Amplification of new physics in the QNM spectrum of highly-rotating black holes

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Based on
PRL 134 (2025) 19, 191401 w/ Marina David
2509.08664, 2510.17962 w/ M. David and Guido Van der Velde

The Dynamics of Black Hole Mergers and Gravitational Wave Generation
IFPU-Trieste, January 12-16 2026

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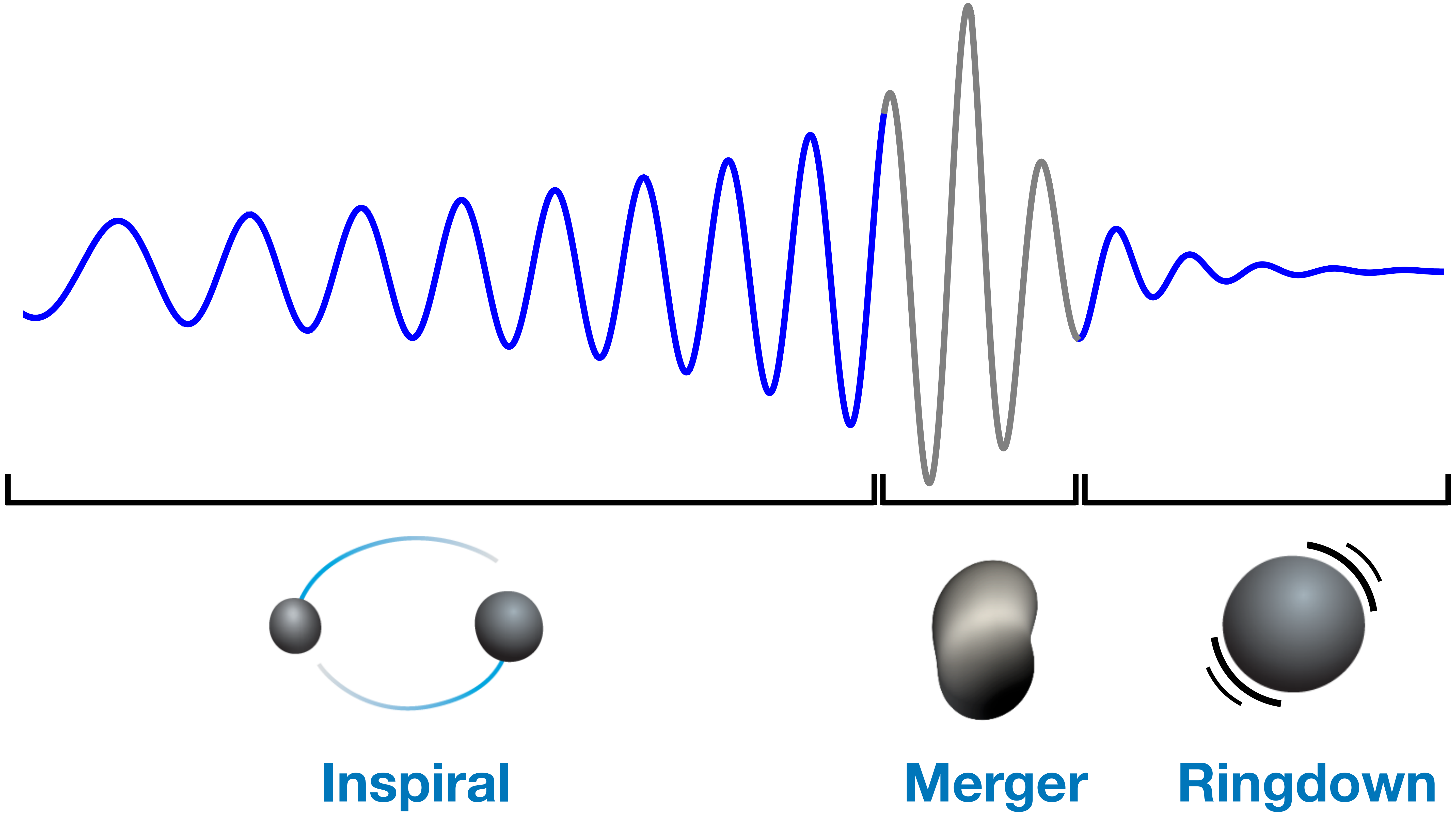
Introduction

Testing GR with black hole binaries



Einstein field equations

$$R_{\mu\nu} = 0\,?$$

Introduction

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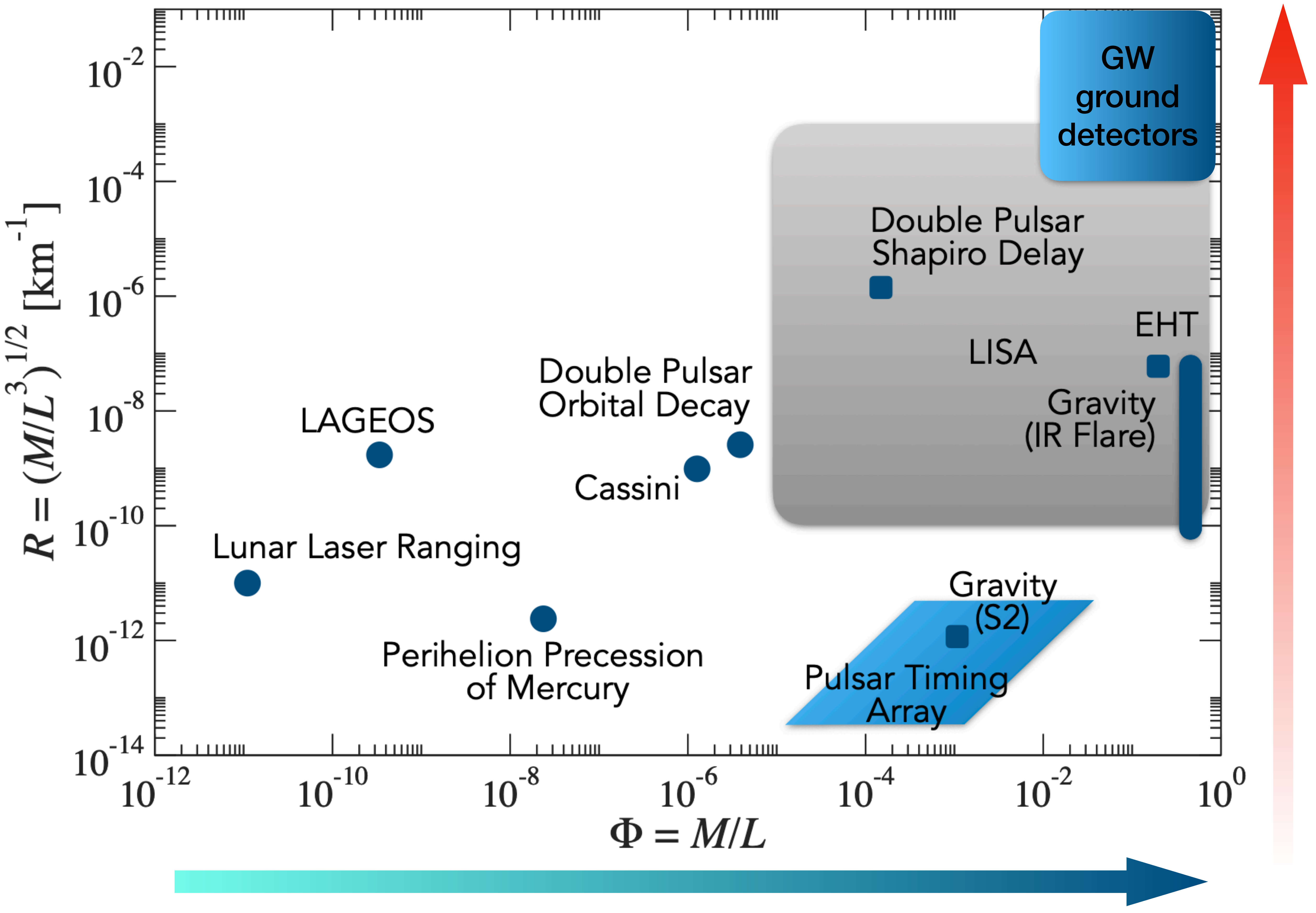
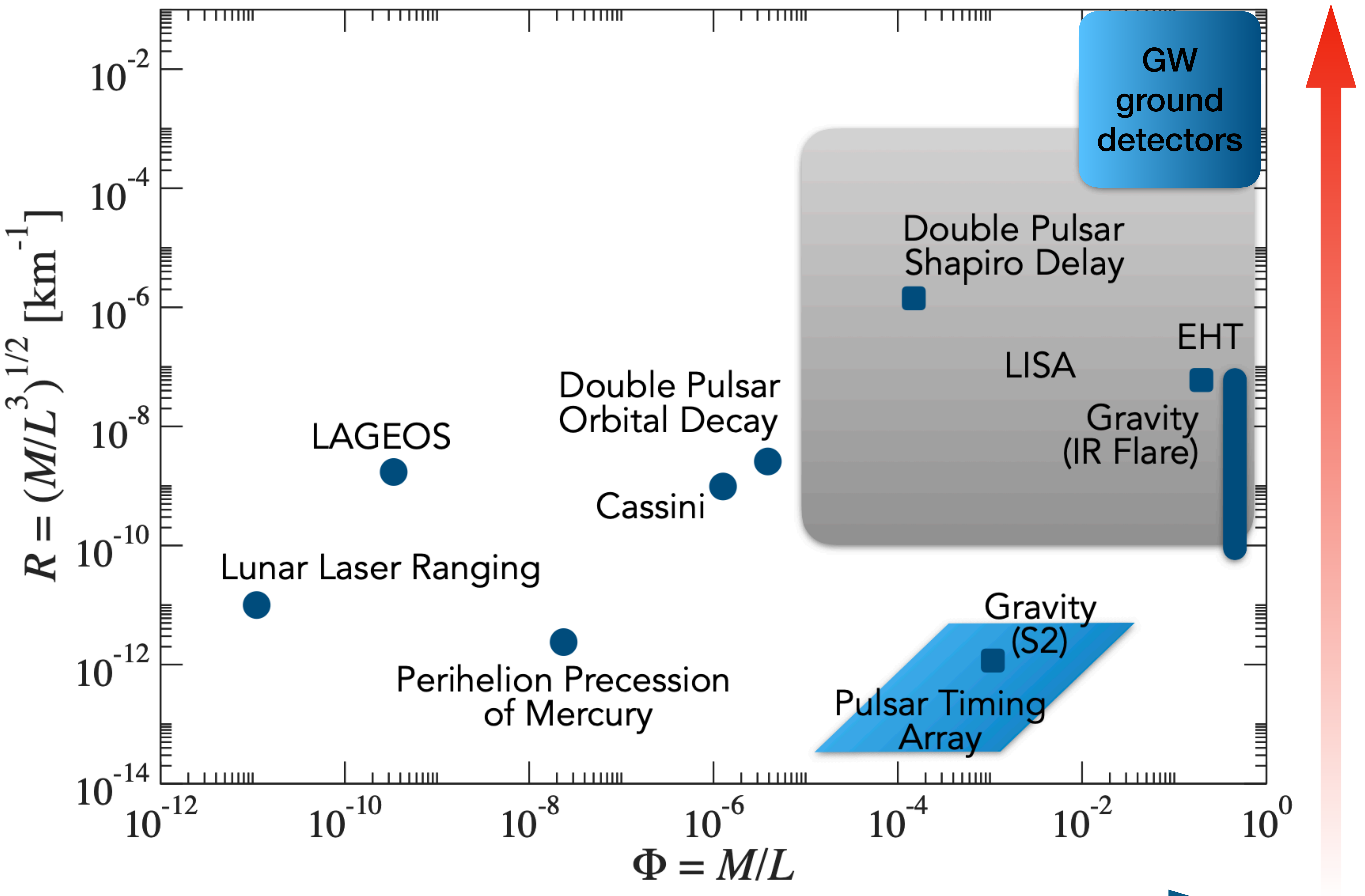


Test GR across different scales

Test the non-linear regime of GR

Introduction

Testing GR with black hole binaries



New physics?

Test GR across different scales

Test the non-linear regime of GR

Introduction

GR as an Effective Field Theory

Agnostic and **universal** approach to include new physics

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{|g|}\,\left[R + \ell^4\mathcal{R}^3 + \ell^6\mathcal{R}^4 + \dots\right]$$

Einstein

Beyond Einstein

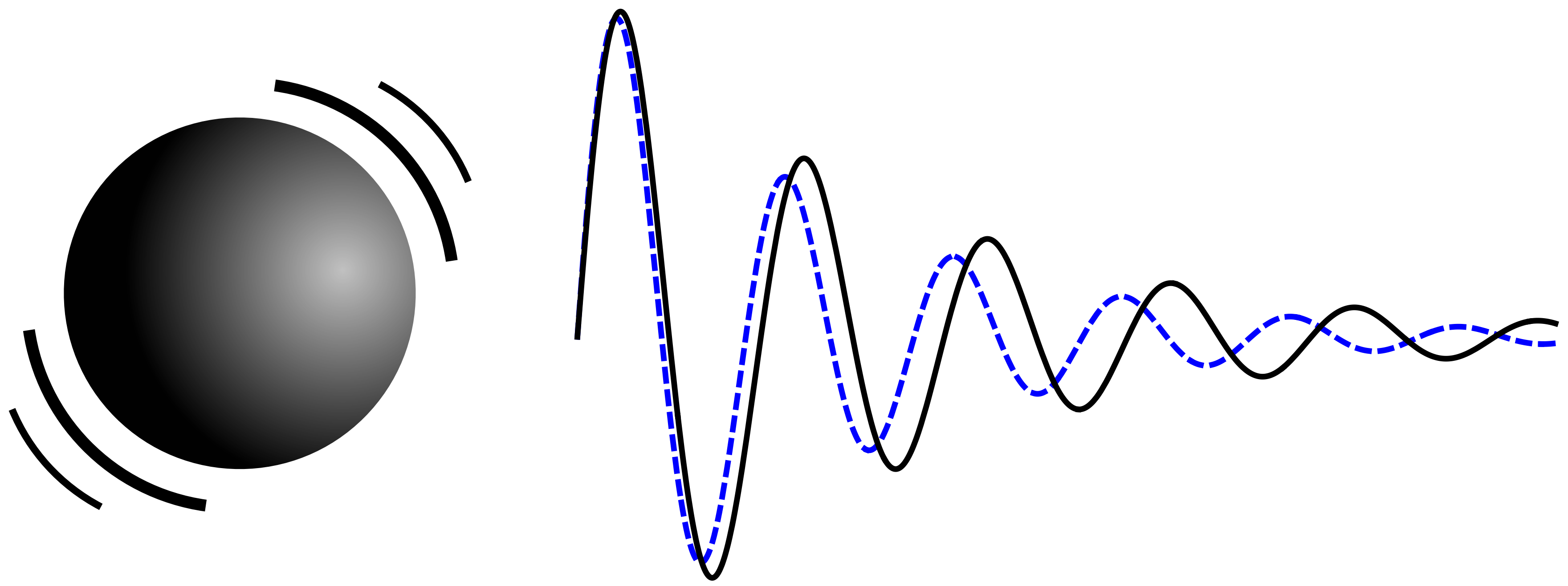
ℓ: scale of new physics

1. It's not ruled out by other experiments
2. It has full predictive power
3. It CAN be tested with GWs

Introduction

Ringdown as a test of new physics

$$\omega = \omega_R + i\omega_I, \qquad \omega_I = -\frac{1}{\tau}$$



$$\Psi = \sum_{l,m,n} A_{lmn} e^{-i\omega_{lmn} t}$$

QNM frequencies ➡ **underlying gravitational theory**

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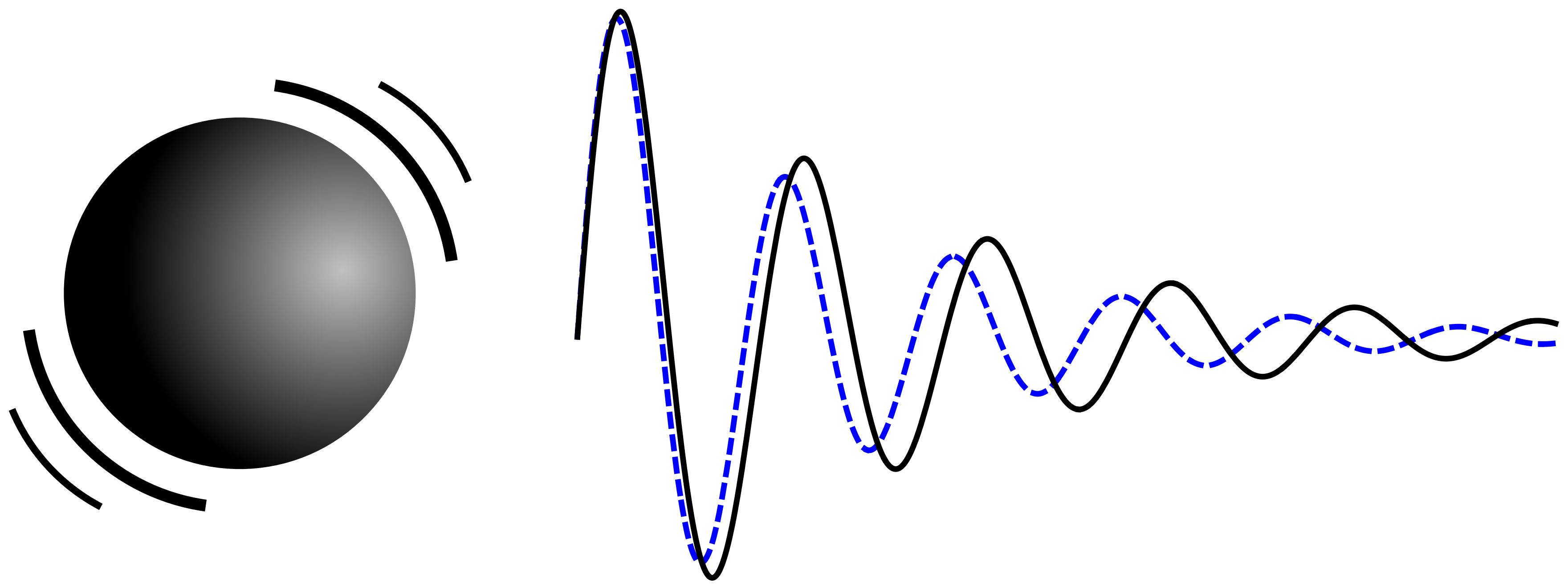

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QNM frequencies ➡ **underlying gravitational theory**

Challenge: QNMs of rotating black holes in theories beyond GR

$$\omega_{lmn} = \omega_{lmn}^{\mathrm{Kerr}} + \delta\omega_{lmn}$$

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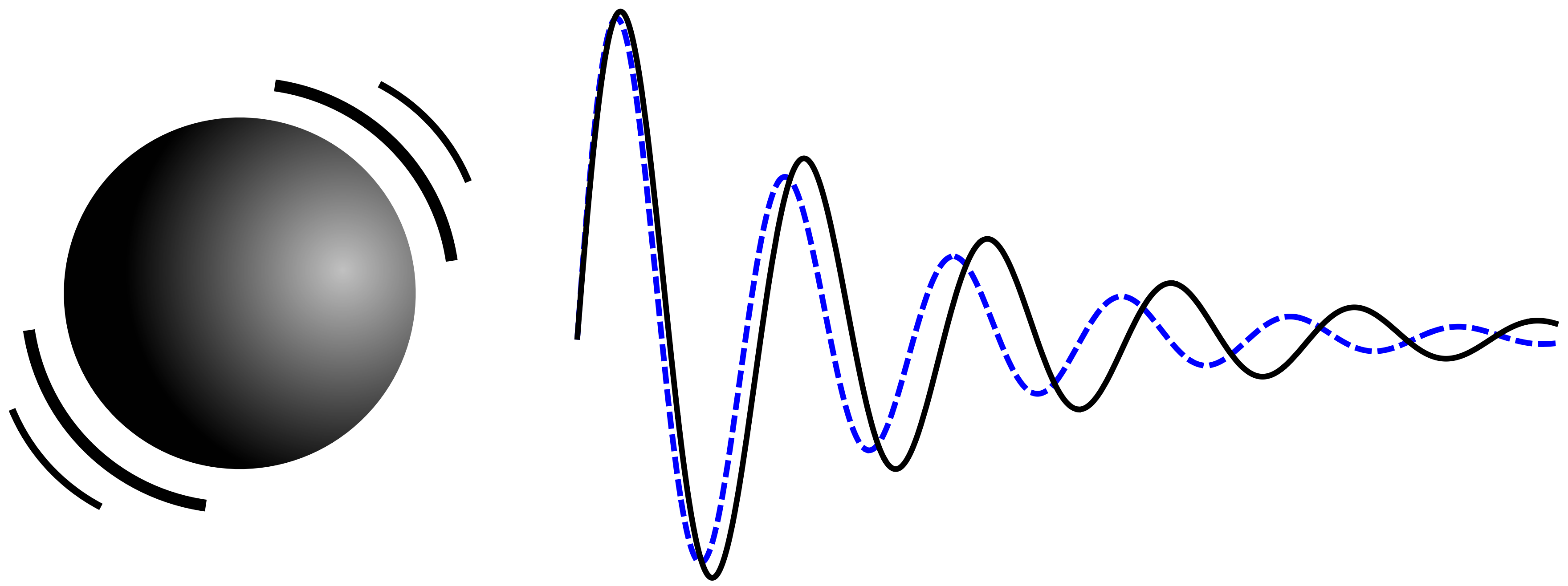

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QNM frequencies → **underlying gravitational theory**

Challenge: QNMs of rotating black holes in theories beyond GR

$$\omega_{lmn} = \omega_{lmn}^{\mathrm{Kerr}} + \delta\omega_{lmn}$$

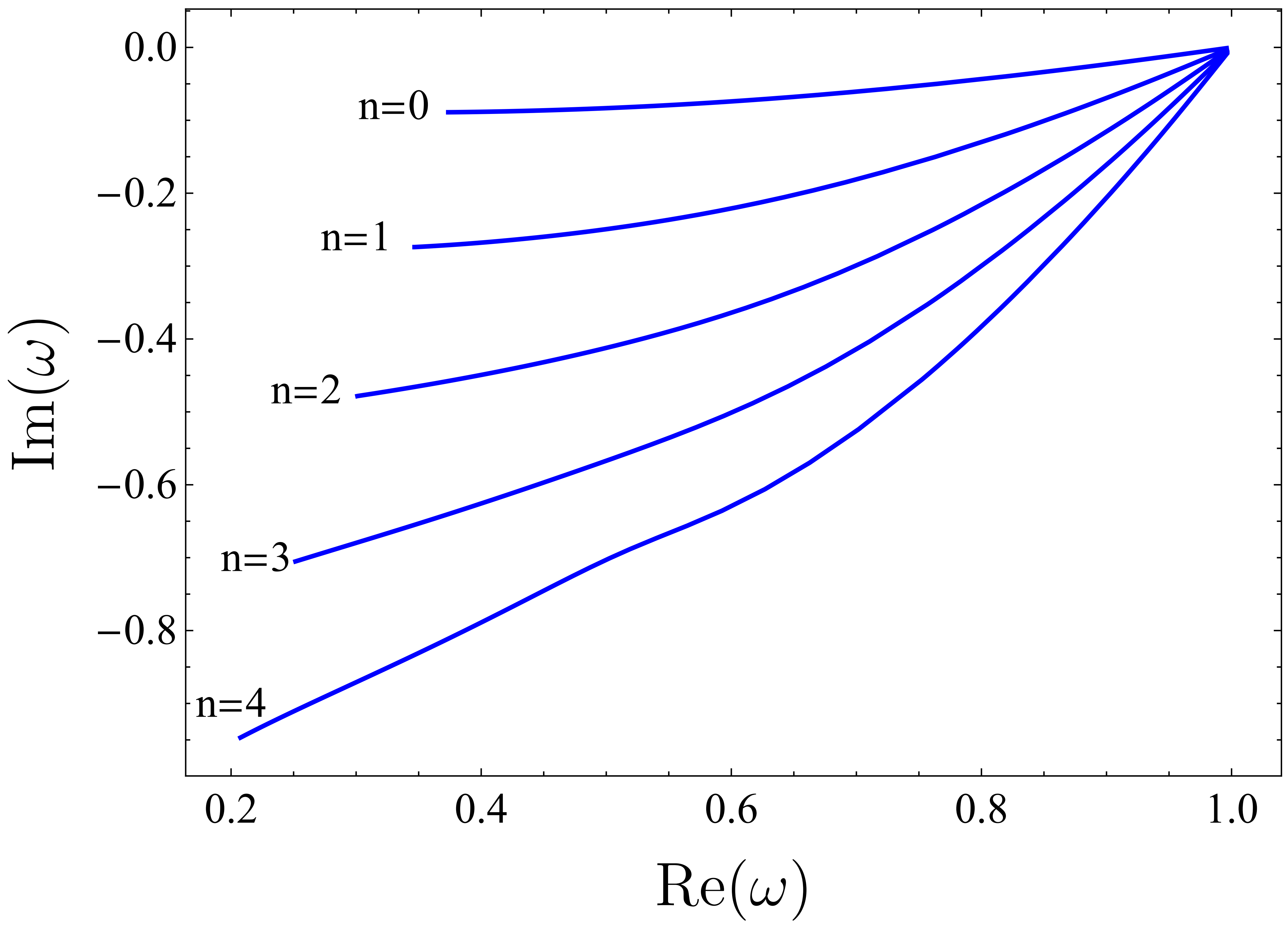
- **Modified Teukolsky equations** [Li, Wagle, Chen, Yunes ’22][Hussain, Zimmerman ’22][PAC, Fransen, Hertog, Maenaut ’23],…
- **Spectral methods** [Chung, Yunes ’24] [Blázquez-Salcedo+ ’24],…
- **No method yet can probe the near-extremal regime**

Introduction

Near-extremal black holes: amplification of new physics?

Classical GR:

- Aretakis instability
- QNM spectrum: **long-lived modes**



Introduction

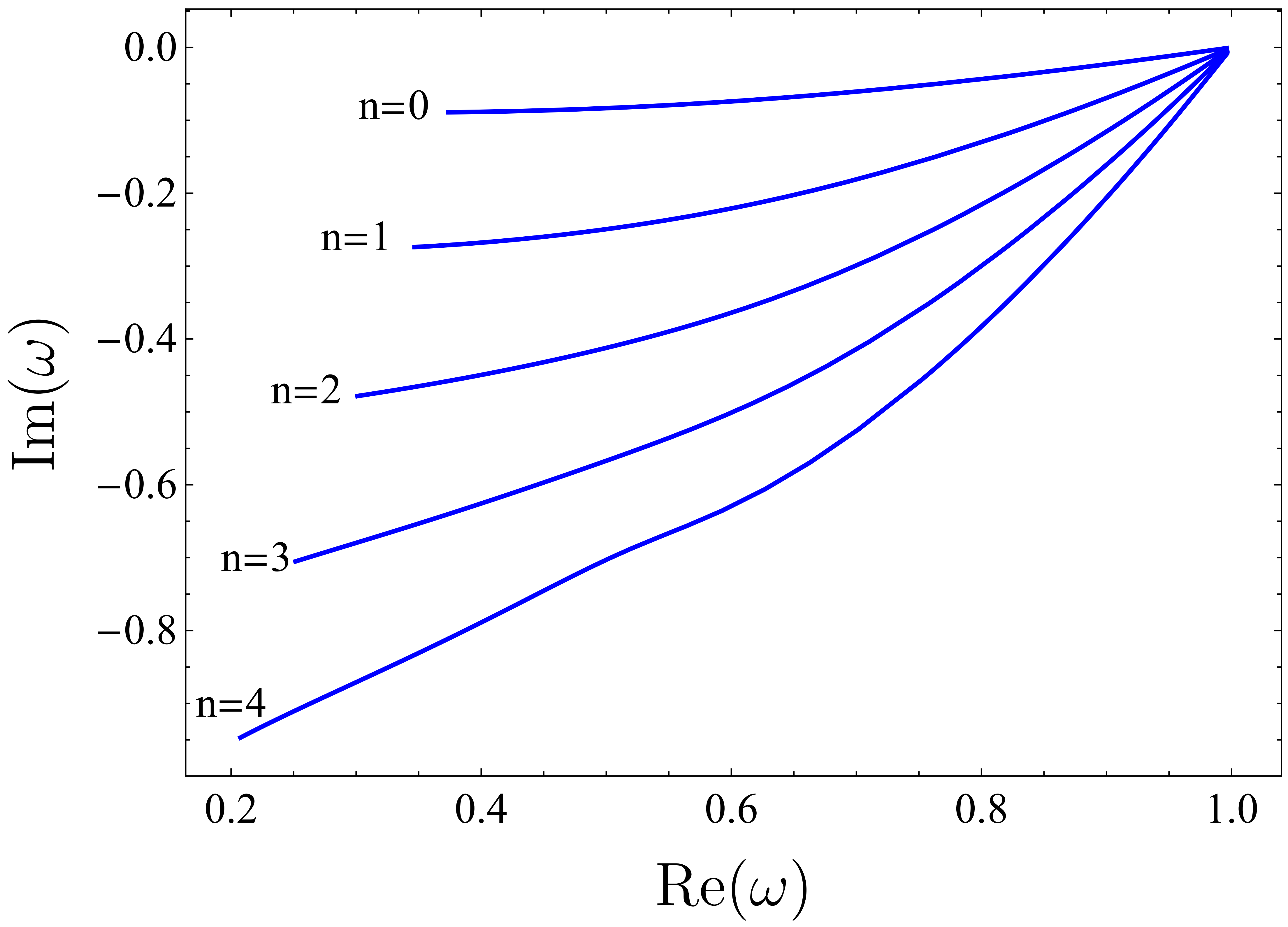
Near-extremal black holes: amplification of new physics?

Classical GR:

- Aretakis instability
- QNM spectrum: **long-lived modes**

New physics:

- Quantum effects [Heydeman, Iliesiu, Turiaci, Zhao]
- Divergence of tidal forces [Horowitz, Kolanowski, Remmen, Santos]
- Singular horizon [Kleihaus, Kunz, Mojica, Radu]
- **QNM spectrum???**



Introduction

Plan of the talk

1. Spectrum of near-extremal Kerr
2. Isospectral EFTs
3. BH perturbations in isospectral EFTs
4. Results for QNMs

Part 1: QNM spectrum of near-extremal Kerr

Spectrum of near-extremal Kerr

Kerr metric

$$ds^2 = -\frac{\Delta}{\Sigma}\left(dt - a\sin^2\theta d\phi\right)^2 + \Sigma\left(\frac{dr^2}{\Delta} + d\theta^2\right) + \frac{\sin^2\theta}{\Sigma}\left((r^2+a^2)d\phi - adt\right)^2 ,$$

$$\Delta = r^2 - 2Mr + a^2, \quad \Sigma = r^2 + a^2\cos^2\theta$$

$M \rightarrow$ mass, $\quad a \rightarrow$ angular momentum per mass

Extremal limit: $a = M$

Near-extremal regime: $\epsilon = 1 - \frac{a}{M} \ll 1$

Spectrum of near-extremal Kerr

Teukolsky equation

Decoupled, 2nd order equation for curvature perturbations on top of Kerr

$$\mathcal{O}(\Psi) = 0\,, \qquad \Psi = \text{component of the Weyl tensor}$$

$$\text{Separable: } \Psi = e^{-i\omega t + im\phi}\ {}_sS_{lm}(\theta; a\omega)\psi_{lm}(r)$$

${}_sS_{lm}(\theta; a\omega) \to$ Spin-weighted spheroidal harmonics (s = spin = ± 2)

$\psi_{lm}(r) \to$ Satisfies master radial equation

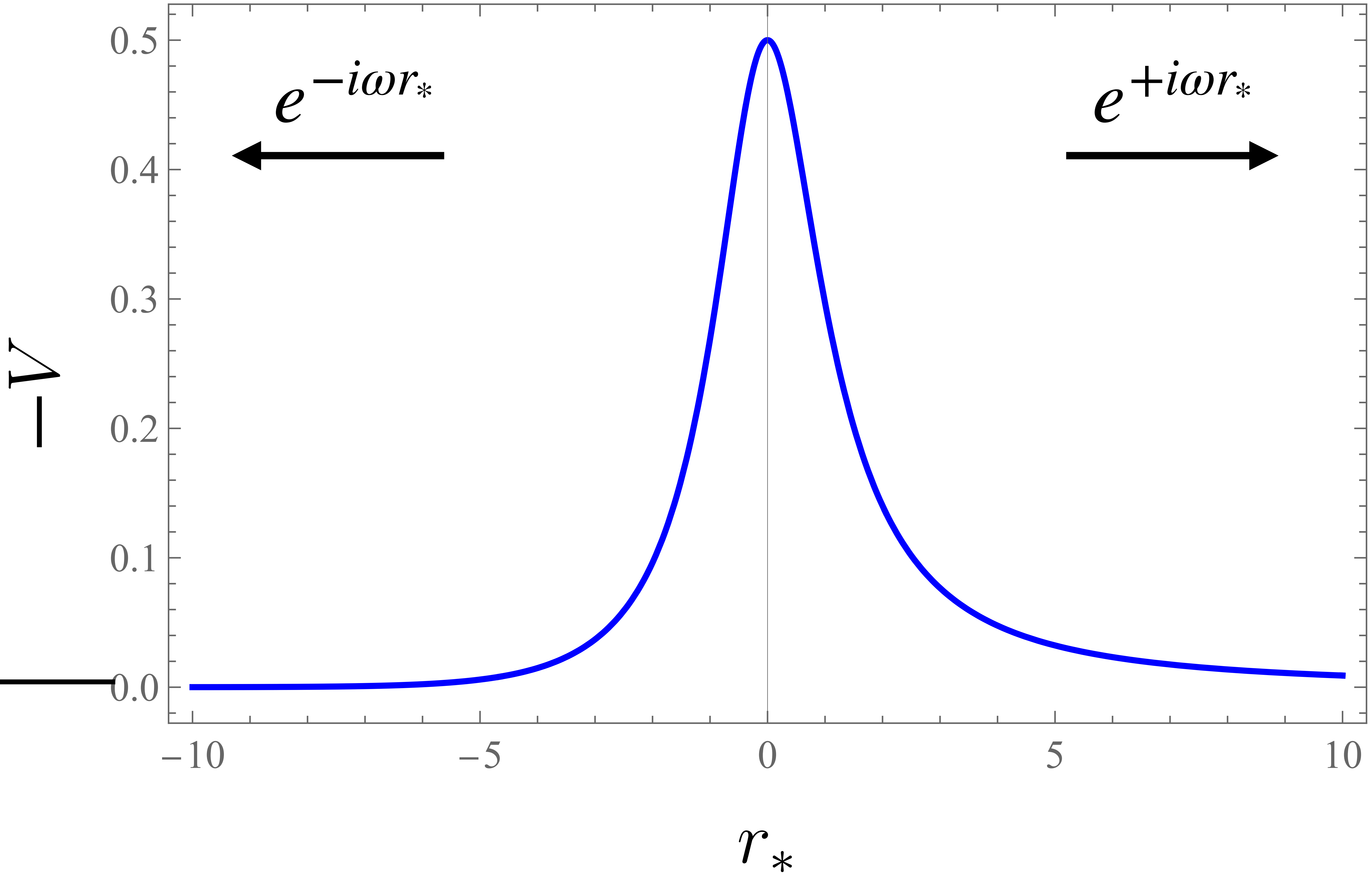
$$\Delta^{s+2}\frac{d}{dr}\left[\Delta^{2-s}\frac{d}{dr}\psi_{lm}\right] + V(r)\psi_{lm}$$

$V(r)$ effective potential

Spectrum of near-extremal Kerr

Definition of QNMs

Horizon



Infinity

No waves coming from the horizon or from infinity

Solutions only for discrete family of complex ω

Spectrum of near-extremal Kerr

Eikonal limit and WKB method

In the eikonal limit $l \to \infty$, QNMs are related to the **maximum of the potential**

$$V = \left[\omega(r^2 + a^2) - am\right]^2 - \Delta\left(A_{lm} - 2ma\omega + (a\omega)^2\right)$$

Angular separation constants

WKB formula:

$$V(r_0) = \left.\frac{dV}{dr}\right|_{r_0,\omega_R} = 0, \qquad \omega_I = -\left(n + \frac{1}{2}\right)\Delta \left.\frac{\sqrt{2\partial_r^2 V}}{\partial_\omega V}\right|_{r_0,\omega_R}$$

Real part of ω

Imaginary part of ω

Modes labeled by the ratio $\mu = \frac{m}{L}$, where $L = l + 1/2$

Spectrum of near-extremal Kerr

Eikonal limit and WKB method

Maximum of the potential in the extremal limit?

Spectrum of near-extremal Kerr

Eikonal limit and WKB method

Maximum of the potential in the extremal limit?

For $\mu > \mu_{\rm cr} \approx 0.74$, $r_0 \to M$ at extremality. Modes live near the horizon and are **long lived**

→ **Zero-damping modes (ZDMs)**

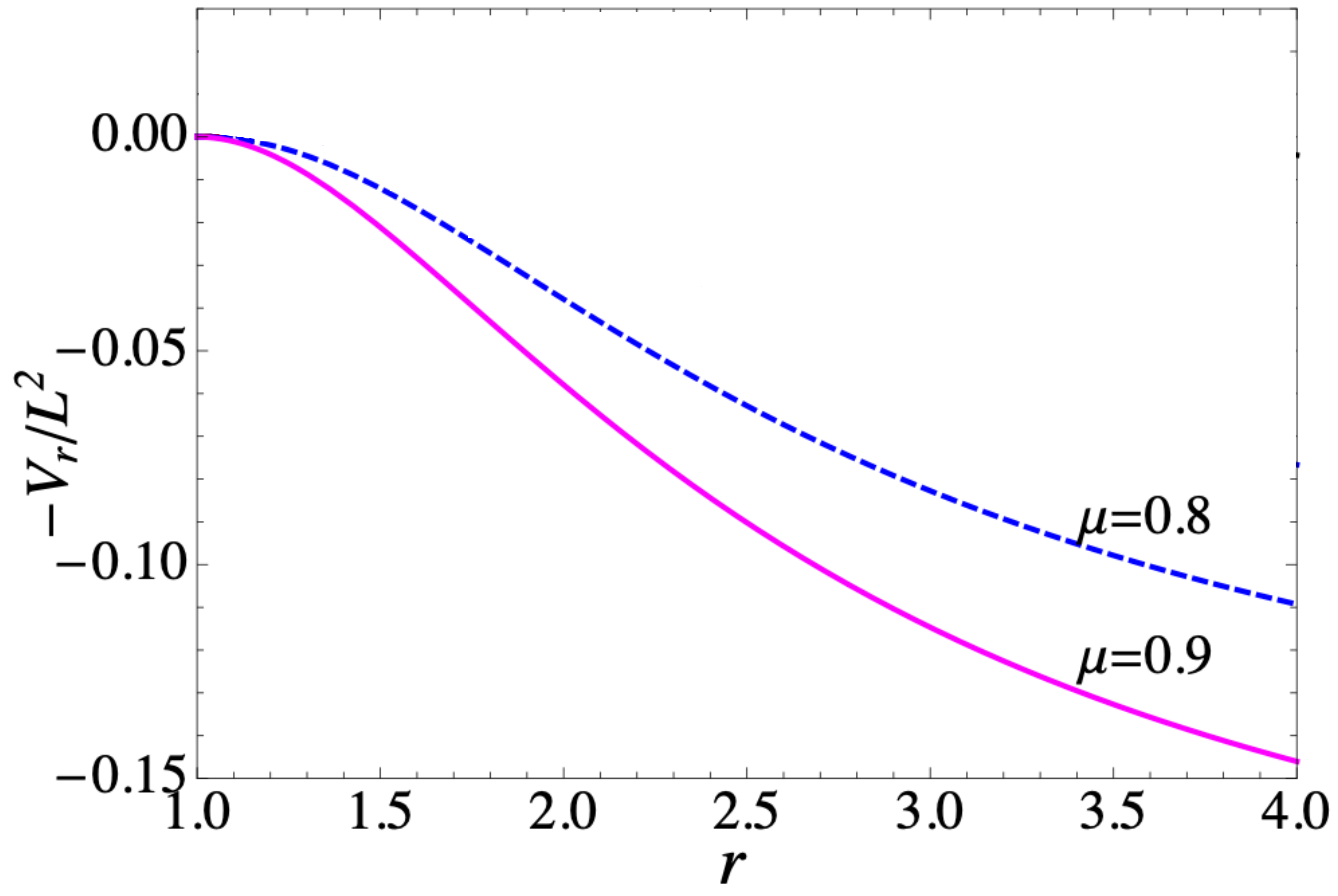


Figure taken from 1212.3271

Spectrum of near-extremal Kerr

Eikonal limit and WKB method

Maximum of the potential in the extremal limit?

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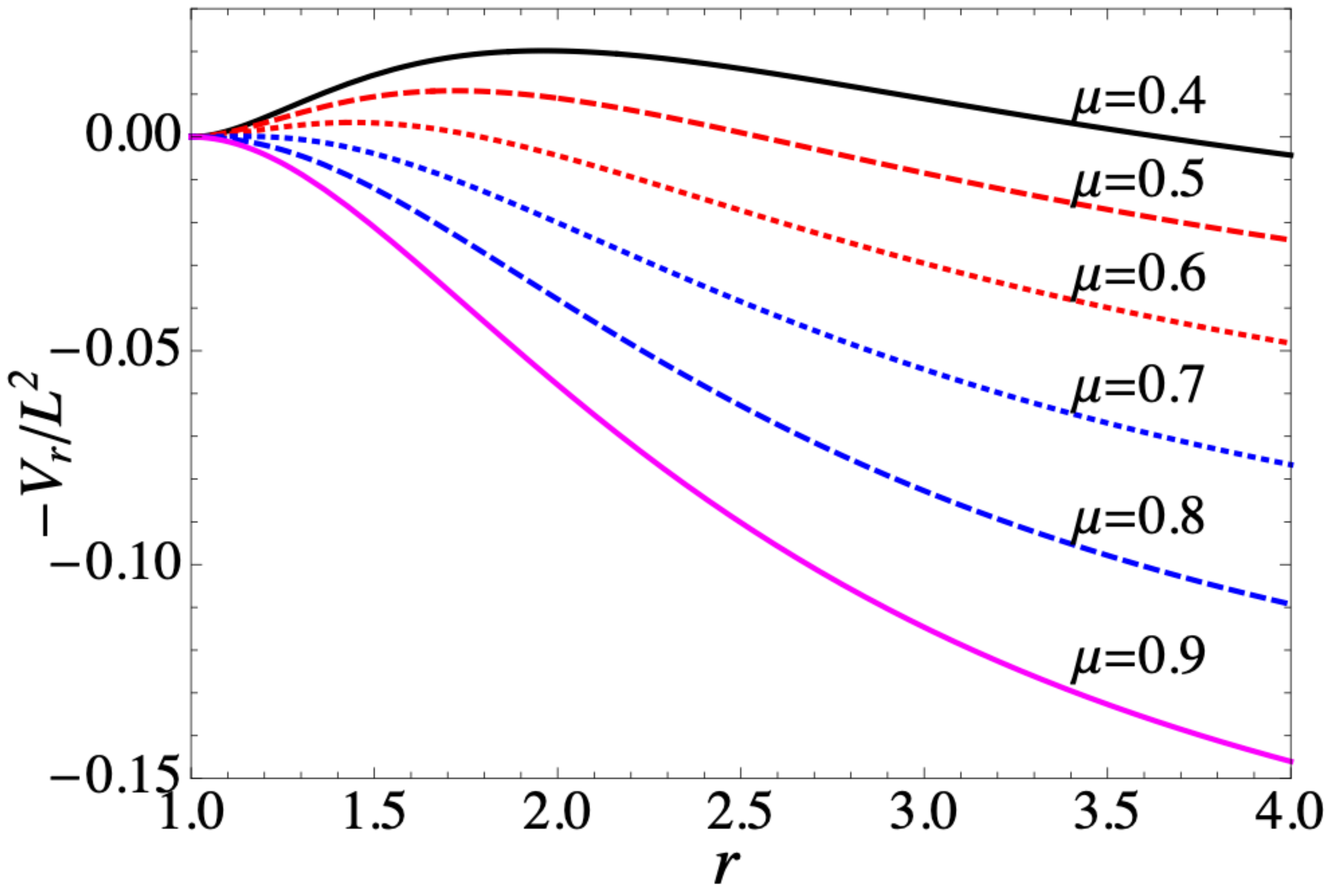
→ **Zero-damping modes (ZDMs)**

For $\mu < \mu_{\rm cr}$, the maximum is located outside the horizon

→ **Damped modes (DMs)**



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Maximum of the potential in the extremal limit?

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→ **Zero-damping modes (ZDMs)**

For $\mu < \mu_{\rm cr}$, the maximum is located outside the horizon

→ **Damped modes (DMs)**

In addition, ZDMs also exist for $0 \leq \mu \leq \mu_{\rm cr}$, but they are unrelated to the maximum of the potential [Yang+ '12, '13]

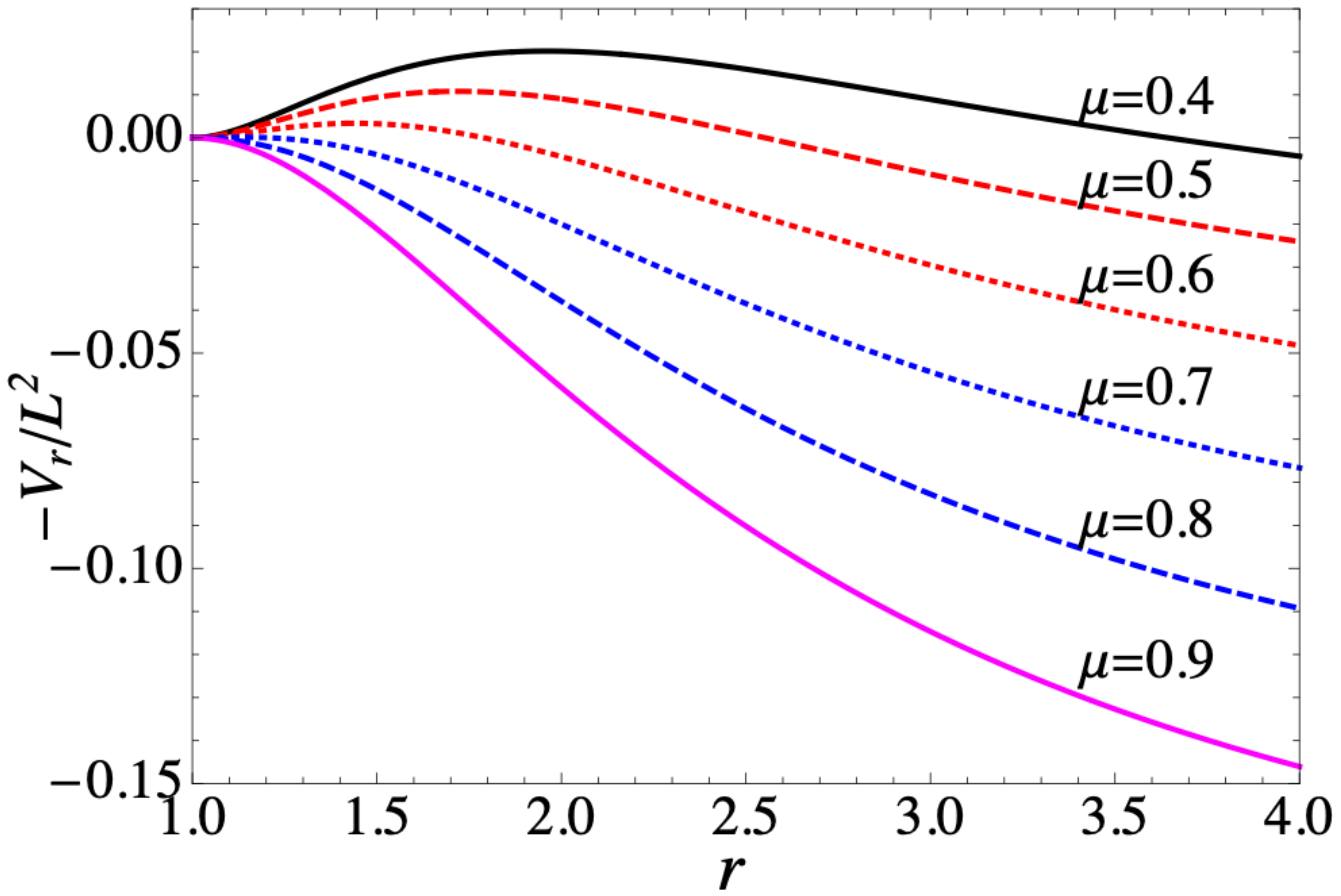


Figure taken from 1212.3271

Spectrum of near-extremal Kerr

Branching of the spectrum

Conclusion: the QNM spectrum of near-extremal Kerr bifurcates in two families of modes [Yang, Zhang, Zimmerman, Nichols, Berti, Chen '12, '13]

Zero-damping modes (ZDMS) (exist for $\mu \geq 0$)

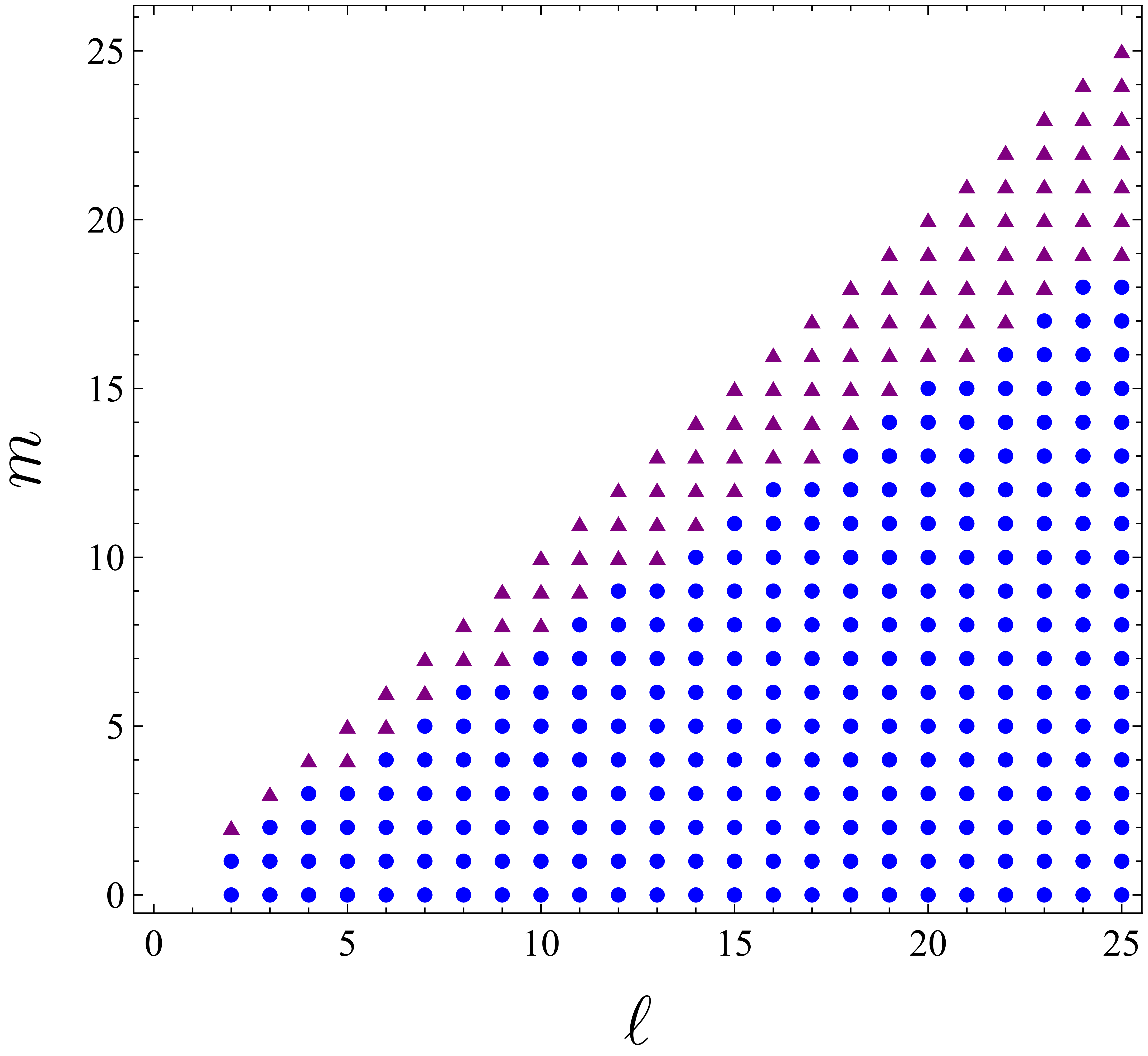
$$\omega = m\Omega - \frac{i}{M}\left(n + \frac{1}{2}\right)\sqrt{\frac{\epsilon}{2}}, \qquad \epsilon = 1 - \frac{a}{M}$$

Infinitely long-lived

Damped modes (DMs) (exist for $\mu \leq 0.744$)→ finite damping times

Spectrum of near-extremal Kerr

Branching of the spectrum

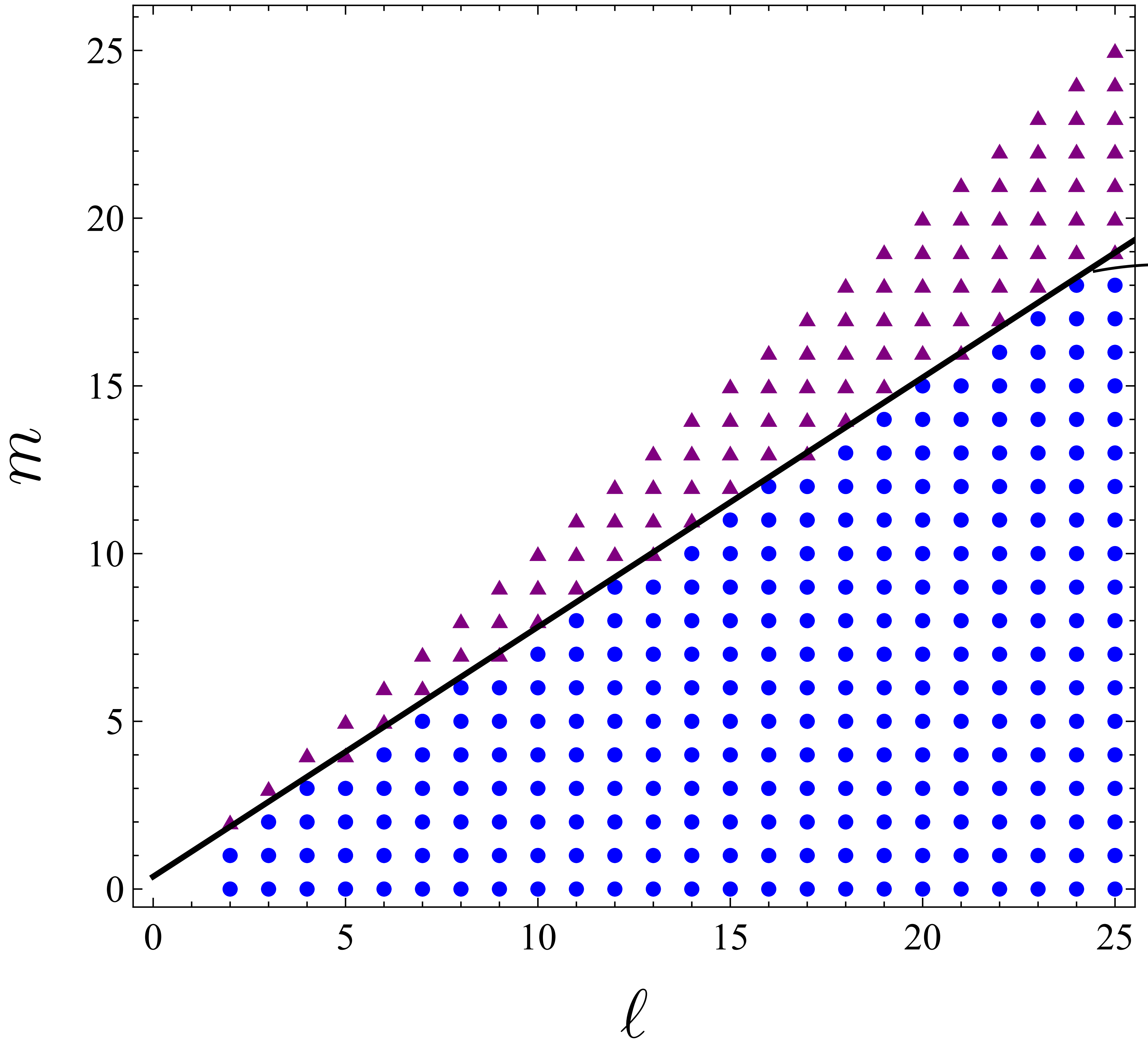


▲ = only ZDMs exist (GR)

● = ZDMs and DMs exist (GR)

Spectrum of near-extremal Kerr

Branching of the spectrum



▲ = only ZDMs exist (GR)

● = ZDMs and DMs exist (GR)

Phase boundary obtained from the eikonal limit

$$\frac{m}{\ell + 1/2} = \bar{\mu}_{\rm cr} \approx 0.744$$

Part 2: Isospectral EFTs

EFT extension of GR

$$S_{\mathrm{EFT}} = \frac{1}{16\pi}\int d^4x\sqrt{|g|}\left[R + \ell^4\left(\lambda_{\mathrm{ev}}R_3 + \lambda_{\mathrm{odd}}\tilde{R}_3\right) + \ell^6\left(\epsilon_1 R_2^2 + \epsilon_2 \tilde{R}_2^2 + \epsilon_3 R_2\tilde{R}_2\right) + \ldots\right]$$

Two cubic invariants: $R_3 = R_{\mu\nu}{}^{\rho\sigma}R_{\rho\sigma}{}^{\delta\gamma}R_{\delta\gamma}{}^{\mu\nu}, \quad \tilde{R}_3 = R_{\mu\nu}{}^{\rho\sigma}R_{\rho\sigma}{}^{\delta\gamma}\tilde{R}_{\delta\gamma}{}^{\mu\nu}$

Three quartic invariants: formed from $R_2 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}, \quad \tilde{R}_2 = R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma}$

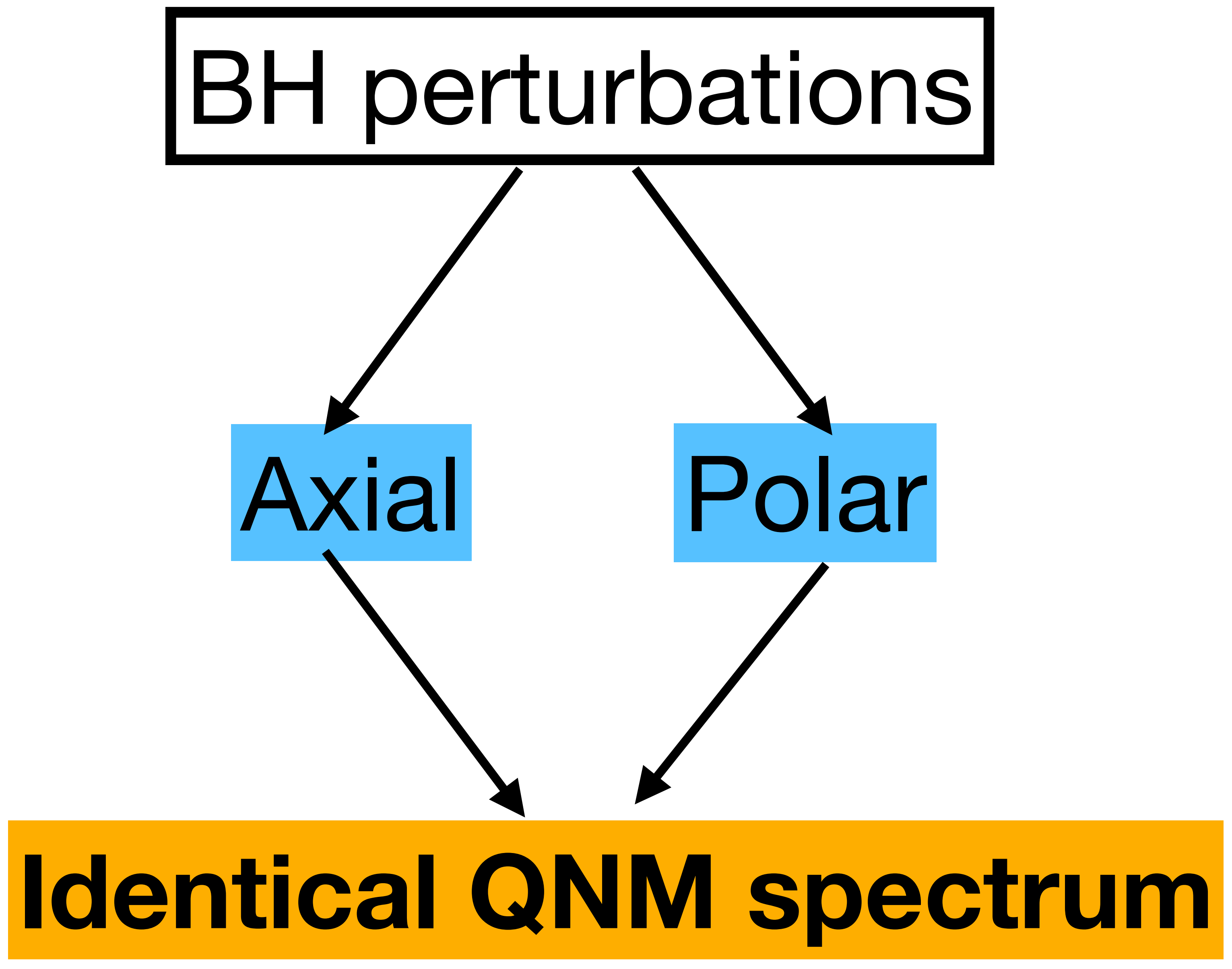
Dual Riemann tensor $\tilde{R}_{\mu\nu\rho\sigma} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}R^{\alpha\beta}{}_{\rho\sigma}$

$(\lambda_{\mathrm{ev}},\ \epsilon_1,\ \epsilon_2)$ even parity $\qquad (\lambda_{\mathrm{odd}},\ \epsilon_3)$ odd parity

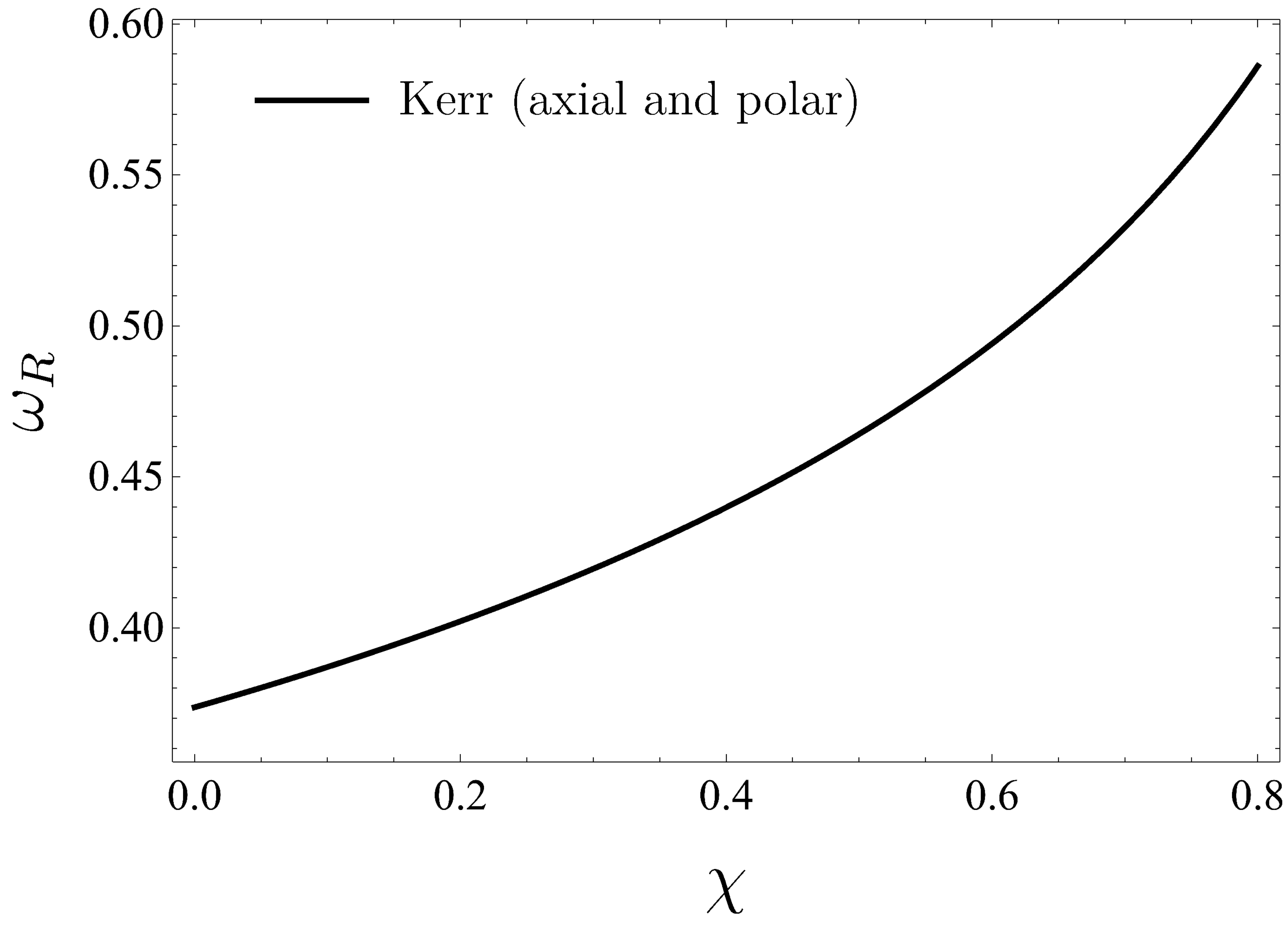
EFT extension of GR

Breaking of isospectrality

Special property in GR: **isospectrality**



Isospectrality in GR



EFT extension of GR

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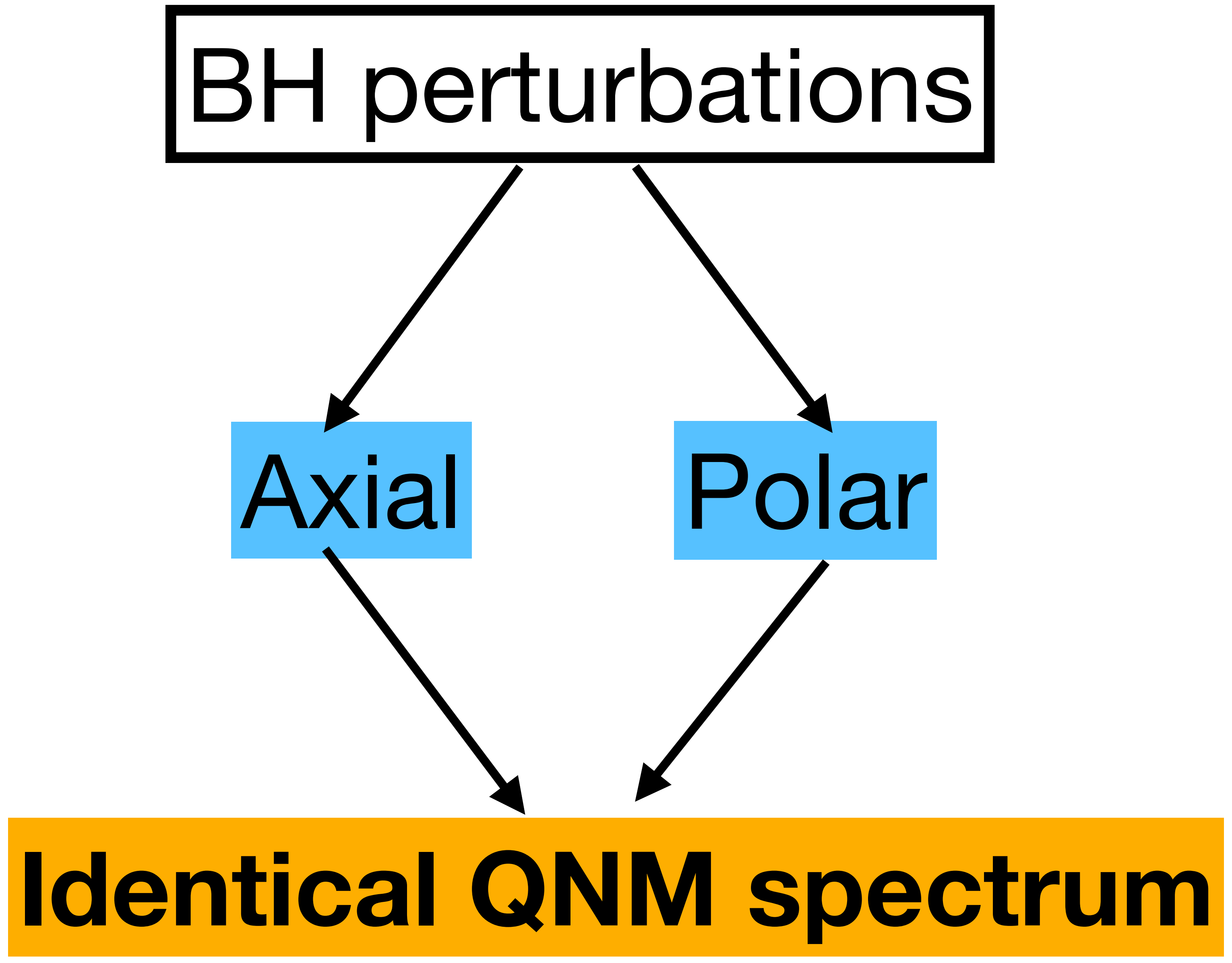
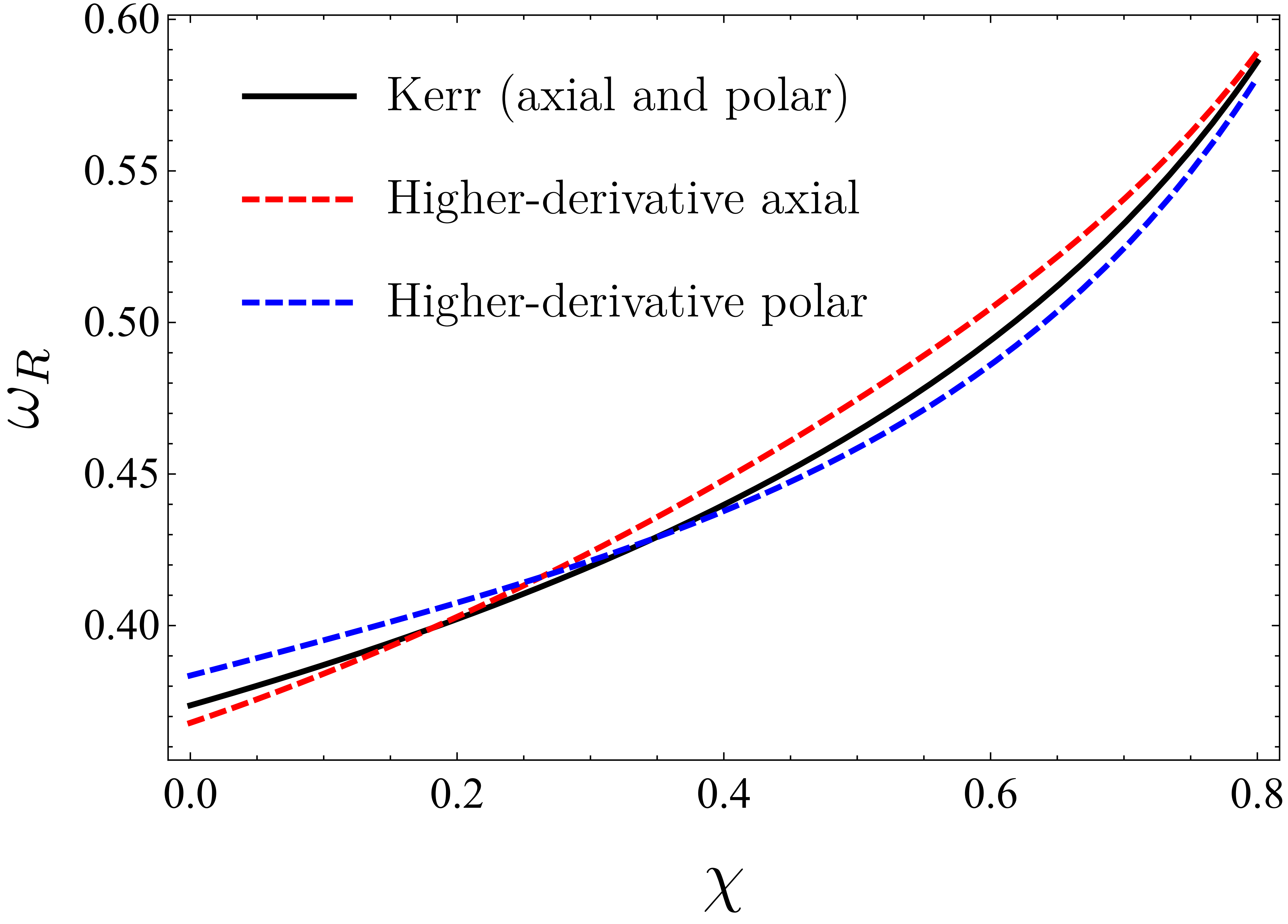


Not true in extensions of GR!

Isospectrality **breaking** beyond GR



Is there an isospectral theory?

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$$S_{\text{iso}} = \frac{1}{16\pi G}\int d^4x\sqrt{|g|}\left[R + \alpha\left(R_2^2 + \tilde{R}_2^2\right)\right]$$

Unique eikonal-isospectral extension of GR to eight derivatives

Is there an isospectral theory?

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Unique eikonal-isospectral extension of GR to eight derivatives

Key feature: dispersion relation for large-momentum GWs is non-birefringent

$$k^2 = 64\alpha R^{\lambda}{}_{\alpha}{}^{\eta}{}_{\beta} R^{\rho\alpha\sigma\beta} k_\lambda k_\eta k_\rho k_\sigma$$

Remark: $k^2 \neq 0 \rightarrow$ GWs no longer follow null geodesics

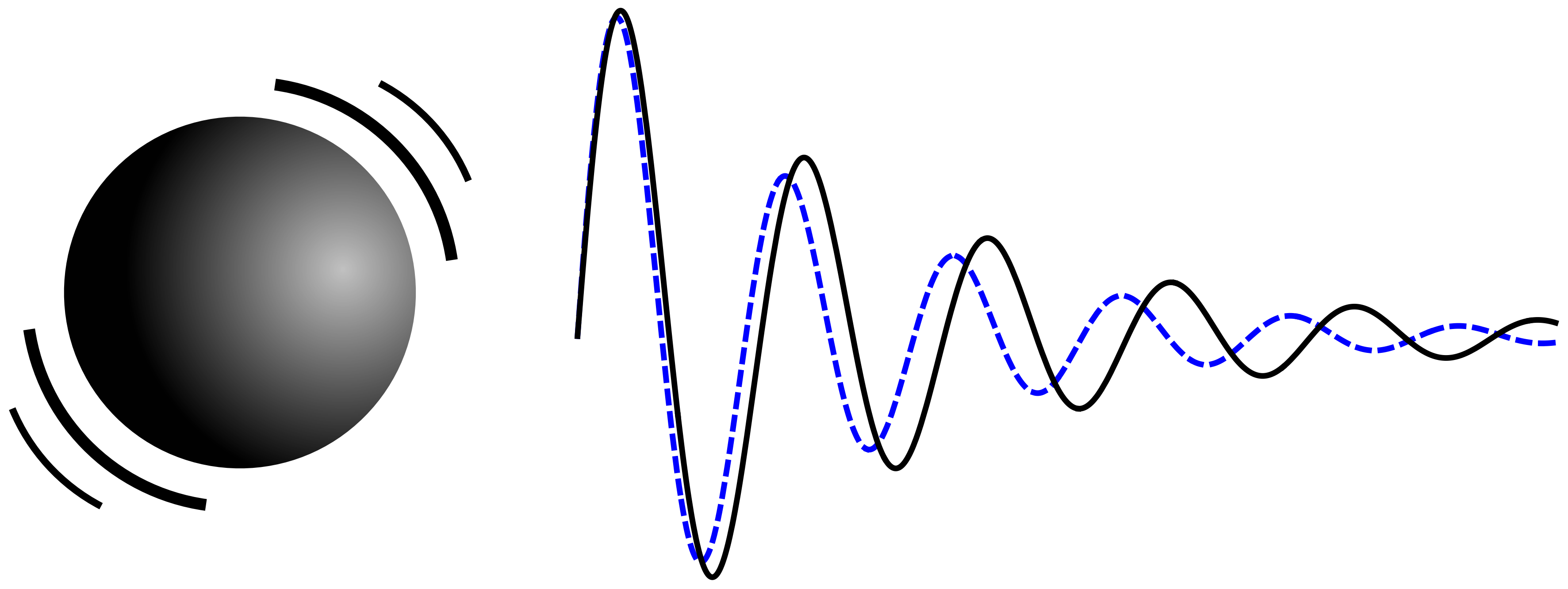
Eikonal QNMs and photon sphere

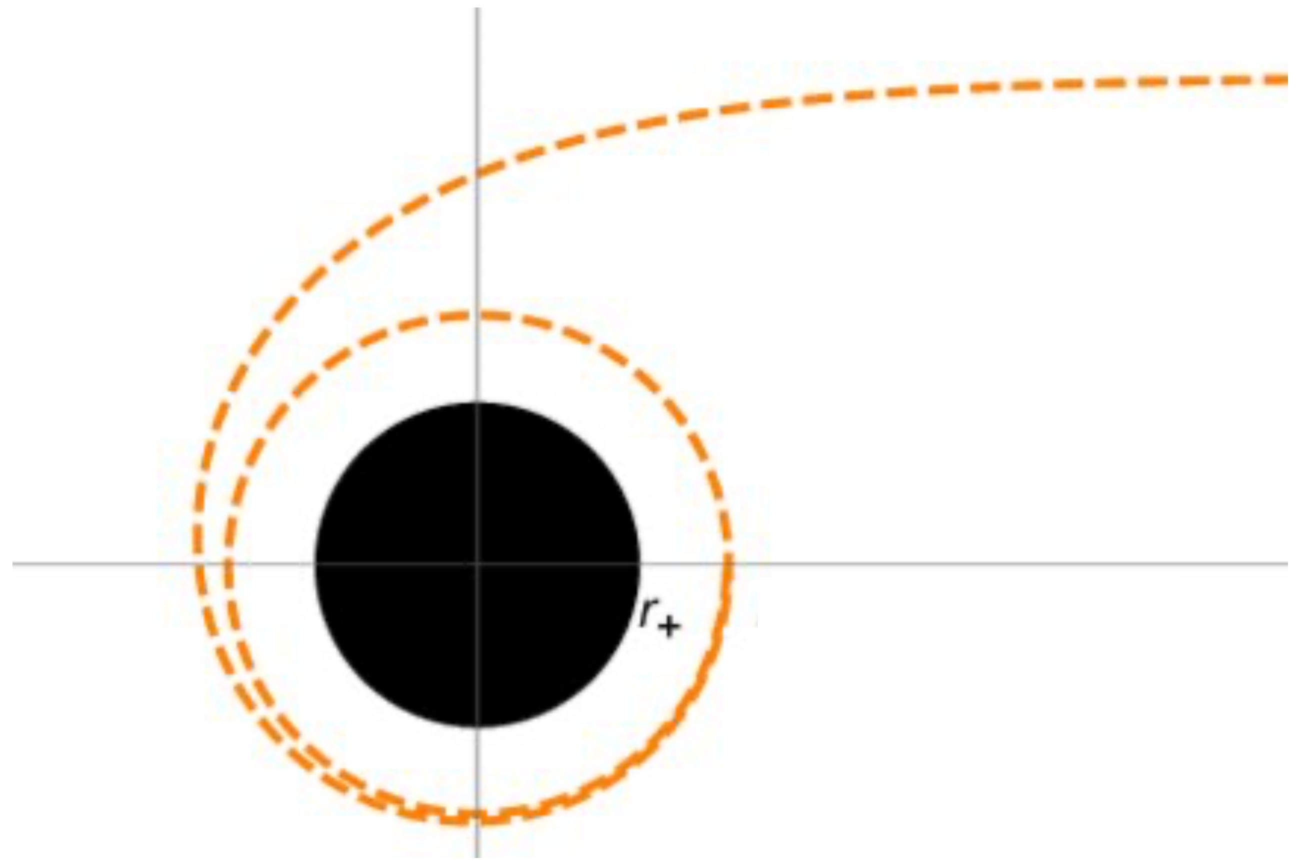
In GR eikonal QNMs are related to **unstable photon sphere geodesics**

[Cardoso+ ’08] [Yang+ ’12]

Real frequency ⟷ Orbital frequency

Damping time ⟷ Lyapunov exponent





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Beyond GR: Generalized correspondence

QNMs ⟷ Unstable **GW *orbits***

(not geodesic!)

Isospectrality   Non-birefringence

Summary

$$S_{\text{iso}} = \frac{1}{16\pi G}\int d^4x\sqrt{|g|}\left[R + \alpha\left(R_2^2 + \tilde{R}_2^2\right)\right]$$

1. Non-birefringent dispersion relation
2. Isospectral eikonal QNMs

} ***Isospectral EFTs***

Generalizable to higher orders

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} ***Isospectral EFTs*** — Generalizable to higher orders

Isospectrality related to String Theory

$$S_{\text{iso}} = S_{II}^{\text{string theory}}, \qquad \alpha = \frac{\zeta(3)}{256}\alpha'^3$$

Supersymmetry? Duality? Born-Infeld-like gravity?

Part 3: BH perturbations in the isospectral EFT

Master equation for perturbations

Dispersion relation for GWs

$$k^2 = 64\alpha R^{\lambda}{}_{\alpha}{}^{\eta}{}_{\beta} R^{\rho\alpha\sigma\beta} k_\lambda k_\eta k_\rho k_\sigma$$

Intuitive idea: **effective scalar equation** that yields the same dispersion relation

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$$\left(\nabla^2 + 64\alpha R^{\lambda}{}_{\alpha}{}^{\eta}{}_{\beta} R^{\rho\alpha\sigma\beta} \nabla_\lambda \nabla_\eta \nabla_\rho \nabla_\sigma \right) \Phi = 0$$

Master equation for perturbations

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More rigorously: $\mathscr{D}^2 h_{\mu\nu}^{\mathrm{TT}} = 0$ (diagonal operator=isospectrality)

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More rigorously: $\mathscr{D}^2 h^{\mathrm{TT}}_{\mu\nu} = 0$ (diagonal operator=isospectrality)

Remark: it is enough to consider the **Kerr background** (w/o corrections)

Solving the equation

Step 1: decompose the field in spheroidal harmonics

$$\Phi = e^{-i\omega t + im\varphi} \left[S_{lm}(x; a\omega)\psi_{lm}(r) + \alpha \sum_{l' \neq l} S_{l'm}(x; a\omega)\psi_{l'm}(r) \right]$$

Solving the equation

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Step 2: project the equation on S_{lm}

$$\int_{-1}^{1} dx S_{lm}(x; a\omega)(r^2 + a^2x^2)D^2\Phi = 0$$

Solving the equation

Step 1: decompose the field in spheroidal harmonics

$$\Phi = e^{-i\omega t + im\varphi}\left[S_{lm}(x;a\omega)\psi_{lm}(r) + \alpha\sum_{l'\neq l} S_{l'm}(x;a\omega)\psi_{l'm}(r)\right]$$

Step 2: project the equation on S_{lm}

$$\Delta\frac{d}{dr}\left[\Delta\frac{d\psi_{lm}}{dr}\right] + \left(V - \hat{\alpha}\Delta U_{lm}\right)\psi_{lm} = 0$$

$$U_{lm} = -1152M^8\Big(\underbrace{A_{lm} - 2ma\omega + (a\omega)^2}_{\lambda_{lm}}\Big)^2\int_{-1}^{1} dx\frac{S_{lm}(x;a\omega)^2}{2\pi(r^2+a^2x^2)^4}$$

Solving the equation

Step 3: simplify the potential

$$U_{lm} = -1152M^8\lambda_{lm}^2 \int_{-1}^{1} dx \frac{S_{lm}(x; a\omega)^2}{2\pi(r^2 + a^2x^2)^4}$$

Solving the equation

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$$U_{lm} = -\frac{576M^8\lambda_{lm}^2}{K(-k)} \int_0^{\pi} \frac{d\theta}{(r^2 + a^2x_0^2 \sin^2\theta)^4 \sqrt{1 + k\sin^2\theta}}$$

$$k = \frac{u^2x_0^2(1 - x_0^2)}{\mu^2 - u^2(1 - x_0^2)}, \quad \mu^2 - (1 - x_0^2)\left(\frac{A_{lm}}{l^2} + u^2x_0^2\right) = 0, \quad \mu = \frac{m}{l}, \quad u = \frac{a\omega}{l}$$

Solving the equation

Step 3 extended. We modify the integrand exploiting a gauge freedom

$$I_{lm} = \frac{1}{2\pi}\int_{-1}^{1} dx S_{lm}(x; a\omega)^2 f(x) = \frac{1}{2\pi}\int_{-1}^{1} dx S_{lm}(x; a\omega)^2 \left[f(x) + \mathcal{F}[h(x)]\right]$$

$\mathcal{F}[h]$ is certain 3rd order differential operator and $h(x)$ any smooth function such that $h(\pm 1) = 0$

Then, I_{lm} is the only constant for which the differential equation

$$f(x) + \mathcal{F}[h(x)] = I_{lm}$$

admits a smooth solution that vanishes on $x = \pm 1$.

In the eikonal regime, $\mathcal{F}[h]$ becomes of first order and the equation can be solved analytically.

Solving the equation

QNMs through the WKB formula

$$\Delta \frac{d}{dr}\left(\Delta \frac{d\psi}{dr}\right) + V_\alpha \psi = 0$$

$$V_\alpha = \left[\omega(r^2 + a^2) - am\right]^2 - \Delta\left(\lambda_{lm} + \hat{\alpha} U_{lm}\right)$$

Real part of the frequency: $V_\alpha(r_0) = \left.\frac{dV_\alpha}{dr}\right|_{r_0,\omega_R} = 0$

Imaginary part of the frequency: $\omega_I = -\left(n + \frac{1}{2}\right)\Delta \left.\frac{\sqrt{2\partial_r^2 V_\alpha}}{\partial_\omega V_\alpha}\right|_{r_0,\omega_R}$

Part 4: Results for QNMs

Results for QNMs

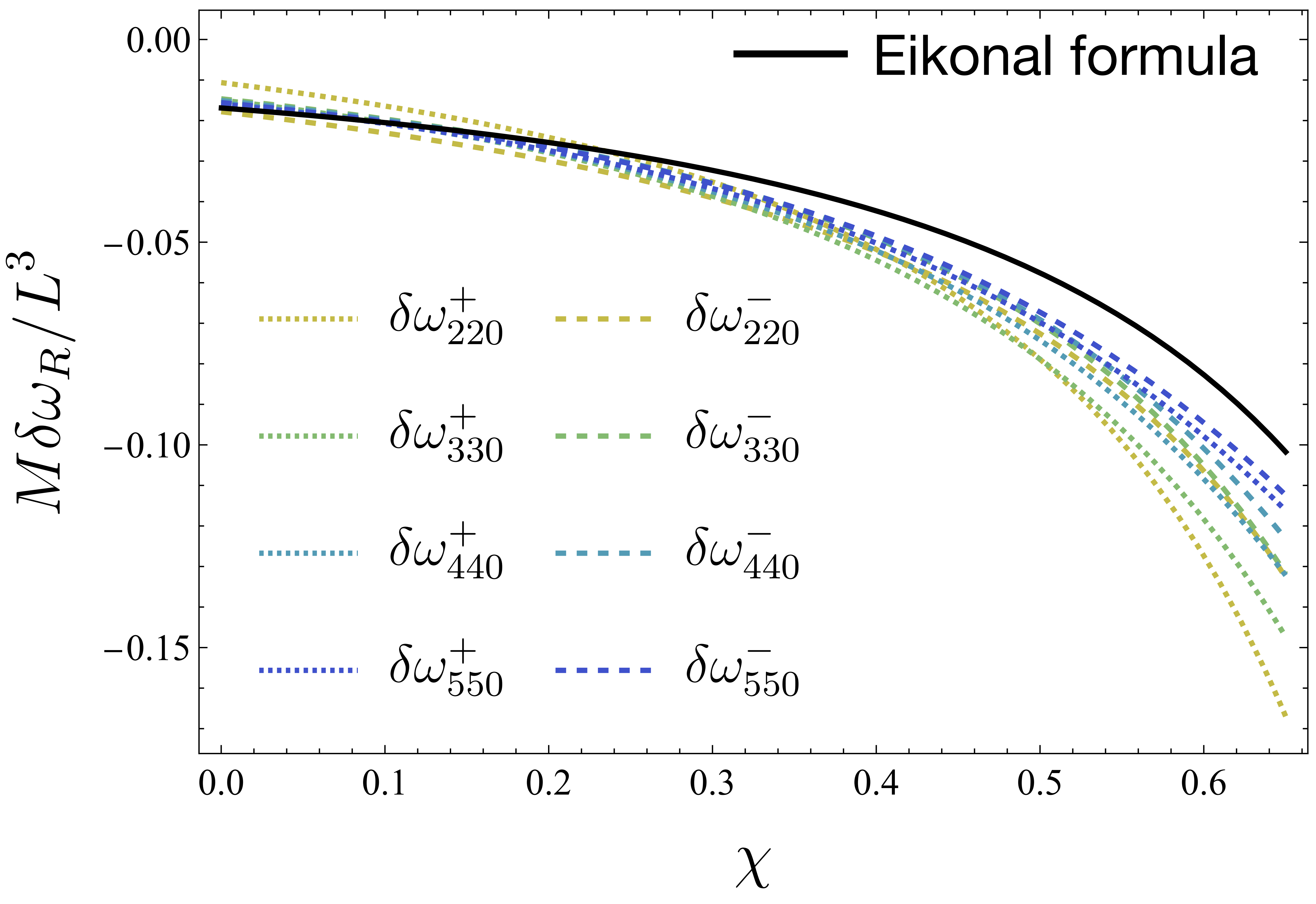
Generalities

- We write $\omega = \omega^{\mathrm{Kerr}} + \hat{\alpha}\delta\omega,$ $\qquad \hat{\alpha} = \frac{\alpha}{M^6}$
- Result depends on $\mu = m/\ell$ and on $\chi = J/J_{\mathrm{max}}$
- For $\mu > \mu_{\mathrm{cr}} \approx 0.74$ we have "**zero damping modes**" in the extremal limit
- For $\mu < \mu_{\mathrm{cr}}$ the modes are damped
- There are also ZDMs for $\mu < \mu_{\mathrm{cr}}$, but these are not captured by the WKB analysis [Yang+ '12, '13]

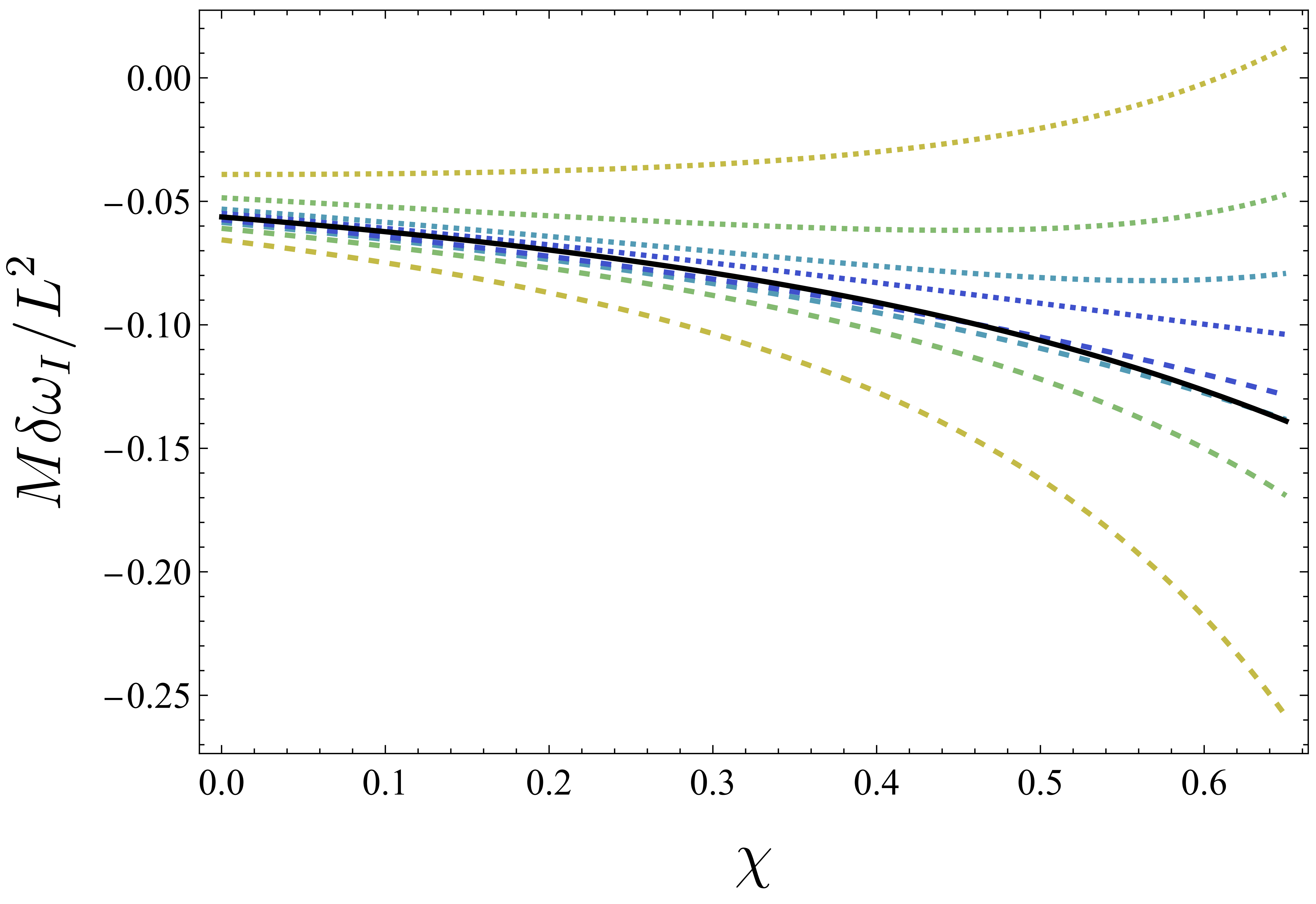
Comparison with modified Teukolsky

Convergence for l=m

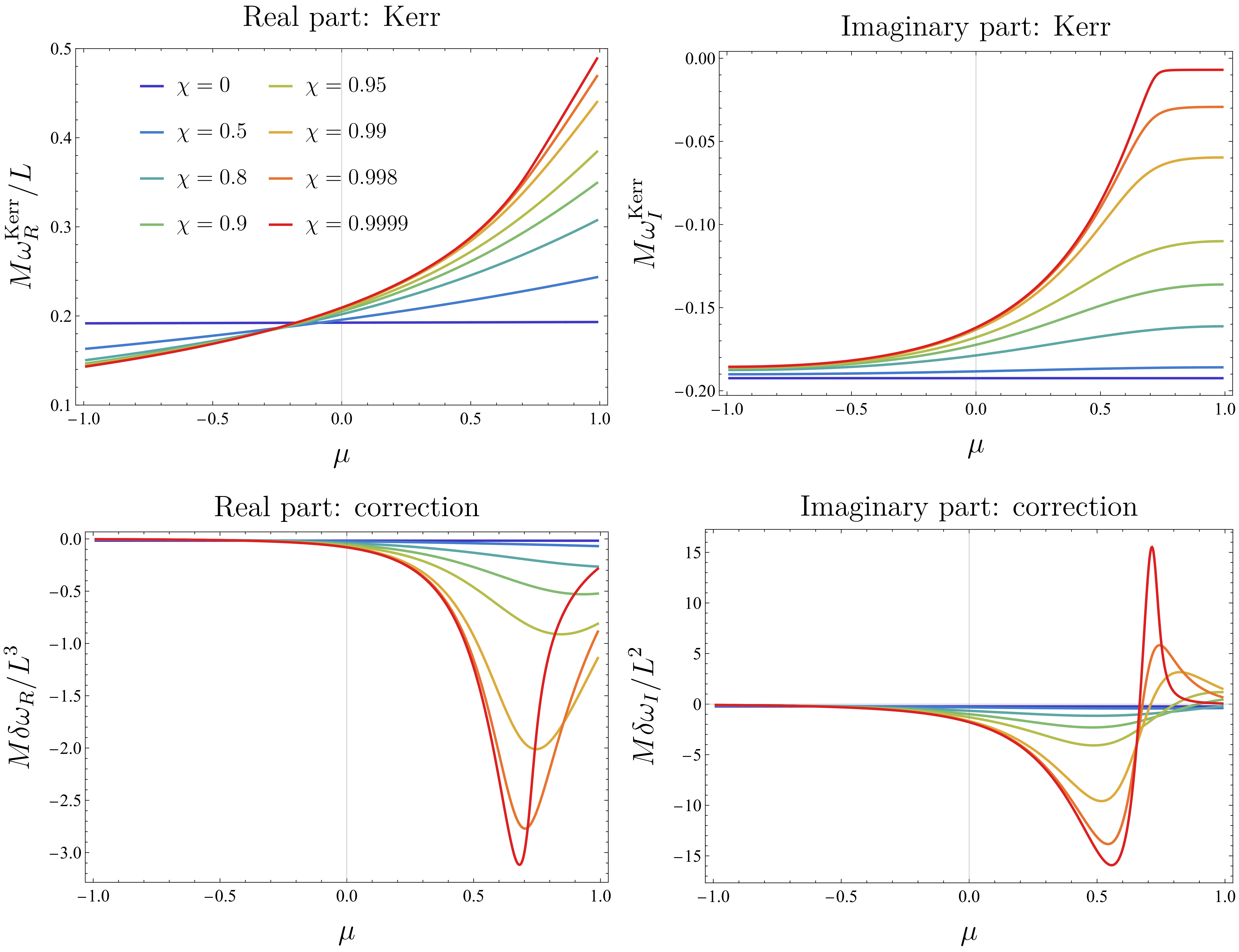
Real part



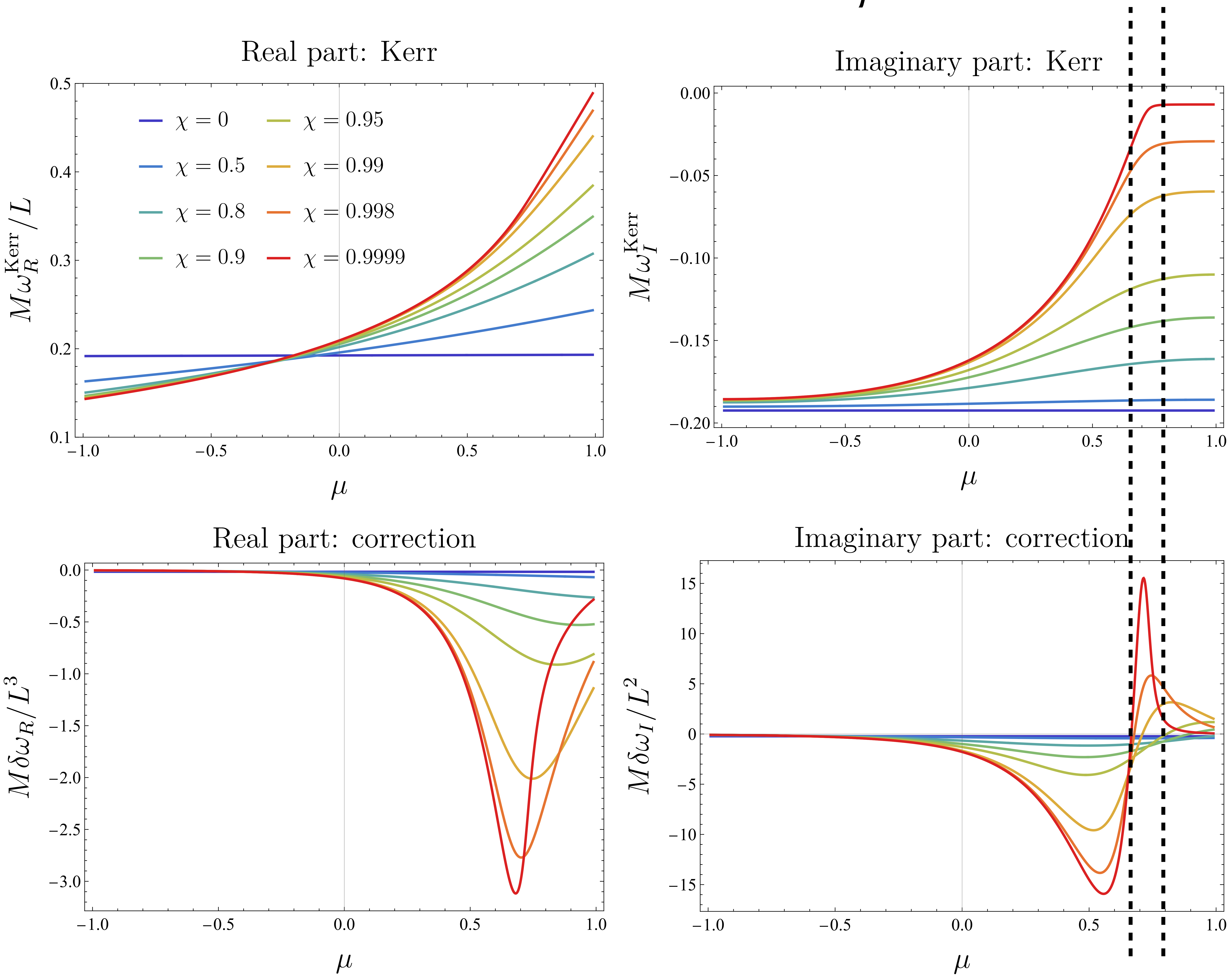
Imaginary part



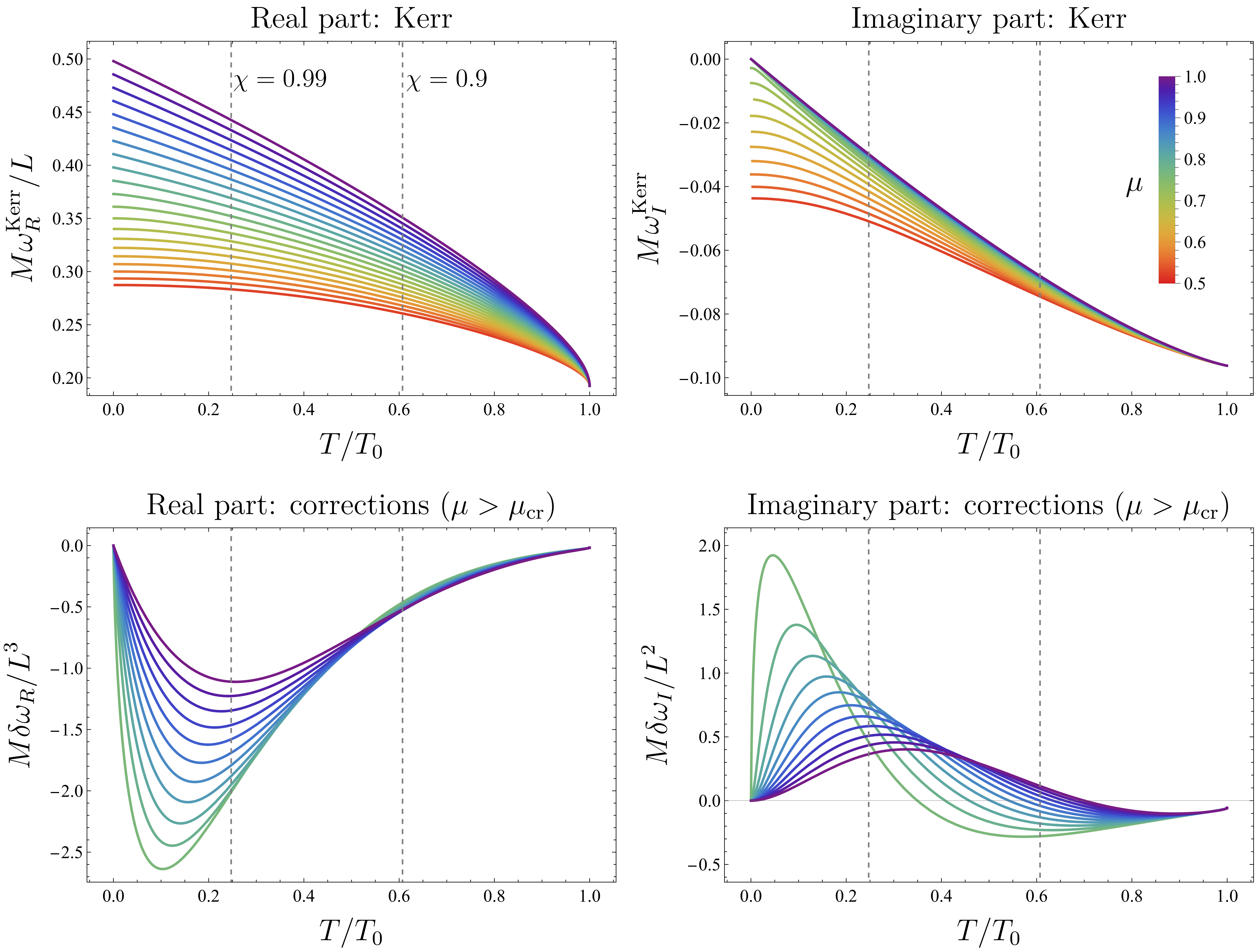
Results as a function of μ



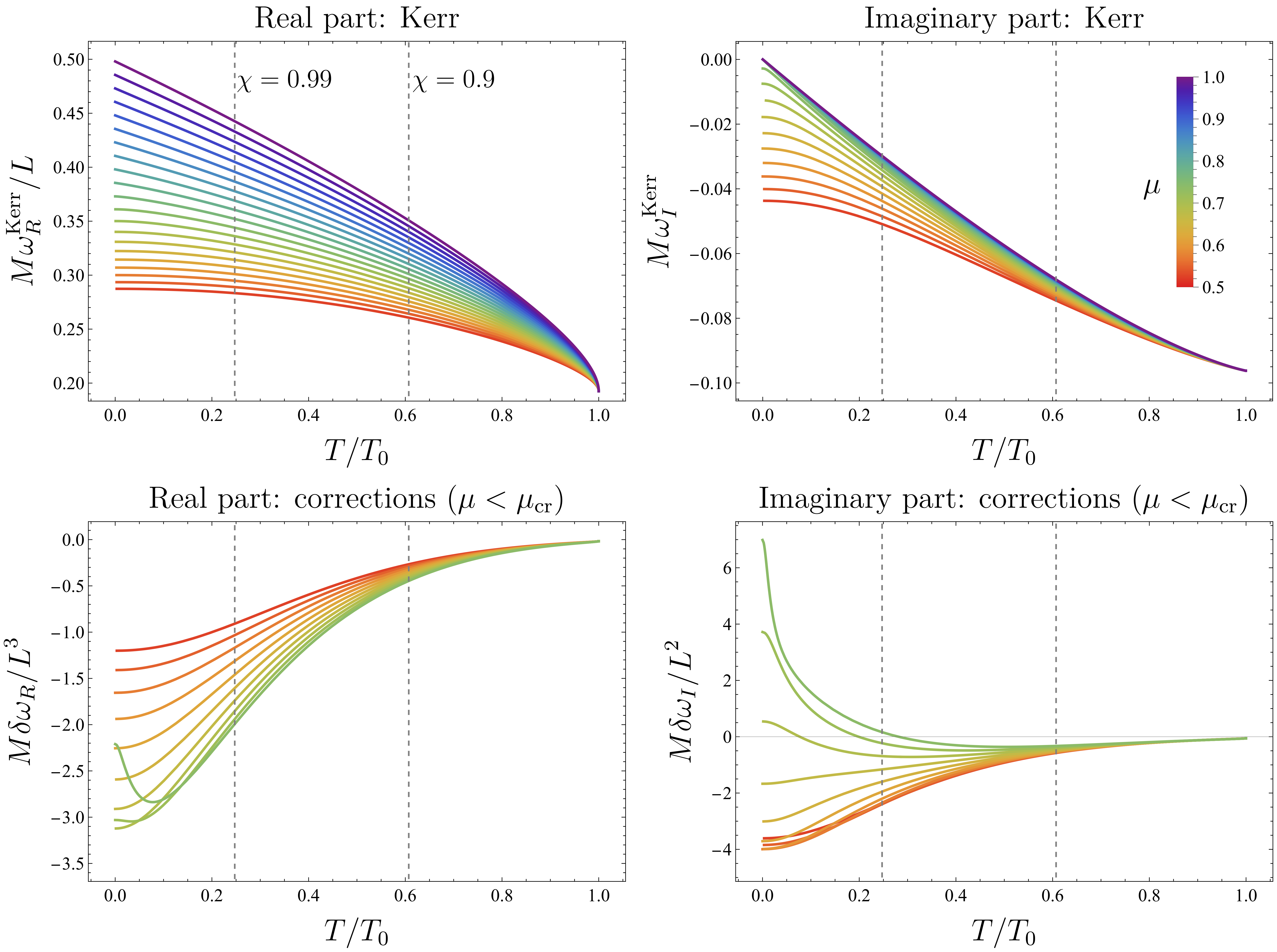
Results as a function of μ



Results as a function of the temperature



Results as a function of the temperature



Expansion near the critical point

Reminder: effective potential

$$\Delta \frac{d}{dr}\left(\Delta \frac{d\psi}{dr}\right) + V_\alpha \psi = 0$$

$$V_\alpha = \left[\omega(r^2 + a^2) - am\right]^2 - \Delta\left(\lambda_{lm} + \hat{\alpha} U_{lm}\right)$$

We look first for perturbative corrections to the Kerr QNMs

$$\omega = \omega^{\mathrm{Kerr}} + \hat{\alpha}\delta\omega$$

We consider $\epsilon \ll 1$ and $|\mu - \mu_{\mathrm{cr}}| \ll 1$

There are three different regimes depending on the relative size of ϵ and $|\mu - \mu_{\mathrm{cr}}|^3$

Expansion near the critical point

Regime I: $\epsilon \ll |\mu - \bar{\mu}_{\rm cr}|^3$, $\mu > \bar{\mu}_{\rm cr}$

$$\omega^{\rm Kerr} = m\Omega\left[1 - \sqrt{2\epsilon}\sqrt{\frac{7}{4} - \frac{A_{lm}(m/2)}{m^2}}\right] - \frac{i}{M}\left(n + \frac{1}{2}\right)\sqrt{\frac{\epsilon}{2}} + \mathcal{O}(\epsilon)$$

$$\delta\omega = \mathcal{O}(\epsilon^{1/2}) + i\mathcal{O}(\epsilon)$$

No qualitative change to the GR prediction

Expansion near the critical point

Regime II: $|\mu - \bar{\mu}_{\rm cr}|^3 \ll \epsilon \ll 1$

$$\omega^{\rm Kerr} = m\Omega \left[1 - \frac{3\epsilon^{2/3}}{2}\right] - \frac{i}{M}\left(n + \frac{1}{2}\right)\sqrt{\frac{3\epsilon}{4}} + \mathcal{O}(\epsilon)$$

$$\delta\omega = \frac{L^3\bar{\mu}_{\rm cr}^3}{4M}\xi\epsilon^{1/3} - i\left(n + \frac{1}{2}\right)\frac{L^2\bar{\mu}_{\rm cr}^2}{8\sqrt{3}M}\xi\epsilon^{1/6} + \ldots, \qquad \xi \approx -601$$

$$\frac{\delta\omega_I}{\omega_I^{\rm Kerr}} \approx \frac{L^2\bar{\mu}_{\rm cr}^2\xi}{12}\epsilon^{-1/3}$$

Expansion near the critical point

Regime III: $\epsilon \ll |\mu - \bar{\mu}_{\rm cr}|^3 \ll 1$, $\mu < \bar{\mu}_{\rm cr}$

$$\omega^{\rm Kerr} = m\Omega \left[1 + \frac{\eta^2(\mu - \bar{\mu}_{\rm cr})^2}{2}\right] - i\left(n + \frac{1}{2}\right)\frac{\eta^{3/2}|\mu - \bar{\mu}_{\rm cr}|^{3/2}}{2M} + \ldots$$

$$\delta\omega = \frac{L^3\bar{\mu}_{\rm cr}^3}{4M}\xi\eta|\mu - \bar{\mu}_{\rm cr}| - i\left(n + \frac{1}{2}\right)\frac{3}{8M}L^2\bar{\mu}_{\rm cr}^2\xi\eta^{1/2}|\mu - \bar{\mu}_{\rm cr}|^{1/2} + \ldots$$

$$\frac{\delta\omega_I}{\omega_I^{\rm Kerr}} \approx \frac{3L^2\bar{\mu}_{\rm cr}^2\xi}{4\eta|\mu - \bar{\mu}_{\rm cr}|}$$

Expansion near the critical point

Summary: amplification of new physics

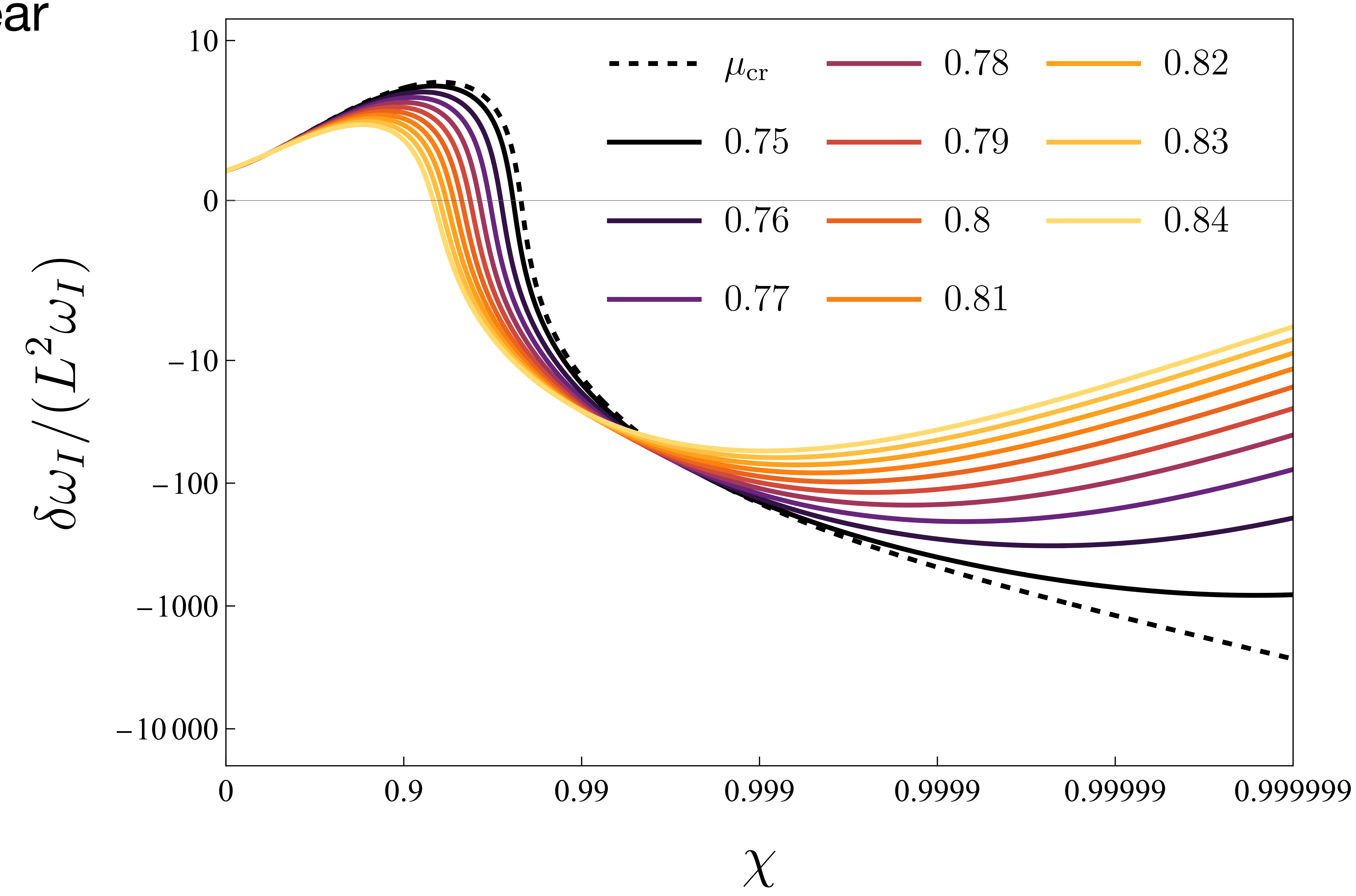
The corrections to ω_I diverge for modes near the critical line

$$\frac{\delta\omega_I}{\omega_I} \propto \hat{\alpha}\epsilon^{-1/3}$$

$$(|\mu - \bar{\mu}_{\rm cr}|^3 \ll \epsilon \ll 1)$$

$$\frac{\delta\omega_I}{\omega_I} \propto \hat{\alpha}\,|\mu - \bar{\mu}_{\rm cr}|^{-1}$$

$$(\epsilon \ll |\mu - \bar{\mu}_{\rm cr}|^3 \ll 1)$$



Non-perturbative analysis

Modification of the critical point

Observation: $r = M$ is always an extremum of V_α for $a = M, \omega = m\Omega$.

Condition for the existence of DMs is

$$E_{lm} \equiv \frac{1}{2}\frac{d^2V_\alpha}{dr^2}\bigg|_{r=M,a=M,\omega=m\Omega} = \frac{7}{4}m^2 - A_{lm} - \hat{\alpha}U_{lm} < 0$$

The phase boundary is modified

$$\mu_{\rm cr} \approx \bar{\mu}_{\rm cr} + \hat{\alpha}\delta\mu_{\rm cr}\,, \quad \delta\mu_{\rm cr} = \frac{L^2\bar{\mu}_{\rm cr}^2}{2\eta}\xi$$

Non-perturbative analysis

Full QNMs

When $|\mu - \bar{\mu}_{\rm cr}| \sim \hat{\alpha}\delta\mu_{\rm cr}$, the perturbative expansion in $\hat{\alpha}$ for the QNM frequencies breaks down.

We need to obtain the exact solution. For instance, in regime III:

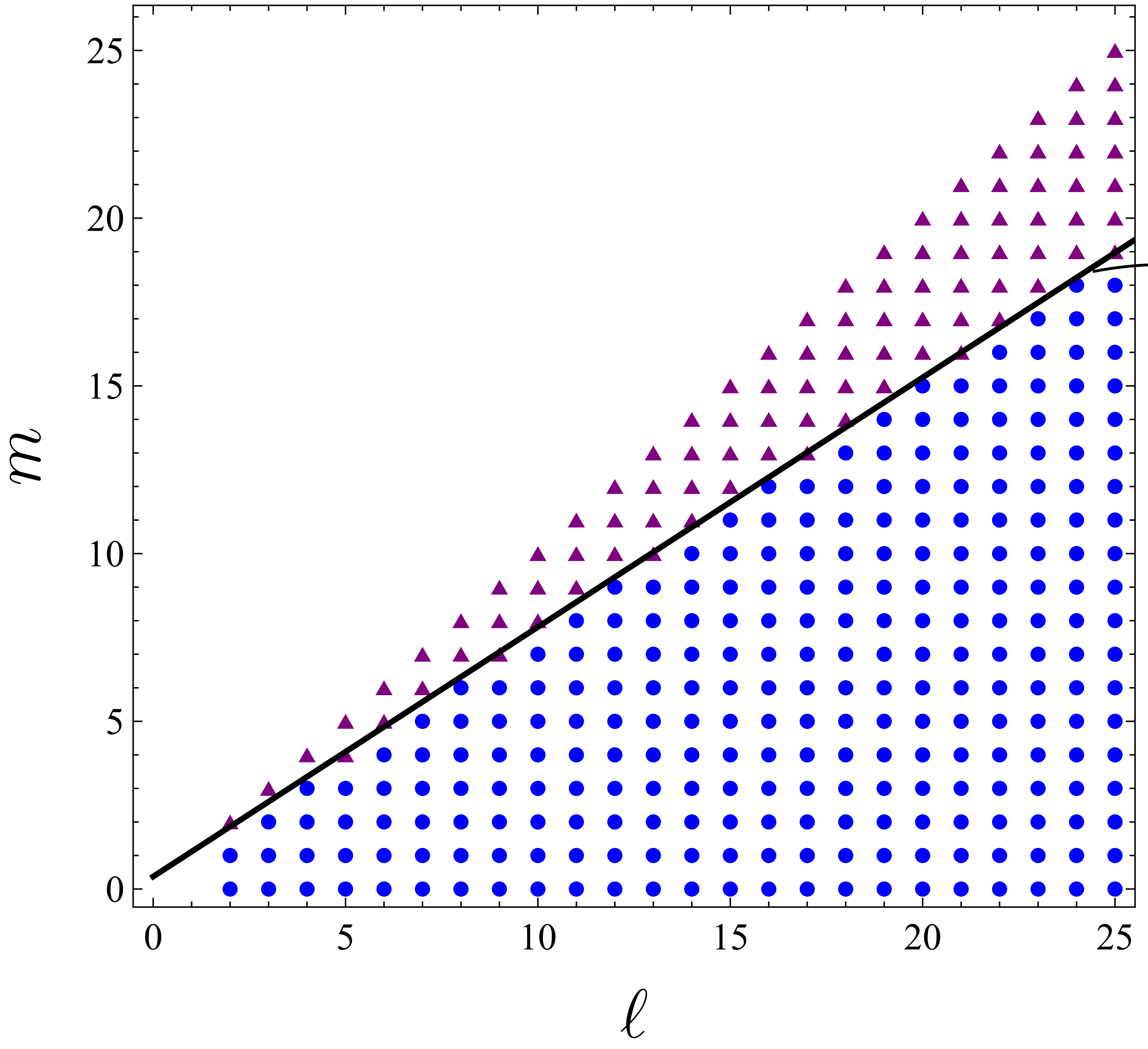
$$\omega_I \approx -(n+1/2)\frac{\eta^{3/2}|\mu - \mu_{\rm cr}|^{3/2}}{2M}\left(1 + \mathcal{O}(\hat{\alpha})\right)$$

$$\frac{\omega_I}{(n+1/2)} \approx -\frac{\eta^{3/2}}{2M}|\mu - \bar{\mu}_{\rm cr}|^{3/2} - \frac{3\eta^{3/2}}{4M}\hat{\alpha}\delta\mu_{\rm cr}|\mu - \bar{\mu}_{\rm cr}|^{1/2} + \dots ,$$

→ **The divergence is a consequence of the change in the critical point**

Non-perturbative analysis

Qualitative change in the spectrum



▲ = only ZDMs exist (GR)

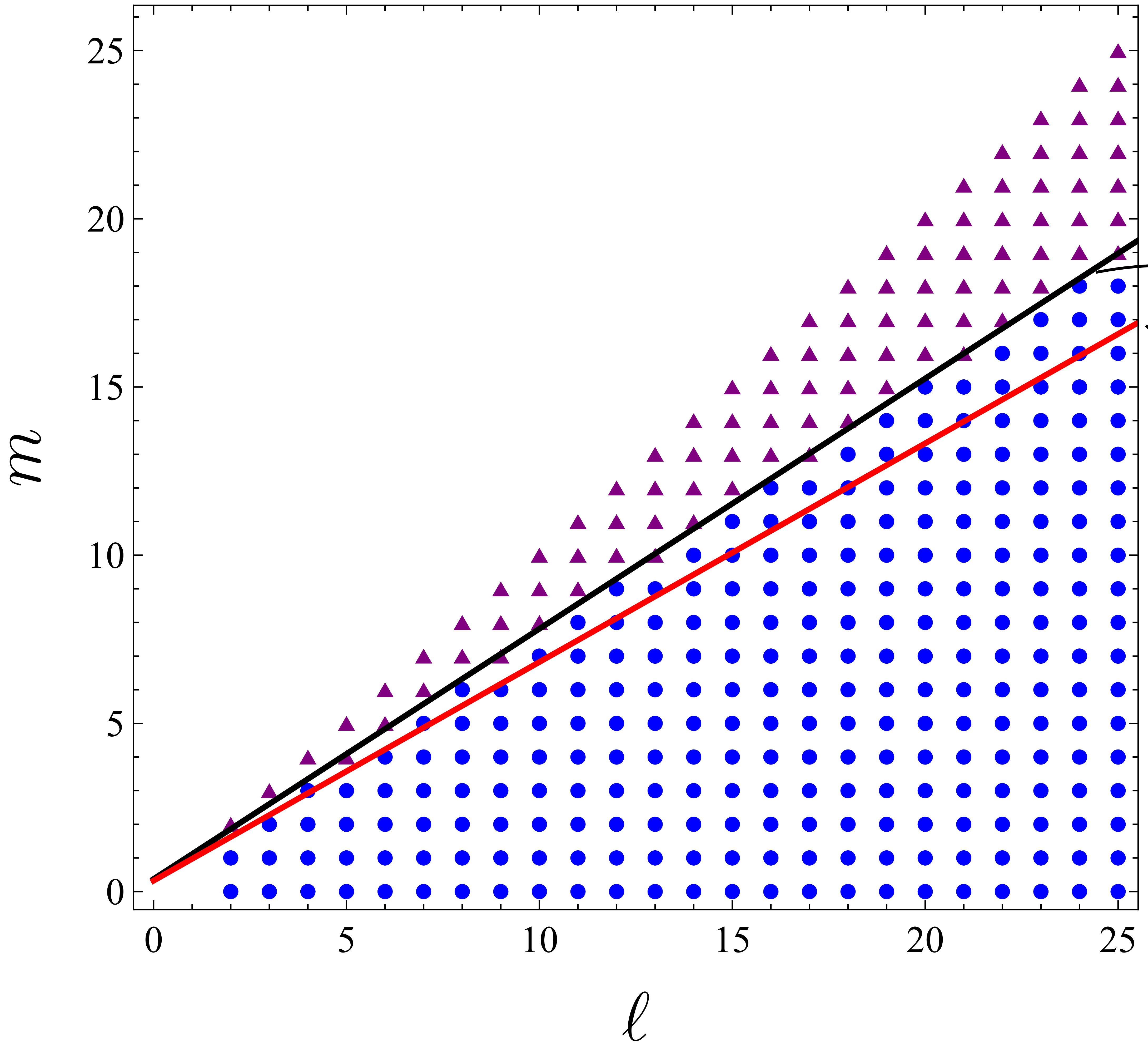
● = ZDMs and DMs exist (GR)

Phase boundary obtained from the eikonal limit

$$\frac{m}{\ell + 1/2} = \bar{\mu}_{\mathrm{cr}} \approx 0.744$$

Non-perturbative analysis

Qualitative change in the spectrum



▲ = only ZDMs exist (GR)

● = ZDMs and DMs exist (GR)

Phase boundary obtained from the eikonal limit

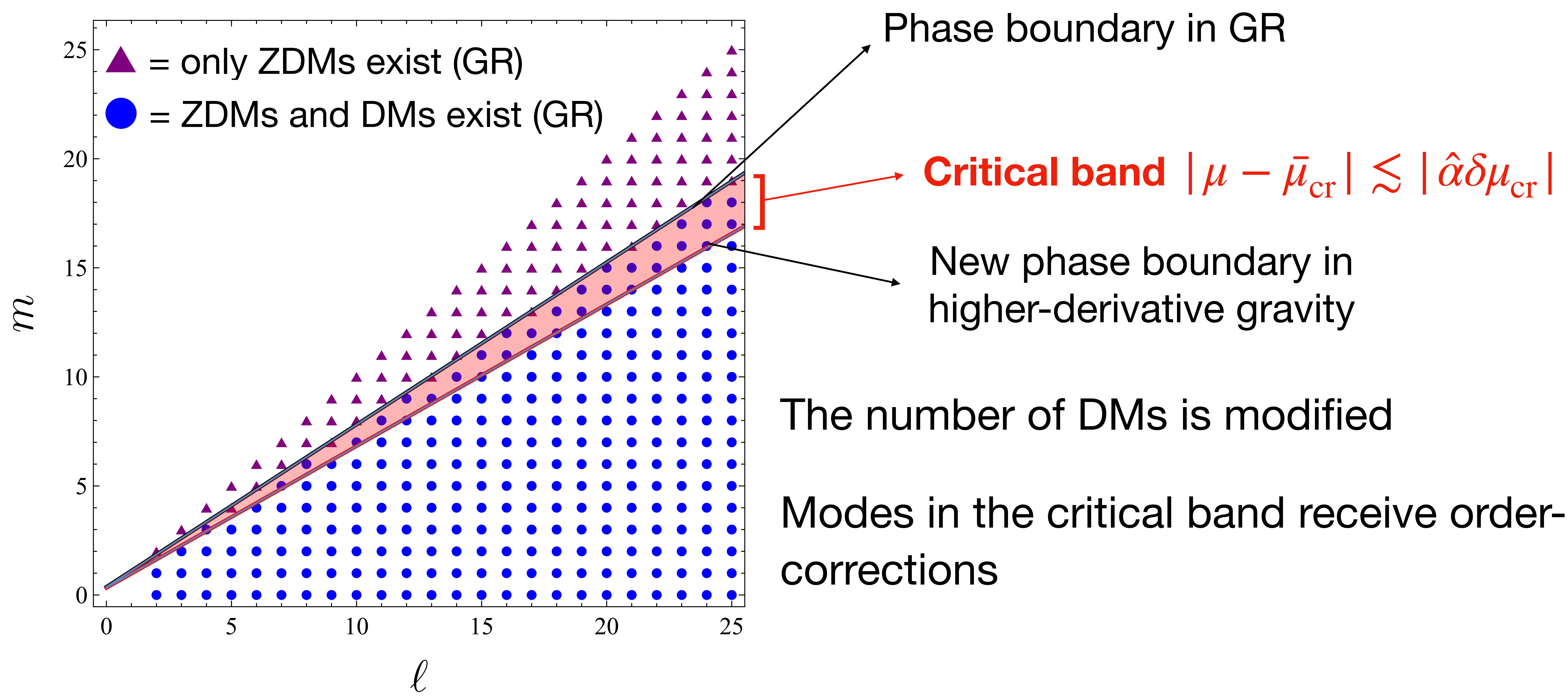
$$\frac{m}{\ell + 1/2} = \bar{\mu}_{\rm cr} \approx 0.744$$

New phase boundary in higher-derivative gravity

$$\mu_{\rm cr} = \bar{\mu}_{\rm cr} + \hat{\alpha}\delta\mu_{\rm cr}$$

Non-perturbative analysis

Qualitative change in the spectrum



The number of DMs is modified

Modes in the critical band receive order-one corrections

Regime of validity

Are there modes in the critical band within the regime of validity of the EFT?

Remark: $\mu = m/L$ is rational

Dirichlet theorem $\Rightarrow$ many modes with $|\mu - \bar{\mu}_{\rm cr}| \sim 1/L^2$

On the other hand the critical band is $|\mu - \bar{\mu}_{\rm cr}| < |\hat{\alpha}\delta\mu_{\rm cr}| \sim 71.4|\hat{\alpha}|L^2$

$$71.4|\hat{\alpha}|L^4 \gtrsim 1$$

Condition for modes in the band

Observe: it can be satisfied even if $|\hat{\alpha}| \ll 1$

Regime of validity

EFT regime 1: Wilsonian point of view

We assume $\alpha \sim \ell_{\rm UV}^6$, with $\ell_{\rm UV} = E_{\rm UV}^{-1}$ related to massive degrees of freedom

The EFT is only reliable for resolving distances grater than $\ell_{\rm UV}$

$$|\hat{\alpha}| = \frac{\ell_{\rm UV}^6}{M^6} \ll 1 \text{ (BH radius larger than } \ell_{\rm UV}\text{)}$$

$$L \ll |\hat{\alpha}|^{-1/6} \text{ (wavelength larger than } \ell_{\rm UV}\text{)}$$

$\Rightarrow 71.4\,|\hat{\alpha}|\,L^4 \ll |\hat{\alpha}|^{1/3} \ll 1 \Rightarrow$ **No modes in the critical band for large L**

Still, modes near the critical line get corrections of order $|\hat{\alpha}|\,L^4 \gg |\hat{\alpha}|\,L^2$

Regime of validity

EFT regime 2: convergence of the higher-derivative expansion

The EFT is classically consistent if additional HD corrections can be neglected

Result: in the regime in which $|\hat{\alpha}| \ll 1$ and $|\hat{\alpha}|L^4 \sim 1$, the effect of any additional HD correction on the QNM frequencies is negligible compared to the $\alpha\mathscr{R}^4$ term

→Large changes in the QNM spectrum can take place consistently

Conclusion: as a classical theory, the EFT is consistent. Whether we can trust these predictions depends entirely on the UV completion

Sensitivity of lower l modes

Exact condition for the phase boundary

[Detweiler '80] [Yang+'12]

$$ {}_sE^{\text{Kerr}}_{lm} = \frac{7m^2}{4} - s(s+1) - {}_sA_{lm}(m/2) > 0 \qquad \text{(No DMs)} $$

This is $V''(r_+)\Big|_{\omega=m\Omega, a=M}$ expressed in a real form of the Teukolsky equation.

Remark: we only need to know the near-horizon extremal Teukolsky equation.

Modified near-horizon Teukolsky equation [PAC, David '24]

$$ {}_sA_{lm} \to {}_sA_{lm} + \hat{\alpha}\delta A^{\pm}_{lm} $$

Sensitivity of lower l modes

Exact condition for the phase boundary

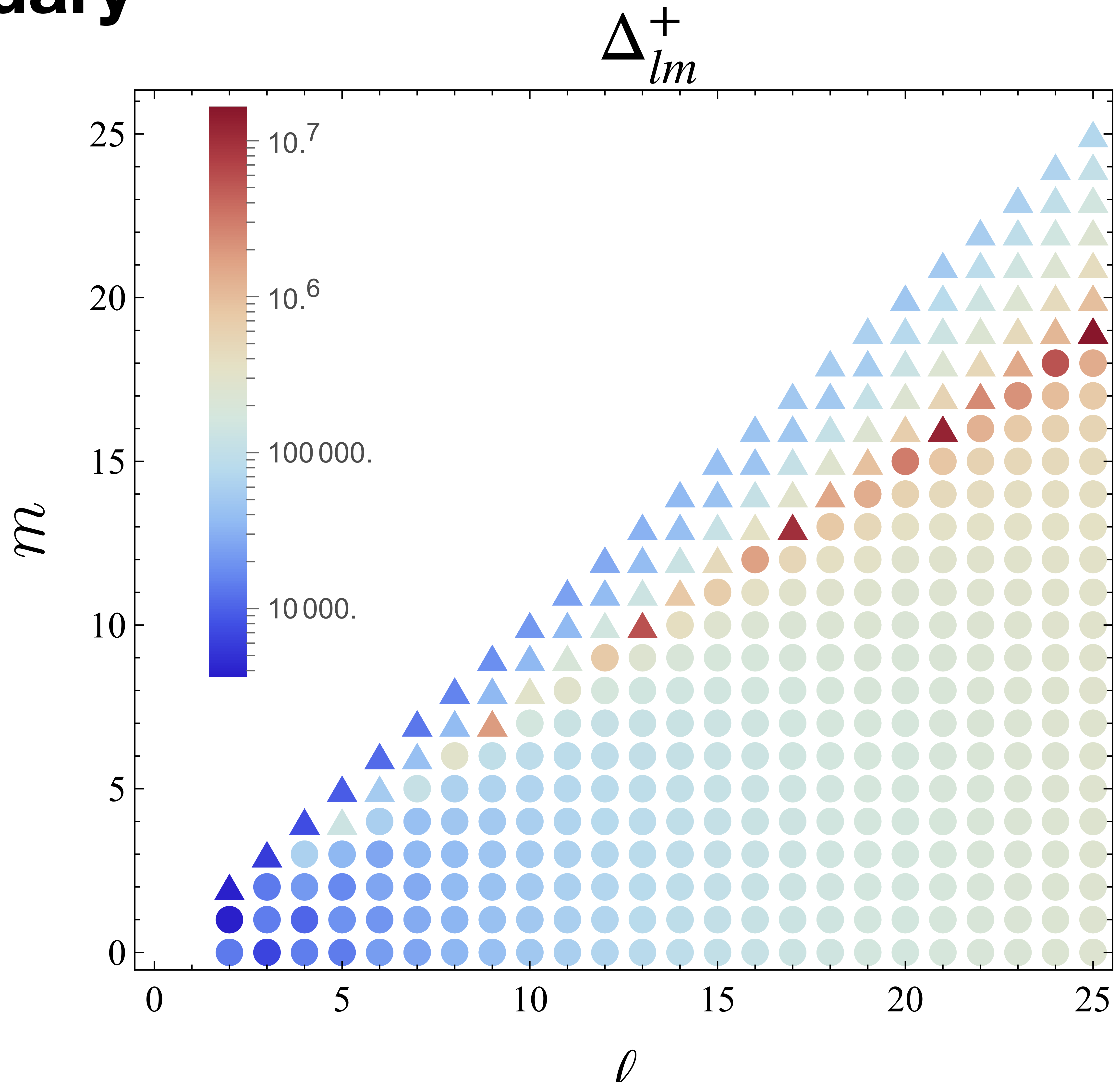
Modified phase boundary

$$ {}_sE_{lm} = \frac{7m^2}{4} - s(s+1) - {}_sA_{lm} - \hat{\alpha}\delta A^{\pm}_{lm} > 0 $$

Relative correction:

$$ \Delta^{\pm}_{lm} = \frac{\delta A^{\pm}_{lm}}{\frac{7m^2}{4} - s(s+1) - {}_sA_{lm}} $$

$$ \omega_I \sim |{}_sE_{lm}|^{3/2} \;\Rightarrow\; \delta\omega_I/\omega_I \sim 3/2\Delta^{\pm}_{lm} $$



Conclusions

Specific remarks

- First computation of **gravitational QNMs with high rotation** beyond GR
- **Key development: effective scalar equation** for eikonal perturbations in "isospectral" theories
- Master equation could have more applications: **time domain simulations?**
- **Future work:** extension for non-isospectral theories

Conclusions

General remarks

- Beyond-GR effects **increase dramatically** for high rotation
- Highly-rotating BHs have long-lived modes: **high-precision spectroscopy**

Highly rotating BHs → Golden events to test new physics

Conclusions

General remarks

- Beyond-GR effects **increase dramatically** for high rotation
- Highly-rotating BHs have long-lived modes: **high-precision spectroscopy**

Highly rotating BHs → Golden events to test new physics

Open questions

- QNM computation for lower l
- Implications for the time domain signal

Thank you

Bonus slides

Why test EFT corrections

EFT is the main hypothesis for beyond-GR physics

Conditions for a theory to be **viable:**

1. It's not ruled out by other experiments
2. It has full predictive power
3. It CAN be tested with GWs

Very few “alternatives" to GR remain. EFT is the best motivated one

Observability of higher-derivative corrections

Relative corrections to GR = Const × Δ

$$\Delta = \frac{\ell^4 (GM)^2}{r^6}$$

$$\Delta_{\text{Sun}} \sim \left(\frac{\ell}{5\times 10^8\text{km}}\right)^4, \quad \Delta_{\text{Earth}} \sim \left(\frac{\ell}{2\times 10^8\text{km}}\right)^4, \quad \Delta_{BH}(10M_\odot) \sim \left(\frac{\ell}{40\text{km}}\right)^4$$

30 orders of magnitude increase

In addition, "Const" can become large in special cases (high rotation)