The Sound of Black Holes

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Tory Island, County Donegal,
Ireland, 1995.
Photograph by Martine Franck

Standard Model not the final word

Torsione, by Giovami Anselmo

Galaxy interactions

Gravitational lensing

Rotation curves

Velocities of galaxies in clusters

Structure formation

...

General Relativity not final word

Theorem (Penrose 1965; 1969):

For “reasonable” matter, trapped surface formation results in “singularity,” where at least one of the following holds:

a. Negative local energy occurs.
b. Einstein's equations are violated.
c. The space-time manifold is incomplete.
d. The concept of space-time loses its meaning at very high curvatures – possibly because of quantum phenomena.

Conjecture (Penrose 1969):

No singularity is visible from future null infinity (weak CCC)
General Relativity is deterministic (strong CCC)

Uniqueness: the Kerr solution

Theorem (Carter 1971; Robinson 1975; Chrusciel, Costa & Heusler 2012):
A stationary, asymptotically flat, *vacuum* BH solution must be Kerr

$$
\begin{aligned}
ds^2 \quad = & \quad \frac{\Delta - a^2 \sin^2\theta}{\Sigma} dt^2 + \frac{2a(r^2 + a^2 - \Delta)\sin^2\theta}{\Sigma} dt d\phi \\
& - \frac{(r^2+a^2)^2 - \Delta a^2 \sin^2\theta}{\Sigma} \sin^2\theta d\phi^2 - \frac{\Sigma}{\Delta} dr^2 - \Sigma d\theta^2
\end{aligned}
$$

$$
\Sigma = r^2 + a^2 \cos^2\theta \, , \quad \Delta = r^2 + a^2 - 2Mr
$$

Describes a rotating BH with mass M and angular momentum J=aM, iff a<M

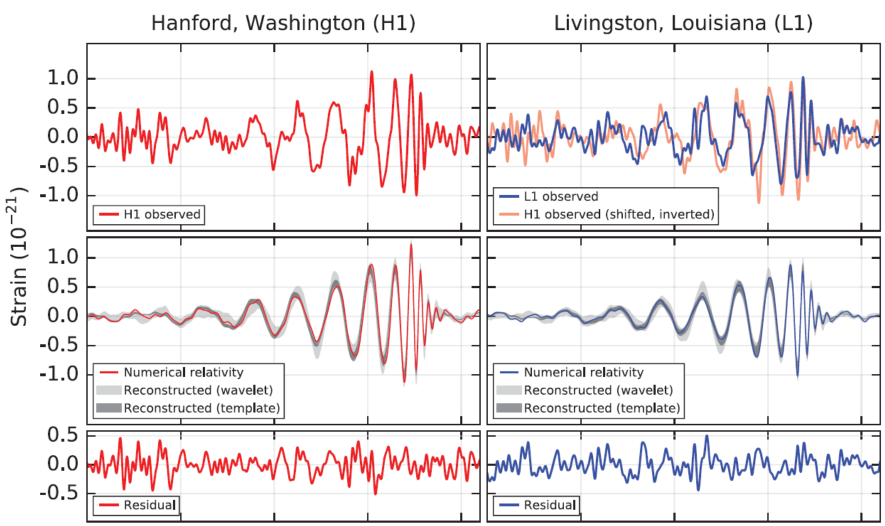
"In my entire scientific life, extending over forty-five years, the most shattering experience has been the realization that an exact solution of Einstein's equations of general relativity provides the *absolutely exact representation* of untold numbers of black holes that populate the universe."

S. Chandrasekhar, The Nora and Edward Ryerson lecture, Chicago April 22 1975

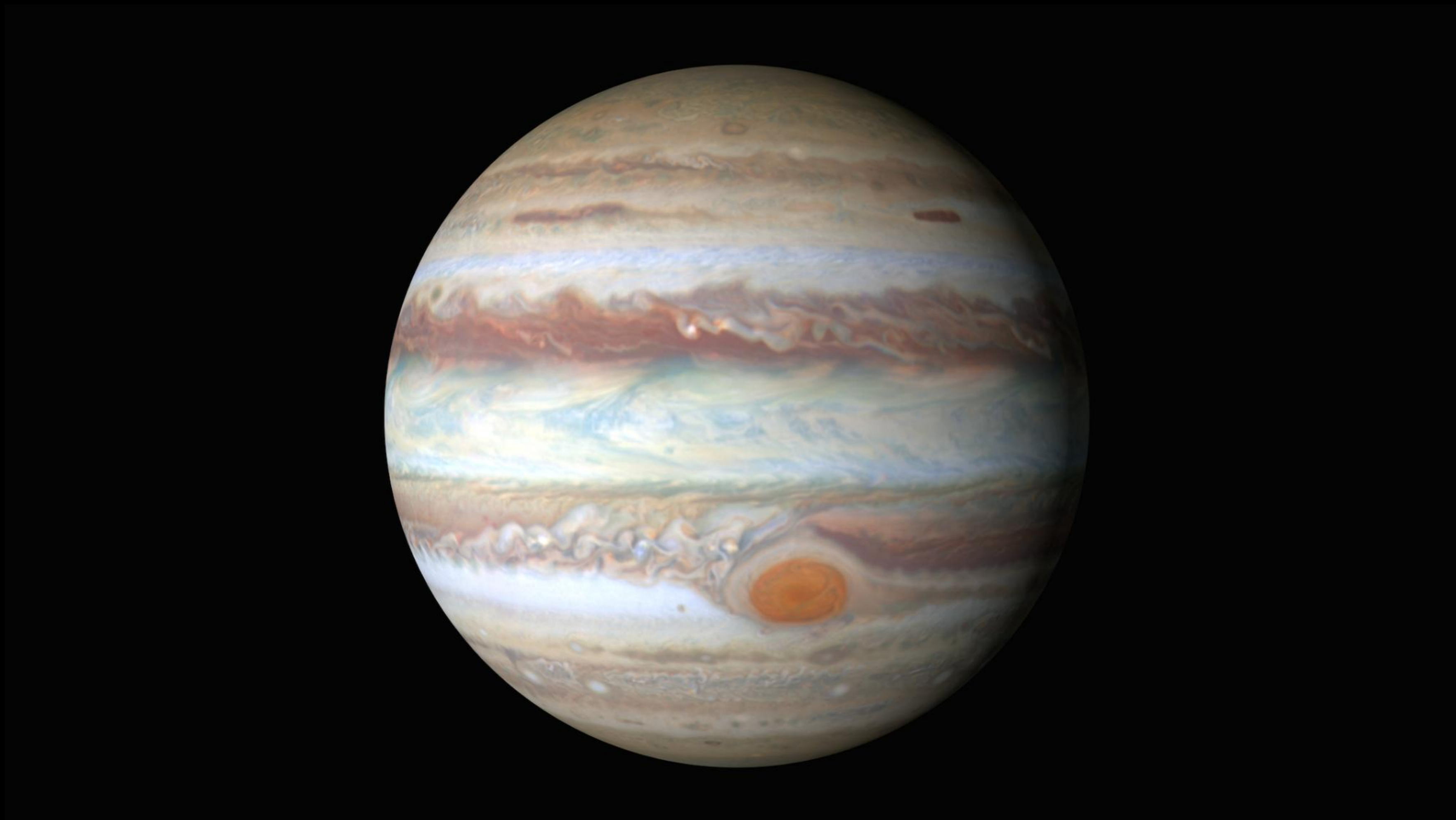
“But a confirmation of the metric of the Kerr spacetime (or some aspect of it) cannot even be contemplated in the foreseeable future.”

S. Chandrasekhar, The Karl Schwarzschild Lecture, Astronomischen Gesellschaft, Hamburg, 18 September 1986

Final state is compact!



Annalen Phys. 529 (2017) 0209



How do we know it's a black hole?

Dark

Massive

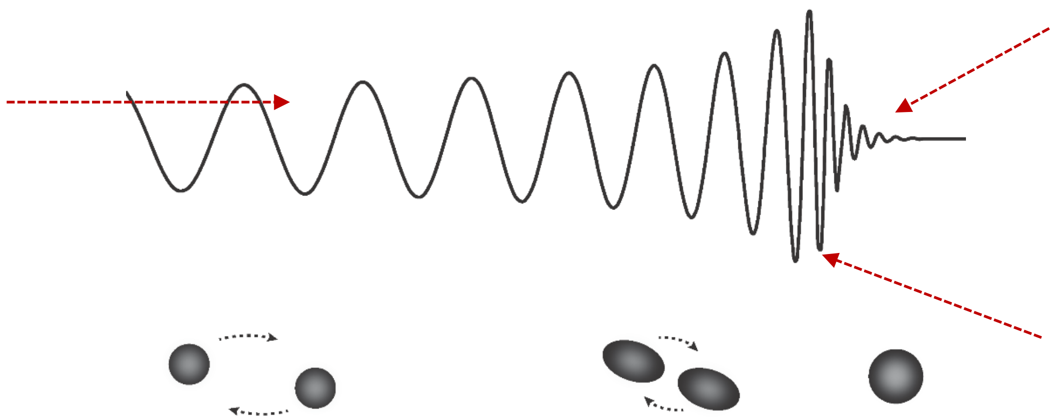
Why is this enough?

Black holes are end-point of gravitational collapse, using the equation of state that is thought to prevail in known matter.

No other massive, dark object has been seen to arise from generic collapse of known matter.

Black holes exist and they emit gravitational waves

Spin-induced multipoles
Tidal heating
Tidal Love number
Environments
Masses, how many, where?



Relaxation of final state
To Kerr?

Horizon formation
Why so simple?

But BHs in GR are simple objects:

Multipolar structure entirely dependent on mass and spin

Tidal Love numbers vanish (black holes don't "polarize")

Relaxation depends only on mass and spin…

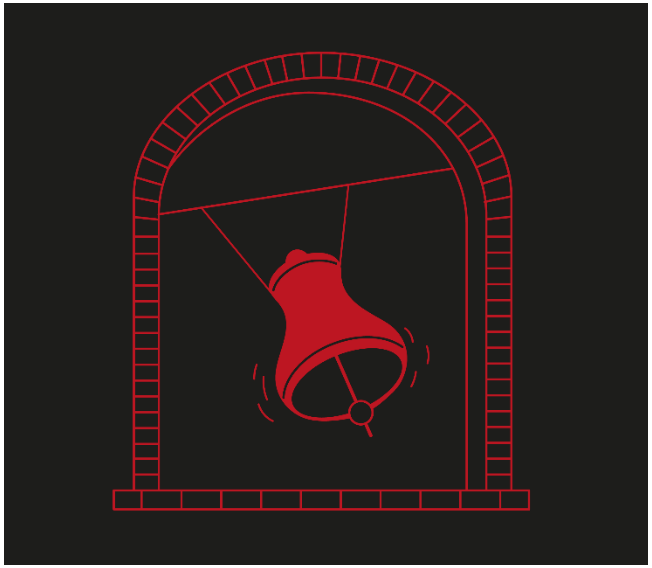
Cardoso & Pani, Nature Astronomy 1: 586 (2017); Living Reviews in Relativity 22: 1 (2019)

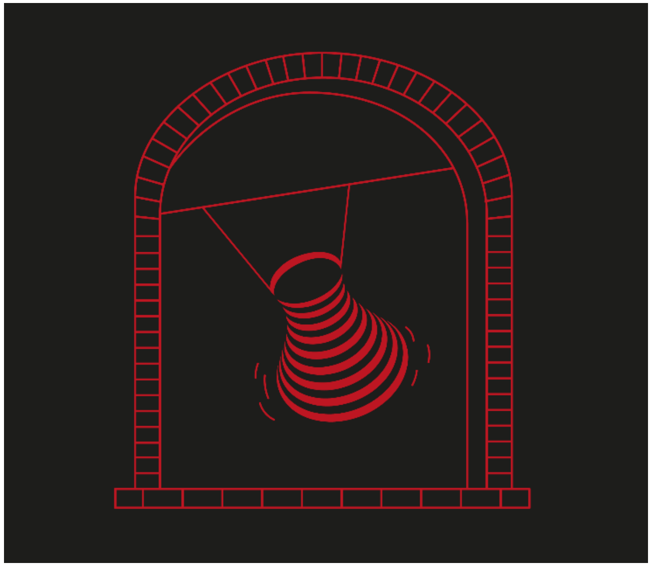
Fundamental questions

Are we really looking at black holes?

Do we understand all the intrincacies of mergers?

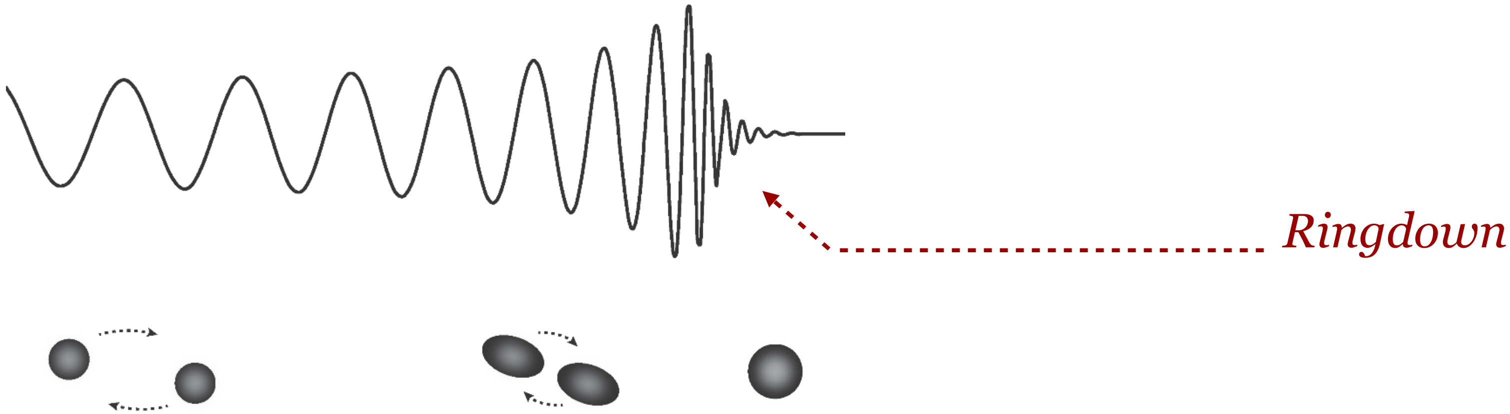
Is it a Kerr black hole? How can we constrain alternatives?





Answer requires understanding of theoretical framework, PDE analysis, precise modelling, observations, challenging simulations & data analysis techniques

The sound of black holes: spectroscopy



$$\frac{\partial^2\Psi}{\partial r_*^2} - \frac{\partial^2\Psi}{\partial t^2} - V(r_*)\Psi = \mathrm{S} \qquad \sim e^{-i\omega t}$$

$$\left(\omega^2 - \mathcal{L}\right)\psi = s \rightarrow \psi = \left(\omega^2 - \mathcal{L}\right)^{-1} s$$

poles = QNMs

$$h = \sum_{n\ell m} A_{n\ell m} e^{-t/\tau_{n\ell m}} \sin(2\pi f_{n\ell m} t) Y_{\ell m}(\theta, \phi)$$

Berti, Cardoso & Starinets CQG26:163001 (2009); Berti + arXiv:2505.23895
Cardoso & Gualtieri CQG33:174001 (2016)

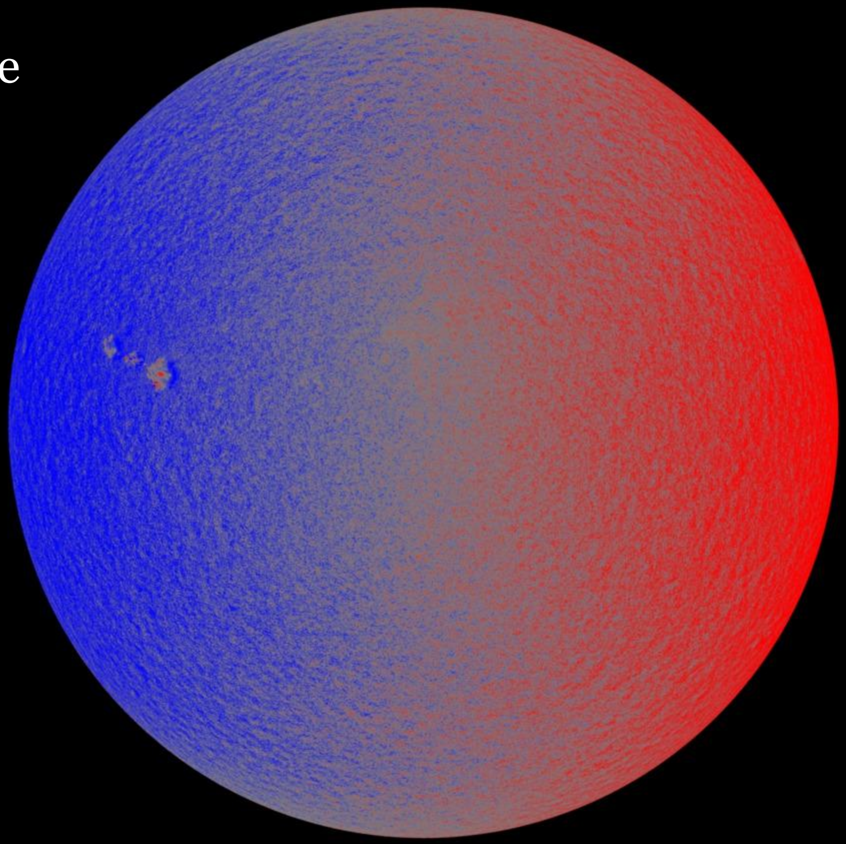
Helioseismology

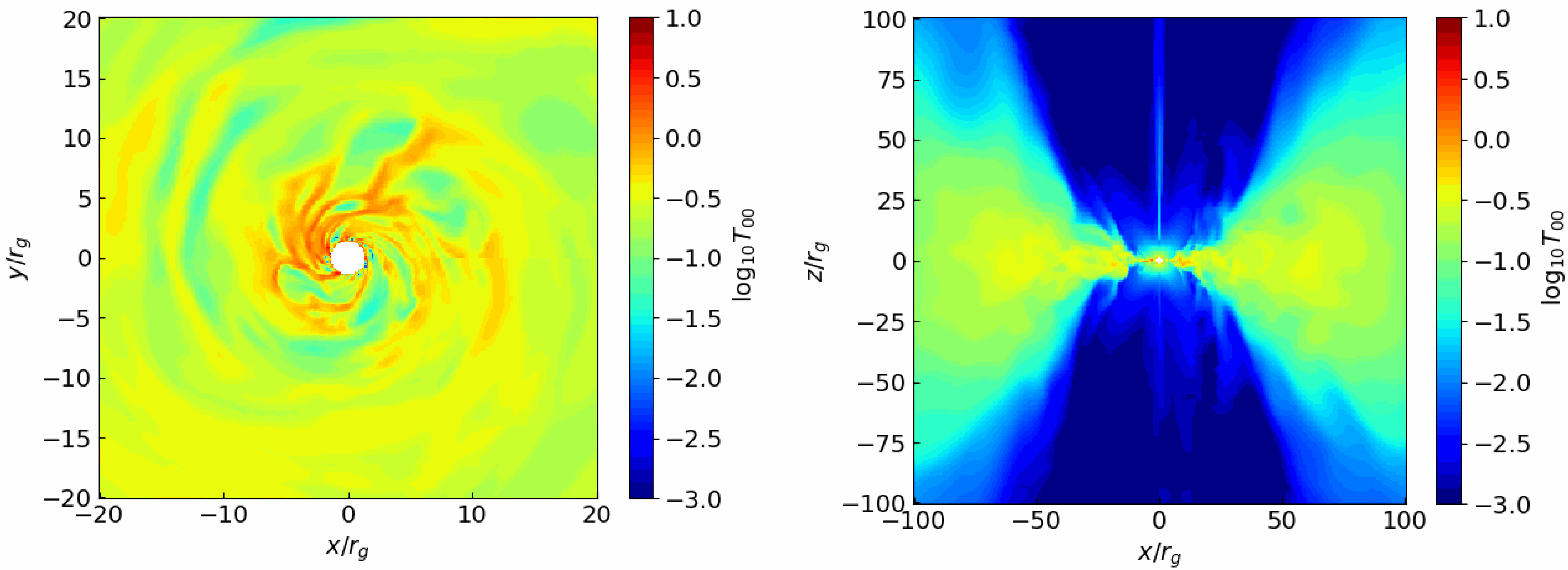
Solar spectrum sensitive to radial structure, opacity, temperature…

More than 10^6 modes with periods between 4 and 6 minutes

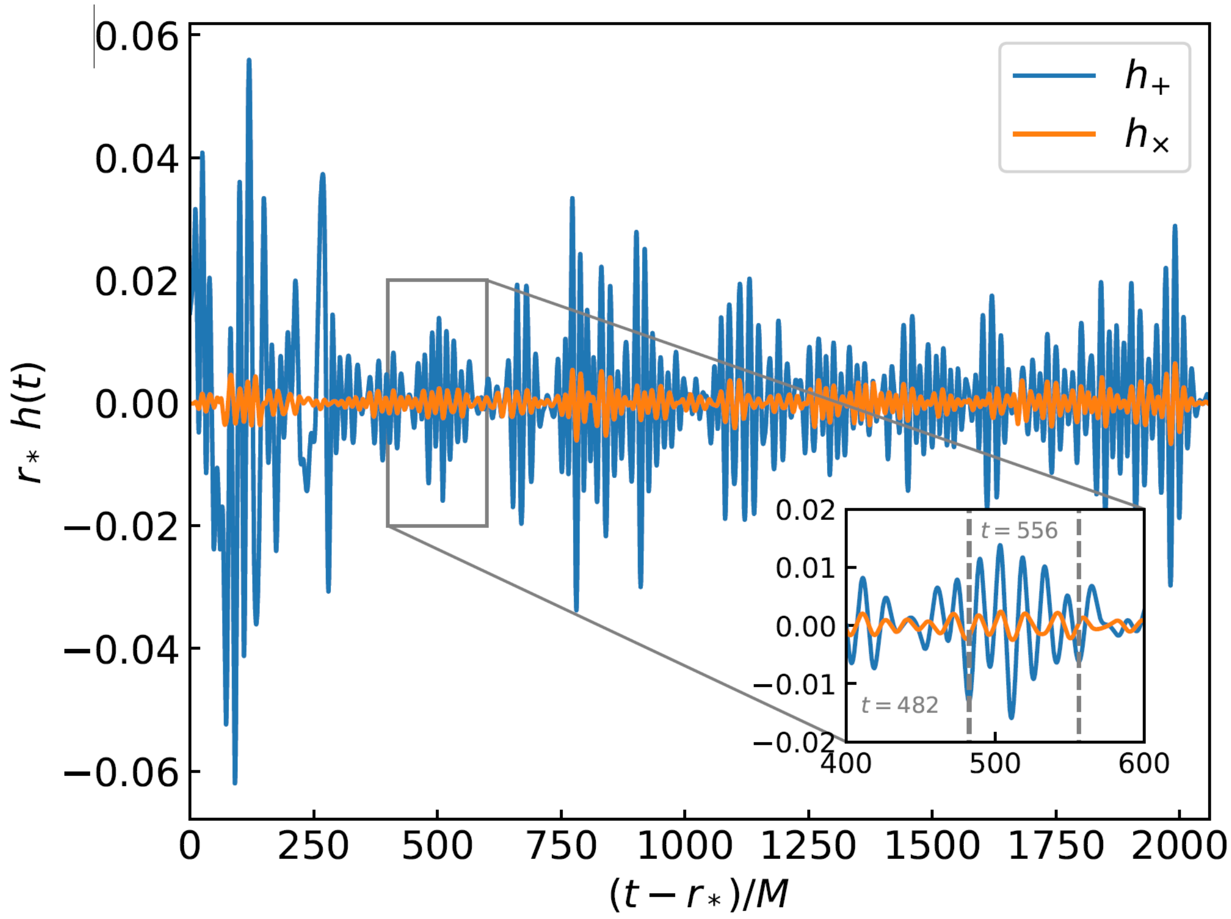
Quality factors of order $10^6 - 10^9$

Driven by turbulence

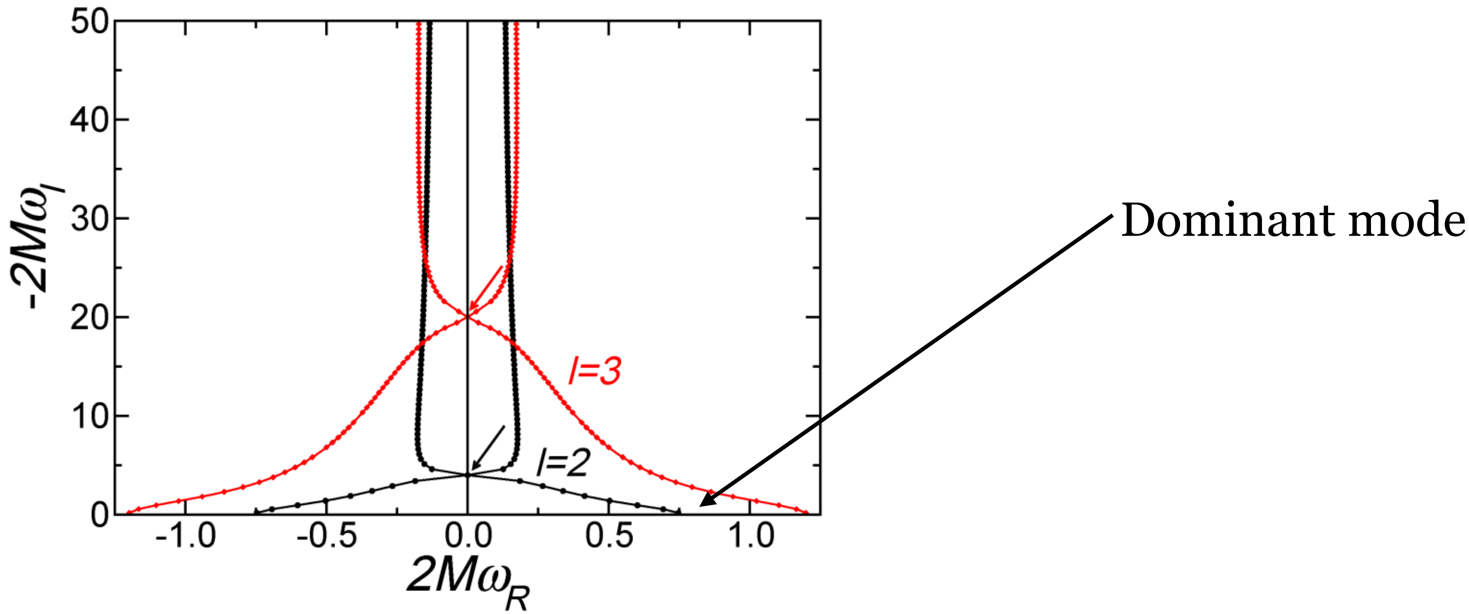




Yuan +, PRD111:063048 (2025)



Yuan +, PRD111:063048 (2025)



$$f = \omega_R / 2\pi = 1.207 \left(\frac{10\ M_\odot}{M} \right) \mathrm{kHz}$$

$$\tau = 1/|\omega_I| = 0.5537 \left(\frac{M}{10\ M_\odot} \right) \mathrm{ms}$$

Quality factors of ~3. Black holes don't ring, they relax with a thud.

Berti, Cardoso & Starinets CQG26:163001 (2009); Berti + arXiv:2505.23895

When is a *linear* ringdown description valid?

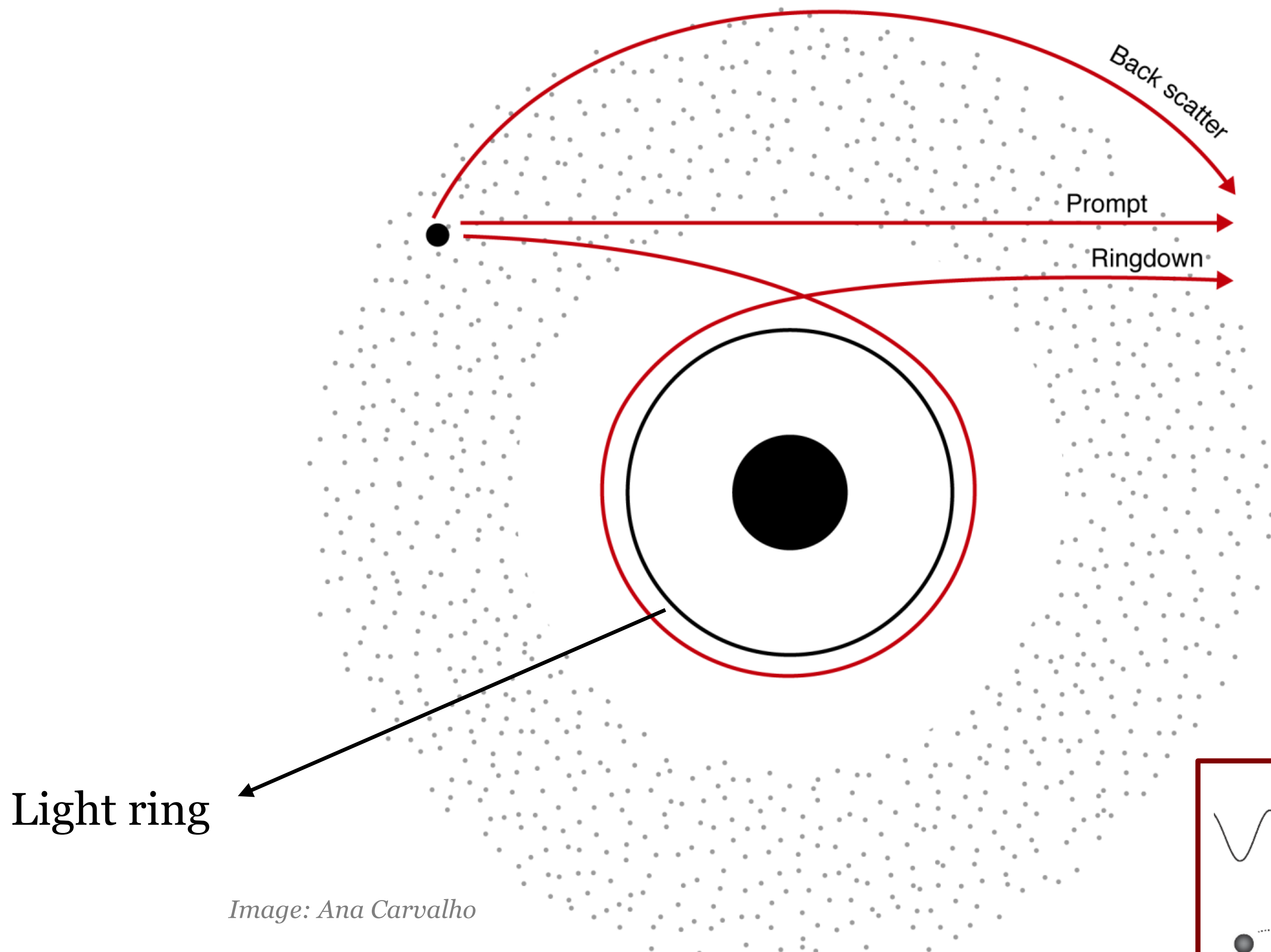
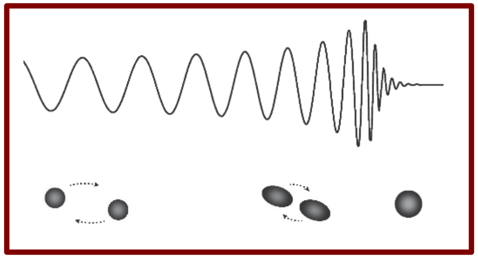
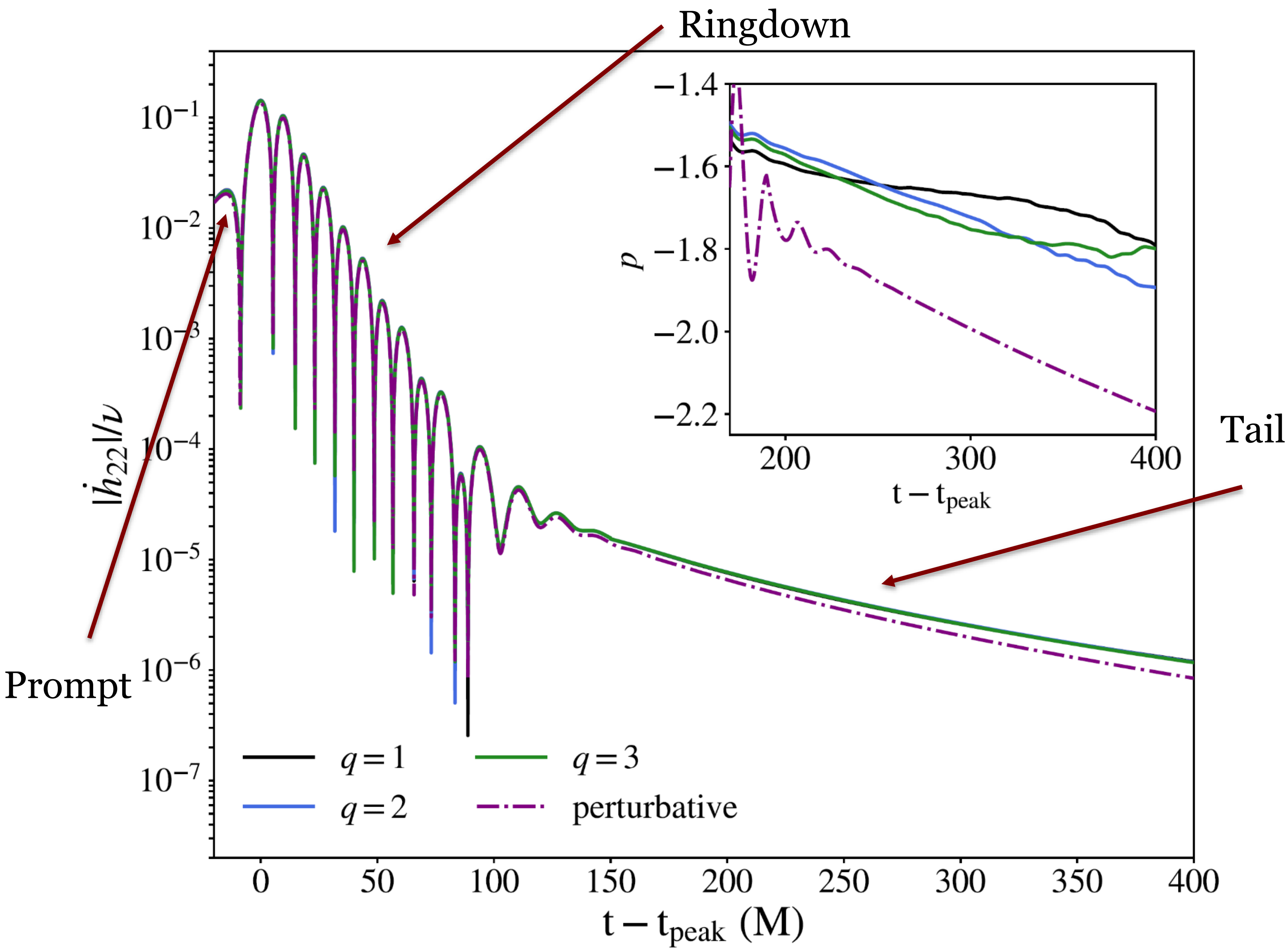


Image: Ana Carvalho

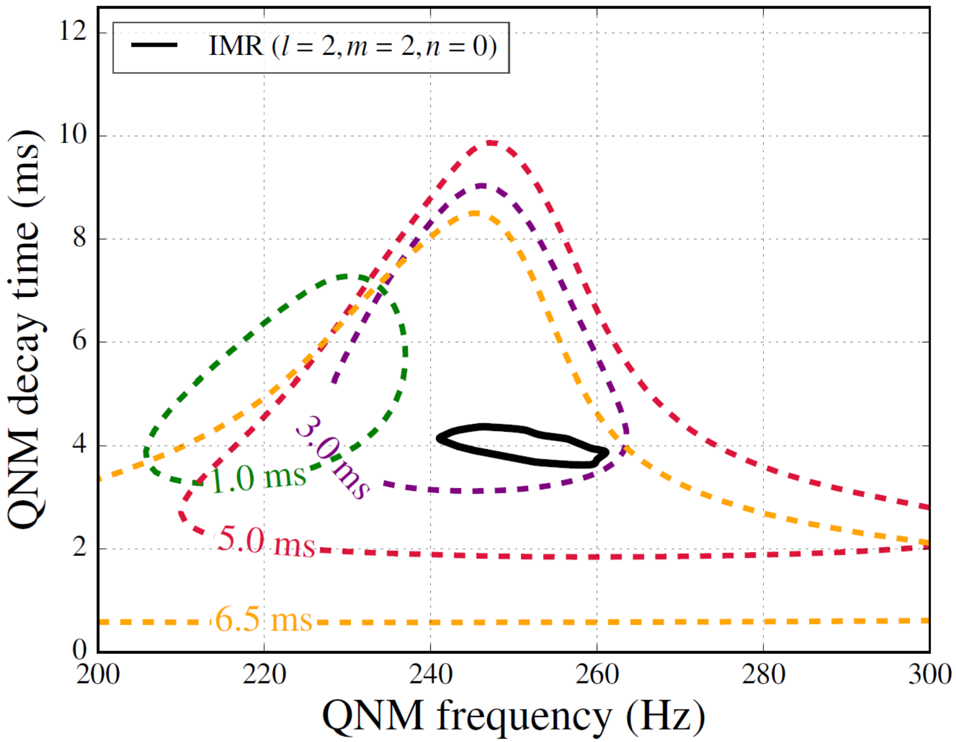


Leaver PRD34:384 (1986); Baibhav + PRD108:104020 (2023)



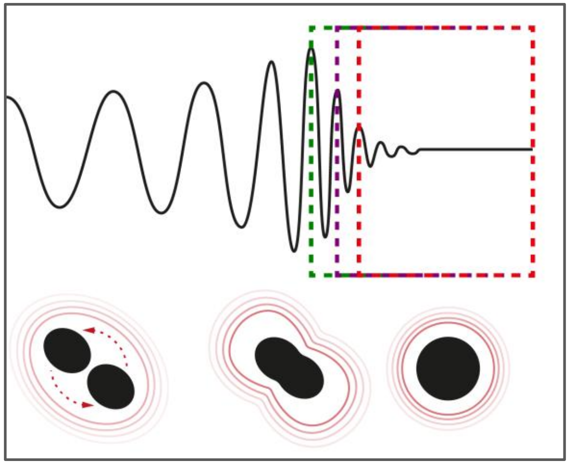
De Amicis + PRL135:171401 (2025); Ma + arXiv:2412.06906

One and two-mode estimates: the start of spectroscopy



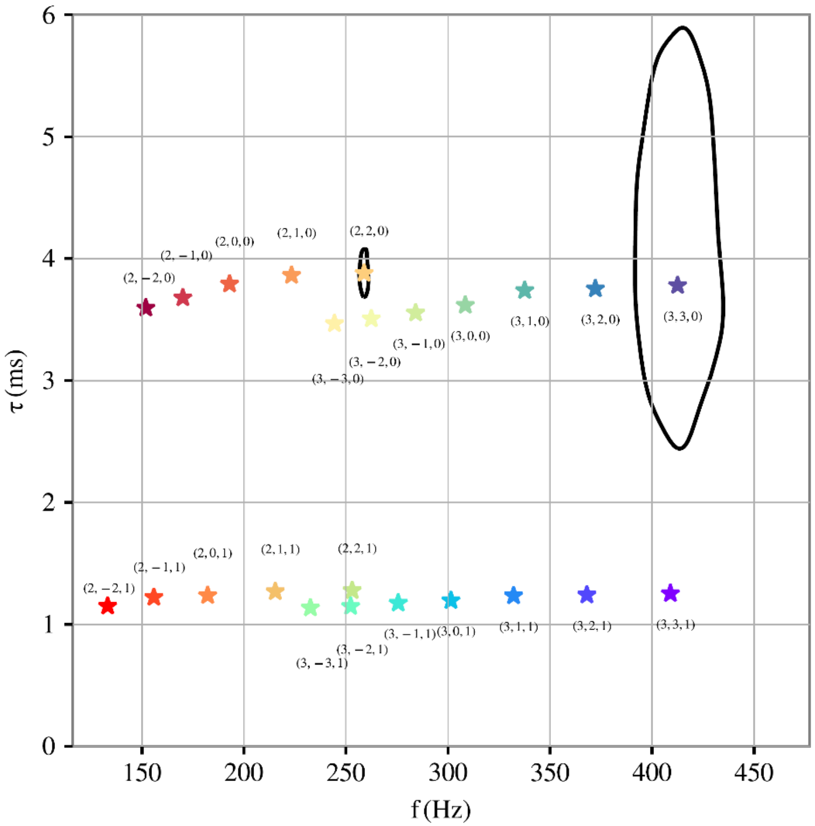
90% posterior distributions.

Black solid is 90% posterior of QNM as derived from the posterior mass and spin of remnant



LSC PRL116:221101 (2016); arXiv:2010.14529;
For future detectors: Berti+ PRL117:10102 (2016)

One and two-mode estimates: the start of spectroscopy



90% posterior distributions.

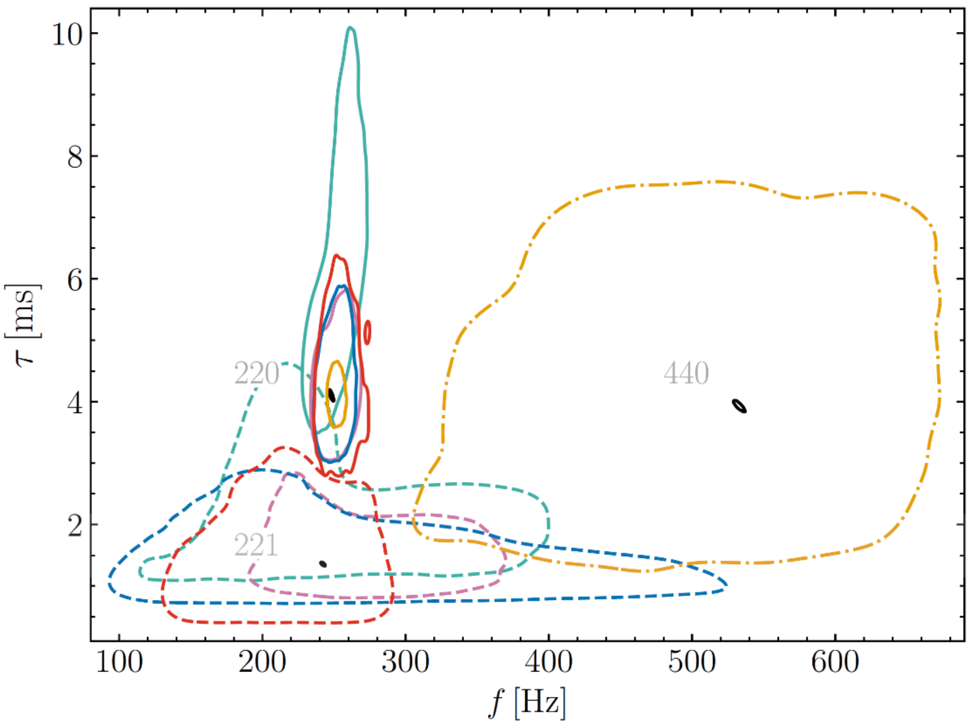
Black solid is 90% posterior of QNM from a future event with SNR=40 in ringdown.

LISA will see SNRs of thousands…

Courtesy of Gregorio Carullo

See also Berti+ PRL117:10102 (2016); Bhagwat+ arXiv:2304.02283

The future is now: GW250114



90% posterior distributions.

Black solid is 90% posterior of QNM from GW250114

LISA will see SNRs of thousands...

Abac + arXiv:2509.08054

Challenges

Strength of the poles ("excitation factors")

Inner products and waveform reconstruction with QNMs ("excitation coefficients")

Norm of resolvent in entire complex plane (pseudospectrum)

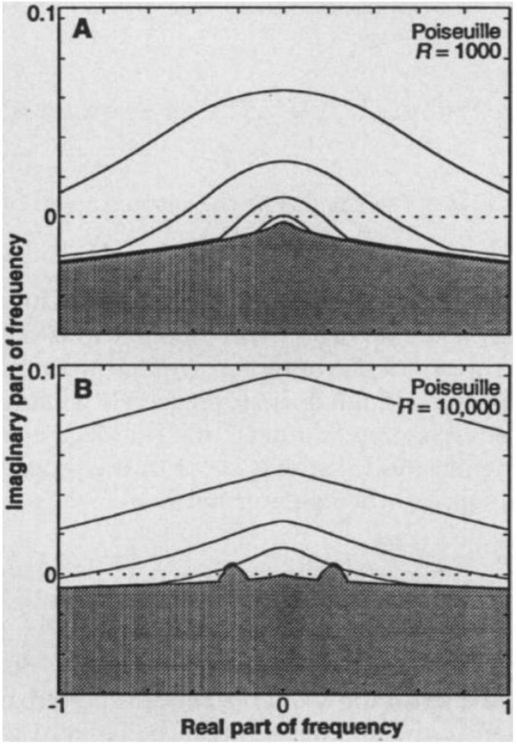
Environments (more soon)

Nonlinearities

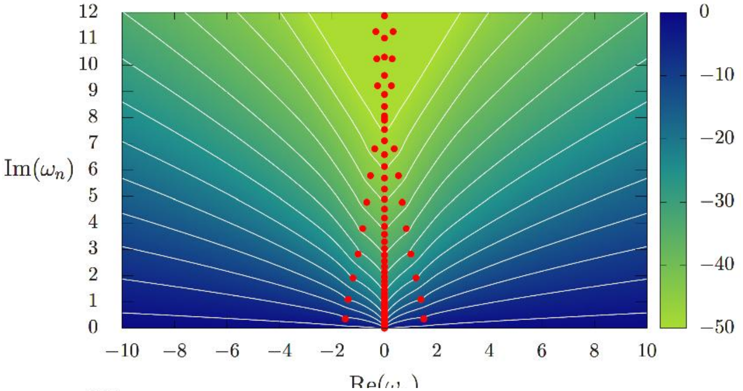
Interlude: pseudospectra

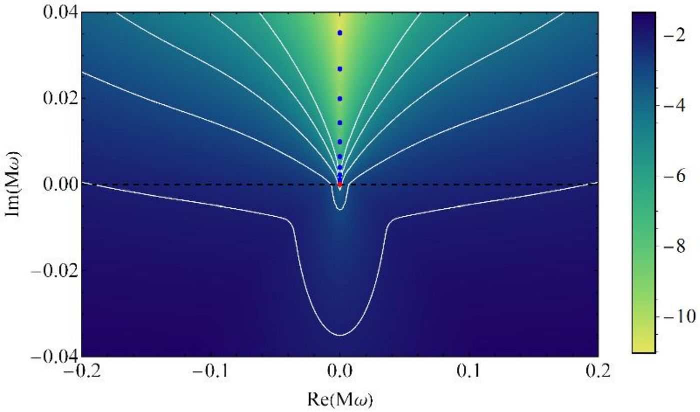
Trefethen+ Science 261: 578 (1993); Jaramillo+ PRX11:031003 (2021)

$$\left\| \left(\omega^2 - \mathcal{L}\right)^{-1} \right\| \geq \epsilon^{-1}$$



(plane) Poiseuille: parabolic velocity profile between static infinite plates





Pseudospectrum of extremal BHs

K. Destounis+ PRD104:084091 (2021)
Also V. Boyanov+ PRD107:064012 (2022)

On the maximum luminosity and nonlinearities

Event	Peak Luminosity
Three Gorges dam	$3\times 10^{-43}\ \mathcal{L}_{\rm P}$
Most powerful laser	$3\times 10^{-38}\ \mathcal{L}_{\rm P}$
Tsar Bomba	$3\times 10^{-27}\ \mathcal{L}_{\rm P}$
Solar luminosity	$1\times 10^{-30}\ \mathcal{L}_{\rm P}$
γ-ray bursts	$1\times 10^{-5}\ \mathcal{L}_{\rm P}$
Inspiralling BHs	$2\times 10^{-3}\ \mathcal{L}_{\rm P}$
High-energy BH collision	$2\times 10^{-2}\ \mathcal{L}_{\rm P}$
Critical collapse	$2\times 10^{-1}\ \mathcal{L}_{\rm P}$
End point of BH evaporation	$\mathcal{L}_{\rm P}$

$$\mathcal{L}_{\rm P} = \frac{c^5}{G} = 3.6 \times 10^{52}\,{\rm W}$$

Conjecture:

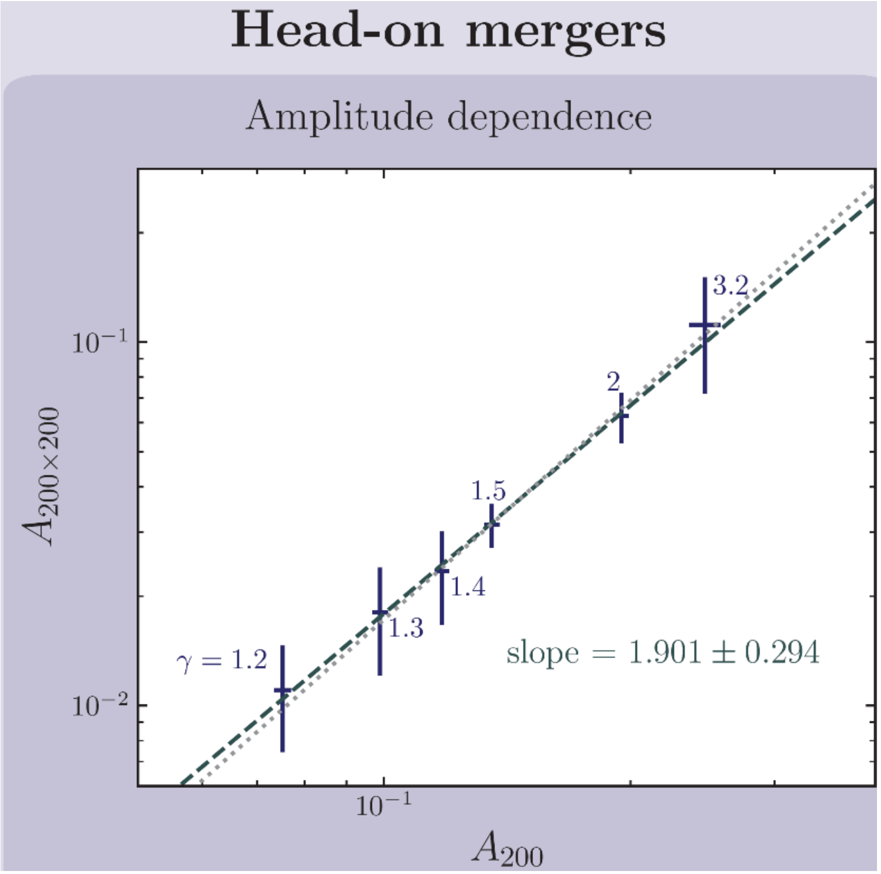
Maximum possible value of luminosity in a past-regular spacetime
is the Planck luminosity $\mathcal{L}_P$

Cardoso + PRD97:084013 (2018)

also Gibbons & Barrow MNRAS 446:3874 (2015); Schiller PRD104:124079 (2021)

Nonlinearities in ringdown

$$\frac{\partial^2 \Psi^{(2)}}{\partial r_*^2} - \frac{\partial^2 \Psi^{(2)}}{\partial t^2} - V\Psi^{(2)} \sim \left(\Psi^{(1)}\right)^2$$



Cheung + PRL130:081401 (2023) ; Redondo-Yuste + PRD109:L101503 (2024); Zhu + PRD109: 104050 (2024)

Nonlinearities in ringdown

$$\frac{\partial^2 \Psi^{(2)}}{\partial r_*^2} - \frac{\partial^2 \Psi^{(2)}}{\partial t^2} - V\Psi^{(2)} \sim \left(\Psi^{(1)}\right)^2$$

$$R_{220\times220} = \frac{A_{220\times220}}{A_{220}^2}$$

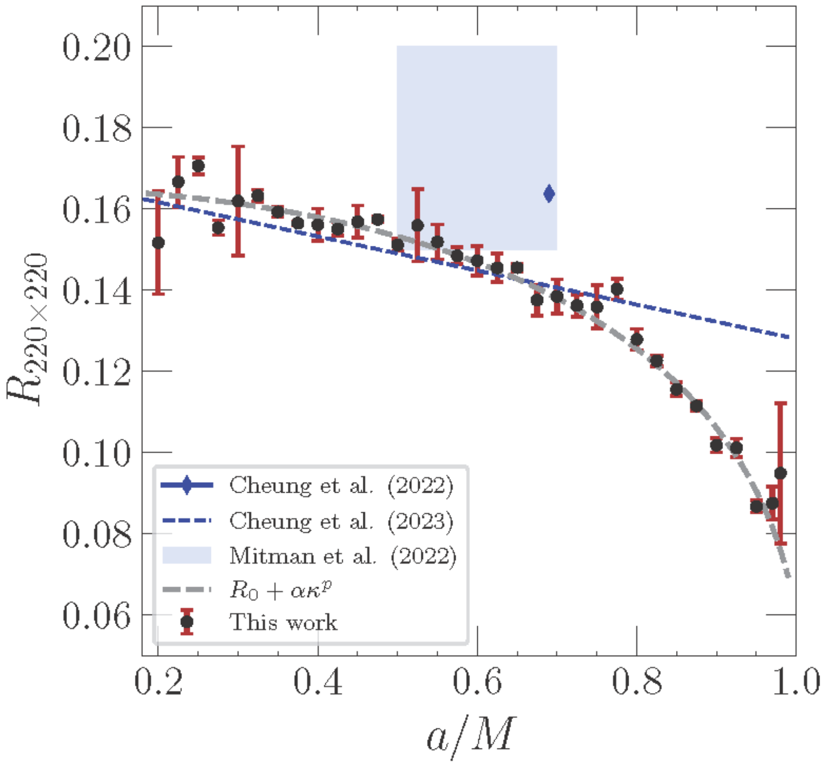
For quasi-circular inspirals of non spinning binaries, from NR:

$$R_{220\times220} = 0.16$$

It is not hopelessly small!

Cheung + PRL130:081401 (2023); Mitman + PRL130:081402 (2023)
Plenty of work, see Bucciotti + (2024, 2025), Bourg + (2024), Kehagias + (2024) etc

Nonlinearities in ringdown: second-order



Mild dependence on spin

Redondo-Yuste + PRD109:L101503 (2024); Zhu + PRD109: 104050 (2024)

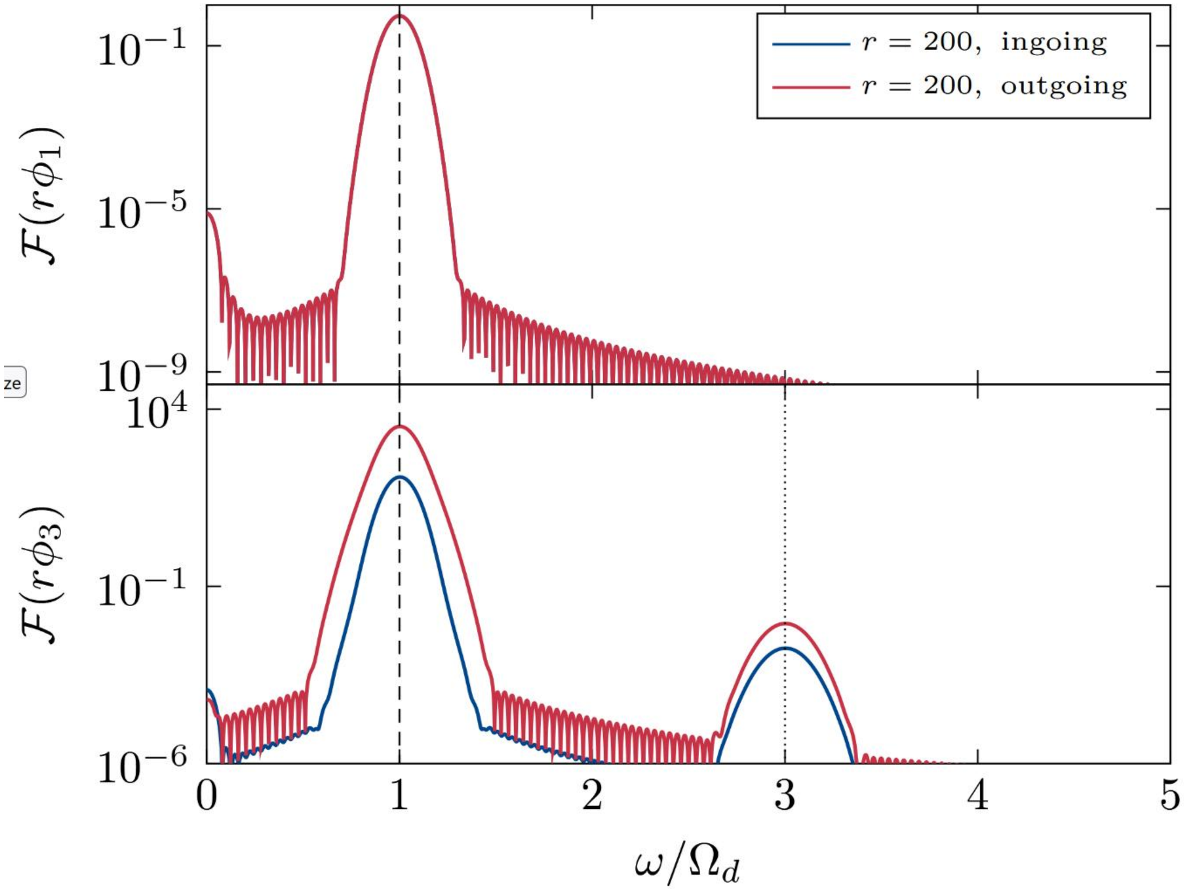
Aeolian Harp, 1976. Built by Douglas Hollis.
Exploratorium, San Francisco

Nonlinearities,
the Tartini tone…

"I heard there was a secret chord,
that David played and it pleased the Lord,
but you don't really care for music, do you?"

— Leonard Cohen, *Hallelujah*

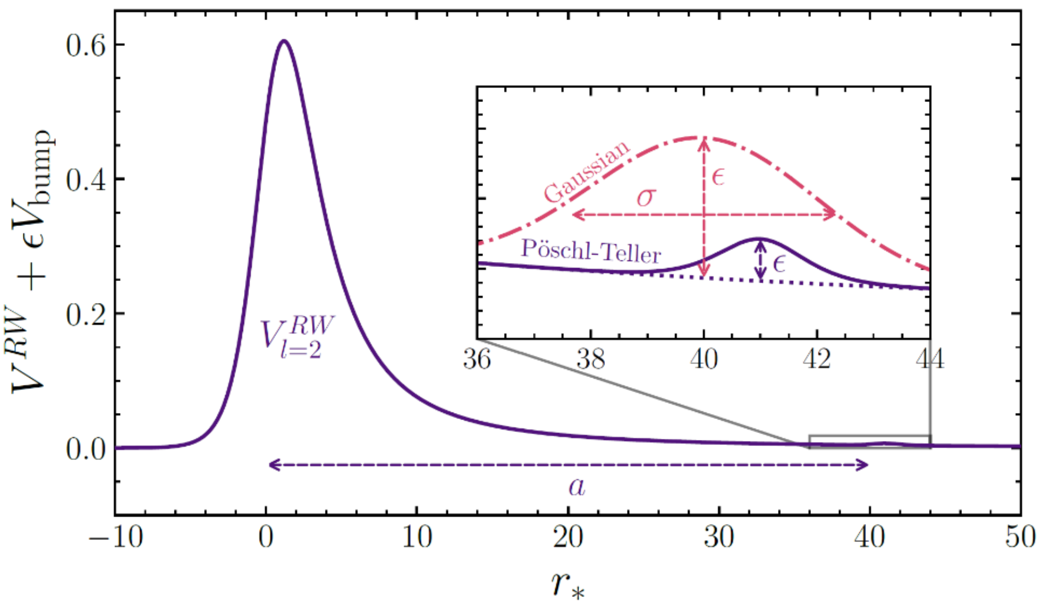
Focusing, spectral broadening, higher harmonics…



V. Cardoso, J. Redondo-Yuste, F. Tuncer, U. Sperhake (2026, ongoing)

Spectral stability: the elephant and the flea

Spectrum is unstable: Nollert gr-qc/9602032; Barausse + PRD89:104059 (2014); Jaramillo+ PRX 11: 031003 (2021); Cheung+ PRL128:111103 (2022); PRD106:084011 (2022); Ianniccari + PRL133:211401 (2024); Oshita + PRD110:084070 (2024); Yang + (2024); Cardoso+ PRD110: 024016 (2024)



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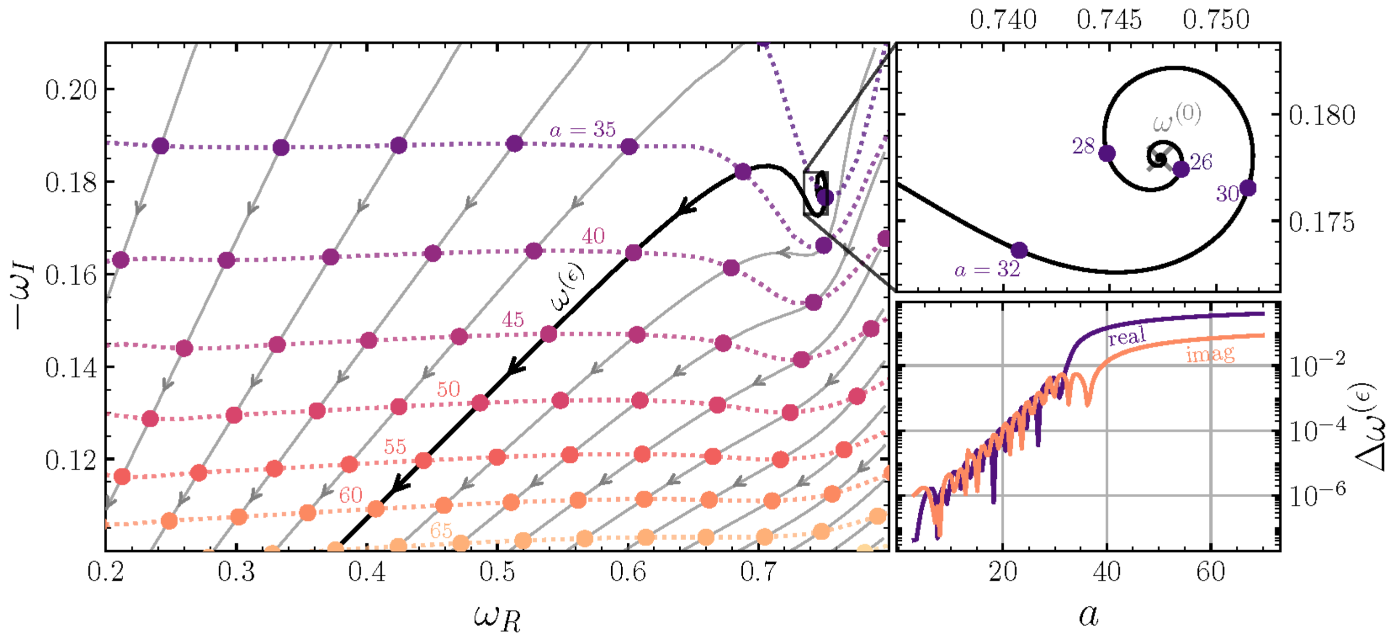
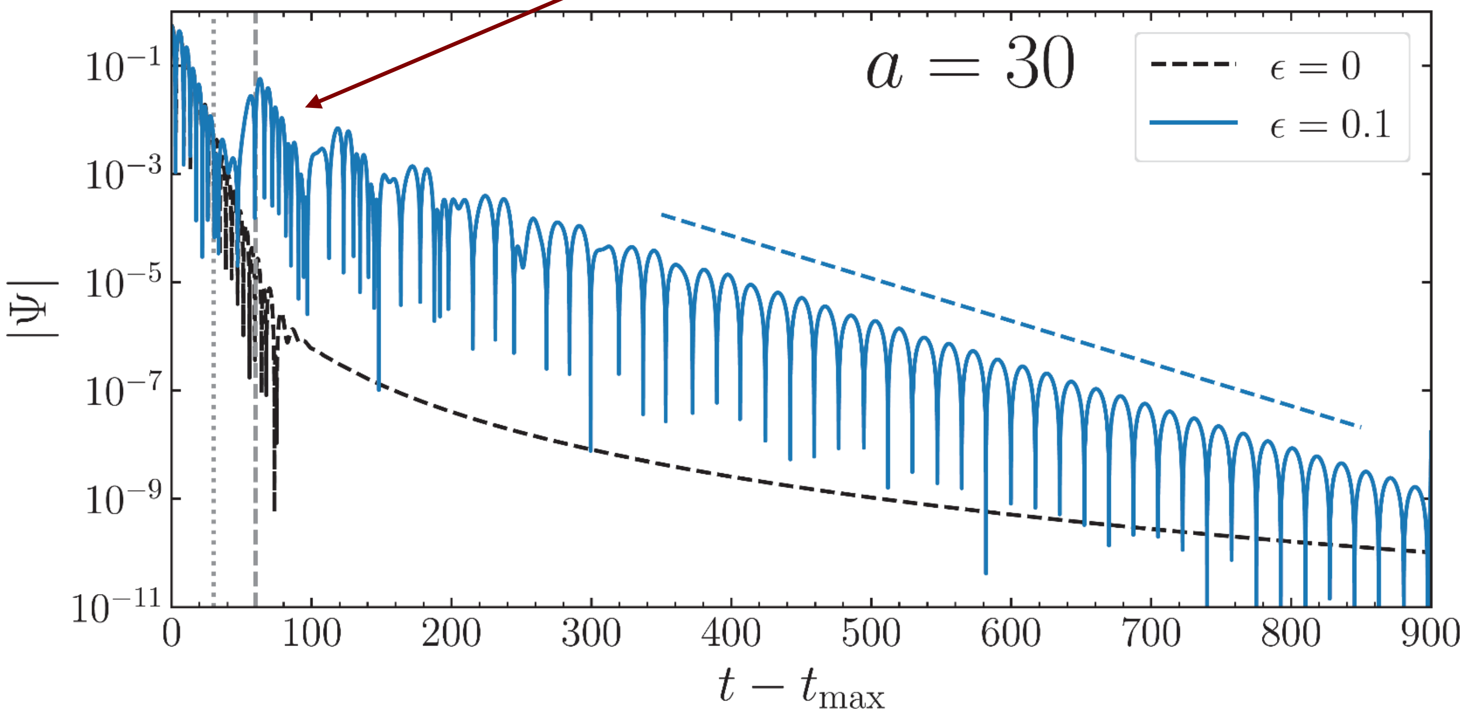


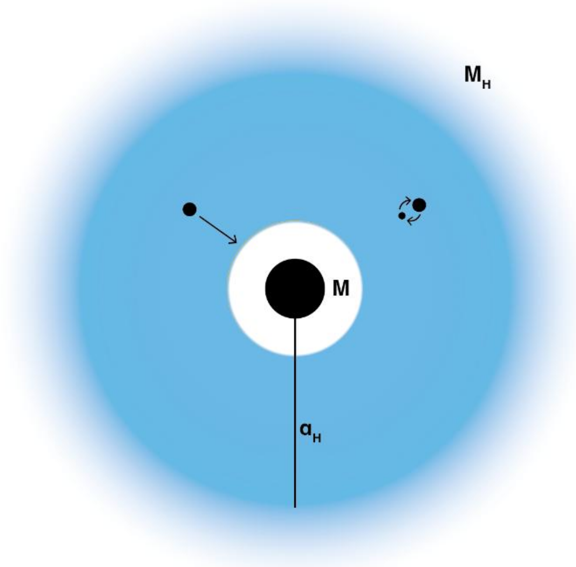
$\varepsilon = 10^{-6}$

Spectral stability: the elephant and the flea

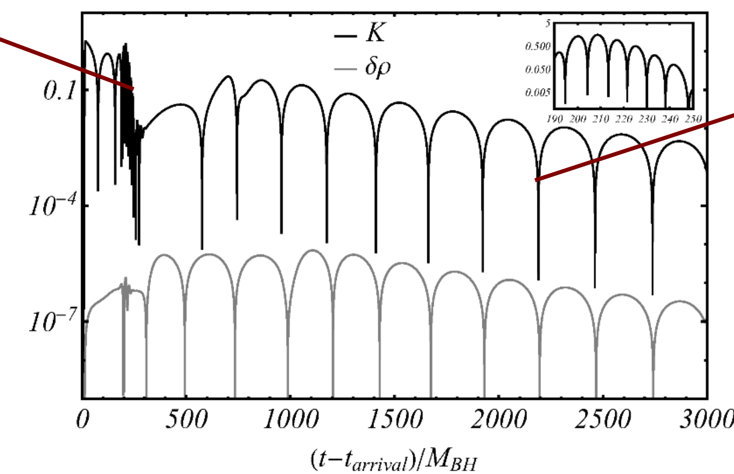
Spectrum is unstable: Cheung+ PRL128:111103 (2022); Berti +PRD106:084011 (2022); Cardoso+ PRD110: 024016 (2024)

Amplitude scales like ε



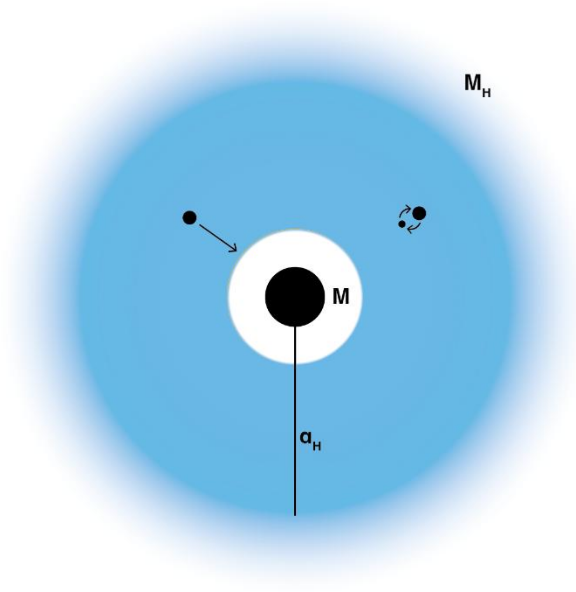


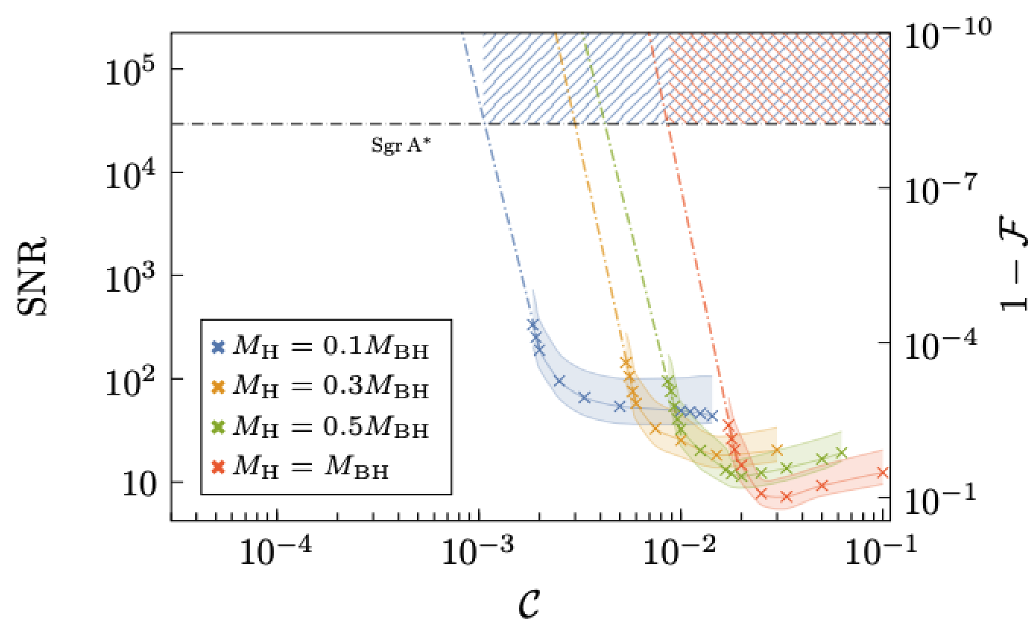
Light ring modes (BH ringdown)



Fluid modes

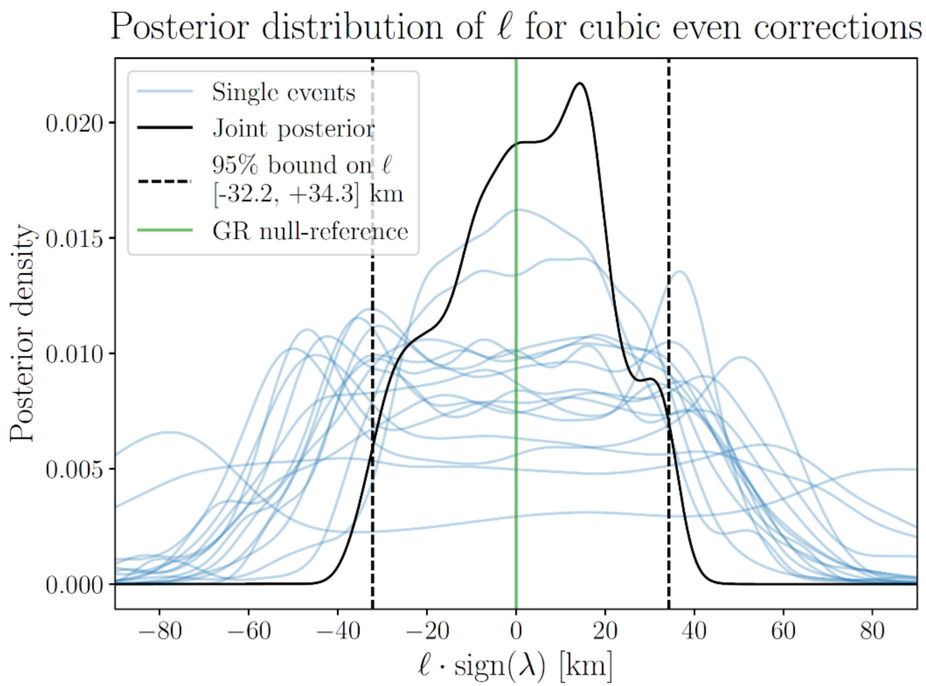
Cardoso+ PRL129:241103 (2022)
Spieksma + PRL134:081402 (2025)





Spieksma + PRL134:081402 (2025)

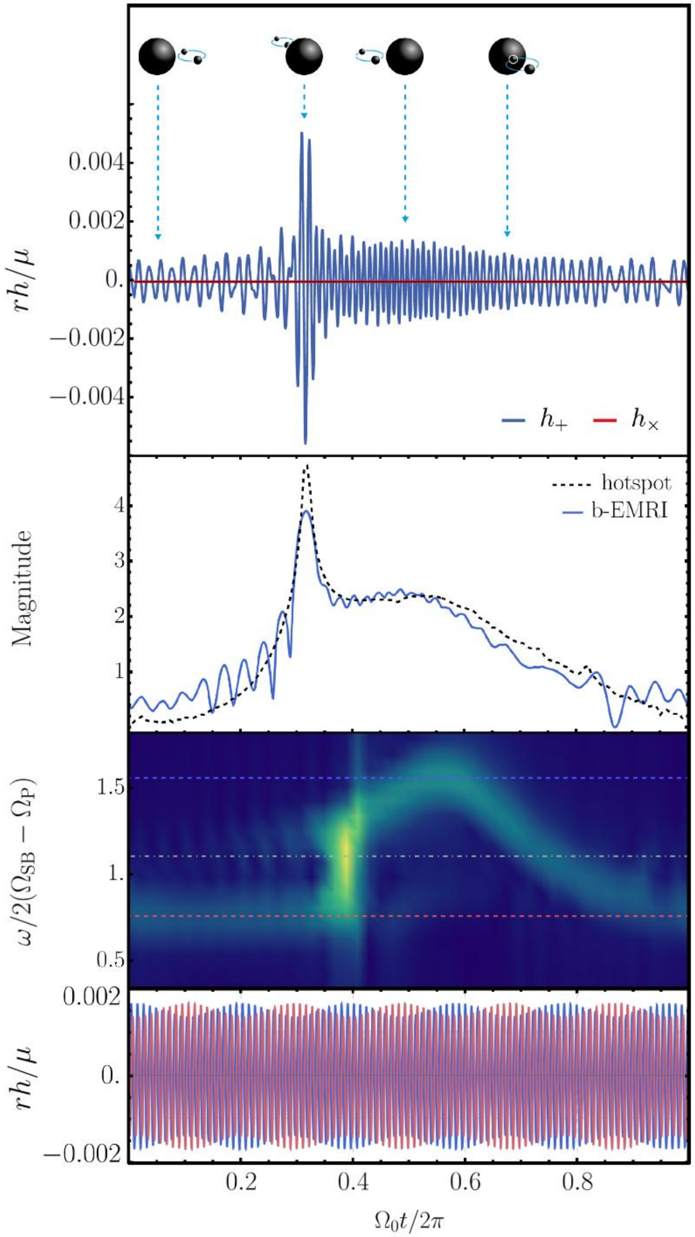
$$S = \int d^4x \frac{\sqrt{|g|}}{16\pi} \left[R + \ell^4 \lambda_{\text{ev}} \mathcal{R}^3 + \ell^6 \left(\lambda_1 \mathcal{C}^2 + \lambda_2 \tilde{\mathcal{C}}^2 \right) \right]$$



Theory	$\text{sign}(\lambda) \cdot \ell$ in km	$\ln(\mathcal{B}_{\text{EFT}}/\mathcal{B}_{\text{GR}})$
Cubic even	[−32.2, +34.3]	[−2.0, +1.0]
Quartic 1	[−24.9, +35.0]	[−2.1, +1.4]
Quartic 2	[−27.0, +38.7]	[−1.7, +0.9]

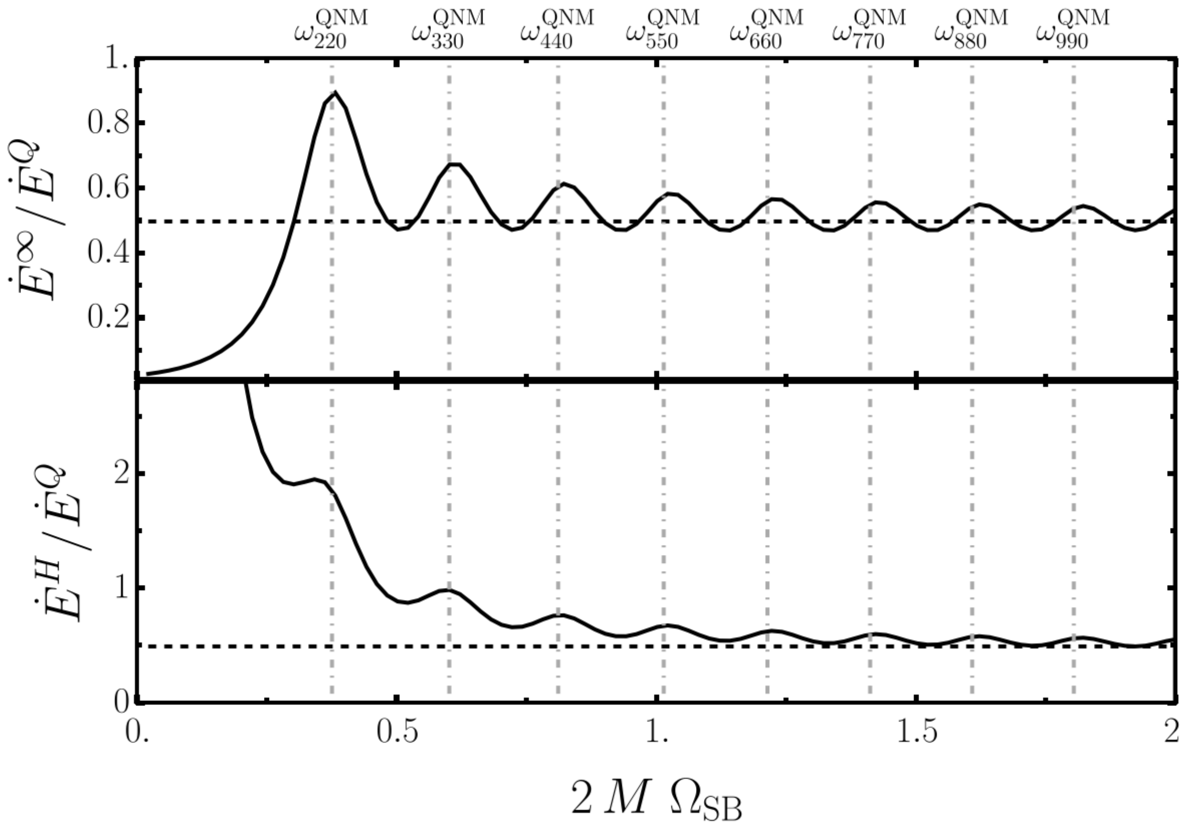
Maenaut + arXiv:2411.17893

Spectroscopy: resonant excitation



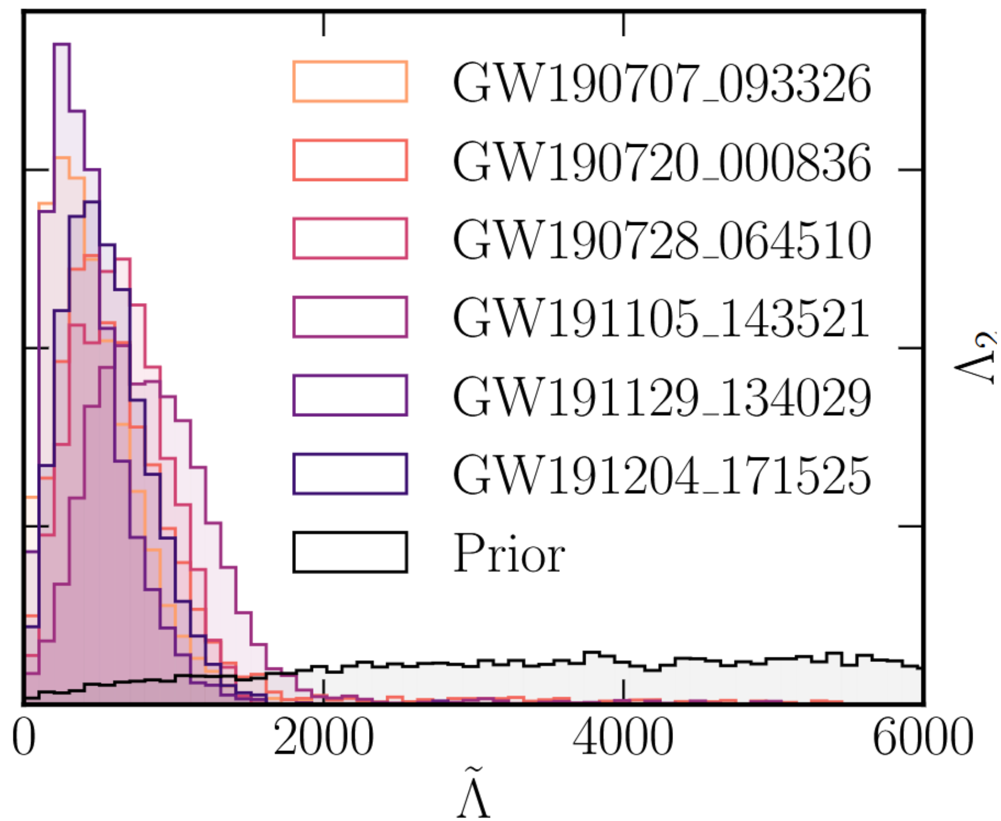
Santos + PRL135: 211402 (2025);
Santos + arXiv:2601.02468

Spectroscopy: resonant excitation



Santos + PRL135: 211402 (2025); Santos + arXiv:2601.02468

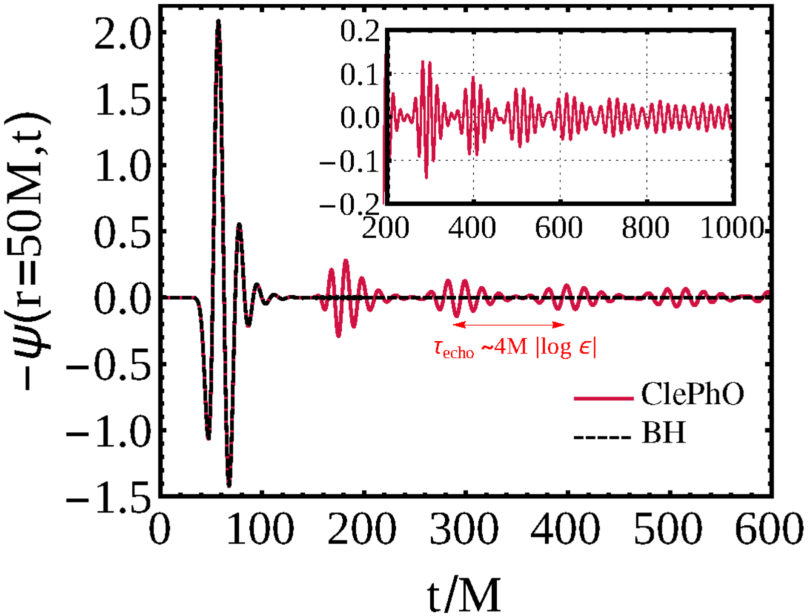
Testing black hole paradigm: no Love

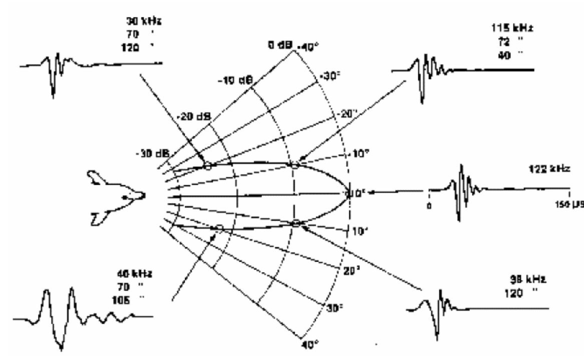


Marginalized Λ posteriors for the six best events. Posteriors are prior dominated at Λ ~ 0, therefore results do not suggest evidence for Λ > 0

Chia + PRD 110:063007 (2024); also Santoro+ 2025

Testing black hole paradigm: echoes





Target echo in reverberation at the dolphin's threshold of detection (from Au [8]).

$$M\omega_R \sim |\log\epsilon|^{-1}$$

$$M\omega_I \sim |\log\epsilon|^{-(2l+3)}$$

Cardoso, Franzin, Pani, PRL116:171101 (2016)
Cardoso, Pani arXiv:1707.03021 (Nature Astronomy 2017)

Au+ (J. Acoust. Soc. Am. 1992)

An informal query

Most important discoveries or outcomes of the gravitational-wave astronomy revolution

Binary black holes exist, merge within a Hubble time

They exist across mass gap

GWs travel at speed of light

Gold and heavy elements produced during NS mergers

Evidence for light-rings

We are probing horizons

“However, I think that the most important discoveries from GW astronomy are still to come”

Strong field gravity is a fascinating topic. From precise maps of Universe to tests of Cosmic Censorship or constraints on dark matter, possibilities are endless & exciting

Thank you

Lessons learned?

Black hole spectroscopy has begun. Relaxation stage informs on final state conjecture, near-horizon physics and environments, and nonlinear effects. Dissecting every part of signal.

Environments are important to understand, current lore is that they are outside precision of detectors or redshift away for large scale galactic potentials. Much more work is necessary to understand extreme environments.

Echoes general feature of near-horizon physics, no fine-tuning

Quantum Surprises?

Bekenstein & Mukhanov 1995
Kleban+2019; Cardoso+ 2019; Agullo+ arXiv:2007.03700

i. Postulate some area quantization

$$A = \alpha l_P^2 N = \alpha \frac{\hbar G}{c^2} N$$

$$\Delta A = \alpha \frac{\hbar G}{c^3} \Delta N = 32\pi \frac{G^2}{c^4} M \Delta M$$

ii. Compute absorbed energy of graviton

$$\Delta M = \alpha \frac{c\hbar}{32\pi G} \frac{\Delta N}{M}$$

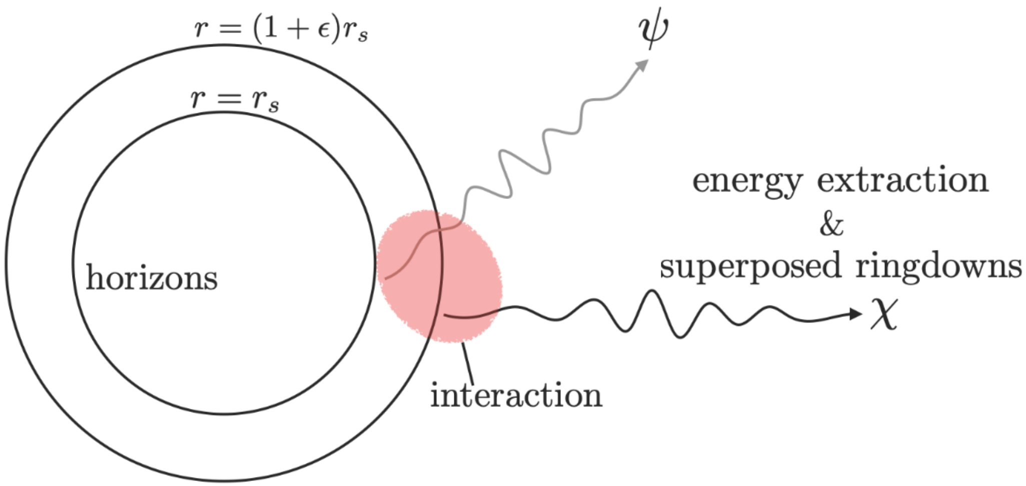
$$\omega_n = \frac{\Delta M c^2}{\hbar} = \frac{n\alpha}{32\pi} \frac{c^3}{MG}$$

Classical! Consequences for ringdown, TLNs, tidal heating

Agullo + PRL126:041302 (2021)
Deppe + arXiv:2411.056045

Double horizons

Cardoso, Oshita, Mukohyama,Takahashi, arXiv:2404.05790



$$\mathcal{L} = -\frac{1}{2} g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - \frac{1}{2} g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi + \frac{\epsilon}{2}(\phi^\mu\partial_\mu\chi)^2 - \frac{1}{2}(\nabla_\mu\phi^\mu)^2(\epsilon_{11}\chi^2 + 2\epsilon_{12}\chi\psi + \epsilon_{22}\psi^2)$$

$\phi^\mu = g^{\mu\nu}\partial_\nu\phi$ with scalar $g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = -1$. Re-write

$$\mathcal{L} = -\frac{1}{2} g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - \frac{1}{2}(\nabla_\mu\phi^\mu)^2(\epsilon_{11}\chi^2 + 2\epsilon_{12}\chi\psi + \epsilon_{22}\psi^2)$$

Inverse disformal metric $\tilde{g}^{\mu\nu} := g^{\mu\nu} - \epsilon\phi^\mu\phi^\nu$:

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + \frac{\epsilon}{1+\epsilon}\phi_\mu\phi_\nu \,.$$

Lessons learned?

Black hole spectroscopy has begun. Relaxation stage informs on final state conjecture, near-horizon physics and environments, and nonlinear effects. Dissecting every part of signal.

Environments are important to understand, current lore is that they are outside precision of detectors or redshift away for large scale galactic potentials. Much more work is necessary to understand extreme environments.

Echoes general feature of near-horizon physics, no fine-tuning

Spectroscopy can be used to test General Relativity or extensions thereof (e.g. Maenaut + 2024 for higher curvature corrections)

HUGE progress in last 10 years, community review ongoing (to appear, 2025)

Thank you

"I heard there was a secret chord,
that David played and it pleased the Lord,
but you don't really care for music, do you?"

— Leonard Cohen, *Hallelujah*

Aeolian Harp, England, 19th century.
The Metropolitan Museum of Art Collection.