The Dynamics of Black Hole Mergers and Gravitational Wave Generation

IFPU, Trieste, 12-16 Jan

THE FELLOWHIP OF THE CURVATURES

$f(\mathcal{R})$ MEETS GAUSS-BONNET

Andrea P. Sanna

Based on: PRD **112** (2025) [arXiv:2510.17965]

In collaboration w/ Fabrizio Corelli & Paolo Pani

INFN ROMA
Istituto Nazionale di Fisica Nucleare
Sezione di Roma

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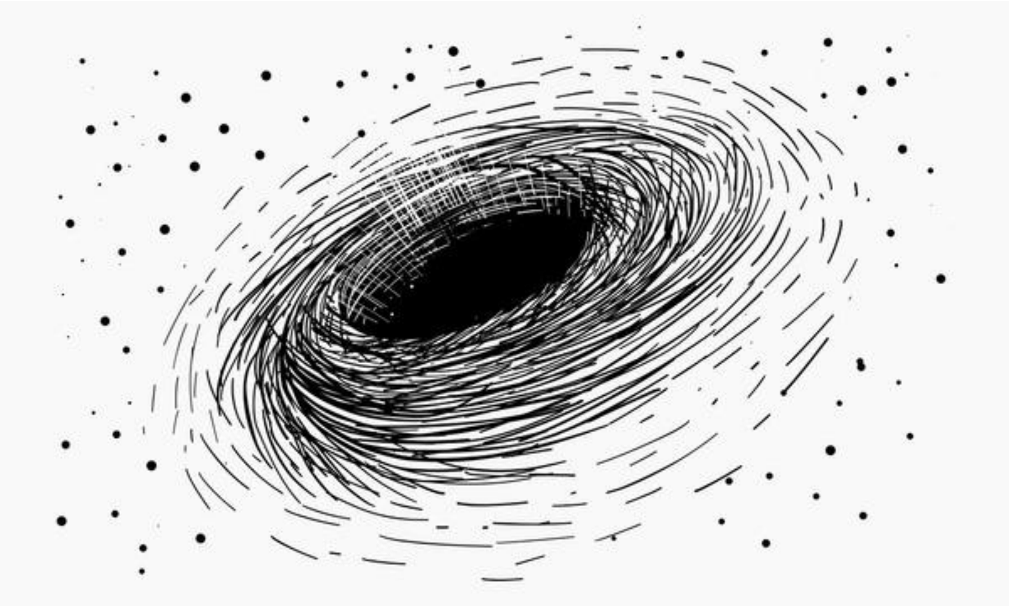
Ministero dell'Università e della Ricerca

Motivations

GR breaks down in the UV (high curvature regime)

↓

Consider higher-curvature terms? [Stelle (1977)…]



Constructing healthy theories implies having **second-order** differential equations

[Ostrogradsky (1850)]

Horndeski Theories

[Horndeski (1974)]

Motivations

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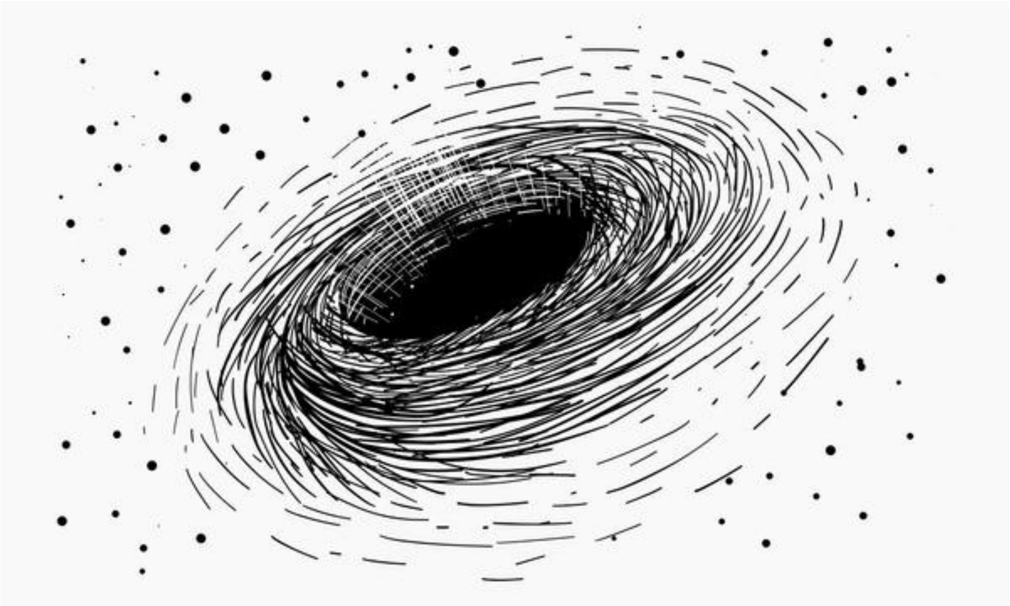


Constructing healthy theories implies having second-order differential equations

[Ostrogradsky (1850)]

$\boldsymbol{f(\mathcal{R})}$

Einstein-dilaton-Gauss-Bonnet

$f(\mathcal{R})$

$$S = \frac{1}{16\pi}\int d^4x\,\sqrt{-g}\,f(\mathcal{R})$$

It shares the same Ricci-constant solutions of GR

For reasonable $f(\mathcal{R})$, vacuum BH solutions are the same as in GR

[Sotiriou & Faraoni (2010)]

Trivial for the UV structure of BHs

Einstein-dilaton-Gauss-Bonnet

$$S = \frac{1}{16\pi}\int d^4x\,\sqrt{-g}\,[\mathcal{R} - (\nabla\phi)^2 + 2\eta(\phi)\mathcal{G}]$$

Quadratic-curvature terms, but second-order equations

Admits black-hole solutions different from GR

But problems lie ahead….

EdGB: nuts and bolts

$$\mathcal{G} = \mathcal{R}^2 - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}$$

$$S = \frac{1}{16\pi}\int d^4x\,\sqrt{-g}\,[\mathcal{R} - (\nabla\phi)^2 + 2\eta(\phi)\,\mathcal{G}]$$

Dilatonic coupling

$$\eta(\phi) = \lambda e^{-\gamma\phi}$$

Low-energy truncation of string theory [Gross & Sloan (1987)]

$$[\lambda] = \ell^2$$

Symmetry under transformations $\boxed{\phi \to \phi + C \qquad \lambda \to \lambda\, e^{\gamma C}}$

Solutions

1. Perturbative expansion in small λ [Mignemi & Stewart (1993)]

2. Nonperturbative methods: numerical solutions [Kanti+ (1996)]

EdGB: static asymptotically-flat black holes (exterior)

$$ds^2 = -\alpha(r)^2 dt^2 + [dr + \alpha(r)\,\zeta(r) dt]^2 + r^2 d\Omega_2^2 \qquad \phi = \phi(r)$$

r_H

$\alpha(r_H)$

$\boldsymbol{\phi(r_H)}$

$\phi'(r_H)$

$\zeta(r_H) = 1$

Regularity at the horizon

r_∞

$\alpha(r) \sim \mathrm{A}$

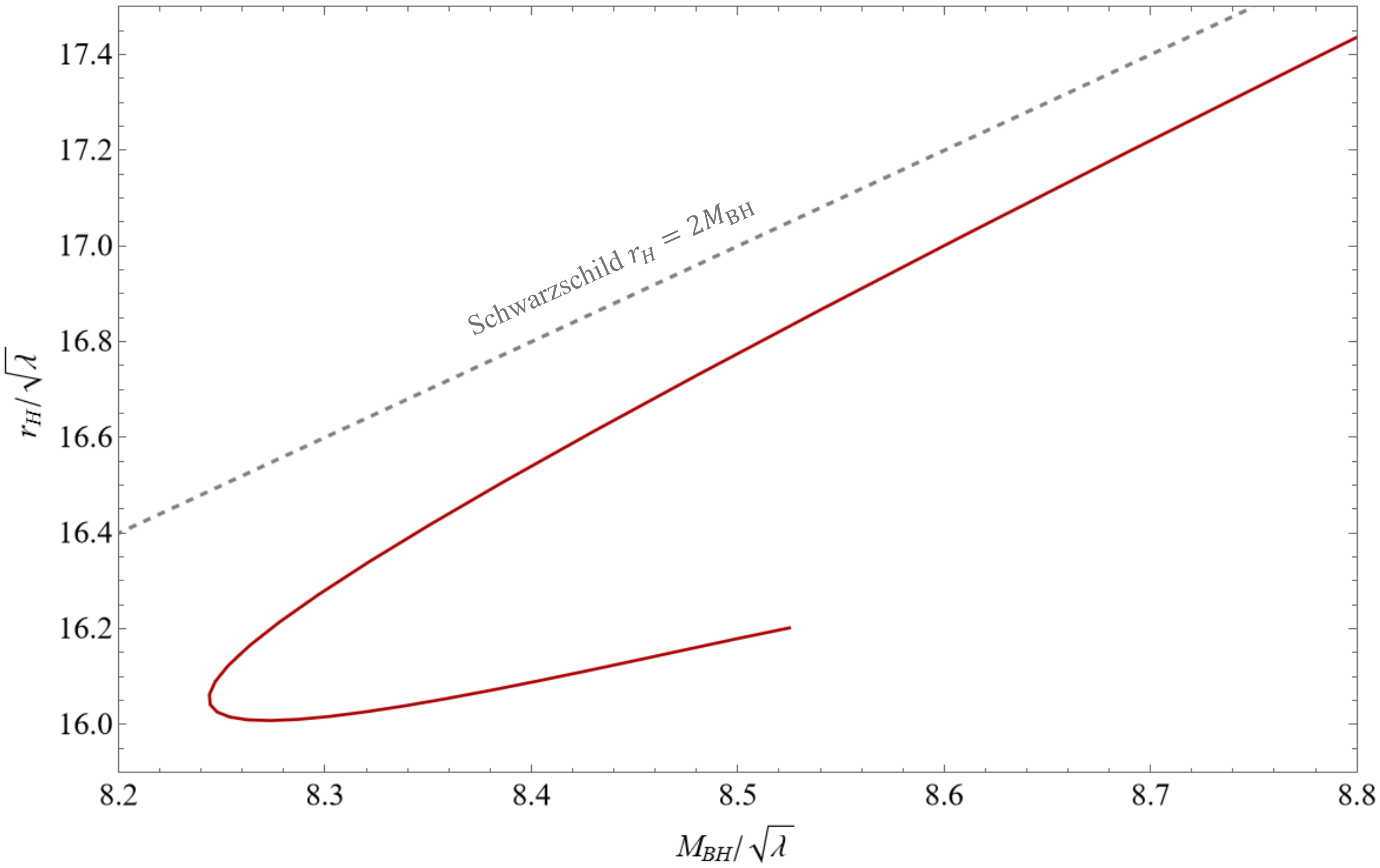
$\phi(r) \sim \frac{\boldsymbol{D}}{r}$

$\zeta(r) \sim \sqrt{\frac{2\boldsymbol{M}_{\mathrm{BH}}}{r}}$

$$\phi \rightarrow \phi + C \qquad \lambda \rightarrow \lambda\, e^{\gamma C}$$

EdGB: mass-radius diagram

$$M_{\rm BH} = \frac{r_\infty}{2}\zeta(r_\infty)^2 \qquad D = -r_\infty^2\,\phi'(r_\infty)$$



No «GR branch»

$\phi = 0$ **not** a solution

$$\Box\phi = -\eta'(\phi)\,\mathcal{G}$$

Always different from zero

[Kanti+(1996), Torii+(1996), Torii & Maeda(1998), Guo+(2008), Ripley & Pretorius(2020), Corelli+(2022, 2023)]

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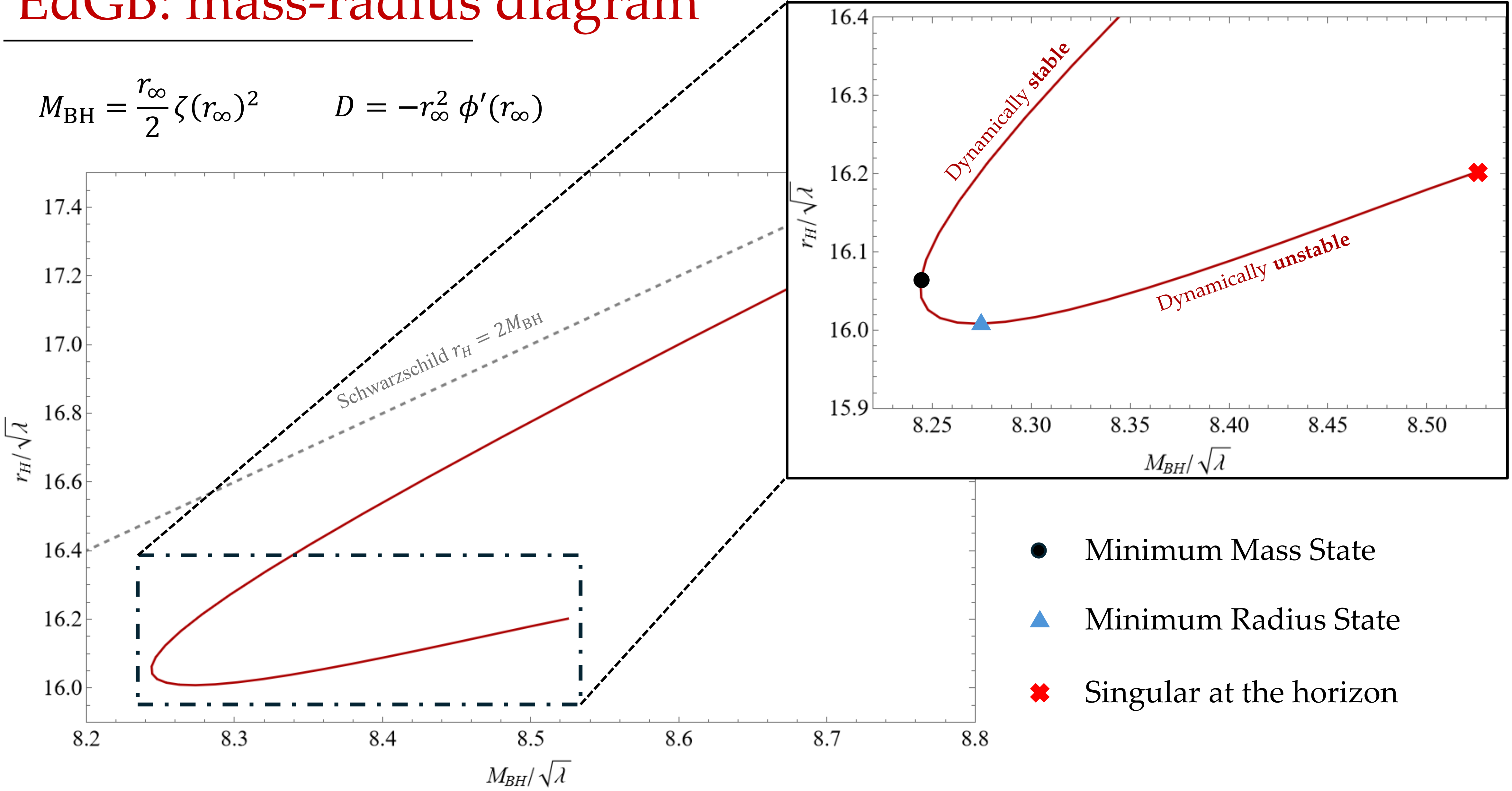

● Minimum Mass State

▲ Minimum Radius State

✖ Singular at the horizon

[Kanti+(1996), Torii+(1996), Torii & Maeda(1998), Guo+(2008), Ripley & Pretorius(2020), Corelli+(2022, 2023)]

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EdGB: static asymptotically-flat black holes (interior)

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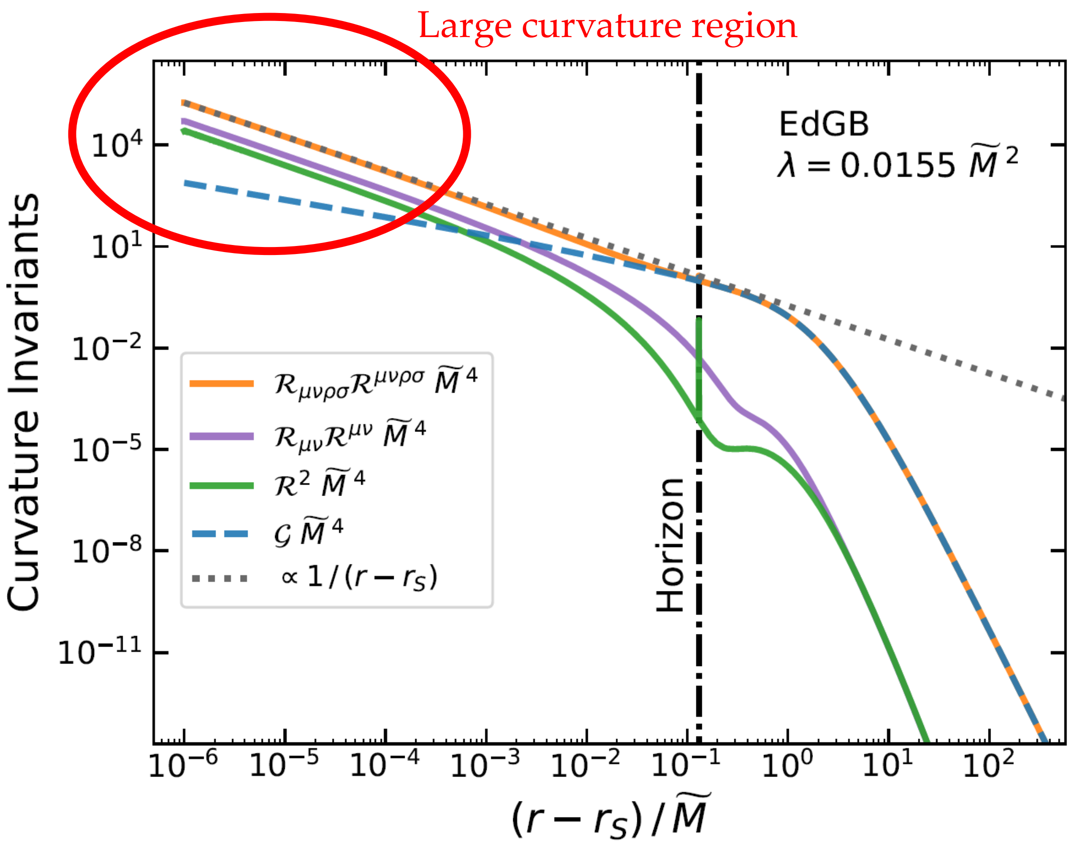

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$\phi(r_H)$

$\phi'(r_H)$

$\zeta(r_H) = 1$

r_∞

$\alpha(r) \sim A$

$\phi(r) \sim -\dfrac{D}{r}$

$\zeta(r) \sim \sqrt{\dfrac{2M_{\rm BH}}{r}}$

[Alexeyev & Pomazanov (1996), Sotiriou & Zhou (2014)]

EdGB: static asymptotically-flat black holes (interior)

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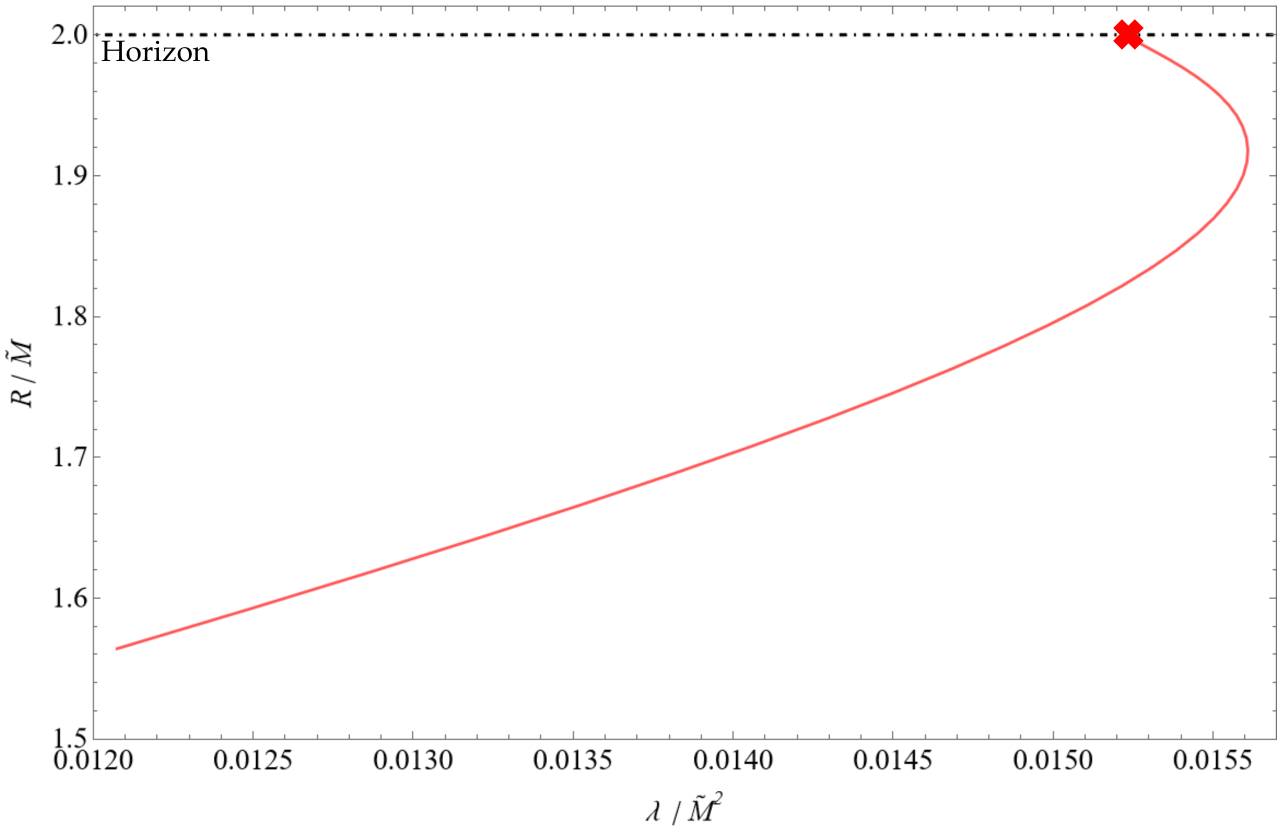

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[Corelli+(2022, 2023)]

EdGB: loss of hyperbolicity

Starting from initial regular data, does the evolution depend continously on the initial data?

The system of equations must be **strongly hyperbolic**

[Sarbach & Tiglio (2012), Hilditch (2013)]

Time-dependent case $[\alpha(r), \zeta(r), \phi(r)] \rightarrow [\alpha(r,t), \zeta(r,t), \phi(r,t)]$

Auxiliary field variable + conjugate momentum

$$Q \equiv \partial_r \phi$$
$$P \equiv \frac{1}{\alpha}\partial_t \phi - \zeta Q$$

$\longrightarrow$ Evolution equations + 2 constraints for α & ζ

Principal symbol $\mathcal{P}_{IJ}(\eta_\mu) = \dfrac{\delta E_{v^I}}{\delta\, \partial_\mu v^J} \eta_\mu$ $\quad v^I = (\phi, Q, P, \alpha, \zeta)$

Strong hyperbolicity $\equiv$ Complete set of η_μ's satisfying $\det \mathcal{P}(\eta_\mu) = 0$

EdGB: loss of hyperbolicity

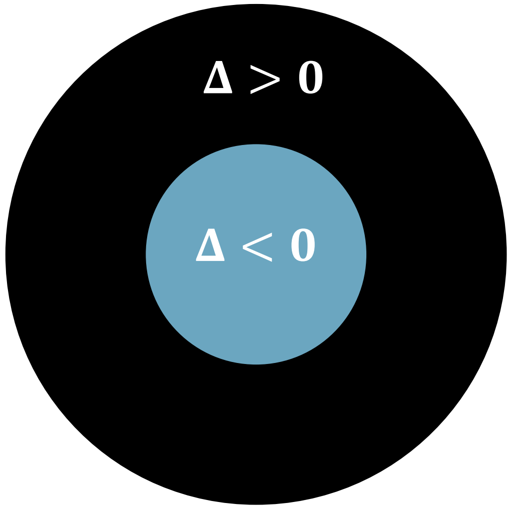
Strong hyperbolicity $\equiv$ Complete set of η_μ's satisfying $\det \mathcal{P}(\eta_\mu) = 0$

$$\det \mathcal{P} \propto \eta_t\, \eta_r^2 \left[\mathfrak{a} \left(\frac{\eta_t}{\eta_r}\right)^2 + \mathfrak{b}\left(\frac{\eta_t}{\eta_r}\right) + \mathfrak{c}\right] = 0$$

$\eta_t = 0$ Redundancy of the equation for $\partial_t \phi$

$\eta_r = 0$ 2 constraints for α and ζ

Real distinct solutions if $\Delta = \mathfrak{b}^2 - 4\mathfrak{a}\mathfrak{c} > 0$

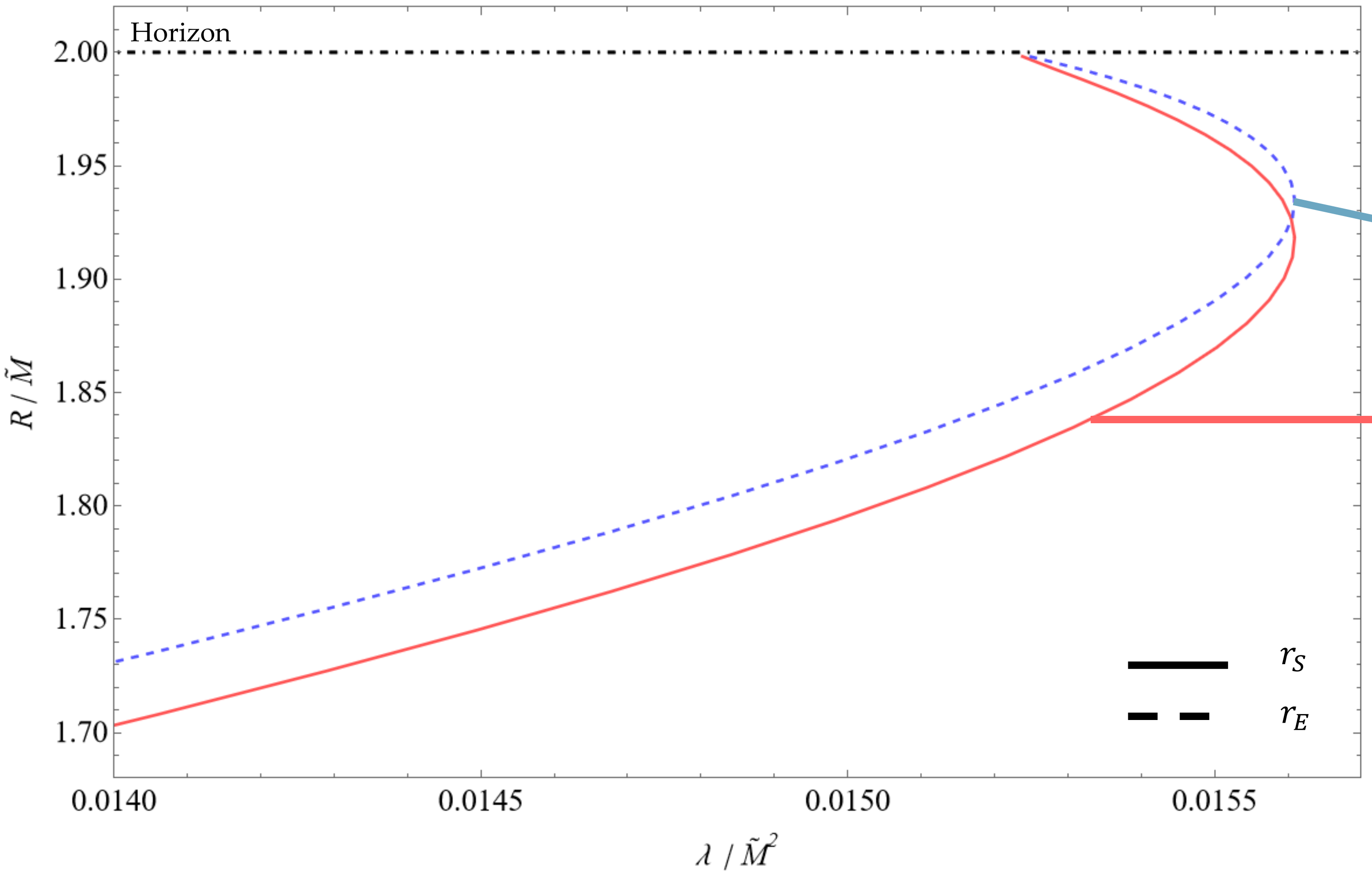


Where $\Delta < 0$, the system is elliptic $\longrightarrow$ Breakdown of predictability

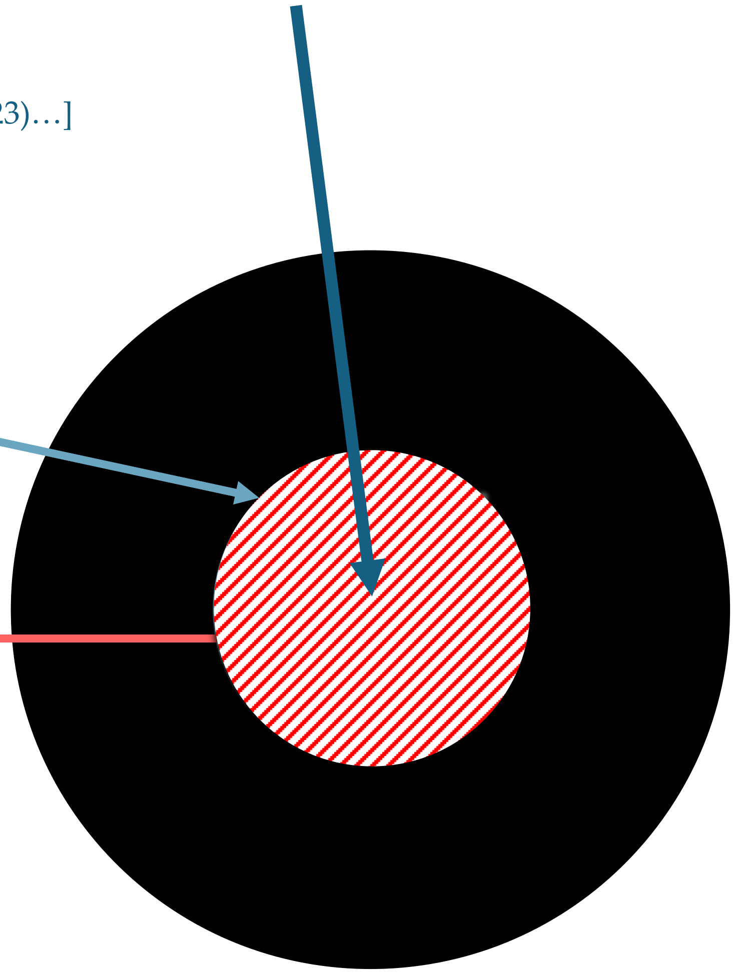
EdGB: black-hole interior structure

Singularity region

Elliptic region [Ripley & Pretorius (2019, 2020), East & Ripley (2021), Corelli+ (2022, 2023), Doneva+ (2023)…]



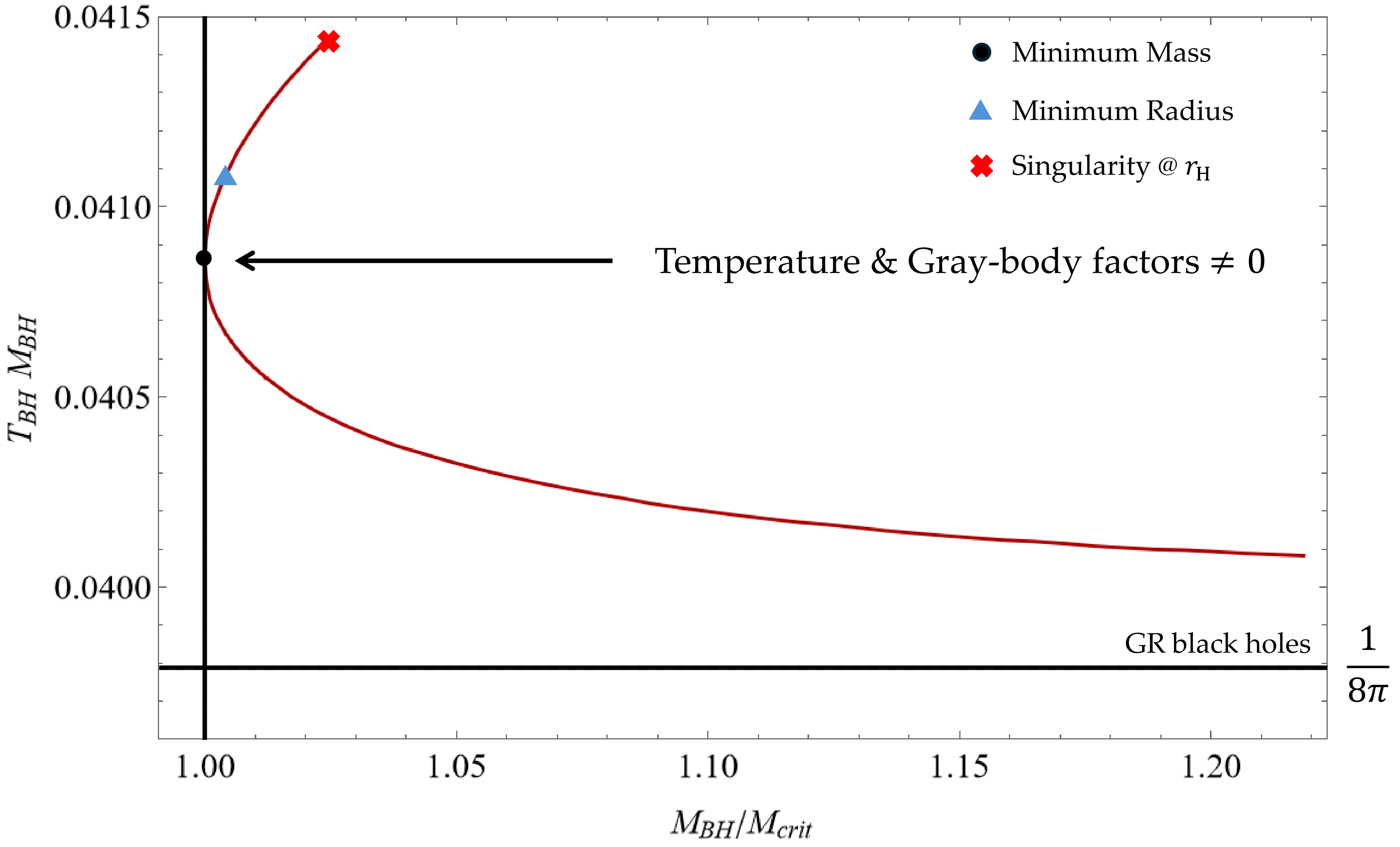
Excised region



Loss of hyperbolicity is **gauge-invariant**

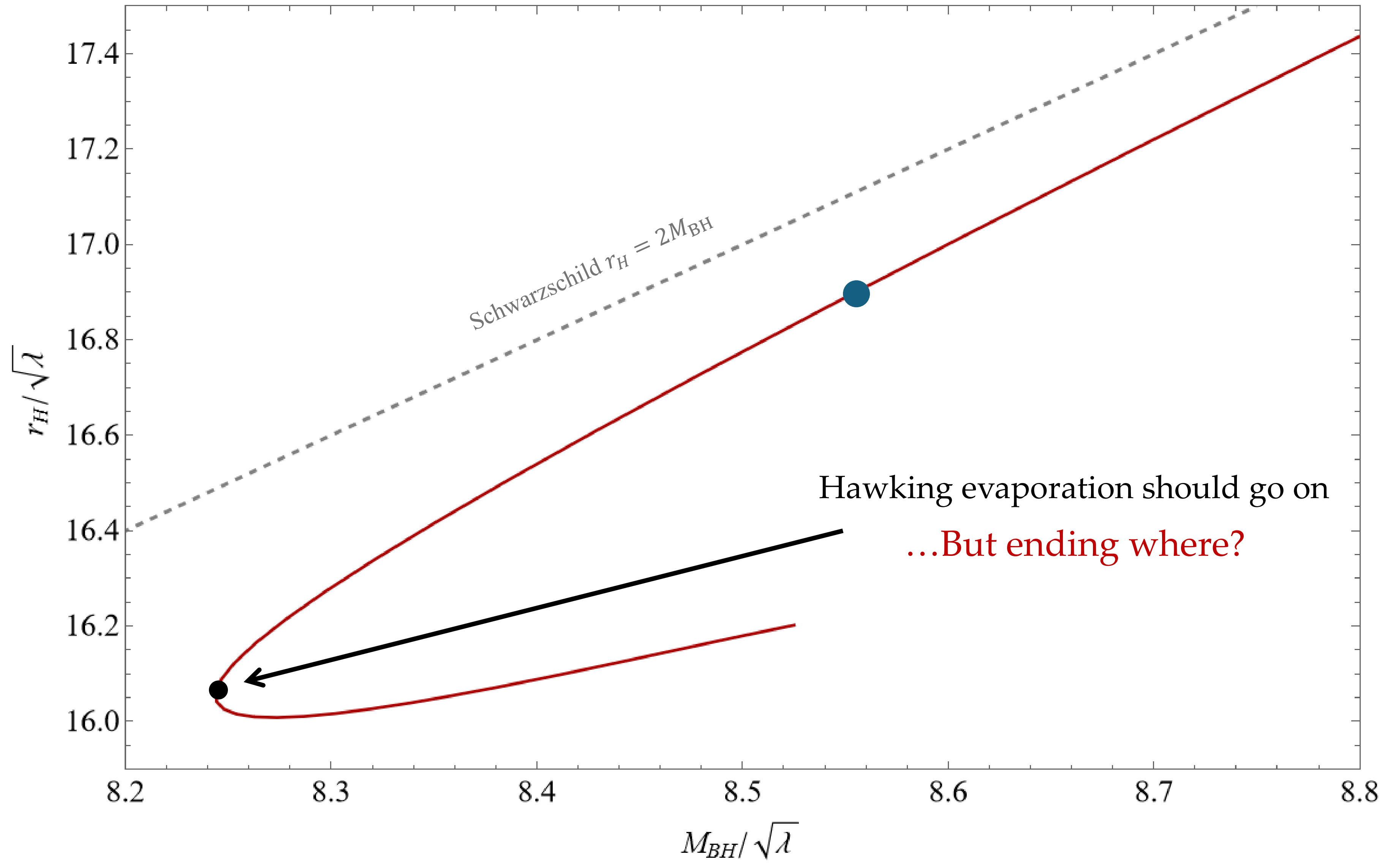
[Reall (2021)]

EdGB: black-hole thermodynamics



[Corelli+(2022, 2023)]

EdGB: Hawking evaporation



[Corelli+(2022, 2023)]

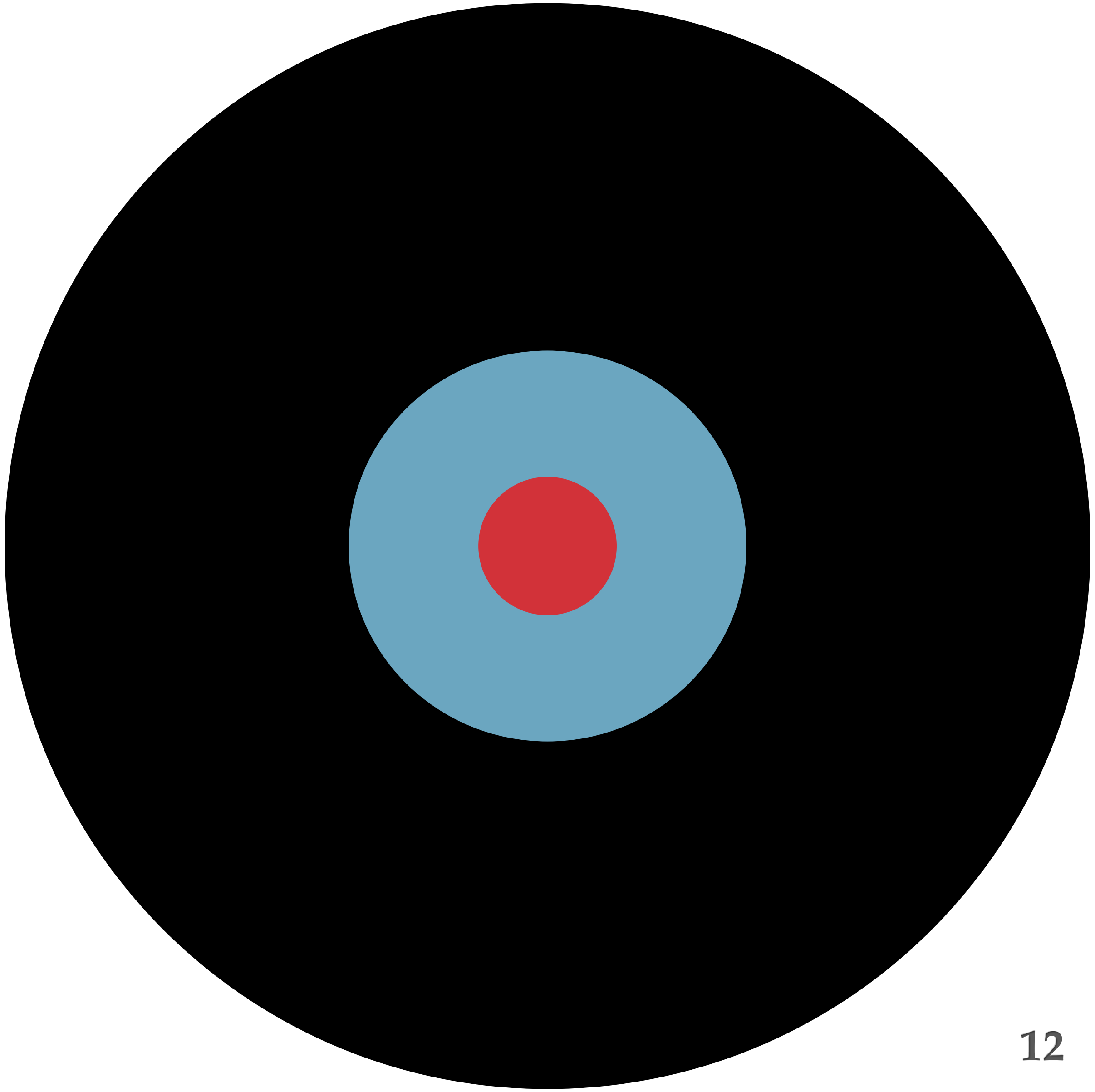
EdGB: loss of hyperbolicity & Hawking evaporation

Hawking radiation emulated with the
coupling with a phantom scalar

[Corelli+(2022, 2023)]

$$S = \frac{1}{16\pi}\int d^4x\,\sqrt{-g}\,[\mathcal{R} - (\nabla\phi)^2 + (\nabla\xi)^2 + 2\eta(\phi)\,\mathcal{G}]$$

Emulate the mass loss during
evaporation *at the classical level*



Black-hole region Elliptic region Singularity region

Some approaches try to address the problem…

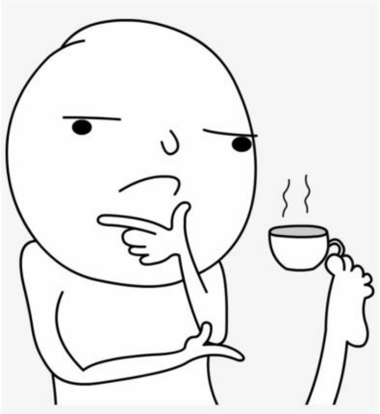
Linear coupling between the scalar field and the Ricci scalar [Thaalba+(2023, 2024)]

$$S = \frac{1}{16\pi}\int d^4x\,\sqrt{-g}\,\left[\mathcal{R} - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2}\left(\frac{\beta}{2}\mathcal{R} - \alpha\,\mathcal{G}\right)\phi^2\right]$$

For some specific values of the constants/couplings, the naked elliptic region can be avoided in simulations…

…but the elliptic region is not eliminated entirely

Add regularising higher-order differential operators [Figueras+(2024, 2025)] EFT approach



$f(\mathcal{R})$ –Dilaton-Gauss-Bonnet theory

PHYSICAL REVIEW D **112,** 124055 (2025)

Gravity with higher-curvature terms and second-order field equations: $f(\mathcal{R})$ meets Gauss-Bonnet

Fabrizio Corelli,[1,2,*] Paolo Pani,[1,2,†] and Andrea P. Sanna[2,‡]
[1]*Dipartimento di Fisica, Sapienza Università di Roma, Piazzale Aldo Moro 5, 00185, Roma, Italy*
[2]*INFN, Sezione di Roma, Piazzale Aldo Moro 2, 00185, Roma, Italy*

$f(\mathcal{R})$ –dGB: combining the best of both worlds?

$$S = \frac{1}{16\pi}\int d^4x\,\sqrt{-g}\,[\ \overset{f(\mathcal{R})}{\mathcal{R}} \quad - (\nabla\phi)^2 + 2\eta(\phi)\,\mathcal{G}]$$

EdGB alters GR vacuum solutions $\longleftrightarrow$ $f(\mathcal{R})$ allows to introduce higher-curvature corrections

$f(\mathcal{R})$ –dGB: the theory

$$S = \frac{1}{16\pi}\int d^4x\,\sqrt{-g}\,[f(\mathcal{R}) - (\nabla\phi)^2 + 2\eta(\phi)\,\mathcal{G}]$$

Reframe it as a scalar-tensor theory $\longrightarrow$ Identify the Ricci scalar as a new self-interacting scalar field χ

[Sotiriou & Faraoni (2010), Jaime+(2011)]

Two scalar fields involved

Bi-scalar extension of Horndeski

$f(\mathcal{R})$ –dGB: the theory

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Two scalar fields involved

$$S = \frac{1}{16\pi}\int d^4x\,\sqrt{-g}\,[f'(\chi)\mathcal{R} - \mathcal{V}(\chi) - (\nabla\phi)^2 + 2\eta(\phi)\,\mathcal{G}]$$

$$\boxed{\mathcal{V}(\chi) = f'(\chi)\chi - f(\chi)}$$

$f(\mathcal{R})$ –dGB: the theory

$$S = \frac{1}{16\pi}\int d^4x\,\sqrt{-g}\,[f'(\chi)\mathcal{R} - \mathcal{V}(\chi) - (\nabla\phi)^2 + 2\eta(\phi)\,\mathcal{G}]$$

$$\boxed{f(\mathcal{R}) = \mathcal{R} + \kappa\, R^n}$$

$$\kappa = \ell^{2(n-1)}$$

$n = 2$ «Quadratic Theory»

$n = 4$ «Quartic Theory»

$$\boxed{\eta(\phi) = \lambda e^{-\gamma\phi}}$$

$$\phi \to \phi + C \quad \lambda \to \lambda\, e^{\gamma C}$$

$$\gamma = 4$$

$f(\mathcal{R})$ –dGB: static asymptotically-flat black holes (exterior)

$$ds^2 = -\alpha(r)^2 dt^2 + [dr + \alpha(r)\,\zeta(r) dt]^2 + r^2 d\Omega_2^2 \qquad \phi = \phi(r) \qquad \chi = \chi(r)$$

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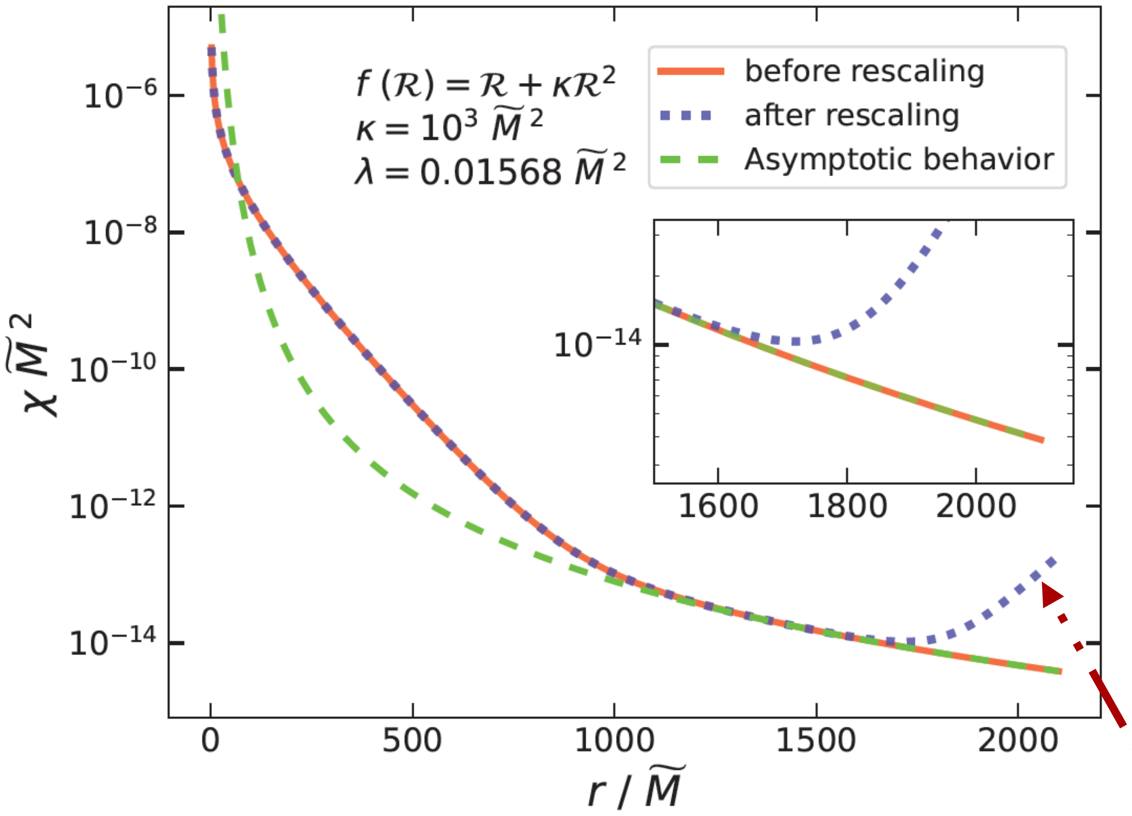
$$\phi \to \phi + C \qquad \lambda \to \lambda\, e^{\gamma C}$$

$f(\mathcal{R})$ –dGB: static asymptotically-flat black holes (exterior)

Quadratic case

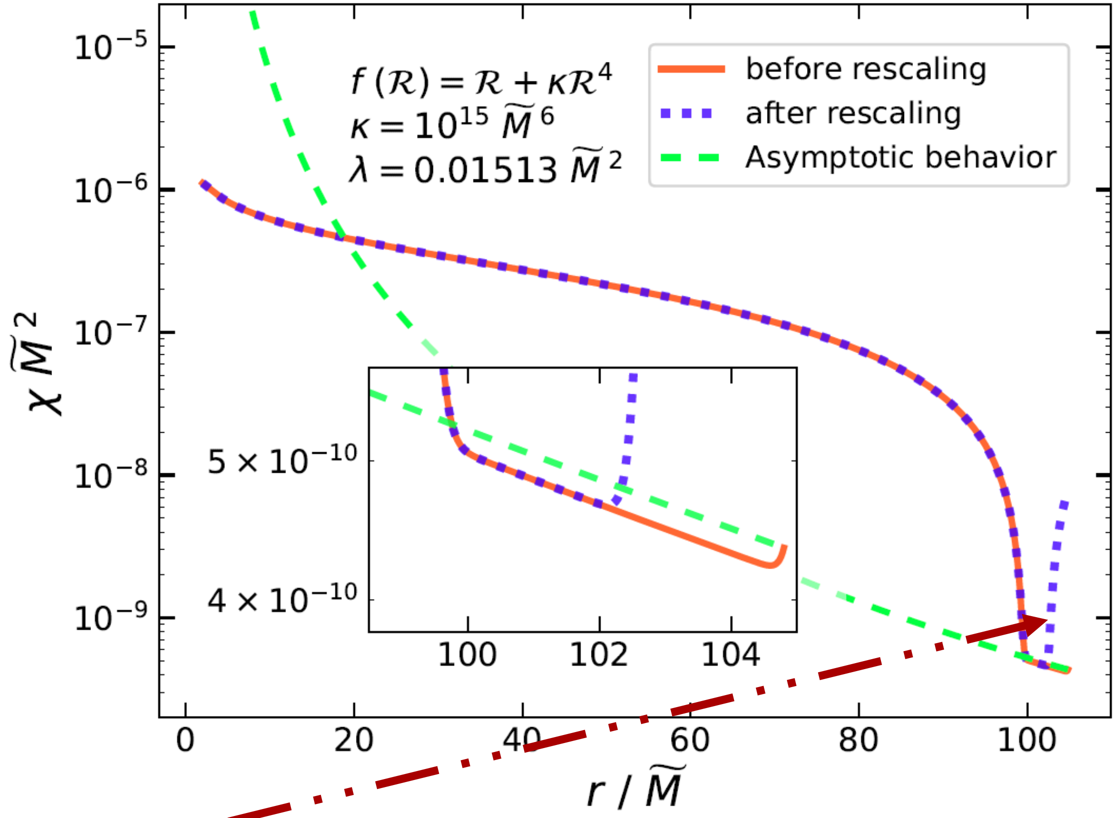
Accuracy: 35 significant figures



Quartic case

Accuracy: 55 significant figures

DANGER!



Numerical errors piling up

$f(\mathcal{R})$ −dGB: static asymptotically-flat black holes (exterior)

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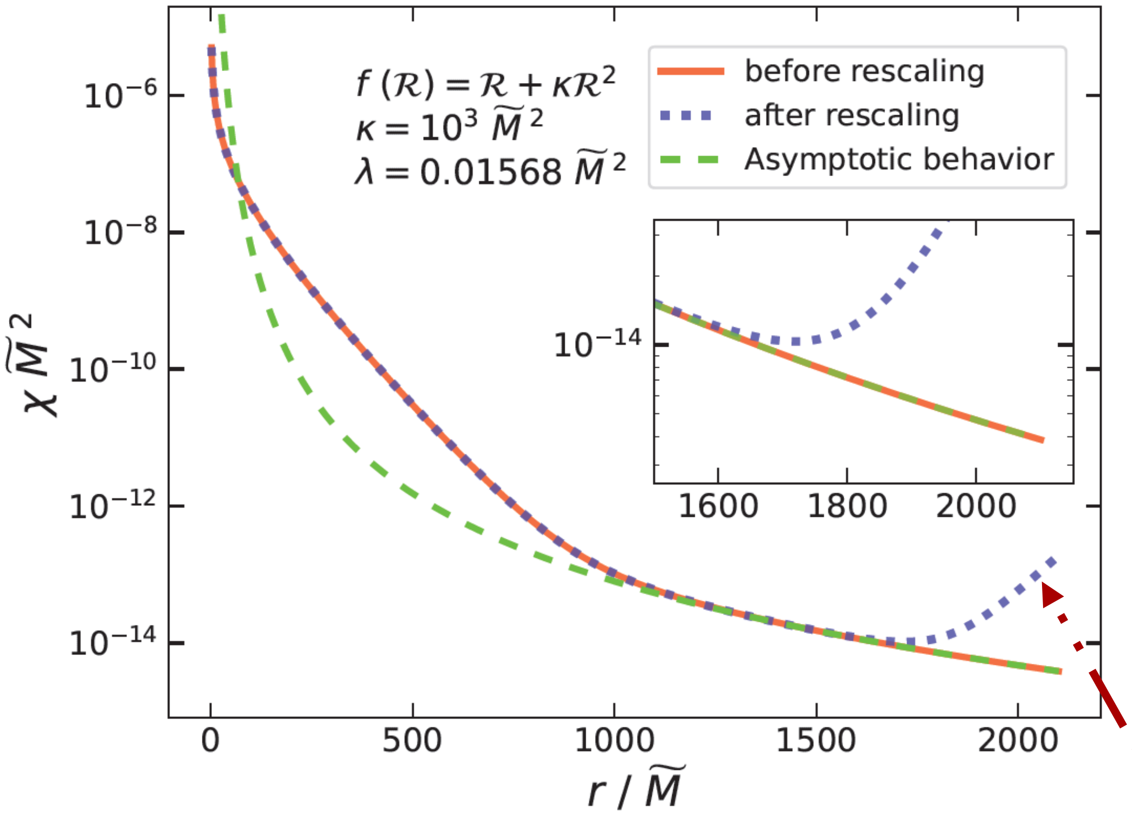

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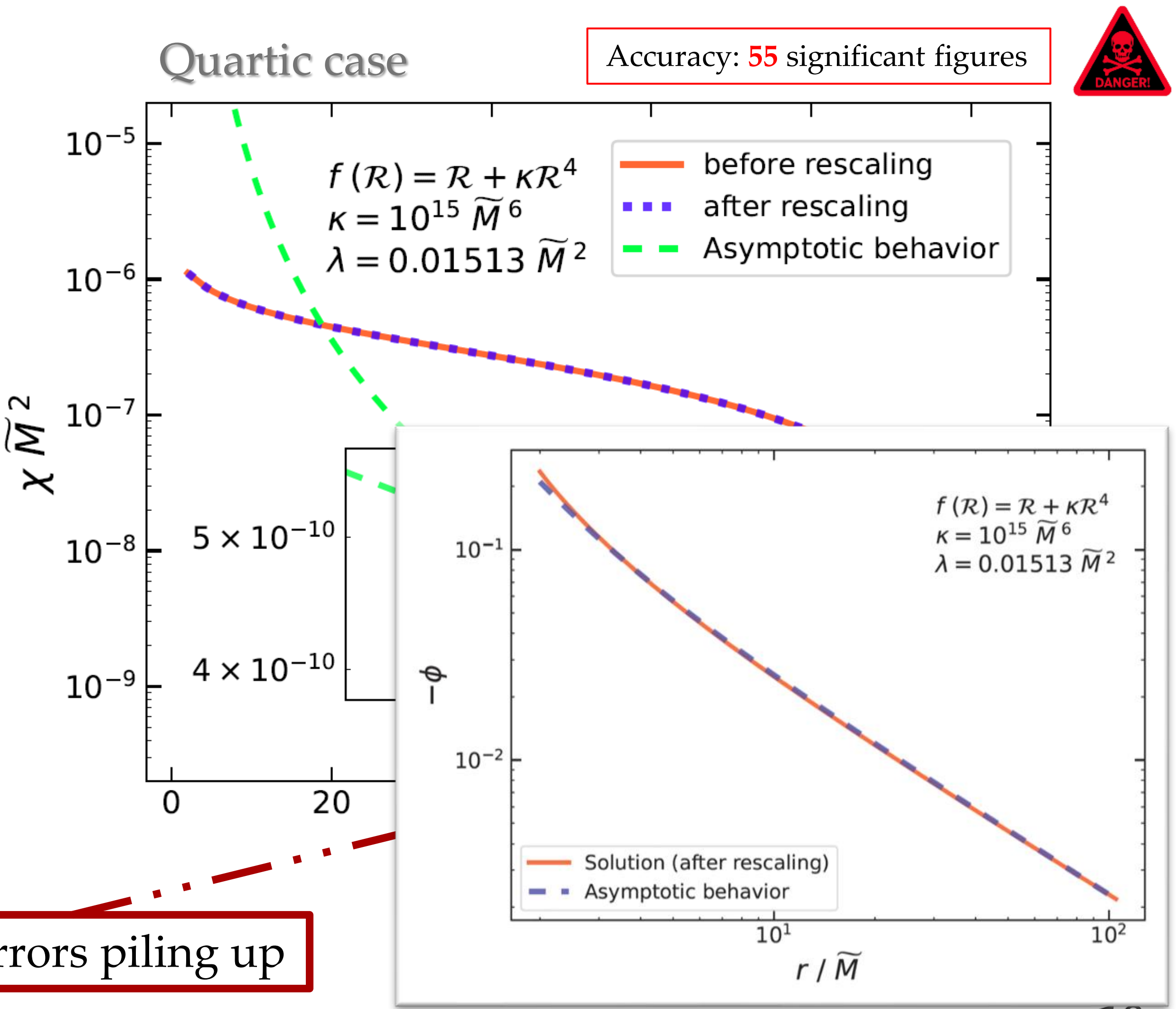
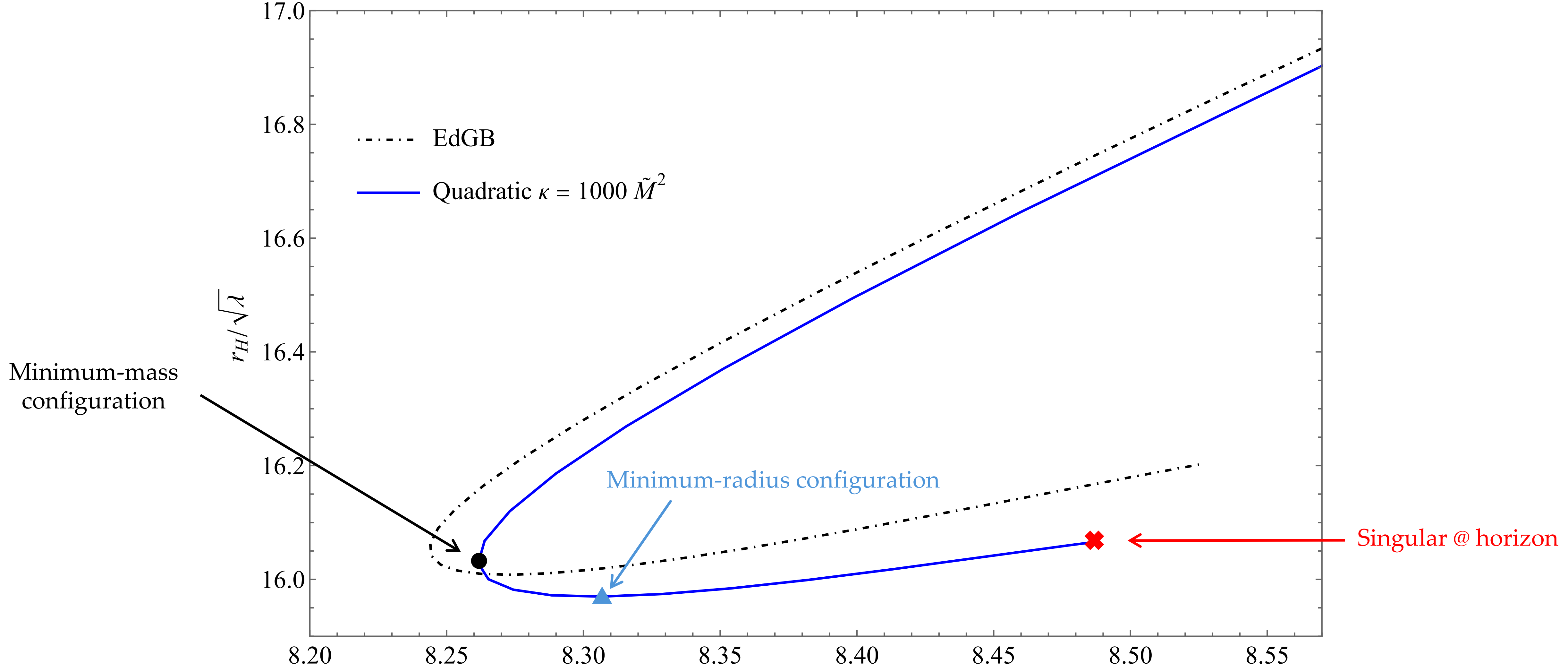


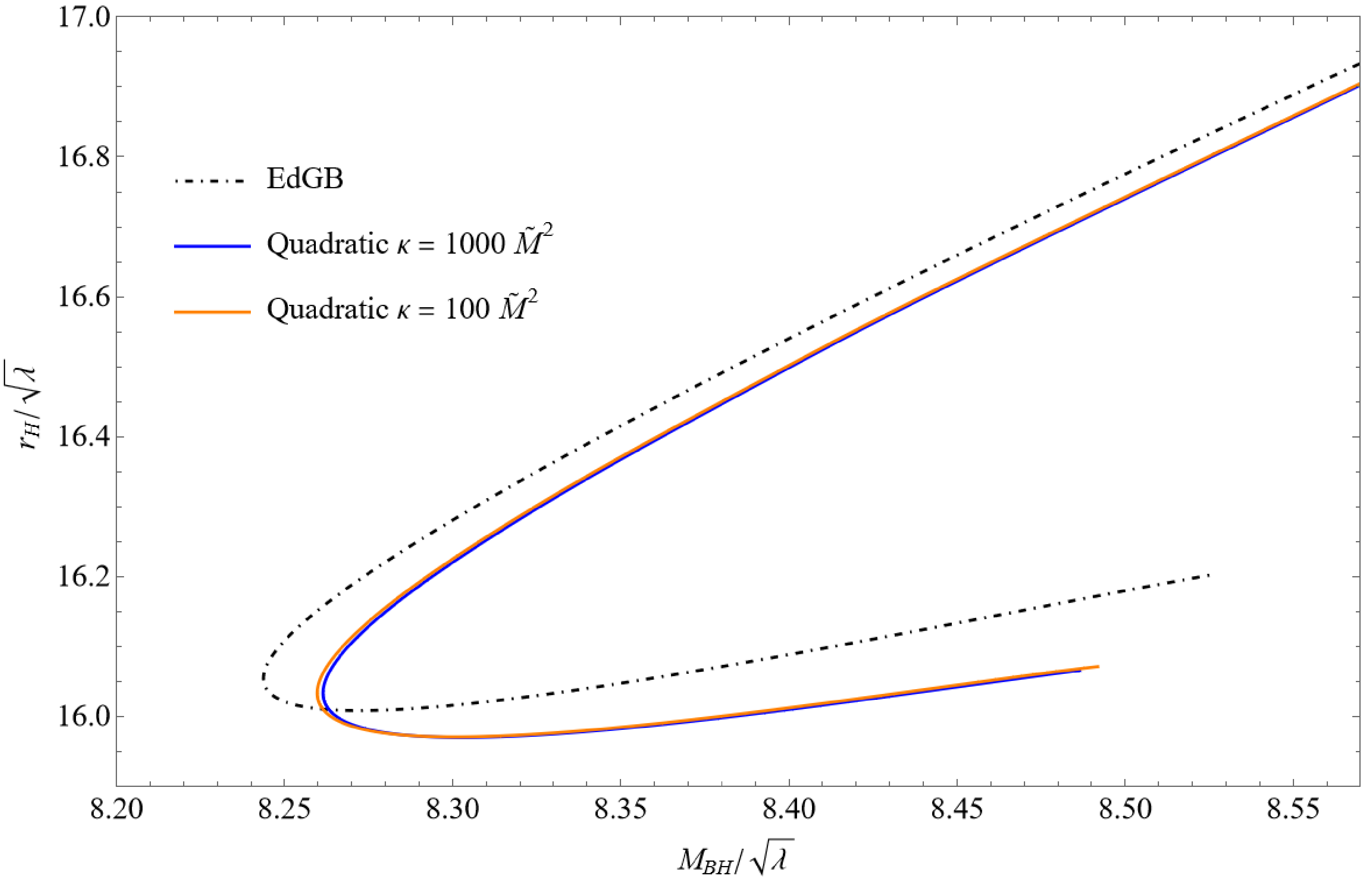
Numerical errors piling up

$f(\mathcal{R})$ −dGB: mass-radius diagrams



Nonperturbative features of EdGB preserved!

$f(\mathcal{R})$ −dGB: mass-radius diagrams



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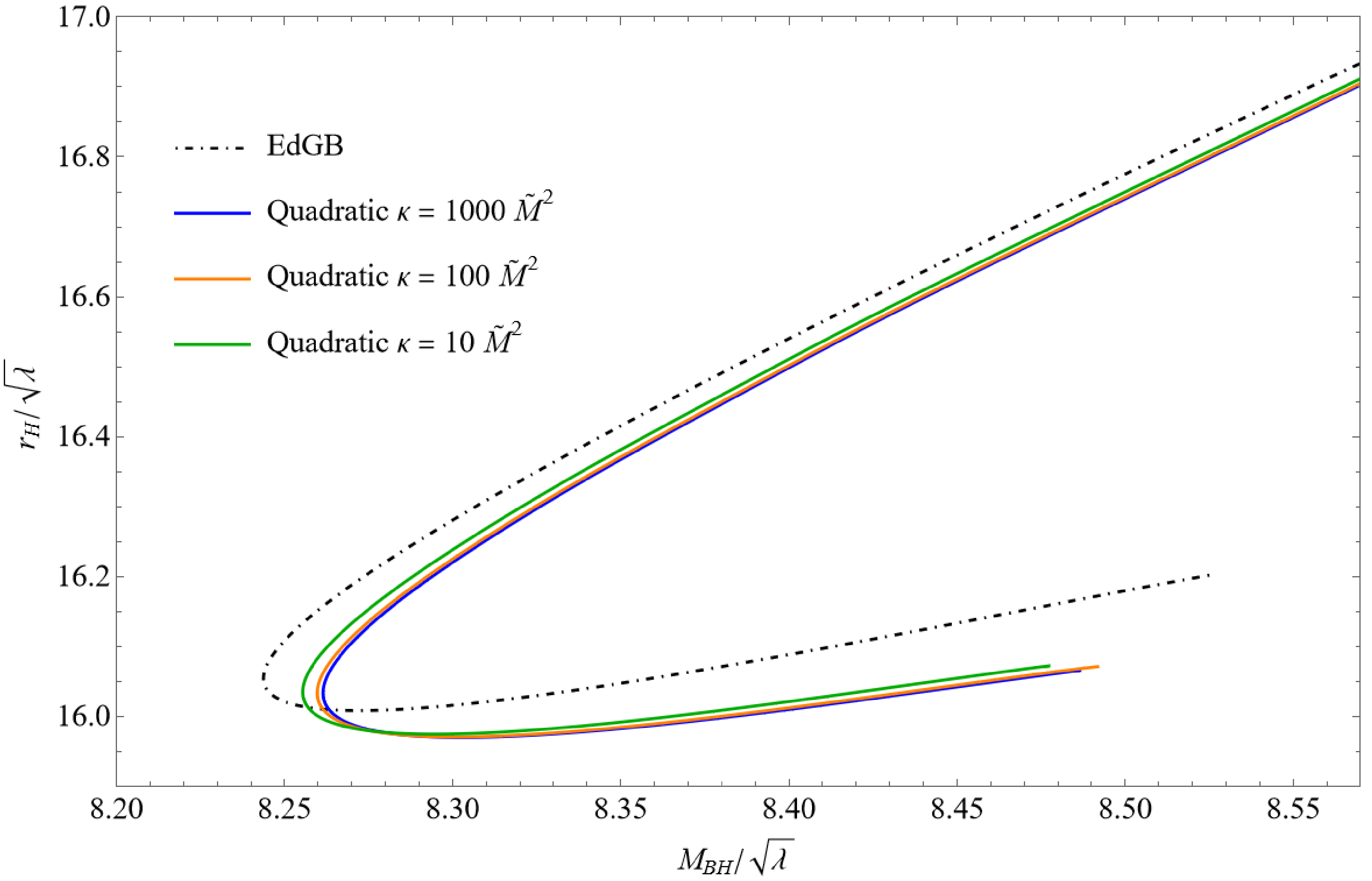

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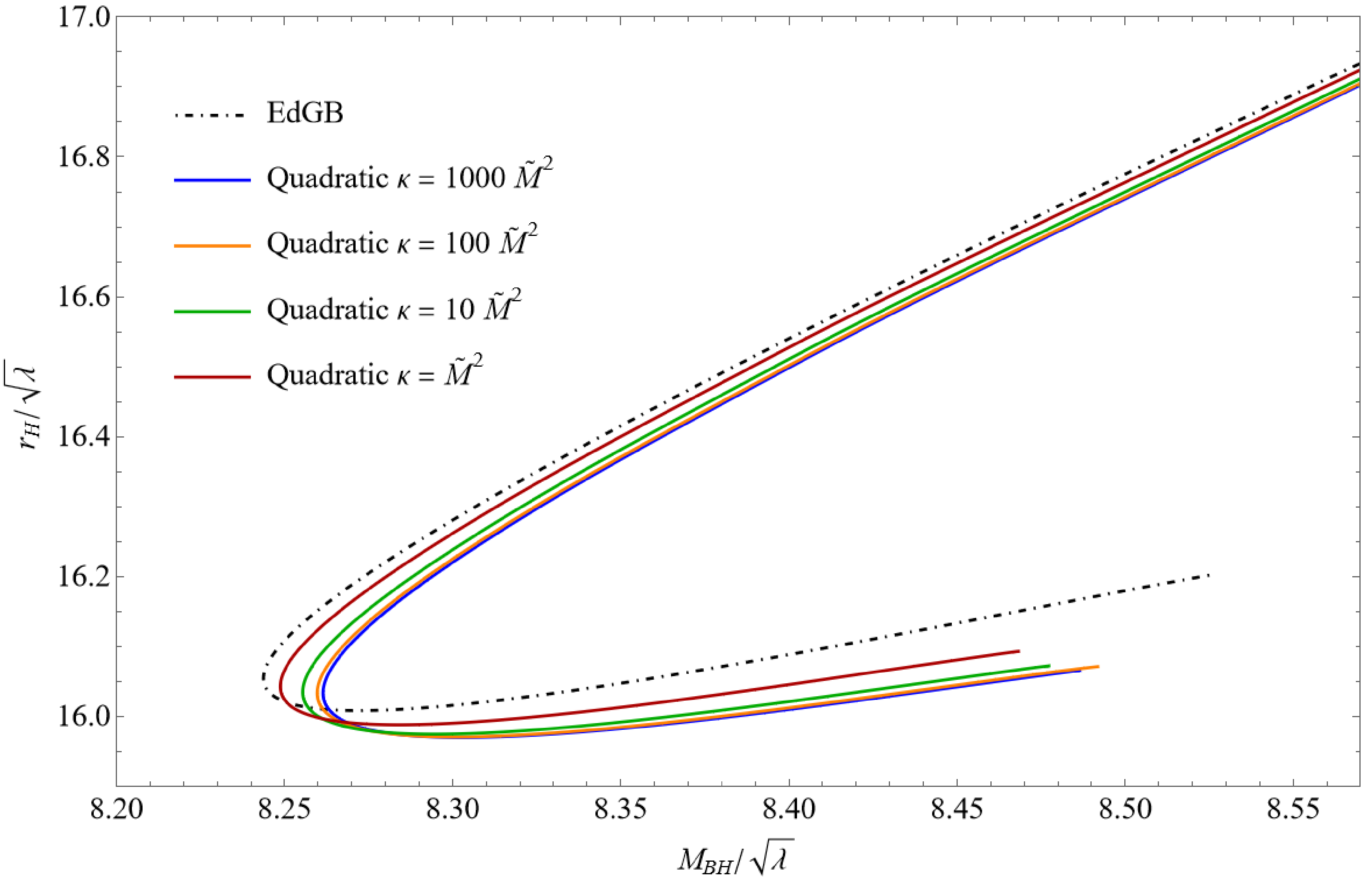

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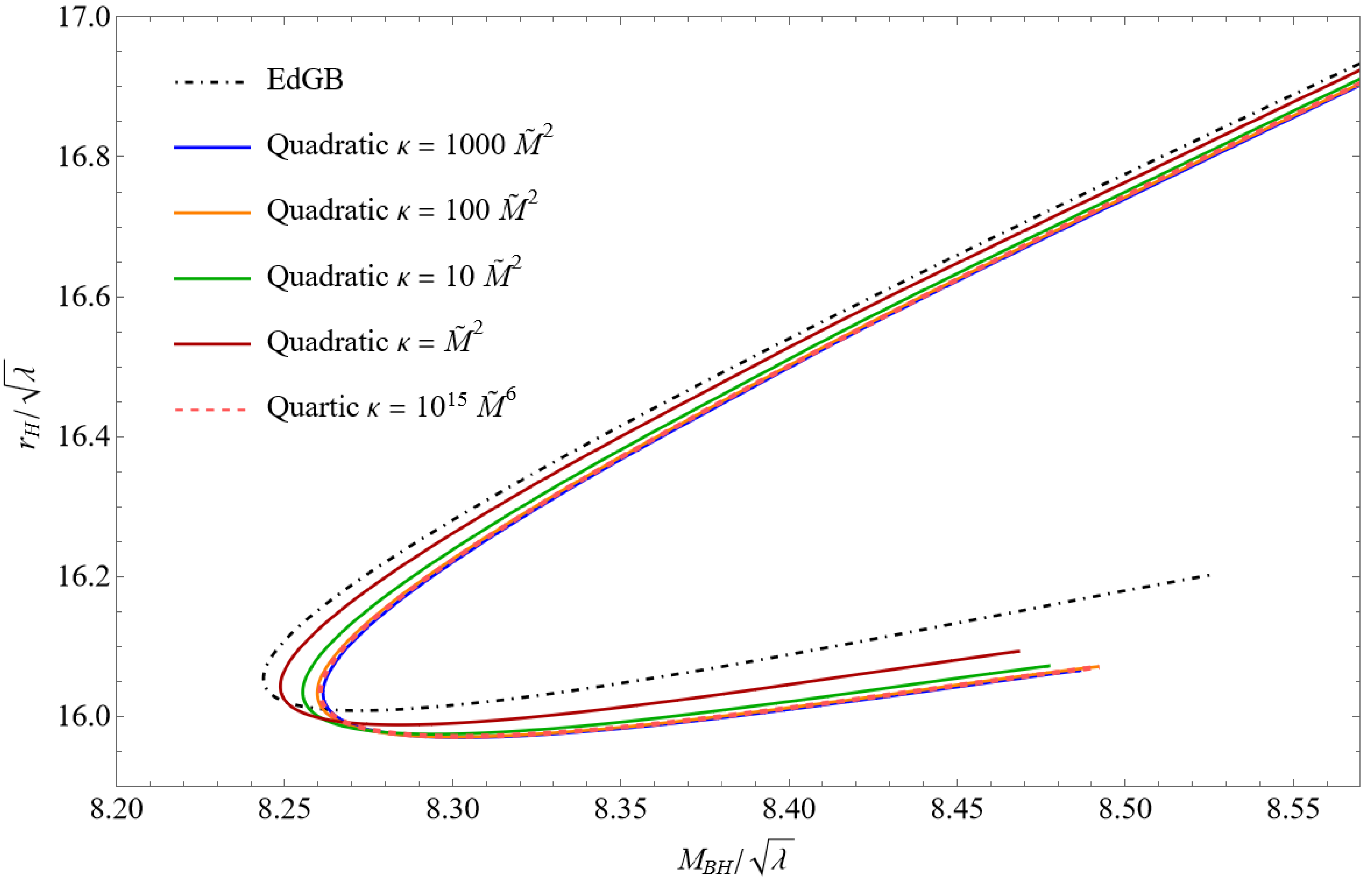

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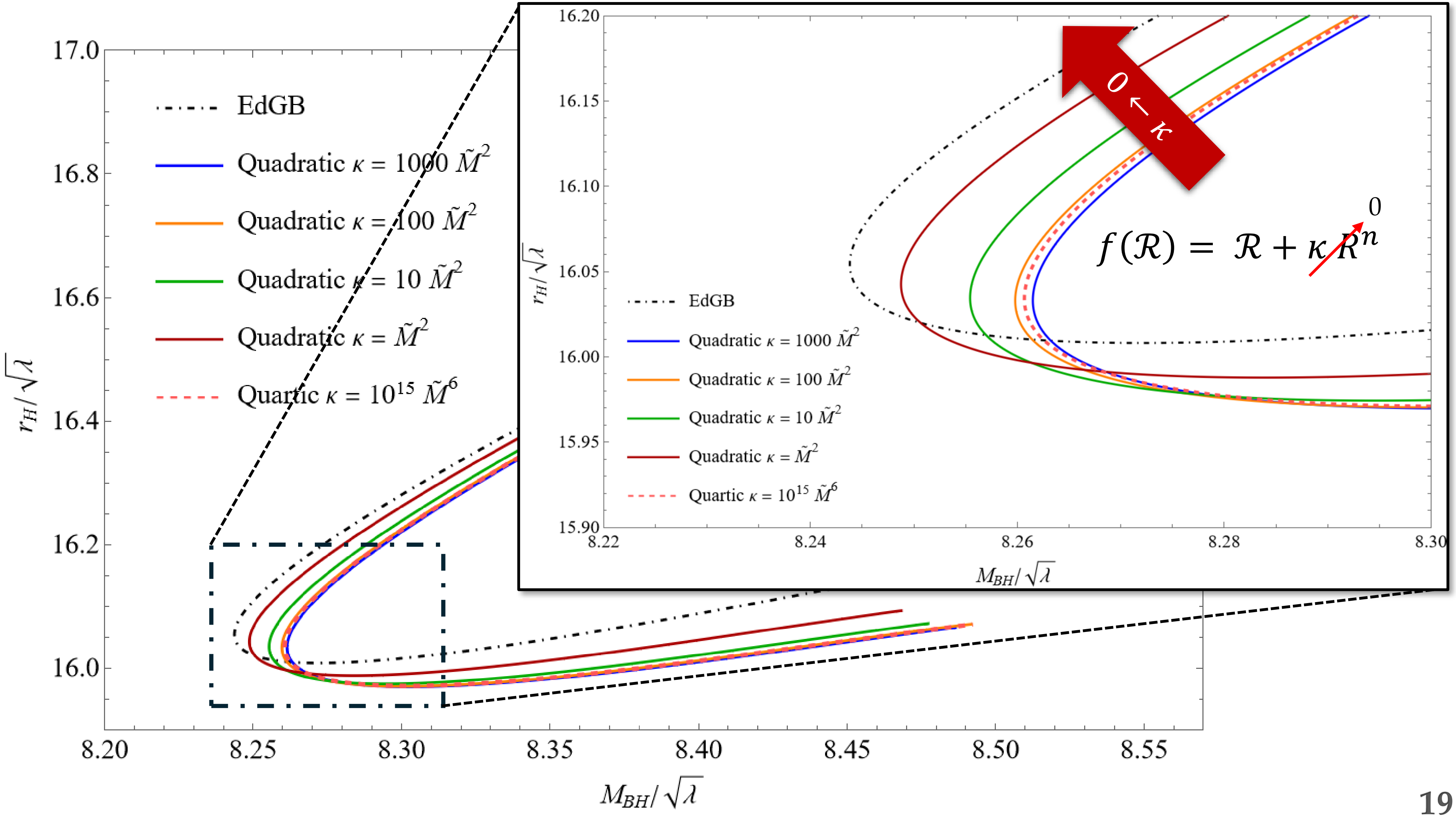

$f(\mathcal{R})$ −dGB: static asymptotically-flat black holes (exterior)

$$ds^2 = -\alpha(r)^2 dt^2 + [dr + \alpha(r)\,\zeta(r)dt]^2 + r^2 d\Omega_2^2 \qquad \phi = \phi(r) \qquad \chi = \chi(r)$$

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$f(\mathcal{R})$ −dGB: static asymptotically-flat black holes (interior)

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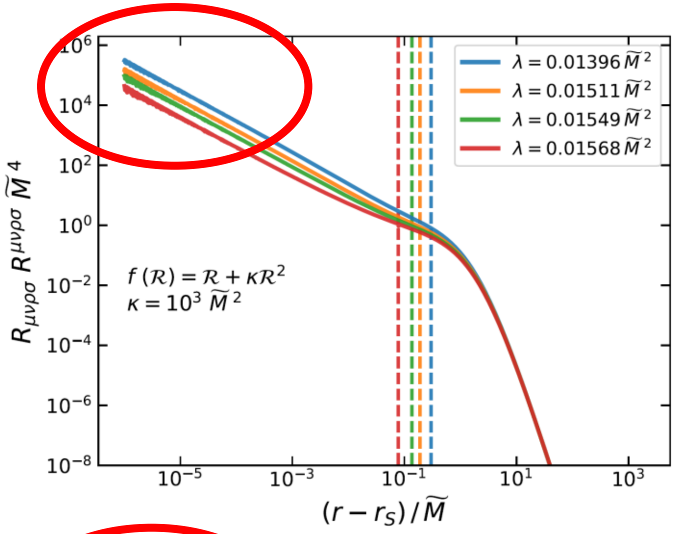
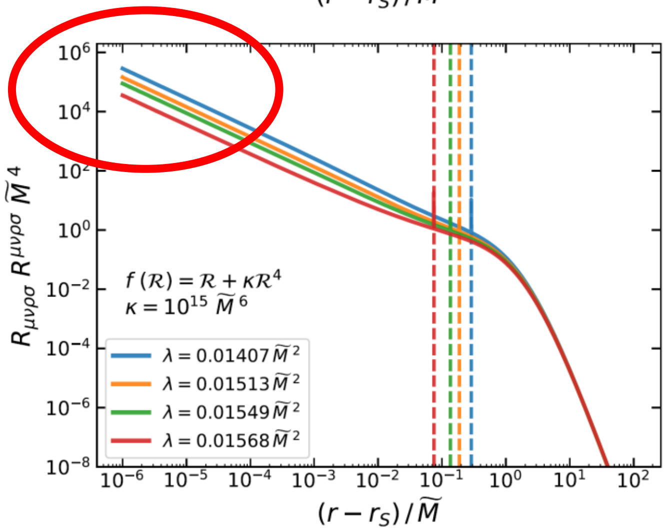
Large curvature region!





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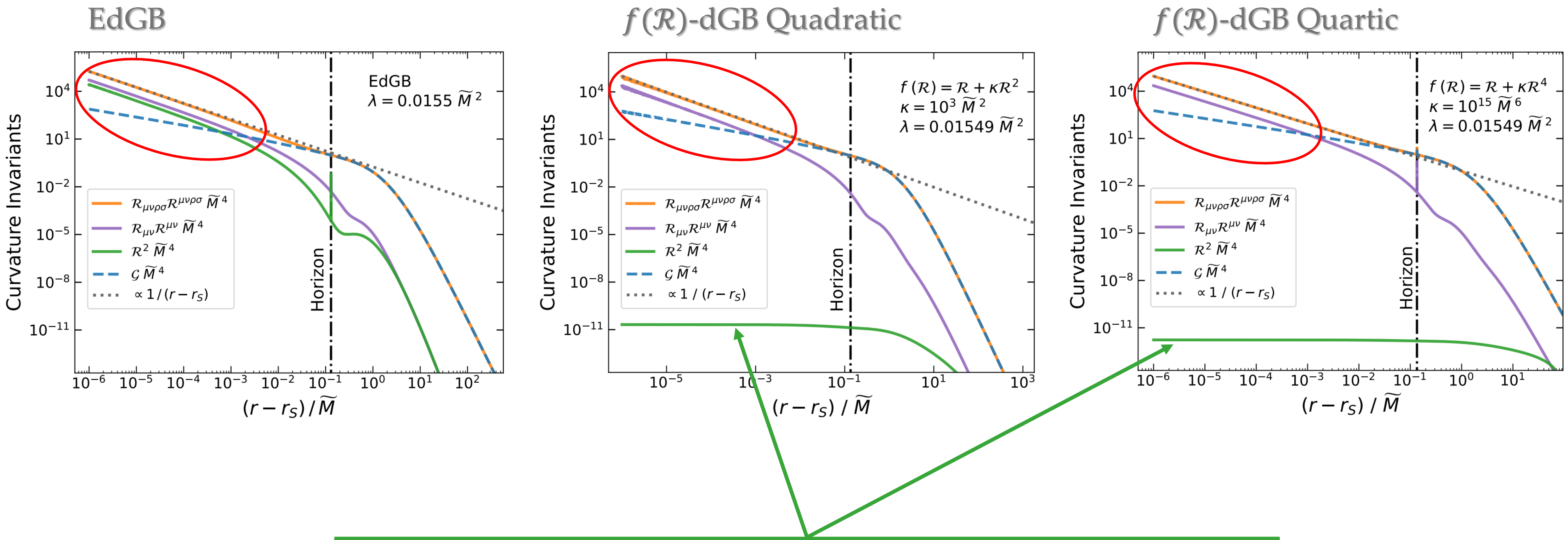
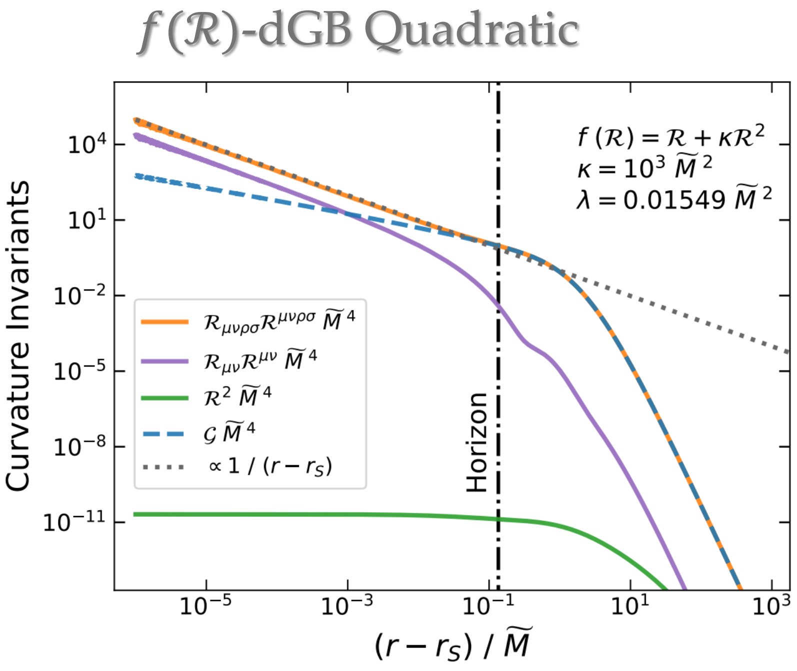
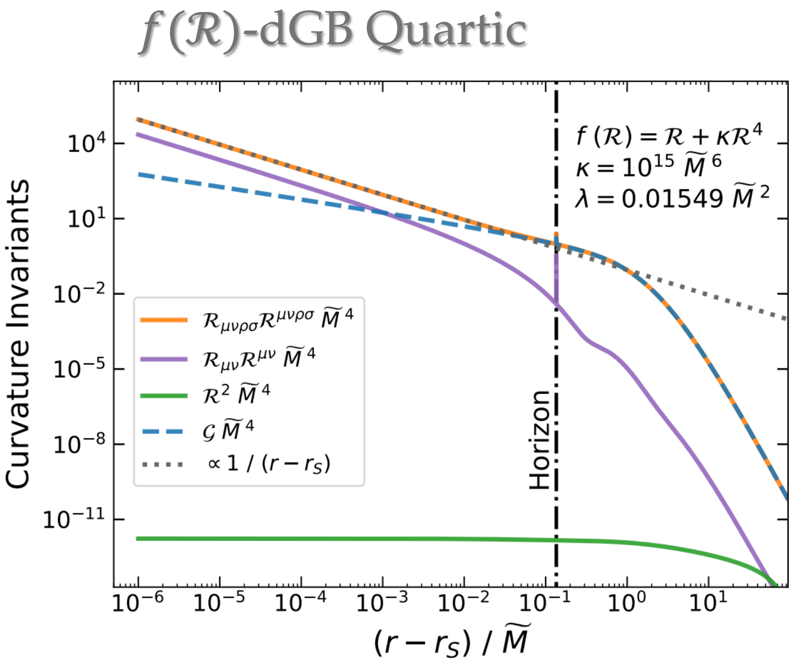
$f(\mathcal{R})$ –dGB: black-hole interior

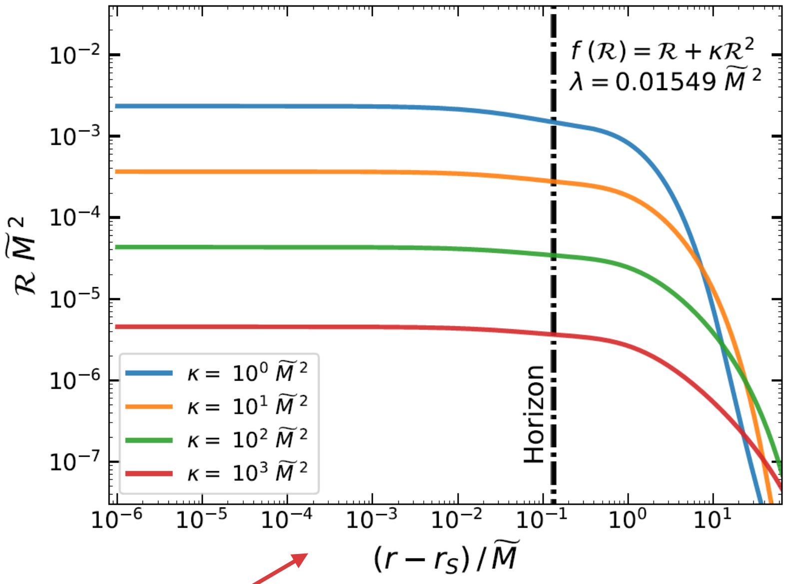


Dynamical mechanism suppressing the Ricci scalar

$f(\mathcal{R})$ –dGB: black-hole interior

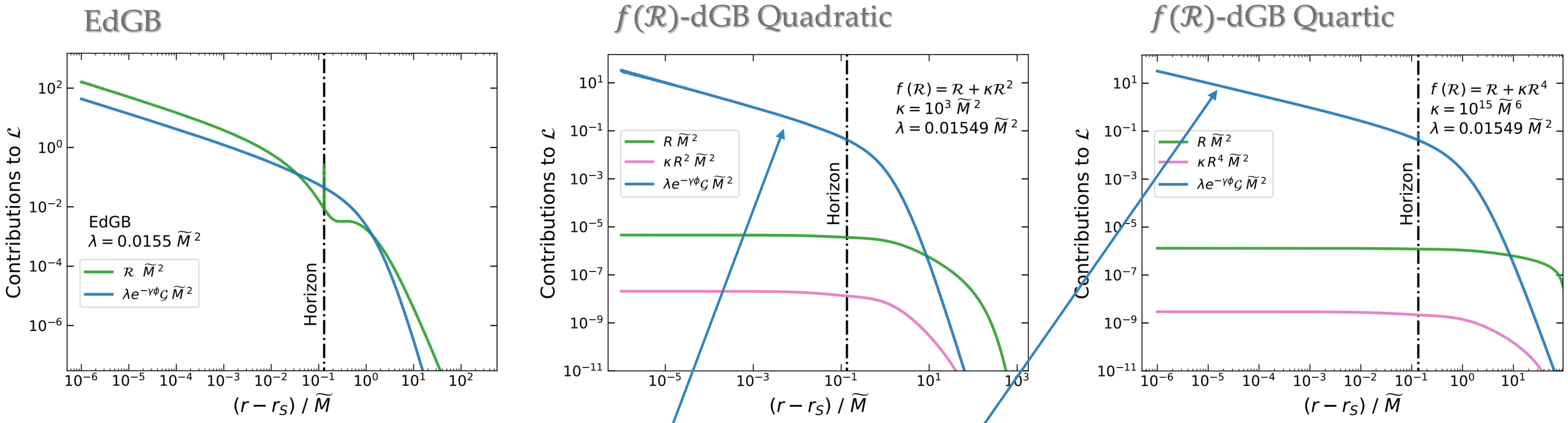






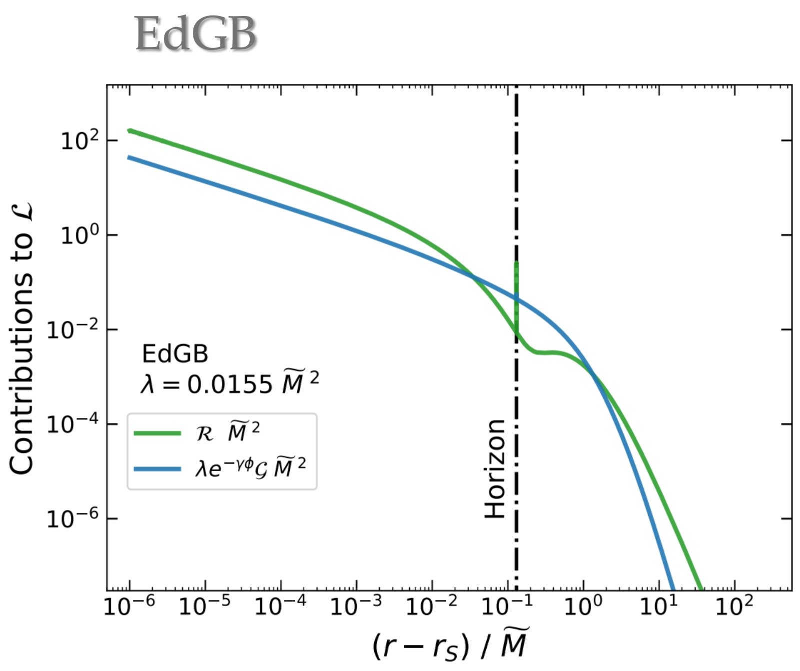
The larger the κ, the more suppressed the contribution

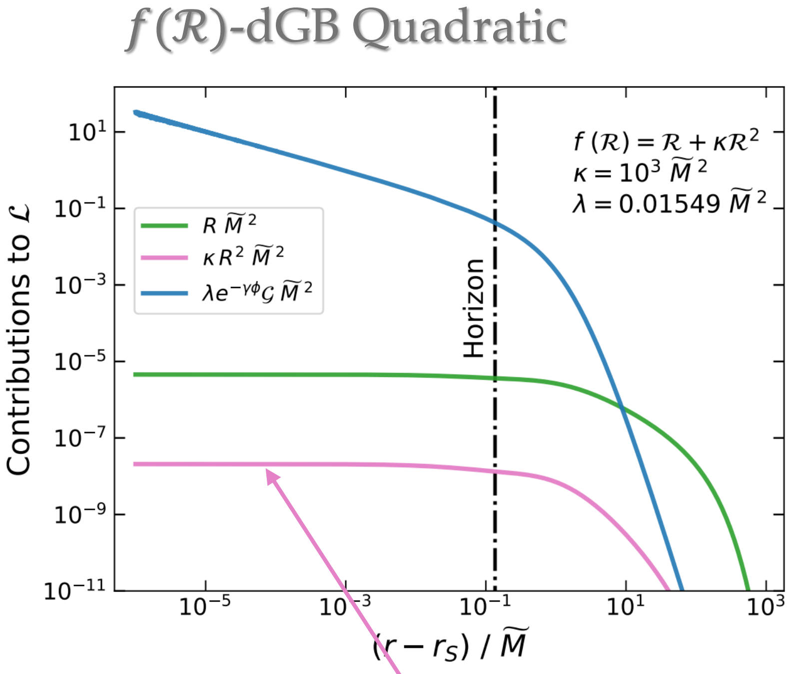
$f(\mathcal{R})$ –dGB: why so similar to EdGB?

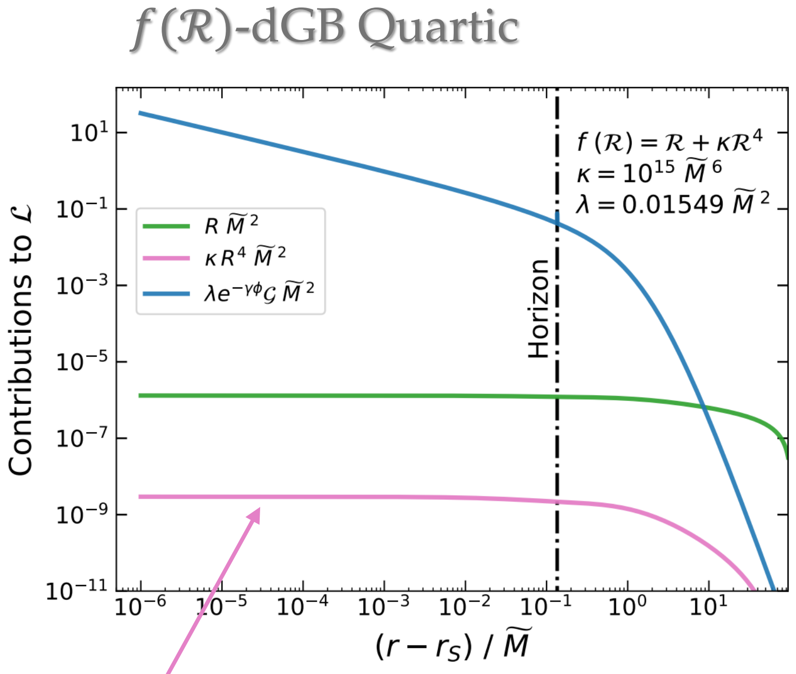


The dGB contribution **always** dominates

$f(\mathcal{R})$ –dGB: why so similar to EdGB?







The larger the n, the more suppressed the contribution

$f(\mathcal{R})$ −dGB: loss of hyperbolicity

Time-dependent case $[\alpha(r), \zeta(r), \phi(r)] \to [\alpha(r,t), \zeta(r,t), \phi(r,t)]$

Auxiliary field variable + conjugate momentum

$$Q \equiv \partial_r \phi$$
$$P \equiv \frac{1}{\alpha}\partial_t \phi - \zeta Q$$
$$\Theta \equiv \partial_r \chi$$
$$\Pi \equiv \frac{1}{\alpha}\partial_t \chi - \zeta \Theta$$

$\longrightarrow$ Evolution equations + 2 constraints for α & ζ

Principal symbol $\mathcal{P}_{IJ}(\eta_\mu) = \dfrac{\delta E_{v^I}}{\delta\, \partial_\mu v^J}\eta_\mu$ $\quad v^I = (\phi, Q, \chi, \Theta, P, \Pi, \alpha, \zeta)$

$$\det \mathcal{P} \propto \eta_t^2\, \eta_r^2 \left[\mathfrak{a}\left(\frac{\eta_t}{\eta_r}\right)^4 + \mathfrak{b}\left(\frac{\eta_t}{\eta_r}\right)^3 + \mathfrak{c}\left(\frac{\eta_t}{\eta_r}\right)^2 + \mathfrak{d}\left(\frac{\eta_t}{\eta_r}\right) + \mathfrak{e}\right] = 0$$

$f(\mathcal{R})$ –dGB: loss of hyperbolicity

$$\det \mathcal{P} \propto \eta_t^2\, \eta_r^2 \left[\mathfrak{a} \left(\frac{\eta_t}{\eta_r}\right)^4 + \mathfrak{b} \left(\frac{\eta_t}{\eta_r}\right)^3 + \mathfrak{c} \left(\frac{\eta_t}{\eta_r}\right)^2 + \mathfrak{d} \left(\frac{\eta_t}{\eta_r}\right) + \mathfrak{e}\right] = 0$$

$\eta_t = 0$ Redundancy of the equation for $\partial_t \phi$ and $\partial_t \chi$

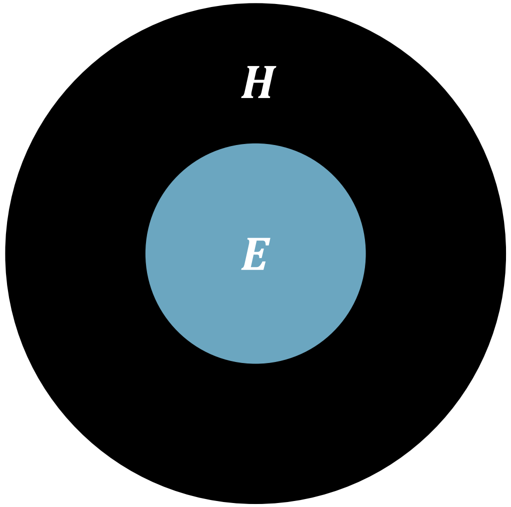
$\eta_r = 0$ 2 constraints for α and ζ

$$\Delta > 0$$

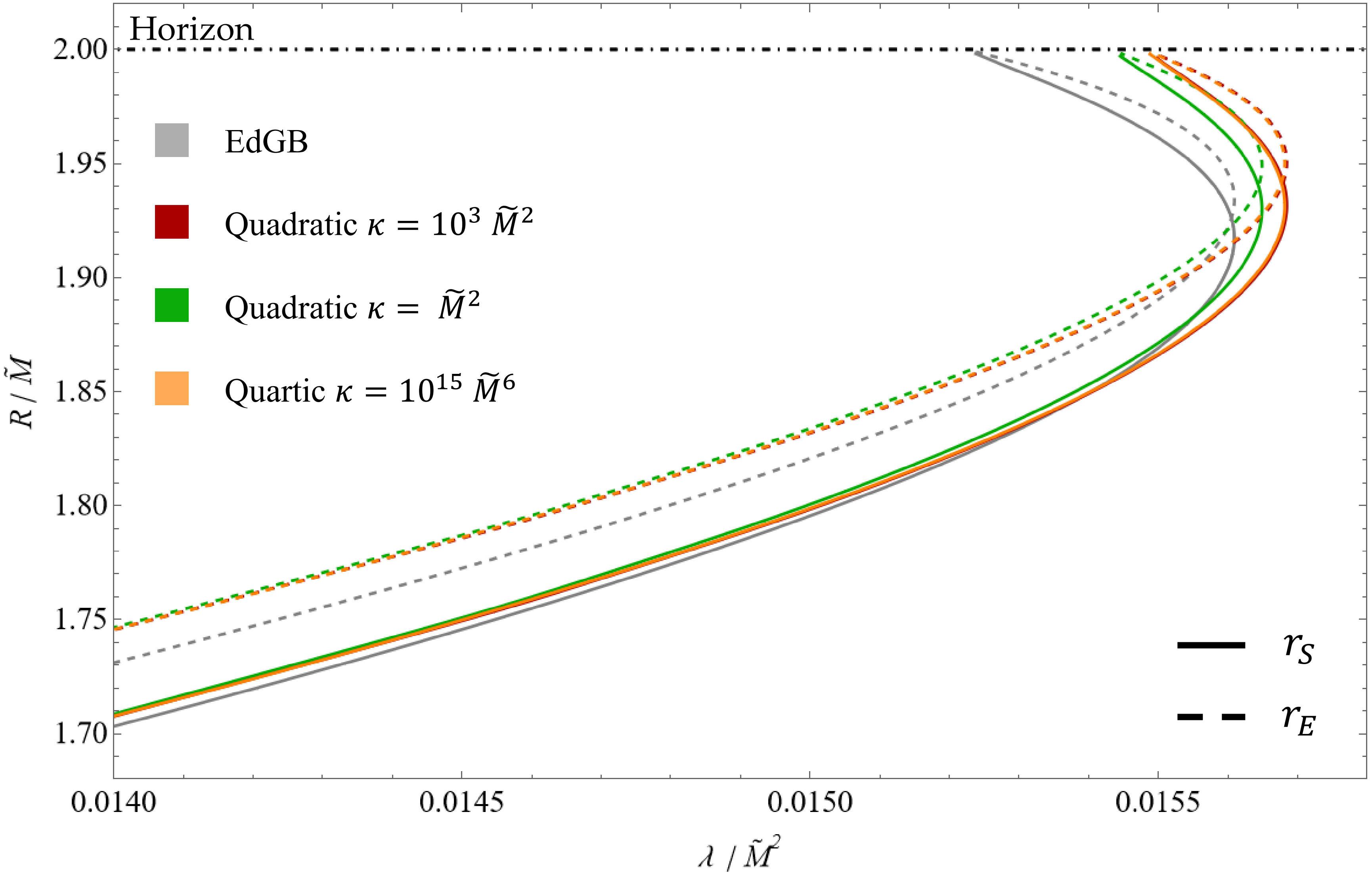
Real distinct solutions if

$$64\mathfrak{a}^3\mathfrak{e} - 16\mathfrak{a}^2\mathfrak{c}^2 + 16\mathfrak{a}\mathfrak{b}^2\mathfrak{c} - 16\mathfrak{a}^2\mathfrak{b}\mathfrak{d} - 3^4 < 0$$

$$8\mathfrak{a}\mathfrak{c} - 3\mathfrak{b}^2 < 0$$



$f(\mathcal{R})$ −dGB: loss of hyperbolicity



Concluding remarks

The inclusion of individual higher-curvature corrections in EdGB reveals qualitatively similar features
…and issues

An infinite tower of higher-curvature terms might be needed [Bueno+ (2025)]

What could be done next? If you are brave enough…

- Description of the formation of these black holes
- Description of Hawking evaporation
- Other numerical simulations (maybe?...)
- Analyze the loss of hyperbolicity in a gauge-invariant way [Reall (2021)]

[Credits: ESA]