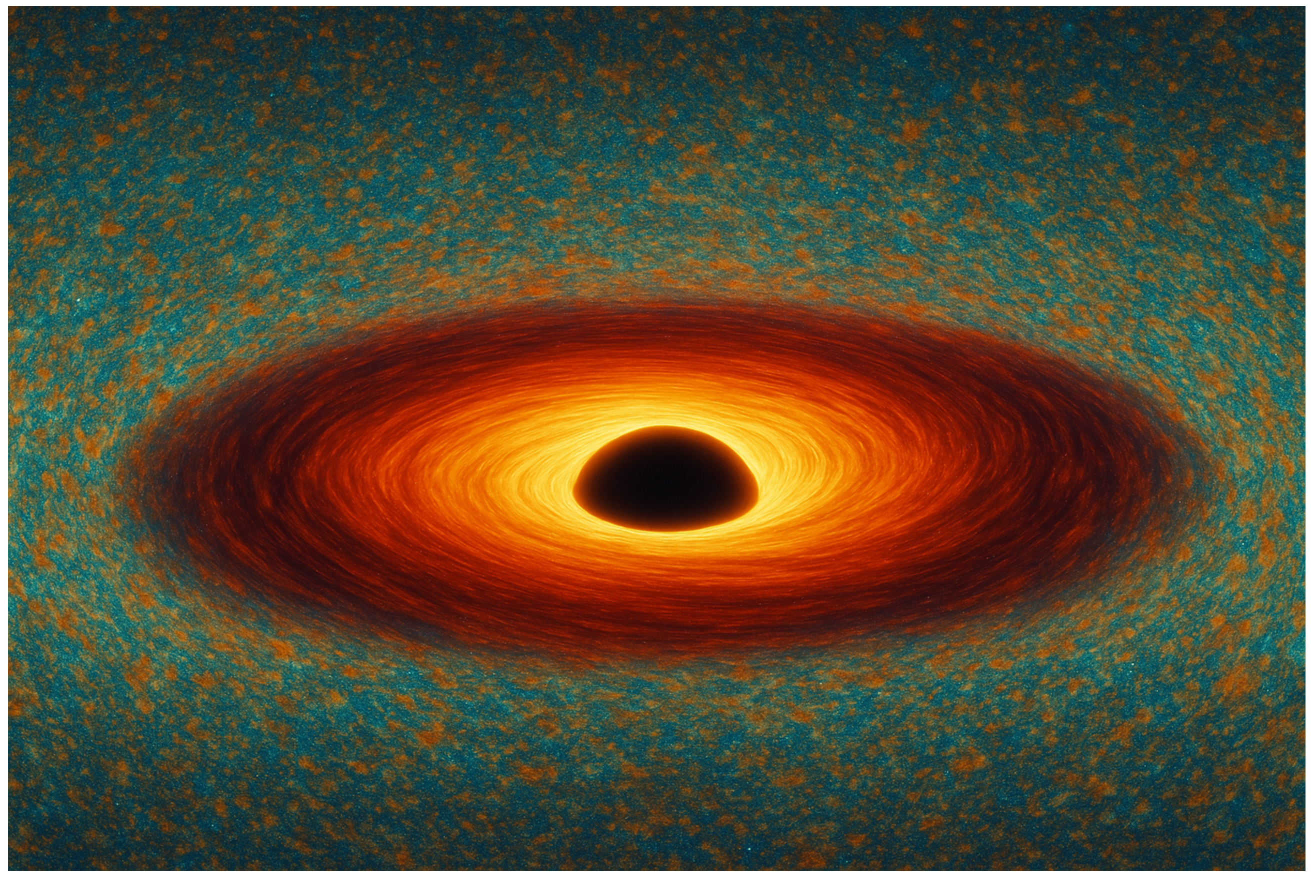
Modeling accretion onto PBHs and its role on (mostly CMB) constraints



For some starting refs:. D. Serpico, arXiv:2406.12489 & V. De Luca and N. Bellomo, arXiv:2312.14097 in "Primordial Black Holes", Springer Series in Astrophysics and Cosmology, 2025, eds. C Byrnes, G. Franciolini, T. Harada, P. Pani and M. Sasaki.

Primordial Black Holes in the Multi-Messenger Era, IPFU Trieste 3-7/11/2025

LAPTh

Pasquale Dario Serpico (LAPTh - Annecy, France)

cnrs

Outline

- Introduction on PBH accretion and its cosmological impact
- Generalities on accretion and luminosity efficiency
- Basic theory in the cosmological setting
- Arguments for disk formation and disk "theory"
- Role of Feedbacks
- *Role of dark matter halo accretion*
- CMB bounds of two motivated benchmarks
- *Cursory considerations on late-universe phenomenology*
- Summary, conclusions, perspectives

Disclaimer

- I have **no PBH-related agenda**, nor preference for tight or loose bounds.
- My goal: to identify models that illustrate **realistic cosmo sensitivities** to PBHs, erring on the conservative side when (realistic) uncertainties are known
- The talk is not constructed for you to accept any number, but to **raise awareness** on (astro)physics issues solved and those which are still uncertain, with the goal of pointing to **directions to improve our knowledge**.

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If we stick to the opinion on stars expressed in the "*Cours de philosophie positive*" (1835)

We understand the possibility of determining their shapes, their distances, their sizes and their movements; whereas we would never know how to study by any means their chemical composition, or their mineralogical structure […] I persist in the opinion that every notion of the true mean temperatures of the stars will necessarily always be concealed from us.

Auguste Comte (1798-1857)

…since I believe that many of the questions I deal with today do (will) admit an answer, I guess that *I am not a positivist.*

Key notions

- Like ordinary BH, PBH can accrete matter → heat & ionise it → radiation
- The associated photon emission can be detected, possibly indirectly
- Potentially relevant effects on the mass and spin evolution of PBHs
- Generic, but quantitatively relevant only for ≳ stellar mass PBH

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Epoch of interest does matter!

- **Late universe ($z \lesssim 20$):** qualitatively similar to signatures of astrophysical BHs (e.g. Galactic X and radio signals), but different abundance, mass function, environmental parameters.
- **Early universe ($z \gg 20$)**: peculiar environmental conditions (homogeneity, photon density…) and observables (notably: CMB anisotropies) suggest a dedicated look

I will focus on the early universe case, with a few comments on the former

Impact of energetic particles injected at high-z

associated to a number of putative processes, like

- Annihilating relics (like WIMP DM)
- Decaying relics such as sterile ν's, Super-WIMP progenitors
- Evaporating (hence "light") PBH
- Accreting (hence "stellar mass or heavier") PBH

Key point

energy of the injected non-thermal particles is not negligible wrt the kinetic energy of the baryonic gas.

Can eventually **heat up** (alter T_M) and especially **ionize the gas** (alter x_e)

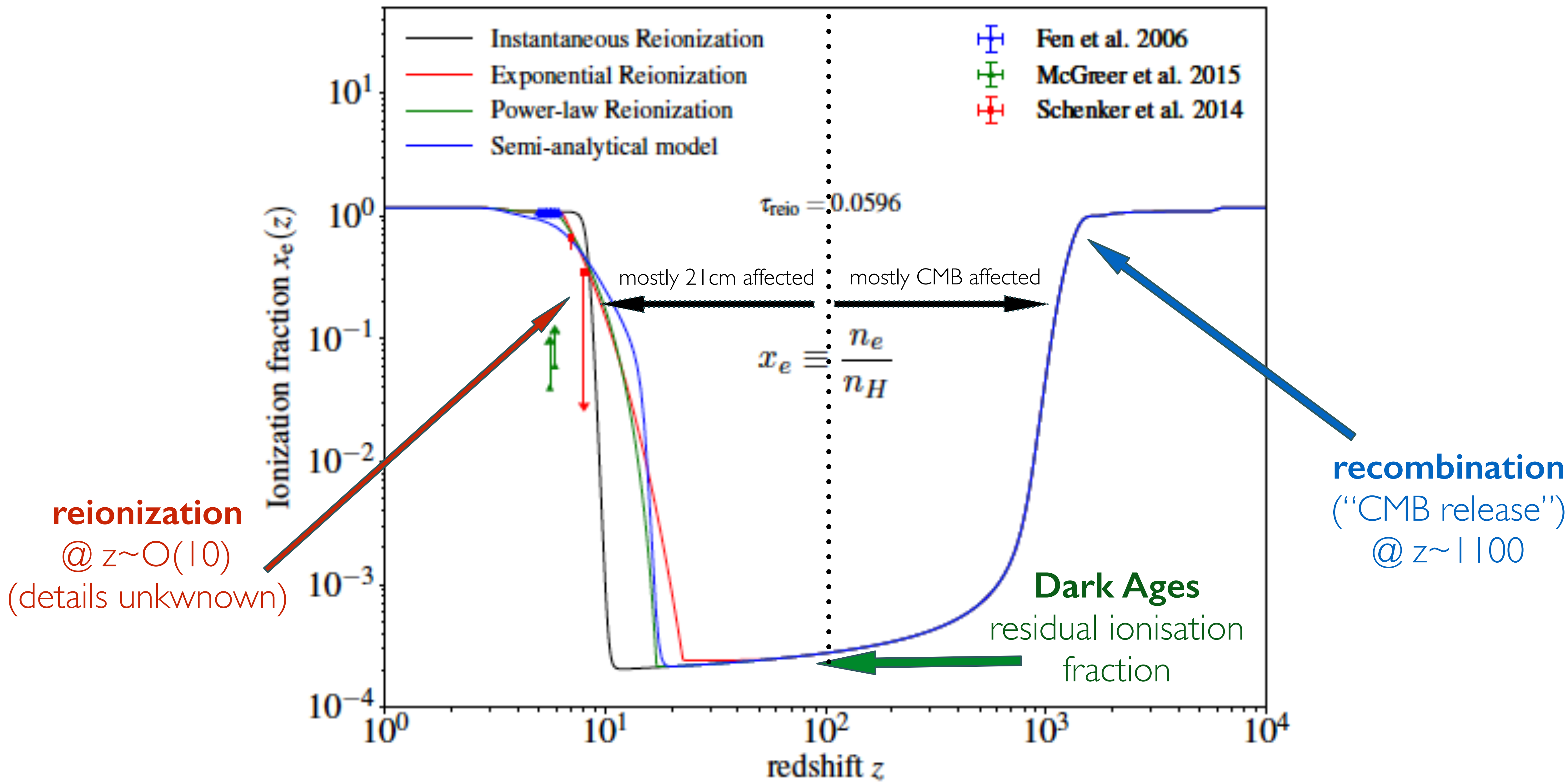
(non-linearly) coupled

➙ **CMB anisotropies are very sensitive to that!** *(How? See Yacine's talk)*

➙ **21cm signal would be even more sensitive!**

The three epochs affected

With respect to the standard **ionisation** evolution, it affects



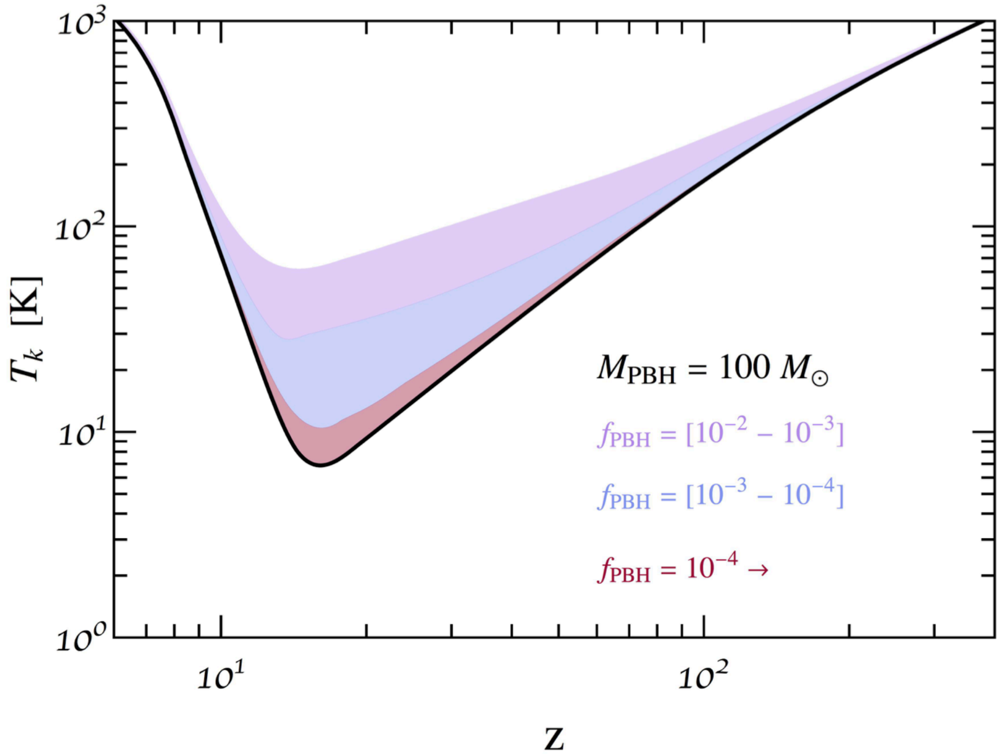
optical depth

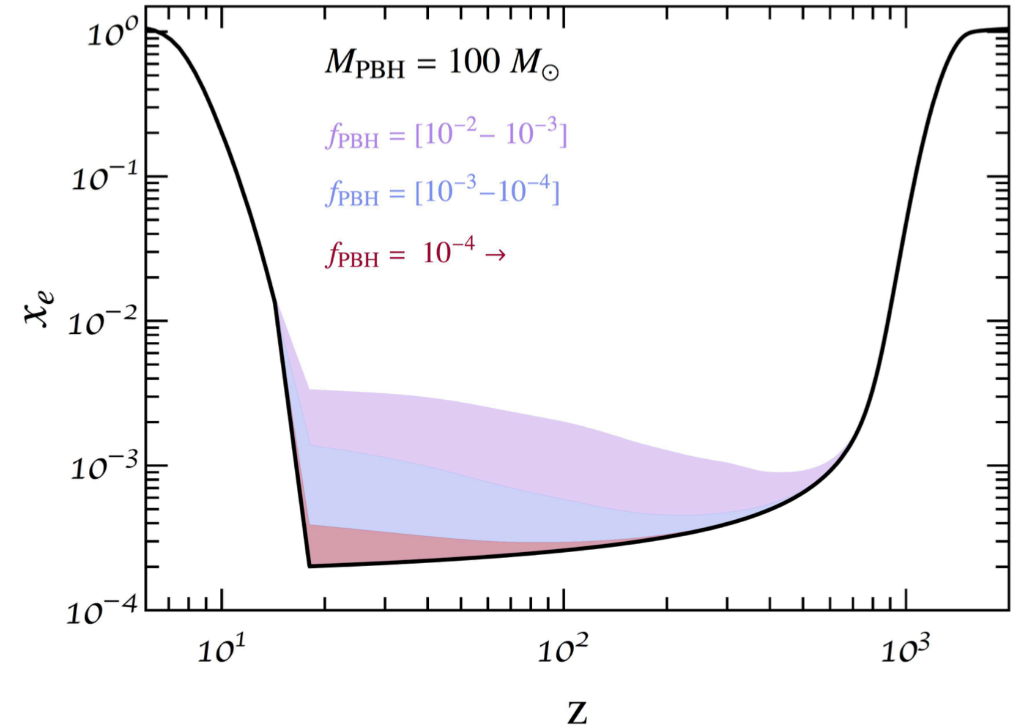
$$\kappa(z) = \sigma_T n_{e,0} \int_0^z \mathrm{d}z' \frac{\mathrm{d}t}{\mathrm{d}z'} (1+z')^3 x_e(z')$$

E-deposition module interfaced via Boltzmann CMB solver dealt with via ExoCLASS *see 1801.01871*

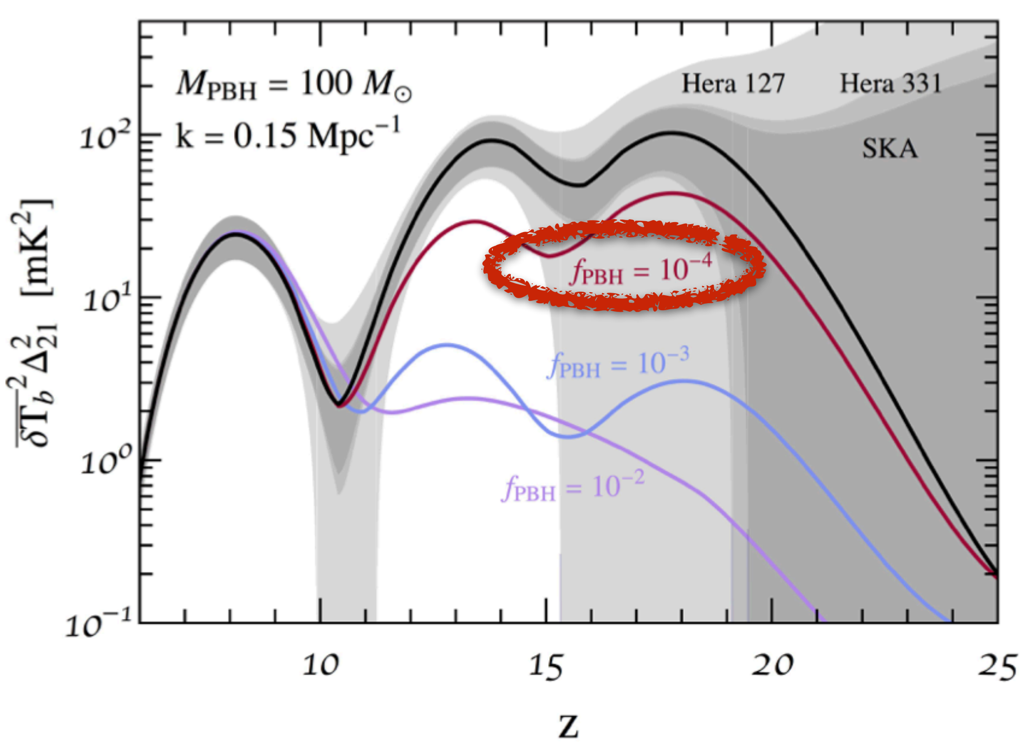
Effects on 21 cm

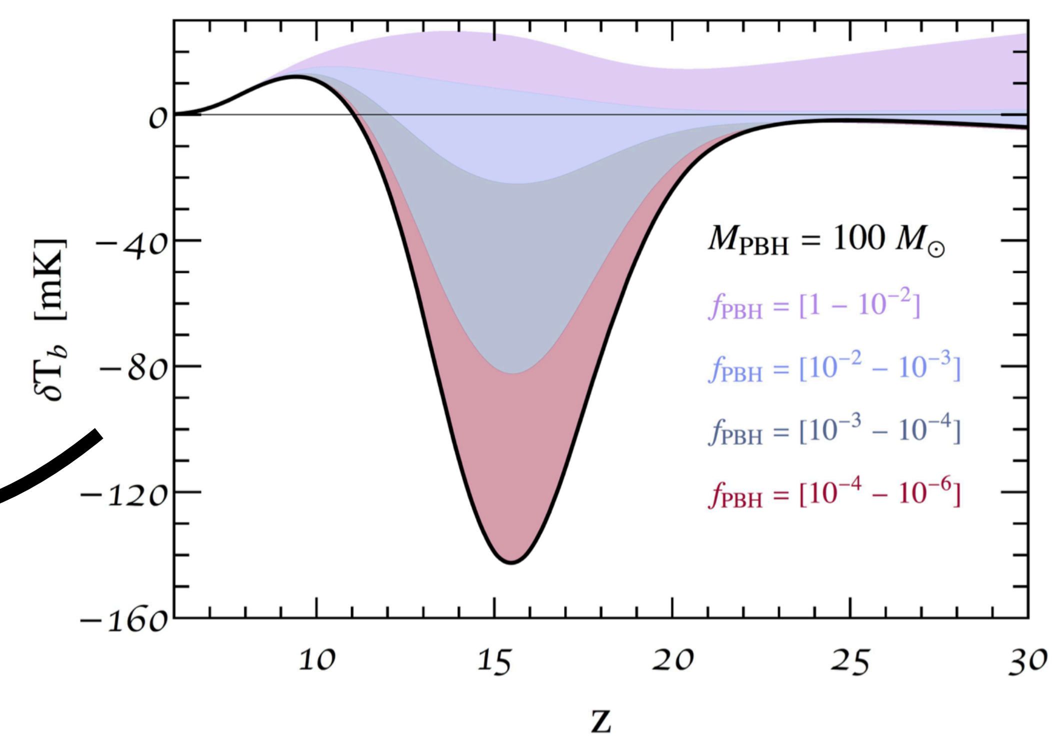
O. Mena, S. Palomares-Ruiz, P. Villanueva-Domingo and S.J. Witte, PRD 100 (2019)043540 [arXiv:1906.07735]





$$\delta T_b(\nu) \simeq 27\, x_{\rm H}\,(1+\delta_b)\left(1-\frac{T_{\rm CMB}}{T_S}\right)\left(\frac{1}{1+H^{-1}\partial v_r/\partial r}\right)\left(\frac{1+z}{10}\right)^{1/2}\left(\frac{0.15}{\Omega_{\rm m}h^2}\right)^{1/2}\left(\frac{\Omega_{\rm b}h^2}{0.023}\right)\,{\rm mK}\,,$$





+ local effects (ionisation bubble) on top of this global signal; but beware of degeneracies with astro!

Accretion rate and luminosity

General notions & bounds

Particle *m* falling from infinity at rest to a distance *R* from point mass *M* acquires kinetic energy

$$\frac{G_N M\, m}{R} = \frac{m}{2}\frac{R_S}{R}$$

Total **luminosity** associated to accretion

$$L = \epsilon \dot{M}$$

fraction **(1-ϵ)** of $\dot{M}$ simply swallowed
→ **mass growth**

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Upper limit to the efficiency

$$\epsilon = \frac{1}{2}\frac{R_S}{R}$$

the closer to the BH a stable orbit can be, the larger ϵ
(benchmark ϵ= 0.1, for max. rotating Kerr ϵ ~0.4)

A too large luminosity will stop further accretion due to radiation pressure **(Eddington limit)**

$$L_{\rm Edd} = \frac{4\pi G\, M\, m_p}{\sigma_T} \simeq 1.3\times 10^{38}\frac{M}{M_\odot}\,{\rm erg/s}$$

Higher masses can sustain higher luminosities!

Largest e.m. luminosity from accretion in <u>spherical symmetry & stationary conditions</u>
Note: *Only limits the visible L, not the accreted mass: we expect ϵ to decrease at high* $\dot{M}$

Accretion, $\dot{M}$

Problem of accretion onto a point mass M is old (but no general solution!)

Under steady state hypothesis

Infinite & cold gas cloud, moving at v_{rel}

$$\dot{M}_{\rm HL} = 4\pi\rho_\infty \frac{(GM)^2}{v_{\rm rel}^3}$$

Hoyle & Littleton '39,'40

Up to a factor 2 smaller in presence of density inhomogeneities/wake account

Bondi & Hoyle '44

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accretion at rest, including pressure

$$\dot{M}_{\rm B} = 4\pi\lambda\rho_\infty \frac{(GM)^2}{c_{s,\infty}^3}$$

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$c_s^2 = \delta P/\delta\rho$

λ~O(0.1-1) accretion eigenvalue comes from solving steady-state problem, depends on equation of state & cooling/drag details

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$\lambda \sim O(0.1\text{-}1)$ accretion eigenvalue comes from solving steady-state problem, depends on equation of state & cooling/drag details

Both parameterised as

$$\dot{M} = 4\pi\lambda_{\rm eff}\rho_\infty v_{\rm eff} r_{B,\rm eff}^2 \quad \text{where} \quad r_{B,\rm eff} = \frac{GM}{v_{\rm eff}^2} \simeq 4.4\times10^{-5}{\rm pc}\frac{M}{M_\odot}\left(\frac{10{\rm km/s}}{v_{\rm eff}}\right)^2$$

key: what are $v_{eff}(c_s, v_{rel})$ & $\lambda_{eff}(c_s, v_{rel})$?

Useful approximations for cosmological applications

(Or why at the high mass end analyses should be modified)

Steady state approximation

For consistency, the system must settle down in the (~Bondi) steady-state fast compared to the cosmological expansion

$$\frac{r_{\rm B}}{v_{\rm eff}} H(z) < 1 \rightarrow M \lesssim 10^{4.5}\, M_{\odot}$$

M. Ricotti, ApJ 662, 53 (2007) [0706.0864]

Homogeneous approximation

A PBH can ionise all the region separating it from the nearest PBH if

$$f_{\rm PBH} > 10^{-15} x_e^3 \frac{M_{\odot}}{M}$$

(always satisfied in our range of parameters)

Up to the maximum multipole used (ℓ~2000), there is more than a PBH in each patch of the CMB

$$N_{\rm PBH} \simeq 5 \times 10^7 \ell^{-1} \left(\frac{f_{\rm PBH} M_{\odot}}{M}\right)^{1/3} > 1$$

PDS, V. Poulin, D. Inman and K. Kohri, Phys.Rev.Res. 2 (2020), 023204

Where does λ come from? Accretion at rest

λ~O(0.1-1) accretion eigenvalue comes from solving spherical steady-state problem

$$\dot{M}_B = 4\pi\lambda\rho_\infty \frac{(GM)^2}{c_{s,\infty}^3}$$

Mass continuity

$$4\pi r^2 \rho |v| = \dot{M} = \text{const}$$

Momentum equation

$$v\frac{\mathrm{d}v}{\mathrm{d}r} = -\frac{GM}{r^2} - \frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}r} - \beta_{\mathrm{drag}} v$$

Equation of state

$$P = \frac{\rho}{m_p}(1+\bar{x}_e)T$$

"Heat equation"

$$v\rho^{2/3}\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{T}{\rho^{2/3}}\right) = \beta_{\mathrm{cool}}(T_{\mathrm{CMB}} - T)$$

In the cosmological context

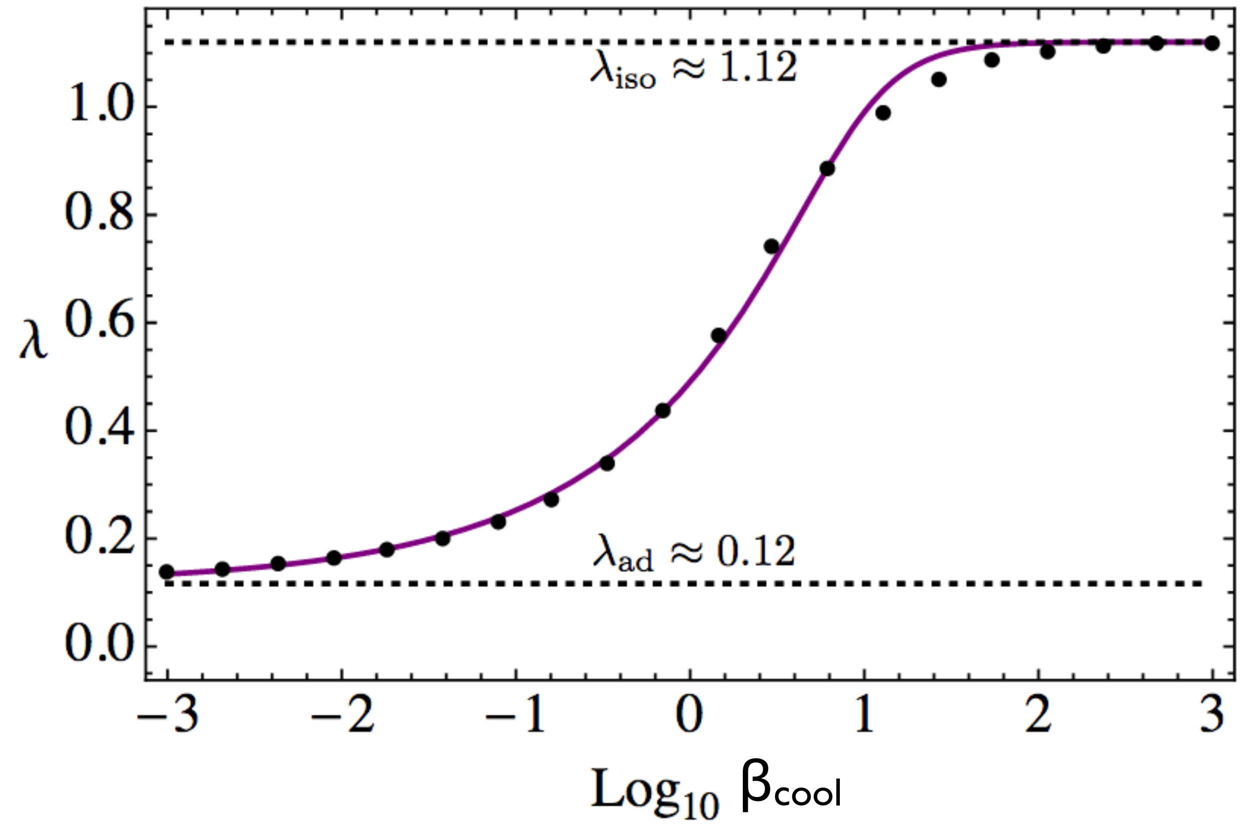
$$\beta_{\mathrm{drag}} = \frac{4}{3}\frac{\bar{x}_e \sigma_T \rho_{\mathrm{CMB}}}{m_p\, c}$$

$$\beta_{\mathrm{cool}} = \frac{2\, m_p}{(1+\bar{x}_e)m_e}\beta_{\mathrm{drag}} \gg \beta_{\mathrm{drag}}$$

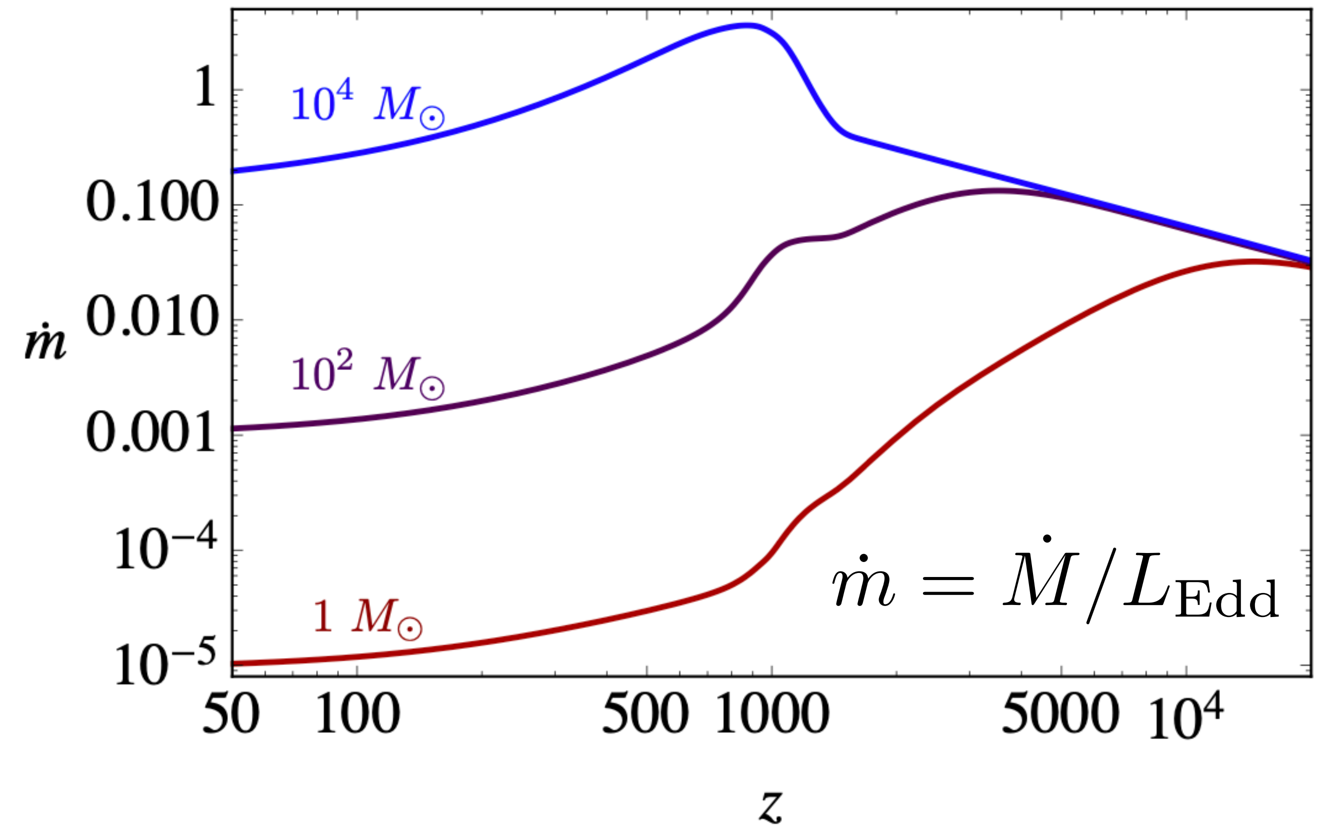
x_e becomes dynamically important in the inner regions

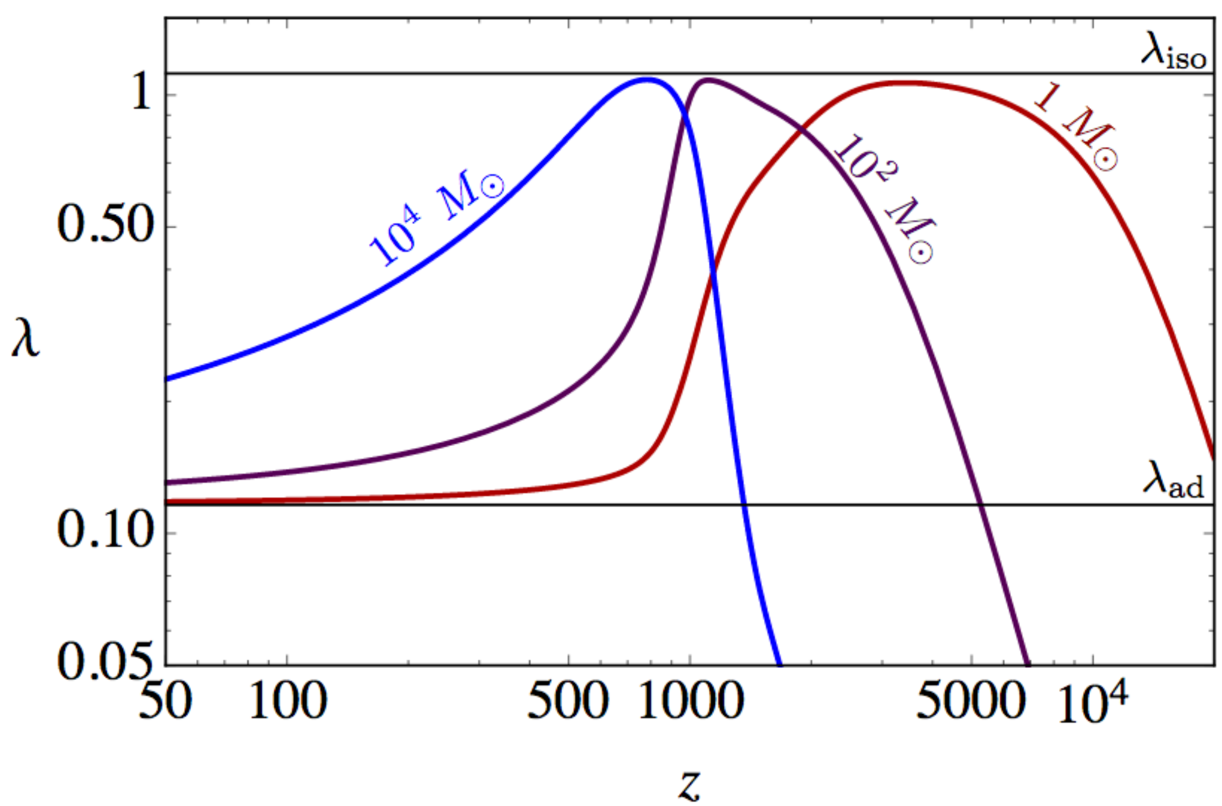
Ali-Haïmoud & Kamionkowski, PRD95 (2017), 043534

Results



(in dimensionless form)





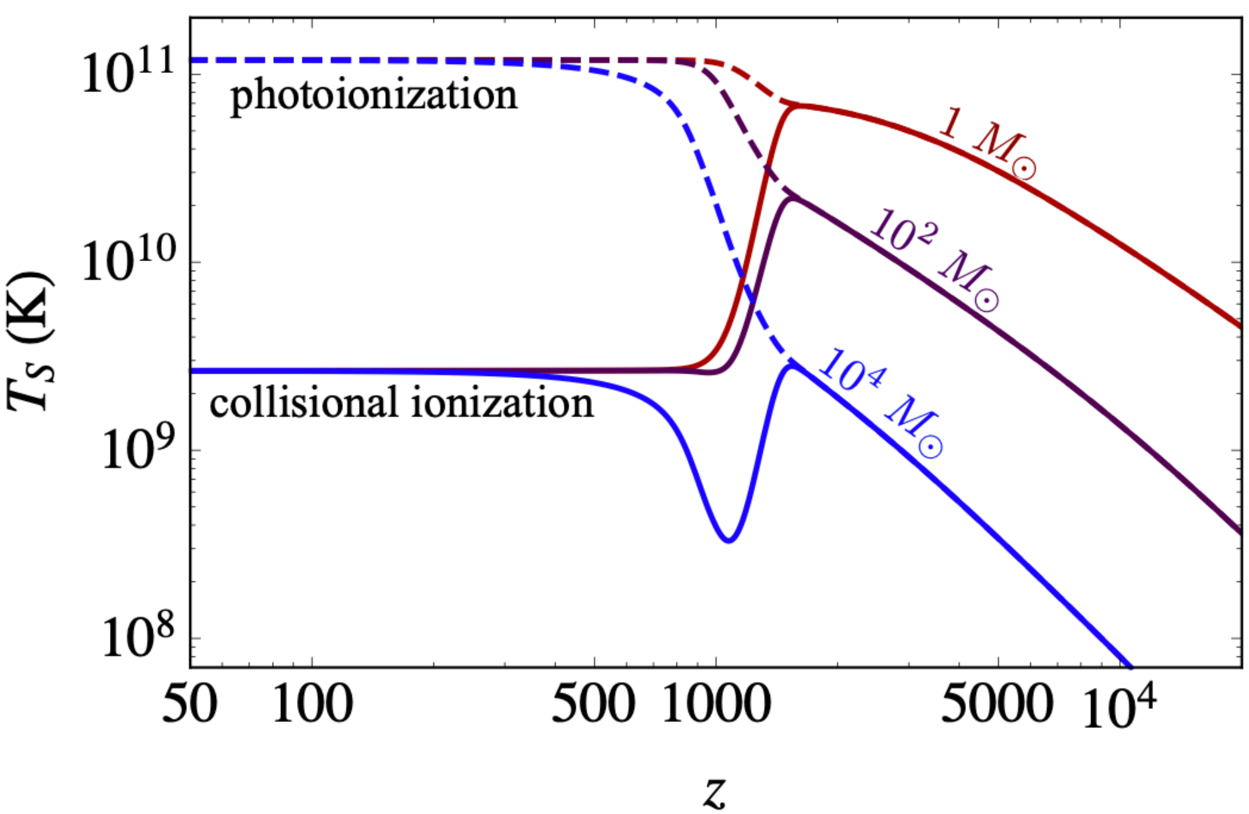
Ali-Haïmoud & Kamionkowski, PRD95 (2017), 043534

Luminosity *(only thing that matters for most pheno)*

Bulk of the emission falls in X-rays/soft gammas from matter heated up to $T \sim 10^9$-10^{11} K
(bremsstrahlung (free-free) radiation near the Schwarzschild radius)

At small *r* (relevant for luminosity) the solution yields

$$n_e = \frac{\dot{M}}{4\pi m_p R_S^2\, c}\left(\frac{r}{R_S}\right)^{-3/2} \qquad T(r) = T_\infty \tau \frac{r_B}{r}$$



bracketed by two cases: if photoionisation is strong enough or matter is ionised via collisions

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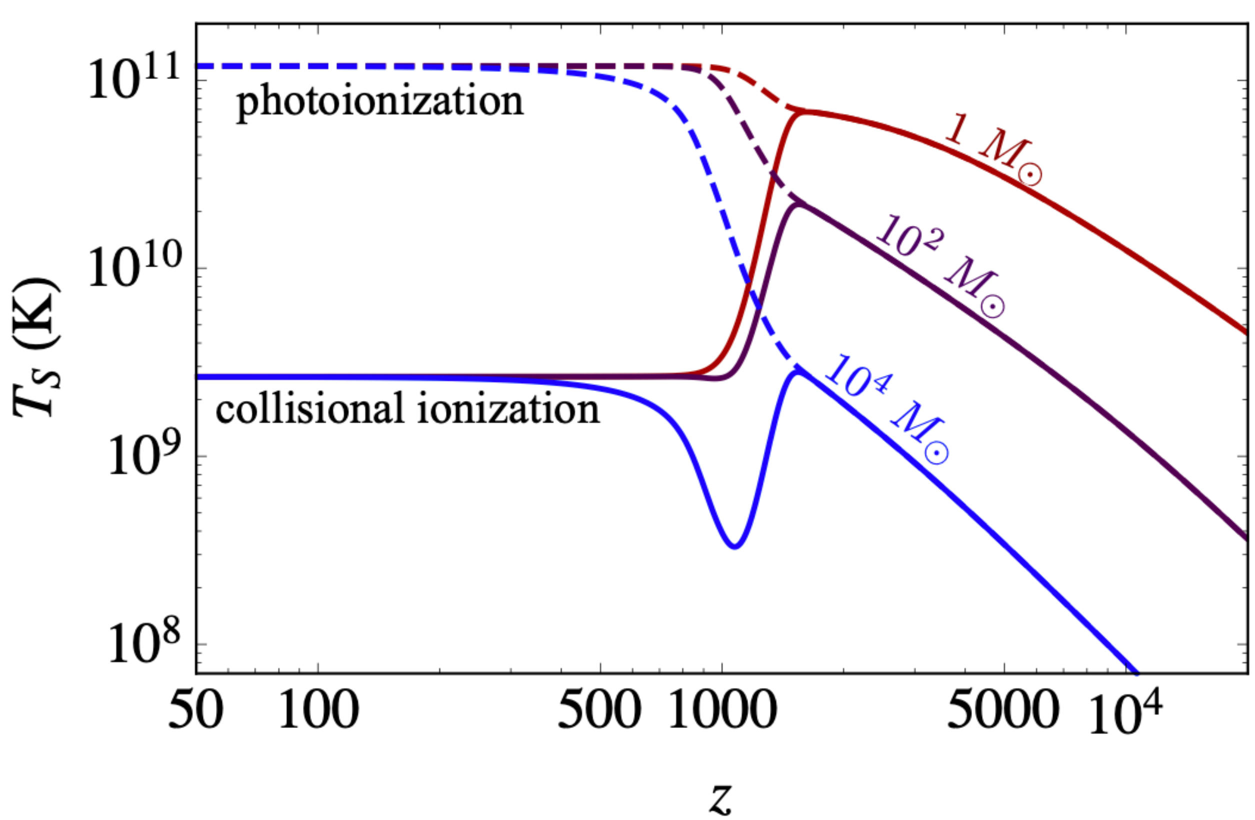
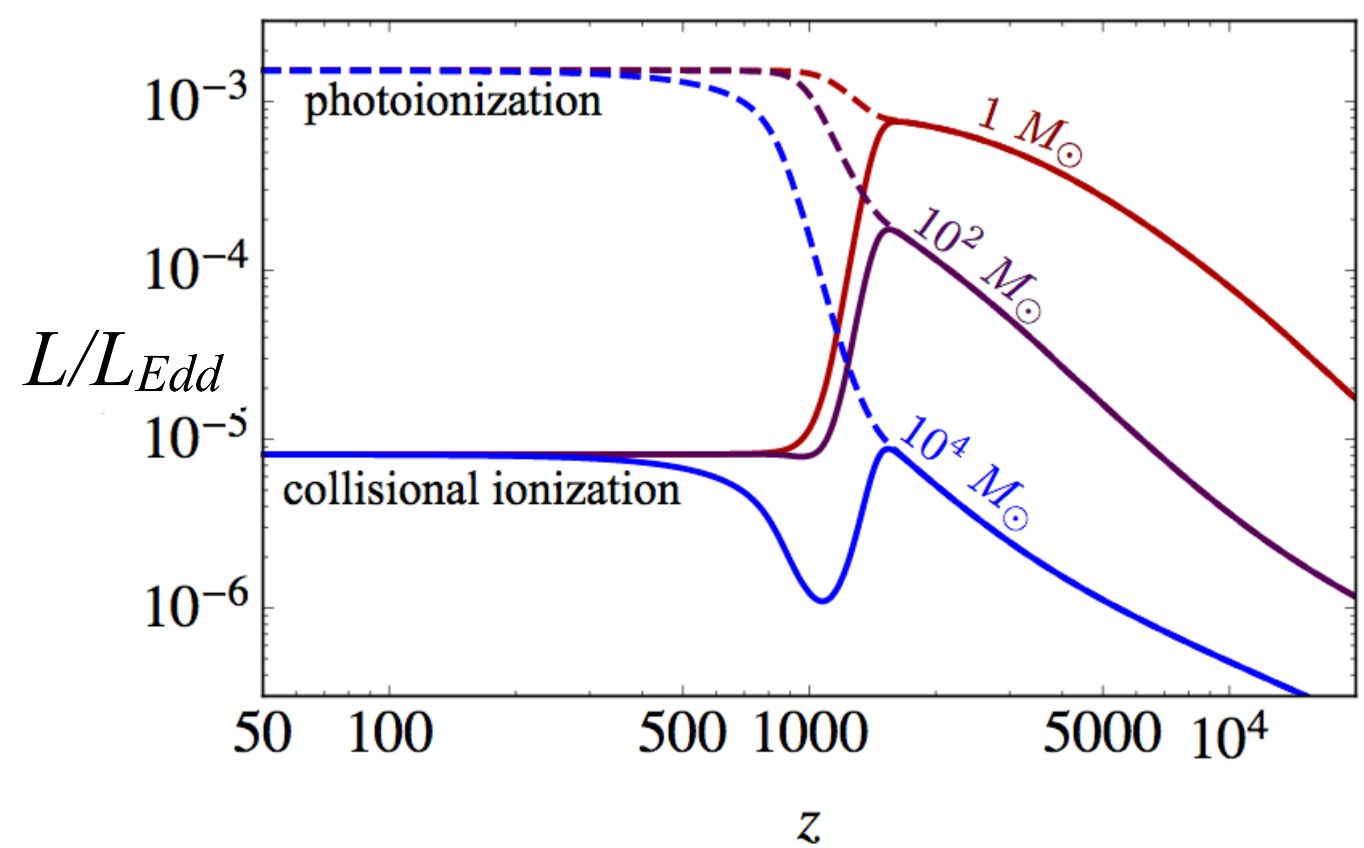
Power per unit volume $j(n_e, T) = \alpha \sigma_T n_e^2\, T \mathcal{J}(T/m_e)$ *if optically thin (OK!)* → $L = \int \mathrm{d}V\, j(n_e, T)$





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The role of gas-PBH velocity

Naive expectation for baryon-DM velocity $v_{bc} \sim c_s$

In this case previous solution should apply

But far from obvious in cosmological setting!

In Newtonian perturbation theory, the relative velocity of DM and baryons evolves as

$$\frac{\partial}{\partial t}\left(a\boldsymbol{V}_{\rm bd}\right) = -\boldsymbol{\nabla}\left(\mathcal{K}_{\rm b} - \mathcal{K}_{\rm d} + c_s^2\delta_b\right) \qquad \boldsymbol{V}_{\rm bd} \equiv \boldsymbol{v}_{\rm b} - \boldsymbol{v}_{\rm d}$$

At matter-radiation equality, DM starts moving ***supersonically*** relative to the coupled baryon-photon plasma. Baryons only start catching up at recombination, eventually driving $V_{bd} \rightarrow 0$

If RHS negligible (true e.g. at rec. scales/early times)

$$\boldsymbol{V}_{\rm bd} \simeq -\boldsymbol{v}_{\rm d}^{\rm rec}\frac{1000}{1+z} \qquad |\boldsymbol{v}_{\rm d}^{\rm rec}| \simeq 30{\rm km/s} \simeq 5\, c_s^{\rm rec}$$

Tseliakhovich & Hirata, Phys. Rev. D 82, 083520 (2010)

(For details and relevant scales, see Yacine's talk)

Note: perturbative (but non-linear) result estimated for (Λ)CDM

A meaningful patch to the theory?

In *Ali-Haïmoud & Kamionkowski 2017*

$$\dot{M} = 4\pi\lambda_{\rm eff}\rho_\infty v_{\rm eff} r^2_{\rm B,eff}$$

the spherical accretion model is extended to $v_{\rm eff} \gg c_s$ just reducing the Bondi accretion according to:

$$r_{\rm B,eff} = \frac{GM}{v^2_{\rm eff}} \qquad v_{\rm eff} = \sqrt{v_L\, c_s}$$

Looking at studies where the hypotheses of the Bondi accretion are relaxed, this is not a too bad approximation for the *average* **accretion rate**. e.g.:

Axisymmetry+inhomog.: *Ruffert, A&A 346 (1999), 861, astro-ph/9903304*
Vorticity: *Krumholz, McKee, Klein, ApJ 618, 757-768 (2005) [astro-ph/0409454]*
Turbulence: *Krumholz, McKee, Klein, ApJ 638, 369-381 (2006) [astro-ph/0510410]*
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More critical for consistency:
With supersonic motion, spherical symmetry breaks down and a disk forms! (see later)

- Both papers assumed spherical accretion for radiative efficiency (as in ROM), but this is not consistent with PBHs moving supersonically at Mach ~5! An accretion disc will form.

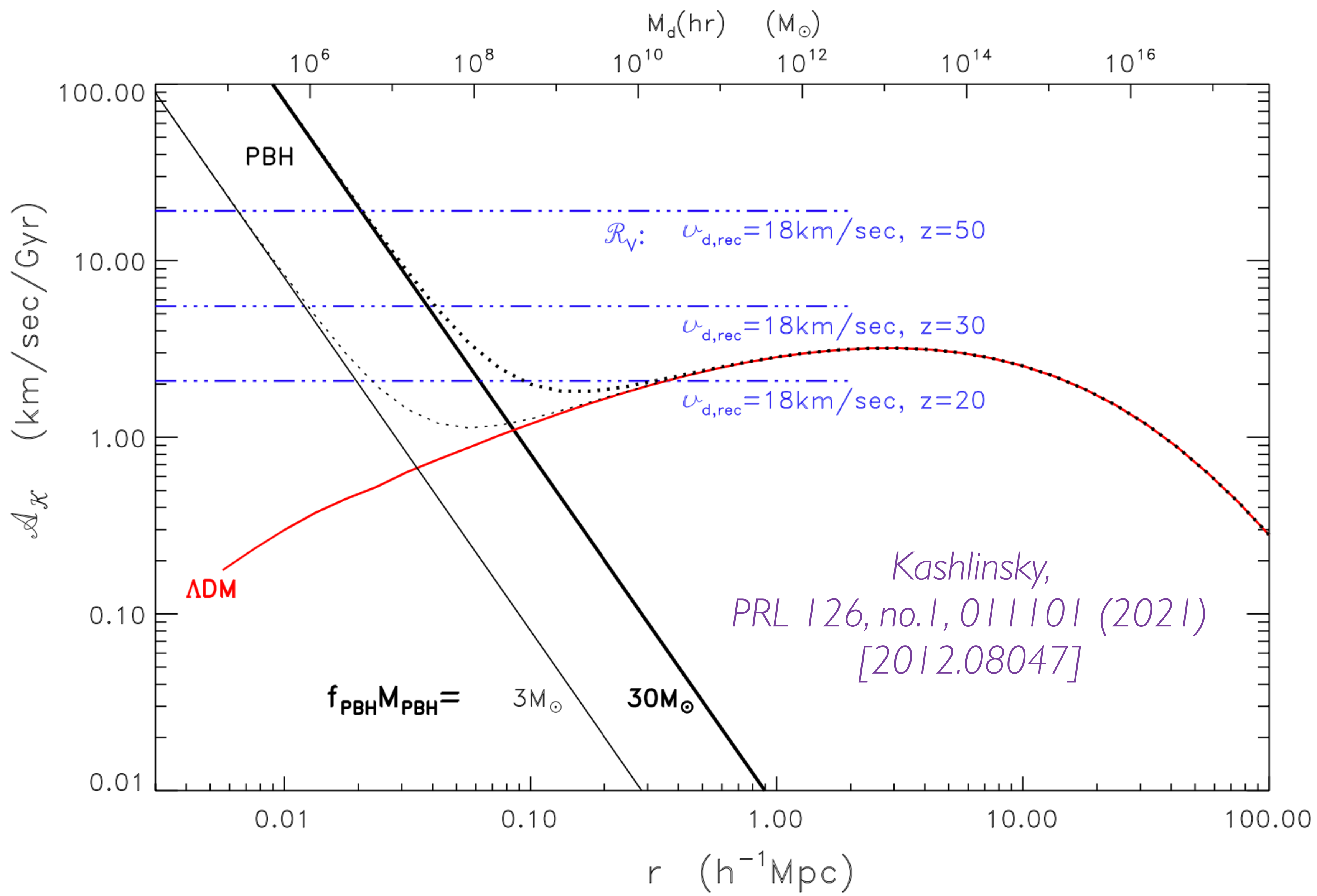
Institute of Cosmos Sciences, University of Barcelona

M. Ricotti's lecture (2017)

The inner characteristics, hence the **luminosity**, are **affected**

PBH cosmology ≠ CDM cosmology!

Due to the **large power spectrum of PBH at small scales ("Poisson noise")** the advective term claimed to be significant and driving velocities to equalisation

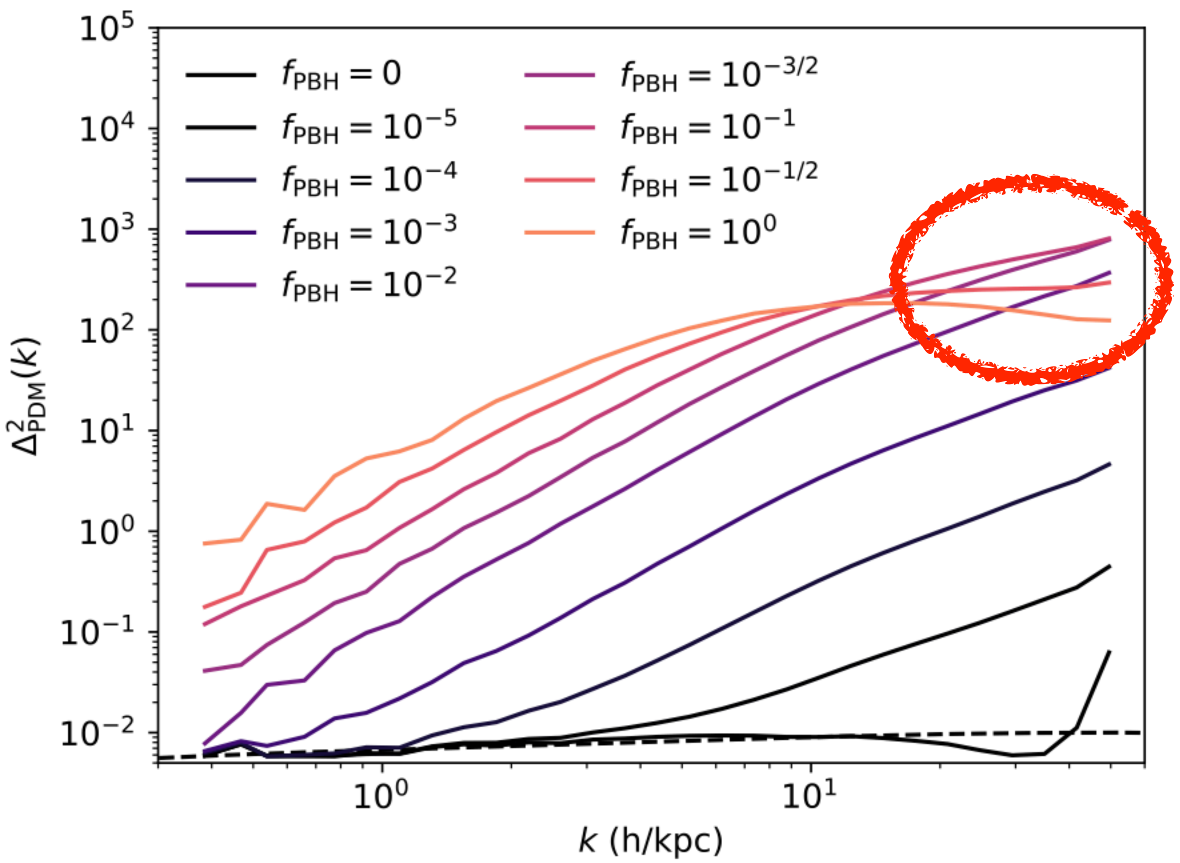


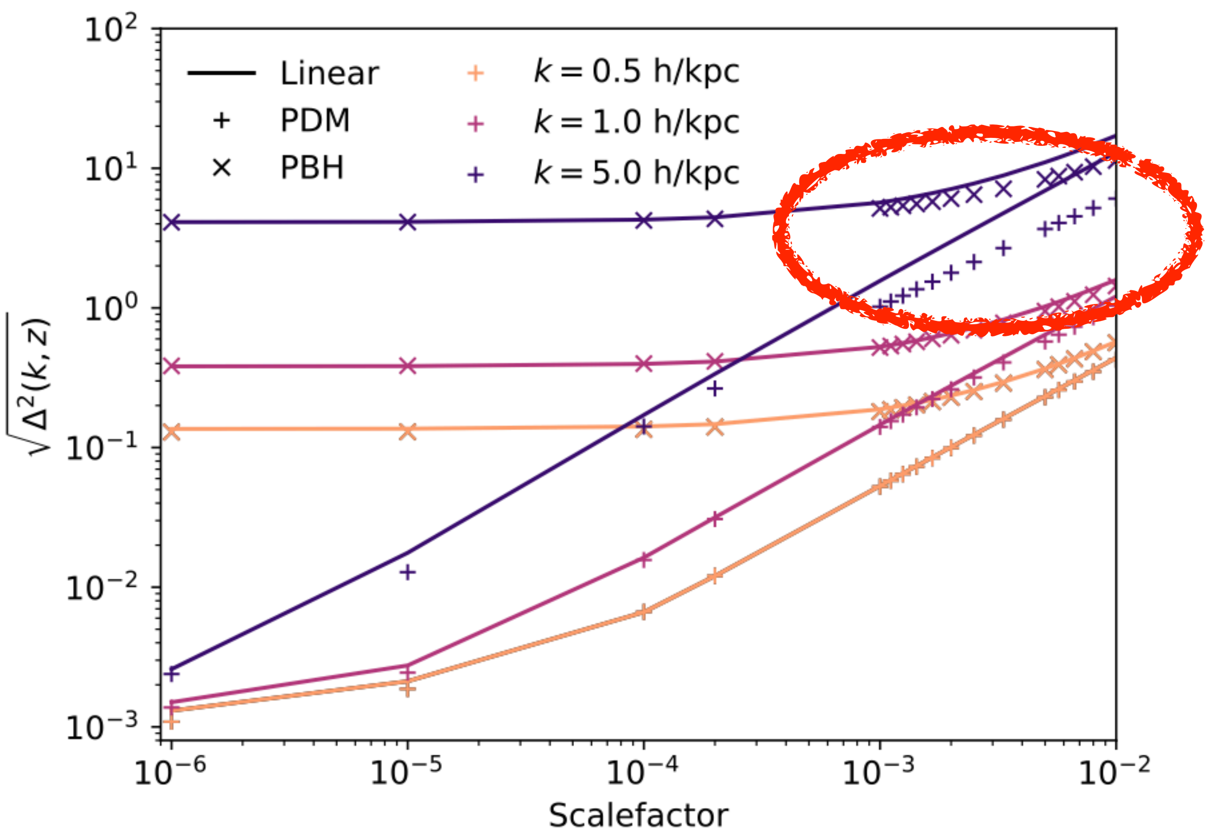
Should one use v_{bc}~c_s?

Not so obvious…

What simulations show

significant non-perturbative effects absent in the ΛCDM model damp the matter power spectrum compared with expectations from perturbation theory.





D. Inman & Y. Ali-Haïmoud, Phys. Rev. D 100 (2019), 083528 [arXiv:1907.08129]

Tests & some consequences

Main effect ≠ PBH-induced random motion disrupting the uniform supersonic motion of baryons in the DM rest frame

Rather, the perturbations due to the granularity of the PBHs accelerate the decoupling of gas from the large-scale flow relative to the underlying DM structures: early enhanced growth of halos with mass $M \leq 10^5 - 10^6\, M_\odot$

B. Liu, S. Zhang and V. Bromm, MNRAS 514 (2022) 2, 2376-2396 [2204.06330]
F. Atrio-Barandela, ApJ. 939 (2022) 2, 69 [2209.04737]

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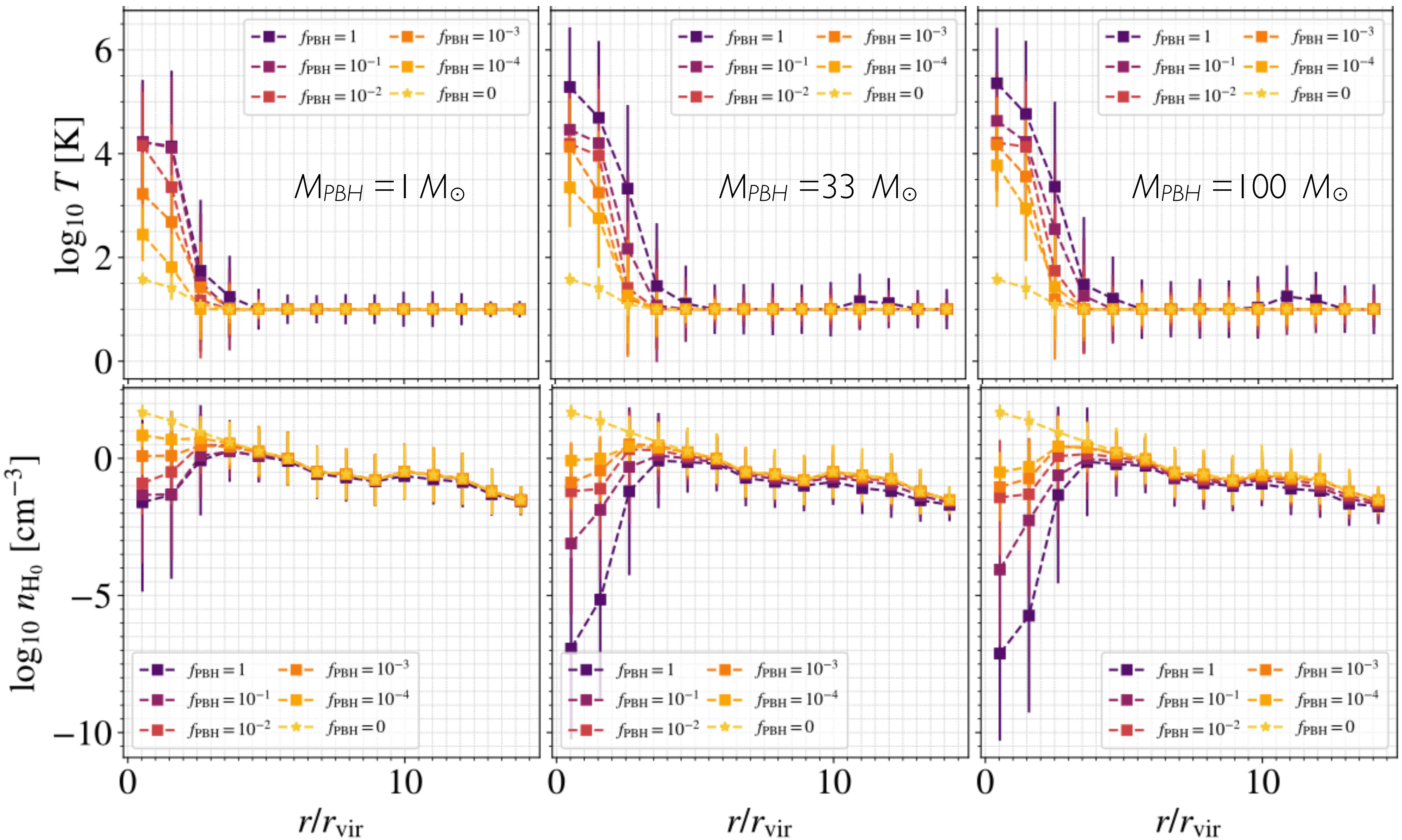
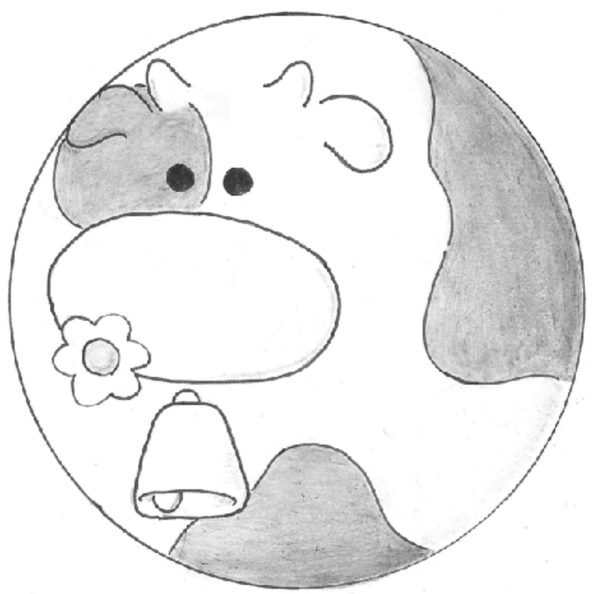


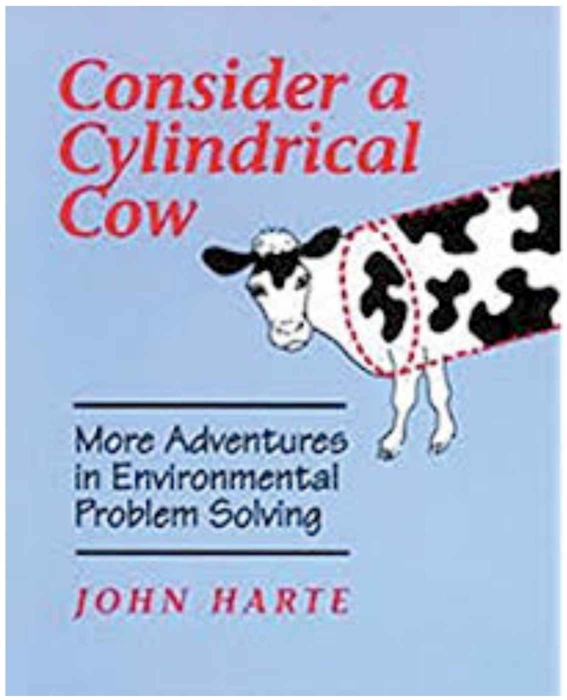
z~23

Alteration expected in the halo baryonic gas properties! *C. Casanueva-Villarreal et al. A&A 688 (2024), 183 [2405.02206]*

disks



by Christopher Berry



Some useful reviews

M. A. Abramowicz & P. C. Fragile, "Foundations of Black Hole Accretion Disk Theory," Living Rev. Rel. 16, 1 (2013) [1104.5499]

F. Yuan and R. Narayan, "Hot Accretion Flows Around Black Holes," Ann. Rev. Astron. Astrophys. 52, 529-588 (2014) [1401.0586]

On disk formation

If the accreted gas has specific angular momentum l, it cannot fall straight onto the BH, but sets in Keplerian motion at distance $r_D(l)$ given by

$$l \simeq r_D v_{Kep}(r_D) \simeq \sqrt{GMr_D}$$

If $r_D \gg 3\, r_{Schw}$ a disk will form (emission dominated by innermost stable orbits)

Shapiro & Lightman 1976; Ipser & Price 1977; Ruffert 1999; Agol & Kamionkowski 2002

In our case

V. Poulin et al. PRD 96, 083524 (2017)

$$l \simeq \left(\frac{\delta\rho}{\rho} + \frac{\delta v}{v_{eff}} \right) v_{eff} r_{HB} \qquad r_{HB} \simeq \frac{GM}{v_{eff}^2}$$

Density gradients perp. to the BH motion

$$\left.\frac{\delta\rho}{\rho}\right|_{k\sim r_{BH}^{-1}} \gg 10^{-4}$$

easy to satisfy because of the enhanced power spectrum on small scales!

Typical velocity dispersion on small scales

$$\delta v \gg 1.5 \left(\frac{1+z}{1000} \right)^{3/2} \mathrm{m/s}$$

Always true e.g. for typical binary PBHs

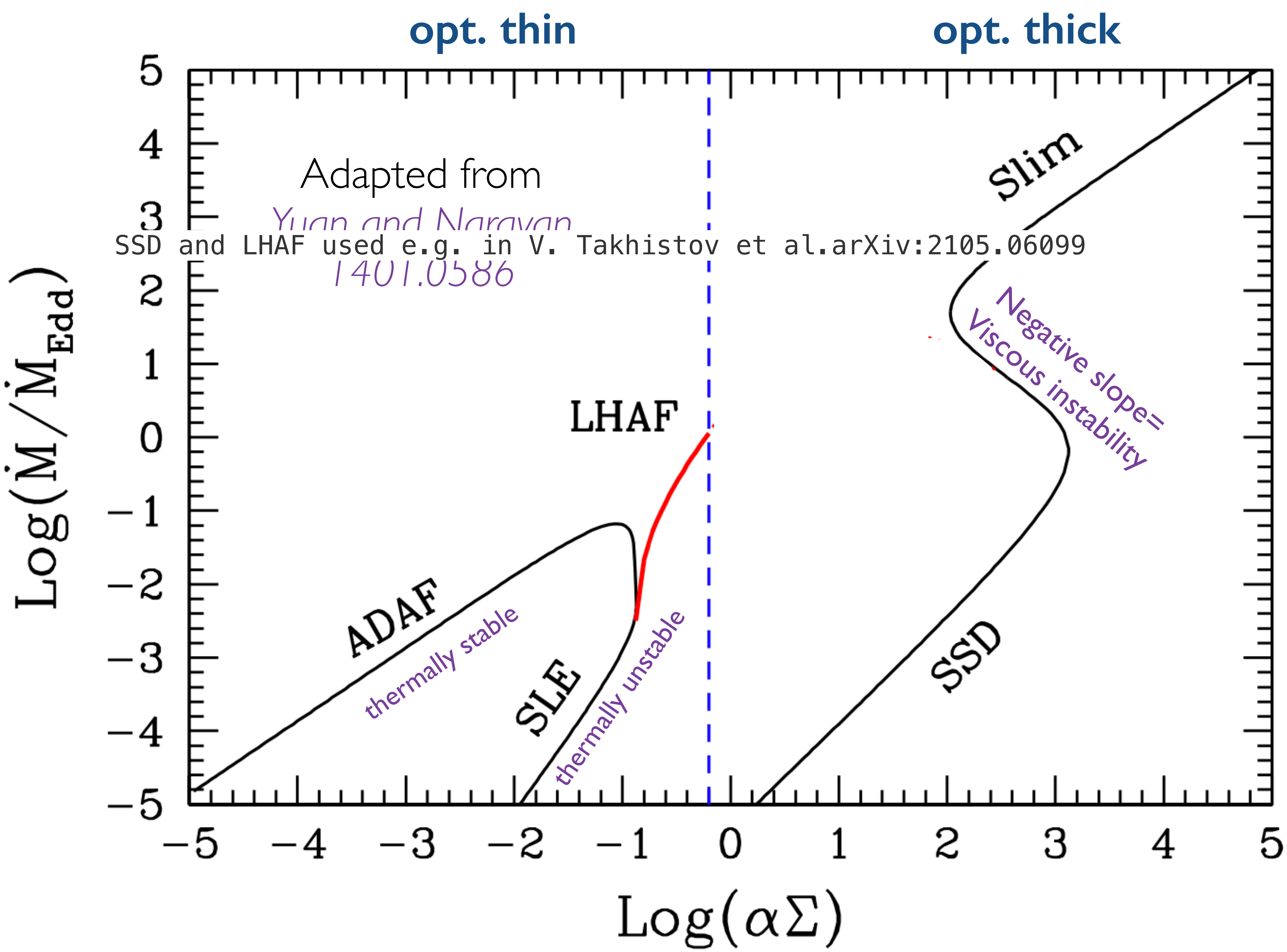
V. De Luca, G. Franciolini, P. Pani and A. Riotto, JCAP 04 (2020), 052, [2003.02778]

These effects vanish however in the limit $f_{PBH} \rightarrow 0$

Disk zoology

No 'complete' or general theory of disks exists, too many variables!
However, a **general enough classifications of different regimes matching most states seen in Nature** has been achieved

Two key parameters control most of the physics: **accretion rate** and **optical depth**



SSD, LHAF & ADAD used e.g. in Takhistov et al. arXiv:2105.06099

High optical depth

SSD or thin disk *Shakura & Sunyaev, A&A, 24, 337–355 (1973)*

- Geometrically thin (and non-gravitating) disk
- Steady state, axisymmetric, hydrostatic balance
- Local radiation balanced by (parametric) viscous heat dissipation

Low accretion

Slim disk (→SSD at low accretion)
or optically thick advection-dominated accretion flow

Katz 1977, Begelman 1979,
Abramowicz et al >80…

accreting gas optically too thick to radiate all the dissipated energy locally; advection kicks in: Radiation is trapped & advected inward with the accretion flow. ϵ lower than ~10%, L capped at few L_{Edd}

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accreting gas optically too thick to radiate all the dissipated energy locally; advection kicks in: Radiation is trapped & advected inward with the accretion flow. ϵ lower than ~10%, L capped at few $L_{\rm Edd}$

High accretion

Both known as **'cold flows'**: the thin disks radiates with blackbody spectrum at temperature

$$L \sim 4\pi R^2 T_b^4 \qquad T_b \simeq \left(\frac{L}{4\pi R^2 \sigma}\right)^{1/4} \simeq 3.8 \times 10^7\,\mathrm{K} \left(\frac{L}{L_{\rm Edd}}\frac{M}{M_\odot}\frac{R_S^2}{R^2}\right)^{1/4}$$

To be compared with the viral temperature

$$T_{\rm vir} \simeq \frac{G\,M\,m_p}{3\,k\,R} \simeq \frac{m_p}{6}\frac{M}{M_\odot}\frac{R_S}{R} = 1.7 \times 10^{12}\,\mathrm{K}\,\frac{M}{M_\odot}\frac{R_S}{R}$$

Low optical depth - hot flows

If optically thin, **thermal equilibrium not achieved.** State typically parameterised via a 2-*T* accreting plasma forming a thick torus, with

$$T_{\rm ions} \sim T_{\rm vir} \gg T_{\rm e}$$

Sizable fraction of accretion energy can go into heating the flow rather than being radiated away

Lower efficiency, but growing function of $\dot{M}$

$$\epsilon(\dot{M}) = 0.1\, f(\dot{M}/L_{\rm Edd})$$

(one may have luminous hot accretion flows, LHAF)

$$f(1) \simeq 1,\ f' > 0$$

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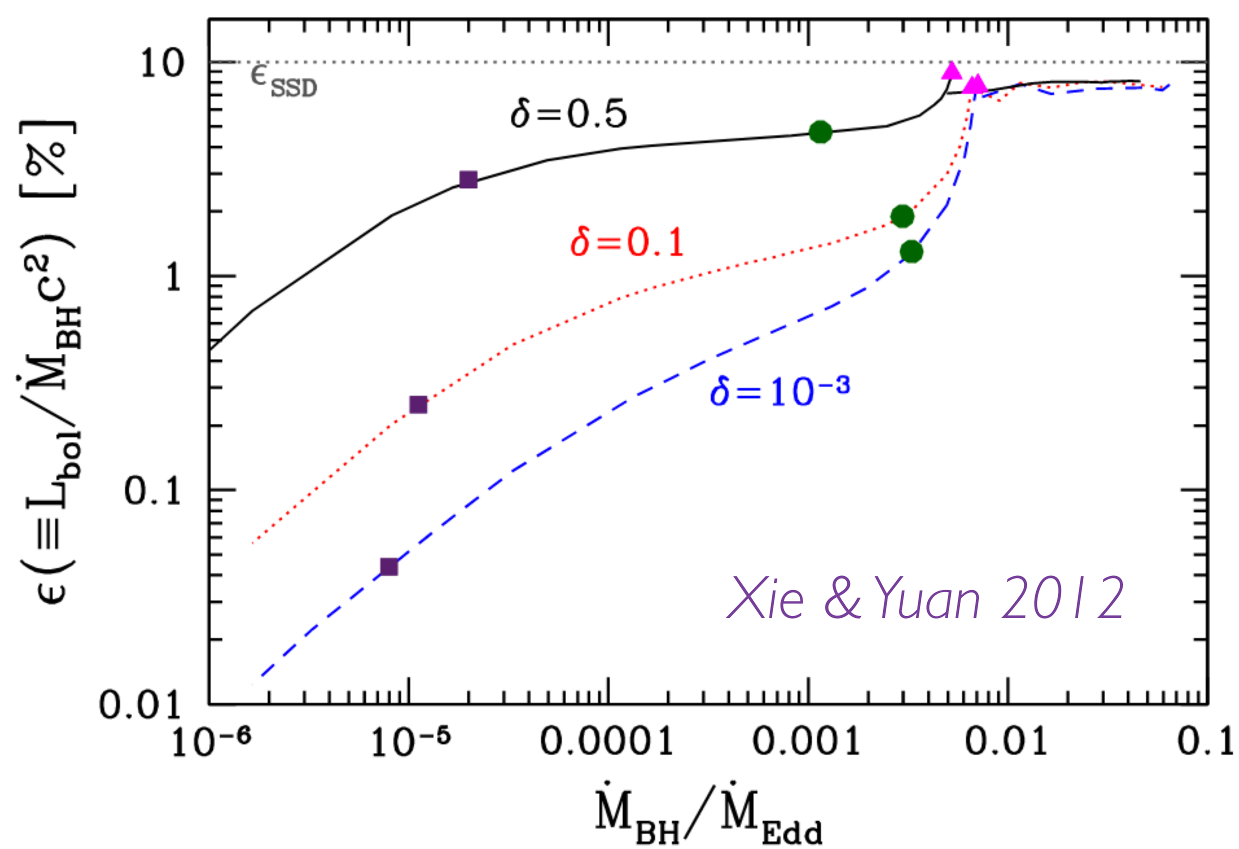
$$f(1) \simeq 1,\ \ f' > 0$$

"useful" (i.e. "visible", but…) **radiation due to electrons, parameterised as**

$$L_e = \delta\, L_{bol} < L_{ions} = (1-\delta)\, L_{bol}$$

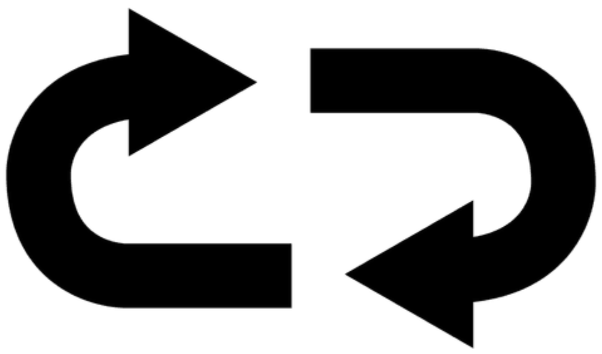
δ=fraction of energy shared by electrons.



Further complications:
Feedbacks

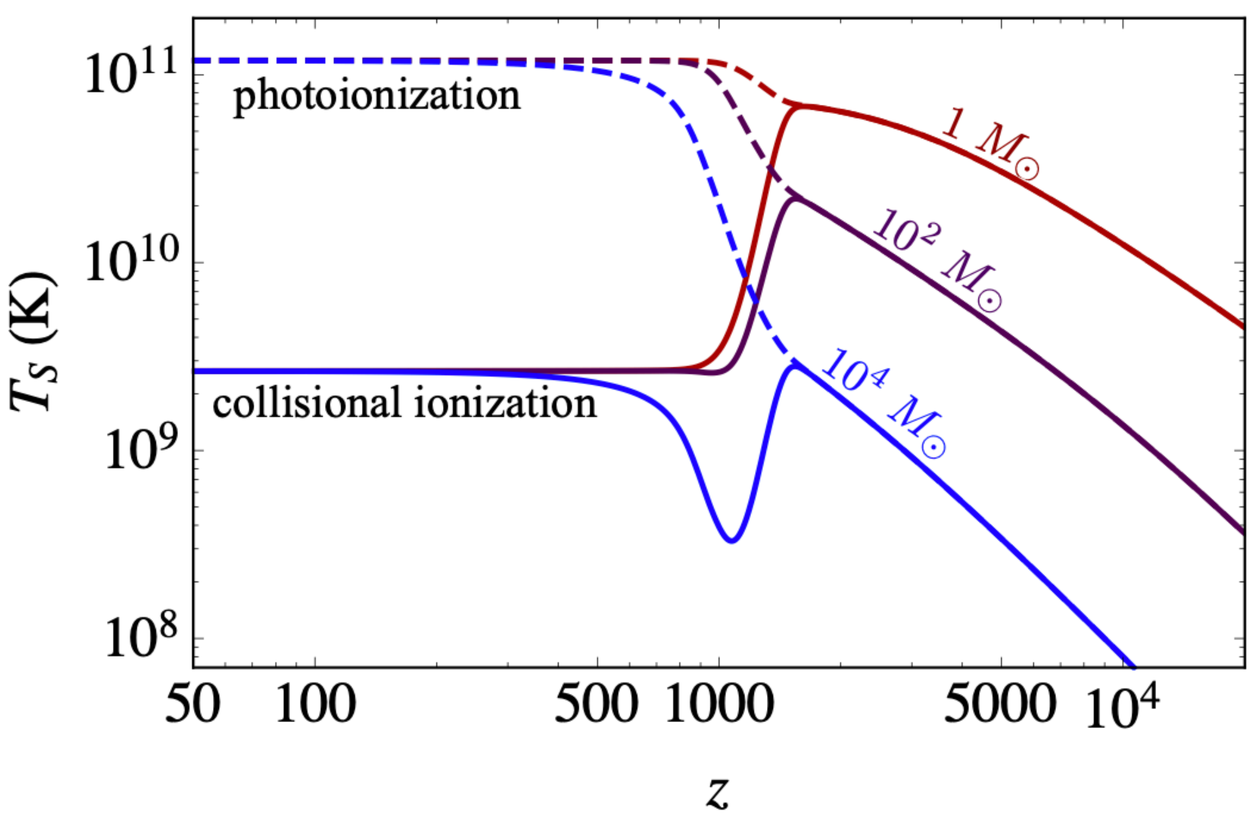
(Thermal and) ionisation feedback

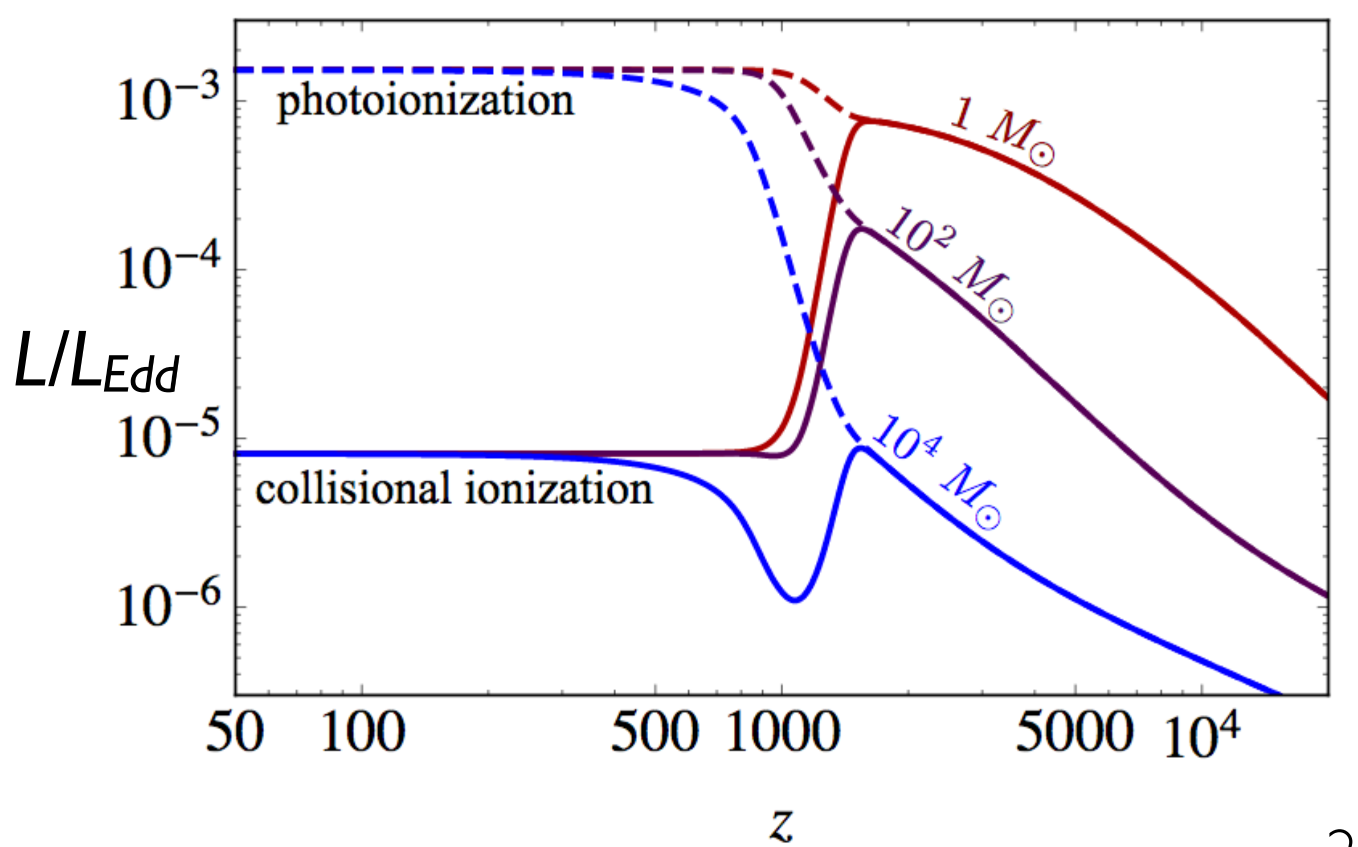
The state of matter falling on the BH depends on the BH luminosity



But the conditions of the innermost, radiating shells depend on that!

Even in spherical case, **exact radiative transfer not available;** bracketed by two cases, depending if photoionisation is strong enough or matter is ionised via collisions





Global accretion feedback

ionising photons from accretion increase gas pressure around the BH, preventing efficient accretion of the surrounding gas, which accumulates ahead of the ionisation front
… until the luminosity drops, depressurised the hot bubble and accretion rises again

The BH caps the accretion rate below Bondi, producing periodic luminosity bursts

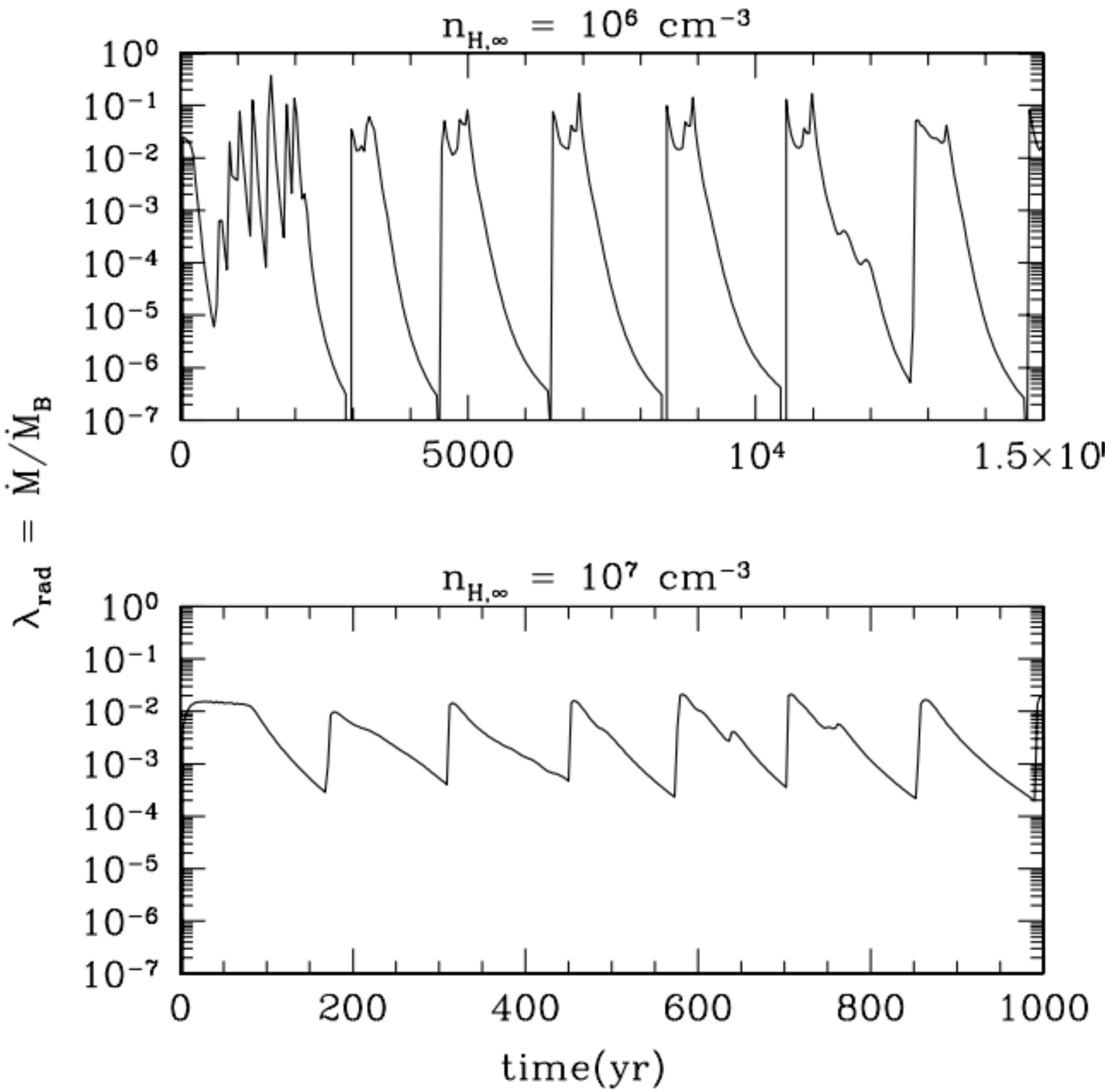
Park & Ricotti ApJ 739,2 (2011)

Park & Ricotti ApJ 747,9 (2012)

spherical symmetry

Mass, density dependence of accretion regimes

That may question steady-state for spherical accretion, once accounting for feedback

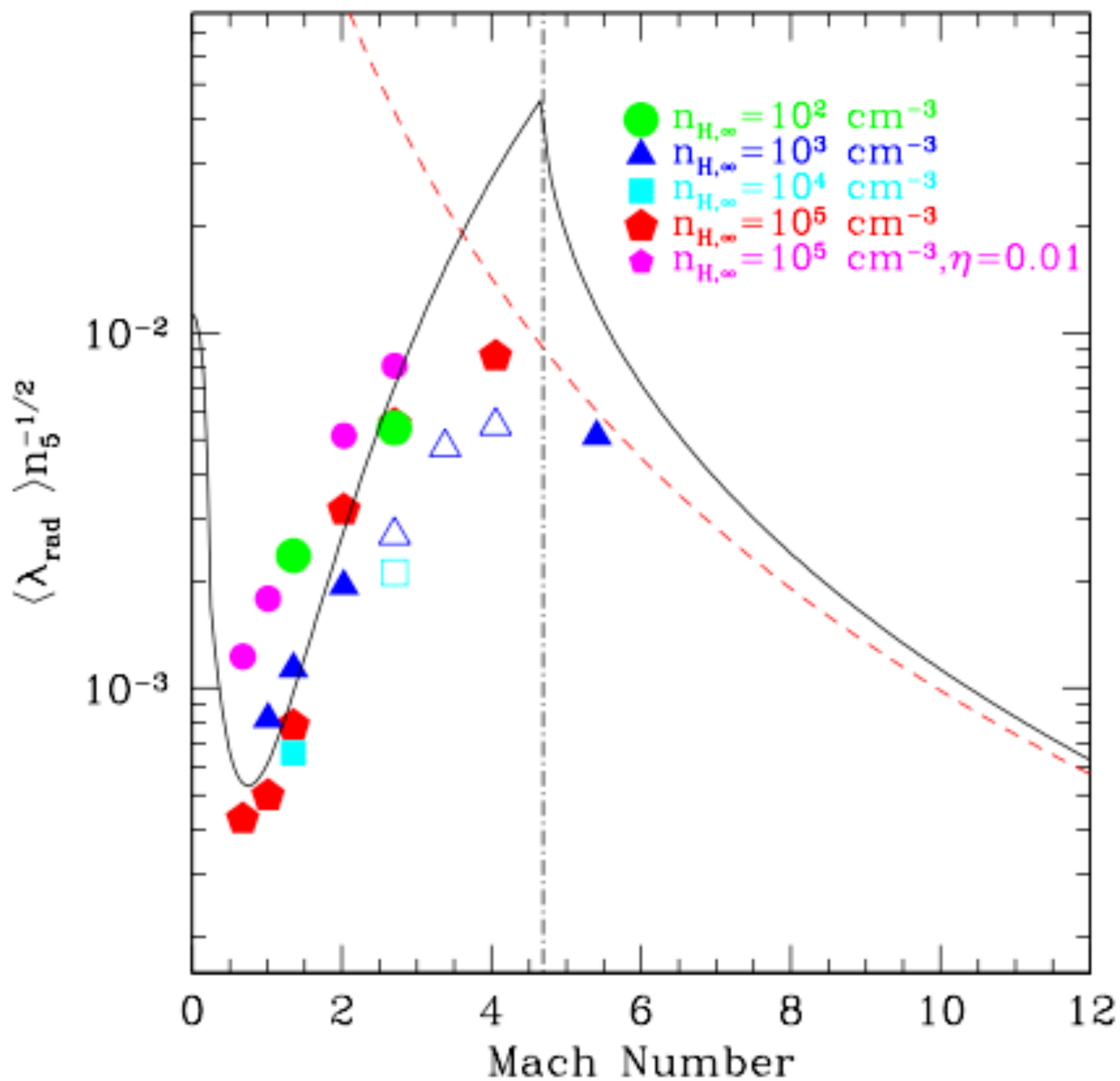


Global accretion feedback - including BH velocity

Park & Ricotti ApJ 767, 163 (2013)

For subsonic motion, cyclic behaviour confirmed, but suppressed accretion

For supersonic motion, failure of spherical symmetric model & steady state typically reached



Significant alteration of accretion rate wrt BHL, non-monotonic behaviour with $\mathscr{M}$

Some caveats

- **Range of parameters!** e.g. spherical case: $n_\infty = 10^5$-10^8 cm^{-3}, M=10^2-10^4 $M_\odot$

A caveat is that our simulations have explored a large but limited parameter space for the masses of the BHs, temperature and density of the ambient gas, etc. So, the proposed scaling relationships [...] should be used with caution for sets of parameters that are significantly different from the range confirmed by simulations *Park & Ricotti 2012*

Even in the moving case, n_∞ most relevant for CMB ~1 o.o.m. below the lowest densities probed ($n_\infty = 10^2$ cm^{-3})

- Instabilities correctly captured? (e.g. 3D vs 2D vs 1D, spherical symmetry of accretion luminosity...)

- Constant & parametric temperature of the ionised region

- Parametric treatment of efficiency (typically ϵ=0.1)

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Park & Ricotti 2012

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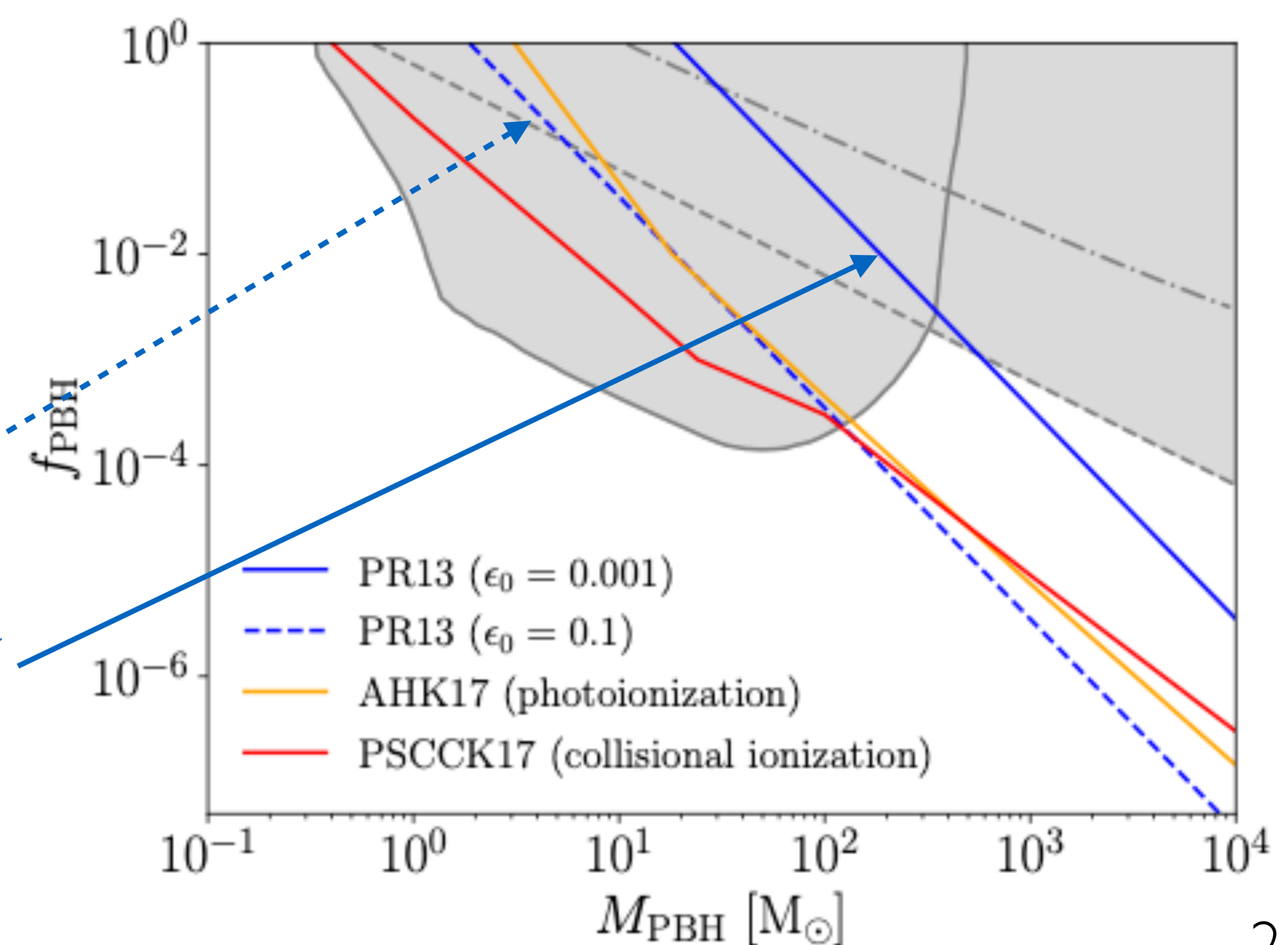
- **Parametric treatment of efficiency (typically ϵ=0.1)**

Am I being unfair? Not unlike other disk models…
Not really: feedback crucially depends on that! E.g.

Facchinetti, Lucca, Clesse, "Relaxing (?) CMB bounds on Primordial Black Holes: the role of ionization fronts" 2022

- The bounds they obtain in the same ballpark of others!
- In order to relax, must tune down by hand the efficiency of a model while leaving accretion suppression due to luminosity untouched… physically inconsistent!



"Morally similar" remarks (details differ!) apply to

D. Agius et al. JCAP 07 (2024), 003 [2403.18895]

Mechanical feedback

Accretion can be reduced via non-relativistic winds or relativistic jets:

- Directly removing material close to the BH
- Exerting pressure onto the medium around the BH

Typically associated with

Magnetic fields **BH spin**

both expected to be absent/small for PBH

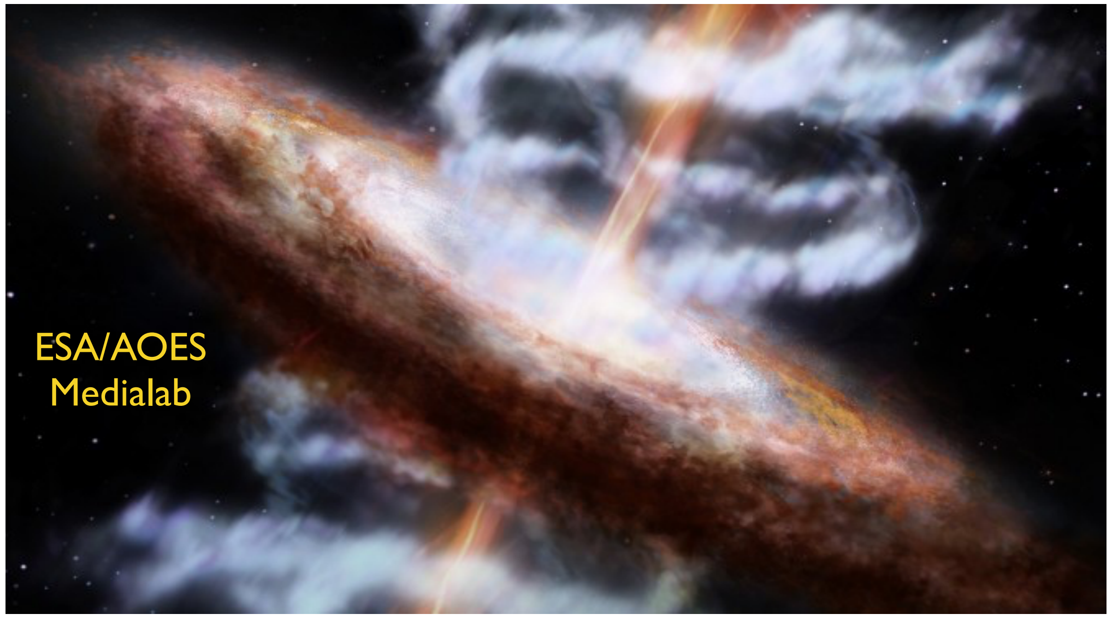
Outflows

$$\dot{M}(r) = \dot{M}_{\rm BH}\left(\frac{r}{R_S}\right)^s$$

$0 \leq s < 1$

Blandford & Begelman 1999

Form inner disk wind (non-rel.) or jet (rel.), matter dominated In extreme cases, "MADs" (magnetically arrested disks)



Jets BH-associated, Poynting-flux dominated

associated to magnetic flux and geometry and BH spin, with power

$$P_{BZ} = \frac{\kappa}{4\pi\, c}\Phi_B^2\Omega_{\rm BH}^2$$

Blandford & Znajek 1977

Extra luminosity from non-thermal (particle acceleration) processes

How important is this extra luminosity, if feedback present?

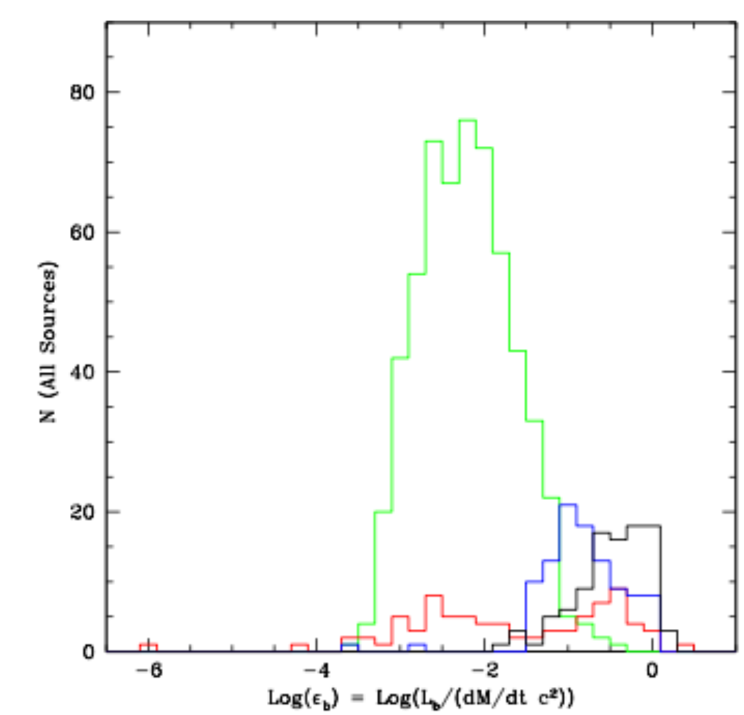
(1) Sample or Source	(2) type	(3) N	(4) Log (dm/dt)	(5) Log (dM/dt) $(M_\odot/yr)$	(6) Log (ϵ_{bol})	(7) Log (ϵ_j)	(8) Log $(\epsilon_j/\epsilon_{bol})$	(9) Log $(\epsilon_{s/d})$	(10) Log (M_{dyn}) $(M_\odot)$
NB16	AGN	576	$-1.67 \pm 0.42(0.20)$	$-2.24 \pm 0.51(0.23)$	$-2.21 \pm 0.56(0.24)$	$-1.07 \pm 0.47(0.28)$	$1.14 \pm 0.73(0.31)$	$1.60 \pm 0.64(0.29)$	$8.07 \pm 0.50(0.35)$
D16/D19	AGN	100	$-0.38 \pm 0.34(0.24)$	$0.09 \pm 0.60(0.26)$	$-0.48 \pm 0.43(0.29)$	$-1.15 \pm 0.38(0.29)$	$-0.67 \pm 0.57(0.40)$	$0.23 \pm 0.51(0.37)$	$9.11 \pm 0.41(0.35)$
M03	AGN	80	$-1.17 \pm 0.84(0.26)$	$-1.97 \pm 0.99(0.30)$	$-1.68 \pm 1.22(0.32)$	$-1.33 \pm 0.71(0.38)$	$0.35 \pm 1.40(0.36)$	$0.83 \pm 1.10(0.35)$	$7.84 \pm 0.81(0.50)$
S15	GBH	103	$-0.74 \pm 0.46(0.12)$	$-8.56 \pm 0.45(0.12)$	$-0.84 \pm 0.52(0.15)$	$-1.34 \pm 0.18(0.14)$	$-0.51 \pm 0.56(0.28)$	$0.40 \pm 0.50(0.25)$	0.83 ± 0.09
GX 339-4	GBH	76	-0.65 ± 0.36	-8.52 ± 0.36	-0.74 ± 0.40	-1.35 ± 0.13	-0.61 ± 0.41	0.31 ± 0.37	0.78
V404 Cyg	GBH	20	-0.88 ± 0.47	-8.53 ± 0.47	-0.99 ± 0.53	-1.20 ± 0.12	-0.21 ± 0.62	0.68 ± 0.56	1.00
J1118+480	GBH	5	-1.08 ± 0.02	-8.85 ± 0.02	-1.22 ± 0.04	-1.86 ± 0.02	-0.63 ± 0.06	0.23 ± 0.04	0.88
A0 6200	GBH	2	-1.85 ± 1.97	-9.68 ± 1.97	-2.08 ± 2.23	-1.21 ± 0.05	0.88 ± 2.28	1.66 ± 2.01	0.82
FRII	W	3	-0.16 ± 0.31	-0.28 ± 0.19	-0.20 ± 0.39	-1.41 ± 0.26	-1.21 ± 0.36	-0.25 ± 0.30	$8.52 \pm 0.24(0.35)$
FRII	Q	29	-0.18 ± 0.21	0.53 ± 0.38	-0.23 ± 0.27	-1.12 ± 0.38	-0.88 ± 0.42	0.07 ± 0.40	$9.36 \pm 0.46(0.40)$
FRII	HEG	55	-0.40 ± 0.31	0.03 ± 0.54	-0.51 ± 0.40	-1.16 ± 0.38	-0.65 ± 0.58	0.24 ± 0.52	$9.07 \pm 0.33(0.35)$
FRII	LEG	13	-0.78 ± 0.32	-0.58 ± 0.52	-0.99 ± 0.41	-1.14 ± 0.37	-0.15 ± 0.53	0.64 ± 0.47	$8.84 \pm 0.31(0.35)$
M03	NS1	7	-0.24 ± 0.31	-1.99 ± 0.63	-0.35 ± 0.45	-1.35 ± 0.49	-1.00 ± 0.61	-0.11 ± 0.54	$6.90 \pm 0.63(0.50)$
M03	Q	13	-0.33 ± 0.20	-0.85 ± 0.46	-0.47 ± 0.29	-1.50 ± 0.91	-1.02 ± 0.79	-0.17 ± 0.82	$8.13 \pm 0.42(0.50)$
M03	S(1-1.9)	17	-0.89 ± 0.66	-2.03 ± 0.71	-1.27 ± 0.95	-1.30 ± 0.76	-0.03 ± 1.07	0.58 ± 0.89	$7.50 \pm 0.46(0.50)$
M03	S2	22	-1.27 ± 0.54	-2.17 ± 0.96	-1.83 ± 0.77	-1.20 ± 0.70	0.63 ± 1.09	1.07 ± 0.92	$7.75 \pm 0.90(0.50)$
M03	L1.9	10	-1.92 ± 0.36	-2.19 ± 0.71	-2.75 ± 0.52	-1.34 ± 0.66	1.41 ± 0.85	1.57 ± 0.76	$8.37 \pm 0.66(0.50)$
M03	L2	10	-2.09 ± 0.40	-2.26 ± 0.65	-3.00 ± 0.58	-1.42 ± 0.74	1.58 ± 1.02	1.66 ± 0.90	$8.47 \pm 0.75(0.50)$

When significant disk luminosity suppressions found, often the jet luminosity is dominant

Ruth A. Daly,

"Black Hole Mass Accretion Rates and Efficiency Factors for over 750 AGN and Multiple GBH,"

MNRAS 500, no.1, 215-231 (2020)

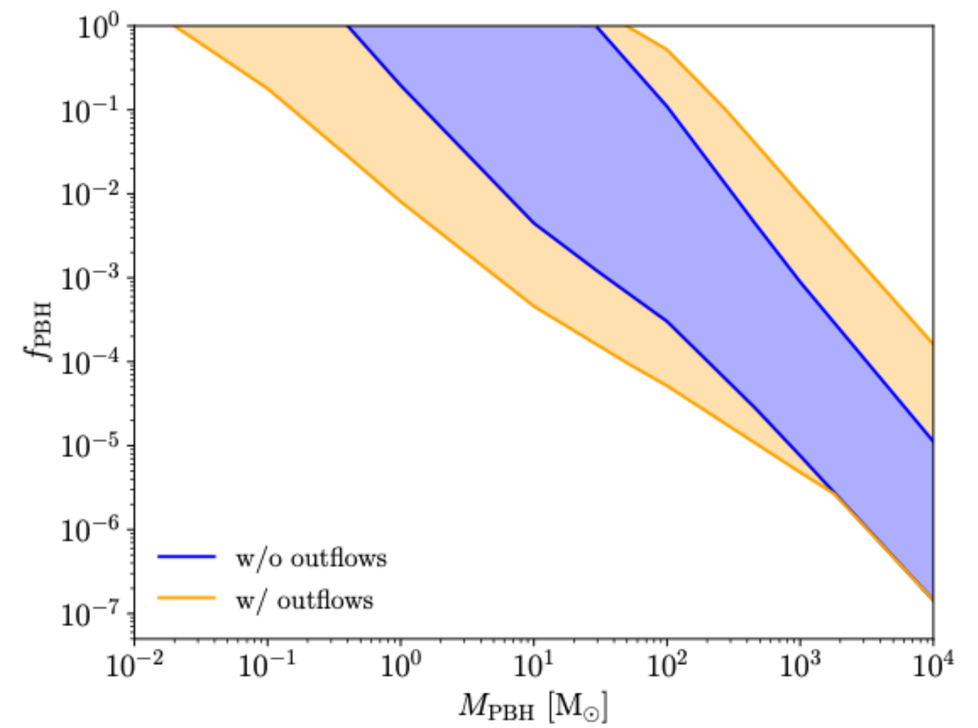


What if, despite all, (mechanical) feedback is present?

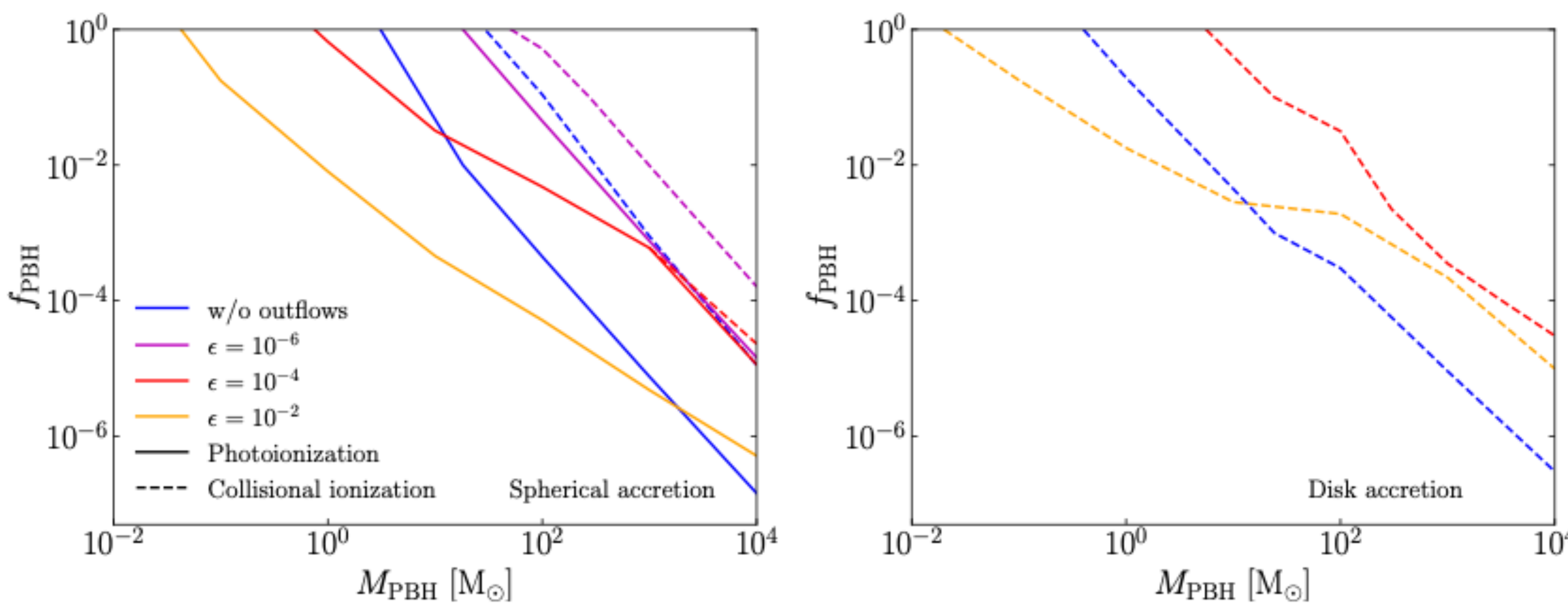
Studied notably in *L. Piga et al. JCAP 12 (2022), 016 [2210.14934]*

e.g. motivated by some studies of hydrodynamical jets from breaking of spherical symmetry

X. Hernandez et al, Rev. Mex. de Astr. y Astrof. 50 (2014) 23 [1103.0250]
A. Aguayo-Ortiz, E. Tejeda & X. Hernandez, MNRAS 490 (2019) 4, 5078 [1909.00884]



Not surprisingly, effect can go either way:
Typically strengthened at low masses and weakened at high masses



Phenomenological input for ADAF models

Numerical solutions of following eqs. In axisymmetric geometry:

Conservation of mass
radial momentum
angular momentum
energy ("heat")
EOS

Parameters typically adjusted to data:
δ, *p, size outflow region, kinematic viscosity, hydro/magnetic pressure*

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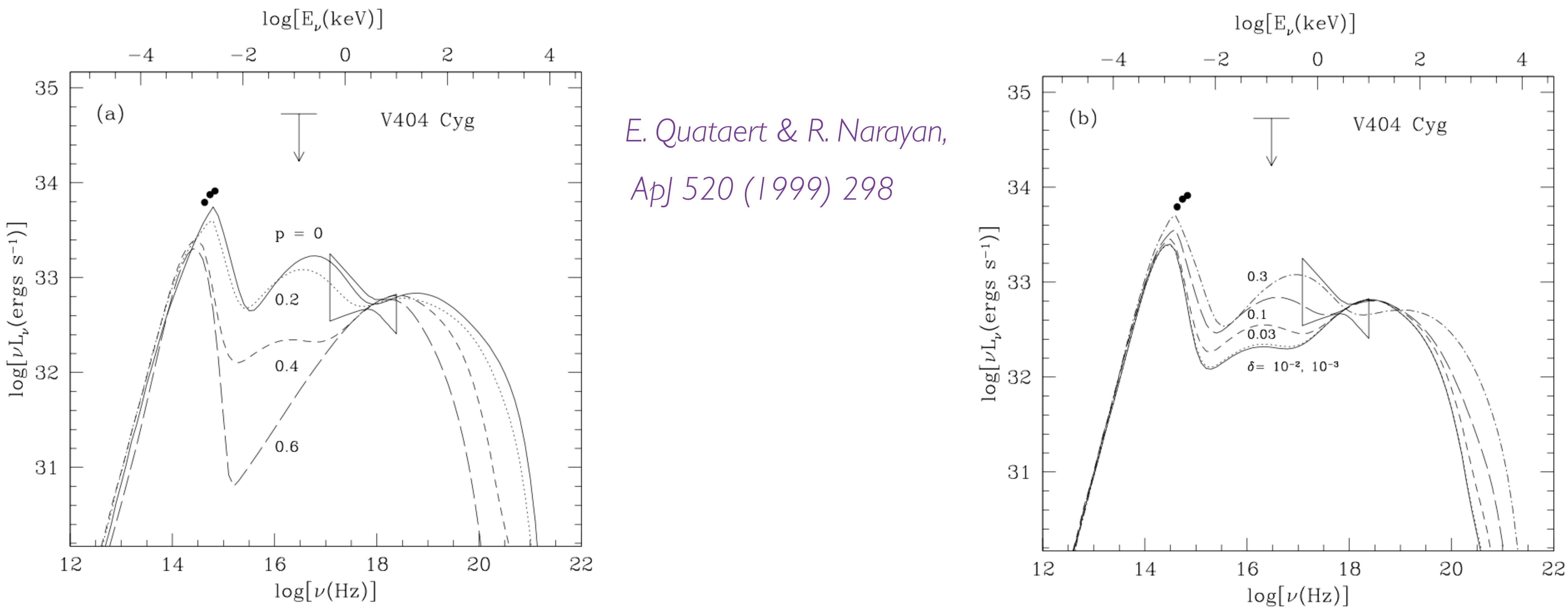

E. Quataert & R. Narayan, ApJ 520 (1999) 298

Anticorrelation between relevance of outflows & energy shared by the electrons δ: cannot put both too low!

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radial momentum
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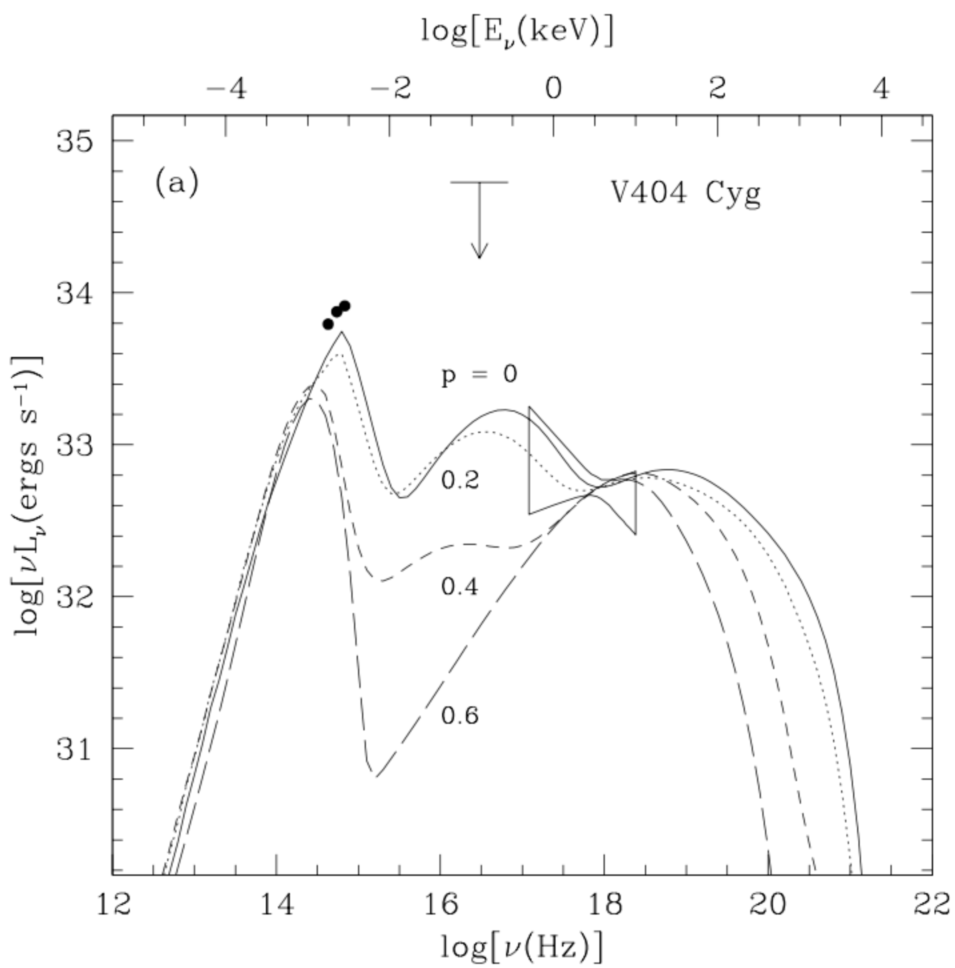
EOS

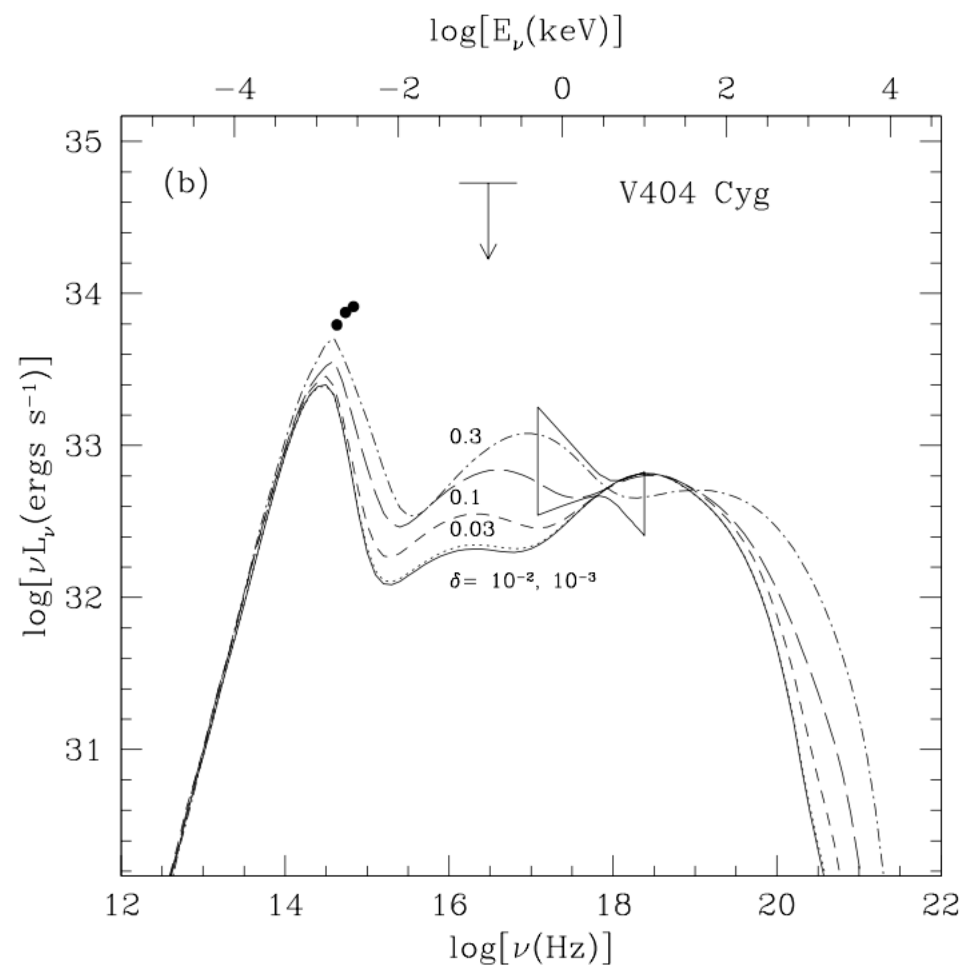
Parameters typically adjusted to data:
δ, p, size outflow region, kinematic viscosity, hydro/magnetic pressure

Theoretical lower limit around δ~0.01
Fits to data suggest 0.1<δ<0.5,
Conservative estimates for δ=0.1

Yuan and Narayan 2014



E. Quataert & R. Narayan, ApJ 520 (1999) 298



Anticorrelation between relevance of outflows & energy shared by the electrons *δ*: cannot put both too low!

Accounting for that in astro vs. cosmo applications

Lessons from astrophysics

- For AGN (most often in ADAF) the inferred value is consistent with λ~0.01 in Bondi

S. Pellegrini, Astrophys. J. 624, 155-161 (2005)

- Even more important for Sgr A* (well-fitted in these extended ADAF models)

Applications in cosmology(?)

- It is doubtful, in my opinion, how much of this is relevant for PBH, which live in a unmagnetised medium and are expected to be ~non-spinning.
- Yet, in the studies I was involved in, we conservatively accounted for the effect via λ~0.01, without including any other ionisation source from the jet.
- Energy is not lost! In these systems, there is an outflow/jet luminosity as well… conceptually disturbing when neglecting it in cases where feedback is invoked

Benchmarks used in studies I took part in

1. **Collisional ionization for spherical case ad v~c_s** *(relatively high $\dot{M}$, low L)*

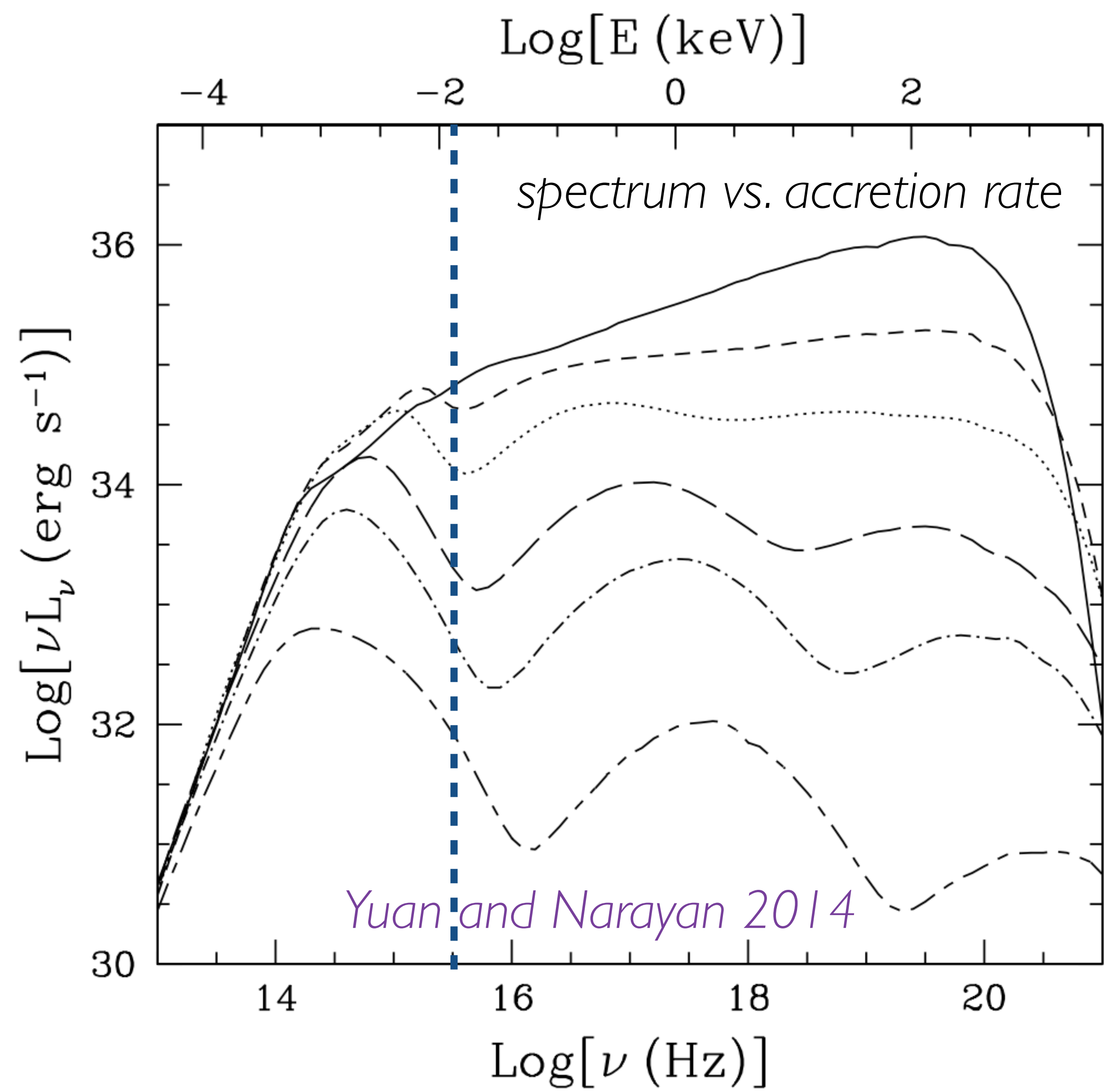
2. **ADAF model with suppressed accretion (λ~0.01) & ε = ε(δ=0.1)**

Disk spectrum parameterised as

$$L_\omega \propto \Theta(\omega - \omega_{\rm min})\omega^{-a}\exp(-\omega/T_s)$$

a~0-0.5 *T_s~O(m_e)*

$\omega_{\rm min}$ accounts for 'useful fraction of the spectrum', $\omega_{\rm min}$~O(10) eV



Effects on the CMB almost bolometric, do not depend much (factor ~2) on the exact *E*-distribution

Role of dark matter accretion

What if PBH do not make all DM?

- A halo of gravitationally bound, collisionless DM will form around PBH
- Even if only a small fraction of the DM halo gets swallowed by the PBH, a baryon at infinity sees a stronger potential, "effectively attracted by a heavier BH''
- Hence we use the same master equation for accretion

$$\dot{M} = 4\pi\lambda_{\rm eff}\rho_\infty v_{\rm eff} r^2_{\rm B,eff}$$

But $r_{\rm B,eff}$ now comes from the solution of

$$\frac{G_N M_{\rm PBH}}{r_{\rm B,eff}} - \Phi_h\left(M_{\rm PBH}, r_{B,\rm eff}, t\right) = v^2_{\rm eff}(t)$$

K. Park, M. Ricotti, P. Natarajan, T. Bogdanovic & J. H. Wise, ApJ 818, 184 (2016)

- Problem reduced to compute the DM halo potential vs. time

Note

The PBH mass remains essentially constant in time over the cosmological epochs of interest (100≲z≲1000), with the most relevant epoch being 300≲z≲600

Analytical expectations

PHB as point-attractor of cold DM moving radially with Hubble flow. A shell at distance r obeys

$$\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} = -\frac{4G_\mathrm{N}}{\pi}3r\left[\frac{3\,M_\mathrm{PBH}}{4\pi\,r^3} + \sum_i \left(\rho_i + 3p_i\right)\right]$$

At any time, the mass of the halo is defined by the 'turn-around radius' satisfying

$$\frac{\mathrm{d}r_\mathrm{t.a.}(t)}{\mathrm{d}t} = 0$$

This leads to the prediction

$$M_\mathrm{halo} \simeq \left(\frac{3000}{1+z}\right) M_\mathrm{PBH}$$

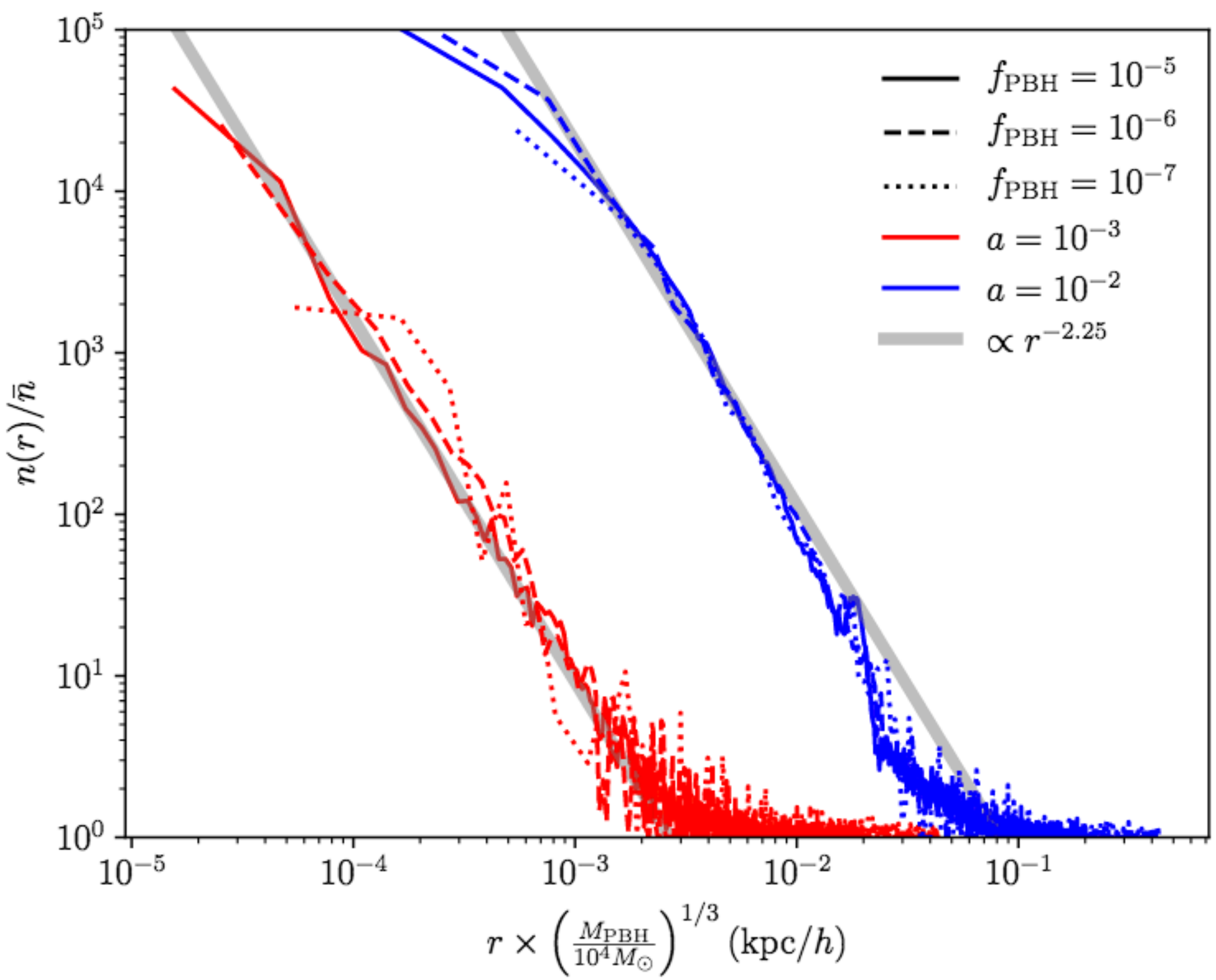
which overshoots more accurate calculations by only a factor 1.6, but leads to a too steep halo profile r^{-3} due to neglecting the angular momentum of DM

Numerical simulations

Self-similar solutions avoiding the free-fall boundary condition at the center and more appropriate for the case at hand suggest a profile $r^{-2.25}$

E. Bertschinger, ApJ 58, 39 (1985)

Our dedicated numerical simulations, with PBH and lighter DM particles, confirm expectations: power-law profile r^{p-3}, with $p \sim 0.75$



PDS, V. Poulin, D. Inman and K. Kohri, Phys.Rev.Res. 2 (2020), 023204

Semi-analytical model

(i.e. "calibrated" to numerical results)

In terms of the (maximal) halo Bondi radius $$r_{B,h} \equiv \frac{G_N M_h}{v_{\rm eff}^2}$$

we can find the analytical solution (if $M_h >> M_{PBH}$, as true in the range of interest)

If $r_{\rm t.a.}(z) < r_{B,h}(z)$

$$r_{B,\rm eff} \simeq r_{B,h}$$

All the halo matters, for the baryon accretion

If $r_{\rm t.a.}(z) \geq r_{B,h}(z)$

$$r_{B,\rm eff} \simeq r_{\rm t.a.} \left[(1-p)\frac{r_{\rm t.a.}}{r_{B,h}} + p \right]^{\frac{1}{p-1}} \leq r_{B,h}$$

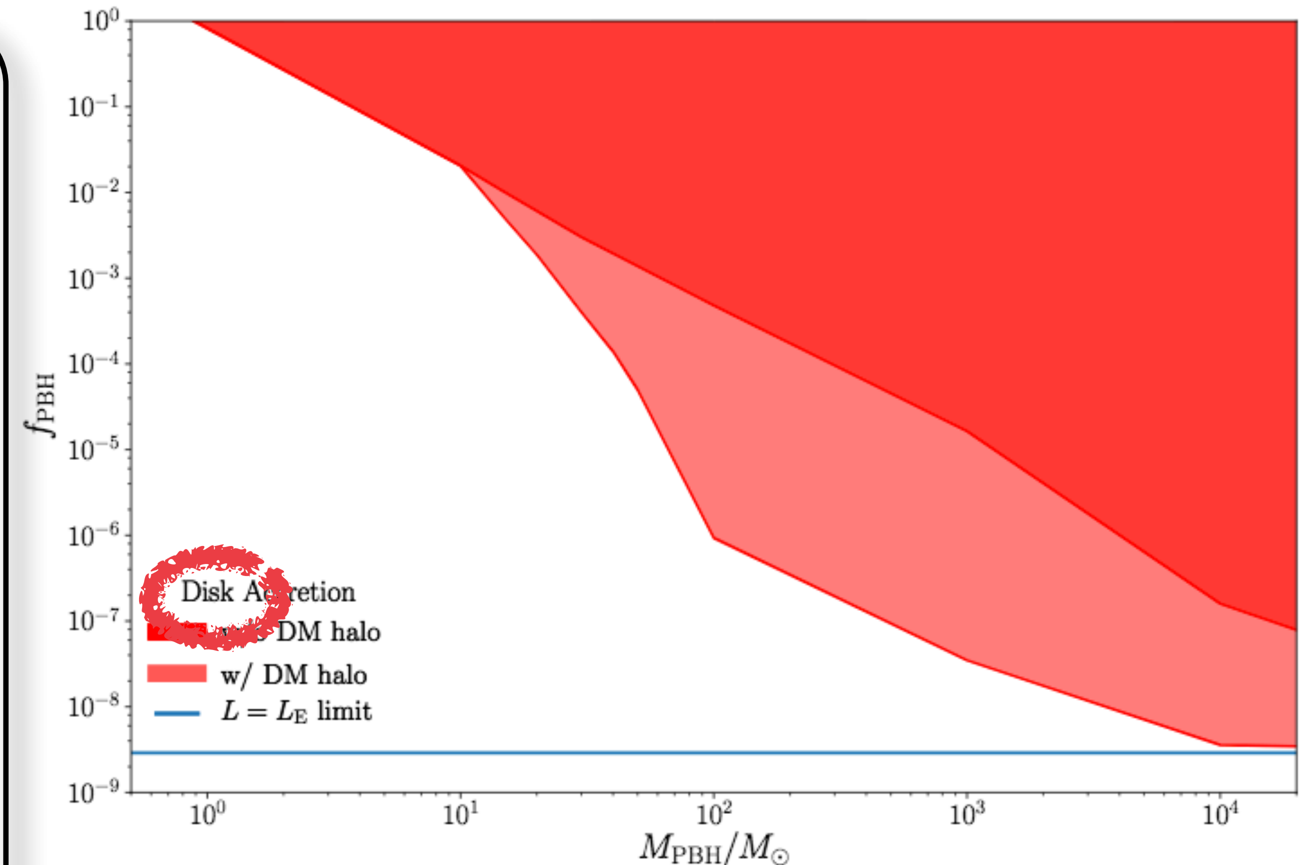
Only a fraction of the halo mass matters, the more the closer p is to 0

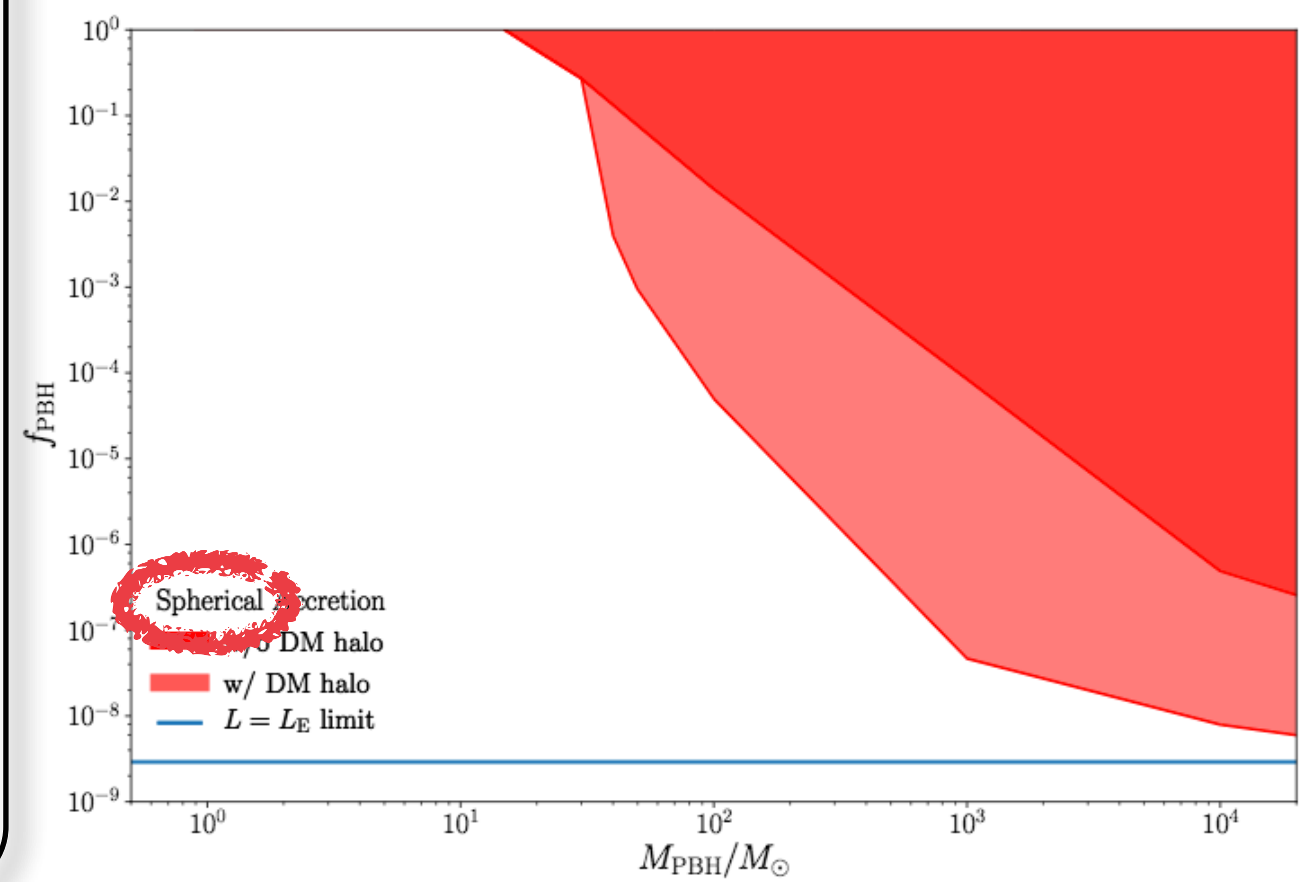
Results: CMB bounds

Results (monochromatic mass function)

- PBH excluded as totality of DM if ***M>15 M***$_\odot$ even for spherical accretion under most conservative case of collisional ionization
- Compared to our results in 2017, factor ~4 improvement due to new & better cosmo data (notably Planck 2018 release with low-ℓ polarization) & better account of t-dependence of E-release/ absorption (via ExoCLASS)
- The DM halos tighten the bound up to ~3 oom.
- Caveat for $0.01 \lesssim f_{PBH} \lesssim 0.1$ (unaccounted modifications of halo profile due to neighboring PBH)
- Spherical and disk case not so different especially at high-M, due to the lower velocity required for spherical case consistency
- Bounds flatten at ***M≳10⁴ M***$_\odot$ since approaching Eddington limit (at which we cap luminosity) for most of the cosmo relevant time

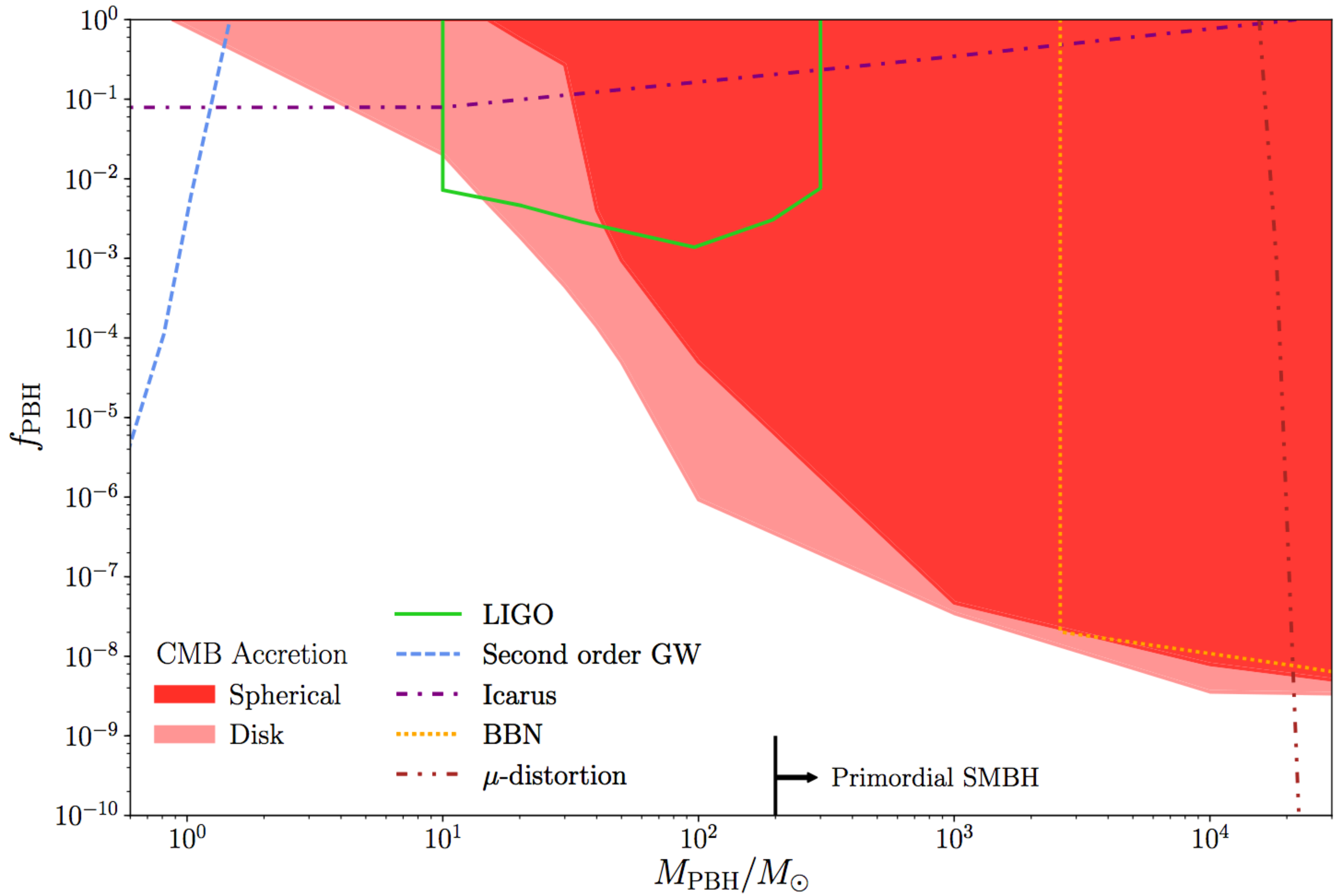
$$f_{\rm PBH} < 2.9 \times 10^{-9} \quad (L_{\rm acc} = L_E)$$





PDS, V. Poulin, D. Inman and K. Kohri, Phys.Rev.Res. 2 (2020), 023204

Comparison with best other bounds



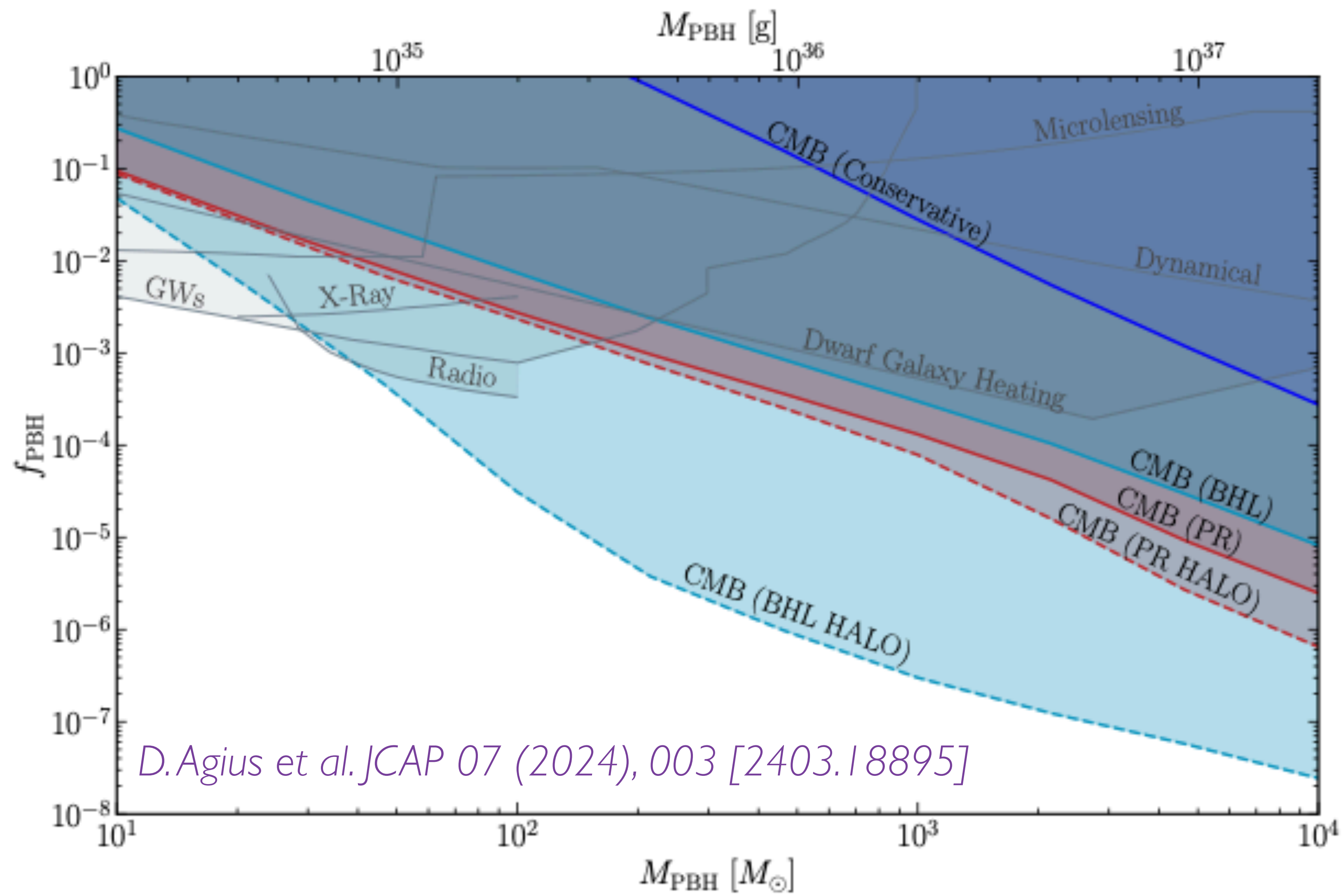
- Compared to the best bounds available, CMB is competitive already at $M \gtrsim 10\ M_{\odot}$ and provides the best bounds for $50\ M_{\odot} \lesssim M \lesssim 2\times10^{4}\ M_{\odot}$
- At least for spherical accretion, marginally compatible with hypothesis that the bulk of "massive" LVK events are due to PBHs

Case with feedback à la P&R 2013

Argument

DM almost irrelevant (especially at low masses) since relevant velocity is controlled by the temperature of the ionised region around the BH.

$$\frac{G_N M_{\rm PBH}}{r_{\rm B,eff}} - \Phi_h\left(M_{\rm PBH}, r_{B,\rm eff}, t\right) = v_{\rm eff}^2(t)$$



D. Agius et al. JCAP 07 (2024), 003 [2403.18895]

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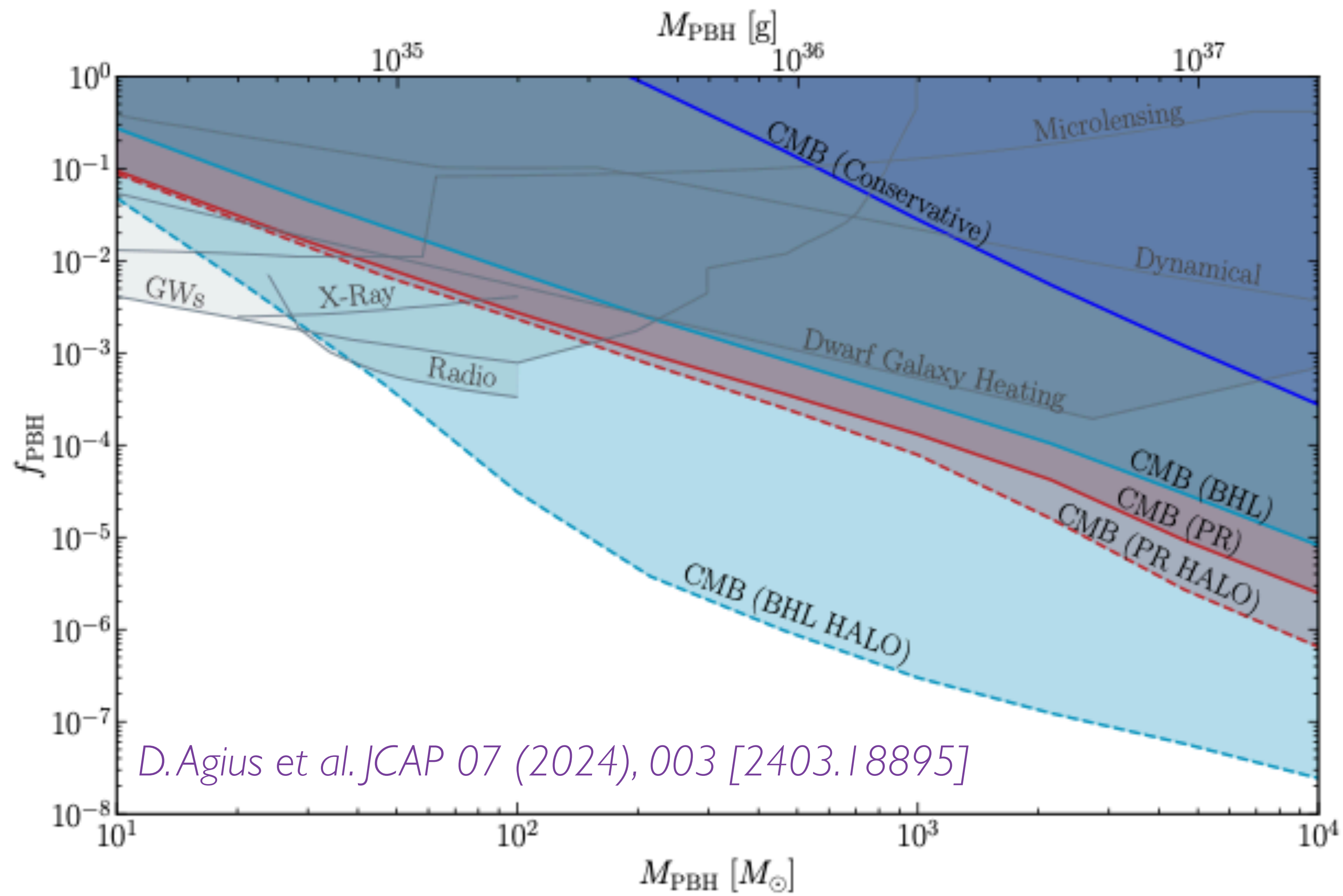


D. Agius et al. JCAP 07 (2024), 003 [2403.18895]

Main criticism (+general caveats):

The inner region temperature is a free parameter! Assumed not to be influenced by the growth of the DM halo, even when size comparable with the ionisation region.
I.e. halo considered on top of unperturbed & parametric solution… hard to trust when crucial role of feedback is invoked. How do extra cold baryons accreted outside affect the solution?

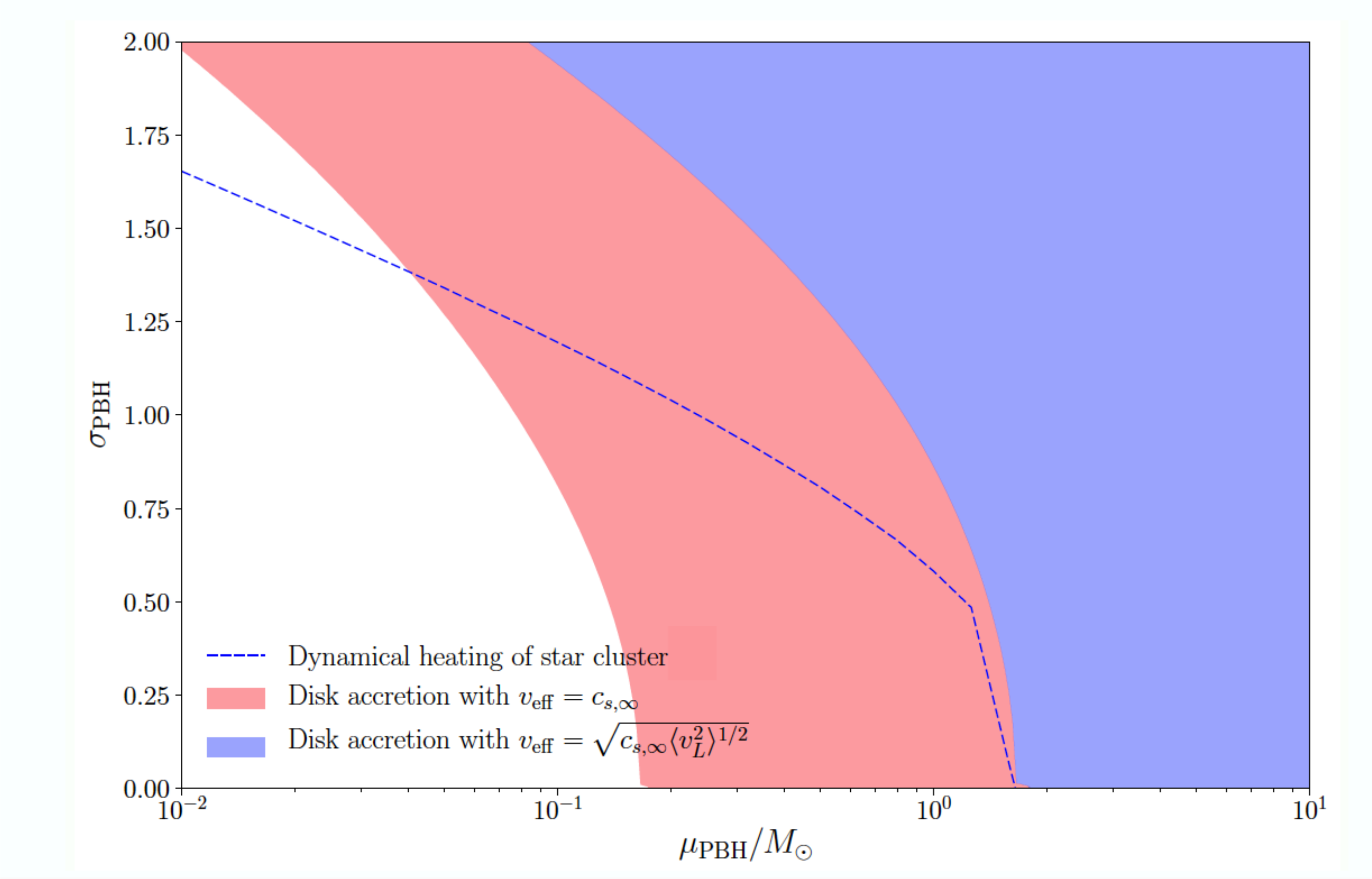
One caveat is that we are neglecting the effects of self-gravity of the gas and the gravitational potential due to the dark matter halo of the host galaxy

Park & Ricotti 2012

What about broad mass functions?

Typically the bounds become stronger for broad mass functions

B. Carr et al. PRD 96, 023514 (2017)
F. Kühnel and K. Freese, PRD 95, 083508 (2017)



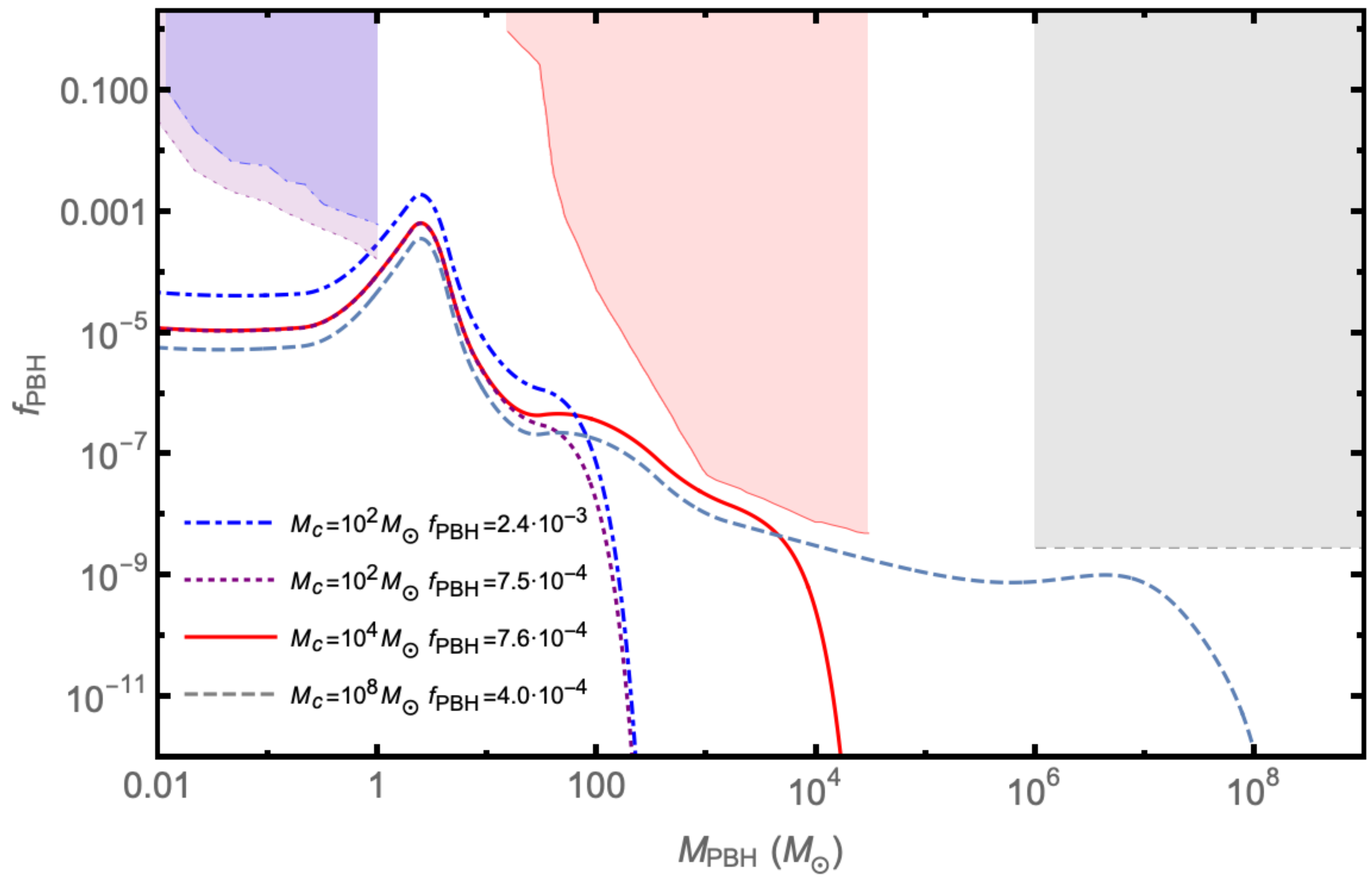
We checked explicitly that this is the case for the CMB bound and a log-normal mass distribution in

V. Poulin et al. Phys. Rev. D 96, 083524 (2017)

True also for complex, broad mass functions

E.g. for a broad MF inspired by QCD crossover change in EOS

J. Iguaz Juan, PDS & G. Franco Abellán, JCAP07(2022)009 [arXiv:2204.07027]



		$f_{\rm PBH}^{\rm max}$
$M_{\rm cut} = 10^2\ M_\odot$	Full	0.129
	Approx	0.177
$M_{\rm cut} = 10^{4.5}\ M_\odot$	Full	1.99×10^{-3}
	Approx	3.09×10^{-3}

Approximated prescription following B. Carr et al. PRD 96, 023514 (2017)

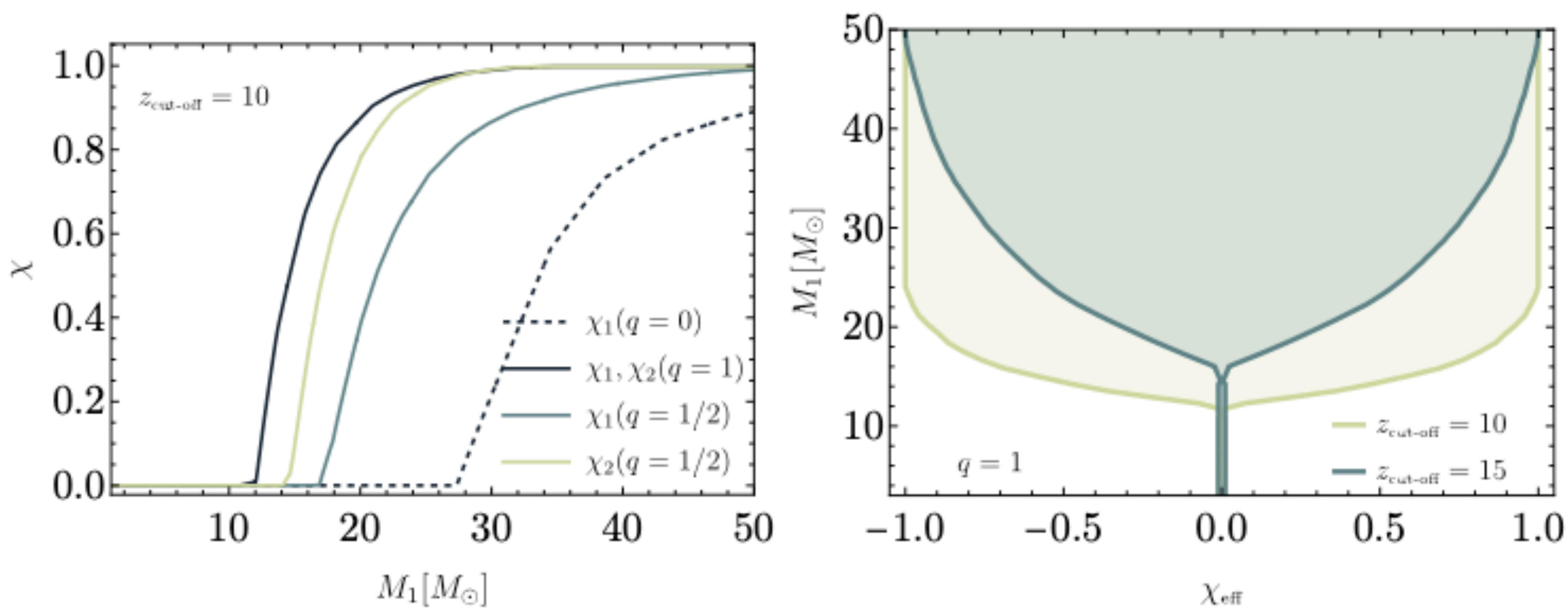
$$\int_{M_{\rm min}}^{M_{\rm max}} {\rm d}M \, \frac{\psi_p(M)}{f_{\rm mono}^{\rm max}(M)} = 1$$

Effects on the PBH in the late universe?

- **Mass growth** Small unless sustained mass growth at low redshifts (z≲10)
- **Spin evolution** Small at $M < 10\, M_\odot$, significant for high M and/or sustained mass growth at z≲10

Qualitative generic expectation:

if significant mass growth via disk accretion, only (P?)BH* above some mass should present large spins



V. De Luca, G. Franciolini, P. Pani and A. Riotto, JCAP 04 (2020), 052, arXiv:2003.02778
V. De Luca, G. Franciolini, P. Pani and A. Riotto, JCAP 06 (2020), 044, arXiv:2005.05641
V. De Luca and N. Bellomo, arXiv:2312.14097

*could be shared by astrophysical BH, unless typically born with large spins even at low masses

Moving to late time universe: caveats & difficulties

- At $z \ll 100$, considerations based on **perturbation theory become unreliable**
- **PBH seed proto-halos much earlier than in ΛCDM,** with low virial velocities. Even if small fraction of gas bound, it may dominate late-time phenomenology
- **Dynamical friction** may become important:

  Massive supersonic PBH in the cosmological baryonic gas slows down in a timescale

$$\tau_{\rm loss}(z)H(z) \simeq 1.8 \times 10^4 \frac{M_\odot}{M}\left(\frac{1+z}{100}\right)^{3/2}\frac{10}{\ln\Lambda}$$

  Ok for CMB to neglect at $M \lesssim 10^{4.5}\, M_\odot$, but important for the physics of PBH in dark ages

- **We expect Galactic magneto-genesis, possibly dynamically acquired PBH spins…**

  We expect accretion theory to complexify, with more and more effects to become relevant

Perhaps more relevant: Need statistical arguments *(e.g. spatial distribution, mass distribution)* to distinguish PBH phenomenology from qualitatively similar astro BH phenomenology

Summary

- If PBH of stellar masses or heavier exist, they should have both cosmological (e.g. CMB, 21cm) and astrophysical effects (e.g. radio, IR, X-ray sources) via their accretion.
- In particular, the CMB & 21cm power spectra are sensitive to the heating & ionisation of the universe due to extra radiation injected by the hot emitting plasma accreting onto PBH.
- key ingredients enter the calculations: **accretion physics** & **PBH cosmo & astrophysics**
- In the stationary spherical accretion & FLRW cosmology, the problem is essentially solved. How realistic that is, it is questionable from both input sides.
- Unless perhaps if f_{PBH} is very small, disks are expected to form notably due to relative movement of the gas with respect to PBH and "large" density perturbations
- If feedbacks can be neglected, general parametric understanding exists of disk phenomenology, with L mildly dependent on some parameters *(share of energy in electrons, details of spectrum…)*

Summary, cont'd

- Feedbacks are generally important for astrophysical systems; their qualitative (*e.g. mechanical ones*) & quantitative (*e.g. ionisation and global modifications*) relevance for cosmological settings is not as clear: current analyses often rely on extrapolations and/or parametric treatment of dynamical effects.

- Despite that, different conservative prescriptions agree on CMB disk bounds within ~1 o.o.m.: PBH cannot constitute 100% of DM below a few $M_\odot$ & must be below ~0.1% at ~100 $M_\odot$.

- Bounds improve for non-monochromatic mass functions and/or accounting for the extra effect of non-PBH DM halo (marginally to ~3 o.o.m., depending on assumptions)

- Especially in models with feedback, where different variables are (non-linearly) coupled, avoid drawing conclusions from changing parameters/adding effects by hand, or ignoring the whole picture (e.g. tuning down efficiency and neglecting non-thermal luminosity when these are at the hearth of the very feedback phenomenon suppressing the accretion)

Some directions for progress

Long-term goal: Understanding the non-perturbative baryon (and DM) halo assembly (densities and velocities) around PBH at very small scales (comparable to $r_{B,eff}$), as a function of PBH IMF (or at least f_{PBH})

After reionisation/star formation seems too ambitious, but perhaps doable in dark ages with dedicated simulations? Naively, I imagine it is simpler than first principle predictions on popIII stars…

Any room left for semi-analytical models?

(Unfortunately, that seems to be required to put "astro" pheno at low-z on firm grounds)

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Long-term goal: Turn parametric treatments of accretion into self-consistent physics ones!

Again, too ambitious in astro settings *(too many parameters, think of metallicity and initial B-fields)*, perhaps realistic to get closer to the goal in dark ages with dedicated simulations?

Examples of sub-goals that may be within reach (at least they look so to me!):

Adding self-consistently the DM halo dynamics in a treatment like *Park & Ricotti 2013*

Covering the relevant cosmo parameter space, avoiding extrapolations of fitting formulae: notable case, the feedback in spherical systems highlighted in *Park & Ricotti 2011-2012* leading to time-dependence…

Turning the inner region temperature in *Park & Ricotti 2013* into a dynamical variable.

Turning ϵ in *Park & Ricotti 2013* into a dynamical variable, ie. solve time-dependent radiative transfer for the spherical problem in cosmo setting

Thank you for your attention!