Exploring the Dynamics of Hairy Black Holes with Numerical Relativity

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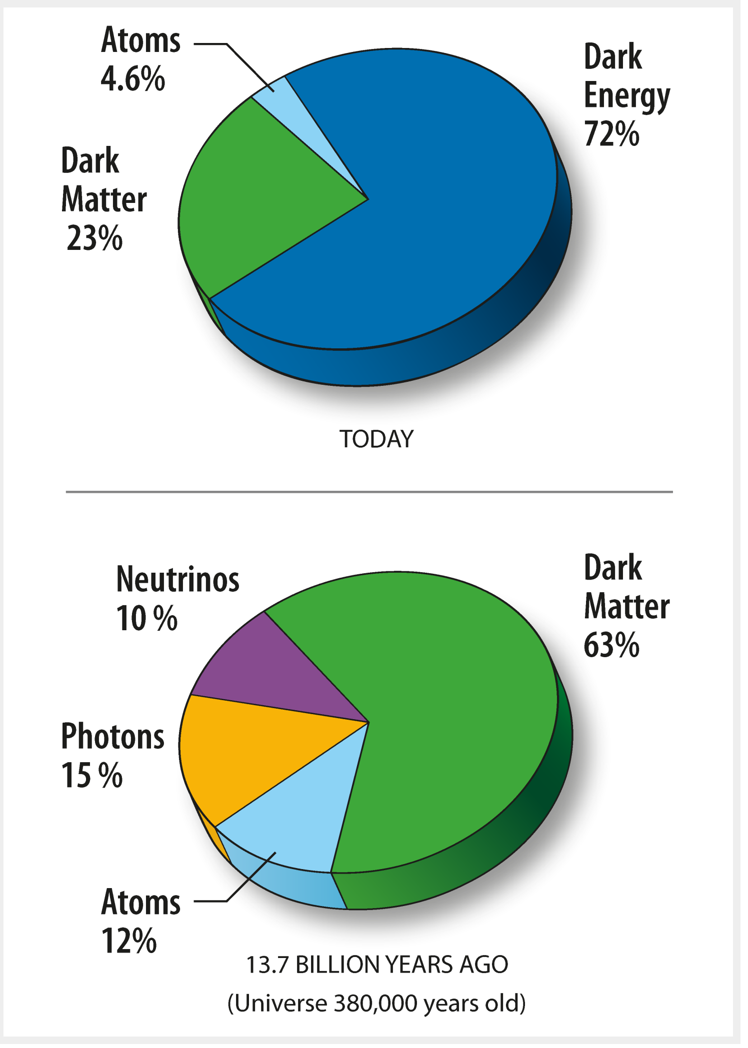
The Dynamics of Black Hole Mergers and Gravitational Wave Generation, January 13 2026

Introduction

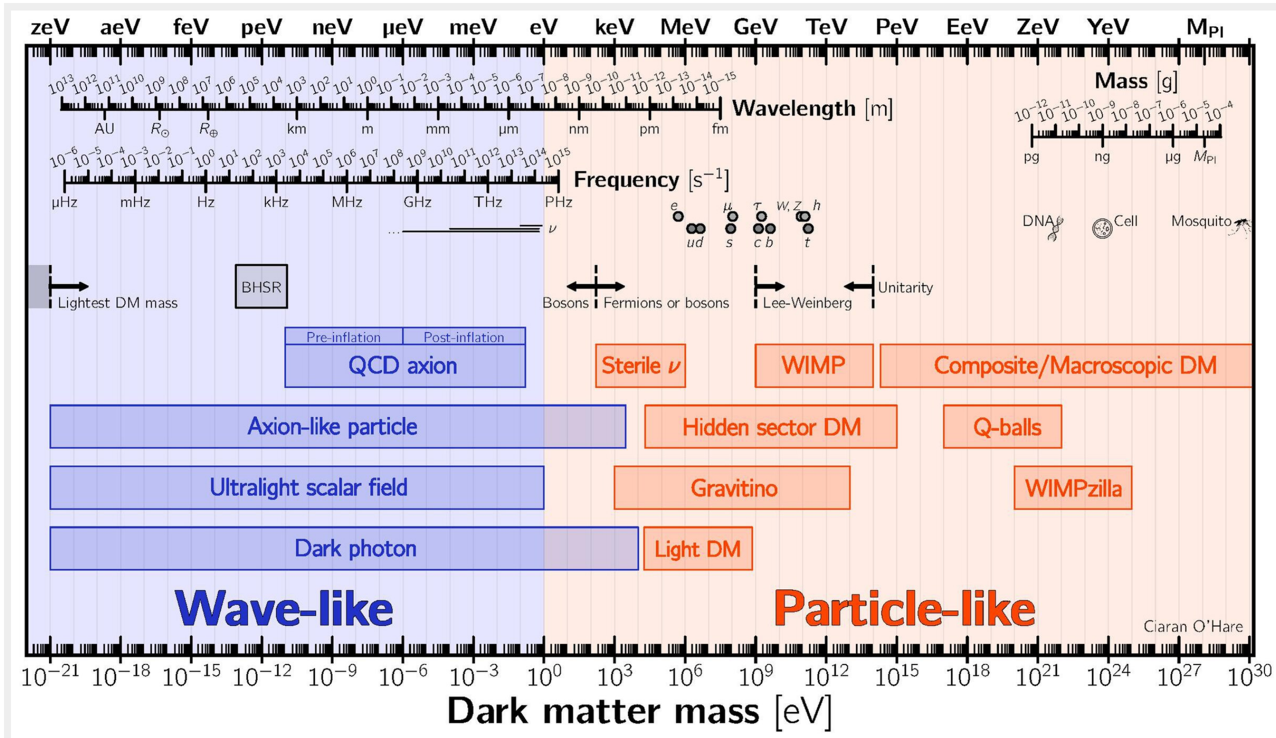
Hidden universe

- Nature of matter making up most of the universe is unknown
- Evidence for existence of dark matter of unknown nature and properties, which interacts gravitationally



(WMAP 2008)

Dark Matter



Source: Wikipedia (created by Ciaran O'Hare)

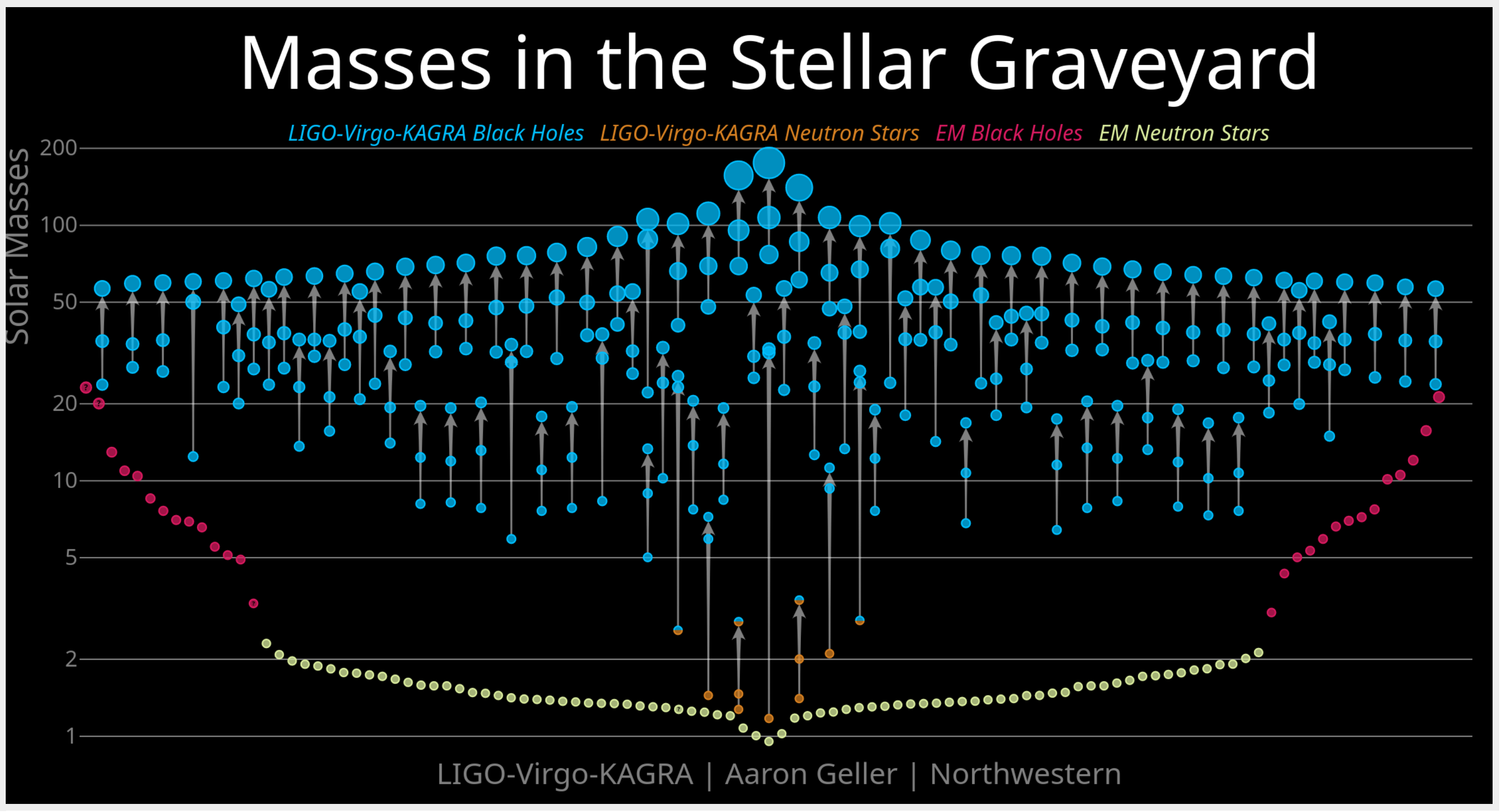
> **Note**
>
> Range of 50 orders of magnitude!

Black Holes

- In recent years, we've witnessed incredible discoveries in astrophysics and BH physics:
  - Detection of gravitational waves from BH binary mergers by the Ligo-Virgo-Kagra collaboration.
  - Direct imaging of supermassive BHs by the Event Horizon Telescope collaboration

Credit: Event Horizon Telescope Collaboration

Stellar graveyard



Credit: LIGO-Virgo-KAGRA / Aaron Geller / Northwestern

Questions

- Are all of these objects really Kerr BHs? Could there be mimickers (exotic compact objects)?
- What is the nature of dark matter and can we probe it by using compact objects?
- Beyond vacuum general relativity, BHs **can** have hair.
  - Could such hairy models be viable alternatives to the Kerr hypothesis?
- Dynamical synchronization is an ubiquitous phenomenon in physical and biological systems.
  - Could hairy BH solutions form and be sufficiently long-lived?

Model: Einstein-scalar

Black Holes with synchronized hair

- Massive complex scalar field Φ minimally coupled to Einstein's gravity

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} - \partial_\alpha \Phi^* \partial^\alpha \Phi - \mu^2 |\Phi|^2\right]$$

- Ansatz

$$ds^2 = -e^{F_0} N dt^2 + e^{2F_1}\left(\frac{dr^2}{N} + r^2 d\theta^2\right) + e^{2F_2} r^2 \sin^2\theta (d\varphi - W dt)^2$$

$$\Phi = \phi(r,\theta) e^{i(m\varphi - \omega t)}$$

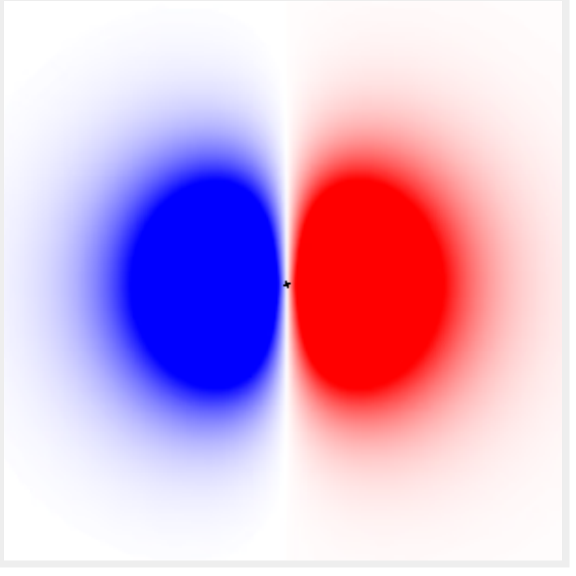
with $N = 1 - \frac{r_H}{r}$.

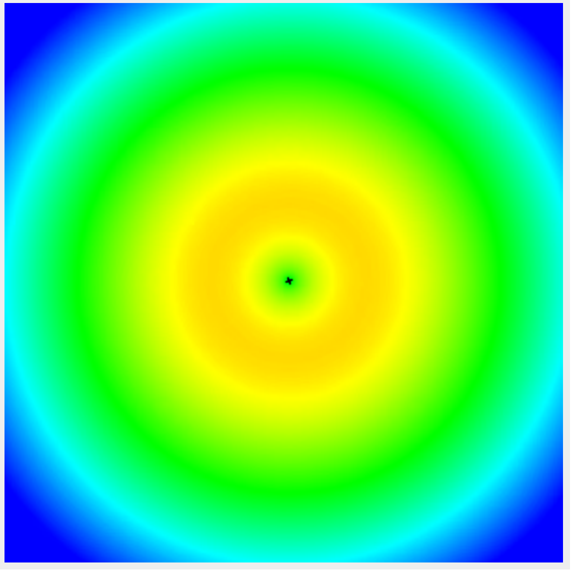
C. A. R. Herdeiro and Radu (2014), C. Herdeiro and Radu (2015)

Black Holes with synchronized hair

Note

- The harmonic dependence breaks the assumptions of no-hair theorems, but still yields a stationary and axisymmetric stress-energy tensor and spacetime!
- The solutions are regular on and outside a horizon, located at $r = r_H$, have equatorial plane symmetry and are asymptotically flat.
- The matter field has to obey the synchronization condition $\omega = m\Omega_H$ where Ω_H is the horizon angular velocity (onset of superradiance).





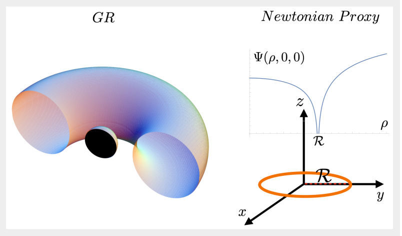
$\Re(\Phi)$ ρ

Dynamical stability

Main question

- Are these solutions dynamically stable? If not, what is the end state?
- The answer may depend on the region of the parameter space

A toy model

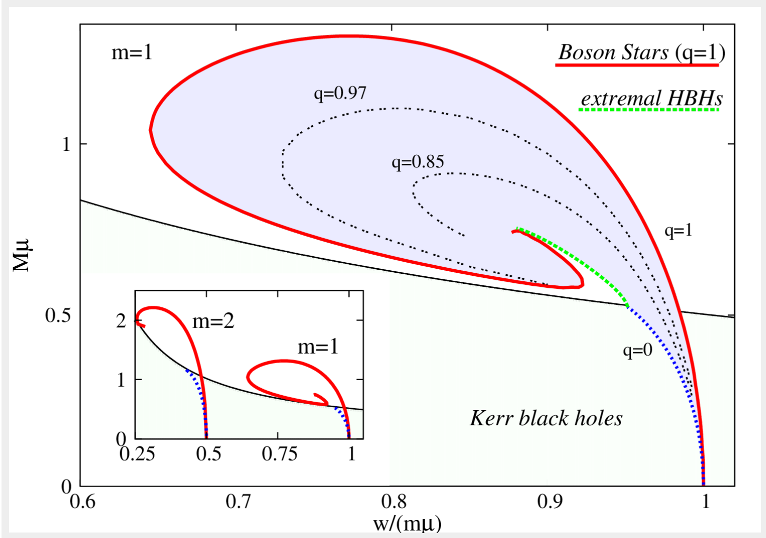


- Newtonian proxy: consider a thin ring of radius $\mathcal{R}$ (we recall that spinning boson stars are toroidal); its gravitational potential (constant mass density set to unity) reads

$$\Psi(\rho, z, \varphi) = -\mathcal{R} \int_0^{2\pi} \frac{d\tilde{\varphi}}{\sqrt{\rho^2 - 2\rho\mathcal{R}\cos(\varphi - \tilde{\varphi}) + \mathcal{R}^2 + z^2}}$$

- Near the origin: $\Psi(\rho, 0, 0) \sim -2\pi - \frac{\pi\rho^2}{2\mathcal{R}^2}$.
  - Thus, the origin is an unstable equilibrium for radial displacements (not in z). Under a small (radial) perturbation, a particle at the origin starts falling towards the ring (away from the center).

Initial data



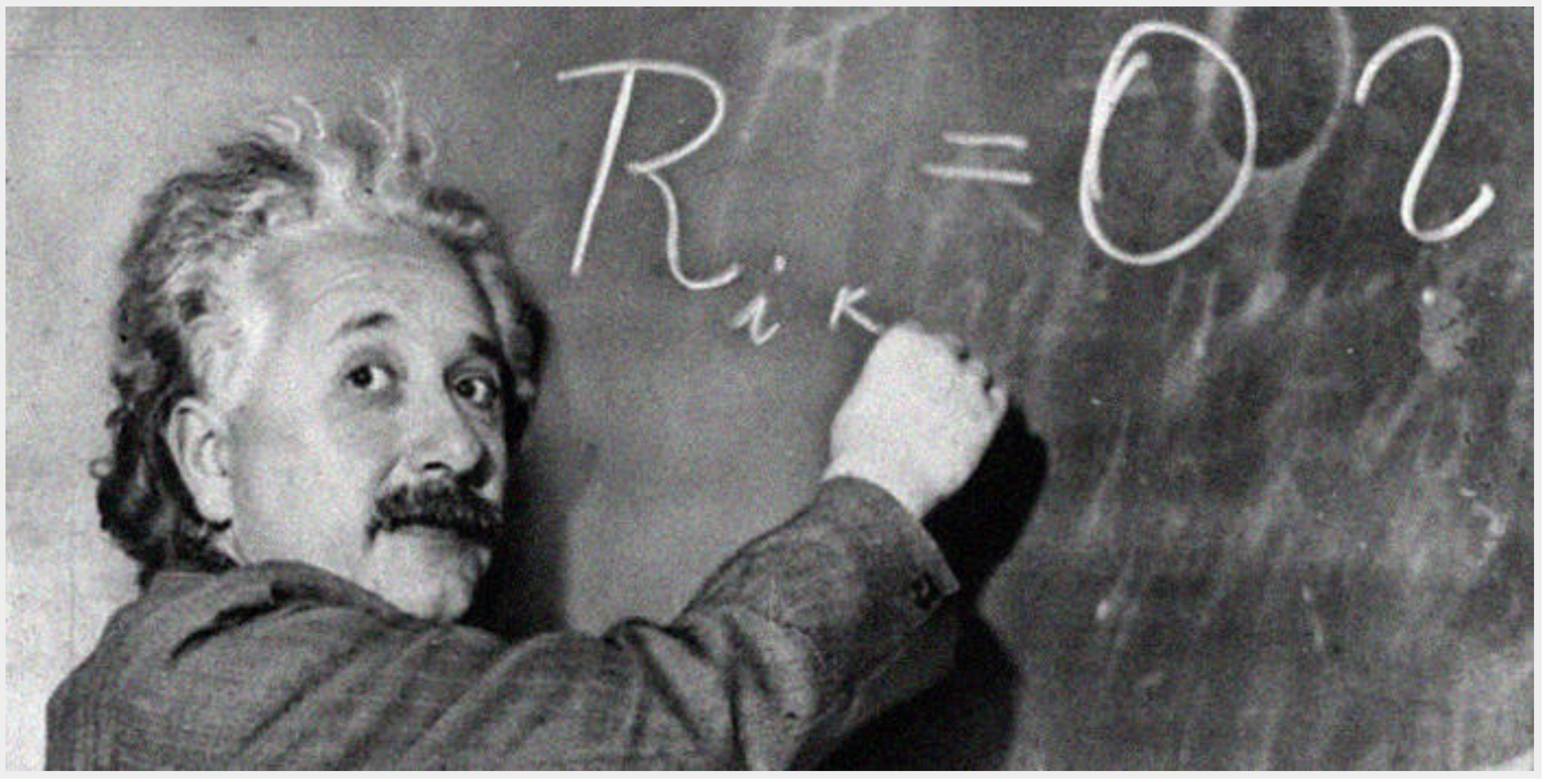
From C. A. R. Herdeiro and Radu (2014)

#	ω	r_H	M	J	M_H/M	J_H/J
1	0.998200	0.179026	0.127692	0.0346921	0.754980	0.0998357
2	0.90	0.2	1.01049	0.91193	0.11754	0.01217

1. In the region where they coexist with Kerr
2. Close to the Boson Star limit (very hairy)

Numerical evolutions

Einstein's equations



$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} \sum_{\delta=t,x^1,\ldots,x^{D-1}} g^{\alpha\delta} \left(\partial_\gamma g_{\delta\beta} + \partial_\beta g_{\delta\gamma} - \partial_\delta g_{\beta\gamma}\right)$$

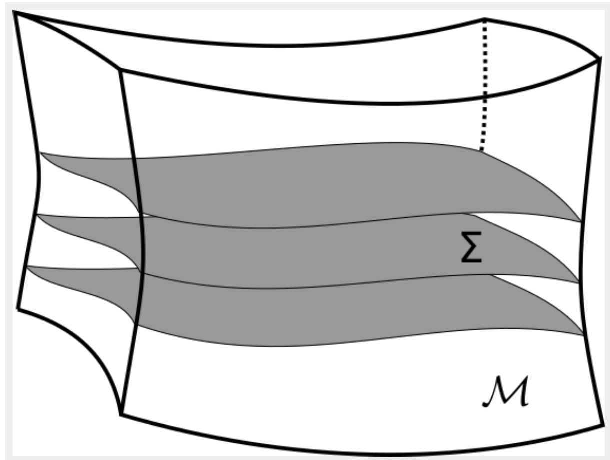
$$\begin{aligned} 8\pi T_{\alpha\beta} &= \sum_\delta \left[\partial_\delta \Gamma^{\delta}_{\alpha\beta} - \partial_\alpha \Gamma^{\delta}_{\delta\beta} + \sum_\gamma (\Gamma^{\delta}_{\alpha\beta}\Gamma^{\gamma}_{\delta\gamma} - \Gamma^{\delta}_{\gamma\beta}\Gamma^{\gamma}_{\delta\alpha})\right] \\ &\quad - \frac{1}{2} g_{\alpha\beta} \sum_{\delta,\gamma} \left\{ g^{\delta\gamma} \sum_\mu \left[\partial_\mu \Gamma^{\mu}_{\delta\gamma} - \partial_\delta \Gamma^{\mu}_{\mu\delta} + \sum_\nu (\Gamma^{\mu}_{\delta\gamma}\Gamma^{\nu}_{\mu\nu} - \Gamma^{\mu}_{\nu\gamma}\Gamma^{\nu}_{\mu\delta})\right]\right\} \end{aligned}$$

- set of 10 2nd order non-linear (coupled) partial differential equations on the metric tensor $g_{\alpha\beta}$

Before numerical evolution…

1. Write Einstein's equations as a well-posed ***Initial Boundary Value Problem***:
   - solution's behaviour depends continuously with the initial data;
   - numerically suitable gauge conditions.
2. **Discretize** resulting PDEs
3. Specify (constraint preserving) physically correct, **boundary conditions**
4. Find a way to deal with singularities

3+1 decomposition



We write the metric as

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij} \left(dx^i + \beta^i dt\right) \left(dx^j + \beta^j dt\right)$$

To learn more

- “3+1 formalism and bases of numerical relativity” (Gourgoulhon 2007)
- “Introduction to 3+1 Numerical Relativity” (Alcubierre 2008)

ADM-York equations

Evolution equations

$$
\begin{aligned}
(\partial_t - \mathcal{L}_\beta)\,\gamma_{ij} &= -2\alpha K_{ij} \\
(\partial_t - \mathcal{L}_\beta)\,K_{ij} &= -\nabla_i \nabla_j \alpha + \alpha\Big[R_{ij} + K K_{ij} - 2K_{ik}K^k{}_j \\
&\qquad + 4\pi\,((S-E)\gamma_{ij} - 2S_{ij})\Big]
\end{aligned}
$$

Constraints

$$
\begin{aligned}
R + K^2 - K_{ij}K^{ij} &= 16\pi E \\
\nabla_j\left(K^{ij} - \gamma^{ij}K\right) &= 8\pi p^i
\end{aligned}
$$

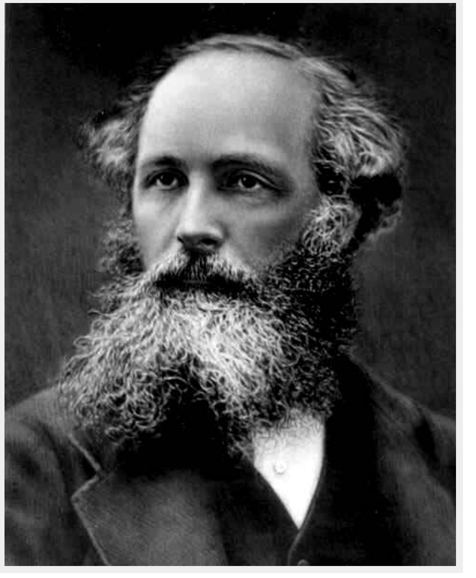
Electromagnetic analogy

Evolution equations

$$-\partial_t \vec{E} + \nabla \times \vec{H} = 4\pi \vec{j}$$
$$-\partial_t \vec{H} + \nabla \times \vec{E} = 0$$

Constraints

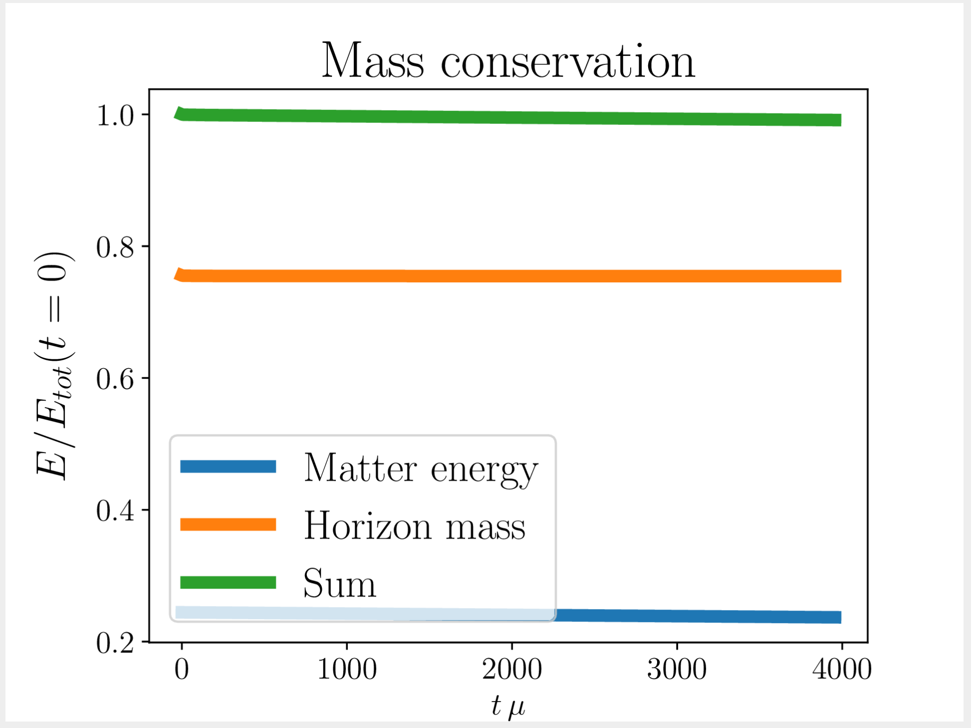
$$\nabla \cdot \vec{E} = 4\pi\rho$$
$$\nabla \cdot \vec{H} = 0$$

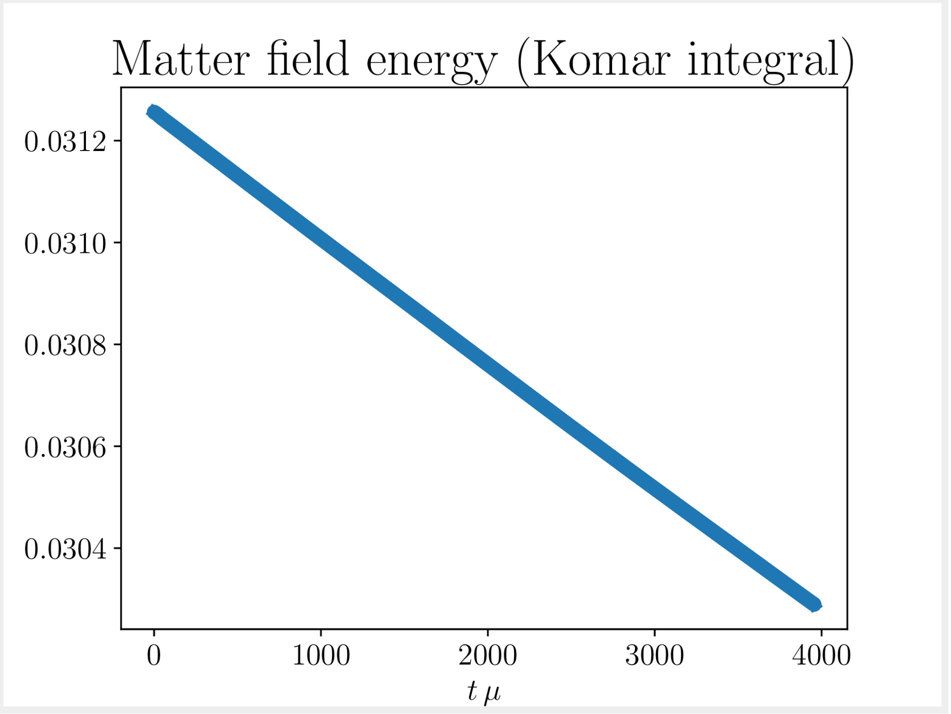


Results

System 1 (not very hairy)

- Hairy BH appears stable on long timescales ($t\mu \sim 4000$)
- Similar results for another Hairy BH in the same region of the parameter space with $M_H/M \approx 0.5$.

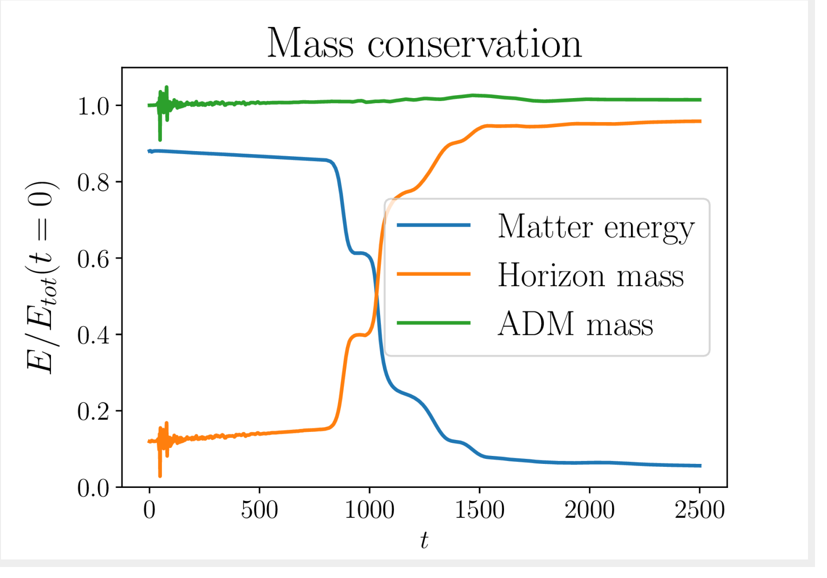




Nicoules et al. (2025)

System 2 (very hairy)

- **Driving phenomenon:** The BH starts to move, in a planar outspiral movement, in the direction of the angular momentum
- When it gets to regions of higher scalar field density, **it perturbs its structure and accretes substantially**
- Similar behavior has been observed for another physical configuration in this region of the parameter space



System 2: scalar field

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Real part of scalar field $\Re(\Phi)$ (Nicoules et al. 2025)

System 2: scalar field density

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Scalar field density ρ (Nicoules et al. 2025)

Model: Einstein-Maxwell-scalar

Model

Einstein-Maxwell-scalar model with the following action (C. A. R. Herdeiro and Radu 2020)

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - (D_\mu\Psi)^* D^\mu\Psi - U(|\Psi|)\right]$$

where $F_{\mu\nu} \equiv \nabla_\mu A_\nu - \nabla_\nu A_\mu$ is the Maxwell tensor, A_μ the 4-potential to which the *gauged* scalar field Ψ couples (minimally) via $D_\mu \equiv \partial_\mu + iqA_\mu$.

- A solution of this model is the Einstein-Maxwell Reissner-Nordström BH and $\Psi = 0$
  - prone to superradiant *scattering* by Ψ when scalar field modes of frequency ω obey $\omega < qU(r_H)$, with $U(r_H)$ the horizon electrostatic potential.

Stationary gauged boson stars

- Free scalar fields: massive scalar potential without self-interaction

$$V(|\Psi|) = \mu^2 |\Psi|^2 \qquad \Psi = \phi(r,\theta) e^{i(\omega t - m\varphi)}$$

m is an integer parameter and ω is the angular velocity of the scalar field

- Line element

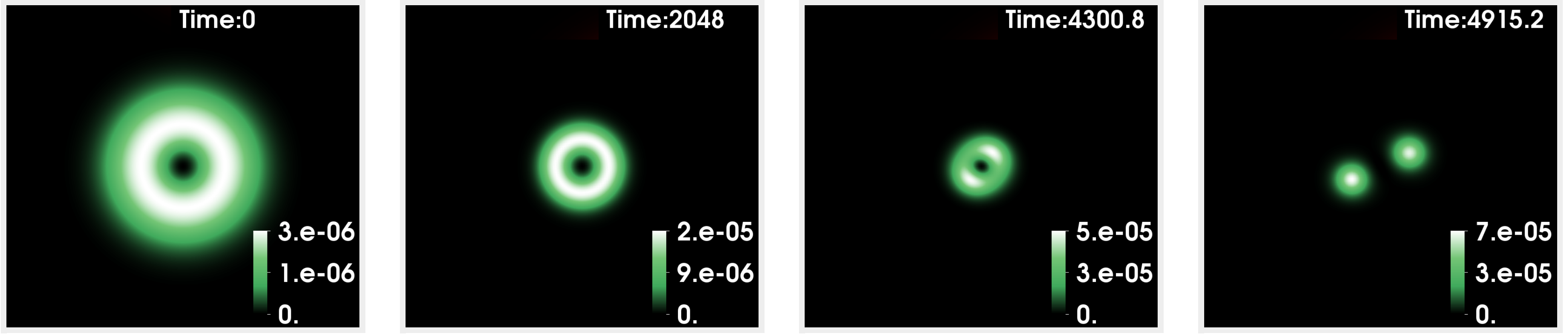
$$g_{\mu\nu} dx^\mu dx^\nu = -e^{2F_0(r,\theta)} dt^2 + e^{2F_1(r,\theta)} \left(dr^2 + r^2 d\theta^2\right) + e^{2F_2(r,\theta)} r^2 \sin^2\theta (d\varphi - W(r,\theta) dt)^2$$

Static charged boson stars

- Achieved when rotation is disabled ($m = 0, \phi = \phi(r)$)
- Equilibrium solutions form sequences characterized by the coupling constant q, which has an upper bound arising from the limit set by Coulomb repulsion
- Simulations in spherical symmetry showed that configurations with densities below the critical value remain stable (López and Alcubierre 2023)

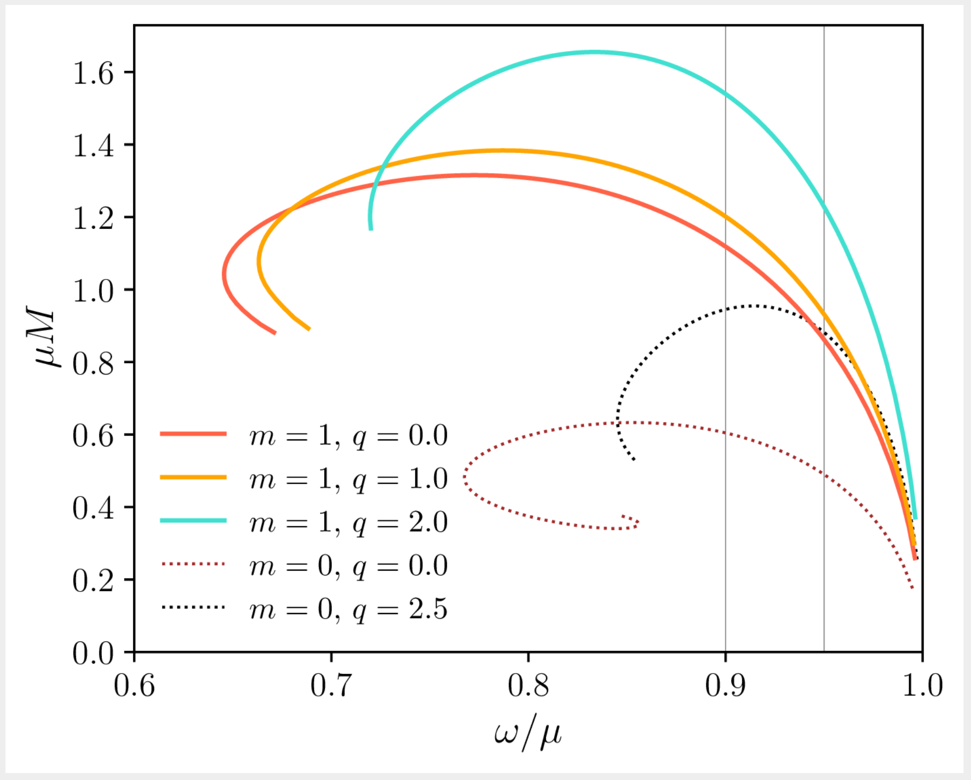
Rotating neutral boson stars

- Rotating neutral ($q = 0$) boson stars are unstable (Sanchis-Gual et al. 2019)
  - Eventually, the system develops a non-axisymmetric instability.

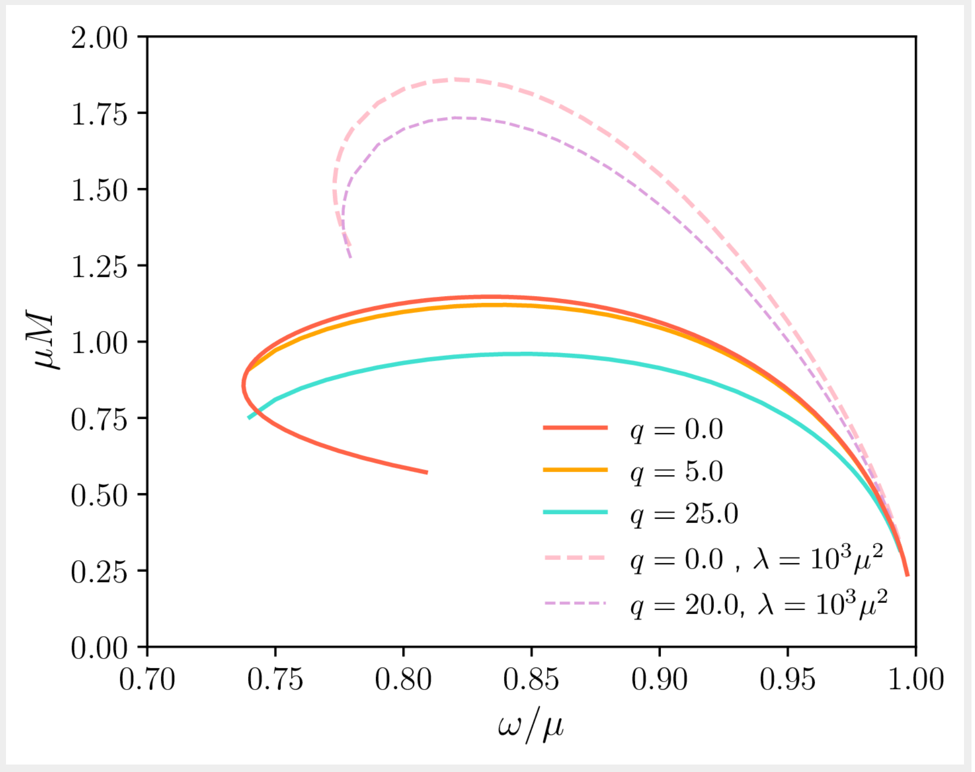


Time evolution of an equatorial cut of energy density of the scalar field

Charged and rotating boson stars



Total mass *vs* scalar field frequency ω for spherical boson stars and rotating ($m = 1$) stars for different values of q.



Mass-to-the-Noether-charge ratio for the $m = 1$ configurations and three different values of the constant q.

Jaramillo et al. (2025)

Black holes with resonant hair

- *Spherical* (and static) solutions found numerically (C. A. R. Herdeiro and Radu 2020)

$$ds^2 = -N(r)\sigma^2(r)dt^2 + \frac{dr^2}{N(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$
$$\Psi = \psi(r)e^{-i\omega t}, \qquad A = V(r)dt$$

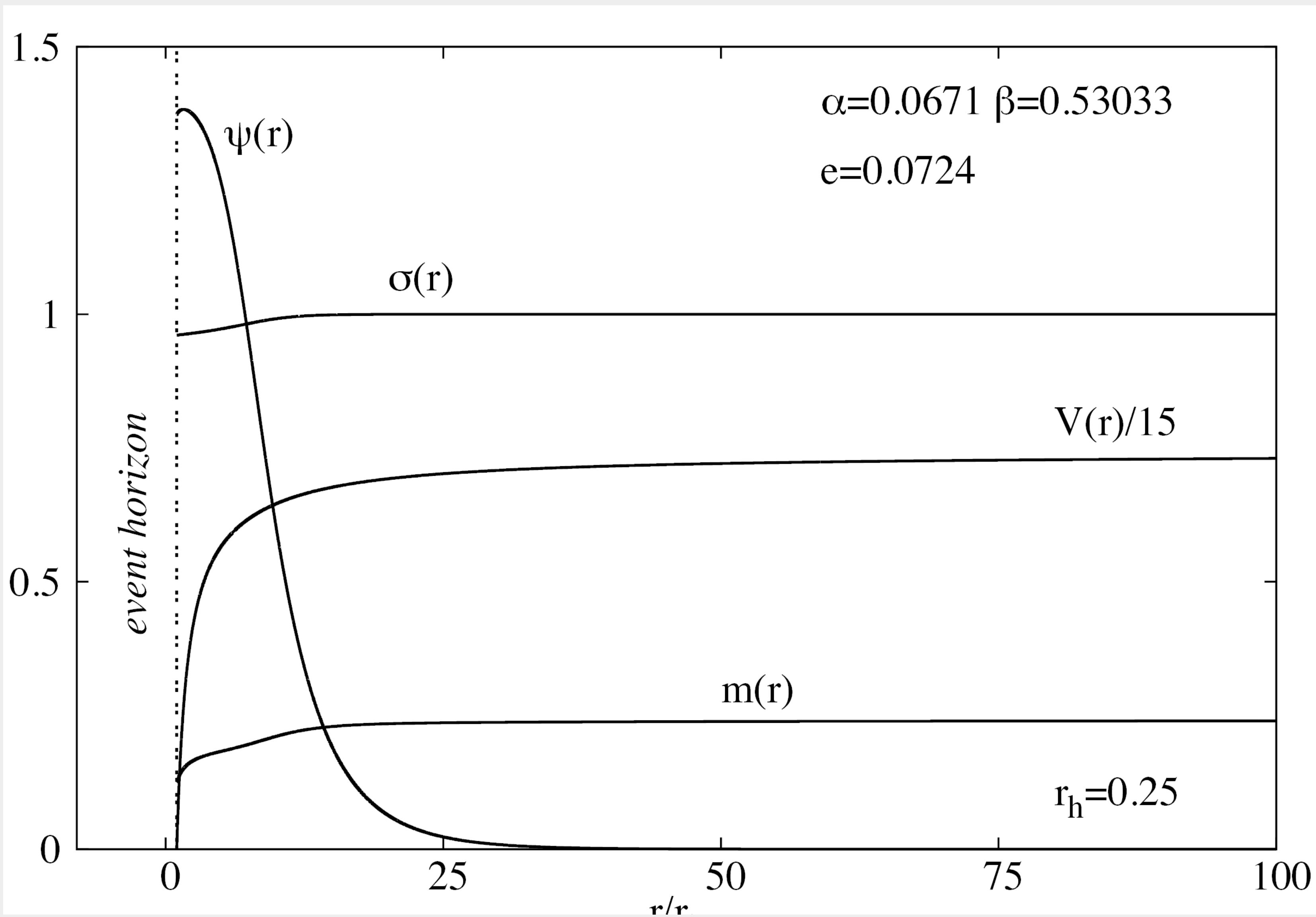
  where $N(r) \equiv 1 - \frac{2m(r)}{r}$

- Regularity at the horizon requires either $\Psi(r_h) = 0$ (trivial) or

$$\omega = qV(r_h)$$

  (*resonance* condition)

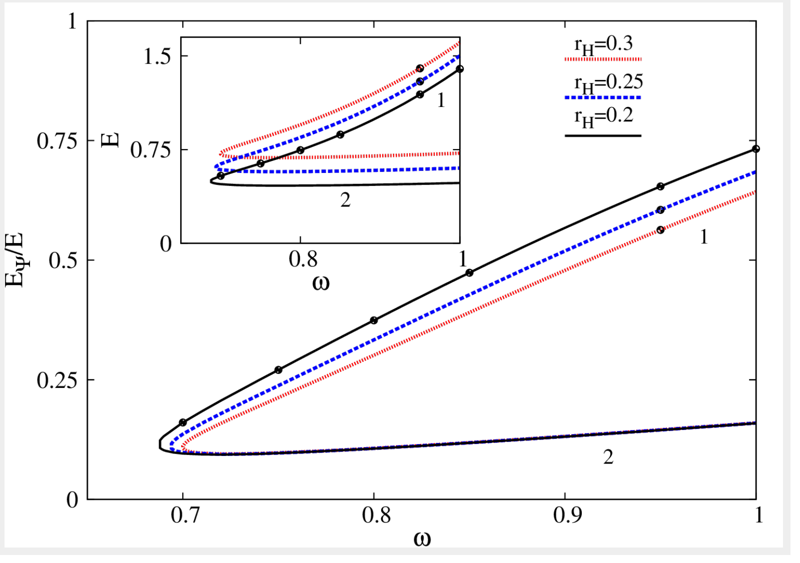
  - charged analogue of the Kerr synchronisation; it enforces vanishing scalar flux through the horizon.



Profile functions of an illustrative BH with gauged scalar hair (C. A. R. Herdeiro and Radu 2020)

Black holes with resonant hair

- A very hairy system can be understood as a small charged particle inside a large (spherical) charged boson star.
  - Newtonian proxy: electrically charged star with mass and charge density ρ_M, ρ_Q near the center, a particle with mass m and charge q is in an unstable equilibrium at the central point if $q\rho_Q > m\rho_M$.

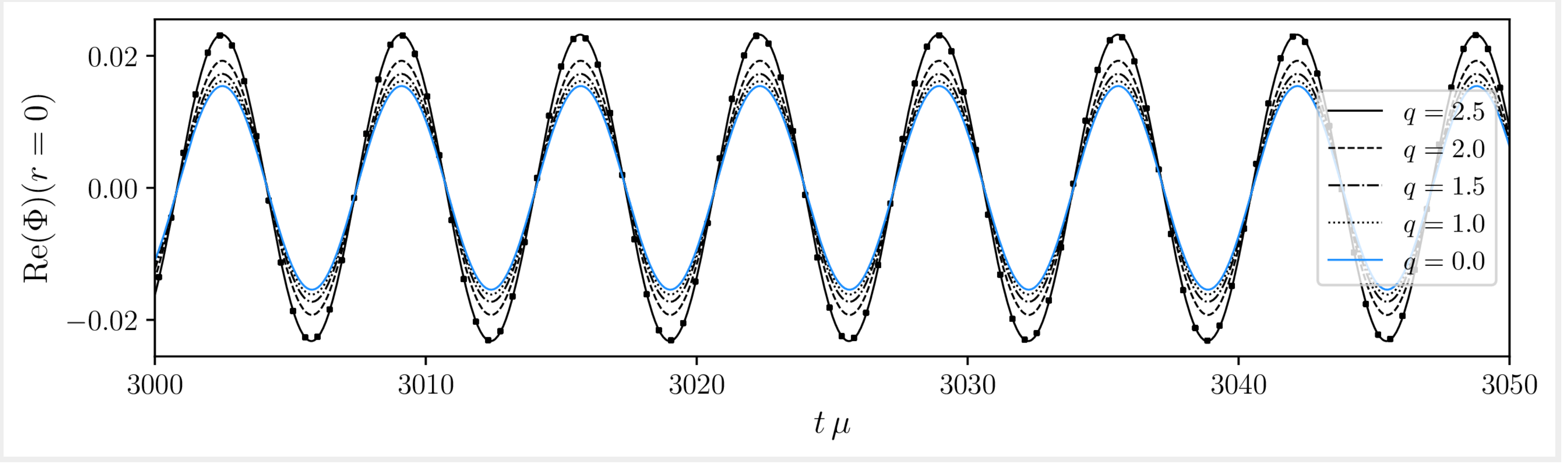


Scalar energy (main panel) and total spacetime energy E (inset) *vs* ω for sequences of solutions with different r_H.

Results

Spherical charged stars

- Stable spherical configuration



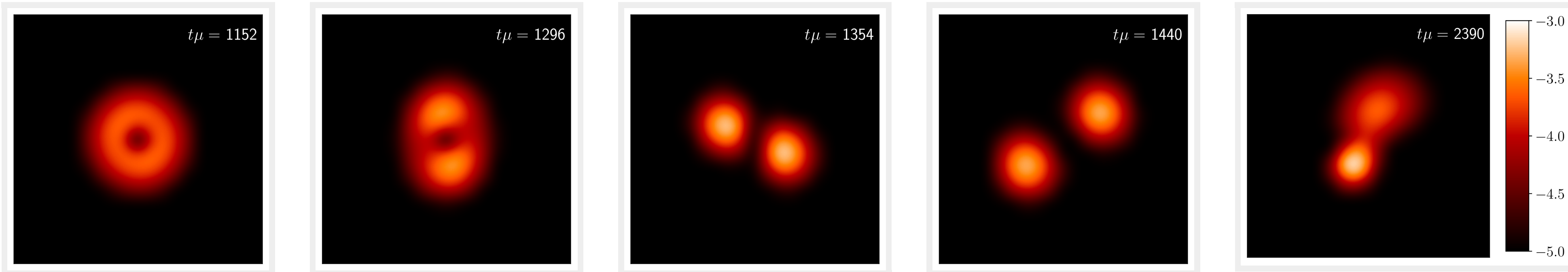
$\Re(\Psi)$ at center of star with $\omega = 0.95\mu$ for several values of the coupling constant q (Jaramillo et al. 2025)

Spinning charged stars

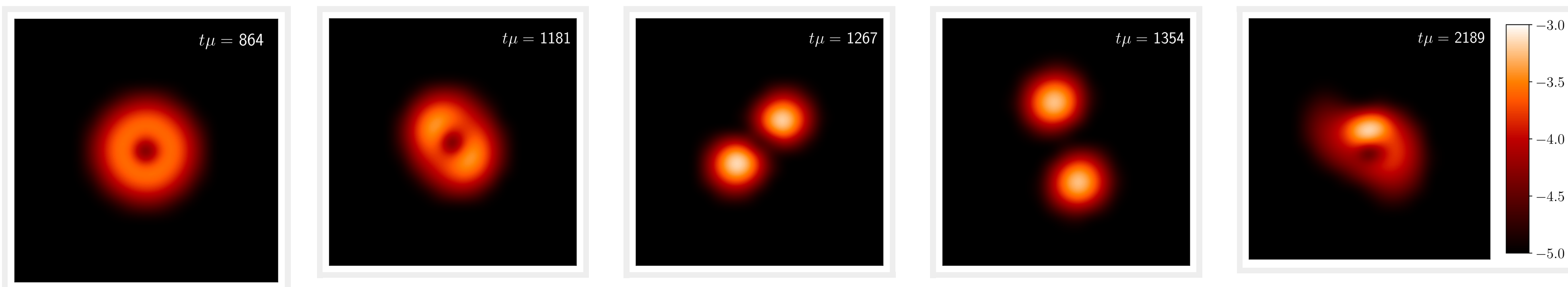
- Configurations with $m = 1, \omega = 0.9\mu$ and $\omega = 0.95\mu$ (Jaramillo et al. 2025)
- Known to be unstable for $q = 0$
  - However, a quartic self-interaction, which induces a positive (repulsive) force, has been shown to stabilize solutions (Siemonsen and East 2021)
  - A charged field, which generates an electric field in these configurations, also has a repulsive effect

$\omega = 0.9\mu$

$q = 0$



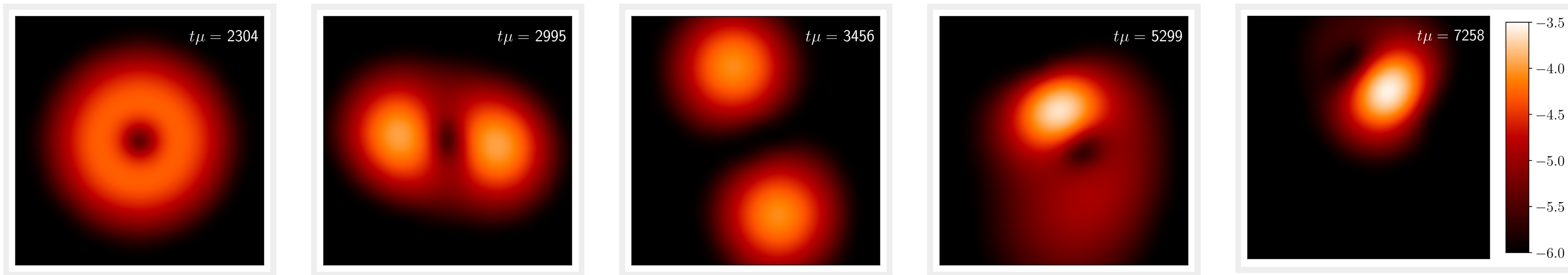
$q = 1.5$



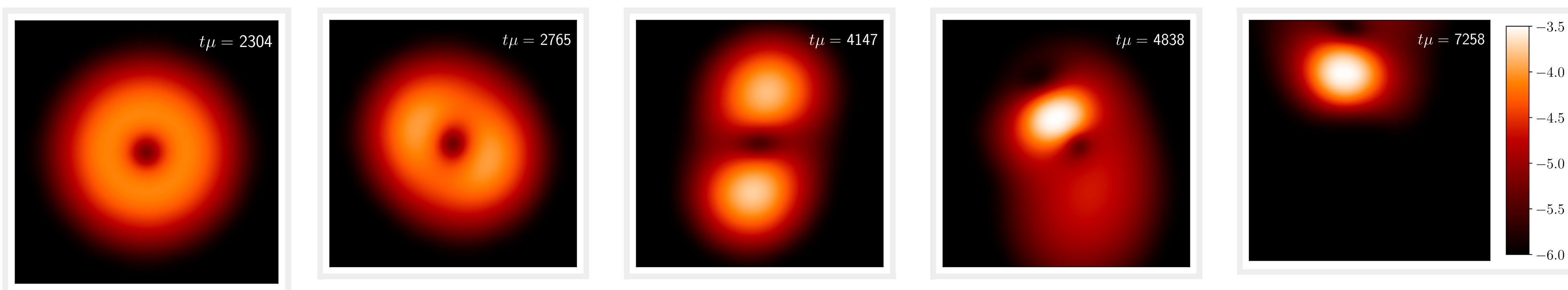
Jaramillo et al. (2025)

$\omega = 0.95\mu$

$q = 0$



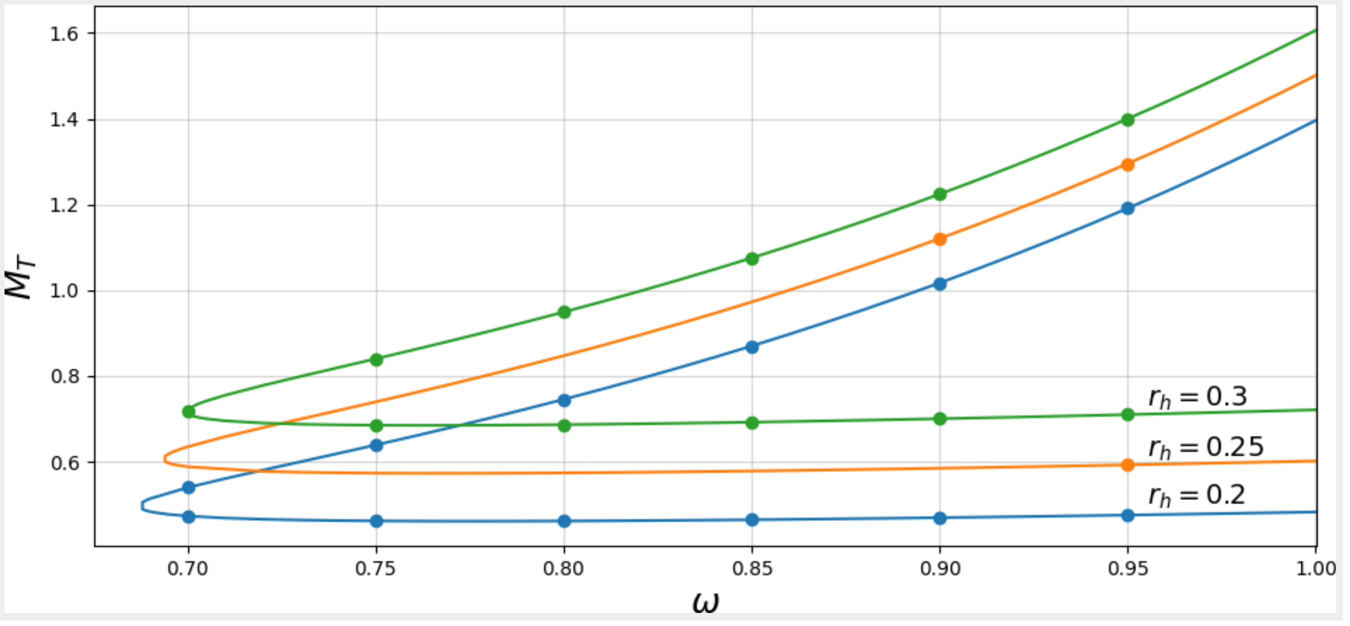
$q = 2$



Jaramillo et al. (2025)

Black holes with resonant hair

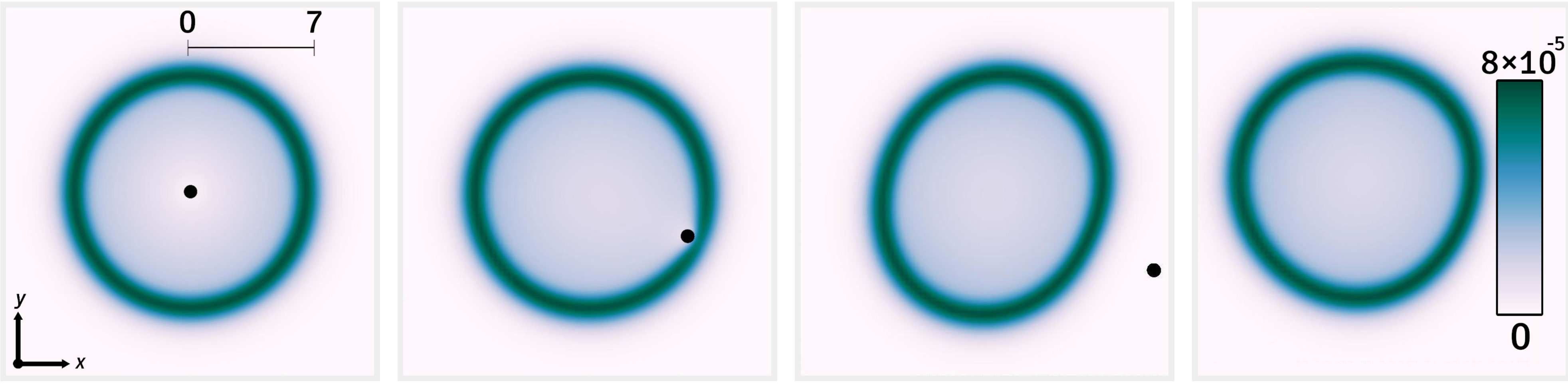
- Dynamical evolutions reveal two qualitatively distinct outcomes (Nicoules et al. 2025), (Ferreira et al. 2025):
  - *absorption* – the hair is absorbed by the BH
  - *fission* – the BH is expelled from the hair resulting in a stable boson star and a hairless BH



Existence curves of solutions for various values of r_h and ω, with $\mu = 1$, $\lambda = 2500$, and $q = 12$.

Fission

- $r_h = 0.2$ and $\omega = 0.7$ (in the upper branch)



Initial configuration | Instability develops | BH is expelled | Final state, BS plus BH

Ferreira et al. (2025)

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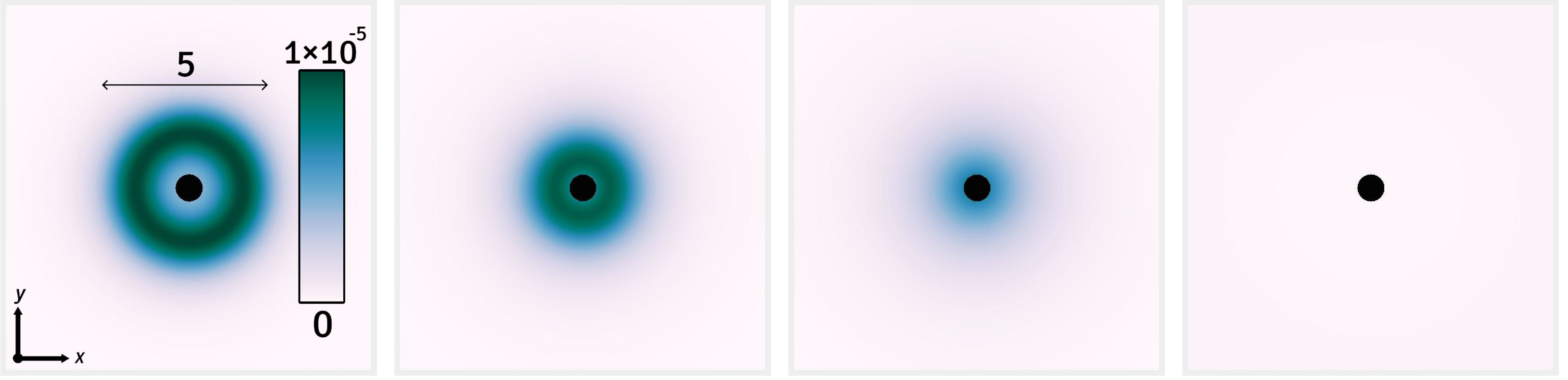
Absorption

- $r_h = 0.2$ and $\omega = 0.95$ (in the lower branch)



Initial configuration | Instability develops | Hair is absorbed | Final state

Ferreira et al. (2025)

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Final remarks

Final remarks

- The (scalar) very hairy BH is unstable in relatively short time scales
  - scalar field is disrupted and its structure changes as the BH reaches higher density regions;
  - (tentatively) related to the stability of the bosonic environment.
- A potentially different case is that of BHs with synchronized Proca hair (C. Herdeiro, Radu, and Runarsson 2016)
  - Proca stars are spheroidal, and the origin can be a stable equilibrium point;
  - dynamical fate of very hairy Proca BHs deserves a detailed analysis (which is underway).

Final remarks

- Spinning $m = 1$ boson stars
  - unstable in the neutral charge limit $q = 0$
  - adding charge q does not act to prevent collapse, but rather accelerates it
  - the evolution of charged $m = 1$ boson stars goes beyond simple analogies of attractive or repulsive forces
  - self-interaction terms do stabilize spinning charged configurations (just as in the uncharged case)
- Black holes with resonant hair
  - end-state depends on model: ejection + boson-star remnant or absorption
  - very-hairy configurations are generically unstable on relatively short timescales

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