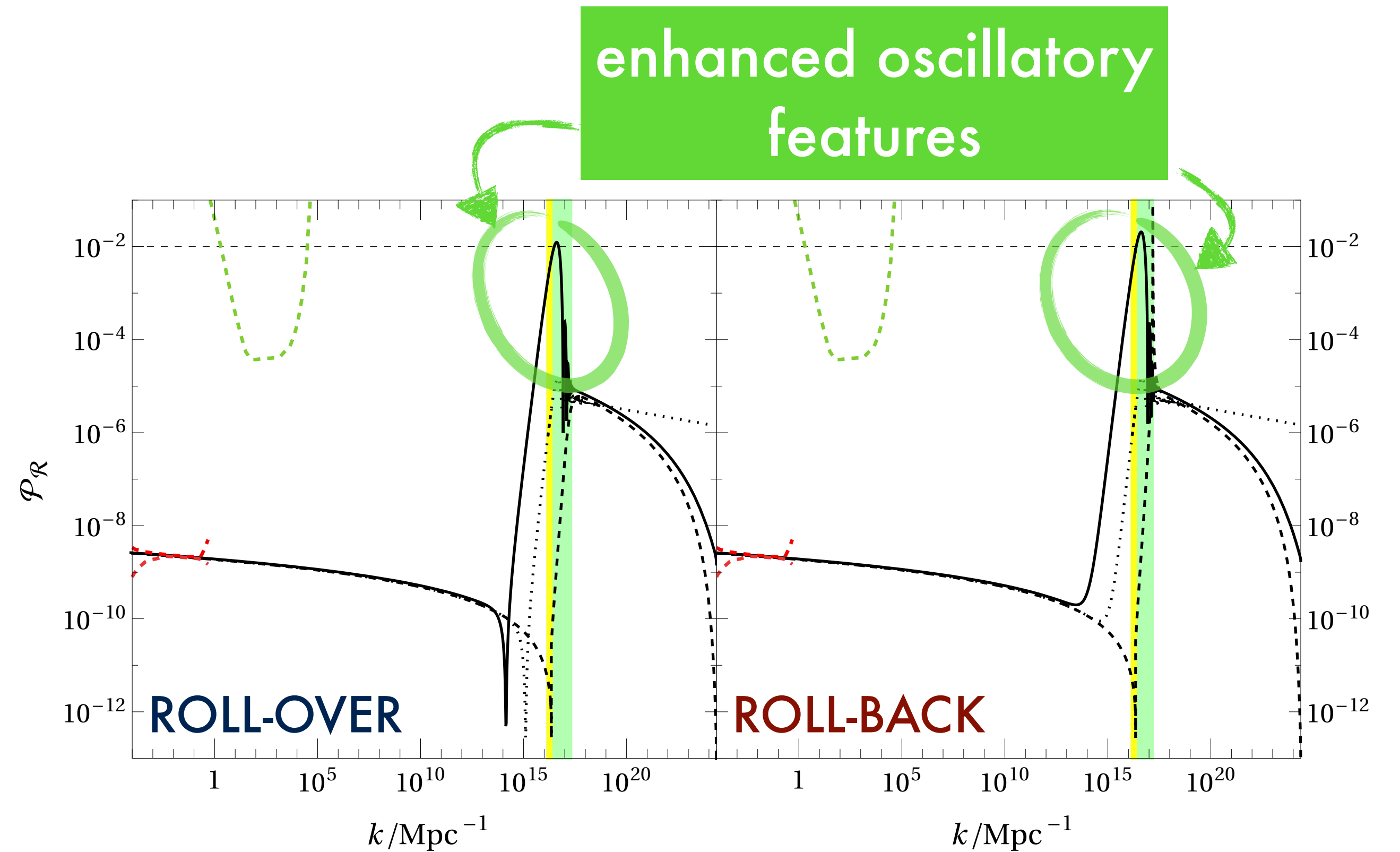
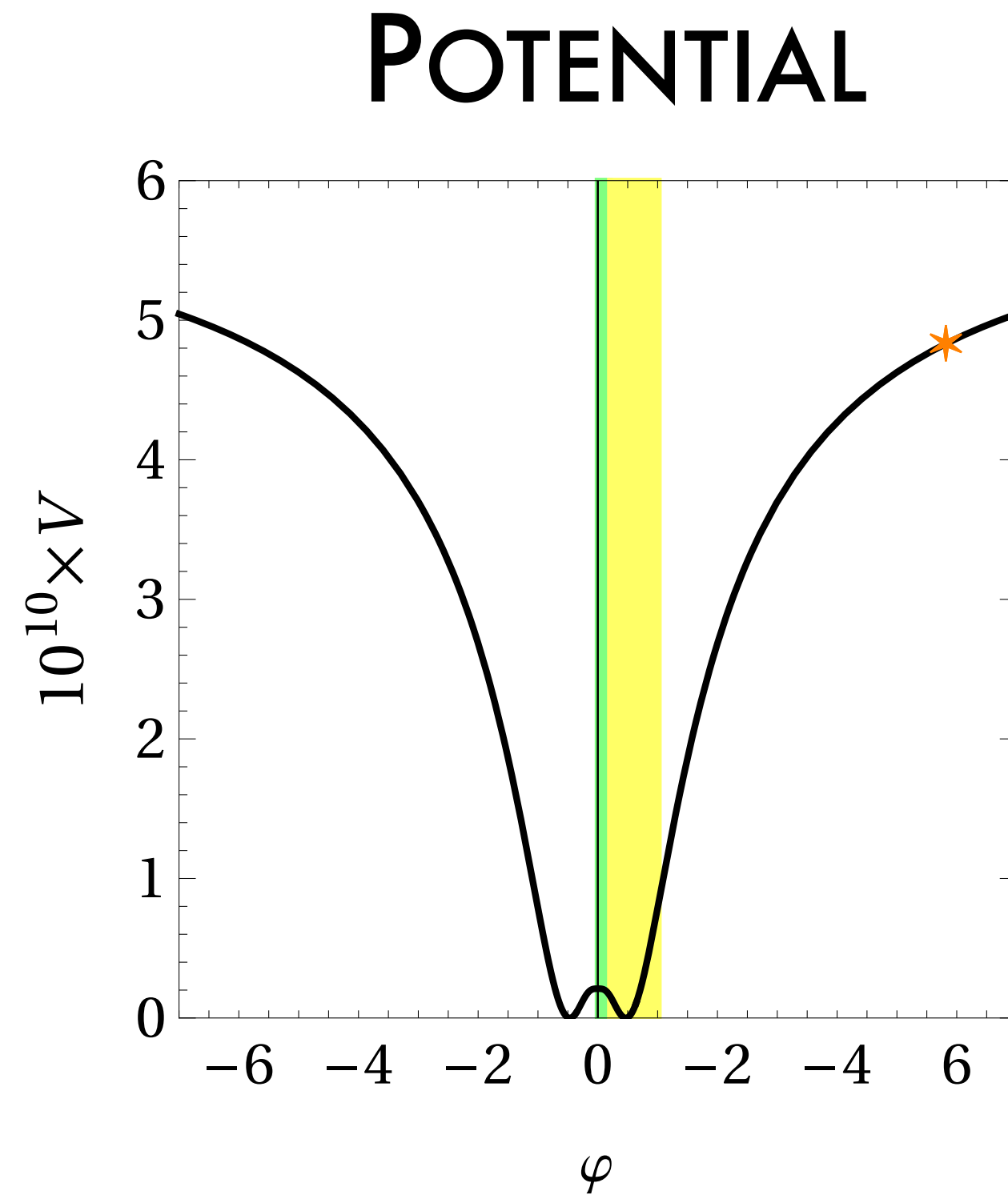
POTENTIAL



POWER SPECTRUM

* same inflationary timeline (SR to USR to SR)

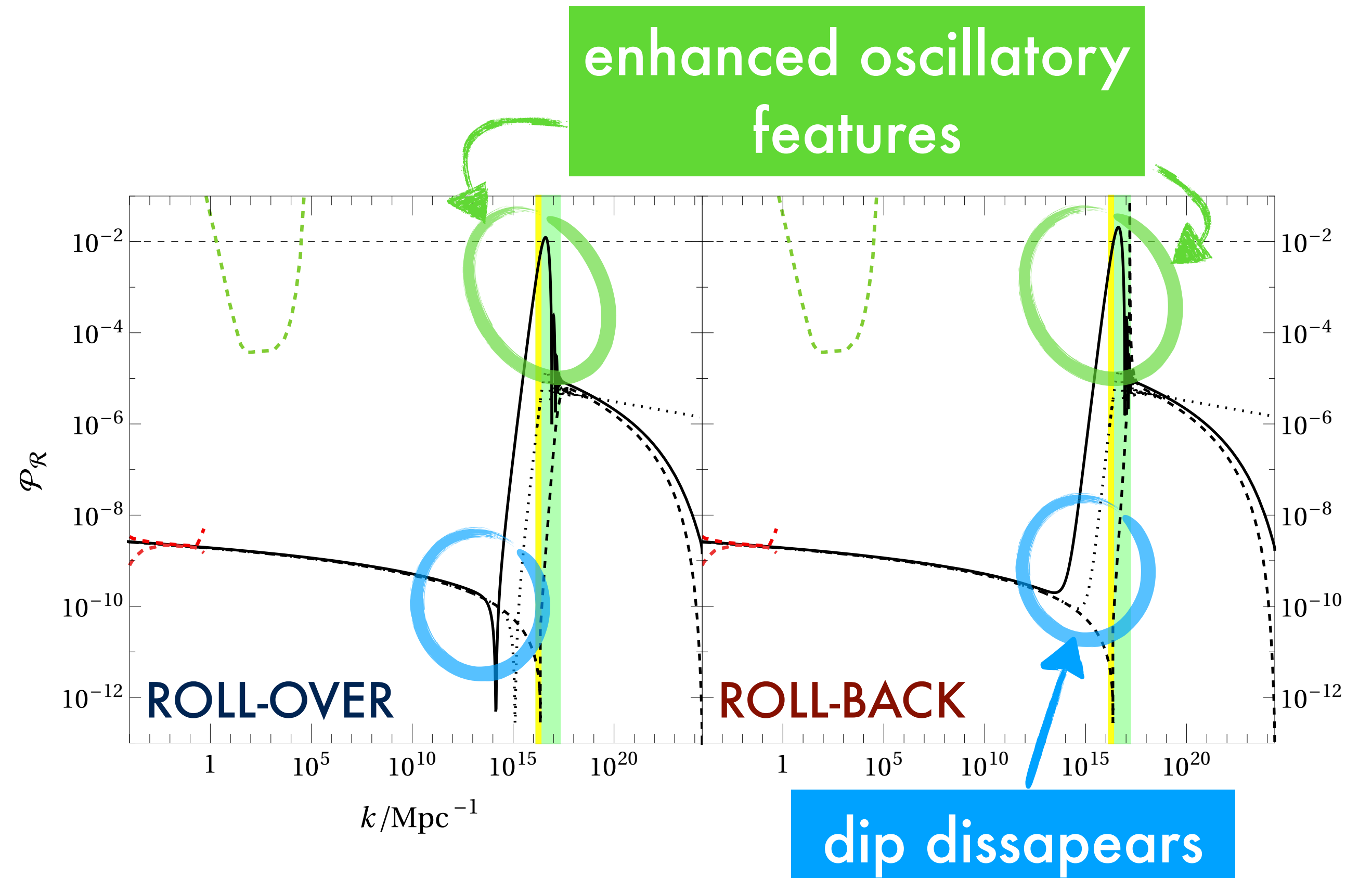
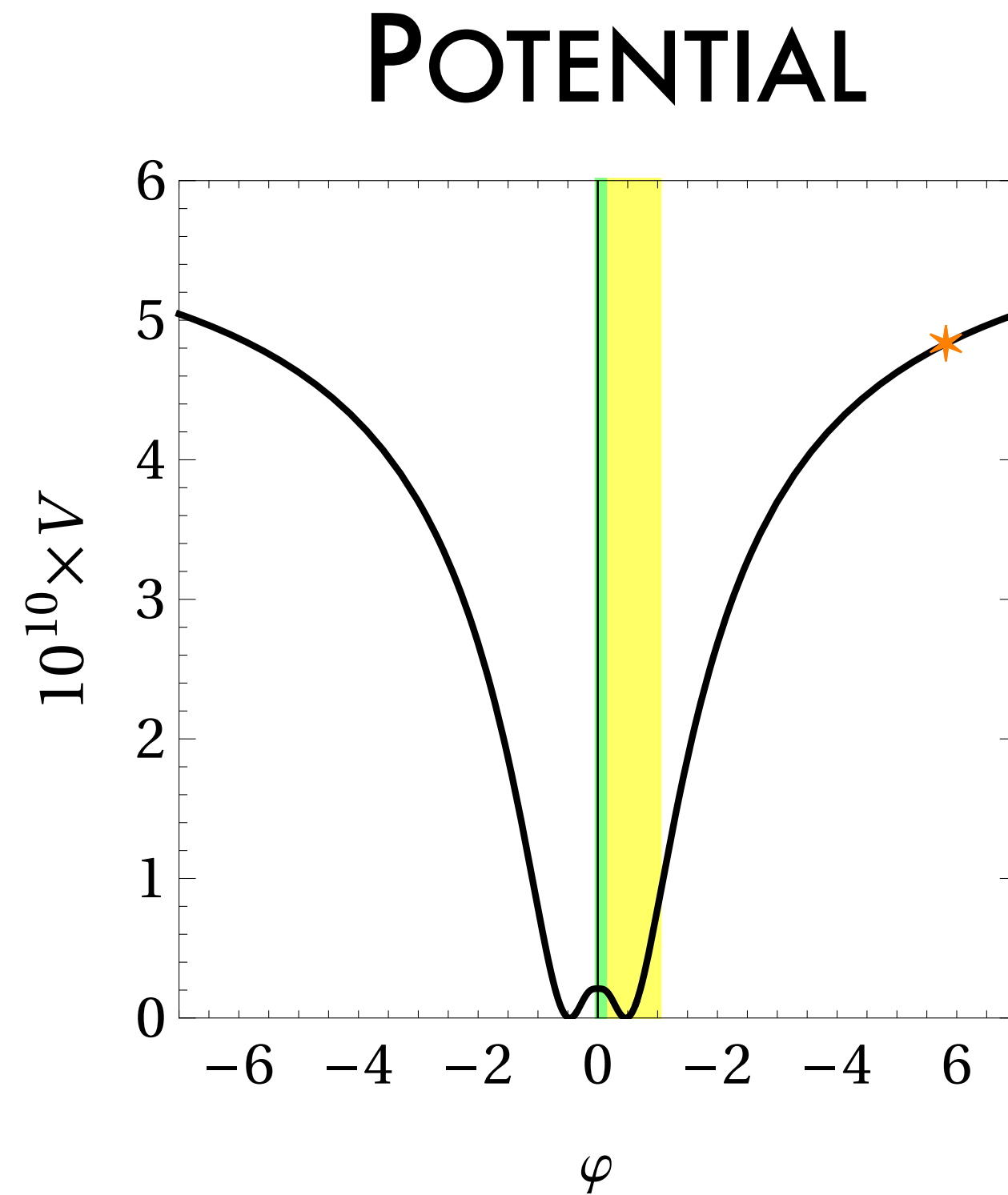
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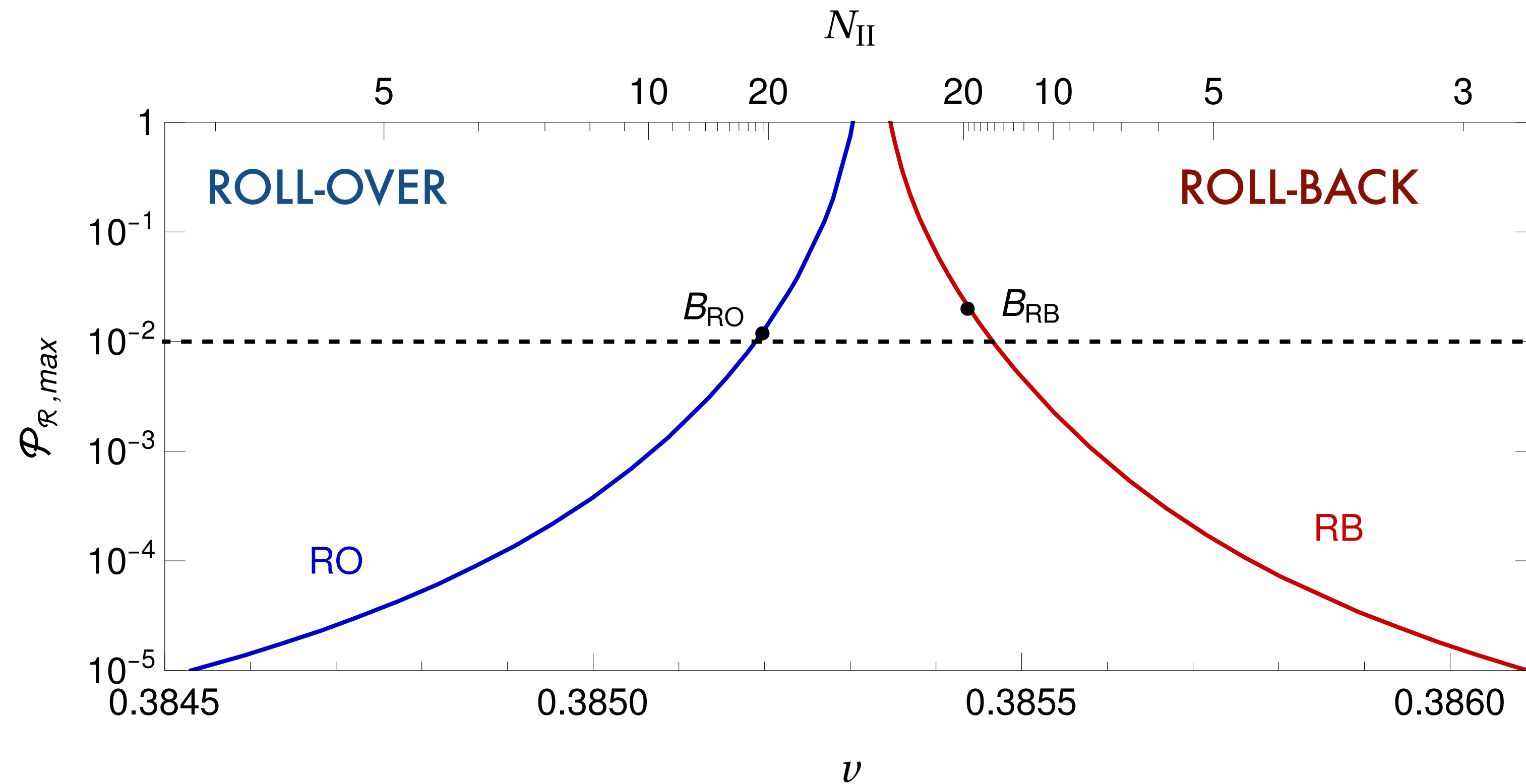


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NESSECITY for
TUNING:

1. abundance *exponentially* sensitive to *peak height*
2. *peak height* sensitive to parameters of the model

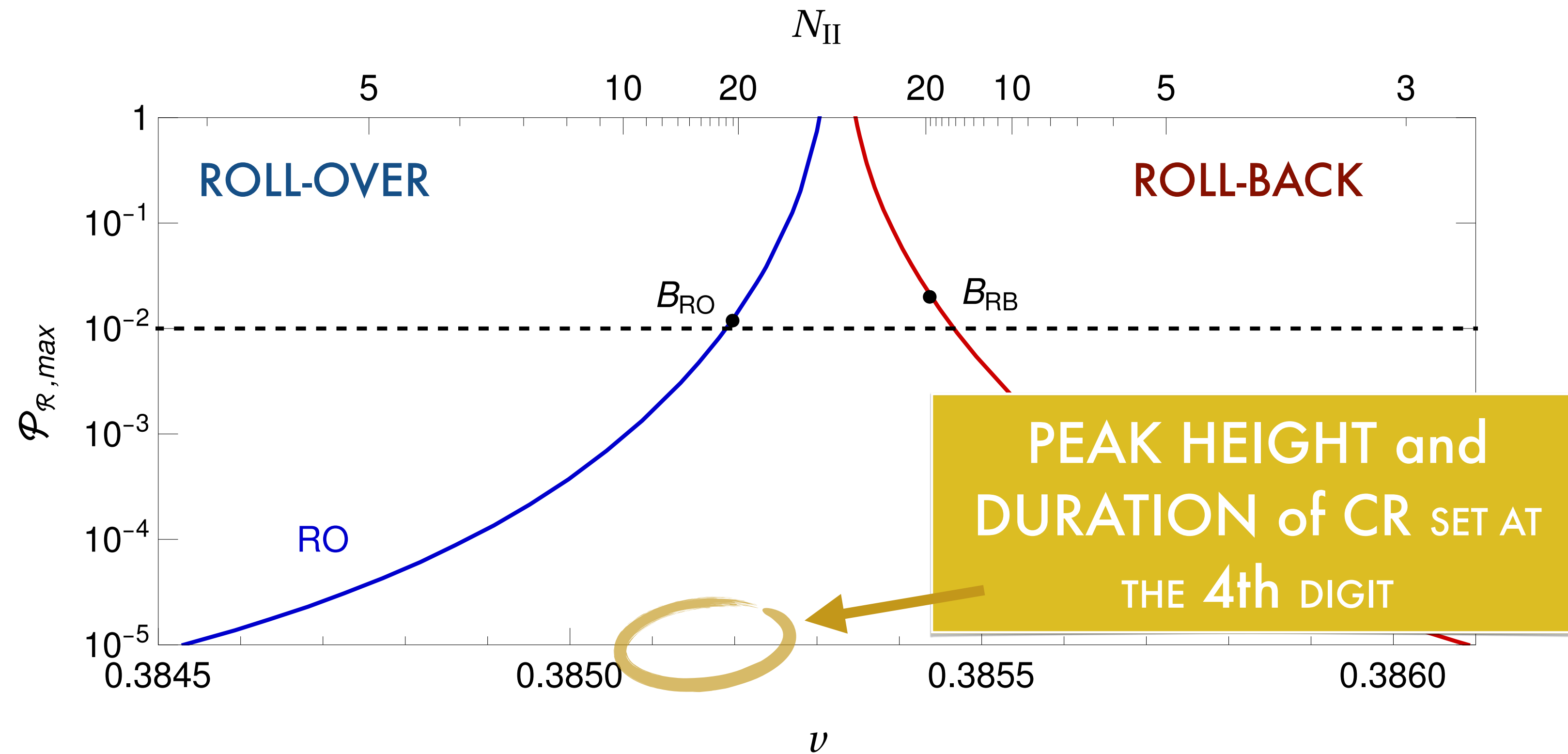
$$\mathcal{P}_{\mathcal{R},\max} \approx 5 \times 10^{-16} |v/v_c - 1|^{-3.9}$$



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SEARCHING FOR MODELS

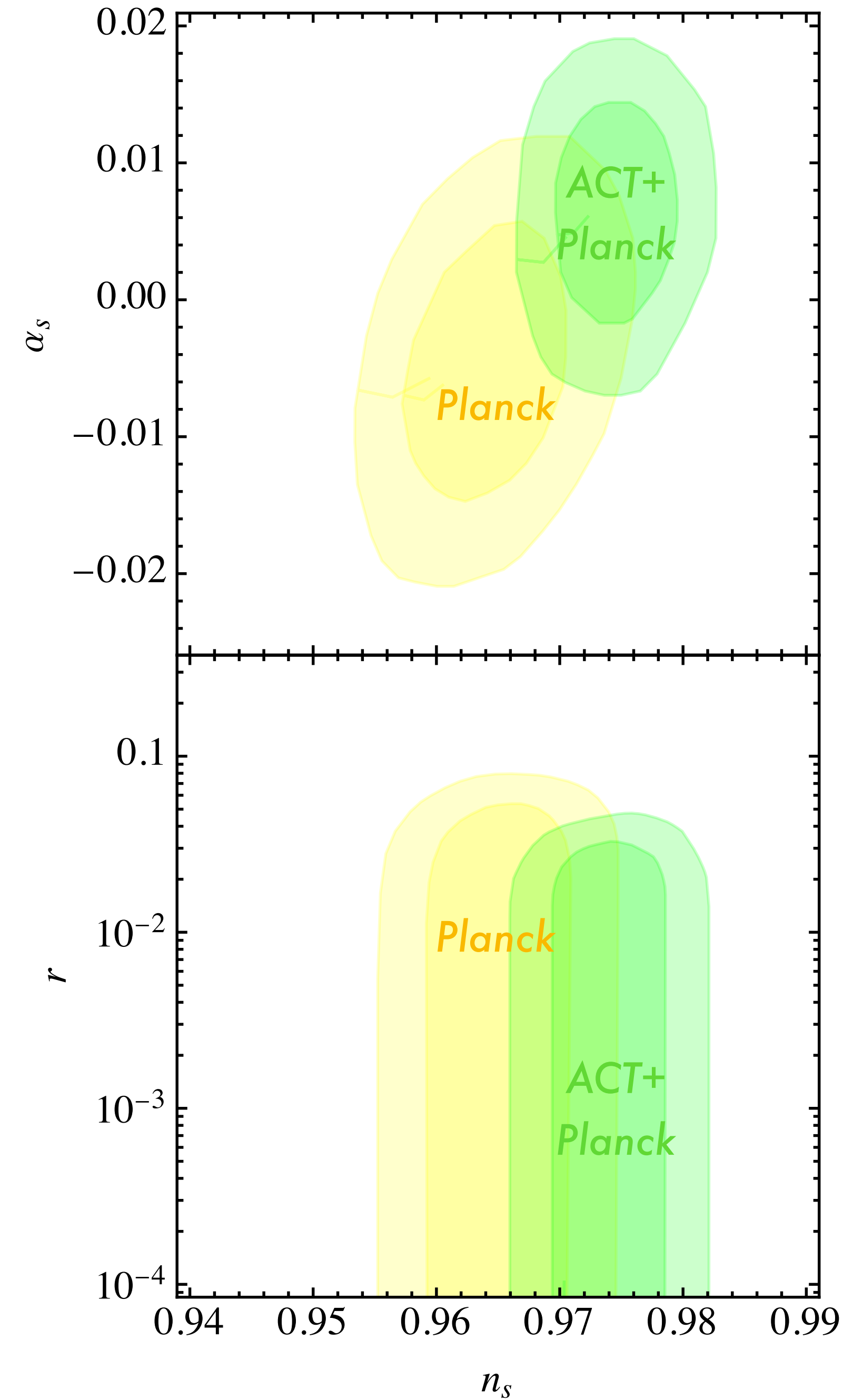
OF SINGLE FIELD INFLATION

SINGLE-FIELD MODELS for CMB and PBHs

$$r \lesssim 0.08 \quad (\text{Planck})$$
$$r \lesssim 0.05 \quad (\text{ACT+Planck})$$

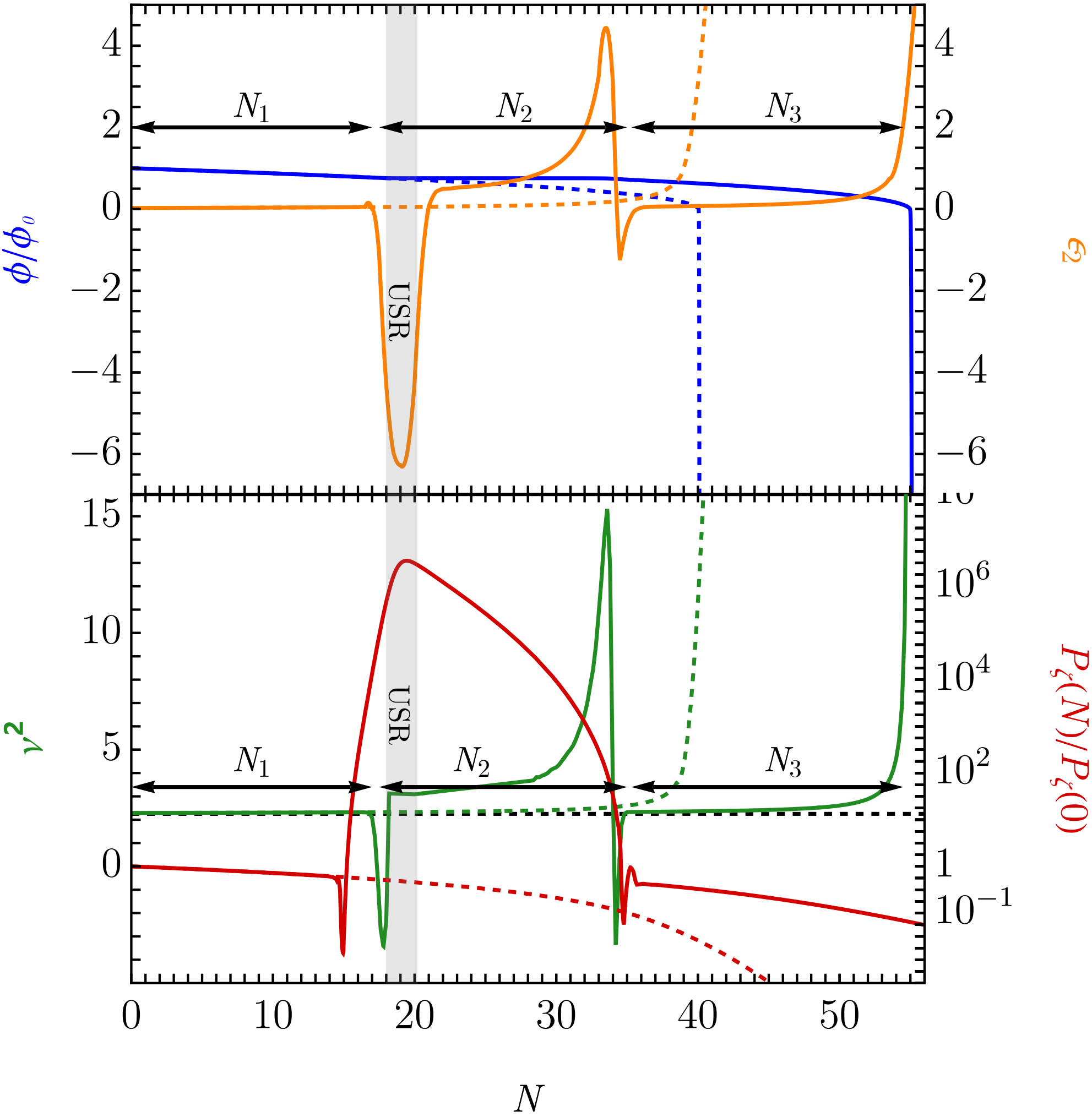
$$n_s = 0.9641 \pm 0.0044 \quad (\text{Planck})$$
$$n_s = 0.9743 \pm 0.0034 \quad (\text{ACT+Planck})$$

$$\alpha_s = -0.010 \pm 0.004 \quad (\text{Planck})$$
$$\alpha_s = 0.0062 \pm 0.0052 \quad (\text{ACT+Planck})$$



SINGLE-FIELD MODELS for CMB and PBHs:
A QUICK GUIDE TO MODEL BUILDING

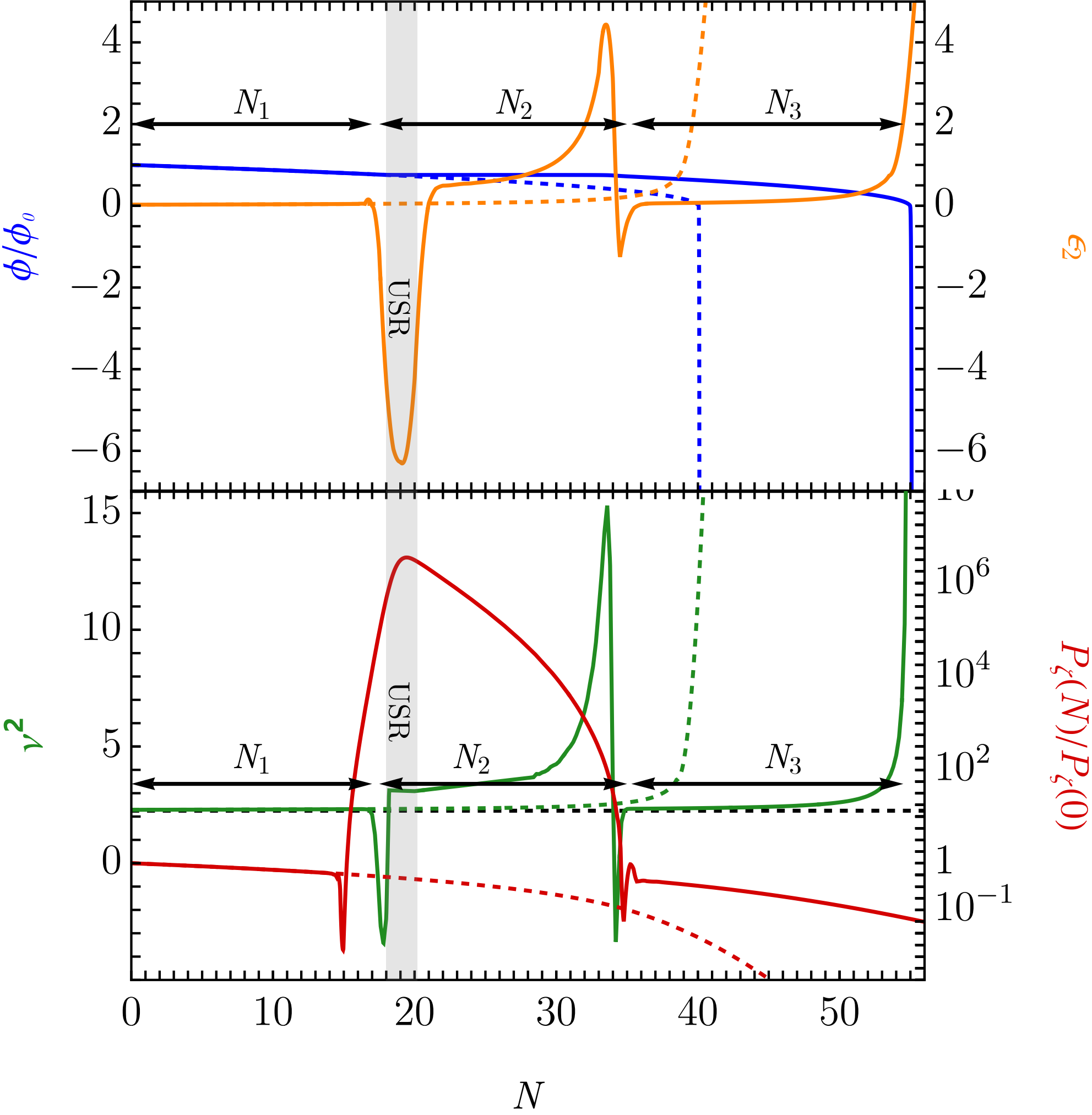
$$N_{\text{tot}} \approx \underbrace{\int_{\phi_2}^{\phi_1} d\phi \frac{V}{\partial_\phi V}}_{=N_1} + \underbrace{\int_{\phi_3}^{\phi_2} \frac{d\phi}{\sqrt{2\epsilon_1}}}_{=N_2} + \underbrace{\int_{\phi_{\text{end}}}^{\phi_3} d\phi \frac{V}{\partial_\phi V}}_{=N_3}$$



SINGLE-FIELD MODELS for CMB and PBHs: A QUICK GUIDE TO MODEL BUILDING

- Find a potential that gives **CMB** with $N_{\text{SR}} \approx \mathcal{O}(20 - 40)$ e-folds.

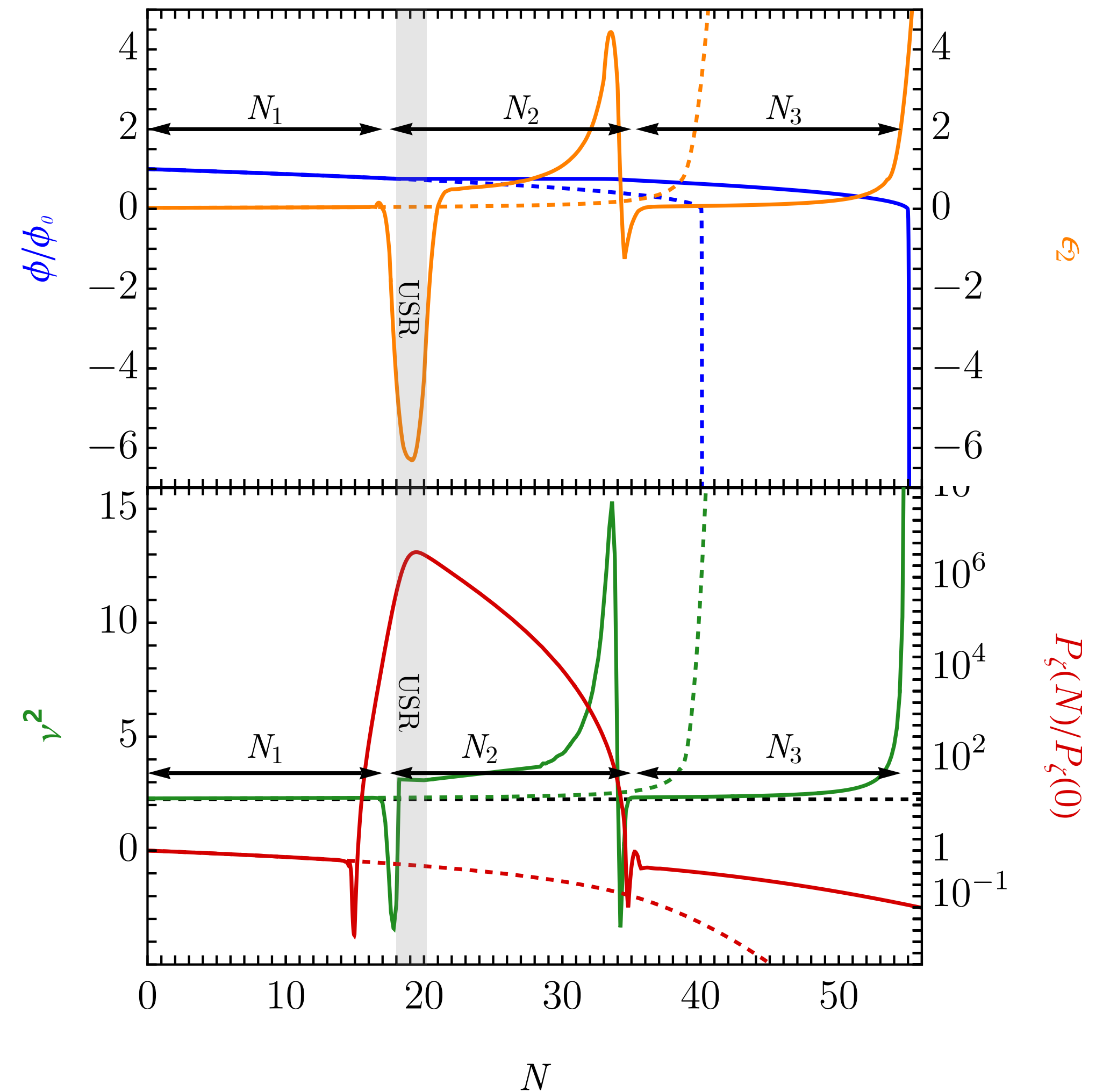
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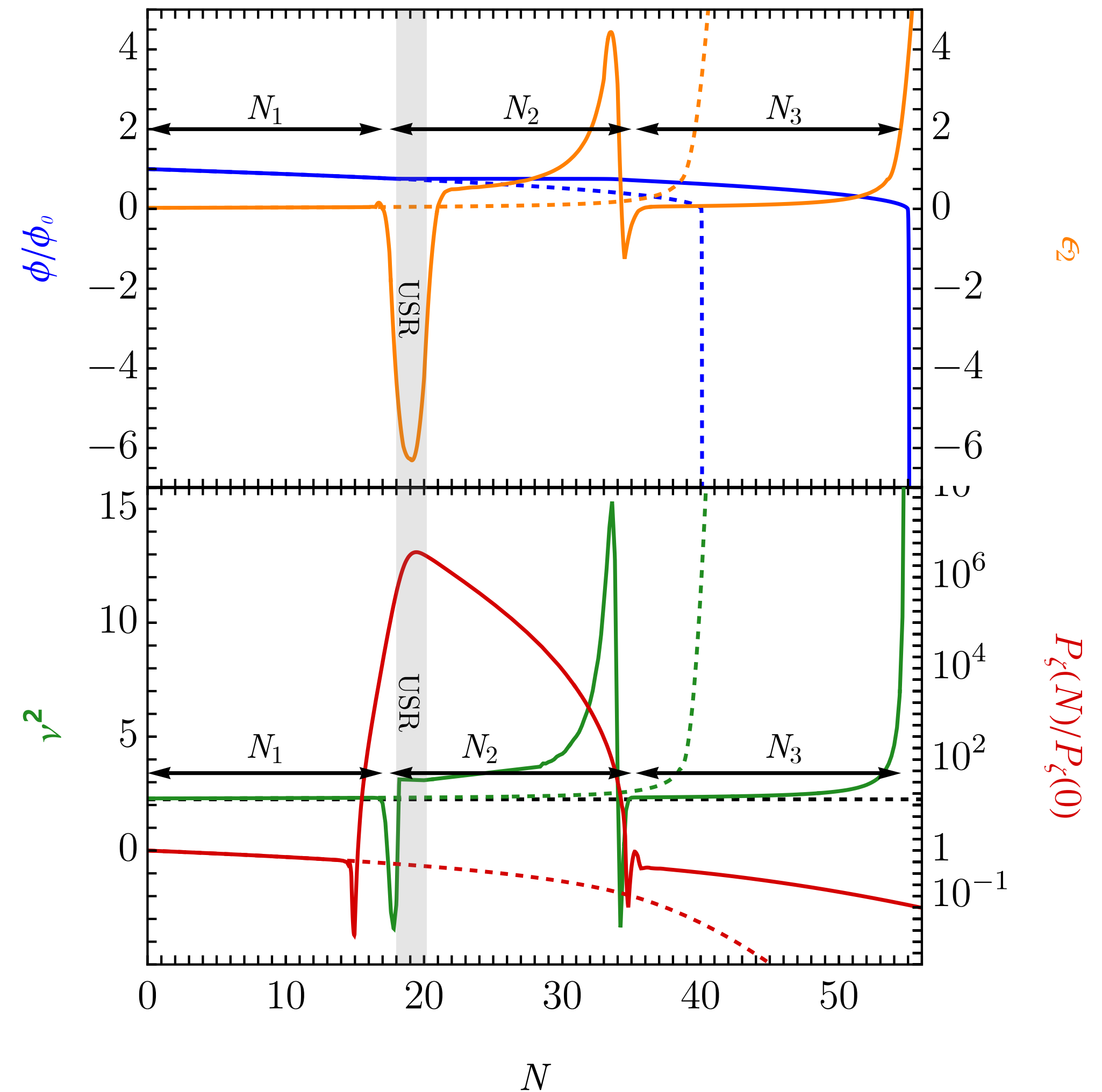
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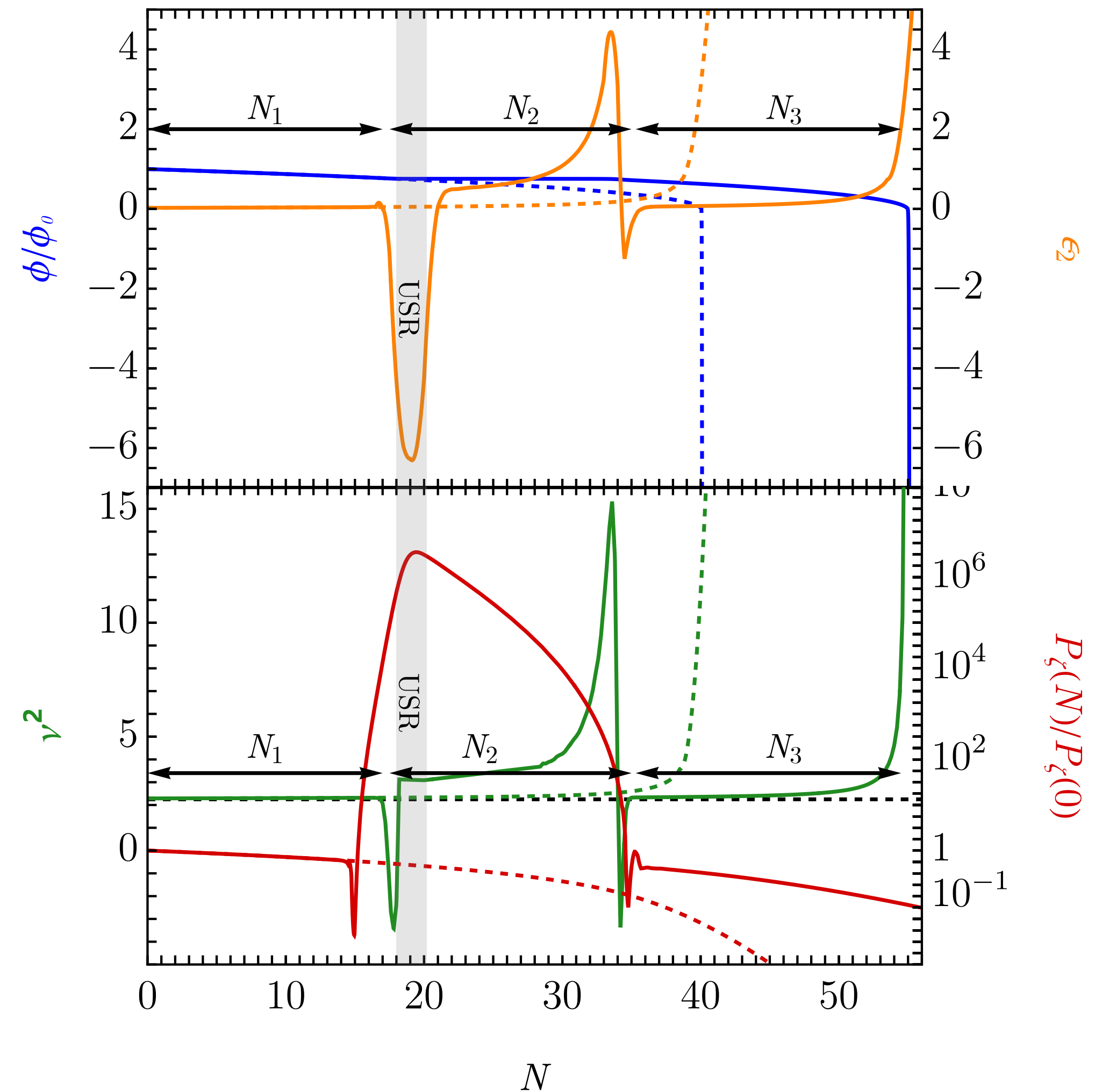


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SINGLE-FIELD MODELS for CMB and PBHs: A QUICK GUIDE TO MODEL BUILDING

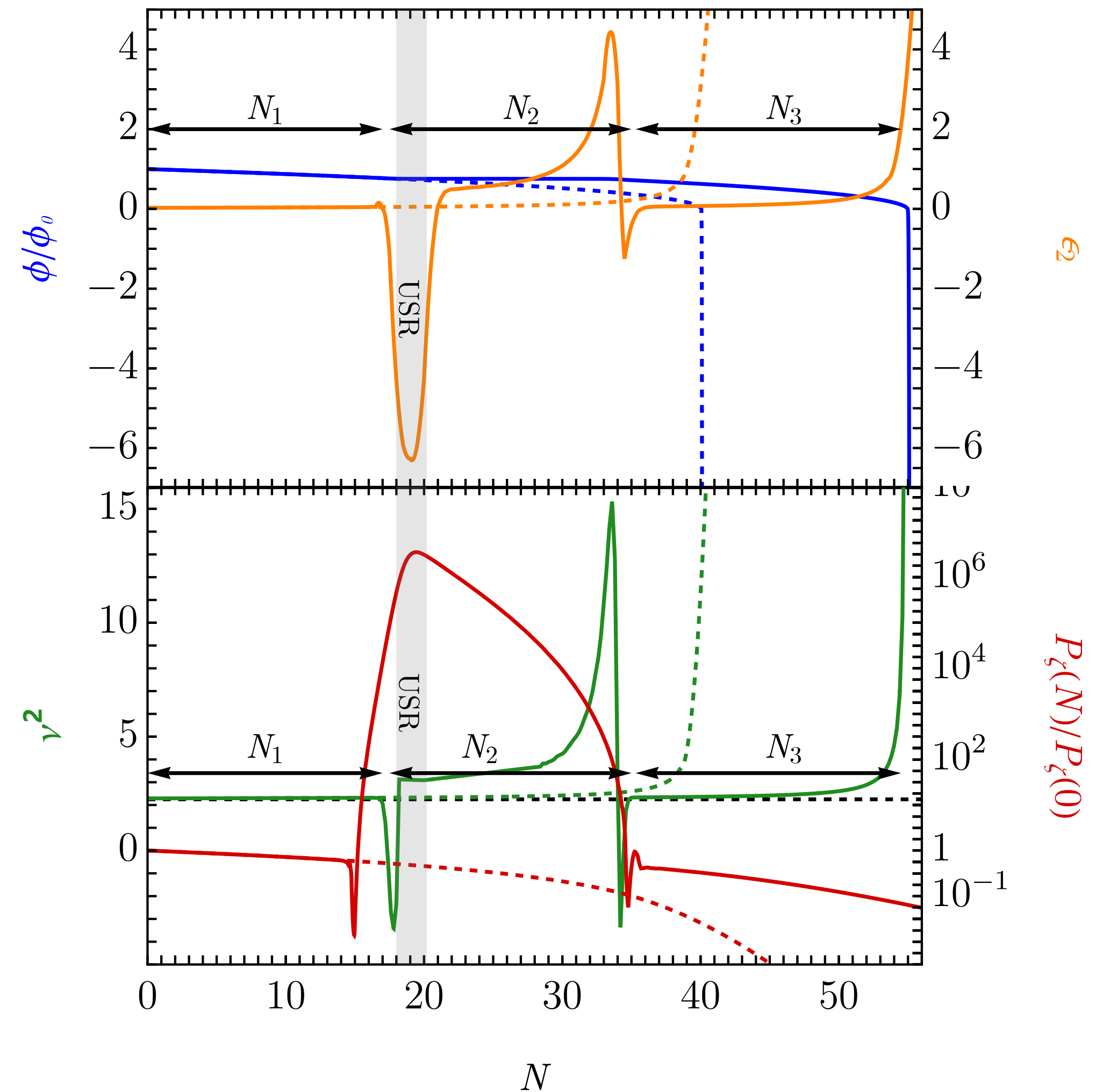
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$$N_{\text{tot}} = N_{\text{SR}} + N_2 \approx 55 \quad (N_{\text{SR}} = N_1 + N_3)$$

- Choose N_1 to fix $M_{\text{PBH}} \simeq 1M_{\odot} \times e^{-2(N_1-18)}$

$$M_{\text{PBH}} \simeq [10^{-17}, 10] M_{\odot} \quad \Leftrightarrow \quad N_1 \in (17, 37)$$

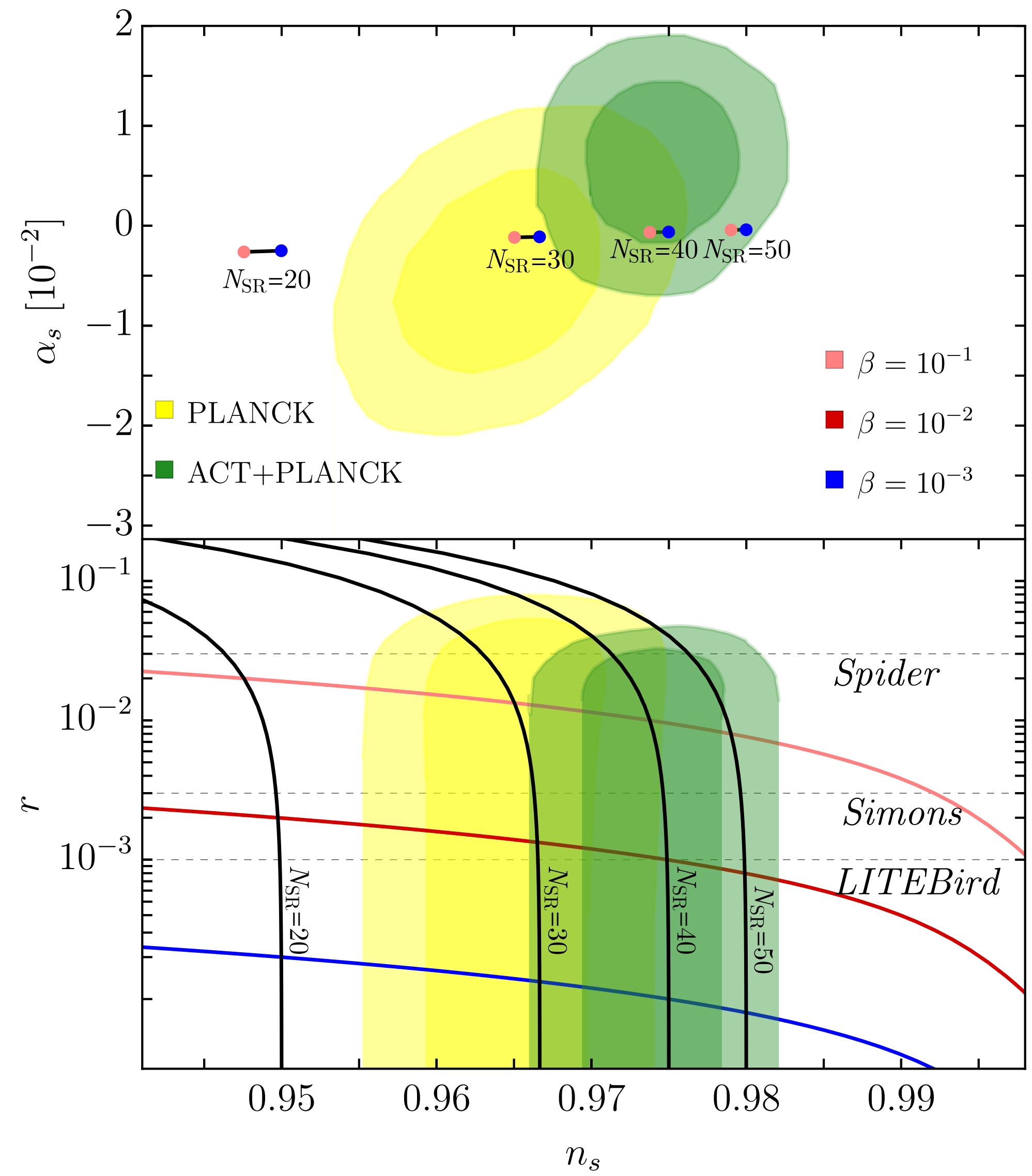
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SINGLE-FIELD MODELS for CMB and PBHs:
A QUICK GUIDE TO MODEL BUILDING

A **toy** model:

$$V(\phi) = V_0 \left(1 + \beta \ln \left(\frac{\phi}{M_{\text{Pl}}} \right) \right) + V_{\text{bump}}(\phi)$$



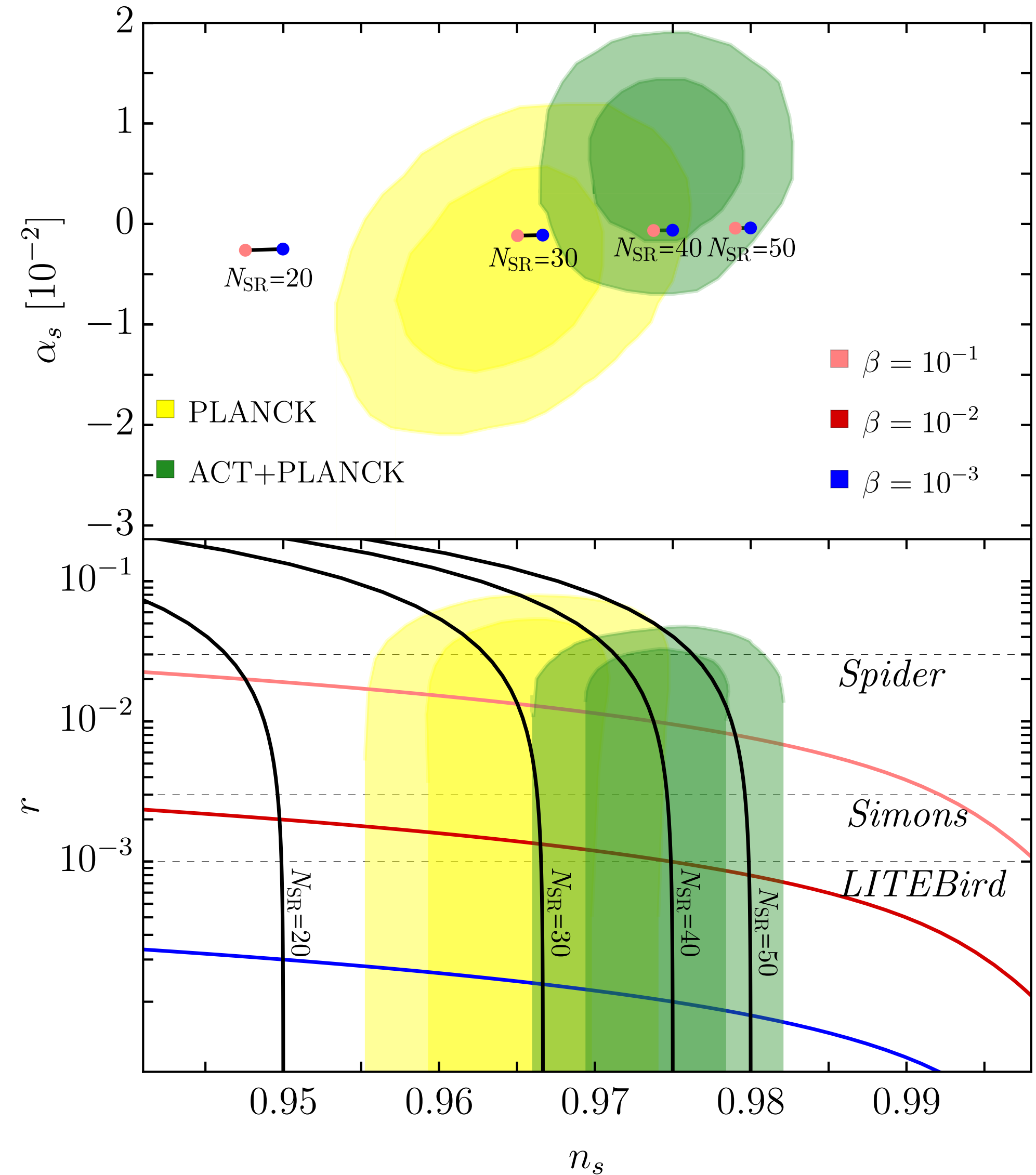
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$$r = \frac{4\beta}{N_{\text{SR}}}, \quad n_s = 1 - \frac{1}{N_{\text{SR}}}, \quad \alpha_s = -\frac{1}{N_{\text{SR}}^2}$$



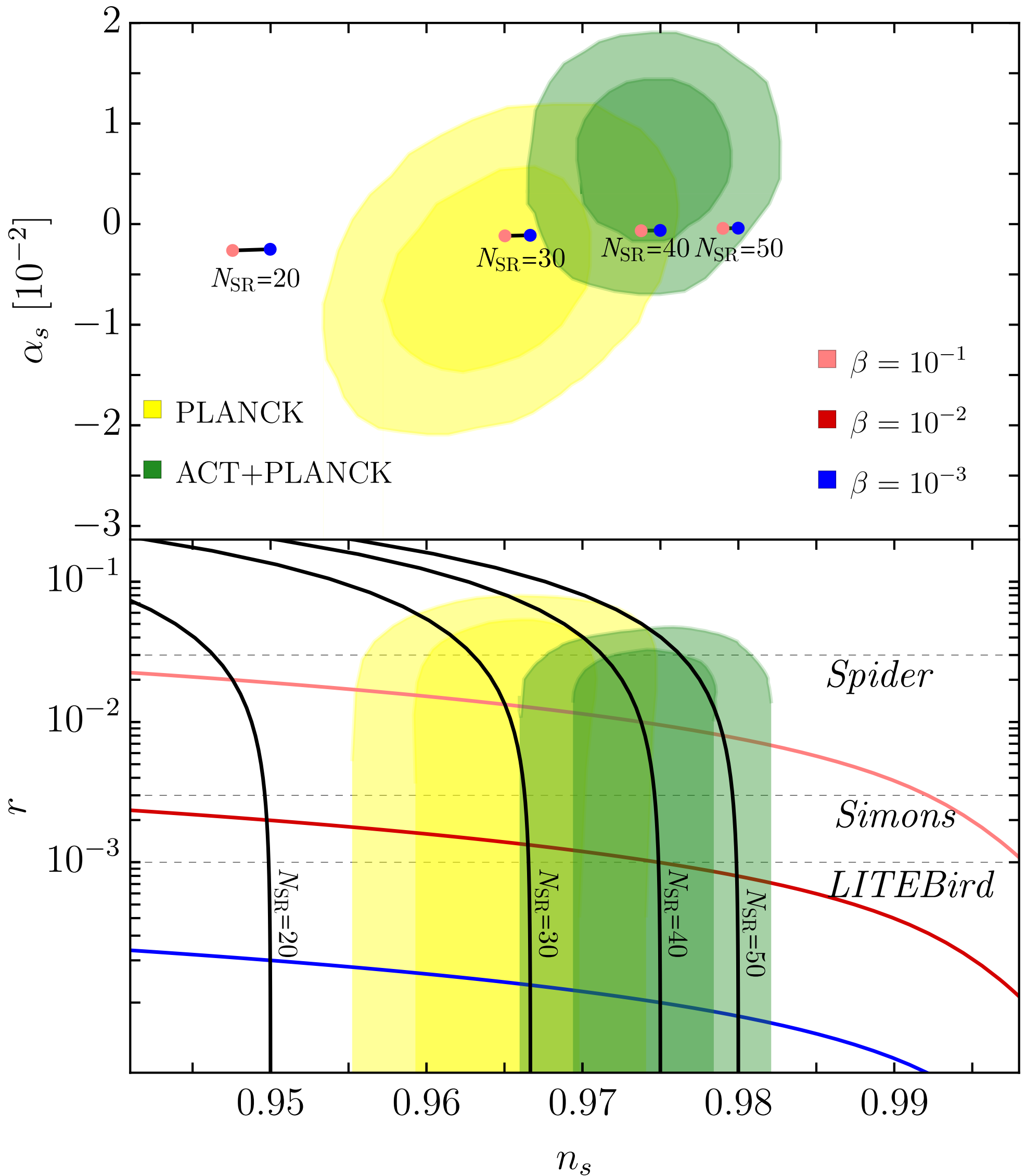
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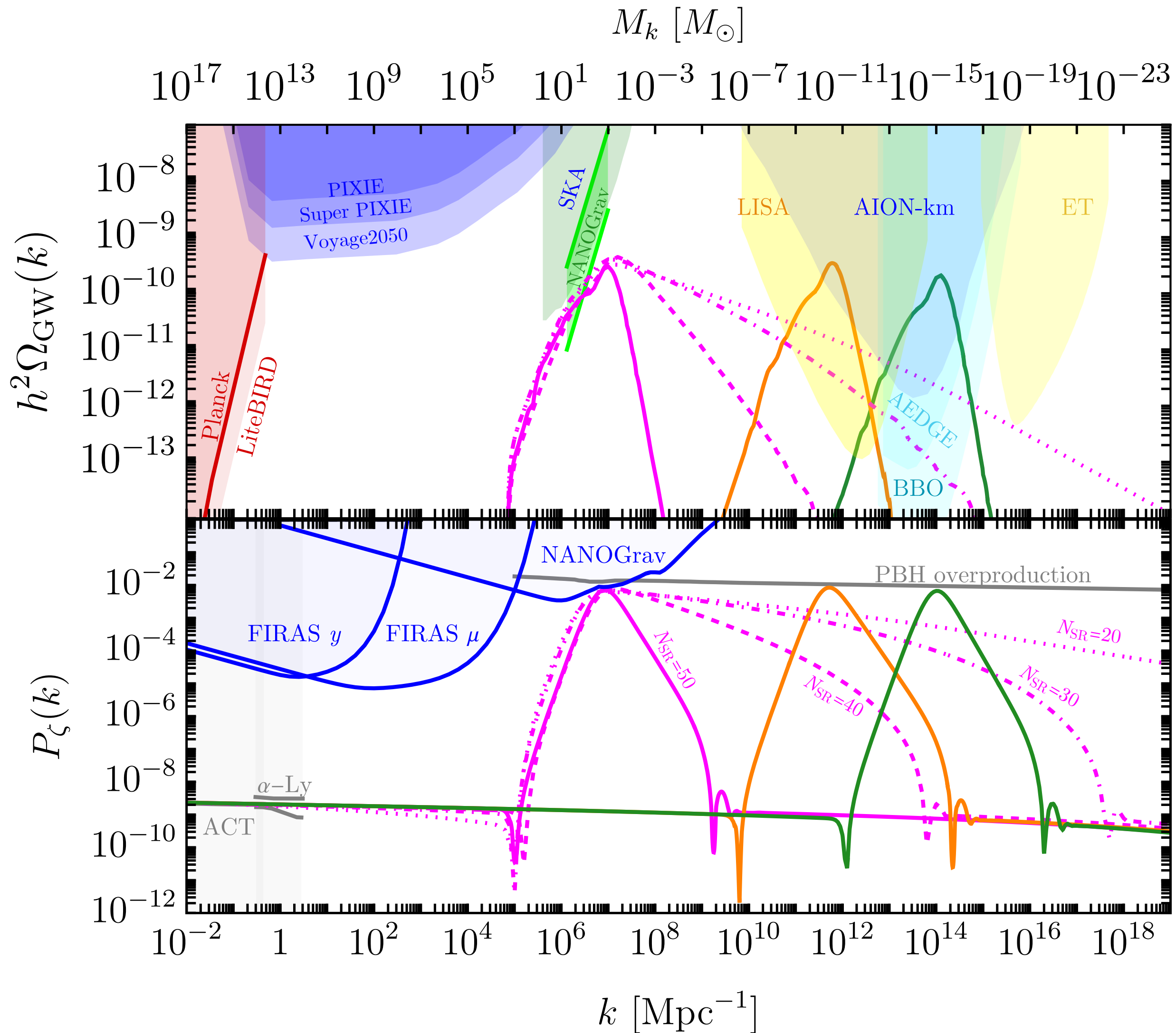
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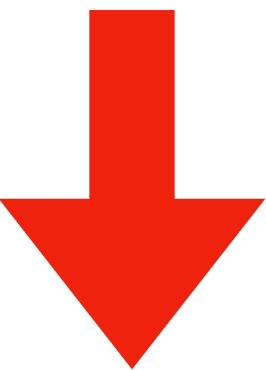
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SINGLE-FIELD MODELS for CMB and PBHs: A QUICK GUIDE TO MODEL BUILDING



$$V_{\text{bump}}(\phi) = V_0 \delta \exp\left(-\frac{(\phi - \phi_0)^2}{2\sigma_\phi^2}\right)$$



$$\phi_0 \approx \phi(N_1)$$

$$n_{s,\text{CR}} - 1 \approx 2\eta_V(\phi_0) \approx -\frac{\delta}{\sigma_\phi^2}$$

NON-MINIMAL POLYNOMIAL INFLATION:

$$S = \int d^4x \sqrt{-g} \left[-\frac{M_{\text{Pl}}^2}{2} \Omega(\phi) R + \frac{1}{2} (\partial\phi)^2 - V(\phi) \right]$$

$$V(\phi) = M_{\text{Pl}}^4 \sum_{n \geq 1} a_n \left(\frac{\phi}{M_{\text{Pl}}} \right)^n$$

up to ϕ^6

$$\Omega(\phi) = 1 + \sum_{n \geq 1} \xi_n \left(\frac{\phi}{M_{\text{Pl}}} \right)^n$$

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NON-MINIMAL POLYNOMIAL INFLATION:

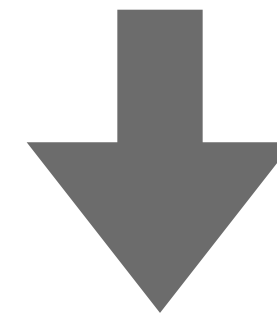
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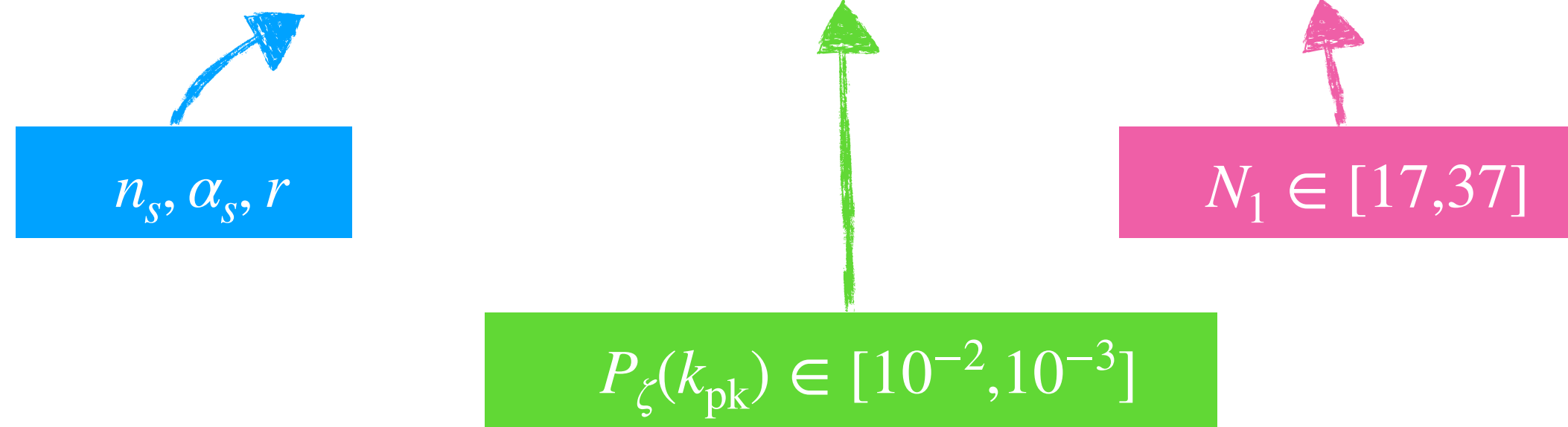


$(7 - 3) + 2 = 6$ parameters

SINGLE-FIELD MODELS for CMB and PBHs: SCANNING for MODELS

MOCK-LIKELIHOOD

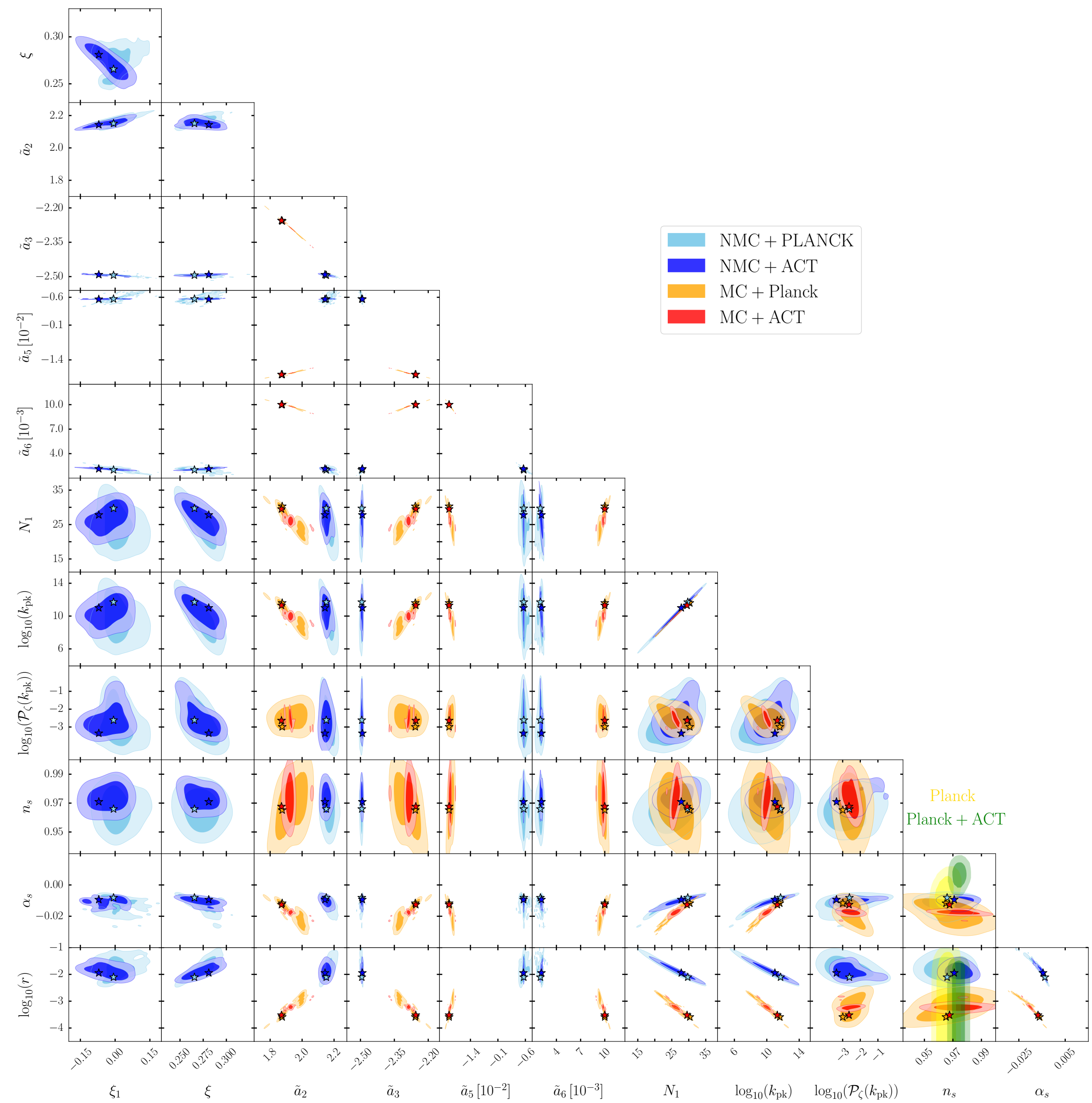
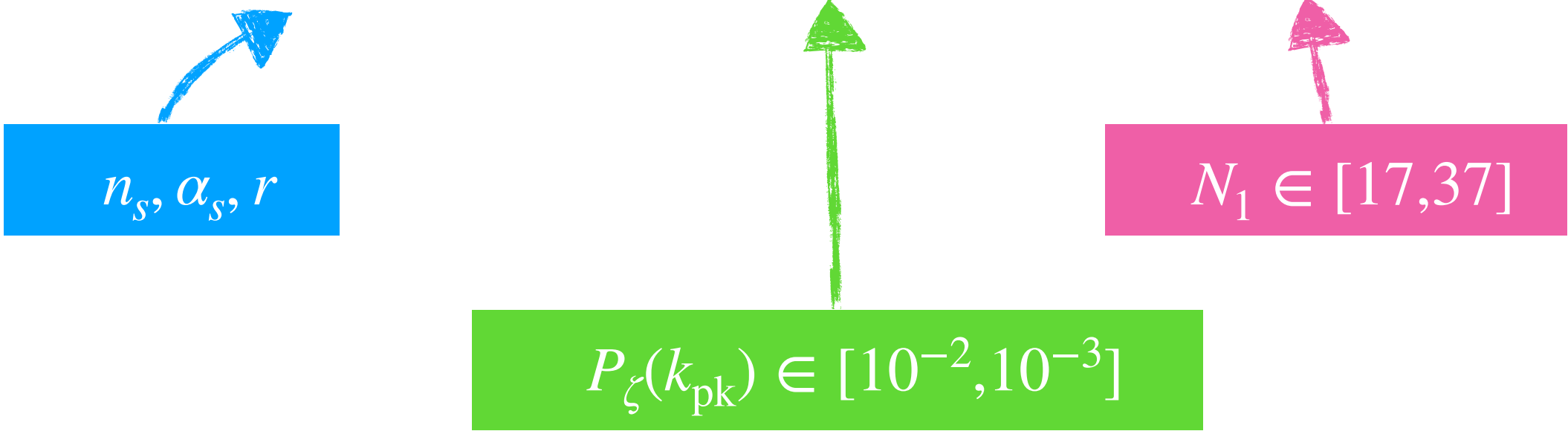
$$\mathcal{L}(D) = \mathcal{L}_{\text{CMB}}(D | \boldsymbol{\theta}) \times \mathcal{L}_{\text{PBH}}(D | P_{\zeta}(k_{\text{pk}})) \times \mathcal{L}(D | N_1)$$



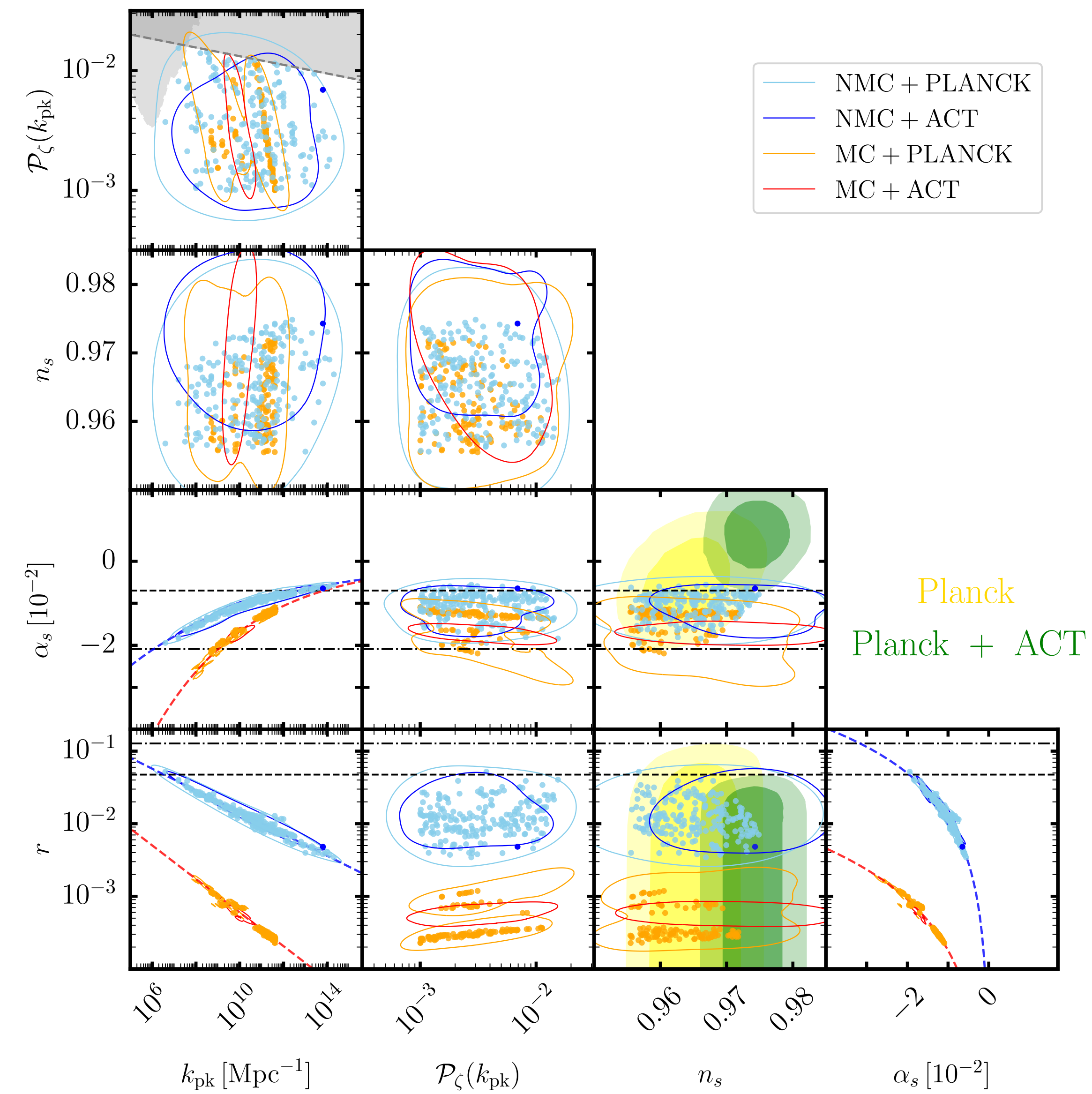
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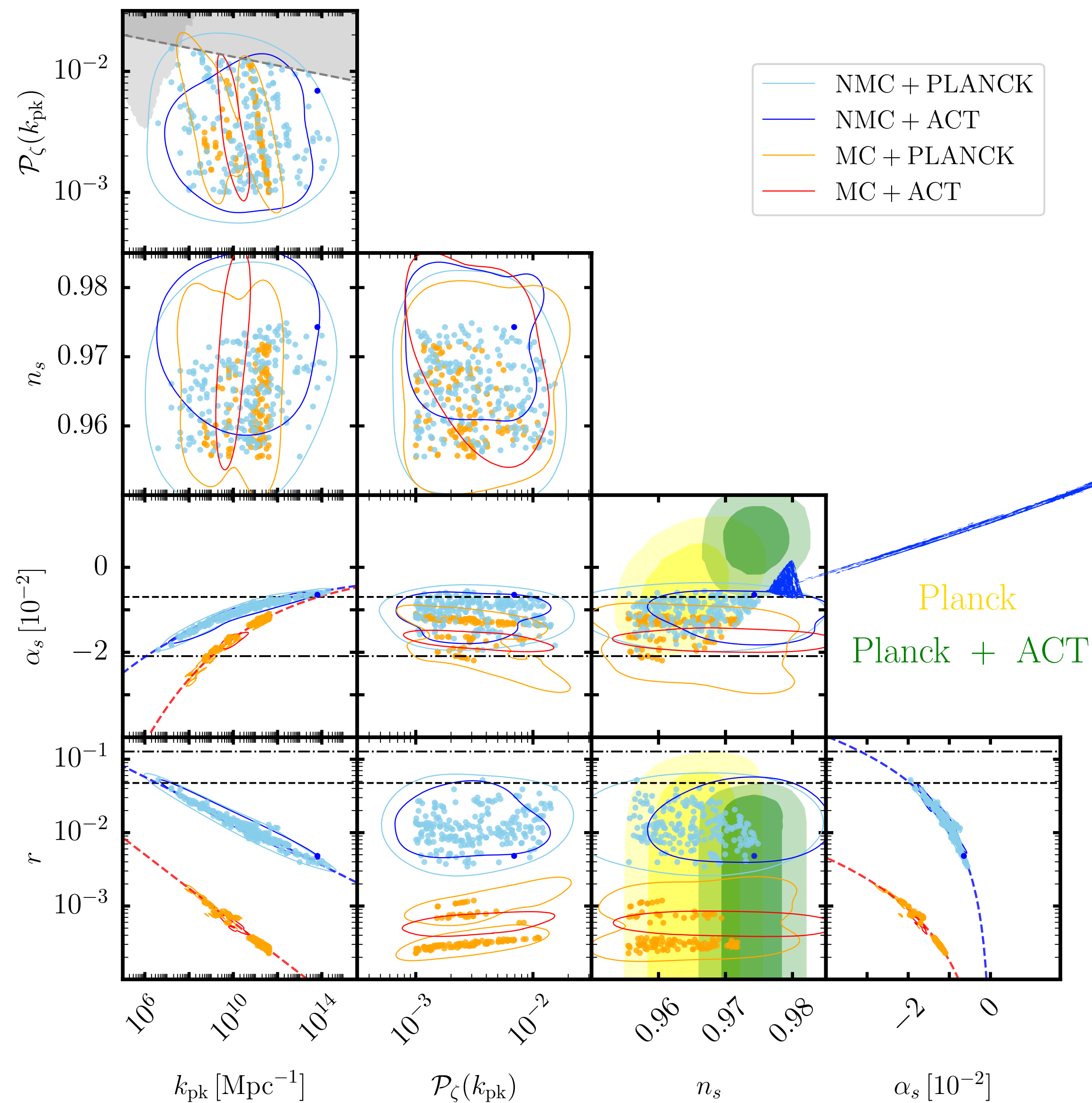
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SINGLE-FIELD MODELS for CMB and PBHs:
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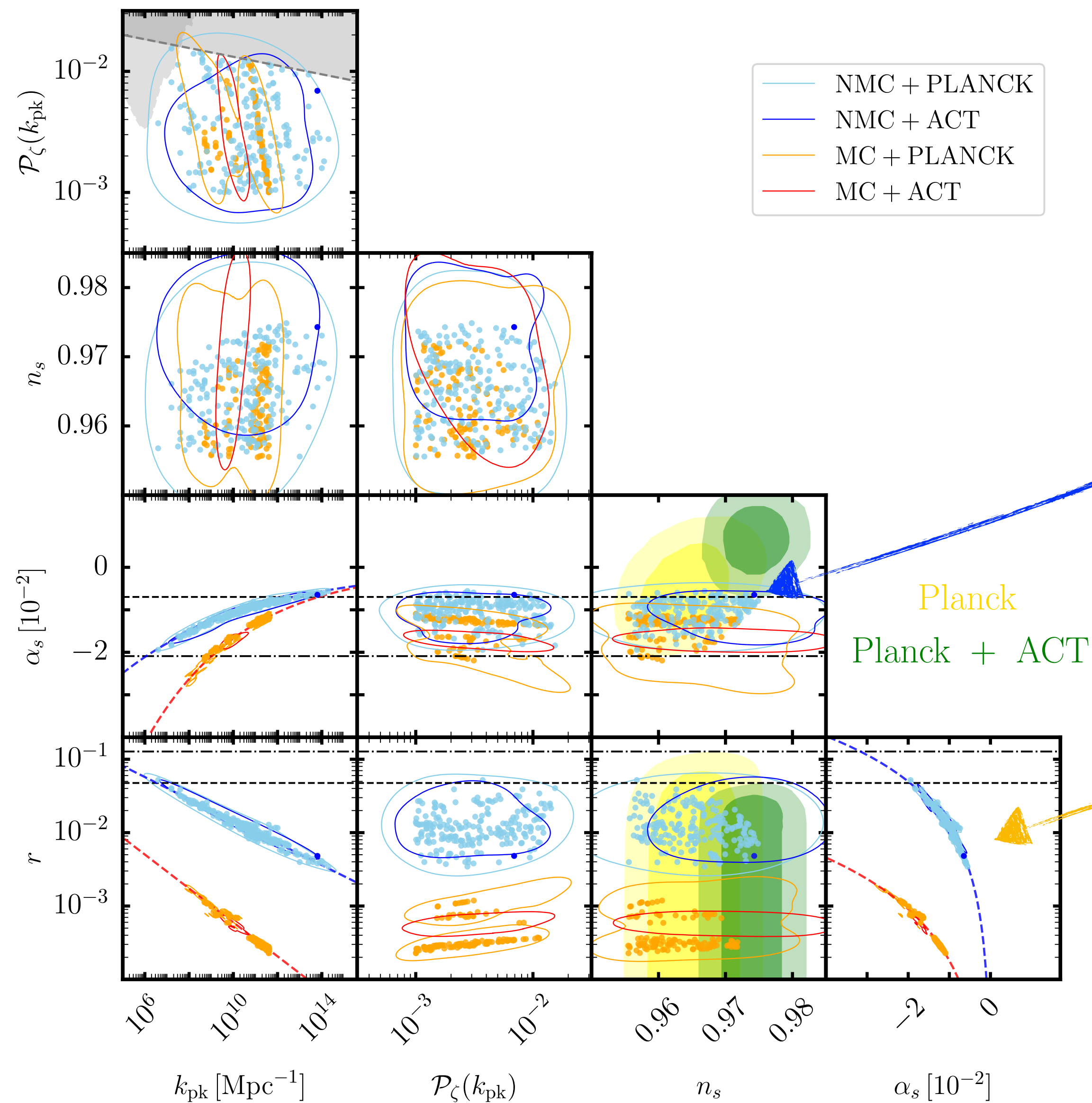


SINGLE-FIELD MODELS for CMB and PBHs: SCANNING for MODELS



one model found for
ACT-Planck

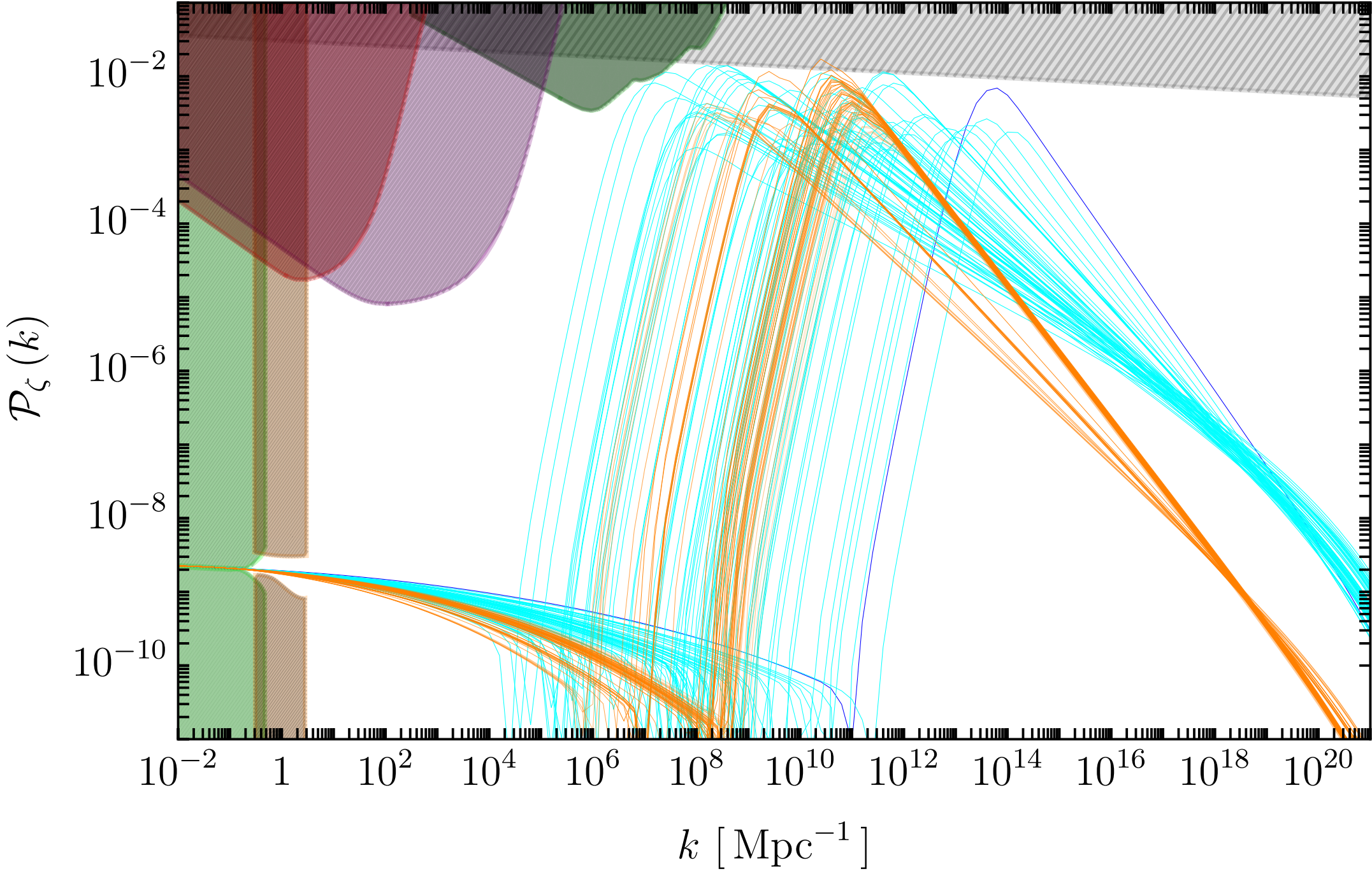
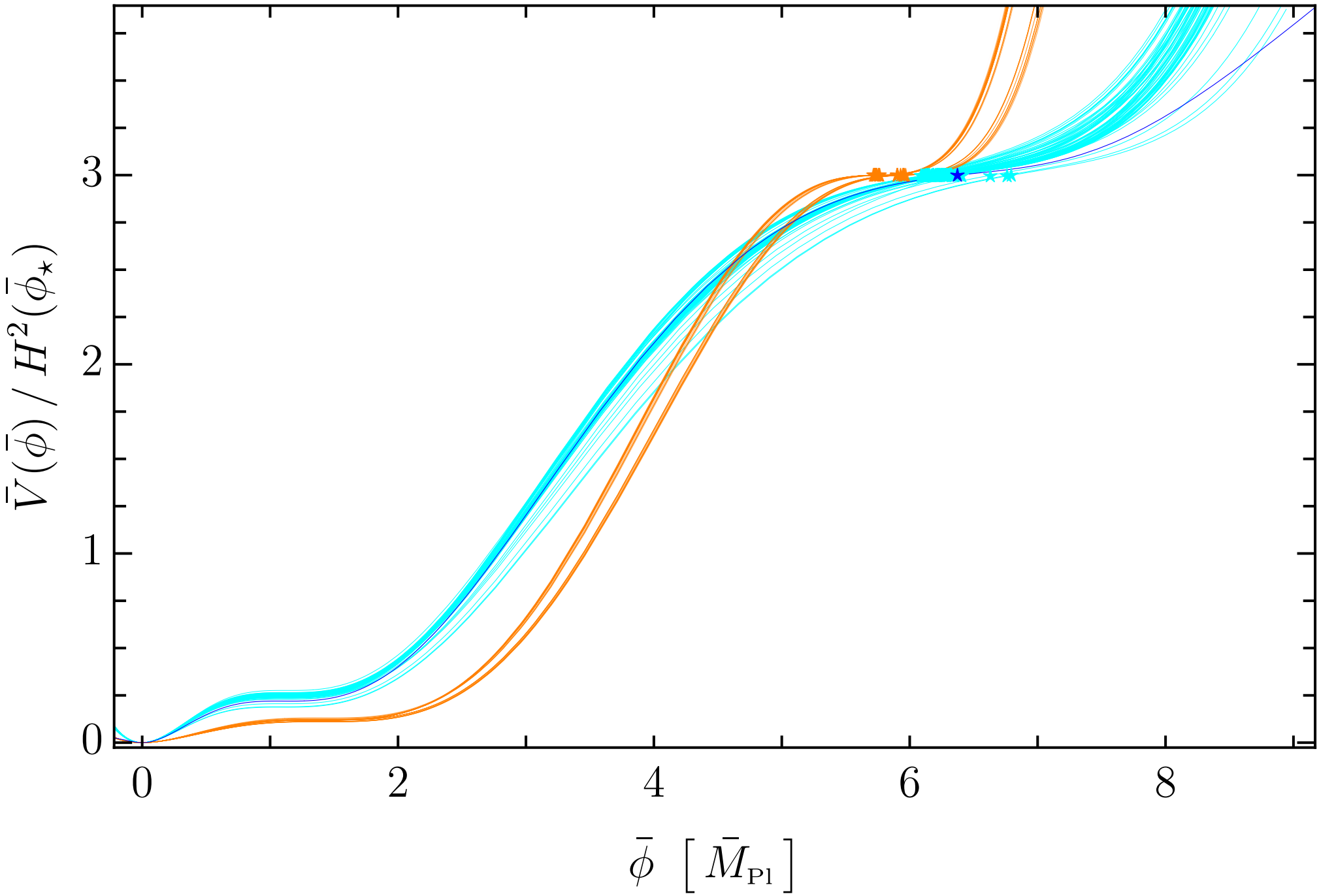
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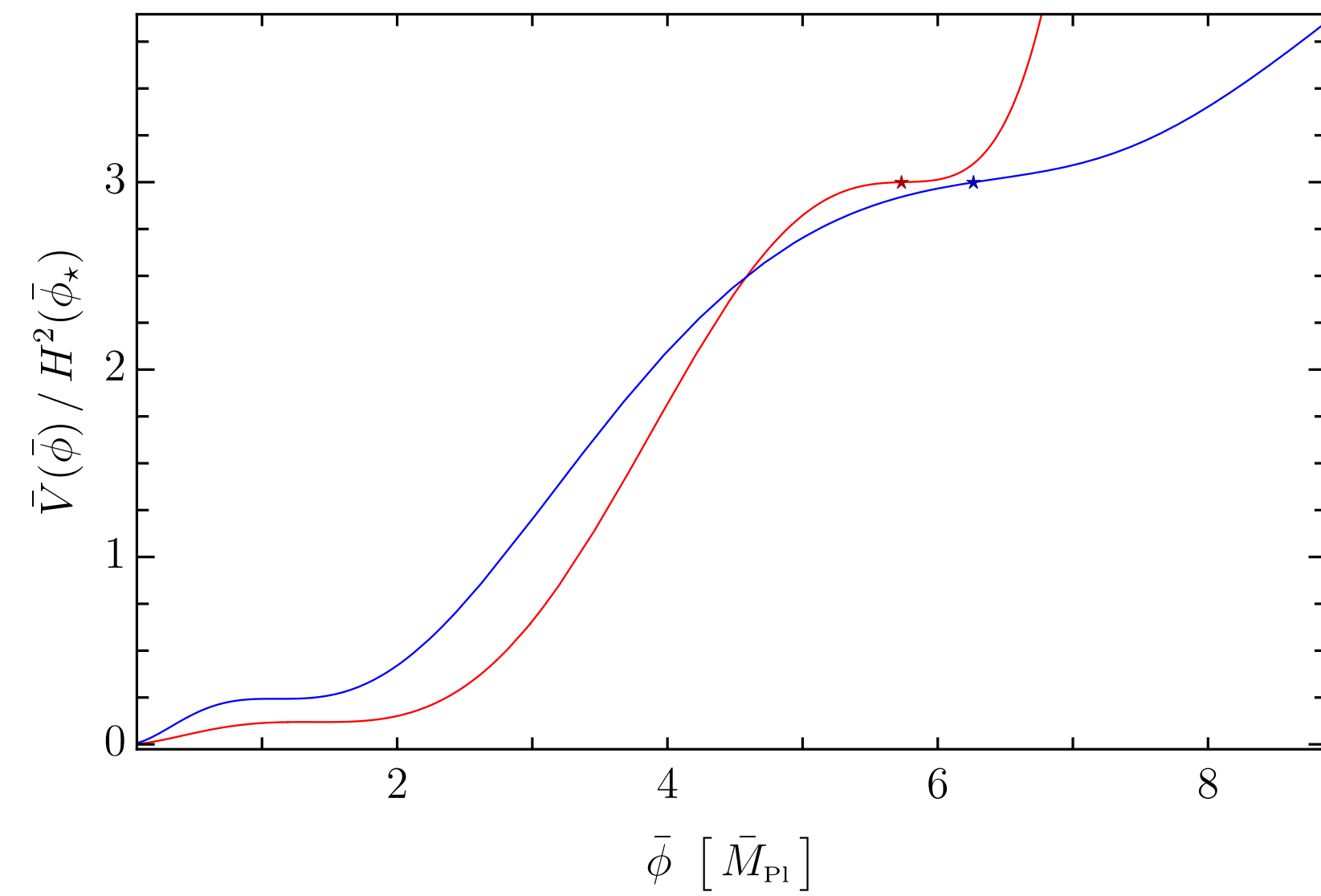
one model found for
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$r \propto -\alpha_s^2 \quad ??$

*SINGLE-FIELD MODELS for CMB and PBHs:
SCANNING for MODELS*

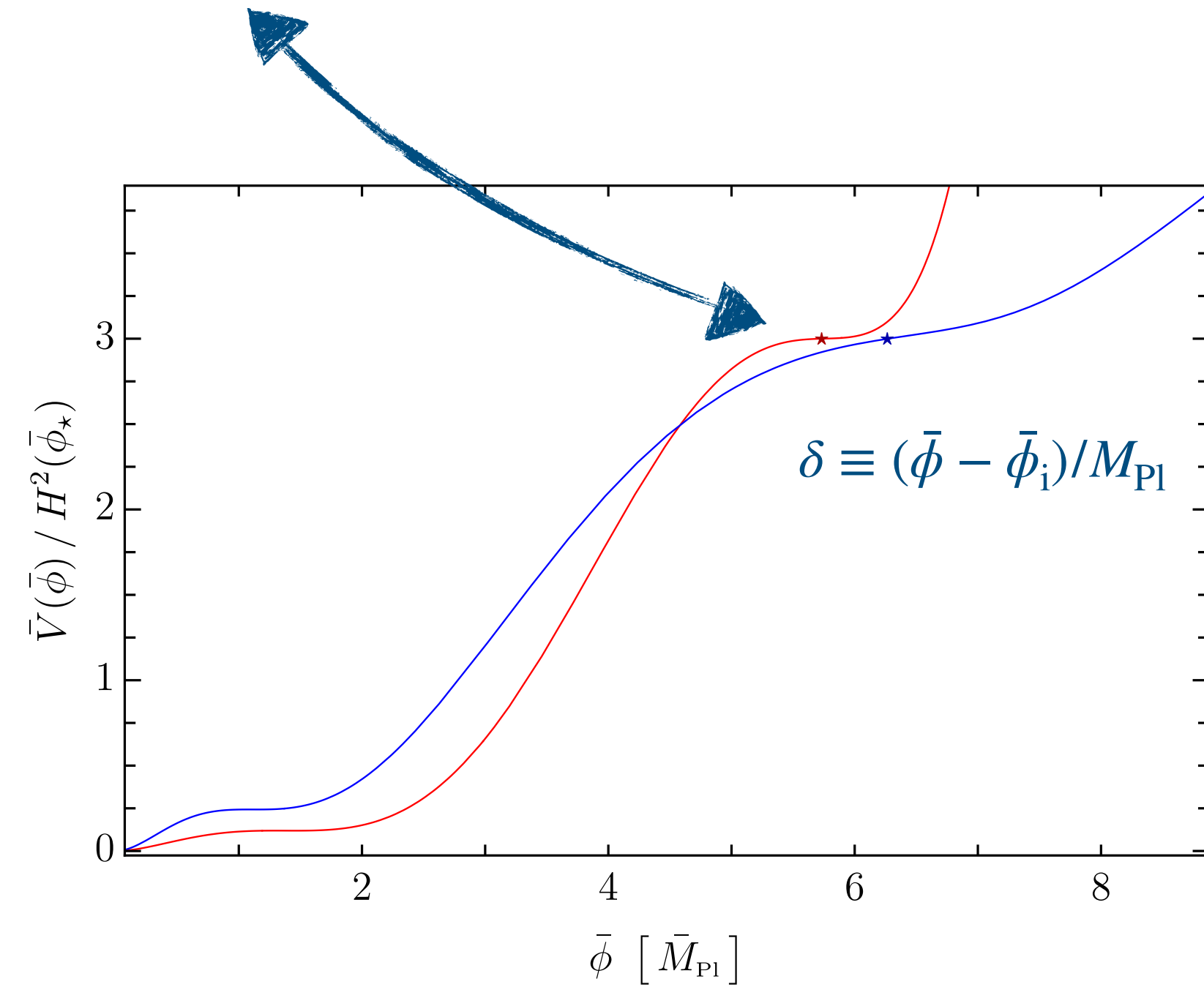


INFLATION AT AN INFLECTION POINT



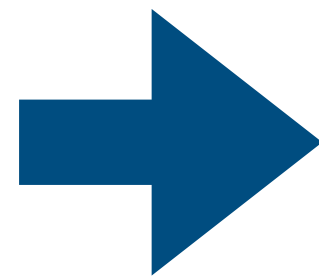
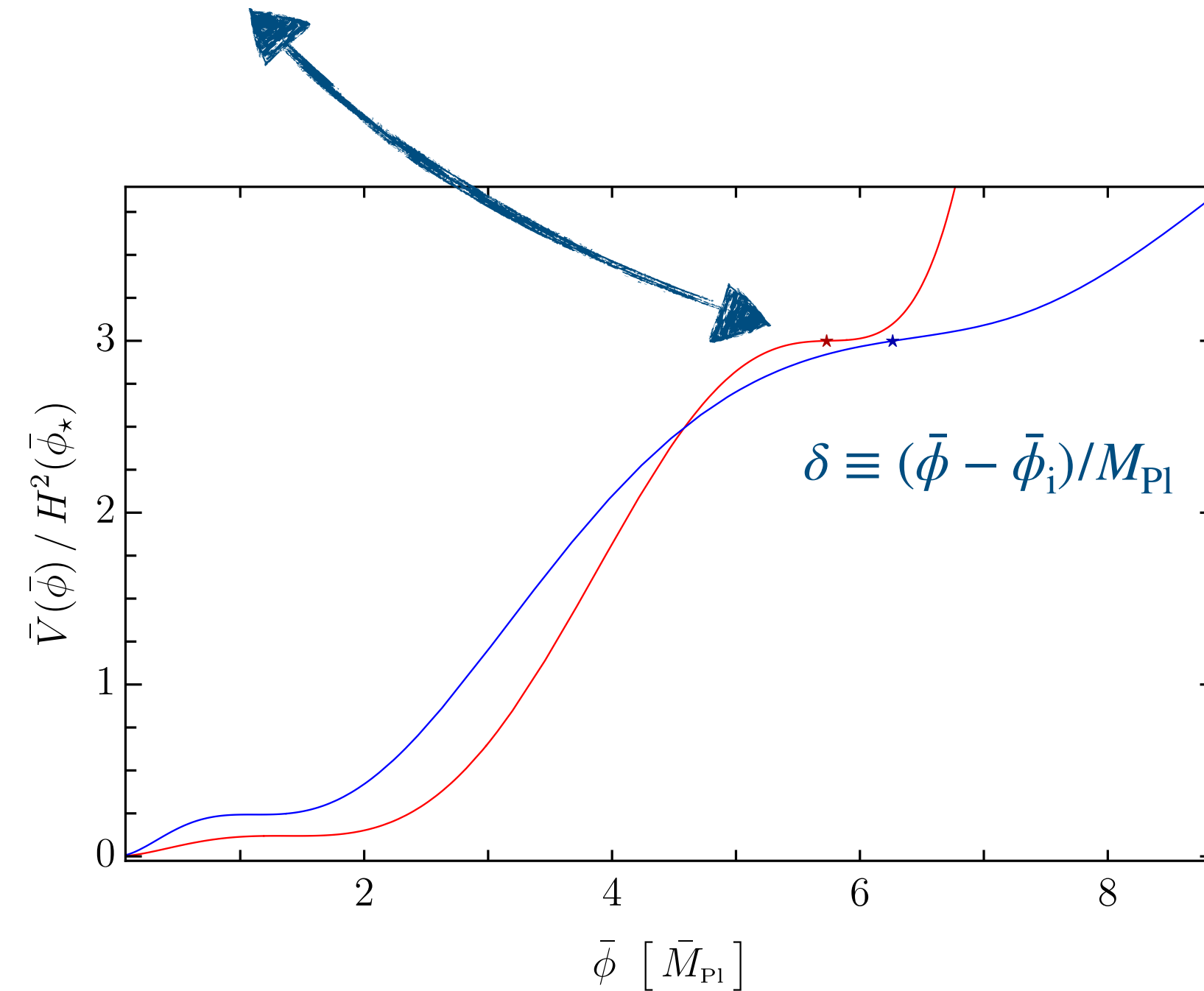
INFLATION AT AN INFLECTION POINT

$$\bar{V}(\delta) = \mu \left(\bar{V}_0 + \beta\delta + \delta^3 \right) + \mathcal{O}(\delta^4)$$



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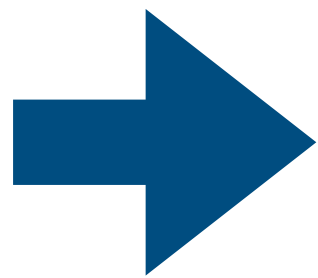
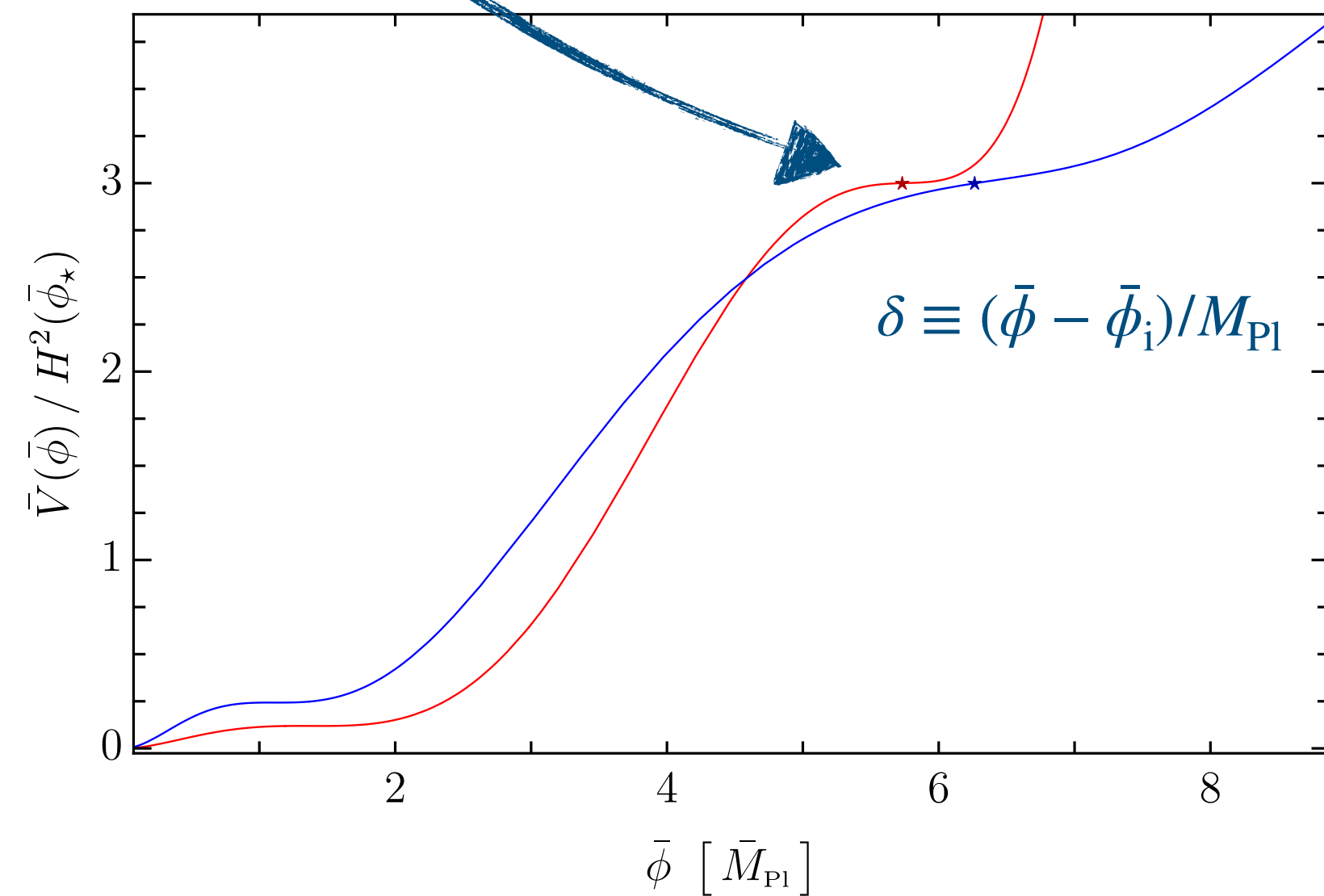
$$n_s \simeq 1 + \frac{12\delta_\star}{V_0} - \frac{(3\beta + 12C - 4)\beta}{V_0^2}$$

$$\alpha_s \simeq -\frac{12\beta}{V_0^2} - \frac{36\delta_\star^2}{V_0^2}$$

$$r \simeq \frac{8\beta^2}{V_0^2} + \frac{48\delta_\star^2\beta}{V_0^2}$$

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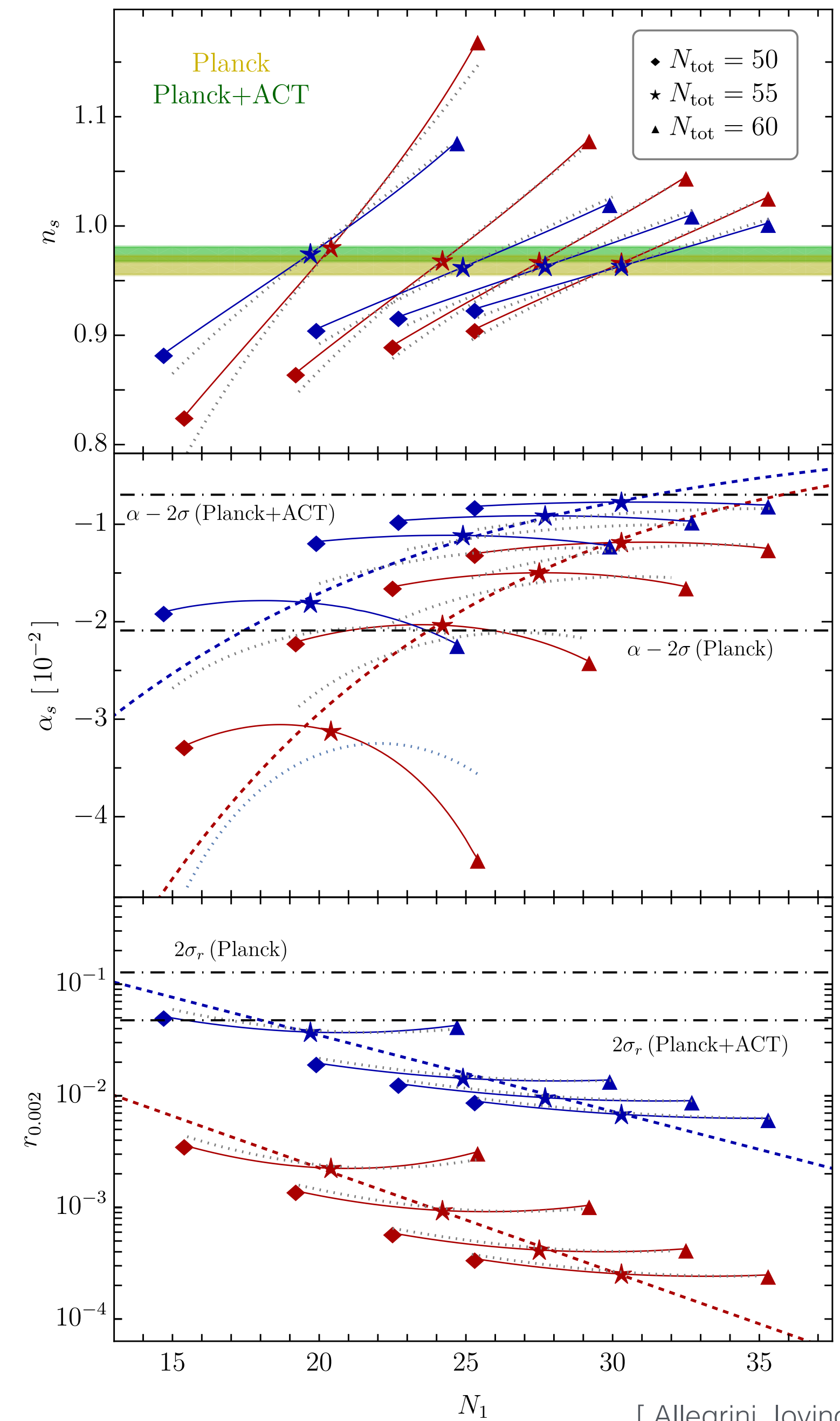
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NON-GAUSSIANITY

IN SINGLE FIELD INFLATION

NON-GAUSSIAN RANDOM FIELDS

often described with the local expansion

$$\zeta(\mathbf{x}) = \zeta_G(\mathbf{x}) + \frac{3}{5}f_{\text{NL}} \left(\zeta_G(\mathbf{x})^2 - \langle \zeta_G(\mathbf{x})^2 \rangle \right) + \frac{9}{25}g_{\text{NL}}\zeta_G(\mathbf{x})^3 + \dots$$

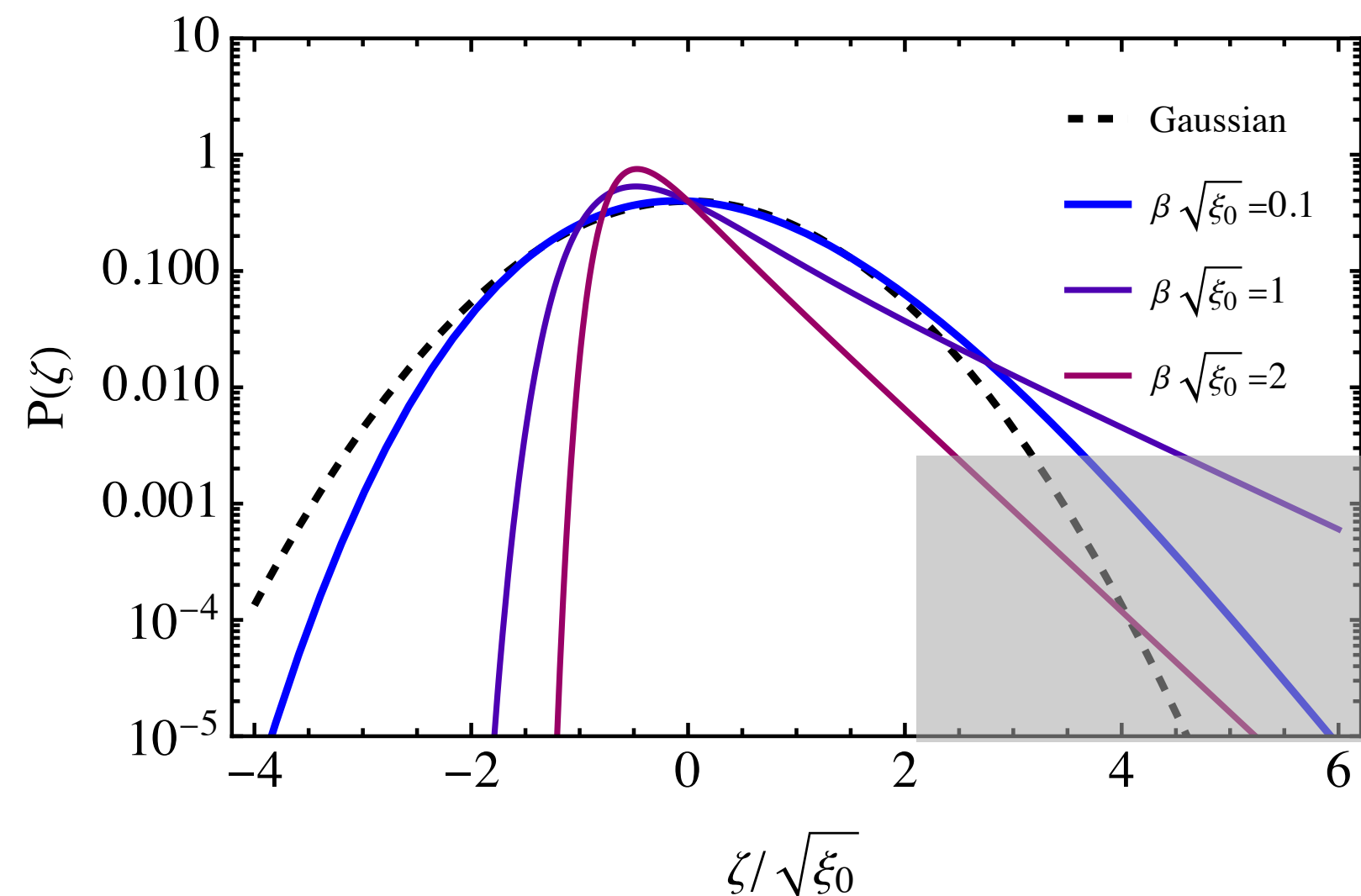
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a non-polynomial relation known in USR models:

Atal, Germani [1811.07857], Taoso, Urbano [2102.03610], Tomberg [2304.10903]



$$\beta = -4\epsilon_2 \approx 4(n_{s,CR} - 1)$$

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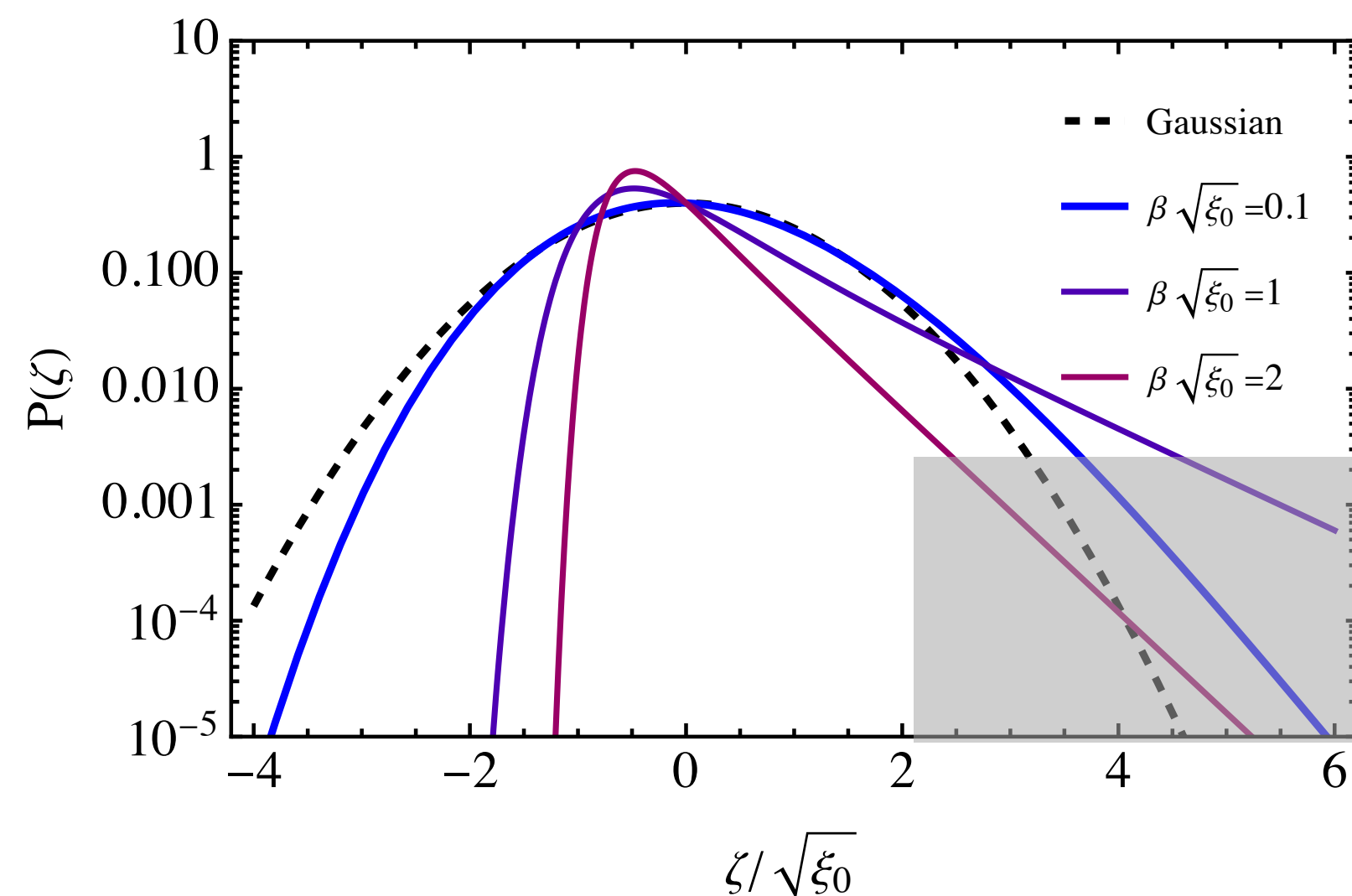
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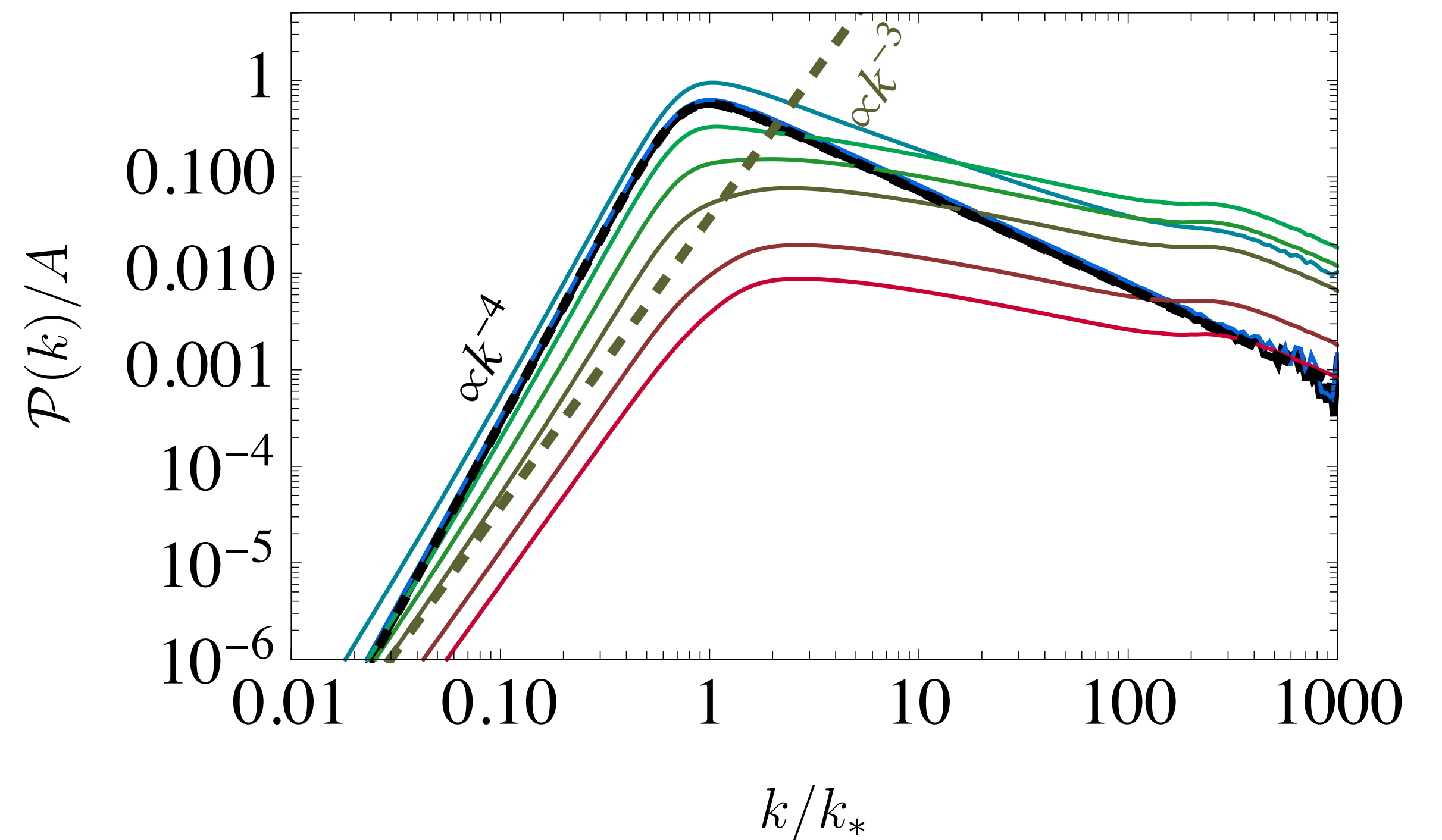
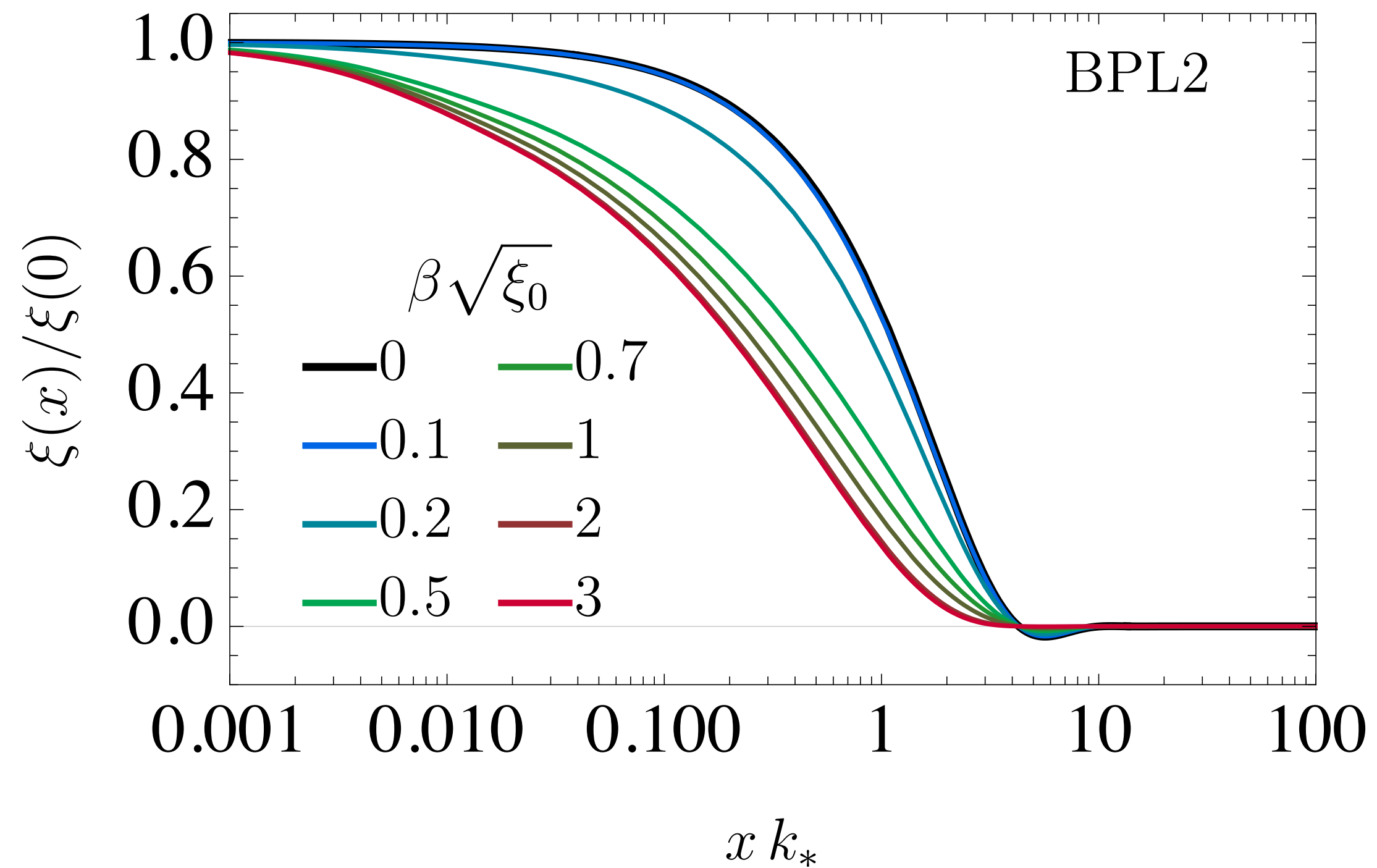
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how to approach it without perturbation theory?

NON-GAUSSIAN RANDOM FIELDS: the 2-point function in USR

$$\zeta(\mathbf{x}) = -\frac{1}{2\beta} \ln |1 - 2\beta\zeta_G(\mathbf{x})|$$



non-Gaussian power spectra from a Gaussian broken power-law

SUMMARY

- Peaks of **inflationary spectra** depend on the **SR-to-USR transition**
- (polynomial) single field models work **only for asteroid mass PBHs?** - need for tools for systematic searches
- strong local non-Gaussianity could be treated **semi-analytically**

