The black hole ringdown and quasinormal modes

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IFPU focus week, January 2026

In this talk:

1. **The dynamics of quasinormal modes**

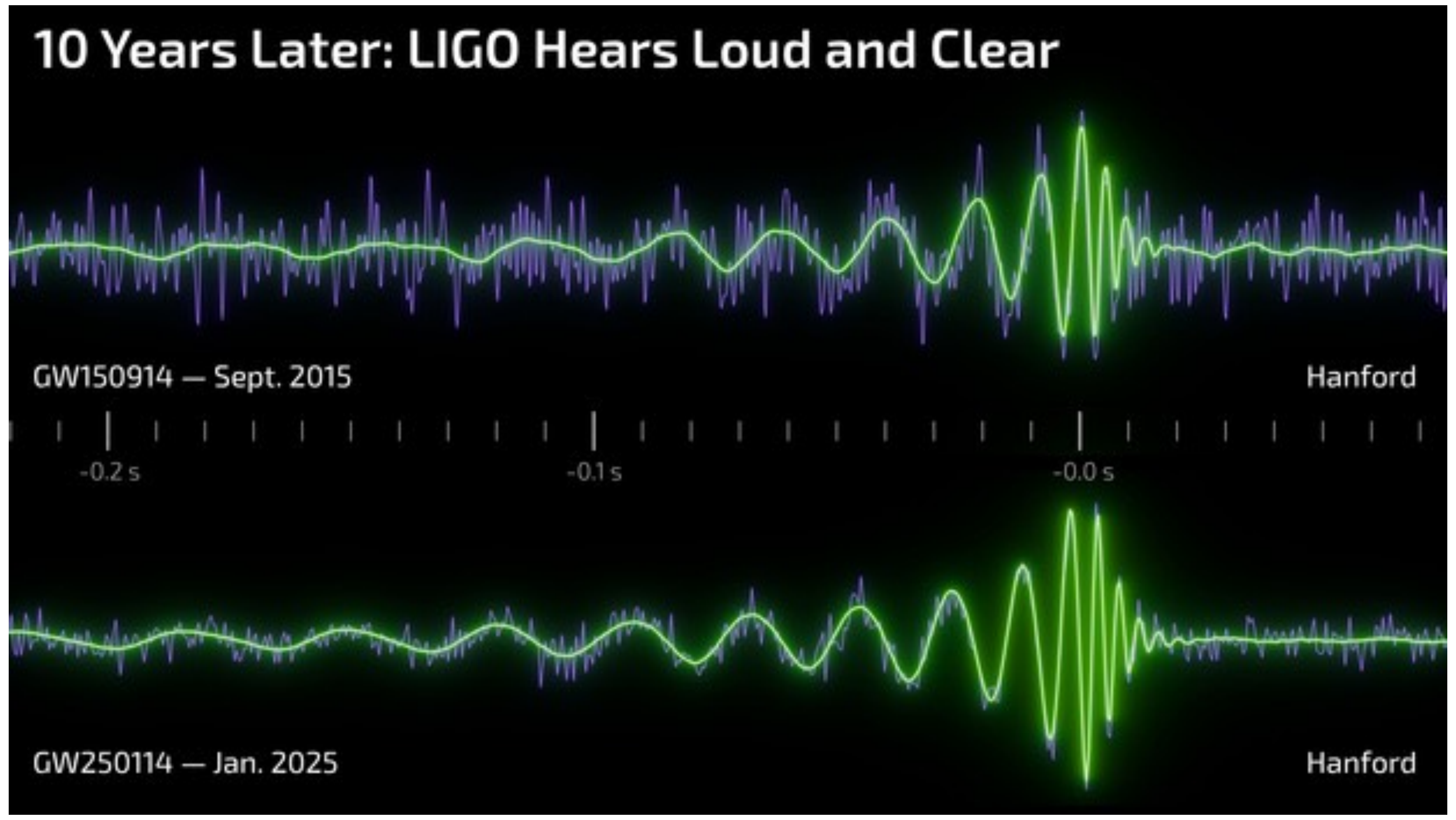
2. **Mathematical properties of black hole modes**

1. The dynamics of quasinormal modes

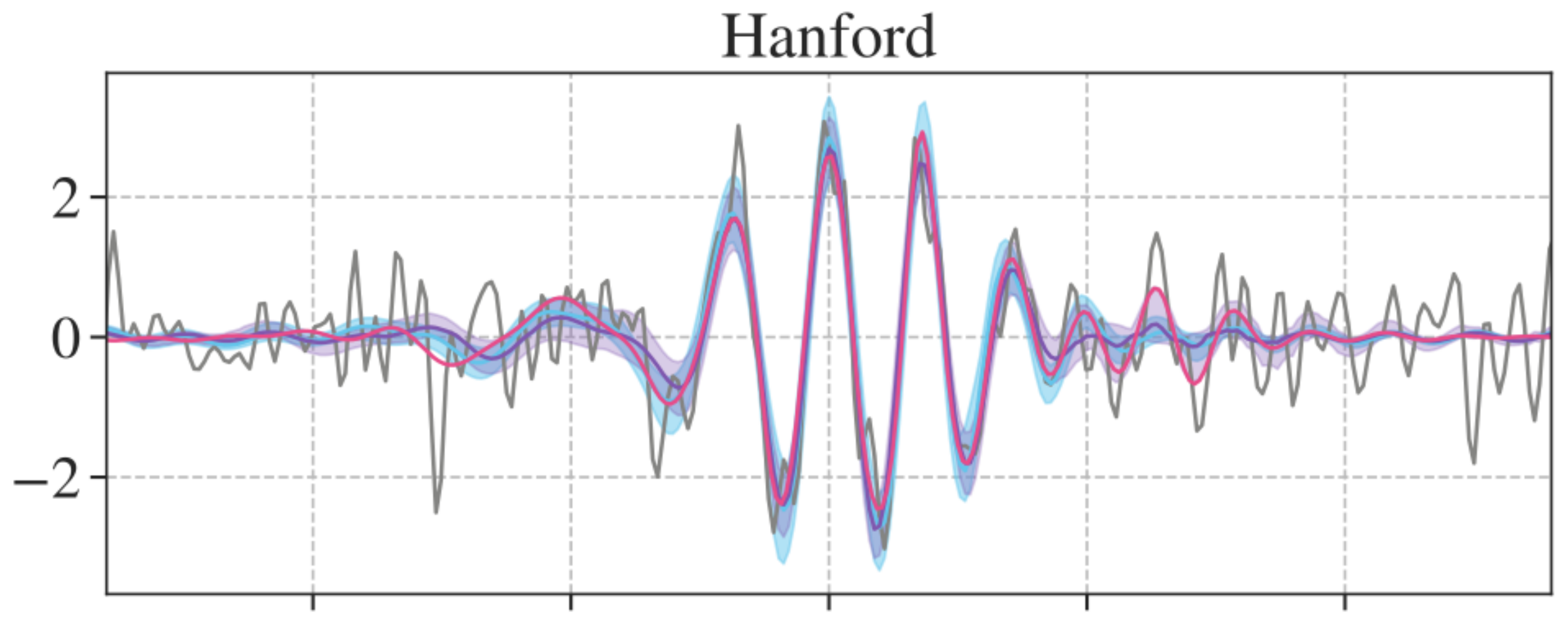


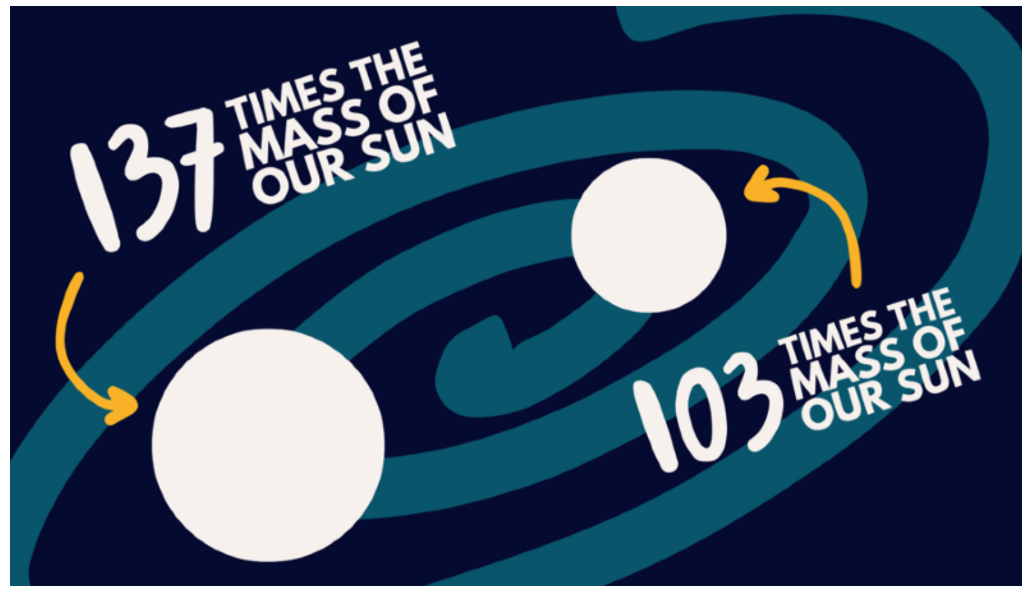
New observations of the black hole ringdown

Loud gravitational wave events (GW250114)



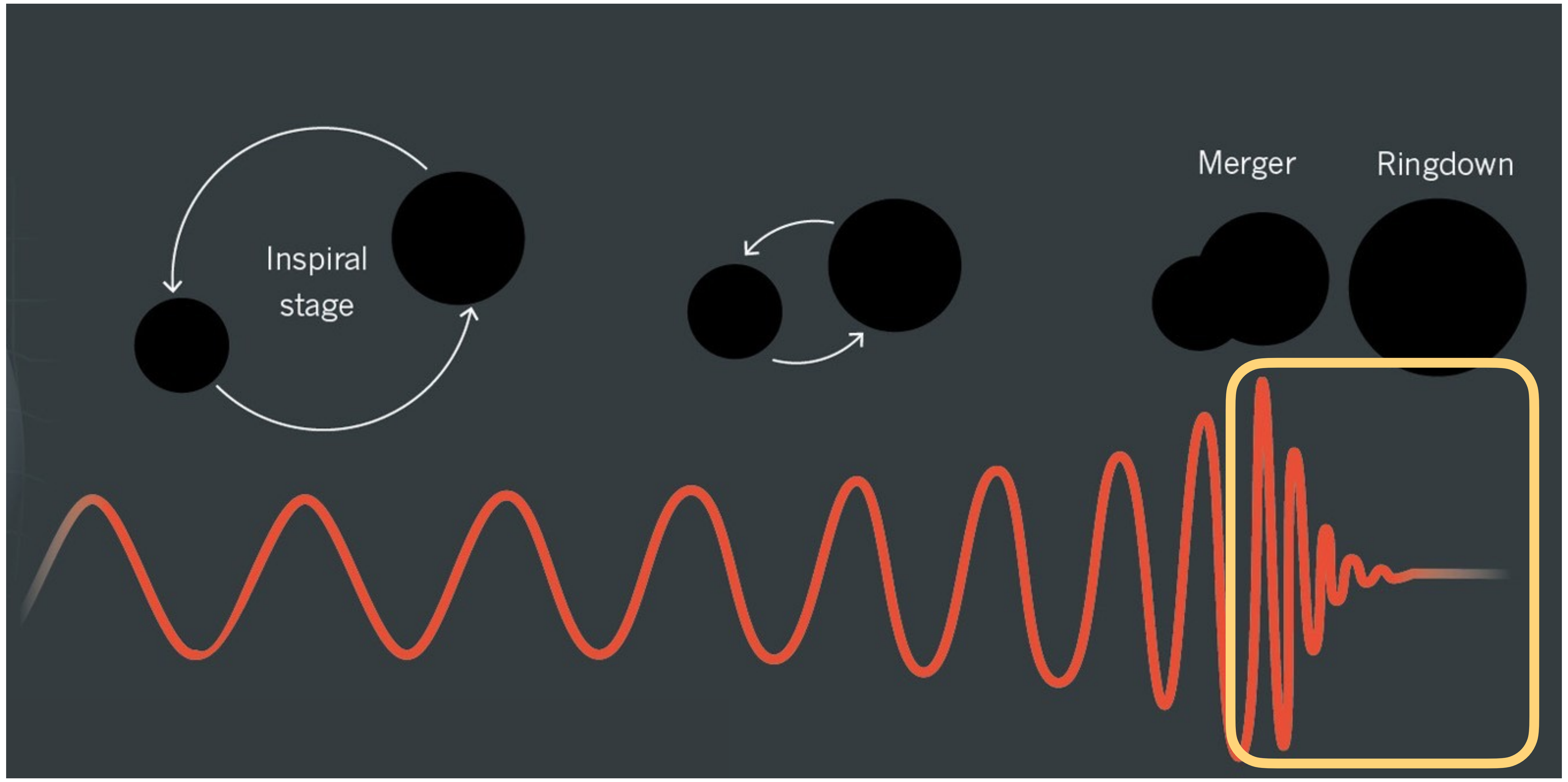
Heavy events (GW231123)





[LVK collaboration]

A ringdown ~~model~~ ansatz from quasinormal modes



[Lucy Reading-Ikkanda]

t_0

A superposition of quasinormal modes

$$\boxed{h \simeq \sum A_{n\ell m} e^{-i\omega_{n\ell m}(t-t_0)+\phi_{n\ell m}}}$$

How many modes?

How do we choose t_0?

How do we **build a more predictive model?**

What error are we making?

How do we predict the A_n, ϕ_ns?

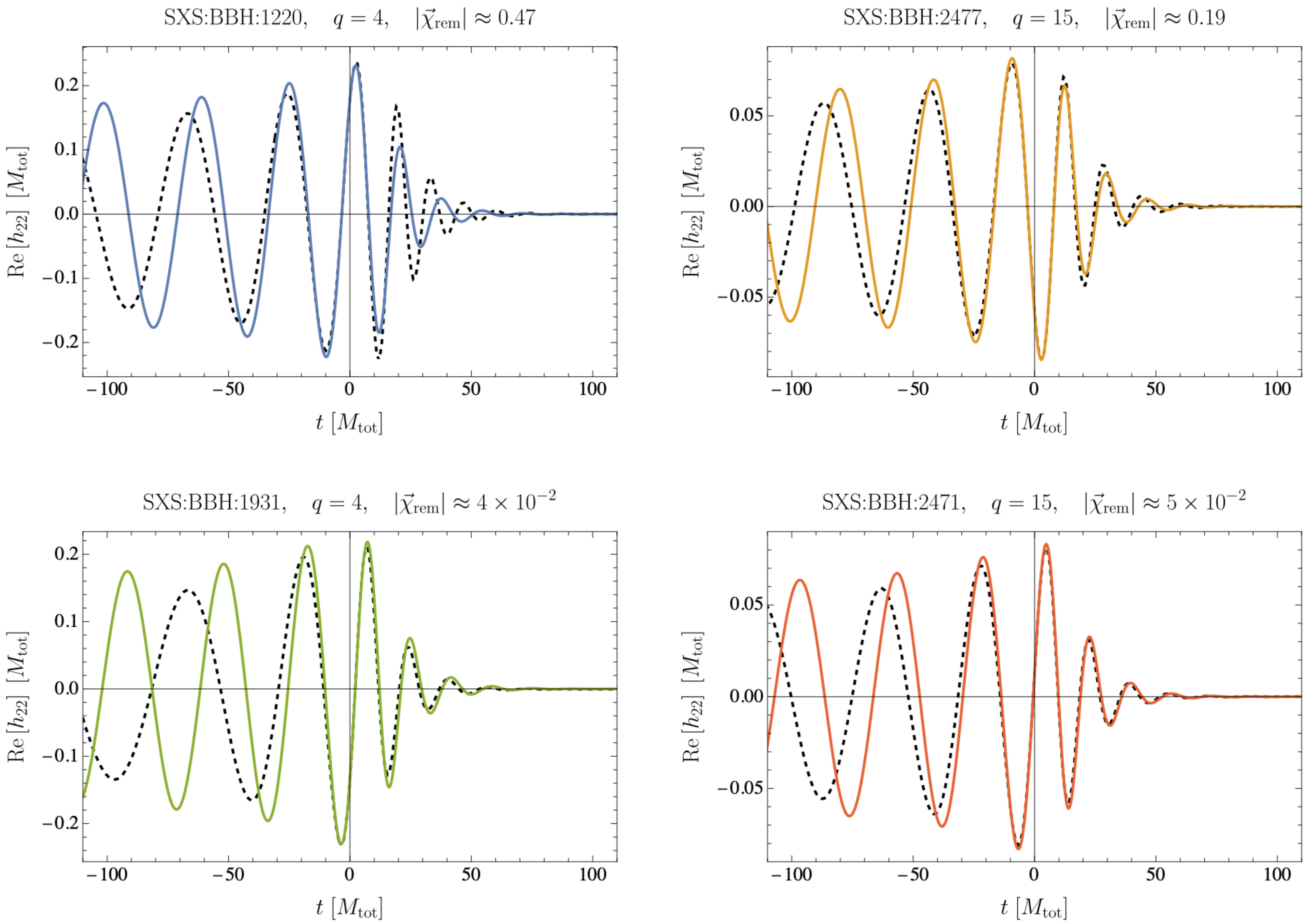
Set up and simplifying assumptions

Simplifications:

1) small mass ratio

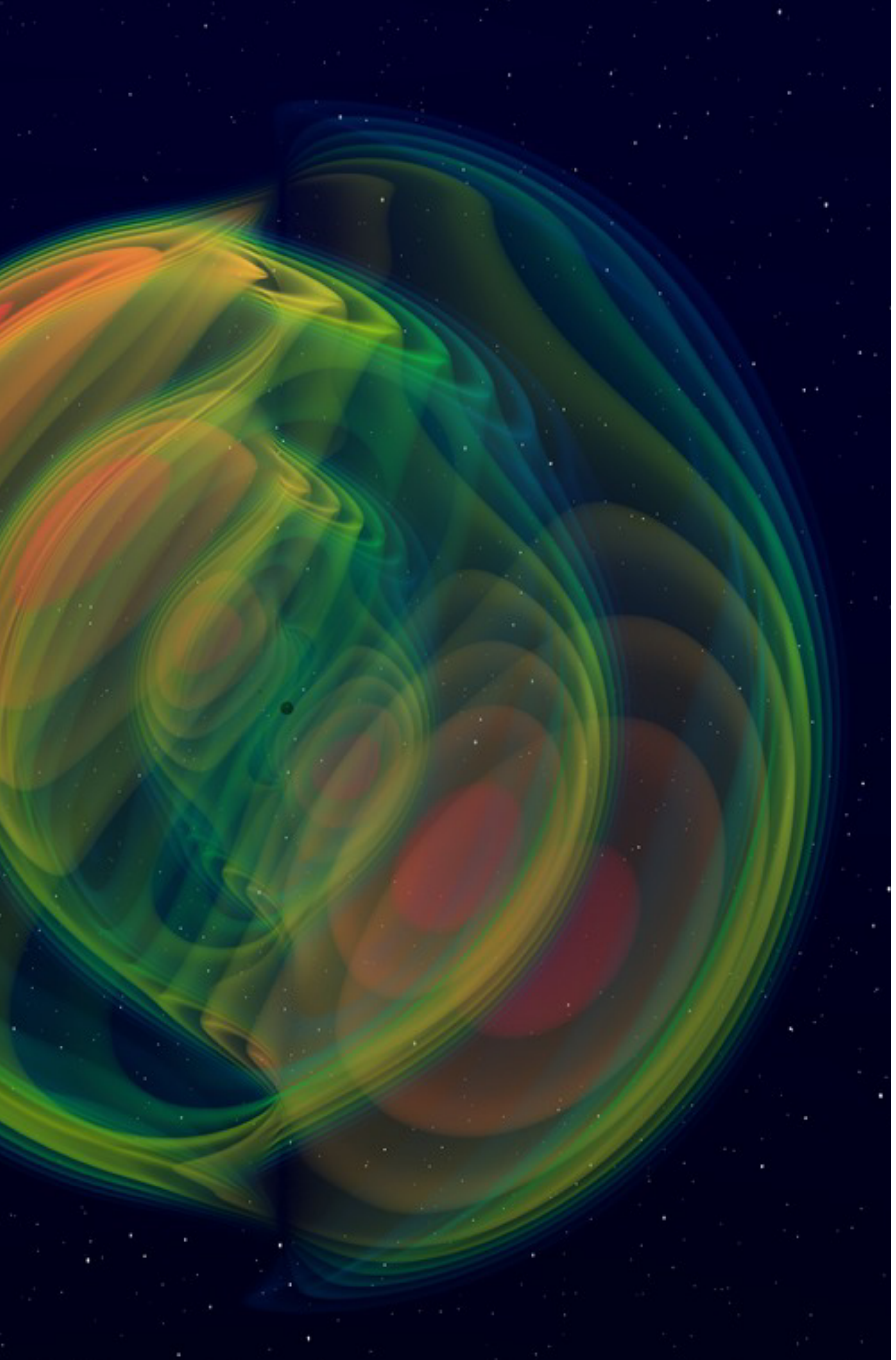
2) not spinning (Schwarzschild)

3) linear order



[Küchler+ 2025]

How do we currently model the ringdown?


[Fisher, Pfeiffer, Buonanno (SXS)]

The technique: black hole perturbation theory

$$g_{\alpha\beta} = \boxed{g^{(0)}_{\alpha\beta}} + \boxed{\epsilon g^{(1)}_{\alpha\beta}} + \dots$$

0th order	1st order
Kerr metric	**Teukolsky equation***

*linearised Einstein equation in vacuum, in terms of Weyl curvature scalars

The dynamics of black hole perturbations

$$\boxed{\left[\partial_t^2 - \partial_{r_*}^2 + V_{\ell m}(r_*)\right] \Psi_{\ell m}(t, r_*) = 0}$$

Green's function

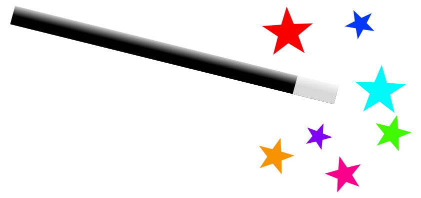
$$\left[\partial_t^2 - \partial_{r_*}^2 + V_{\ell m}(r_*)\right] G_{\ell m}(t, t'; r_*, {r_*}') = \delta(t - t')\delta(r_* - {r_*}')$$

[Leaver 1986]

First order black hole perturbations

$$\left[\partial_t^2 - \partial_{r_*}^2 + V_{\ell m}(r_*)\right] \Psi_{\ell m}(t, r_*) = 0$$

***Frequency domain* Green's function**



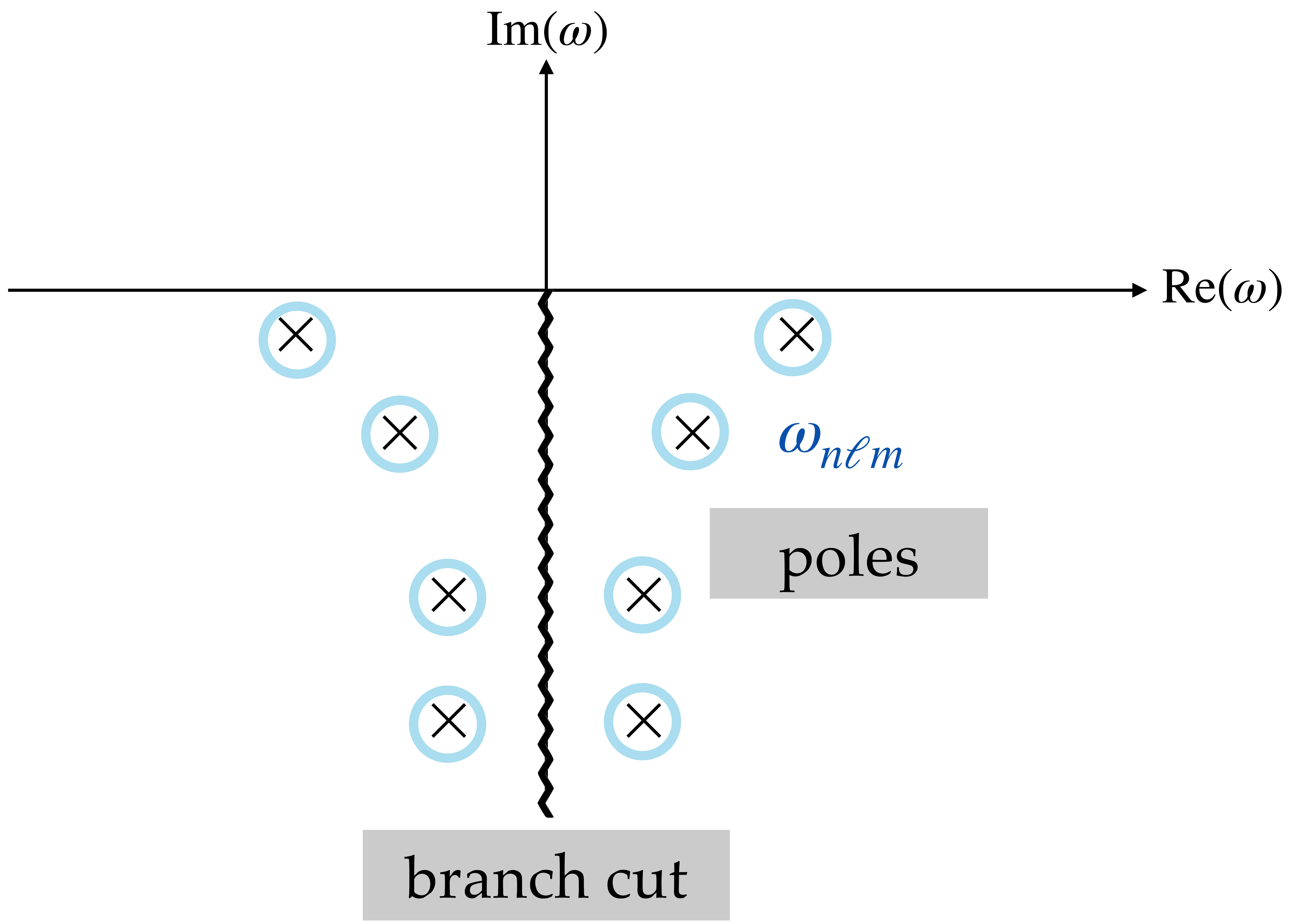
$$\left[-\partial_{r_*}^2 - \omega^2 + V_{\ell m}(r)\right] \tilde{G}_{\ell m}(\omega; r_*, r'_*) = \delta(r_* - r'_*)$$

[Leaver 1986]

The analytic structure of the black hole Green's function

$$\tilde{G}(\omega; r_*, r'_*) = \tilde{G}_-(\omega; r_*, r'_*) + \tilde{G}_+(\omega; r_*, r'_*)$$

Only a branch cut!

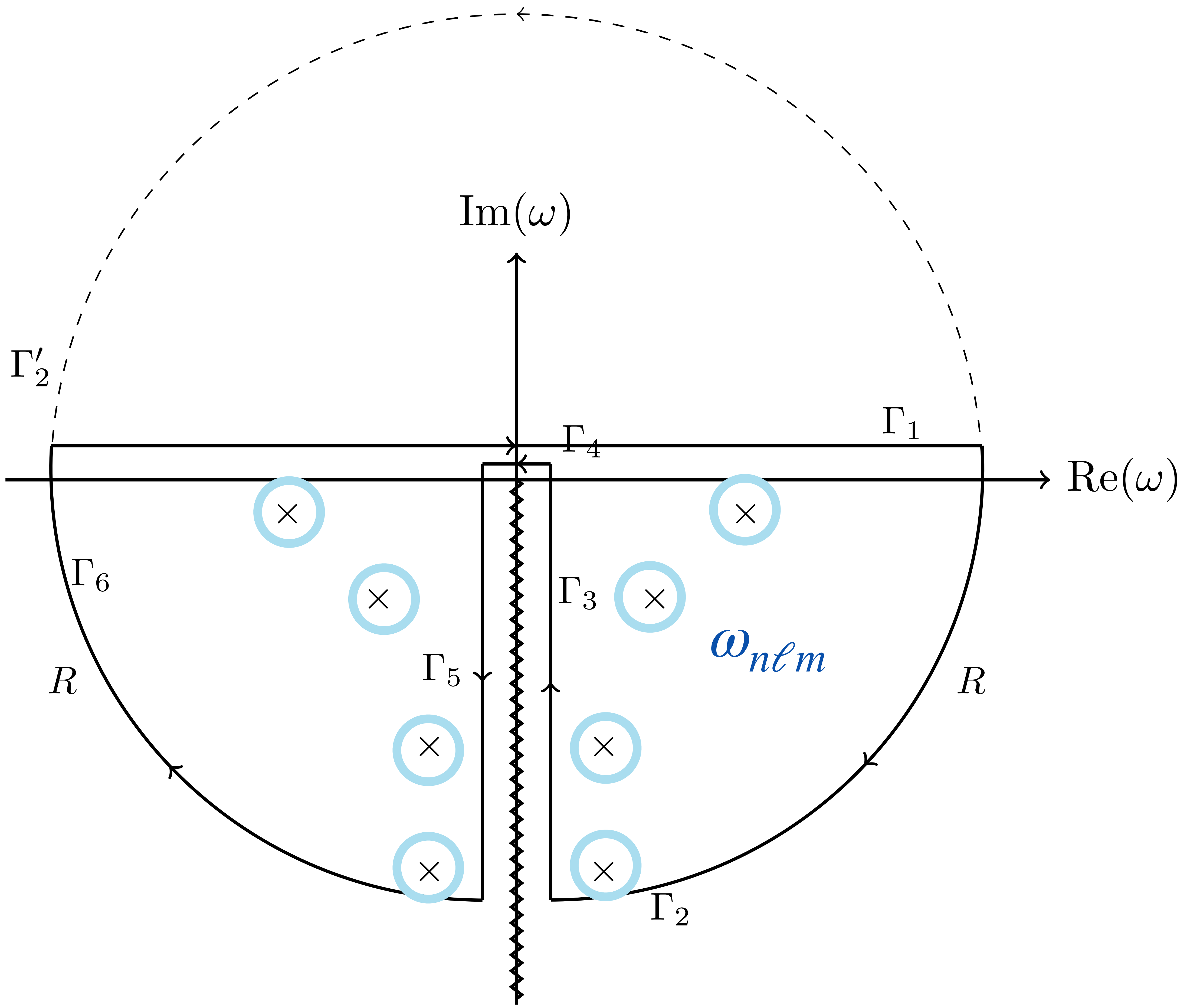


[Leaver 1986]

The QNM Green's function

$$\int_{-\infty+i\epsilon}^{+\infty+i\epsilon} d\omega\, e^{-i\omega t}\tilde{G}_+(\omega; r_*, r_*')$$

$$= \underbrace{\boxed{\sum \mathrm{Res}_{\omega_{n\ell m}} \left\{e^{-i\omega(t-r')}\tilde{G}_+(\omega; r_*, r_*')\right\}}}_{G_{\mathrm{QNM}}(t-t'; r_*, r_*')} + \text{branch cut} + \text{arc}$$



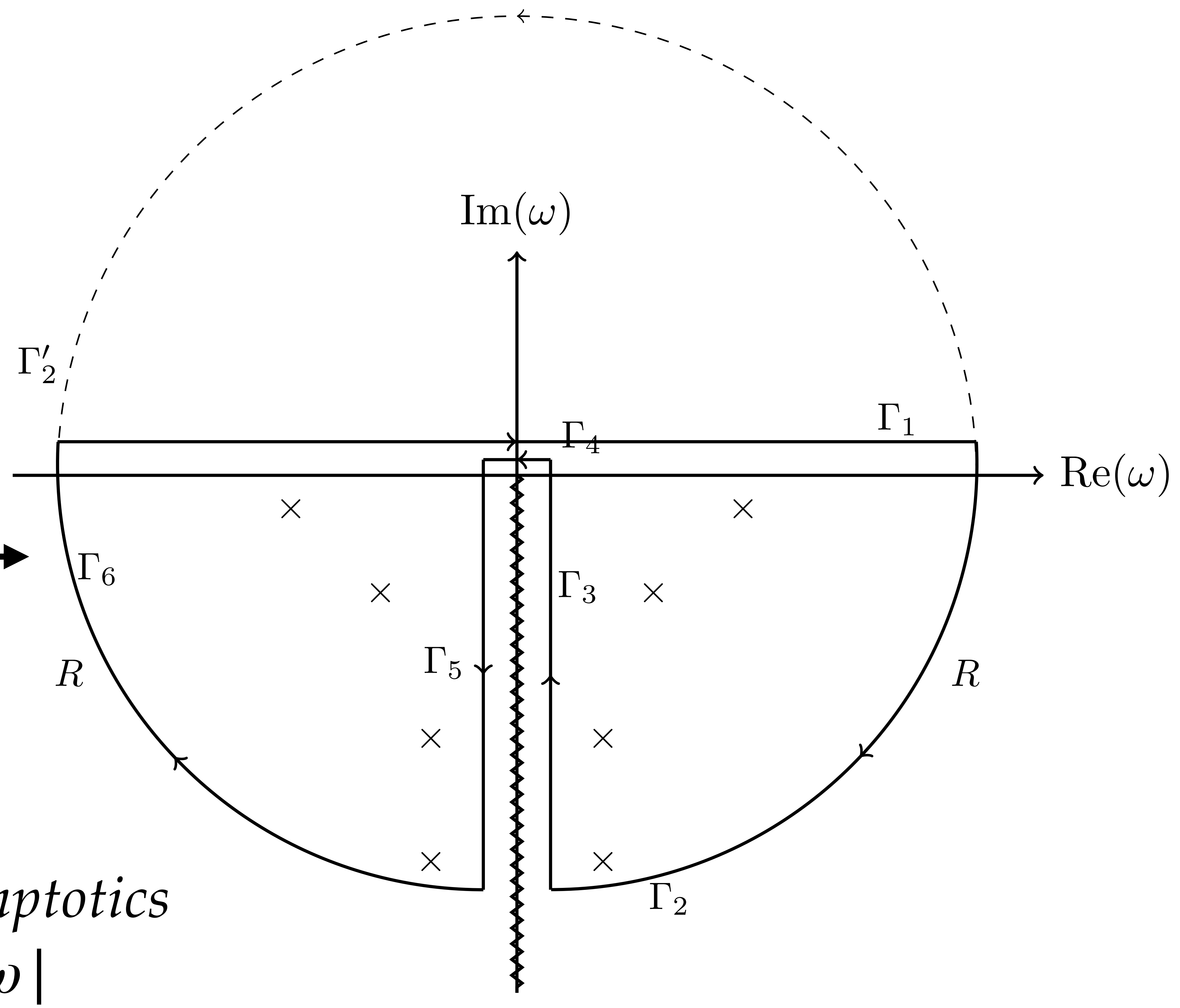
[Leaver 1986]

What about the arc?

I should close the contour in the lower-half plane when the ***arc*=*0***.

$$t - r_* \geq t' + r'_* - 2r_+ \log \frac{r' - r_+}{r'}$$

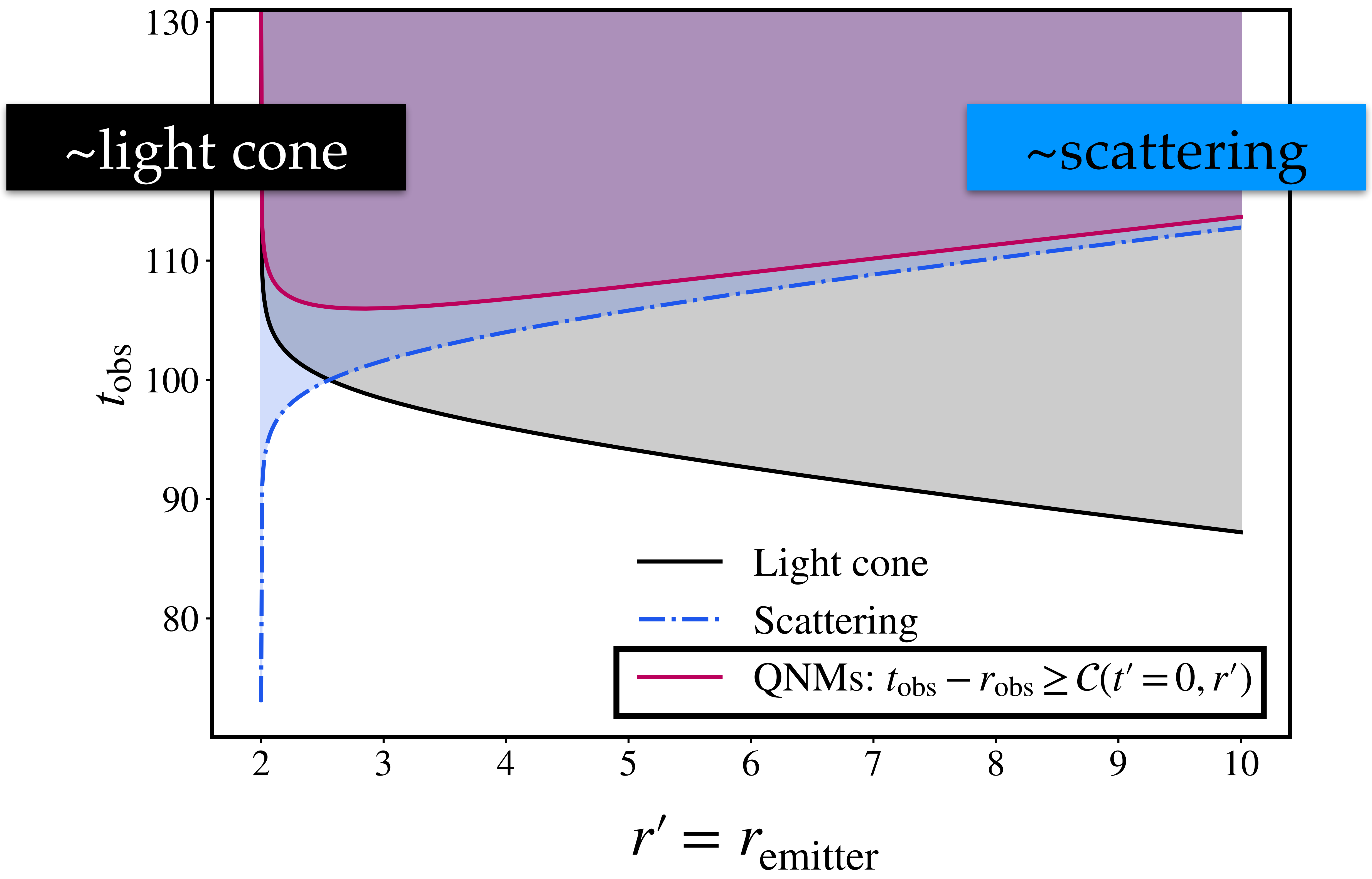
QNM “**causality**”

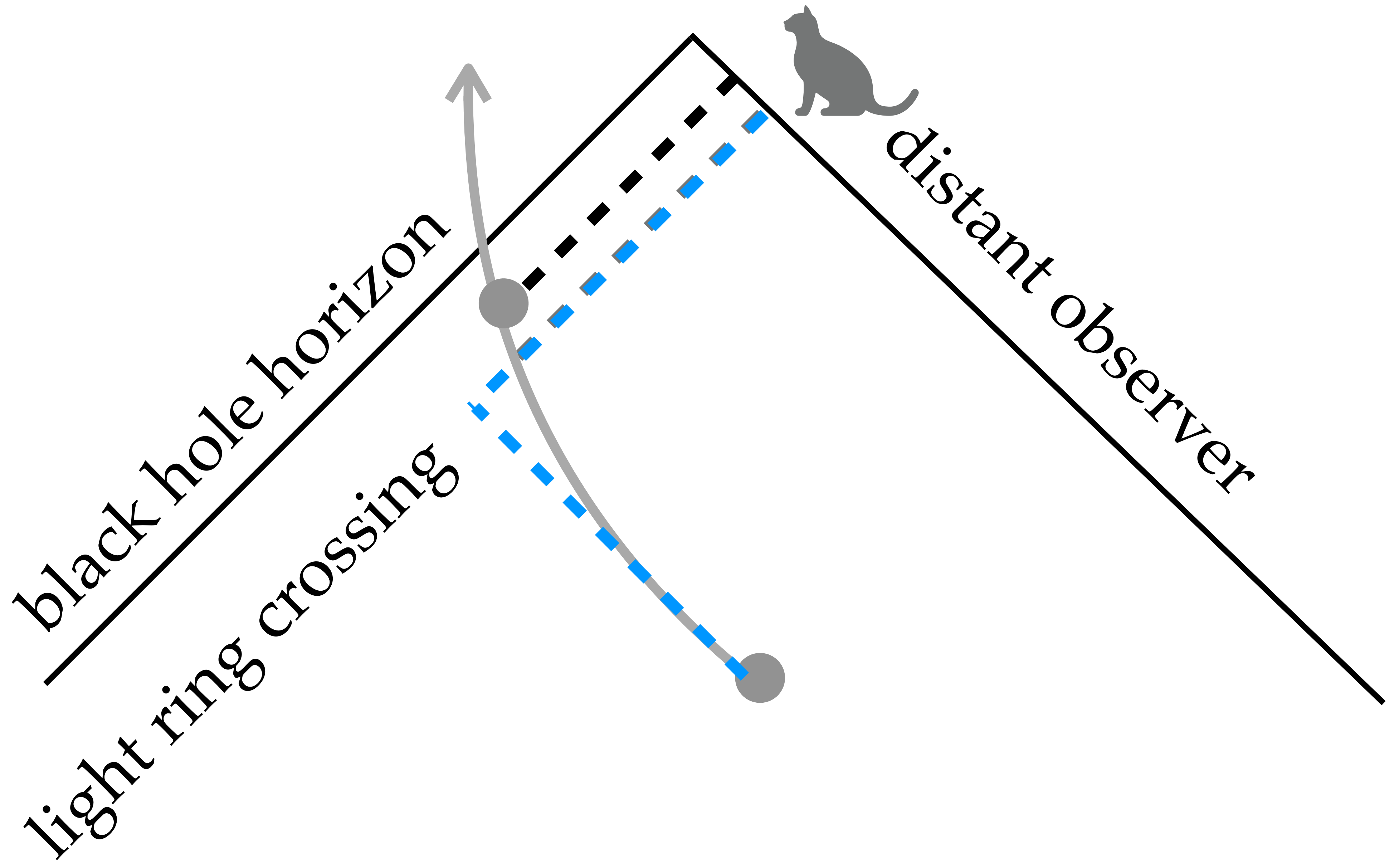


How: look at the asymptotics of $\tilde{G}_+$ at large $|\omega|$

[De Amicis Cannizzaro Carullo LS 2025; see also: Chavda Lagos Hui 2024; Kuntz 2025; Arnaudo Carballo Withers 2025]

Quasinormal mode causality





[De Amicis Cannizzaro Carullo LS 2025; see also: Chavda Lagos Hui 2024; Kuntz 2025; Arnaudo Carballo Withers 2025]

Causality in the QNM Green's function

$$G^{\mathrm{QNMs}}(t-t';r_*,r_*') = \theta\left[t - r_* - \mathscr{C}(t',r_*')\right] \cdot \sum_n \mathrm{Res}_{\mathrm{n}} \psi_n(r')\, e^{-i\omega_n(t-r_*-\mathscr{C})}$$

QNM "**causality**"

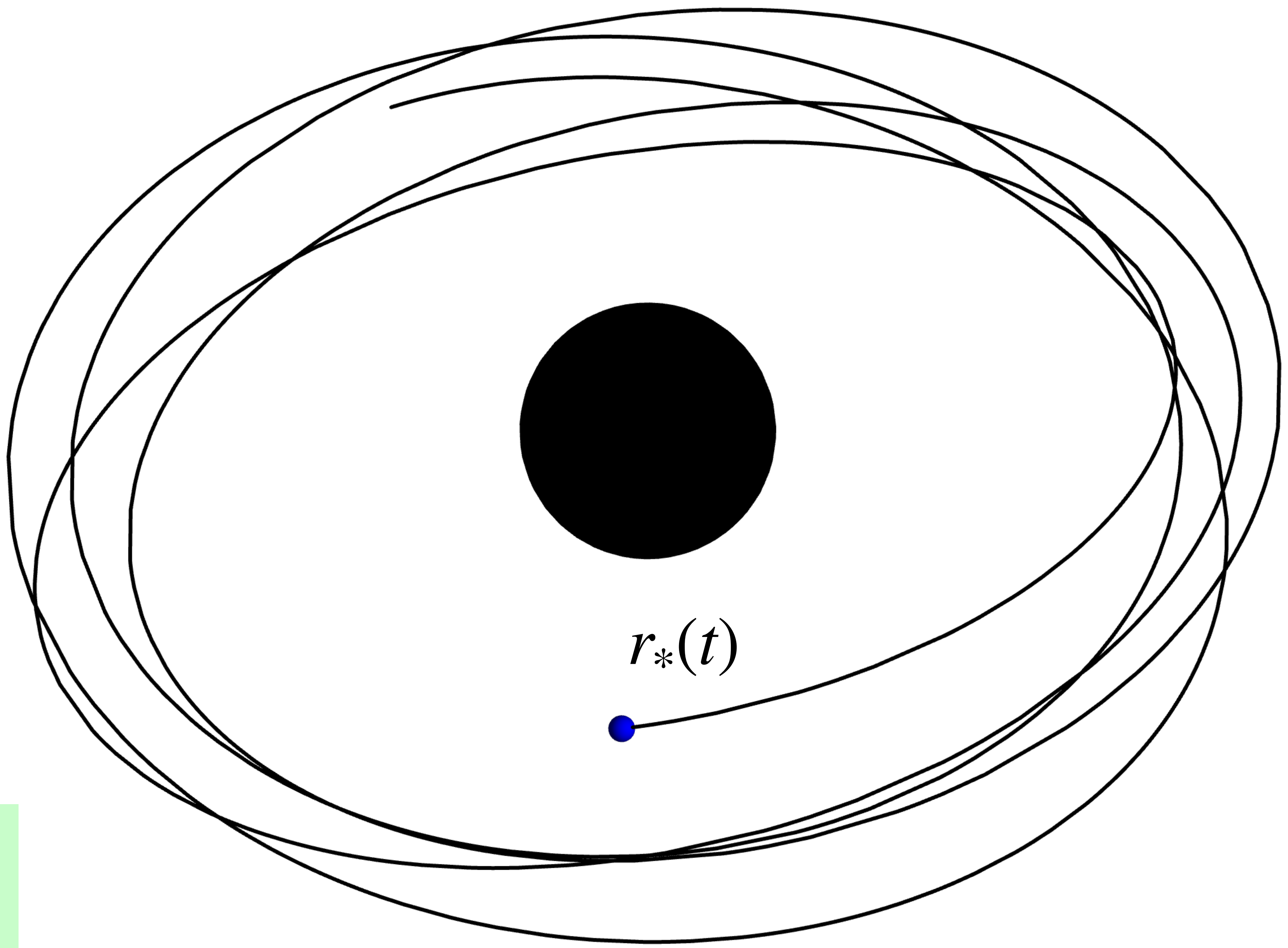
[De Amicis, Cannizzaro, Carullo, LS 2025]

Even more dynamics: a companion's source

$$\left[\partial_t^2 - \partial_{r_*}^2 + V_{\ell m}(r_*)\right] \Psi_{\ell m}(t, r_*) = \mathcal{S}_{\ell m}(t, r_*)$$

source (test particle limit)

$$S_{\ell m}(t, r_*) \sim e^{-im\varphi(t)}\left[f_{\ell m}(t, r_*)\, \delta(r_* - r_*(t)) + g_{\ell m}(t, r_*)\, \partial_{r_*}\delta(r_* - r_*(t))\right]$$



Two types of source contributions

$$\Psi^{\mathrm{QNM}}(t, r_*) = \int_{-\infty}^{\infty} dt' \int_{-\infty}^{\infty} dr'_*\, G^{\mathrm{QNM}} \times \mathcal{S}$$

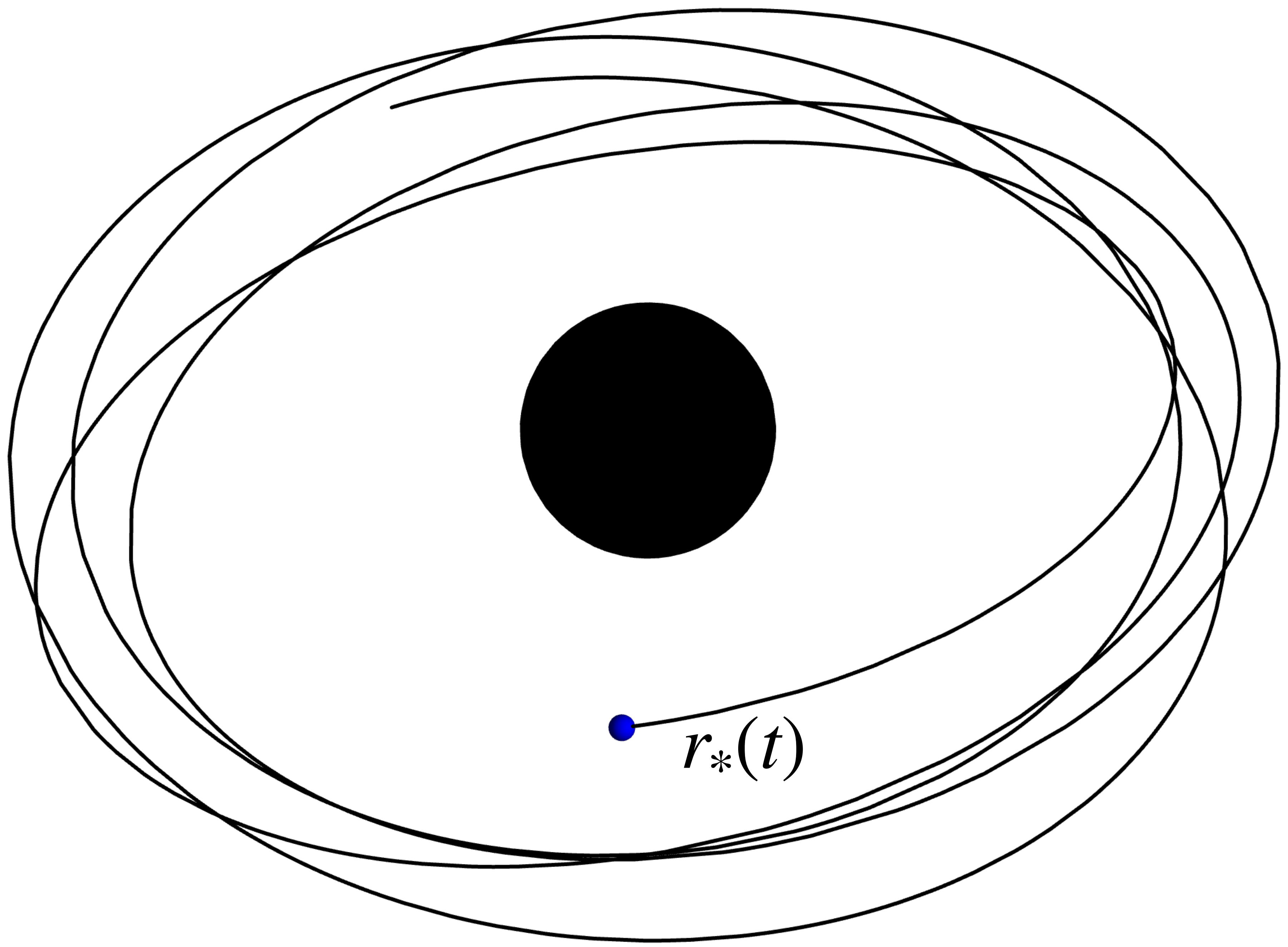
$$f\,(t, r_*)\,\delta\,(r_* - r_*\,(t)) + g\,(t, r_*)\;\partial_{r_*}\delta\,(r_* - r_*\,(t))$$

~position

~speed

retarded,
non-local in time

instantaneous



Convolution with the source

$$\Psi^{\rm QNM}(t,r_*) = \int_{-\infty}^{\infty} dt' \int_{-\infty}^{\infty} dr'_*\, G^{\rm QNM} \times \mathcal{S} \;=\; \sum_{n,\pm} e^{-i\omega_{\ell mn\pm}(t-r_*)}\, B_{\ell mn\pm} \left[\boxed{c_{\ell mn\pm}(t-r_*)} + \boxed{i_{\ell mn\pm}(t-r_*)}\right]$$

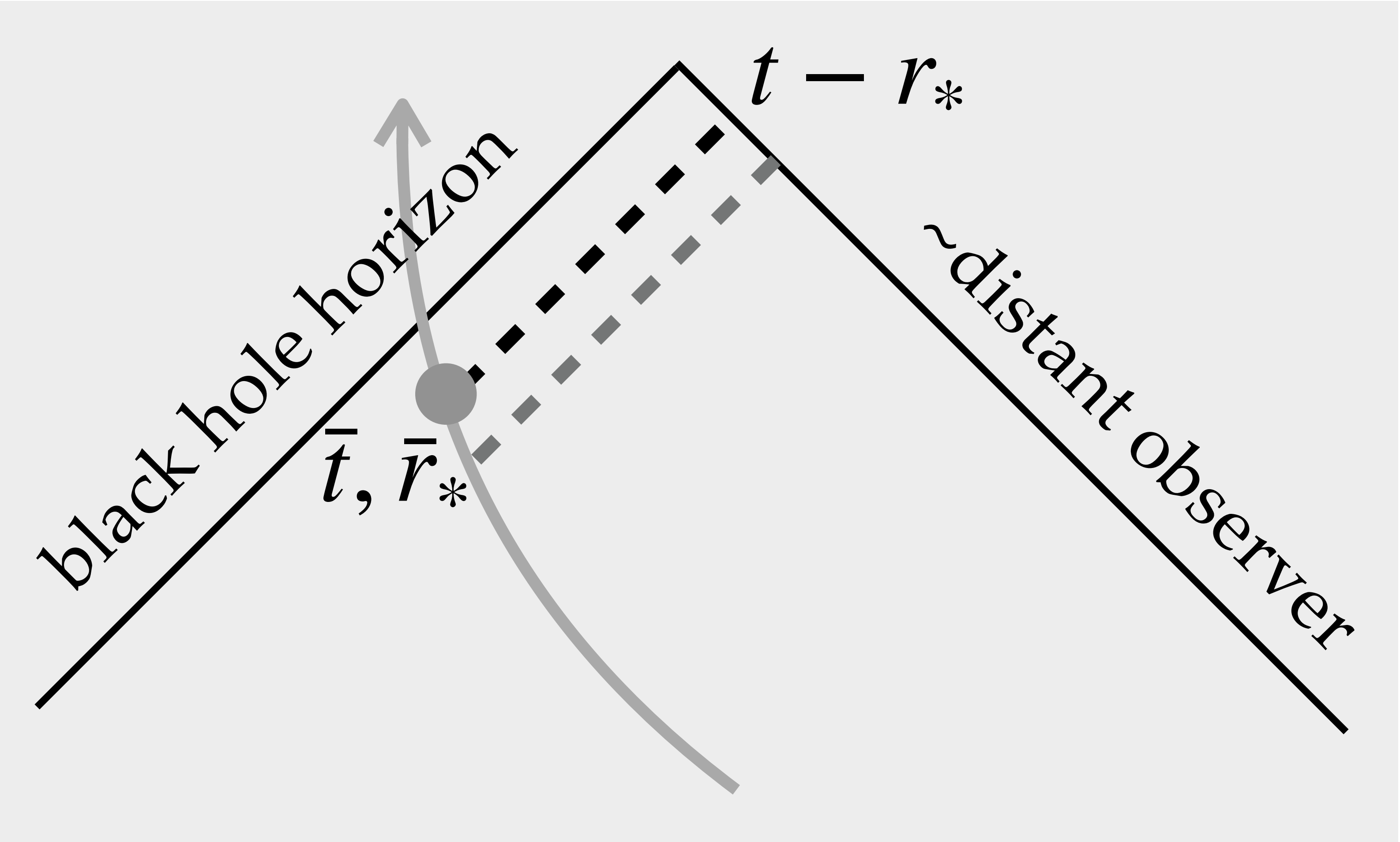
"Excitation" coefficient:

u: QNM mode function

$$c_{\ell mnp}(t-r_*) = -\int_{\bar{r}_*(t-r_*)}^{\infty} dr'_* \frac{1}{\dot{r}_*(t(r'_*))}\Big[u_{\ell mnp} f_{\ell m} - \partial_{r'_*}(u_{\ell mnp} g_{\ell m})\Big]_{(t(r'_*),r'_*)}$$

"Impulsive" coefficient:

$$i_{\ell mnp}(t-r_*) = \frac{\left[\bar{r}^2 - 8\right] u_{\ell mnp}(\bar{t},\bar{r}_*) g_{\ell m}(\bar{t},\bar{r}_*)}{\dot{\bar{r}}_* \left[2r_h^2 - \bar{r}^2\right] - \bar{r}^2}$$



[De Amicis, Cannizzaro, Carullo, LS 2025]

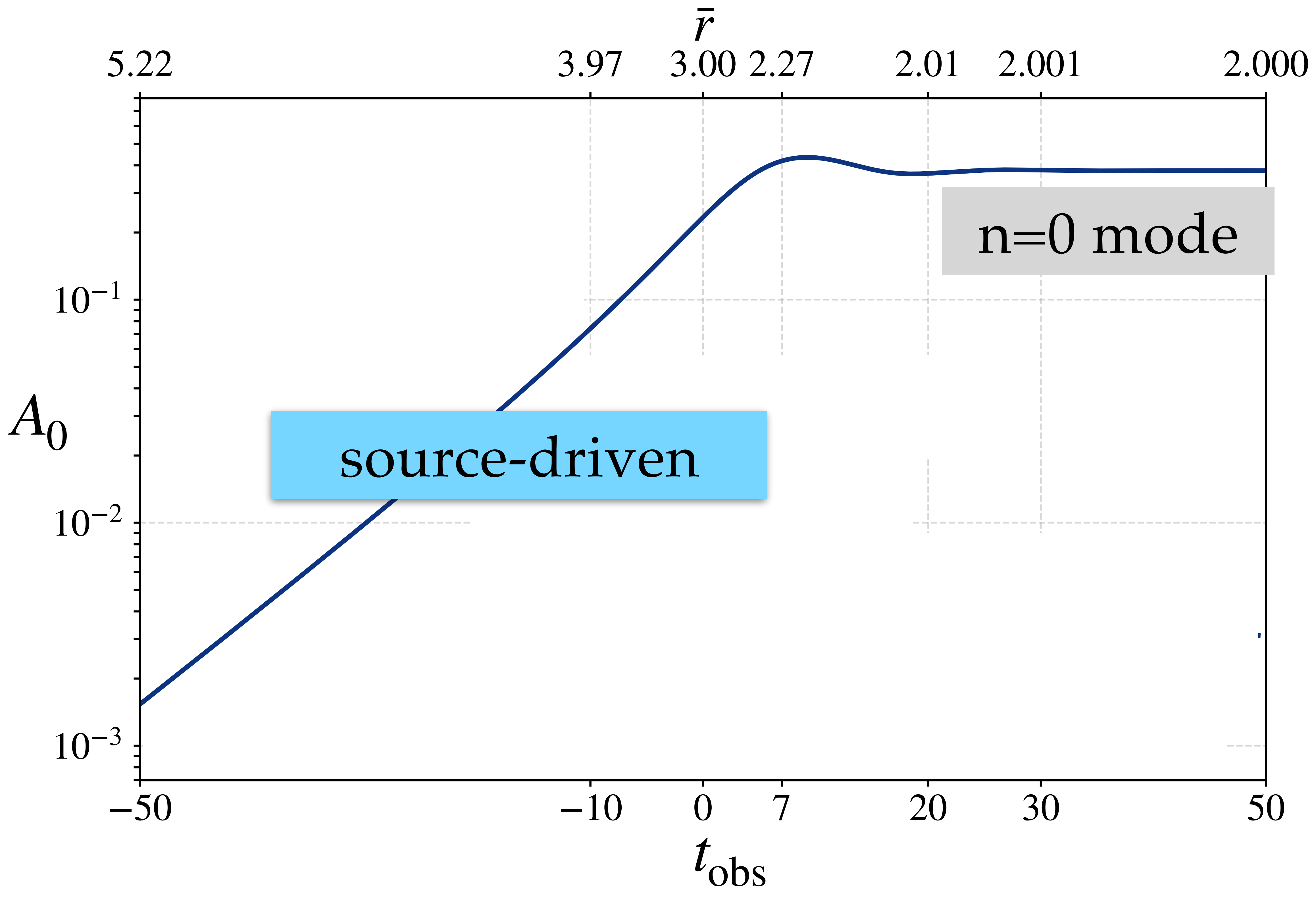
And now…some nice plots

The dynamical ringdown

The standard model: constant amplitudes

Our model: dynamical coefficients

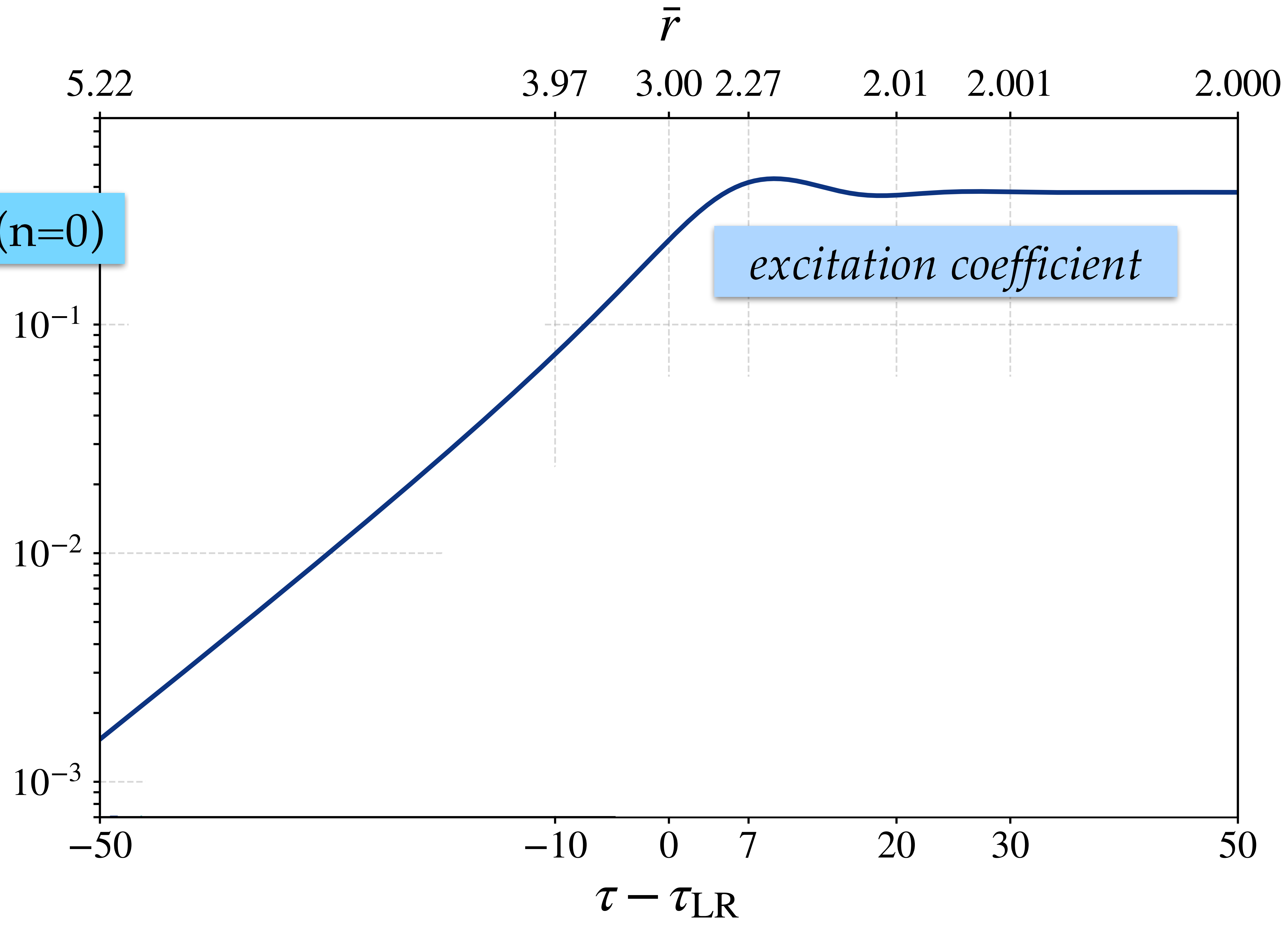
$$h_{\text{ringdown}} = \sum A_{n\ell m} e^{-i\omega_{n\ell m}(t-t_0)+\phi_{n\ell m}}$$



[De Amicis, Cannizzaro, Carullo, LS 2025]

Activation and impulsive coefficients

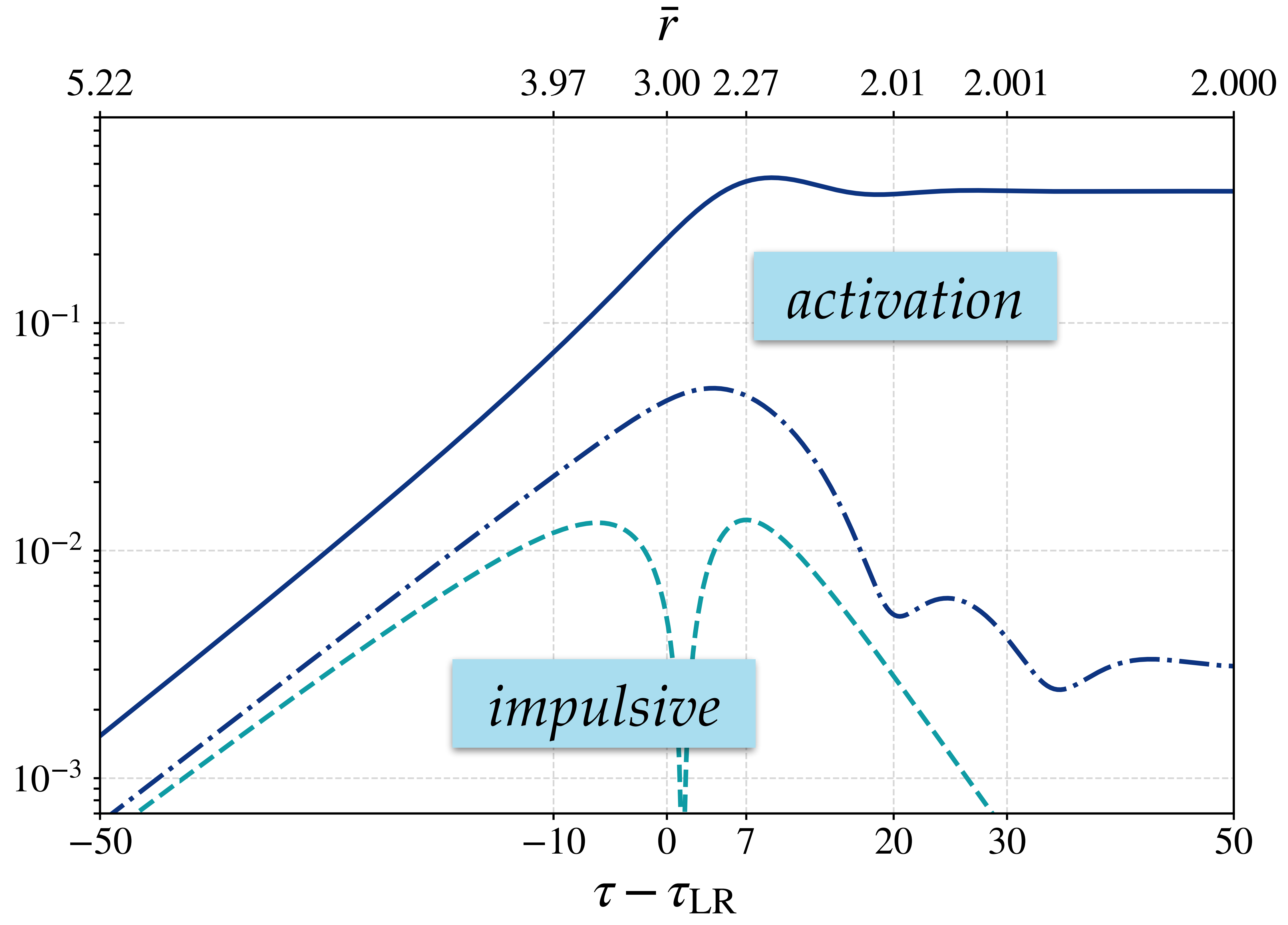
Fundamental mode (n=0)



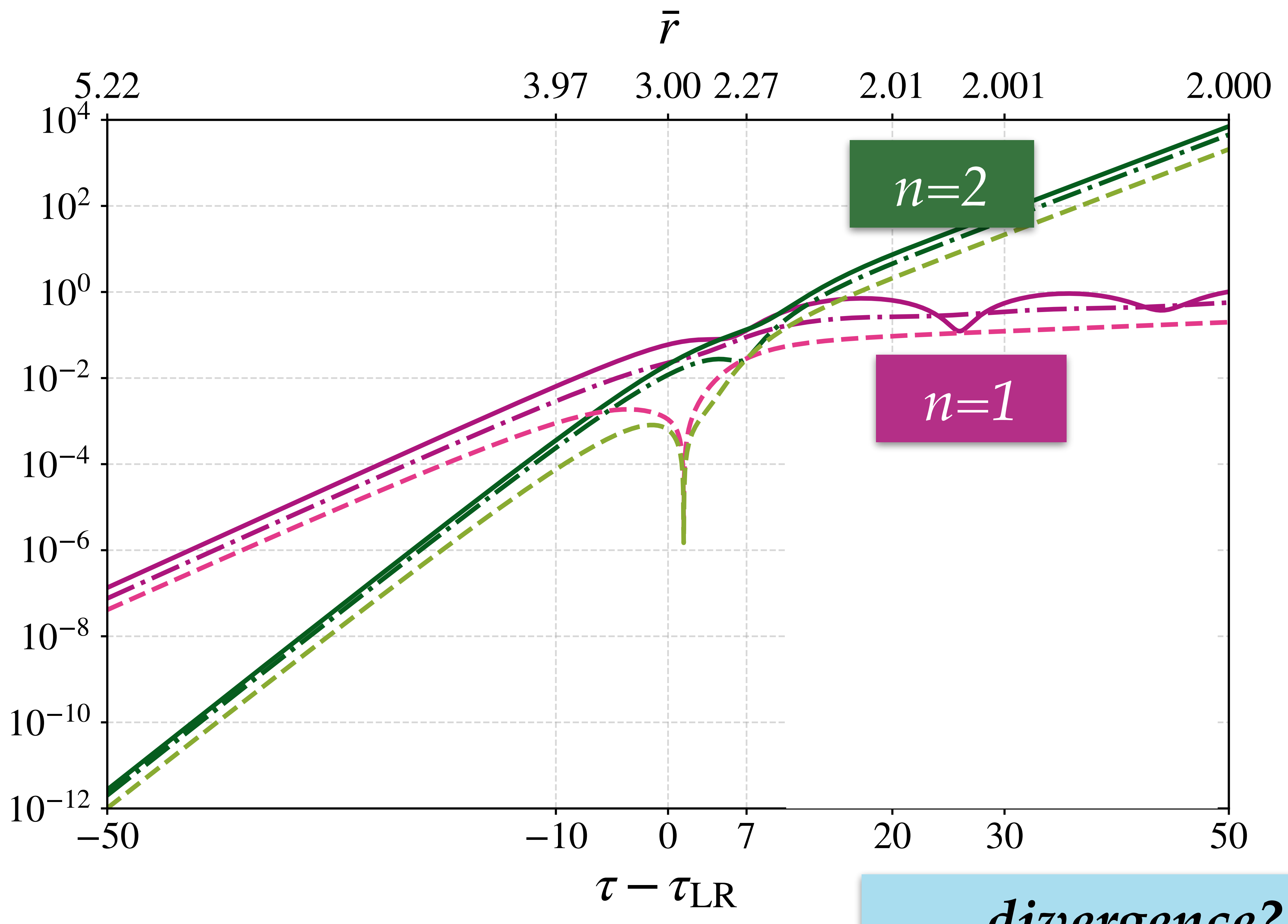
[De Amicis, Cannizzaro, Carullo, LS 2025]

Activation and impulsive coefficients

Fundamental mode



Higher modes



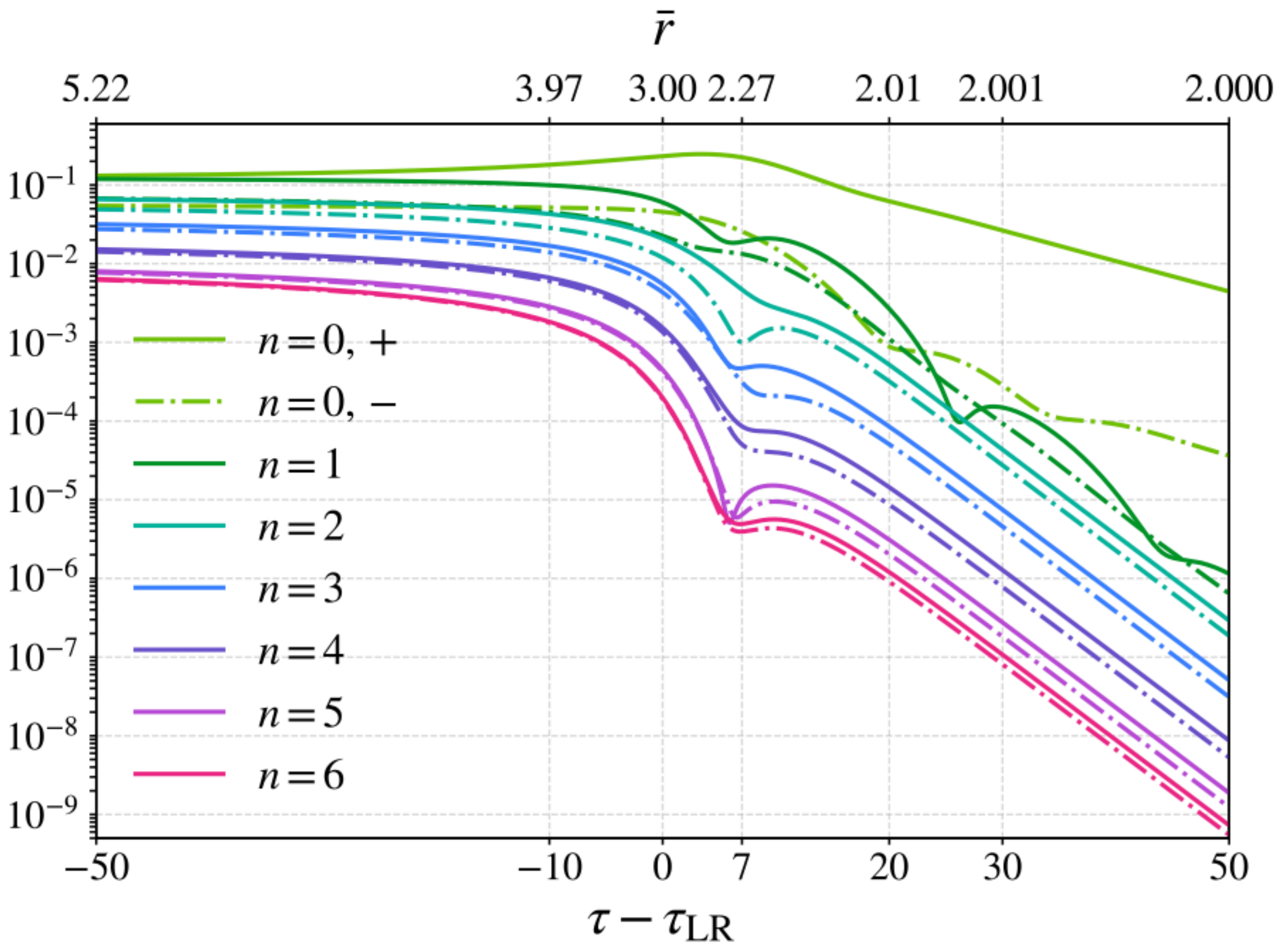
…divergence?

No divergence :)

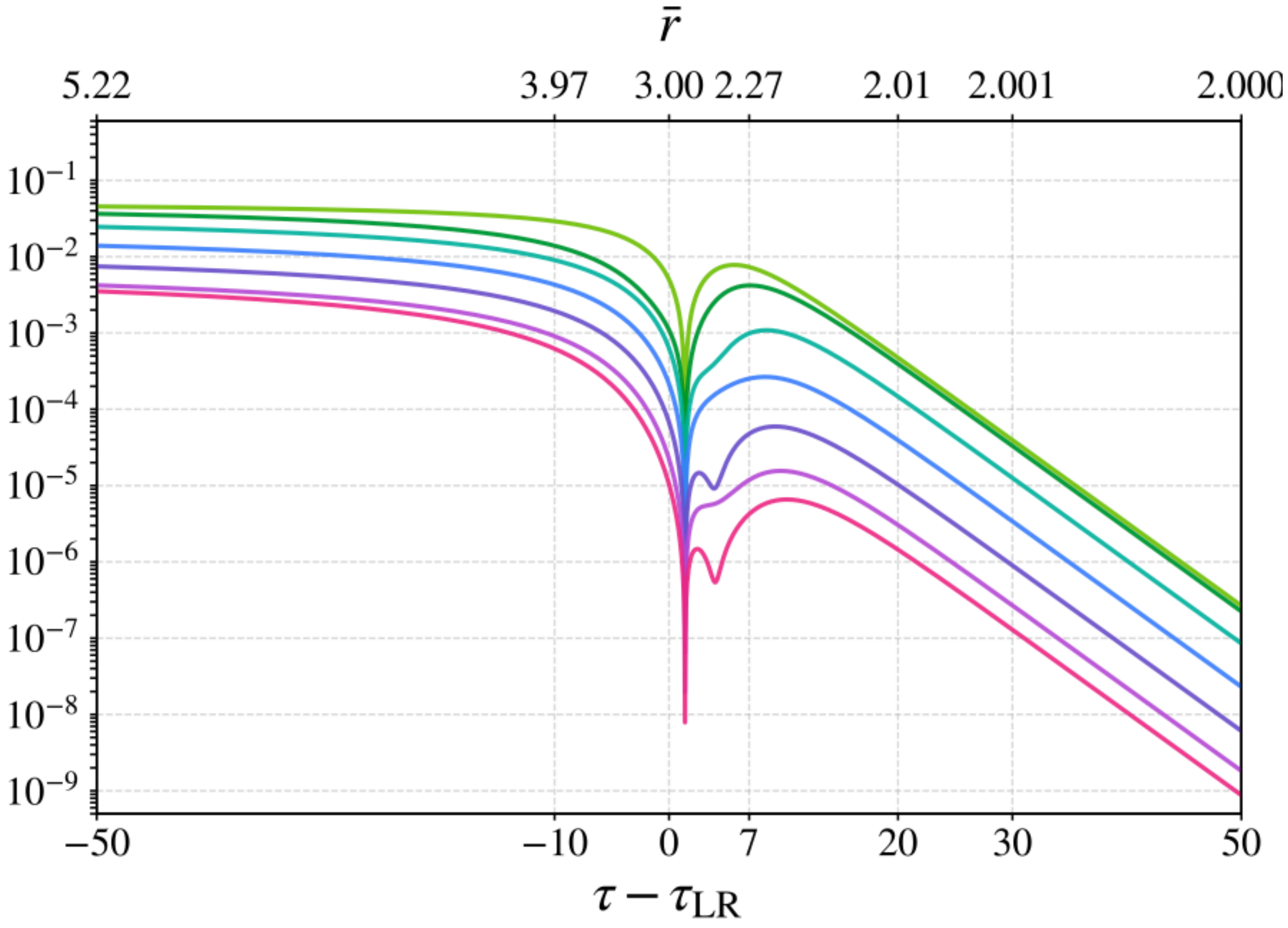
Activation and impulsive…waveform

$$\Psi^{\mathrm{QNM}}_{\ell mn\pm} = e^{-i\omega_{\ell mn\pm}(t-r_*)}\, B_{\ell mn\pm}\left[c_{\ell mn\pm}(t-r_*) + i_{\ell mn\pm}(t-r_*)\right]$$

activation



impulsive

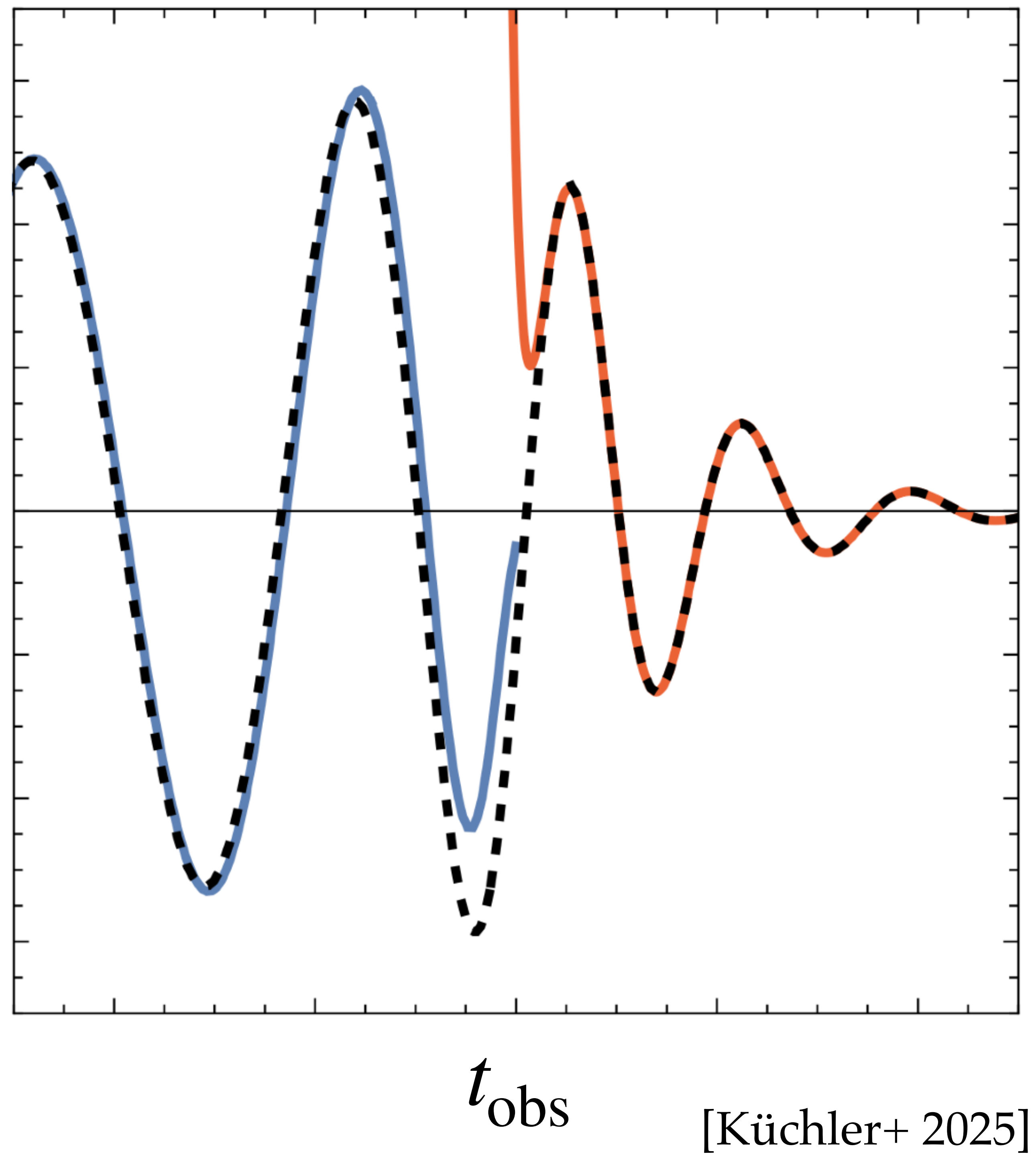


The dynamical ringdown

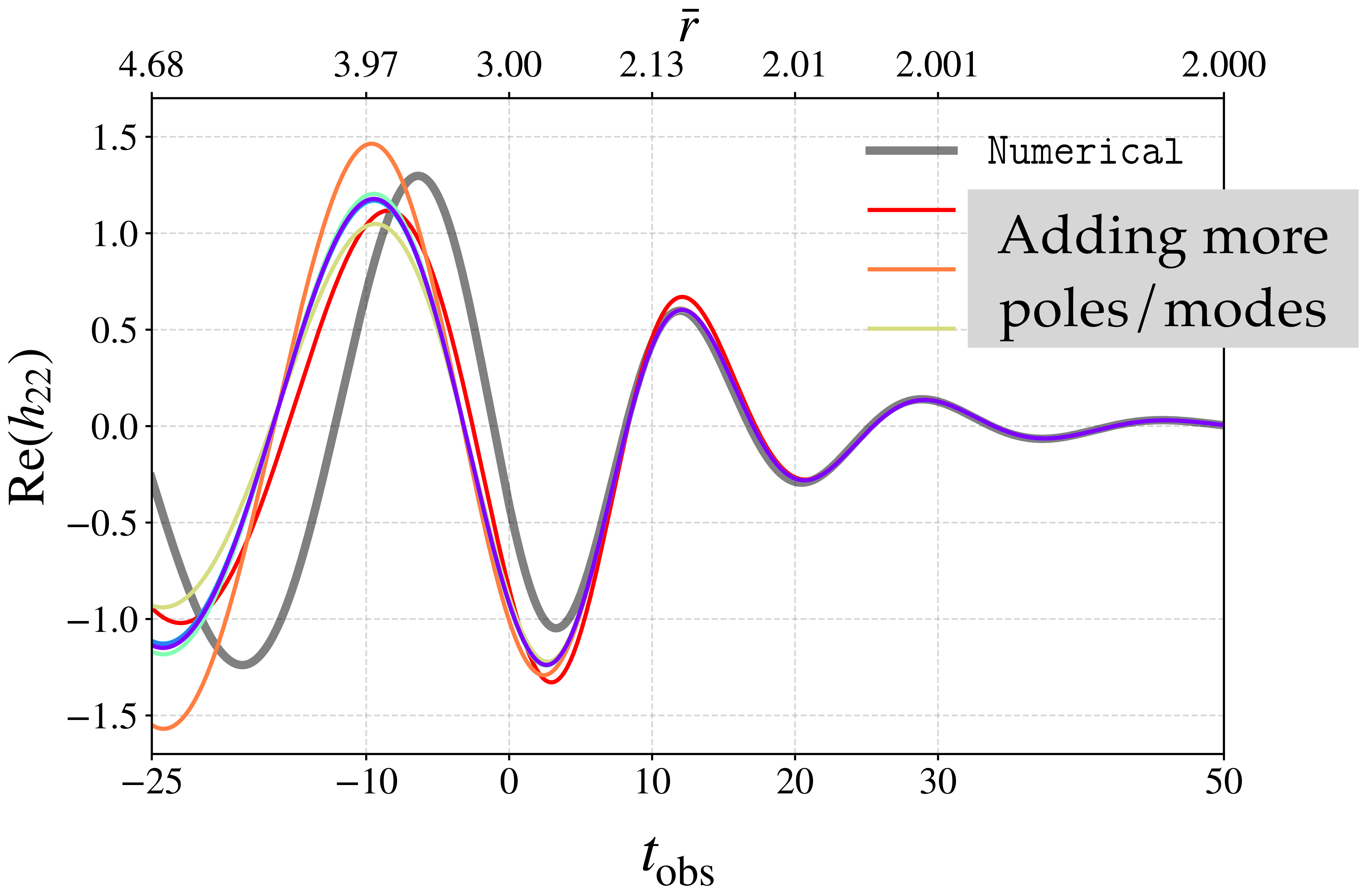
The standard model: divergence

This model: smooth

$$h_{\text{ringdown}} = \sum A_{n\ell m} e^{-i\omega_{n\ell m}(t-t_0)+\phi_{n\ell m}}$$

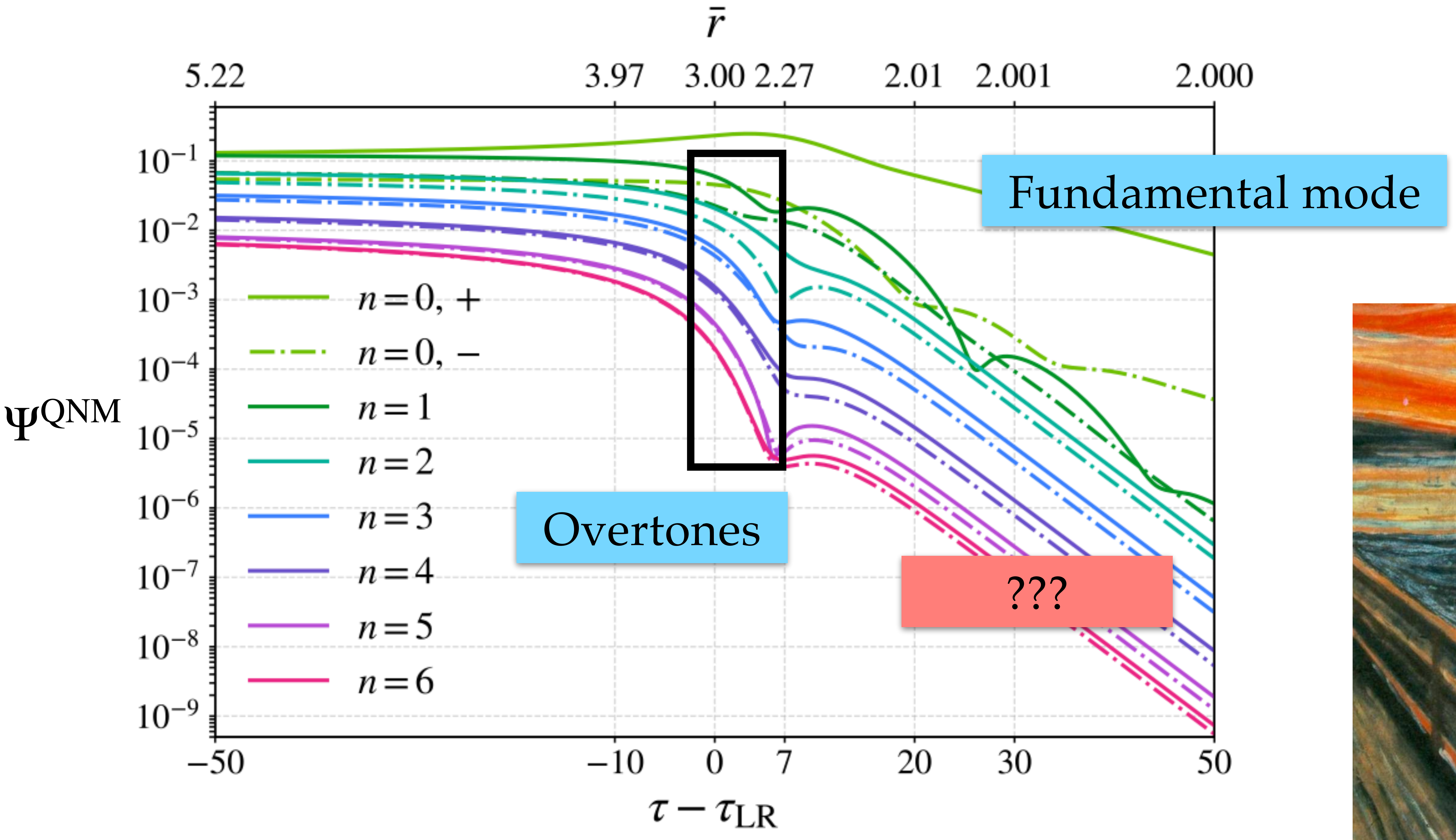


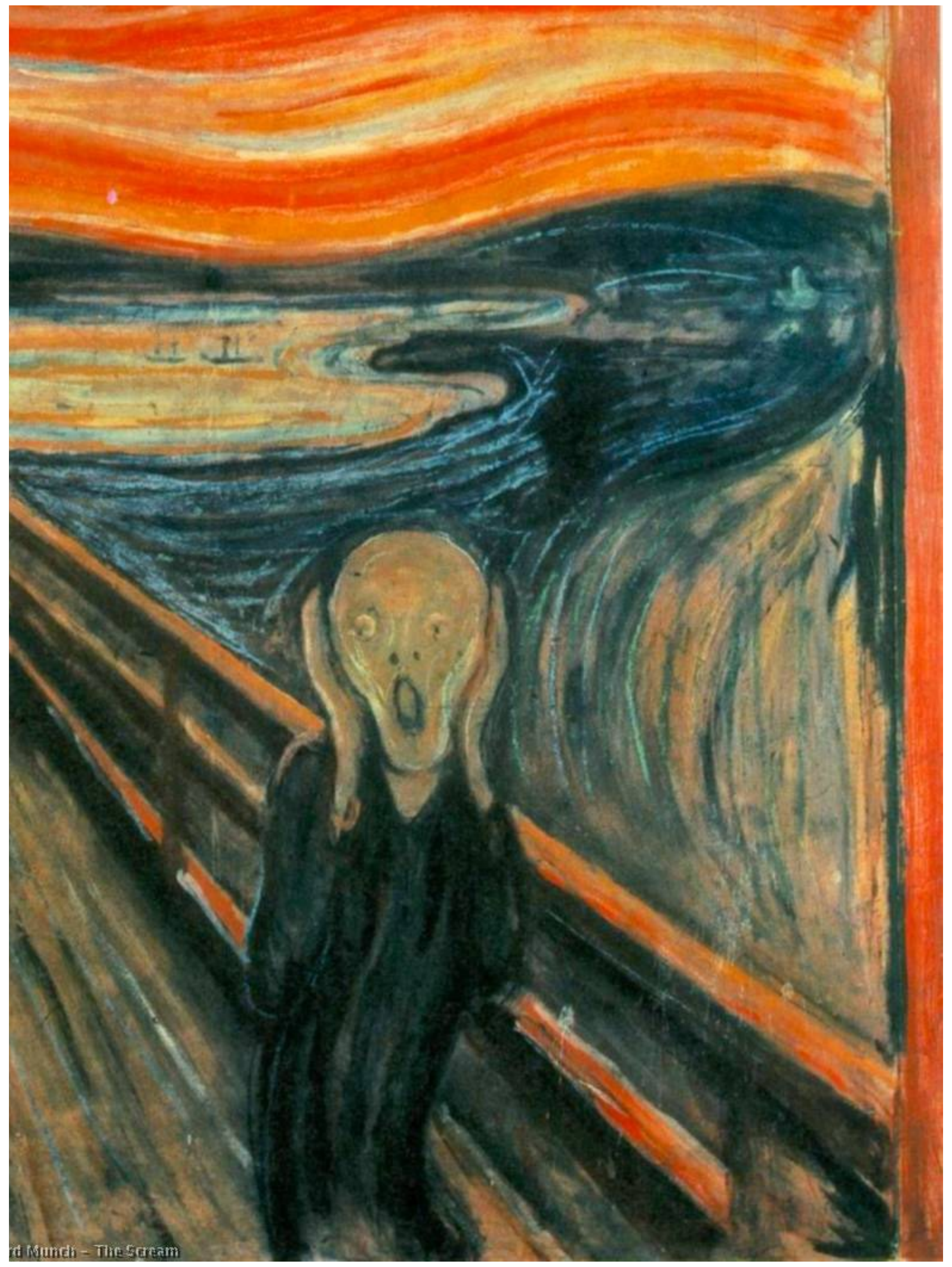
[Küchler+ 2025]



[De Amicis, Cannizzaro, Carullo, LS 2025]

Wait…what's happening at late times?





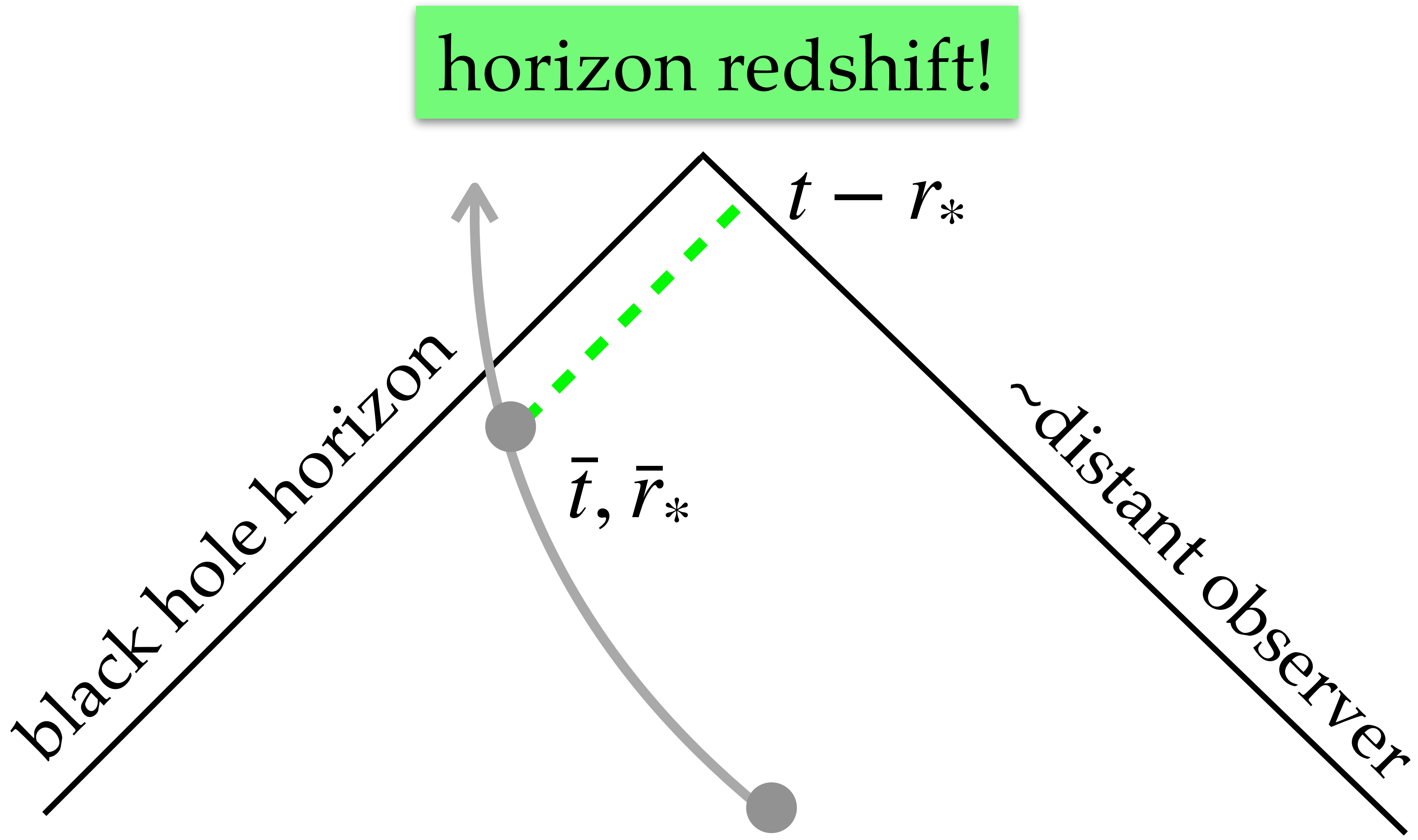
Near horizon limit

$$\Psi^{\mathrm{QNM}}(t, r_*) = \int_{-\infty}^{\infty} dt' \int_{-\infty}^{\infty} dr'_* \, G^{\mathrm{QNM}} \times \mathcal{S}$$

$$\rightarrow \sum_{n=0} e^{-i\omega_n (t-r_*)} + e^{-\kappa_H (t-r_*)} \sum_{k=0} e^{-\kappa_H k (t-r_*)}$$

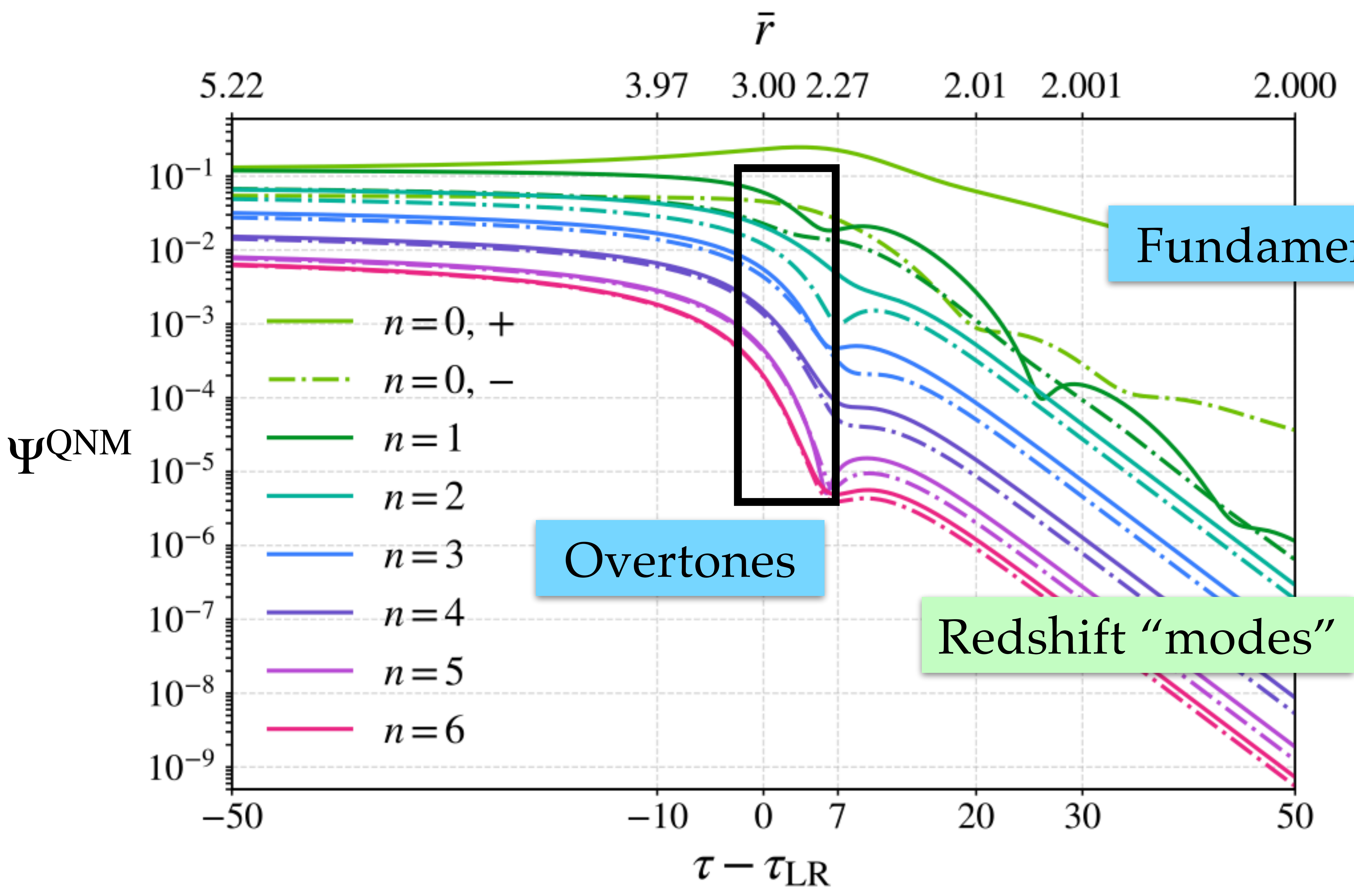
quasinormal modes

redshift modes (κ_H = horizon surface gravity)



[De Amicis, Cannizzaro, Carullo, LS 2025; see also Price 1972, Mino and Brick 2008, Zimmerman and Chen 2011]

Horizon/redshift "modes" in the waveform?



$$\rightarrow \sum_{n=0} e^{-i\omega_n(t-r_*)} + e^{-\kappa_H(t-r_*)} \sum_{k=0} e^{-\kappa_H k(t-r_*)}$$

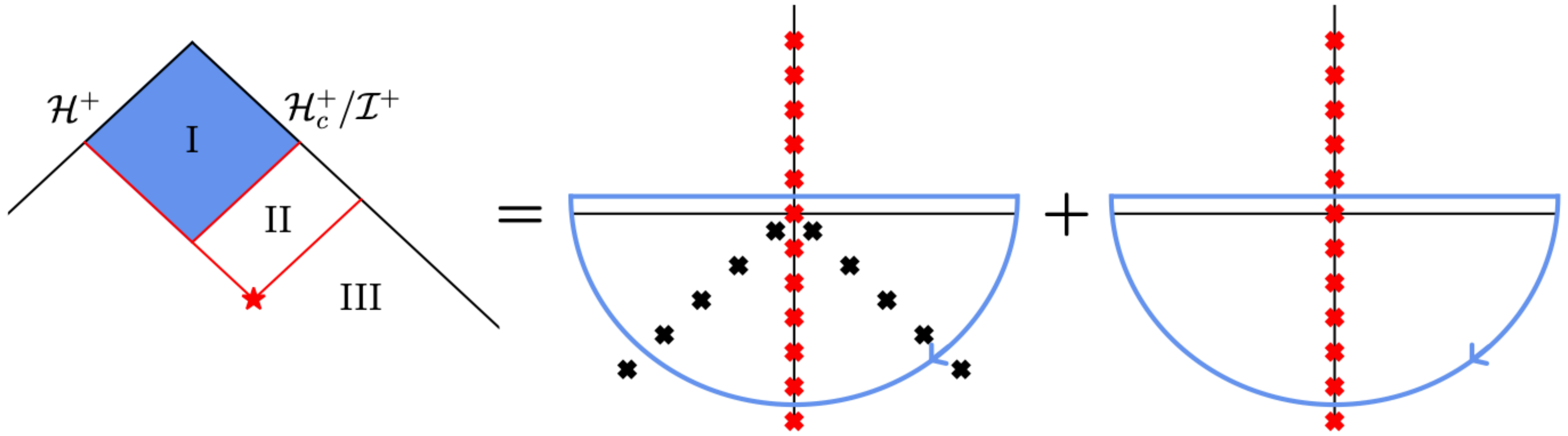
Higher modes are dominated by the **redshift "modes"** **<u>at late times</u>**

[however, see Zimmerman and Chen 2011, Oshita+ 2025]

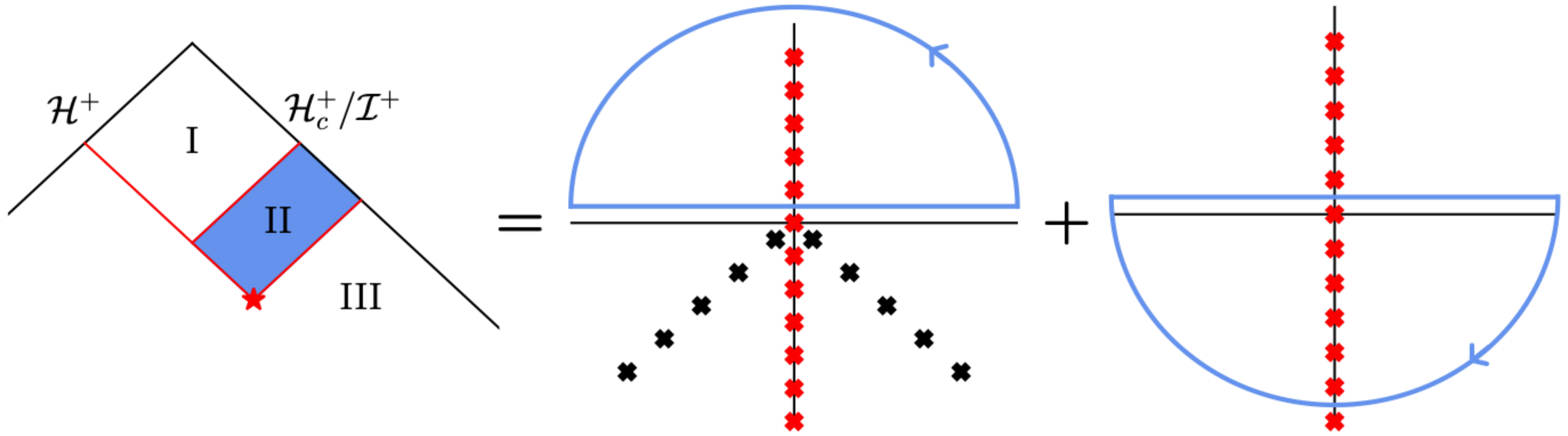
Other parts of the Green's function?

The "prompt response"?

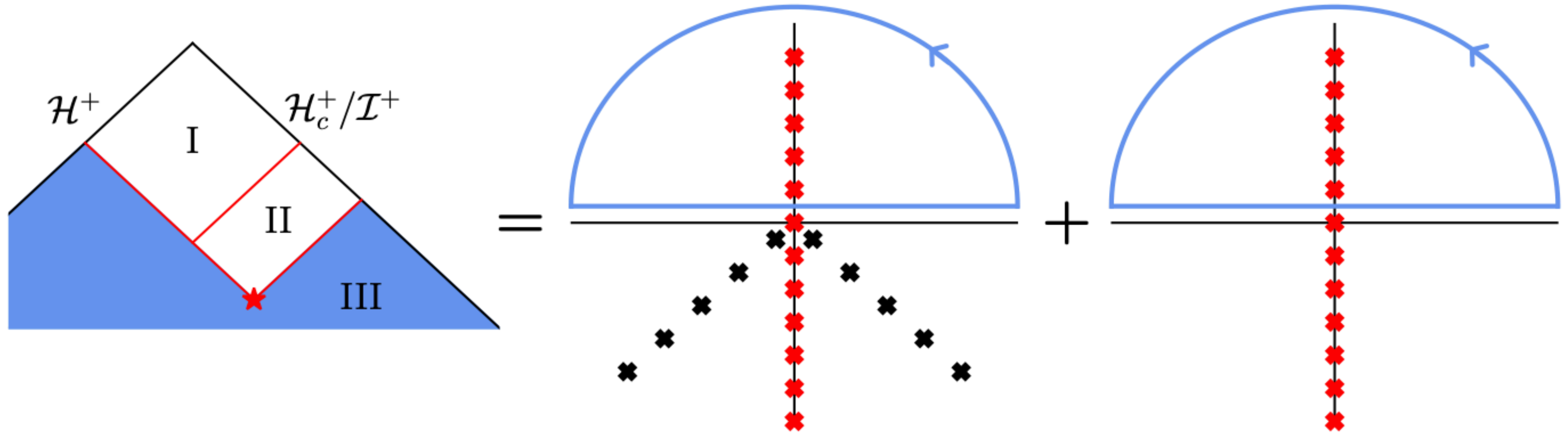
$$G(\omega; r_*, r_*') = G_+(\omega; r_*, r_*') + G_-(\omega; r_*, r_*')$$



= QNM sum
+ branch cut



= branch cut / real line



= 0 (causality/
UHP analyticity)

[Kuntz 2510.17954, Arnaudo, Carballo, Withers 2510.18956]

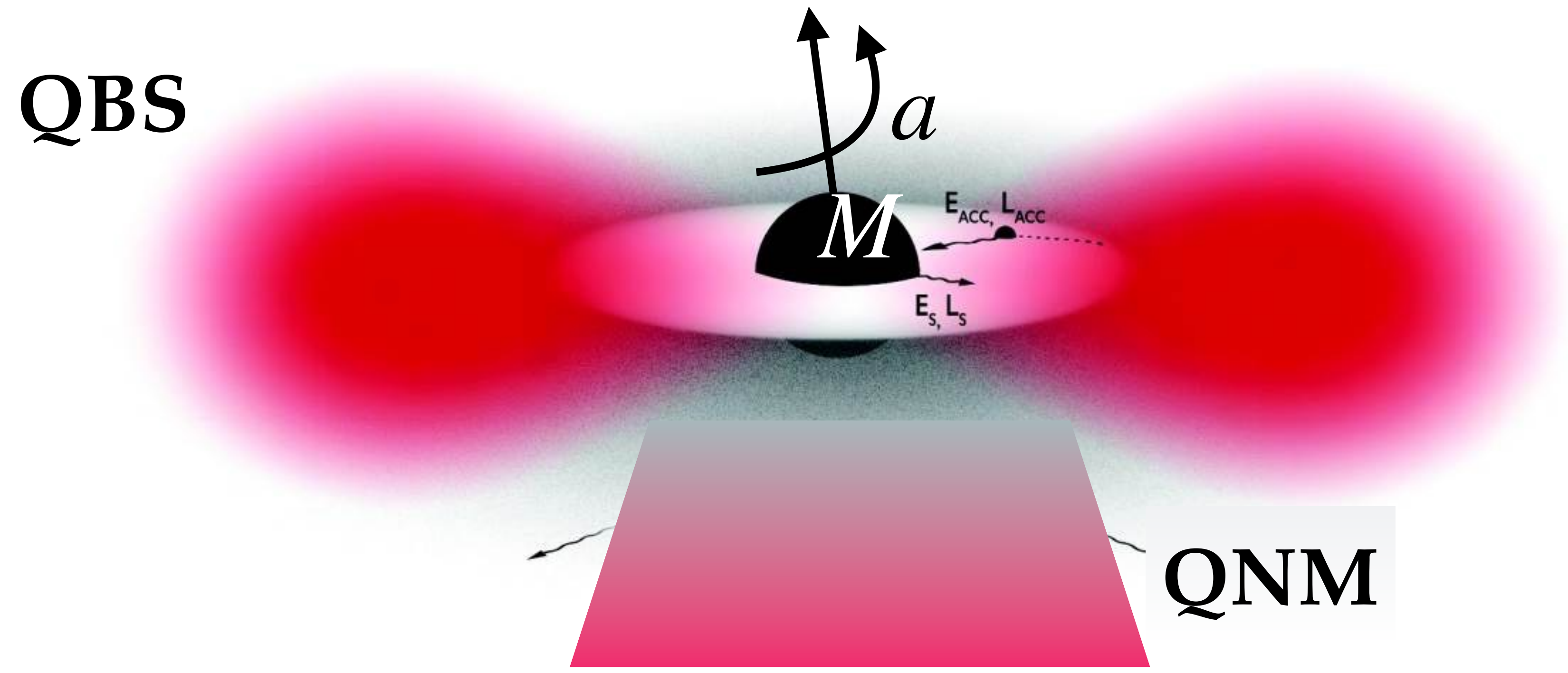
2. The mathematical properties of black hole modes

[reviews: Berti Cardoso Carullo+ (including LS) 2025; Nollert 1999; Berti Cardoso Starinets 2009]

The properties of black hole modes

	Bound states (e.g. hydrogen atom)	Quasinormal/bound
discrete spectrum	✓	✓
Hermitian	✓	✗ $\omega_I < 0$
orthogonal	✓ inner product	✓ non-positive product [Leung+ 1994, Green, LS+ 2022]
spectral stability	✓	✗
complete	✗	✗

Black hole modes of massive scalars: quasibound & quasinormal modes

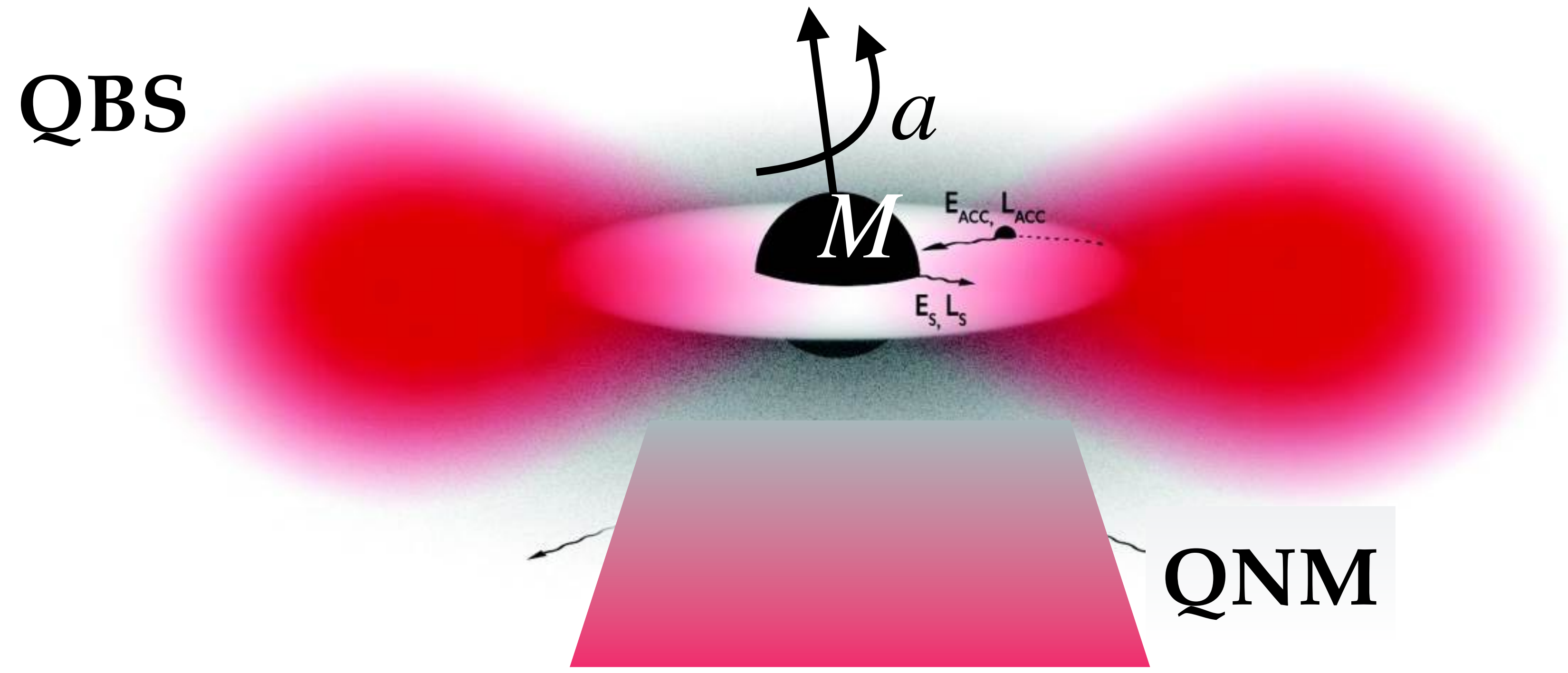


$$\Box_{g_{\mu\nu}}\Phi + \mu^2\Phi = 0 \longrightarrow \Phi(r) \sim e^{-i\omega t - ikr}$$

with $k = \pm\sqrt{\omega^2 - \mu^2}$

[figure adapted from A. Sousa's]

Other black hole modes: quasibound states



The QBS be **unstable** around spinning black holes ($\omega_I > 0$): **superradiance**

boson clouds

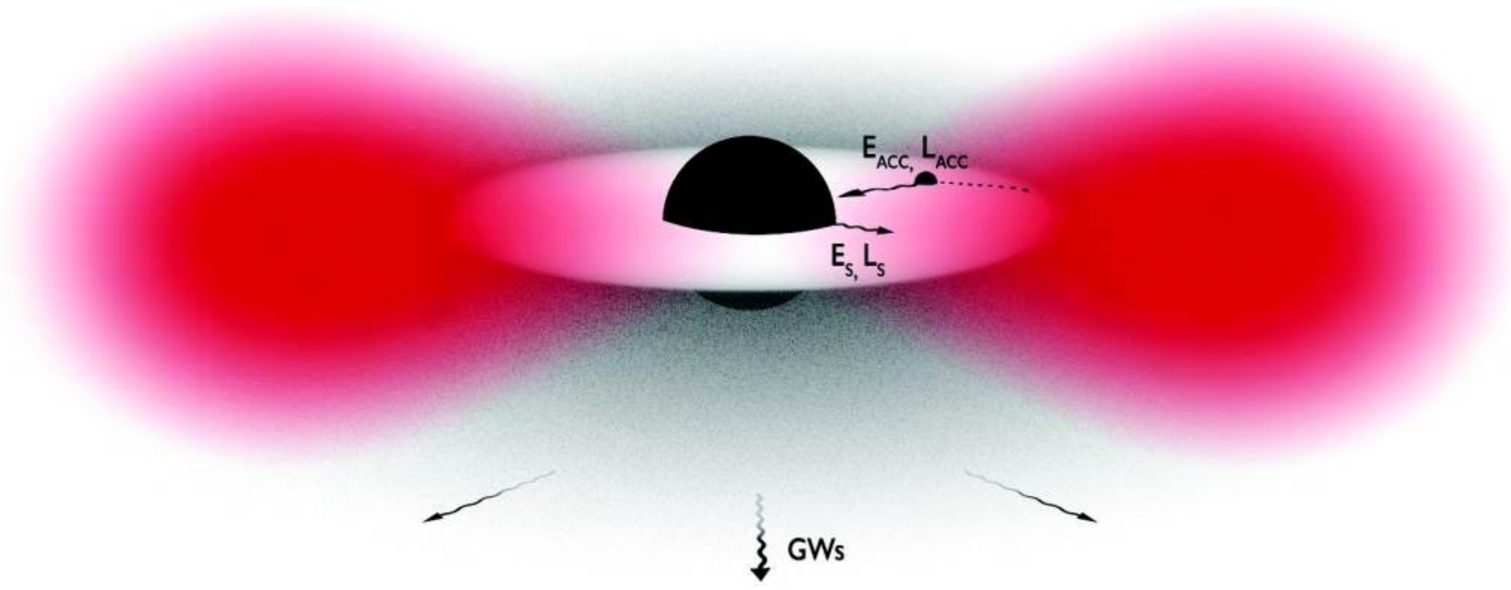
Superradiant clouds

Gravitational Atoms

Non-relativistic limit $M\mu \ll 1$: **hydrogen atom spectrum**

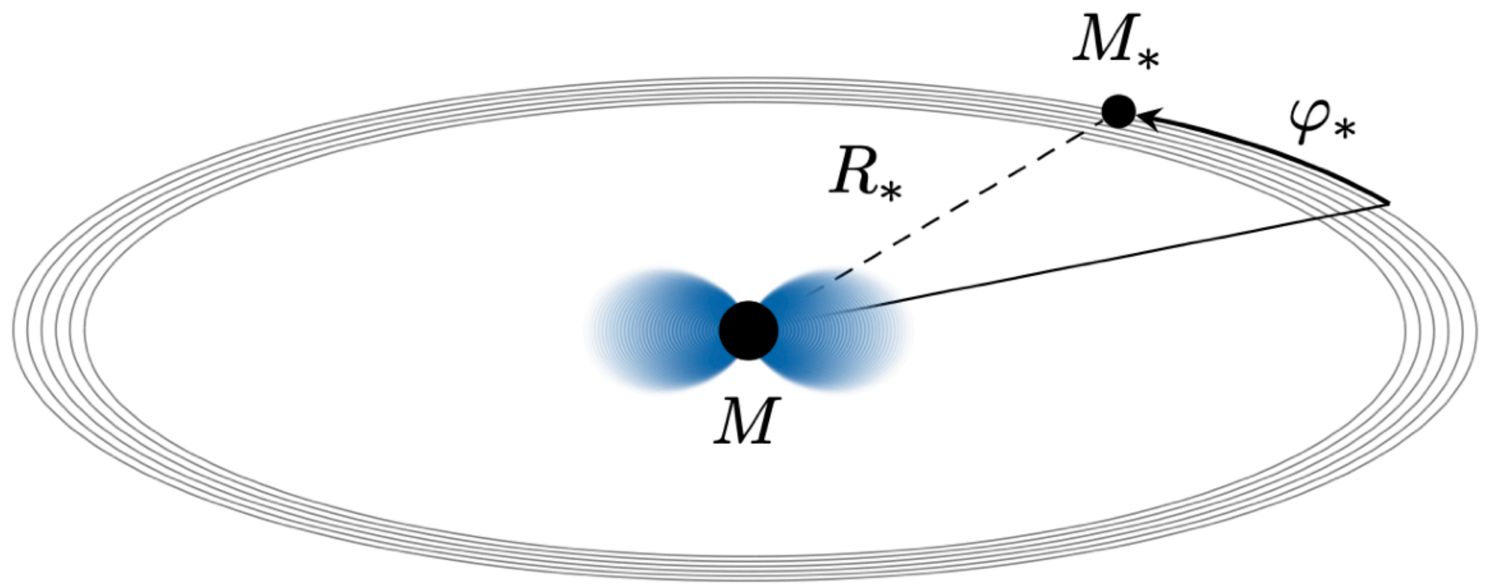
[figure adapted from A. Sousa's]

Observational signatures of other black hole modes



[Ana Sousa]

Direct gravitational wave emission



[Baumann+ 2022]

Environmental effects

Applications of mode products

2) Time-independent perturbation theory

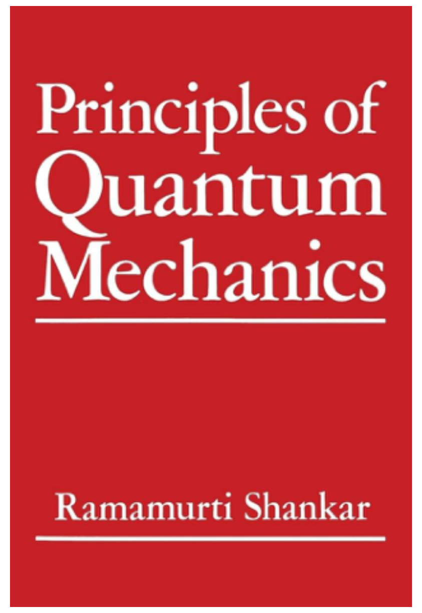
Perturbed problem:

$$\mathscr{O}^{(0)}\psi + \epsilon \mathscr{O}^{(1)}\psi = 0$$

Mode ansatz:

$$\psi = e^{-i(\omega^{(0)}+\epsilon\omega^{(1)})t}\left(\psi^{(0)} + \epsilon \psi^{(1)}\right)$$

$$\omega_{\mathbf{n}}^{(1)} = i \frac{\int \mathrm{d}\Sigma_a t^a (\Psi_2^{4/3} \mathscr{J} \psi_{\mathbf{n}}^{(0)}) \mathscr{O}^{\dagger(1)} \psi_{\mathbf{n}}^{(0)}}{\langle\langle \psi_{\mathbf{n}}^{(0)}, \psi_{\mathbf{n}}^{(0)} \rangle\rangle}$$

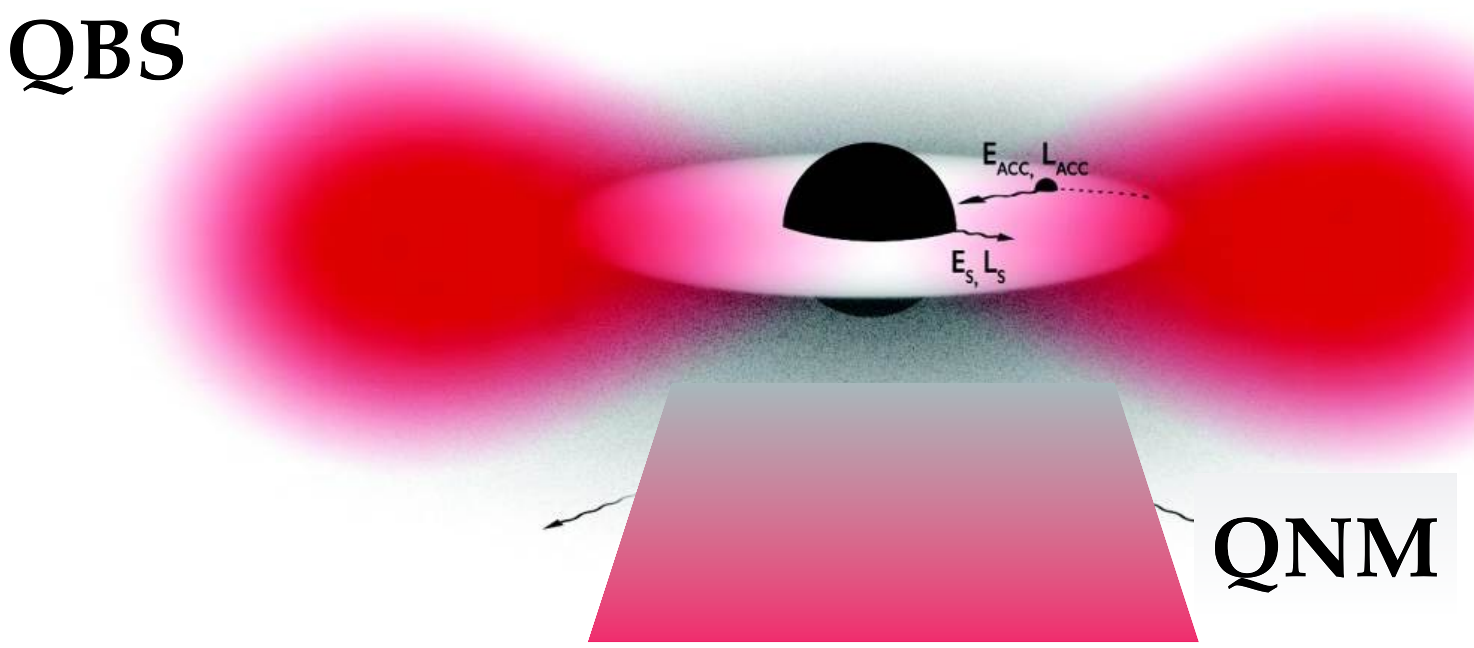


$$\sim E_n^{(1)} = \langle n^{(0)} | V | n^{(0)} \rangle$$

p.s. also time-*dependent* perturbation theory:
see Cannizzaro, LS+ 2023; Green+ 2024
("Extremal black hole weather")

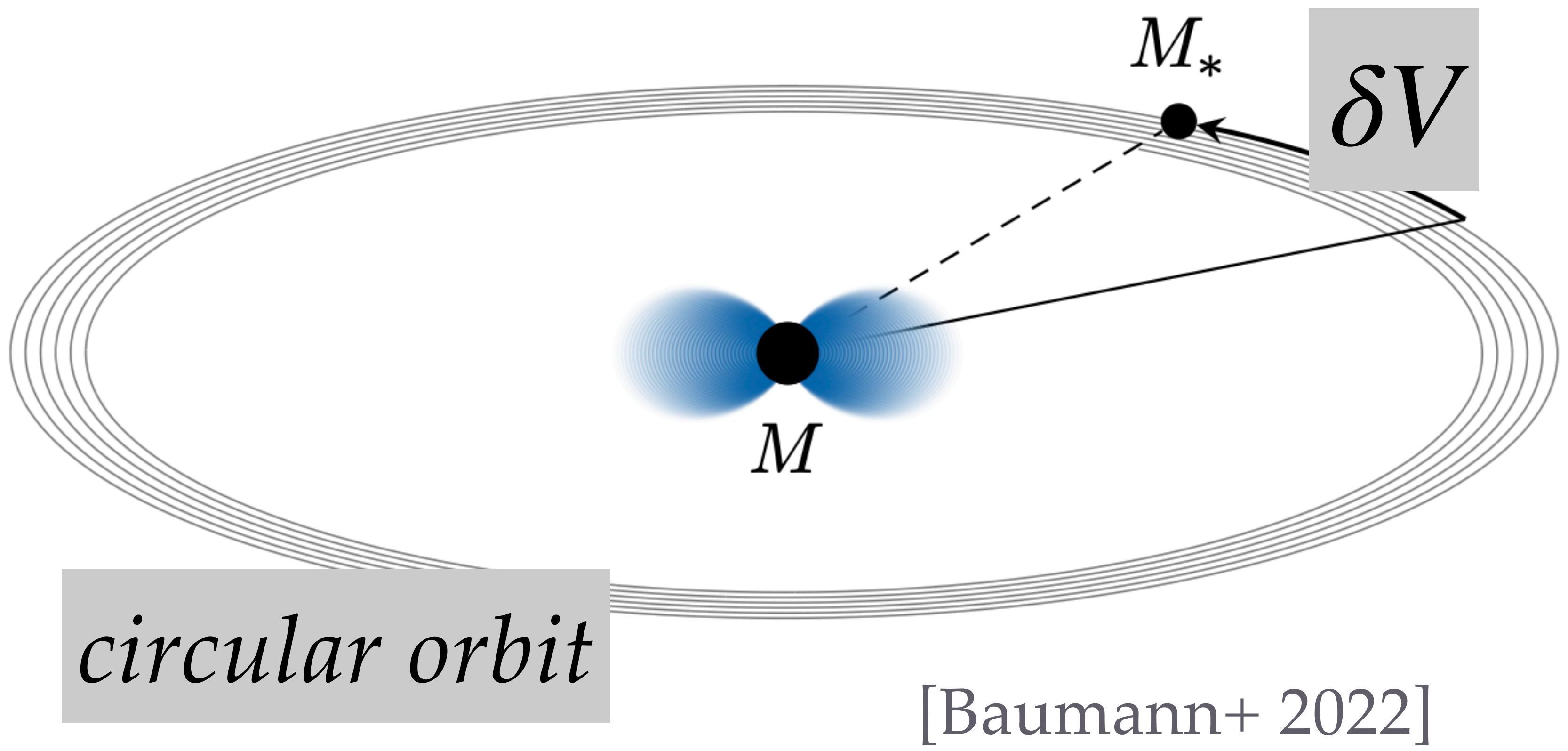
[Cannizzaro, LS, Green, Hollands (PRL) 2023; Lestingi+, to appear; see also Yang+ 2014, Mark+ 2014, Zimmerman+ 2014]

Application 2: Quasinormal-quasibound transitions



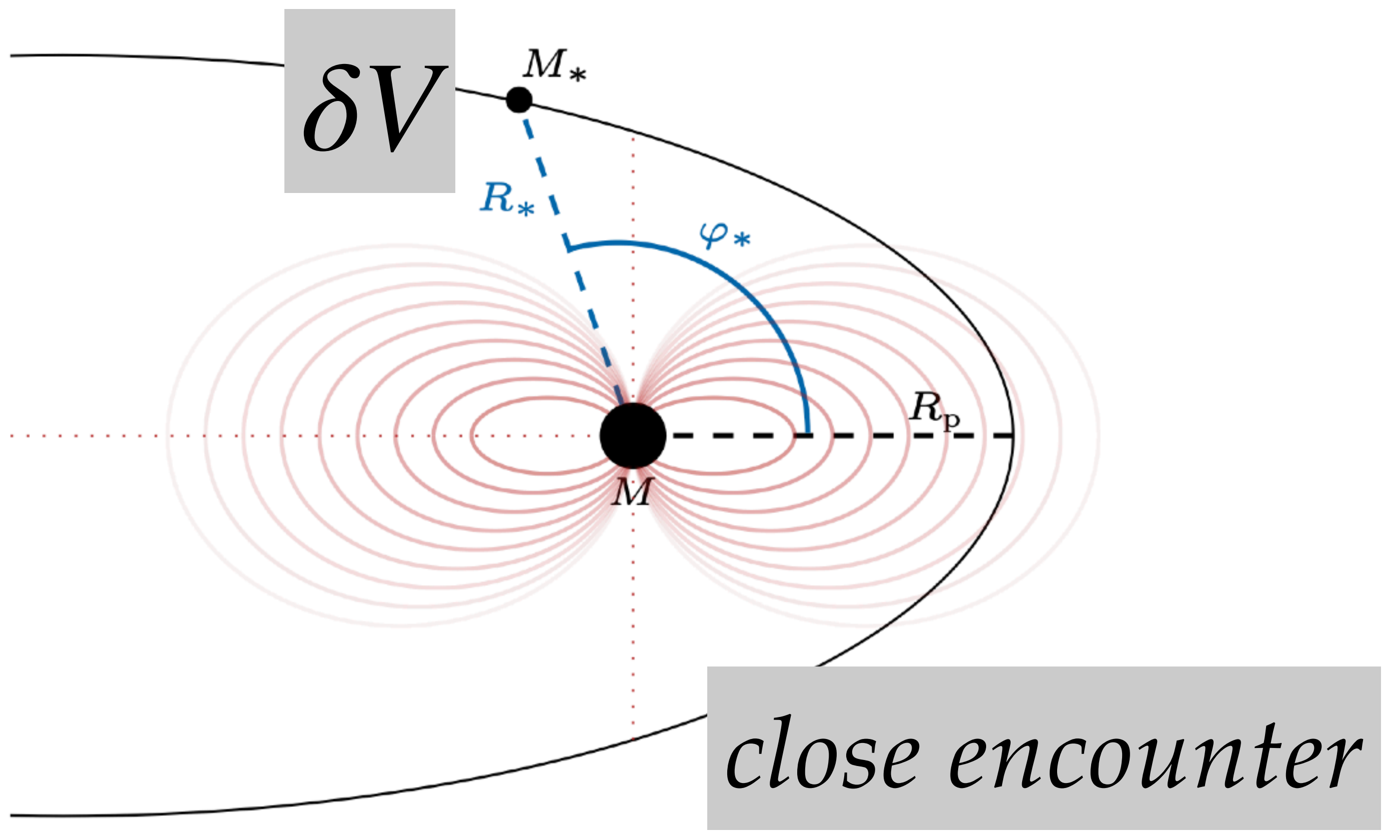
$$c_{\text{QNM}} \sim \frac{\langle\langle \Phi_{\text{QNM}}, \delta V \Phi_{QBS} \rangle\rangle}{\langle\langle \Phi_{\text{QNM}}, \Phi_{QNM} \rangle\rangle}$$

Resonant excitation: suppressed



[Baumann+ 2022]

Non-resonant excitation: enhanced

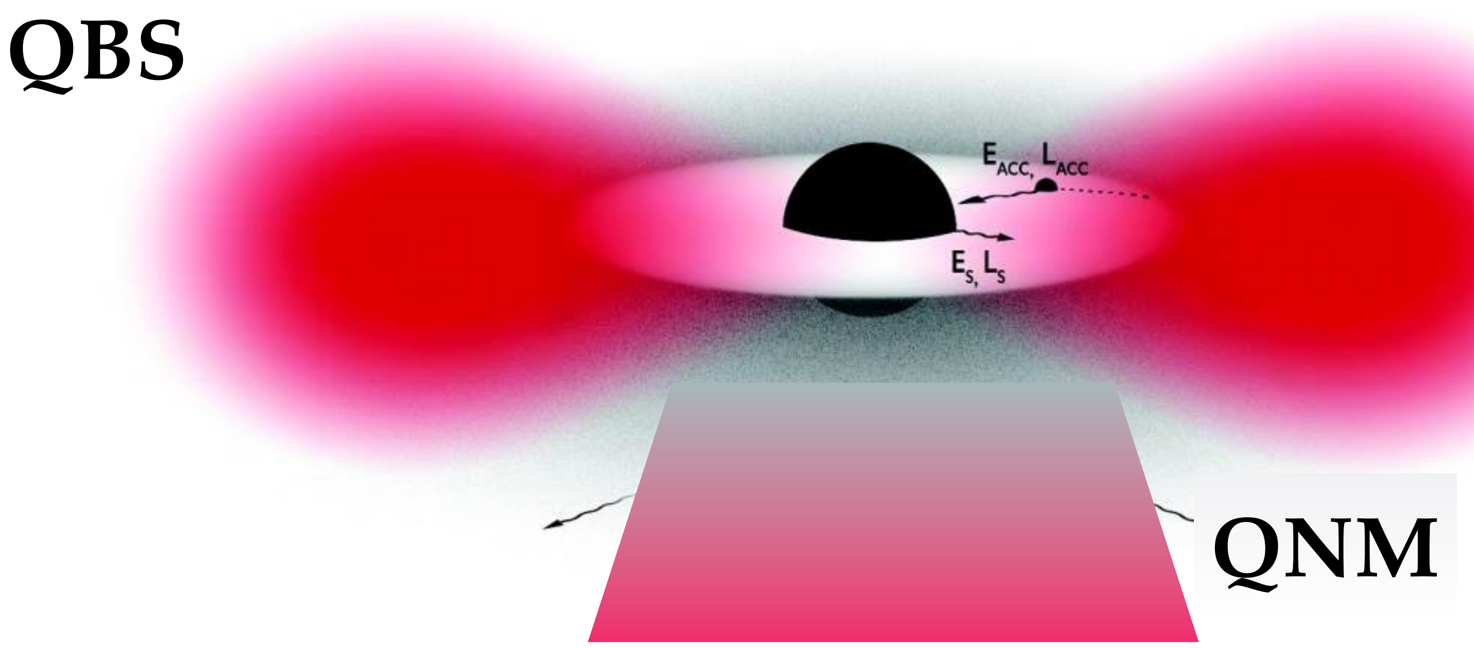


[Tomaselli, Spieksma, Bertone 2024]

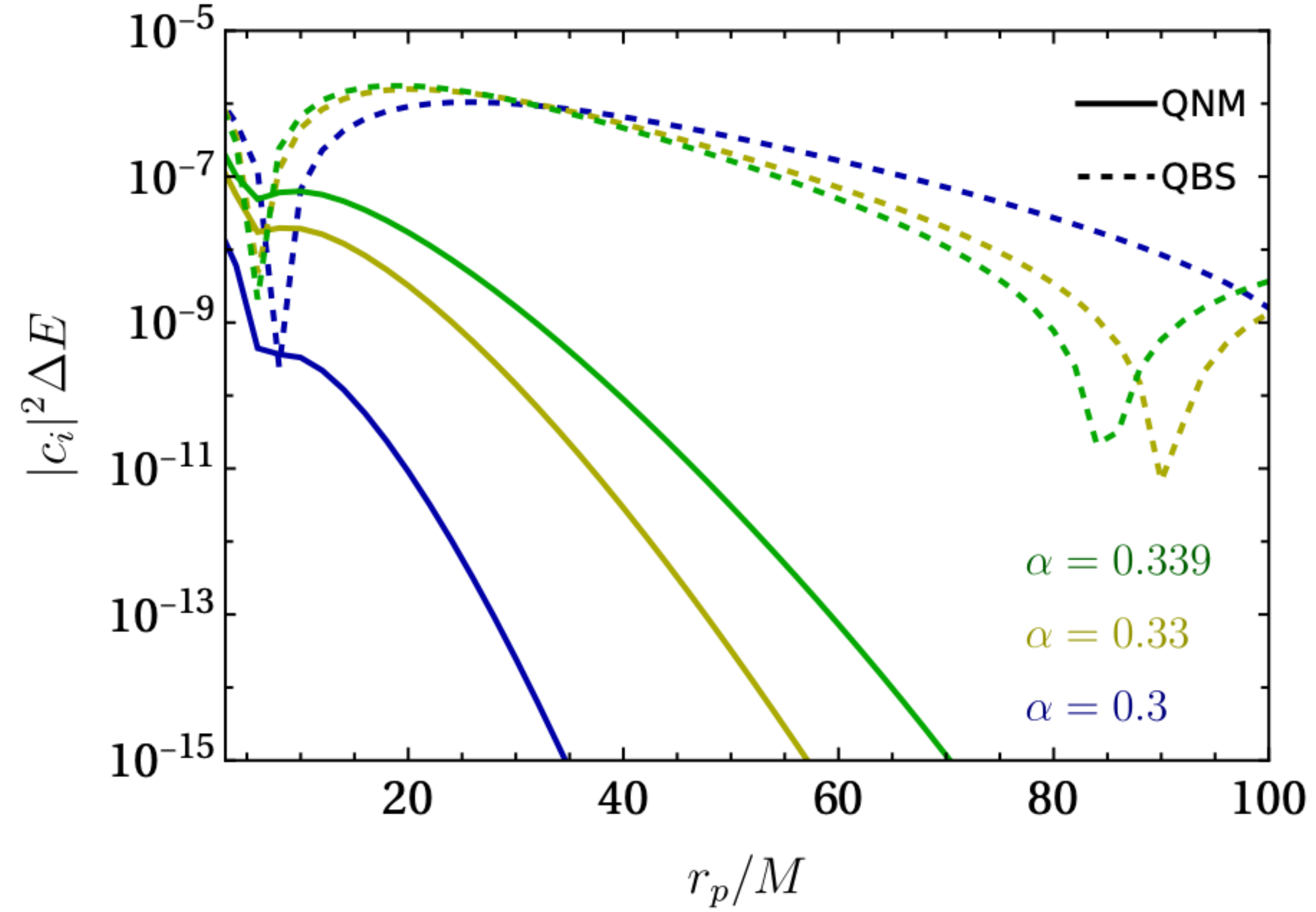
[Cannizzaro, Palleschi, LS, Green, Brito, 2512.15878]

[figure adapted from A. Sousa's]

Application 2: Quasinormal-quasibound transitions

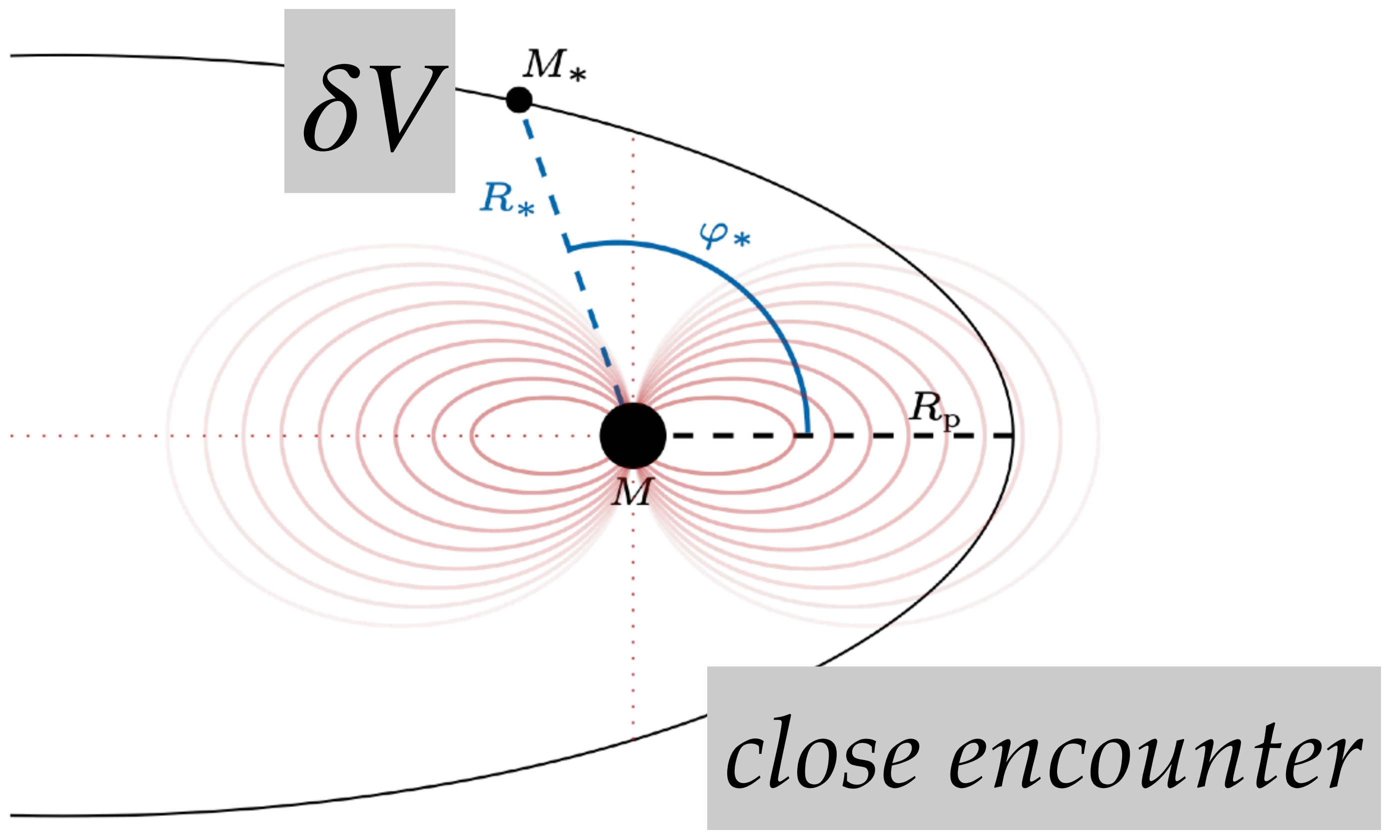


$$c_{\mathrm{QNM}} \sim \frac{\langle\langle \Phi_{\mathrm{QNM}}, \delta V \Phi_{QBS} \rangle\rangle}{\langle\langle \Phi_{\mathrm{QNM}}, \Phi_{QNM} \rangle\rangle}$$



[Cannizzaro, Palleschi, LS, Green, Brito, 2512.15878]

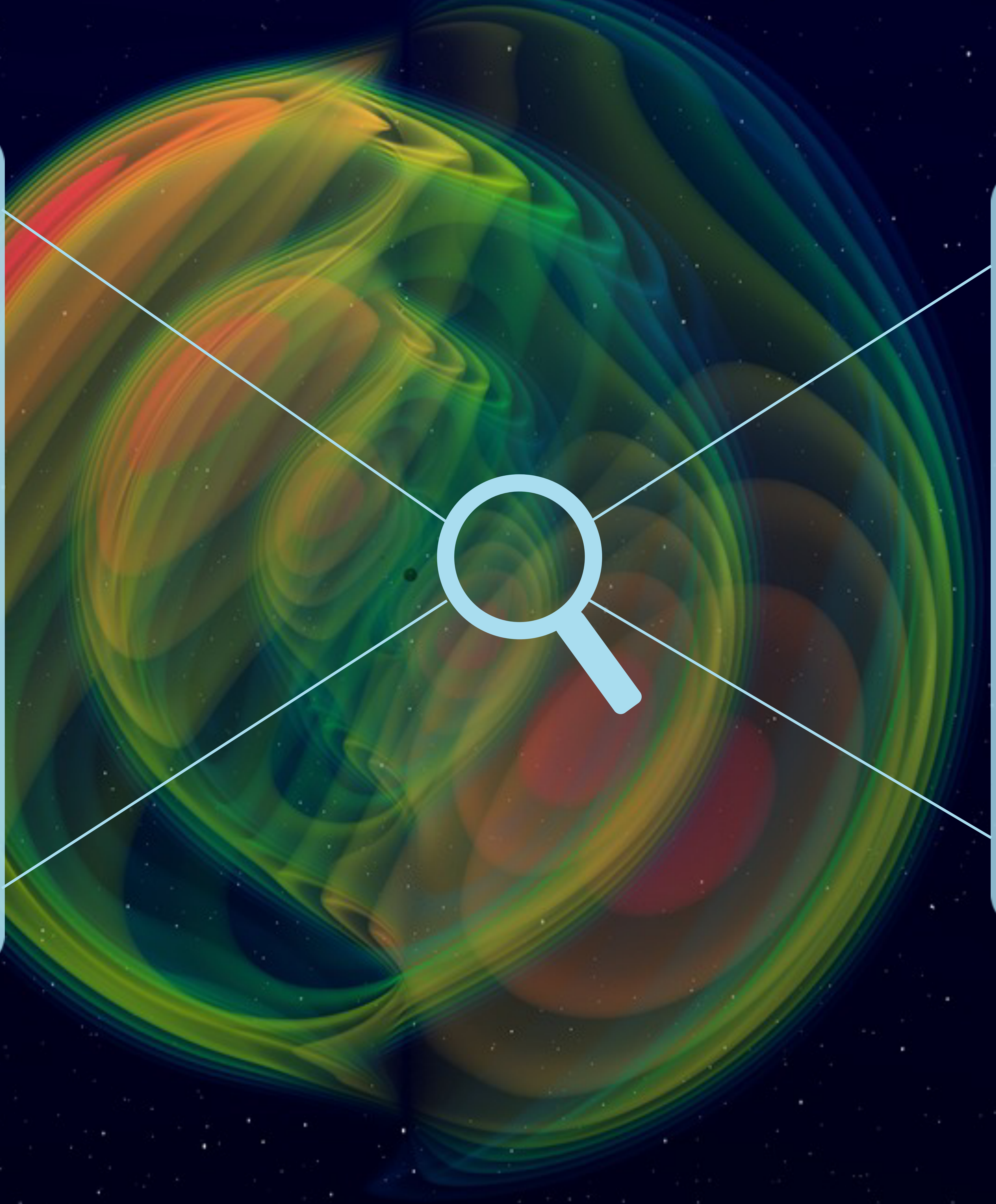
Non-resonant excitation: enhanced



[Tomaselli, Spieksma, Bertone 2024]

[figure adapted from A. Sousa's]

The black hole ringdown: outlook



Tools and properties:

- **mode product & applications**
- Other mode products
- Hyperboloidal slices
- Completeness
- Spectral instability

Dynamics:

- **dynamical QNMs**
- Other components of the Green's function
- Higher perturbative orders
- Horizon modes?

[Figures: L. Reading-Ikkanda/Simons Foundation; Fisher, Pfeiffer, Buonanno (SXS)]

To conclude…a fun fact:

George Green lived (and owned a mill) in Nottingham

Laura Sberna
Anne McLaren Fellow,
University of Nottingham