Black hole tomography

Based on work with Badri Krishnan, Ariadna Ribes Metidieri, John Ostor, Patrick Bourg, …..

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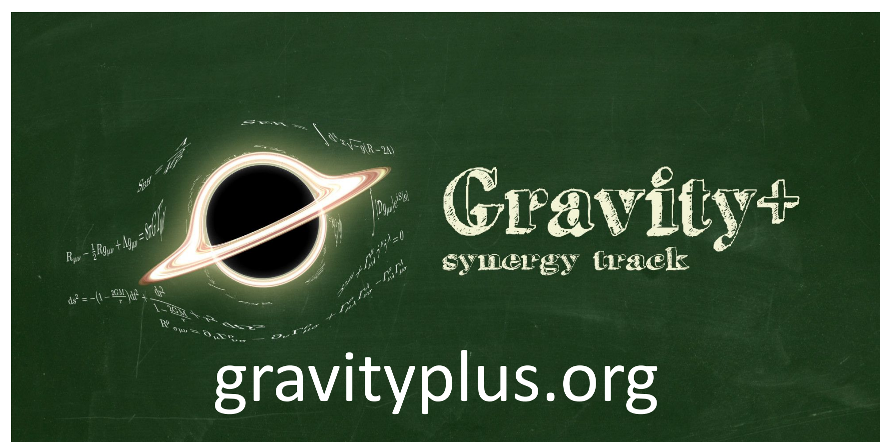
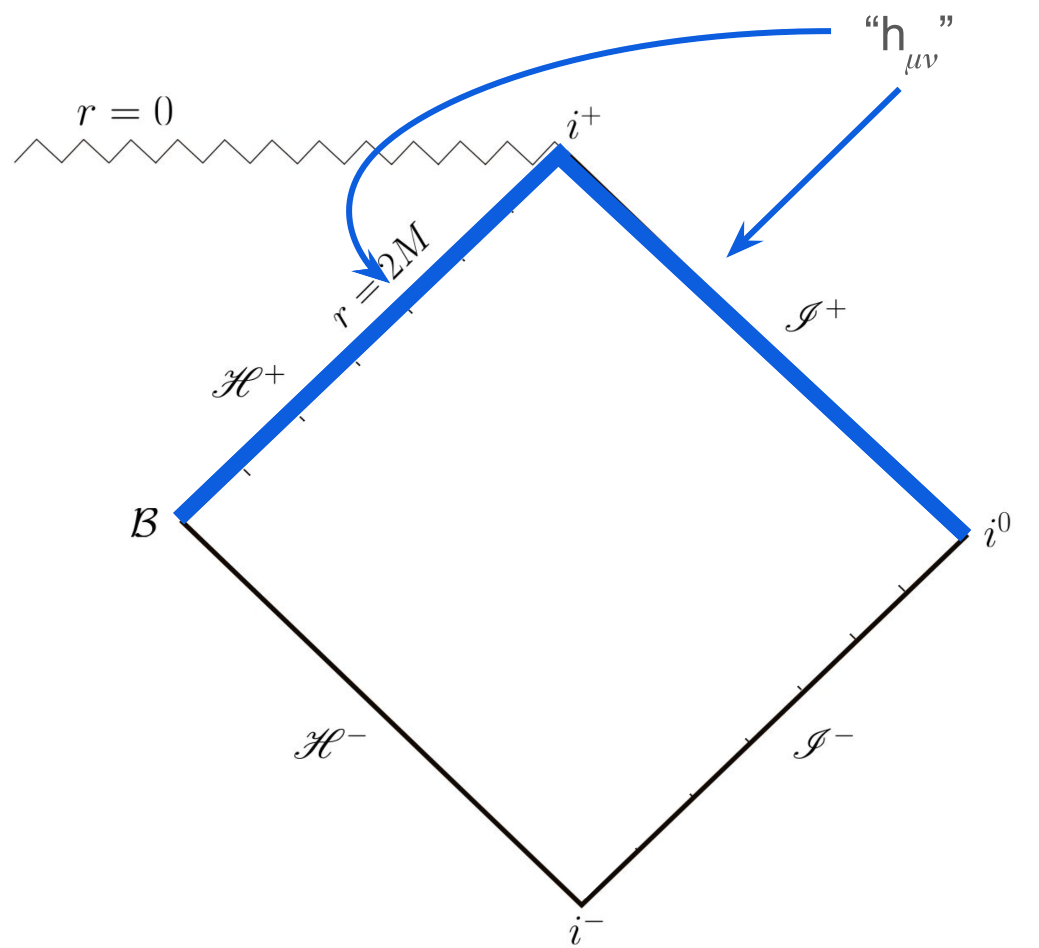


Black hole tomography

Reconstruct horizon dynamics from gravitational wave observations at null infinity



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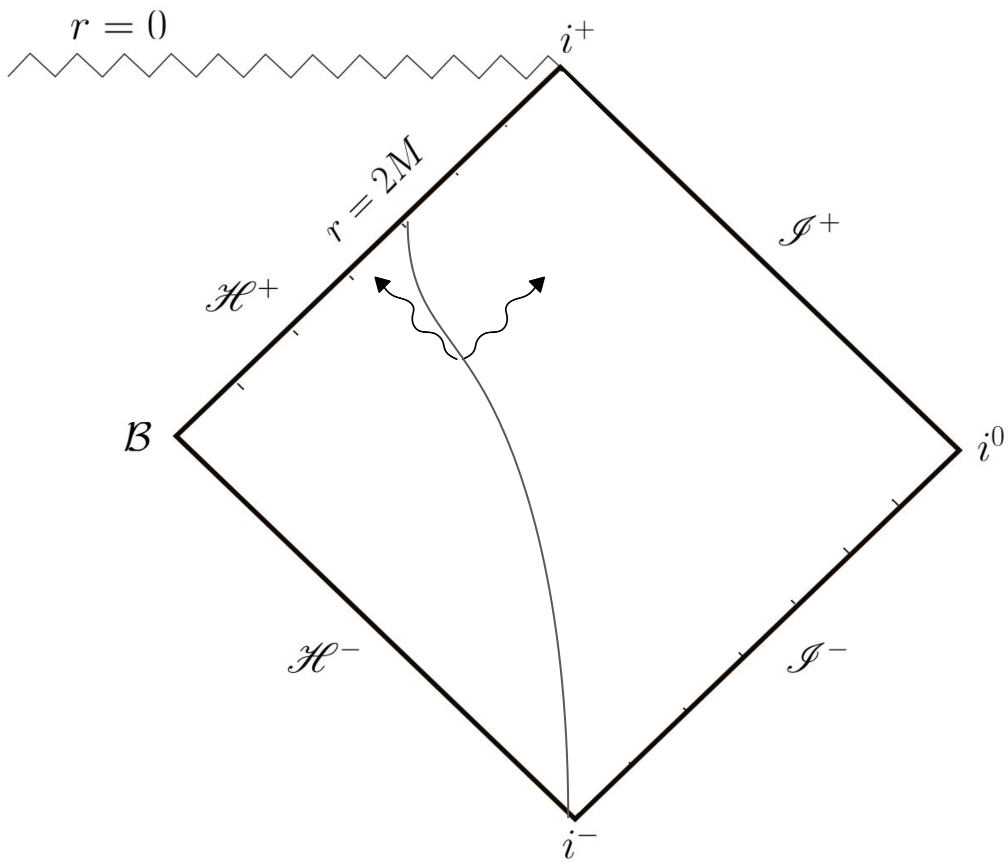
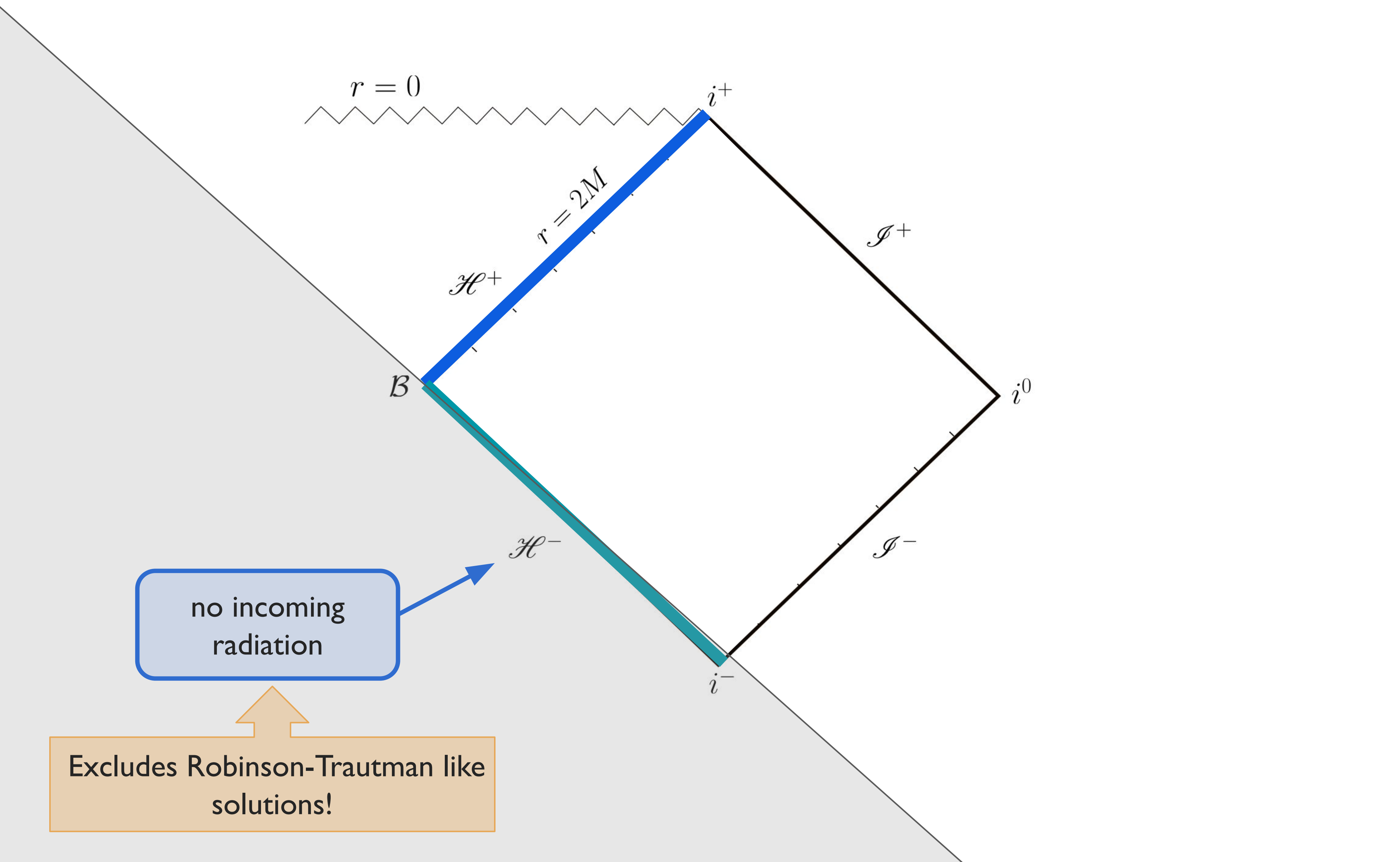
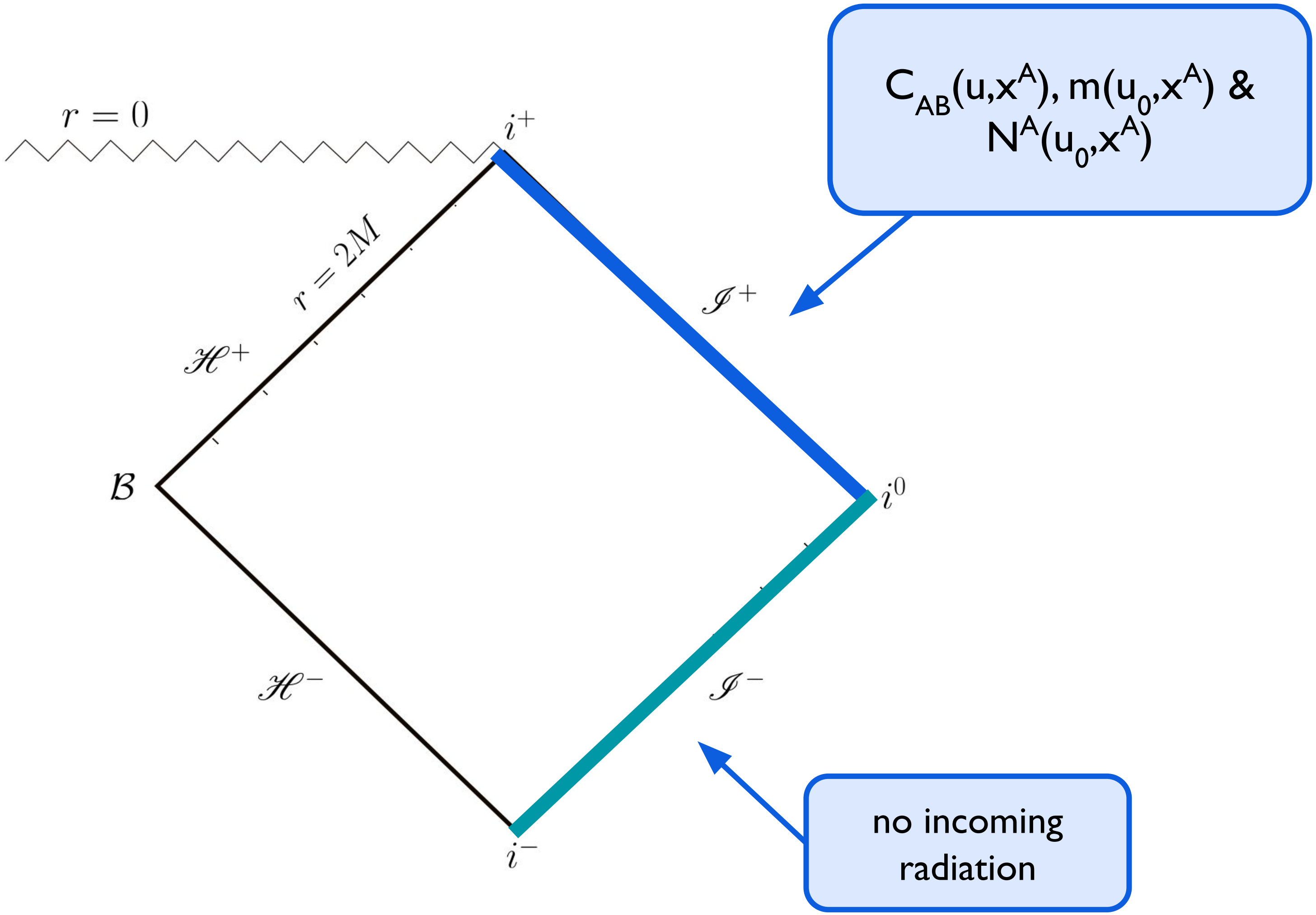


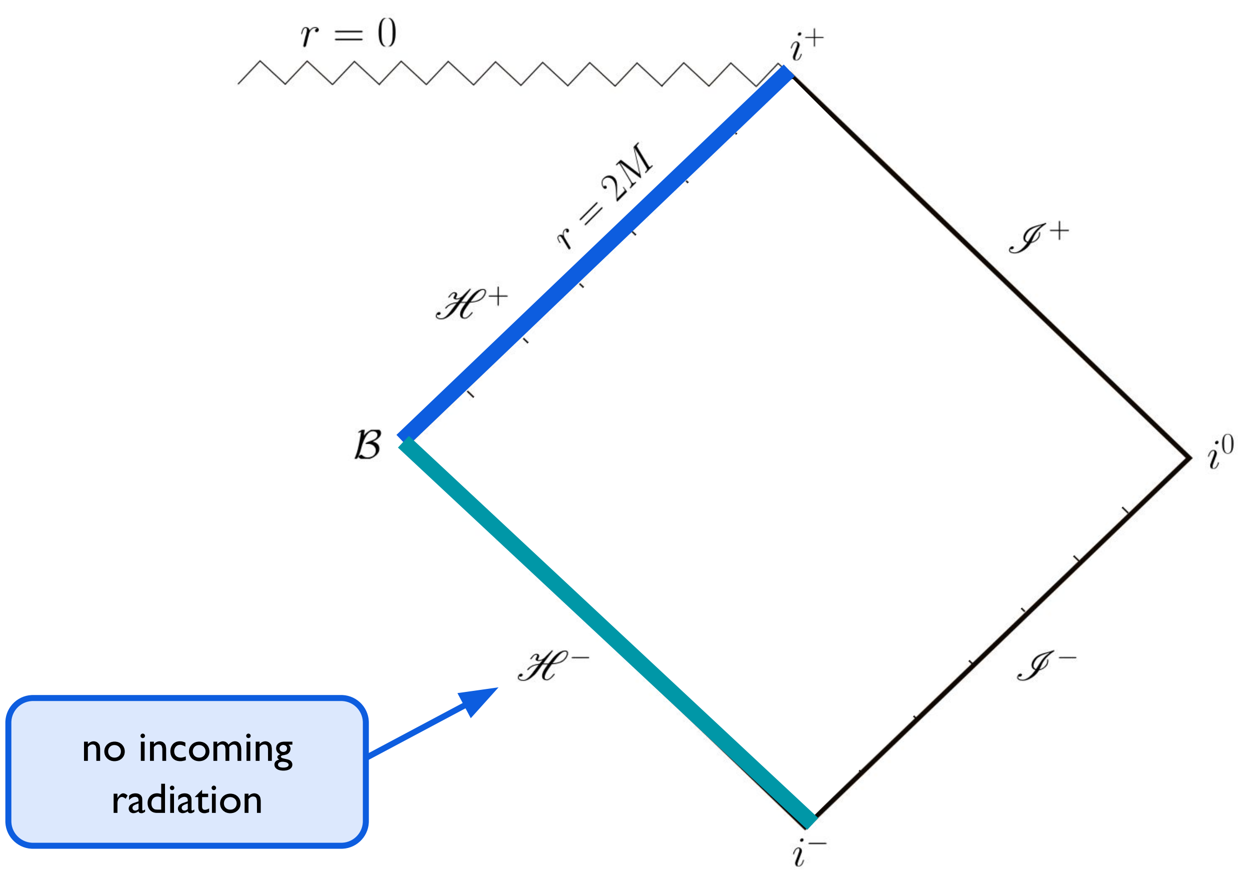
Our goal: BH tomography for QNMs



Similar to Bondi-Sachs framework



Our goal: BH tomography for QNMs



Perturbed isolated horizon

Tidal deformations	Dynamic perturbations
Time-independent perturbations $\Psi_2 \rightarrow$ horizon geometry Note: θ_l and σ_l remain zero	Tidal heating, QNMs, etc. $\Psi_0 \rightarrow$ radiation Note: θ_l and σ_l no longer zero (but at linear order θ_l is)

Overview scheme

1. Specify all data on (non-rotating) horizon:
   Ψ_0 = freely specifiable

   Assume Ψ_0 separable & Fourier mode decomposition
2. Using the "radial" equations obtain the solution everywhere
3. Impose "boundary" conditions:
   - require analyticity → QNMs
   - stability towards the future → no growing modes

Result
QNM solutions everywhere satisfying the standard boundary conditions
Relation Ψ_0 at horizon and Ψ_4 at null infinity

Horizon data

1. Gauge choices adapted to horizon
2. Bianchi identities + Einstein's equations

$$D\widetilde{\sigma} - \kappa_{(l)}\widetilde{\sigma} \triangleq \widetilde{\Psi}_0 ,$$

$$D\widetilde{\Psi}_1 - \kappa_{(l)}\widetilde{\Psi}_1 \triangleq \bar{\eth}\widetilde{\Psi}_0 ,$$

Ψ_1 is nonzero!

$$D\widetilde{\Psi}_2 \triangleq \bar{\eth}\widetilde{\Psi}_1 ,$$

$$D\widetilde{\pi} \triangleq \bar{\widetilde{\Psi}}_1 ,$$

$$D\widetilde{\mu} + \kappa_{(l)}\widetilde{\mu} \triangleq \widetilde{\Psi}_2 + \bar{\widetilde{\Psi}}_2 ,$$

$$D\widetilde{\lambda} + \kappa_{(l)}\widetilde{\lambda} \triangleq \bar{\eth}\widetilde{\pi} + \mu\bar{\widetilde{\sigma}} ,$$

$$D\widetilde{\Psi}_3 + \kappa_{(l)}\widetilde{\Psi}_3 \triangleq \bar{\eth}\widetilde{\Psi}_2 + 3\widetilde{\pi}\Psi_2 ,$$

Ψ_3 is nonzero!

$$D\widetilde{\Psi}_4 + 2\kappa_{(l)}\widetilde{\Psi}_4 \triangleq \bar{\eth}\widetilde{\Psi}_3 - 3\widetilde{\lambda}\Psi_2 ,$$

$$D\widetilde{a} \triangleq -\bar{a}\bar{\widetilde{\sigma}} - \bar{\widetilde{\Psi}}_1 ,$$

$$D\widetilde{\xi^{\bar{z}}} \triangleq \widetilde{\sigma}\bar{\xi}^{\bar{z}} .$$

Horizon solution

$$\widetilde{\Psi}_0 \triangleq \frac{1}{2\pi}\sum_{l,m}\left(\int \mathrm{d}\omega a^-_{lm} e^{-i\omega v} + \int \mathrm{d}\bar{\omega} a^+_{lm} e^{i\bar{\omega} v}\right) {}_2Y_{lm} \,,$$

$$\widetilde{\sigma} \triangleq -\frac{1}{2\pi}\sum_{l,m}\left(\int_{-\infty}^{\infty} \mathrm{d}\omega \frac{a^-_{lm}}{\kappa_{(l)} + i\omega} e^{-i\omega v} + \int_{-\infty}^{\infty} \mathrm{d}\bar{\omega} \frac{a^+_{lm}}{\kappa_{(l)} - i\bar{\omega}} e^{i\bar{\omega} v}\right) {}_2Y_{lm} \,,$$

$$\widetilde{\Psi}_1 \triangleq \frac{1}{2\pi}\sum_{l,m}\left(\int_{-\infty}^{\infty} \mathrm{d}\omega \frac{a^-_{lm}\sqrt{(l+2)(l-1)}}{\sqrt{2}c(\kappa_{(l)} + i\omega)} e^{-i\omega v} + \int_{-\infty}^{\infty} \mathrm{d}\bar{\omega} \frac{a^+_{lm}\sqrt{(l+2)(l-1)}}{\sqrt{2}c(\kappa_{(l)} - i\bar{\omega})} e^{i\bar{\omega} v}\right) {}_1Y_{lm} \,,$$

$$\widetilde{a} \triangleq \frac{1}{2\pi}\left(-\int \mathrm{d}\omega \sum_{l,m} \frac{(-1)^m \bar{a}^+_{lm} e^{-i\omega v}}{i\omega(\kappa_{(l)} + i\omega)} (\bar{a}\, {}_{-2}Y_{l\,-m} + \frac{\sqrt{(l+2)(l-1)}}{\sqrt{2}c} {}_{-1}Y_{l\,-m}) \right.$$
$$\left. + \int \mathrm{d}\bar{\omega} \sum_{l,m} \frac{(-1)^m \bar{a}^-_{lm} e^{i\bar{\omega} v}}{i\bar{\omega}(\kappa_{(l)} - i\bar{\omega})} (\bar{a}\, {}_{-2}Y_{l\,-m} + \frac{\sqrt{(l+2)(l-1)}}{\sqrt{2}c} {}_{-1}Y_{l\,-m})\right) \,,$$

$$\widetilde{\xi}^{\bar{z}} \triangleq \frac{\bar{\xi}^{\bar{z}}_0}{2\pi}\sum_{l,m} {}_2Y_{lm}\left(\int \mathrm{d}\omega \frac{a^-_{lm} e^{-i\omega v}}{i\omega(\kappa_{(l)} + i\omega)} - \int \mathrm{d}\bar{\omega} \frac{a^+_{lm} e^{i\bar{\omega} v}}{i\bar{\omega}(\kappa_{(l)} - i\bar{\omega})}\right) \,,$$

$$\widetilde{\Psi}_2 \triangleq \frac{1}{2\pi}\sum_{l,m} \frac{\sqrt{(l+2)(l+1)l(l-1)}}{2c^2}\left(\int_{-\infty}^{\infty} \mathrm{d}\omega \frac{a^-_{lm}}{i\omega(\kappa_{(l)} + i\omega)} e^{-i\omega v} - \int_{-\infty}^{\infty} \mathrm{d}\bar{\omega} \frac{a^+_{lm}}{i\bar{\omega}(\kappa_{(l)} - i\bar{\omega})} e^{i\bar{\omega} v}\right) Y_{lm} \,,$$

$$\widetilde{\pi} \triangleq \frac{1}{2\pi}\sum_{l,m}(-1)^m \frac{\sqrt{(l+2)(l-1)}}{\sqrt{2}c}\left(\int_{-\infty}^{\infty} \mathrm{d}\omega \frac{\bar{a}^+_{lm}}{i\omega(\kappa_{(l)} + i\omega)} e^{-i\omega v} - \int_{-\infty}^{\infty} \mathrm{d}\bar{\omega} \frac{\bar{a}^-_{lm}}{i\bar{\omega}(\kappa_{(l)} - i\bar{\omega})} e^{i\bar{\omega} v}\right) {}_{-1}Y_{l,-m} \,,$$

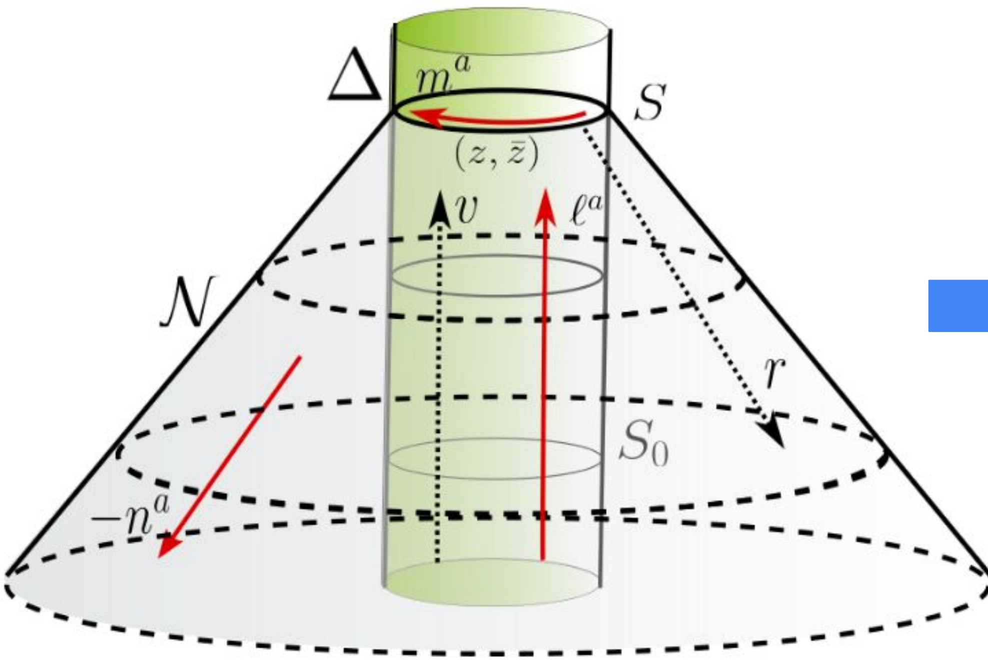
$$\widetilde{\mu} \triangleq \frac{1}{2\pi}\sum_{l,m} \frac{\sqrt{(l+2)(l+1)l(l-1)}}{2c^2}\left(\int_{-\infty}^{\infty} \mathrm{d}\omega \frac{[a^-_{lm} Y_{lm} + (-1)^m \bar{a}^+_{lm} Y_{l,-m}] e^{-i\omega v}}{i\omega(\kappa^2_{(l)} + \omega^2)} - \int_{-\infty}^{\infty} \mathrm{d}\bar{\omega} \frac{[a^+_{lm} Y_{lm} + (-1)^m \bar{a}^-_{lm} Y_{l,-m}] e^{i\bar{\omega} v}}{i\bar{\omega}(\kappa^2_{(l)} + \bar{\omega}^2)}\right) \,,$$

$$\widetilde{\lambda} \triangleq \frac{1}{2\pi}\sum_{l,m}(-1)^m\left(\int_{-\infty}^{\infty} \mathrm{d}\omega \frac{\bar{a}^+_{lm} e^{-i\omega v}}{2ic^2\omega(\kappa^2_{(l)} + \omega^2)} (2ic\omega - (l+2)(l-1)) + \right.$$
$$\left. \int_{-\infty}^{\infty} \mathrm{d}\bar{\omega} \frac{\bar{a}^-_{lm} e^{i\bar{\omega} v}}{2ic^2\bar{\omega}(\kappa^2_{(l)} + \bar{\omega}^2)} (2ic\bar{\omega} + (l+2)(l-1))\right) {}_{-2}Y_{l,-m} \,,$$

$$\widetilde{\Psi}_3 \triangleq -\frac{1}{2\pi}\sum_{l,m} \frac{\sqrt{(l+2)(l-1)}}{2\sqrt{2}c^3}\left(\int_{-\infty}^{\infty} \mathrm{d}\omega \frac{[a^-_{lm}\, {}_{-1}Y_{lm}(l+1)l + 3(-1)^m \bar{a}^+_{lm}\, {}_{-1}Y_{l,-m}] e^{-i\omega v}}{i\omega(\kappa^2_{(l)} + \omega^2)} \right.$$
$$\left. - \int_{-\infty}^{\infty} \mathrm{d}\bar{\omega} \frac{[a^+_{lm}(l+1)l\, {}_{-1}Y_{lm} + 3(-1)^m \bar{a}^-_{lm}\, {}_{-1}Y_{l,-m}] e^{i\bar{\omega} v}}{i\bar{\omega}(\kappa^2_{(l)} + \bar{\omega}^2)}\right) \,,$$

$$\widetilde{\Psi}_4 \triangleq \frac{1}{8\pi c^4}\sum_{l,m}\left(\int_{-\infty}^{\infty} \mathrm{d}\omega \frac{[a^-_{lm}(l+2)(l+1)l(l-1)\, {}_{-2}Y_{lm} + 6c(-1)^m i\omega \bar{a}^+_{lm}\, {}_{-2}Y_{l,-m}] e^{-i\omega v}}{i\omega(\kappa^2_{(l)} + \omega^2)(2\kappa_{(l)} - i\omega)} \right.$$
$$\left. - \int_{-\infty}^{\infty} \mathrm{d}\bar{\omega} \frac{[a^+_{lm}(l+2)(l+1)l(l-1)\, {}_{-2}Y_{lm} - 6c(-1)^m i\bar{\omega} \bar{a}^-_{lm}\, {}_{-2}Y_{l,-m}] e^{i\bar{\omega} v}}{i\bar{\omega}(\kappa^2_{(l)} + \bar{\omega}^2)(2\kappa_{(l)} + i\bar{\omega})}\right) \,.$$

Spacetime solution



Ψ_4 and Ψ_0 satisfy Teukolsky equation (+Teukolsky-Starobinsky identities)

Analytic solutions

$$X_{lm}^{(4)}(r) = k_1 H_c[(l-2)(l+3)+5\iota, 5\iota, 3-\iota, 3, \iota, -\frac{r}{c}]$$
$$+k_2\left(-\frac{r}{c}\right)^{\iota-2} H_c[l(l+1)+\iota^2, \iota(3+\iota), \iota-1, 3, \iota, -\frac{r}{c}]$$

Compare with Leaver:

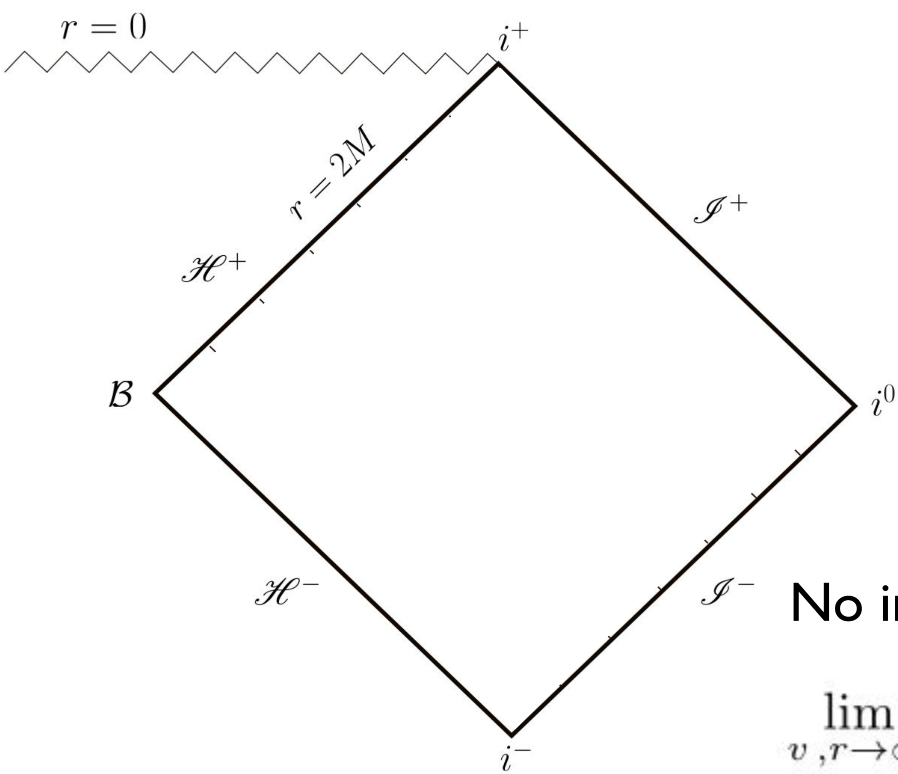
$$X_{lm}^{(4,1)} = e^{2i\omega r}\left(\frac{r+c}{c}\right)^{-1+2ic\omega}\sum_{k=0}^{\infty} a_k \left(\frac{r}{r+c}\right)^k$$

$$0 = \beta_0 - \frac{\alpha_0\gamma_1}{\beta_1 - \frac{\alpha_1\gamma_2}{\beta_2 - \ldots}}$$

$$\alpha_k = (k+1)(3+k-2ic\omega)$$
$$\beta_k = 3 - l(l+1) - 2k^2 + 4ic\omega + 8c^2\omega^2 + k(-2+$$
$$\gamma_k = (k-2ic\omega)(k-2-2ic\omega)\,.$$

QNM solutions

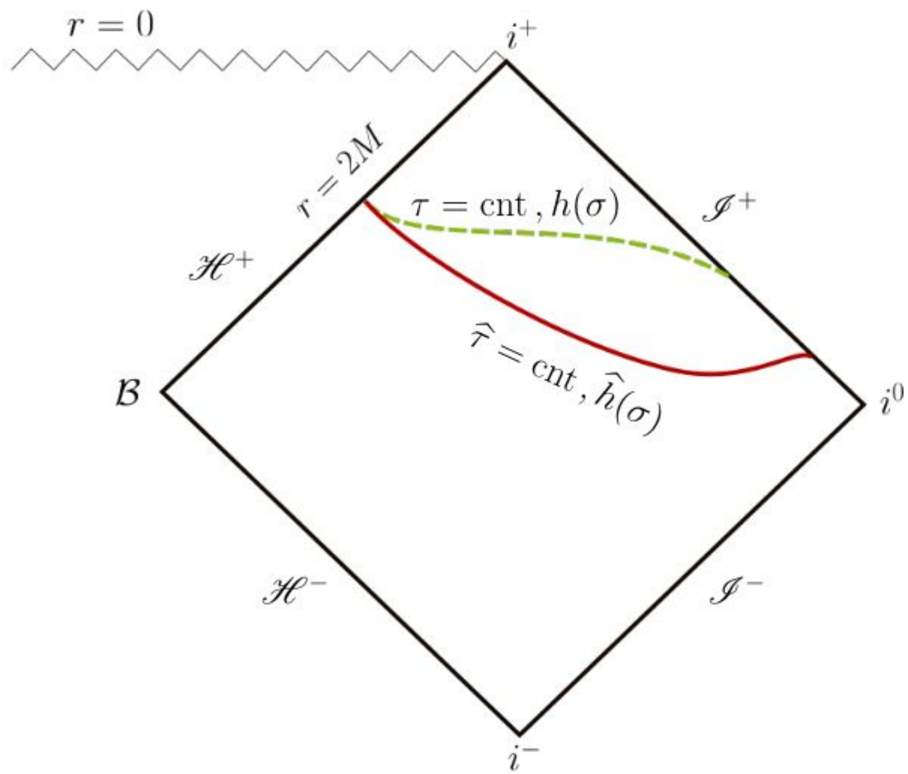
Separability, Fourier mode decomposition, analyticity, stability towards future → QNMs



No incoming radiation: $\Psi_0 \sim r^{-3}$ and $\Psi_1 \sim r^{-2}$

$$\lim_{v\,,r\to\infty} \widetilde{\Psi}_0 \propto \frac{1}{r^5} a_{lmn}^{-(1)} e^{i\omega_{lmn}^{\mathrm{QNM}}(-v+2r+2c\ln r/c)} + \frac{1}{r^5} a_{lmn}^{+(1)} e^{-i\bar{\omega}_{lmn}^{\mathrm{QNM}}(-v+2r+2c\ln r/c)}$$

Connecting horizon to scri



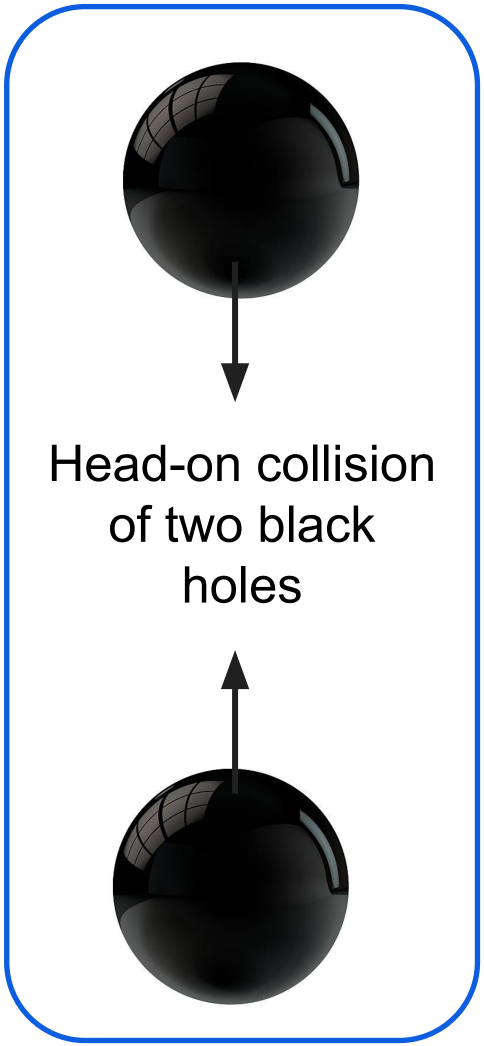
$$\Psi_{4\,,l0n}^{\mathscr{I}^+\,,-} = \frac{K_l^2 \Psi_{0\,,l0n}^{H\,,-} + 6ic\omega_{l0n} \bar{\Psi}_{0\,,l0n}^{H\,,+}}{4c^4 i\omega_{l0n}(\kappa_{(\ell)}^2 + \omega_{l0n}^2)(2\kappa_{(\ell)} - i\omega_{l0n})} \mathscr{F}_{l0n}$$

Natural extensions

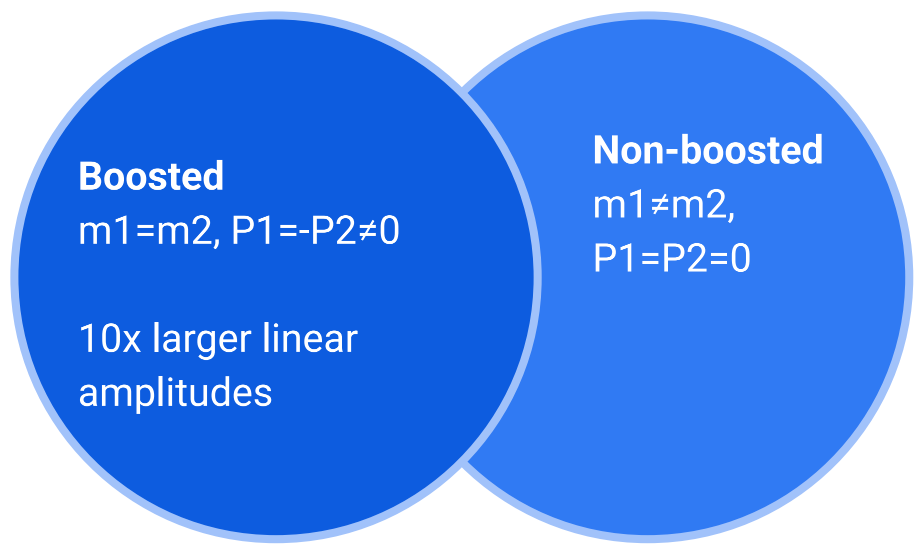
1. Kerr
2. Go to second order probing the dynamic horizon regime
3. Study horizon multipole moments

… go fully non-linear!

Two sets of simulations using the Einstein Toolkit



(1) Resulting BH is non-rotating
(2) Axisymmetric simulations ⟶ only m=0 modes
(3) High resolution near horizon (but poor near infinity)



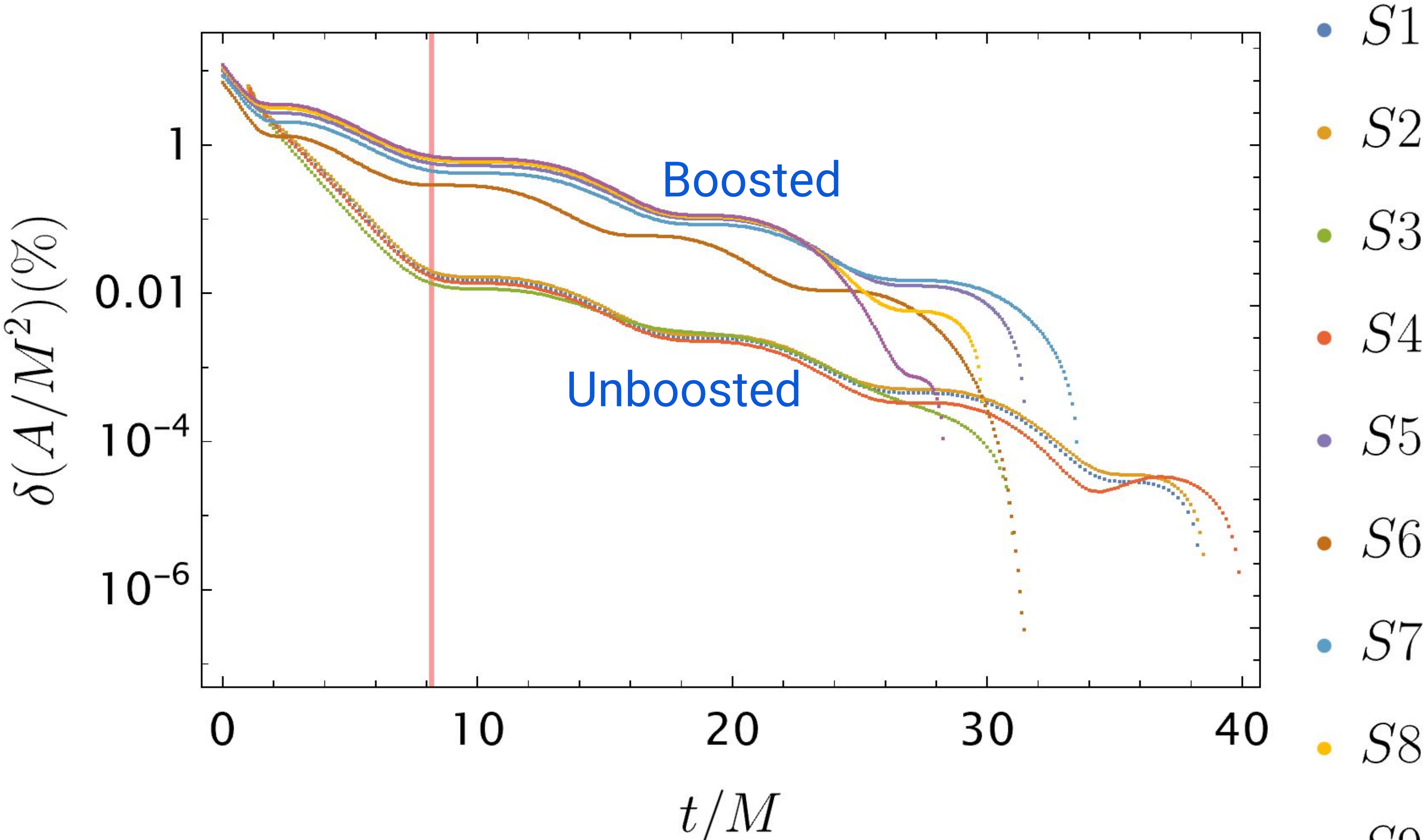
Choice of time

Time ⟷ Definition of frequency

Disclaimer: We simply use the simulation time.

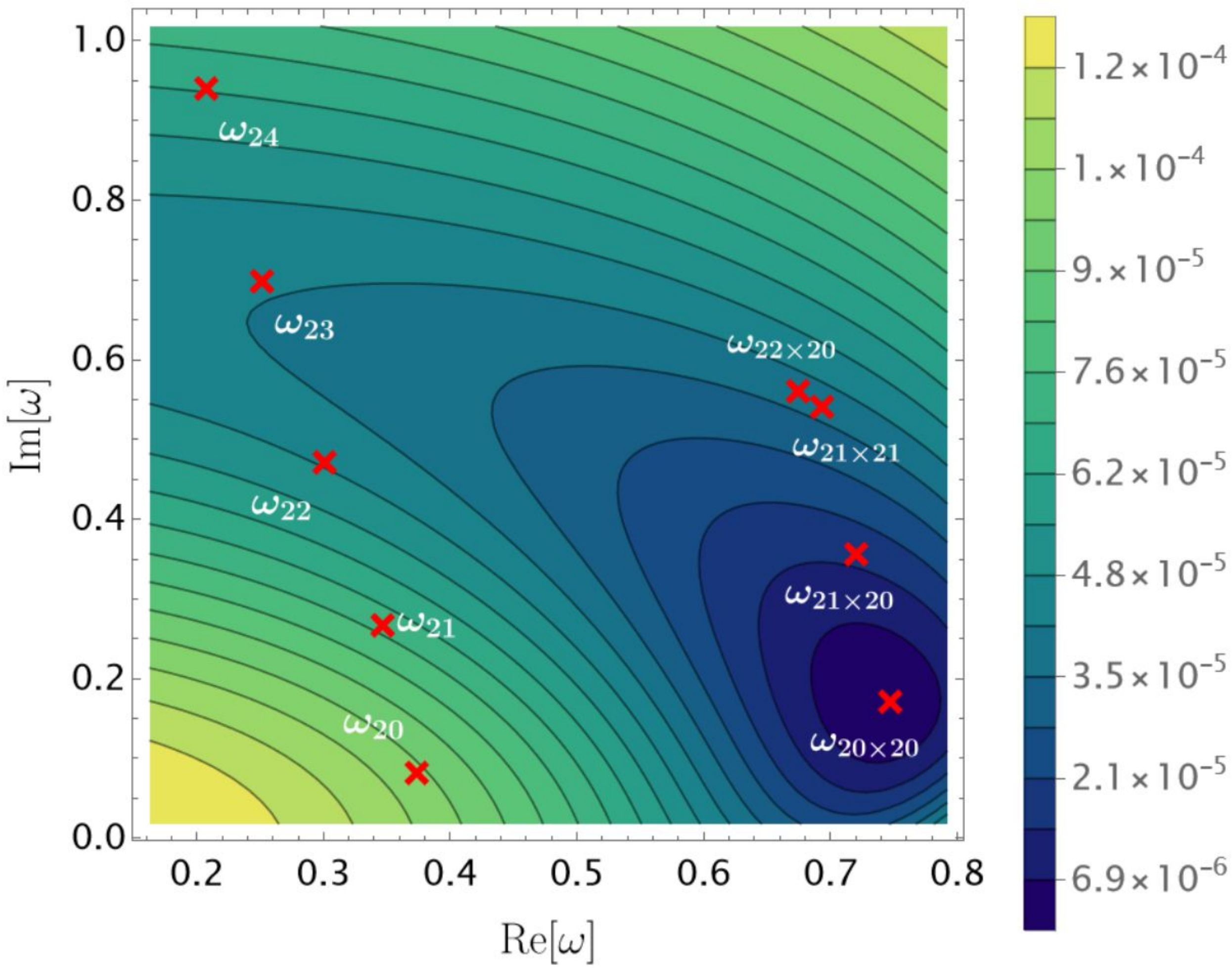
Same issue at infinity!

Ringdown: Mass changes ≤ 1 %

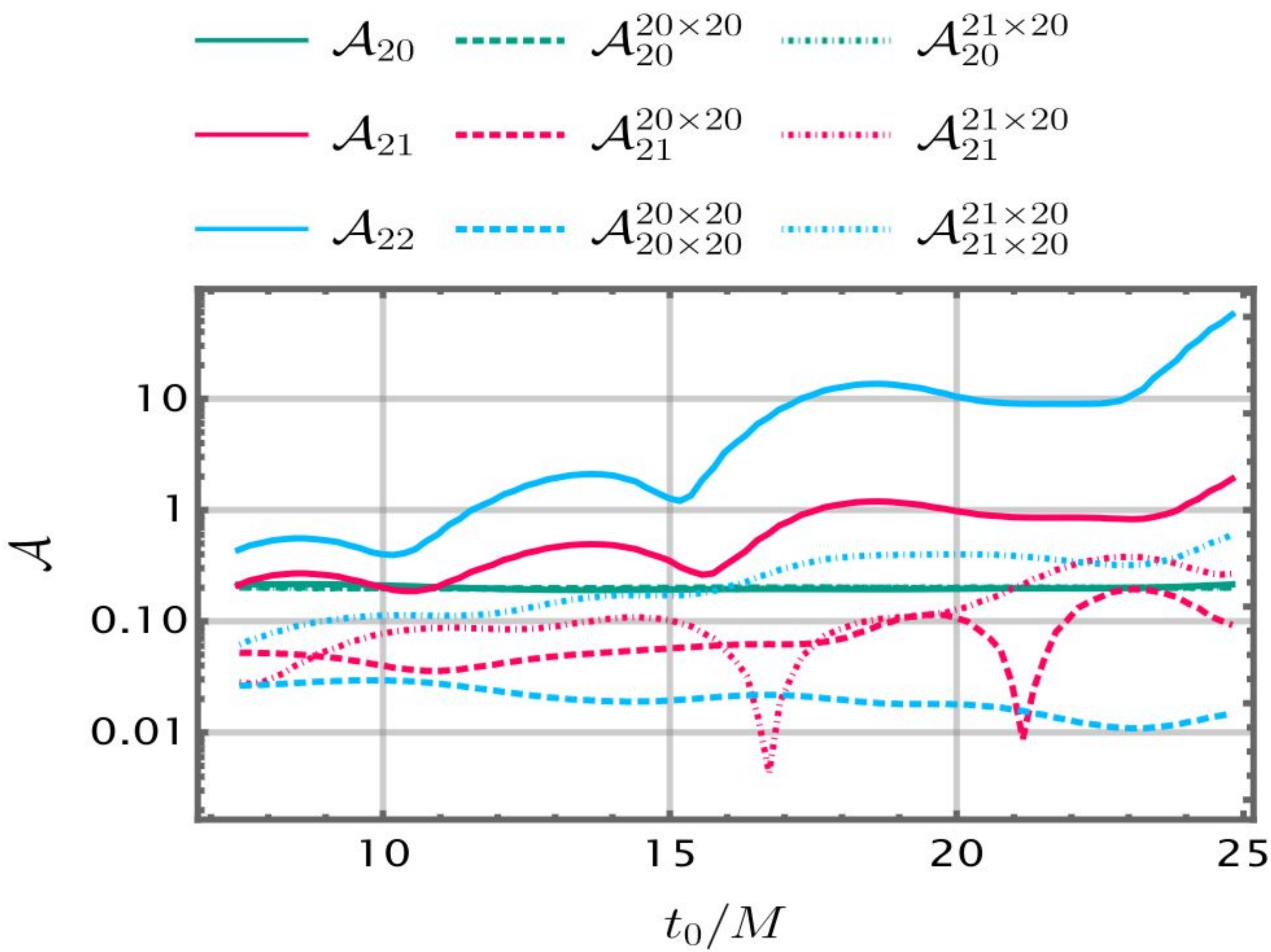


We take $t_{ringdown}$ = 8.2 M

Mismatch after fixing ω_{200} and ω_{201}

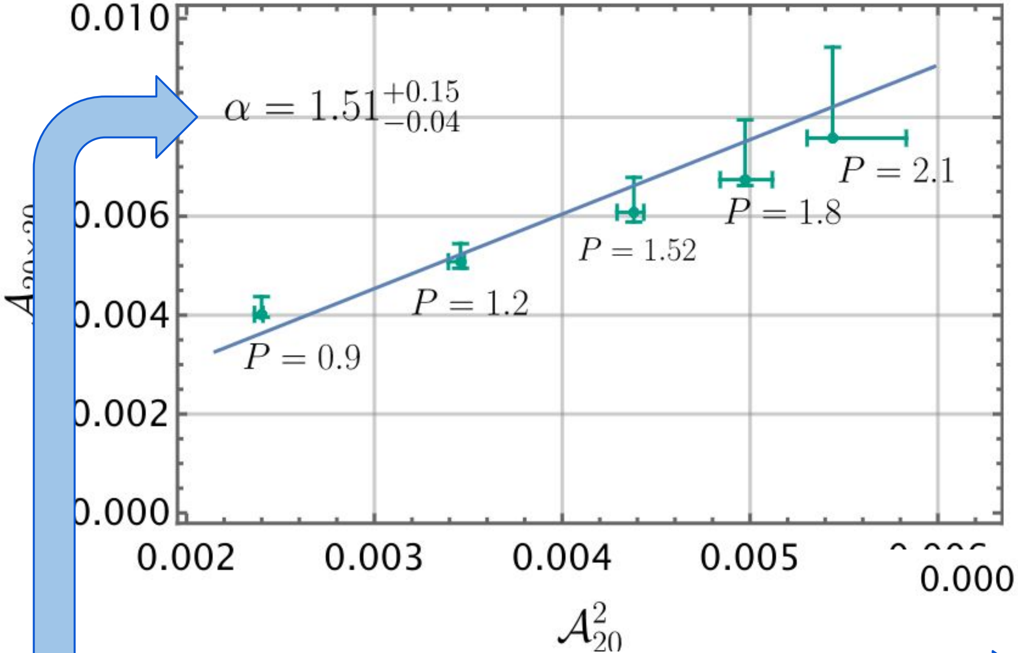


Stability amplitude

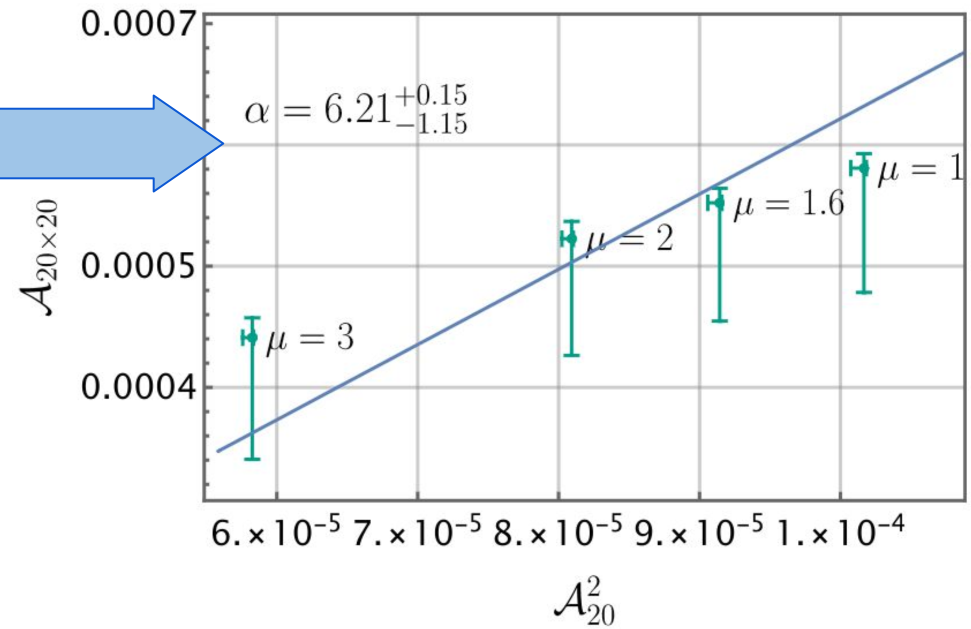


Amplitude relation

Boosted



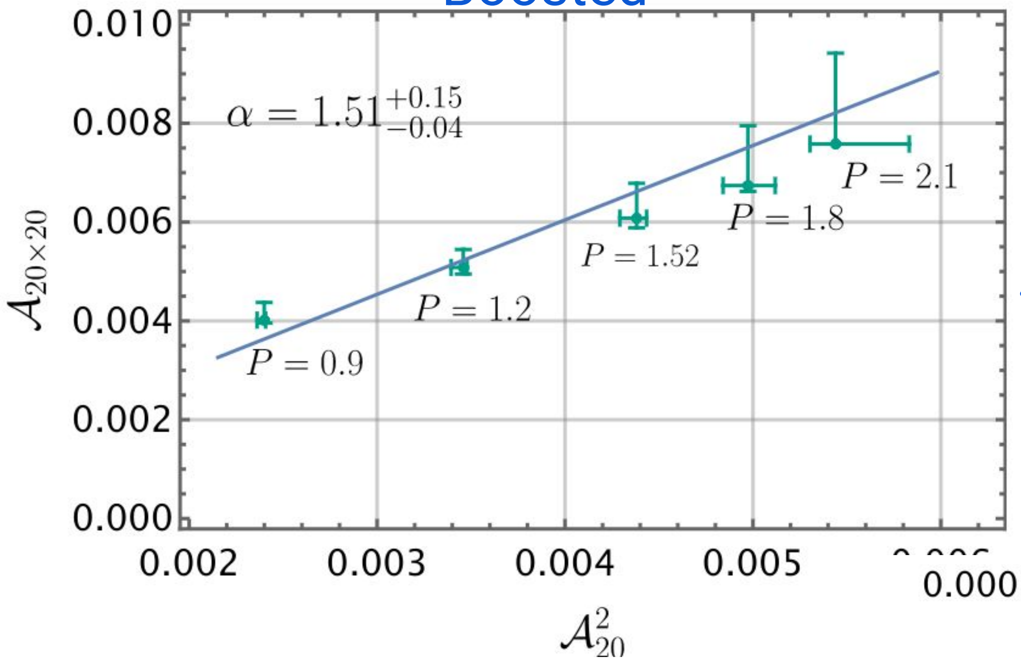
Unboosted



Puzzle: Why are these slopes different?

Amplitude relation explained?

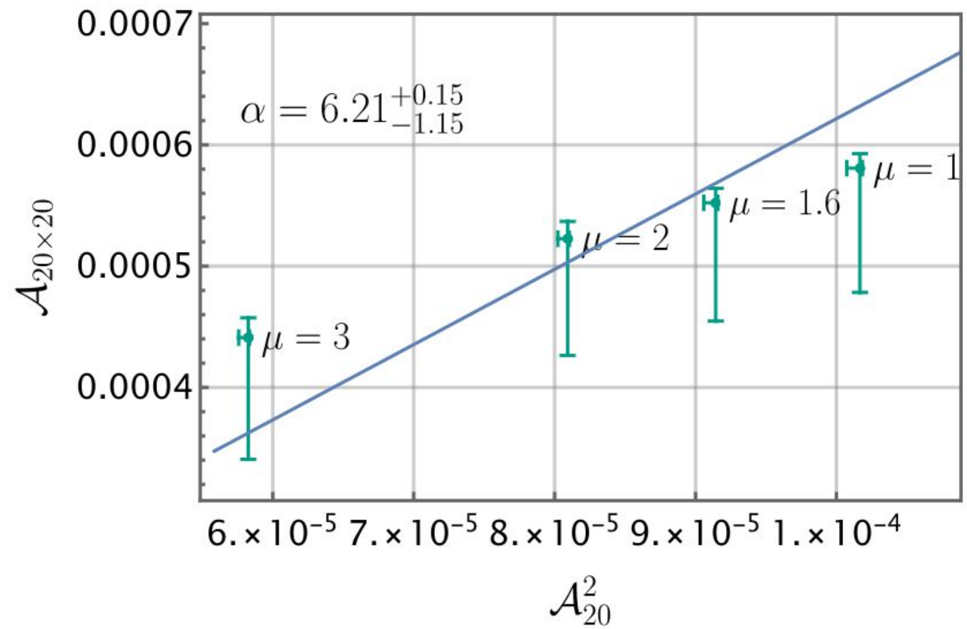
Boosted



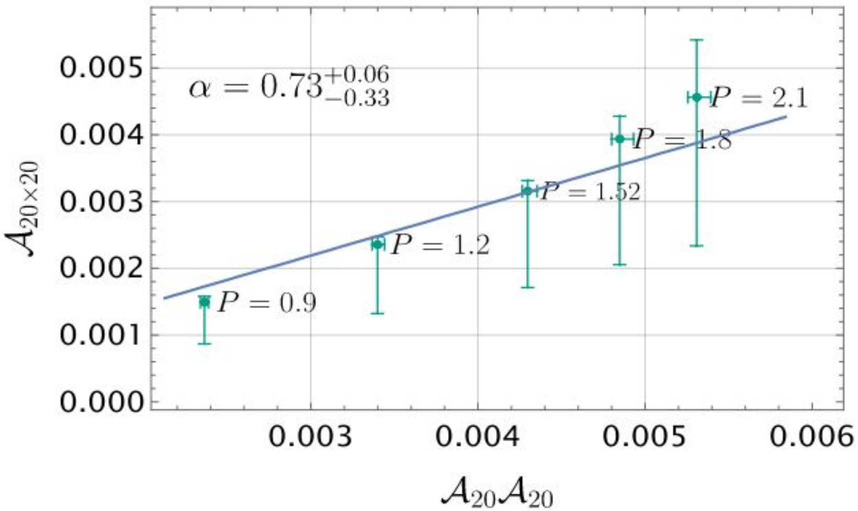
Up-down symmetry
no odd modes!

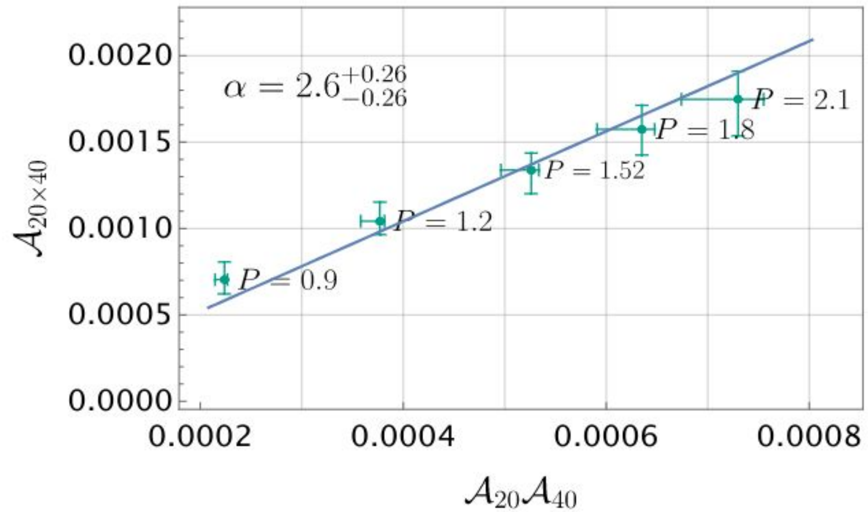
No up-down symmetry
odd and even modes excited

Unboosted



Amplitude relation l=4





Data prefers model with fundamental tone + 2 quadratic modes!

Non-linear black hole tomography?

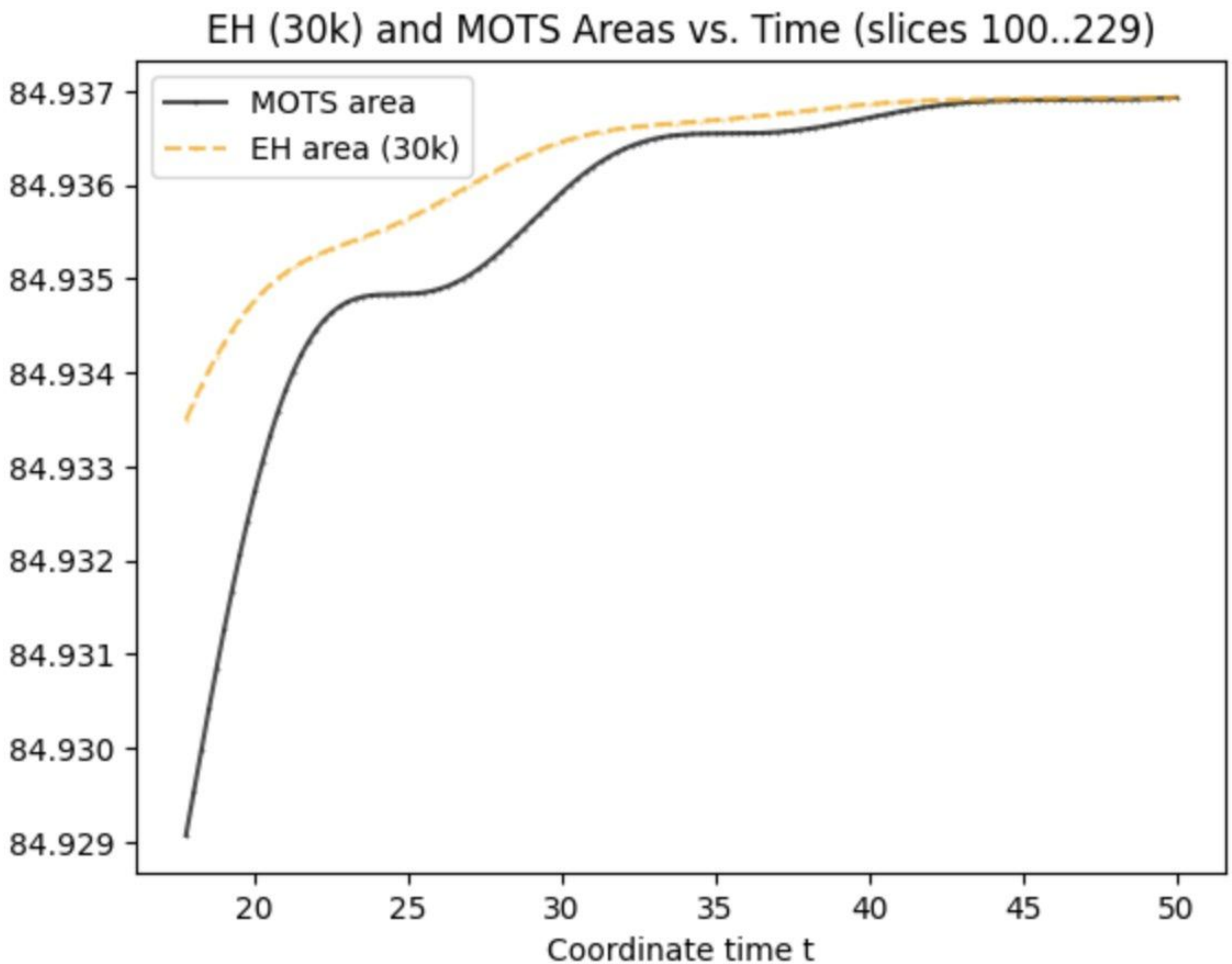
- Same modes found at infinity in similar simulations (with poor resolution near horizon)

- Comparing amplitudes at horizon and infinity not (yet) feasible

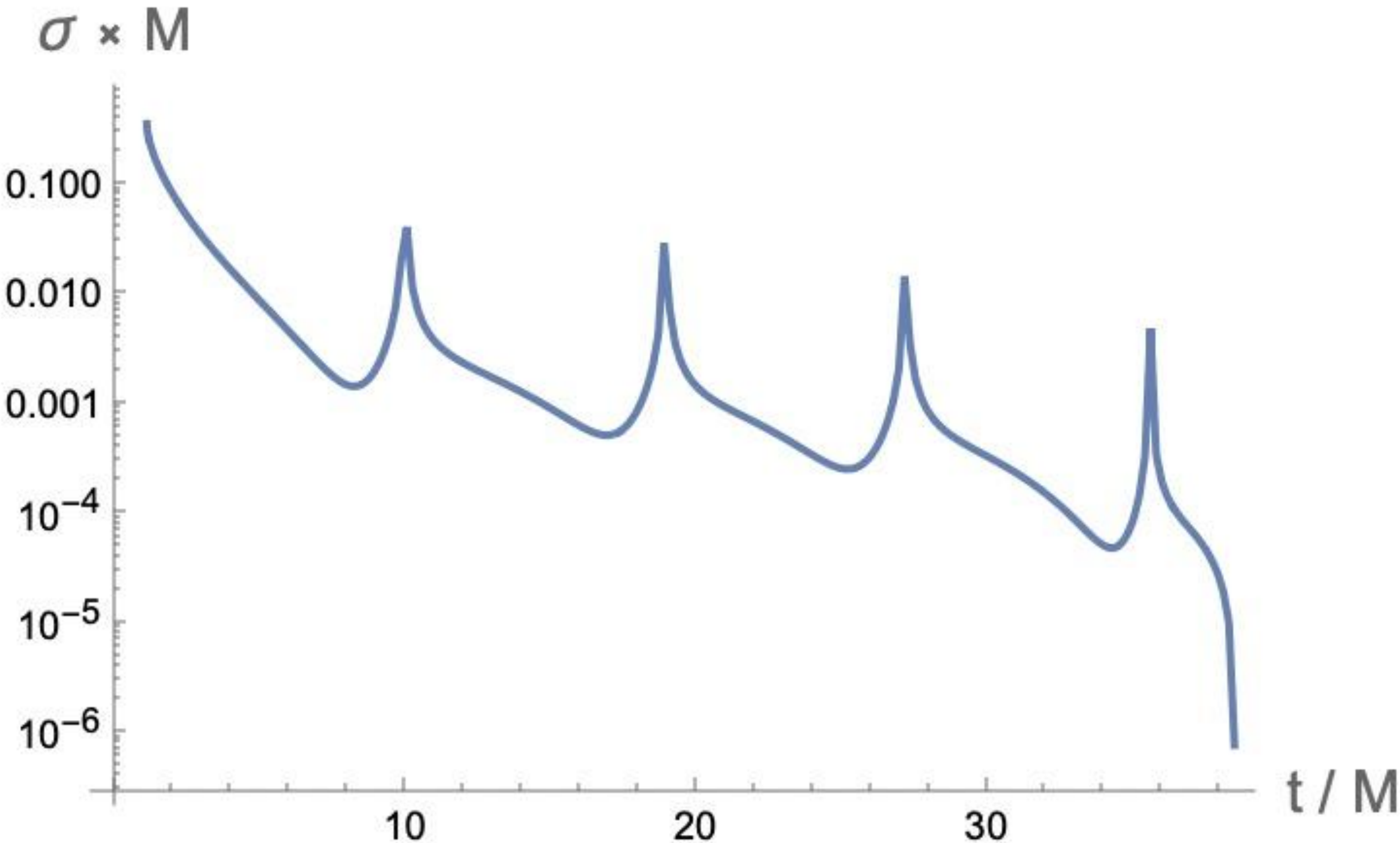
Challenges in comparing NR to BHPT

	NR	Black hole perturbation theory
location	apparent horizon	event horizon
quantity	shear modes	Ψ_0
gauge	1+log & Γ-driver shift	IRG

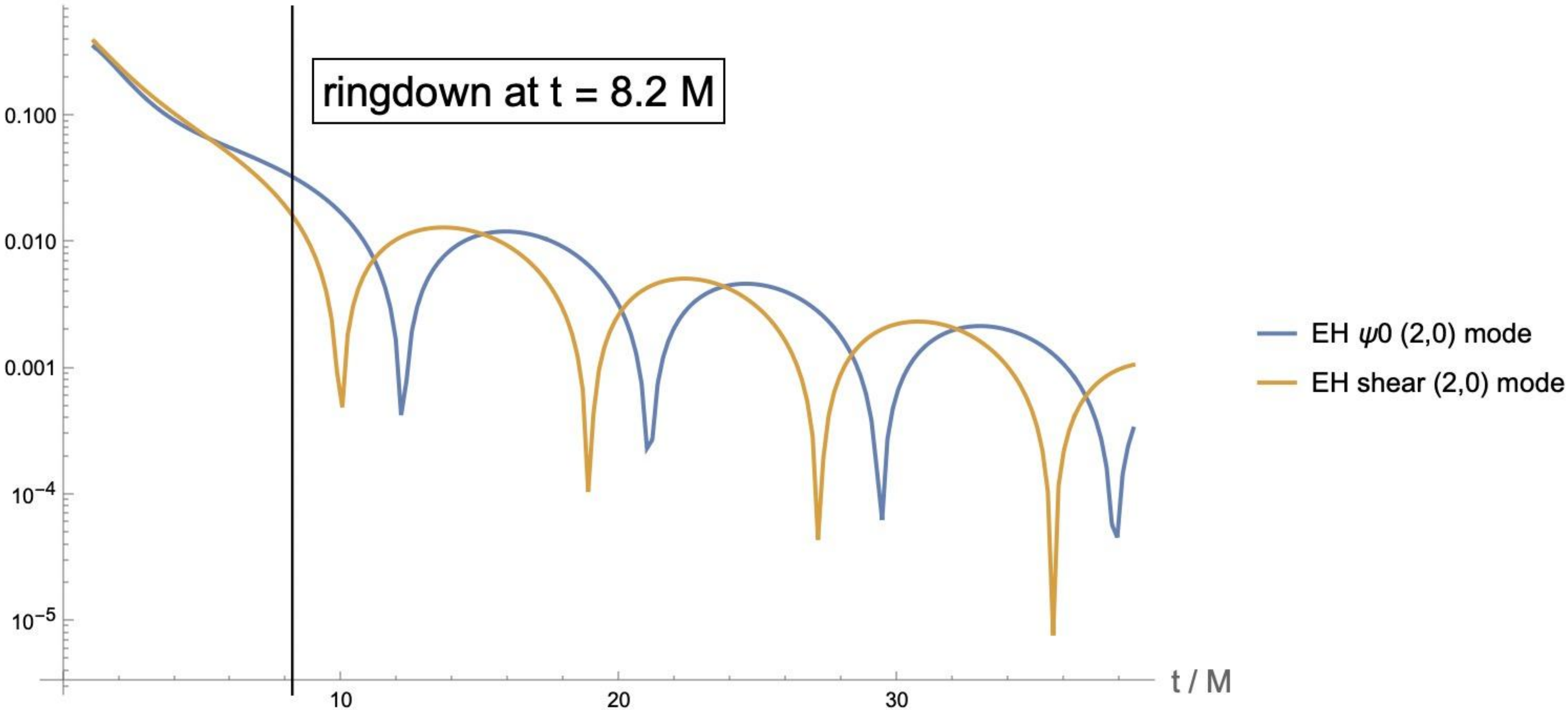
Preliminary: area EH vs MOTS



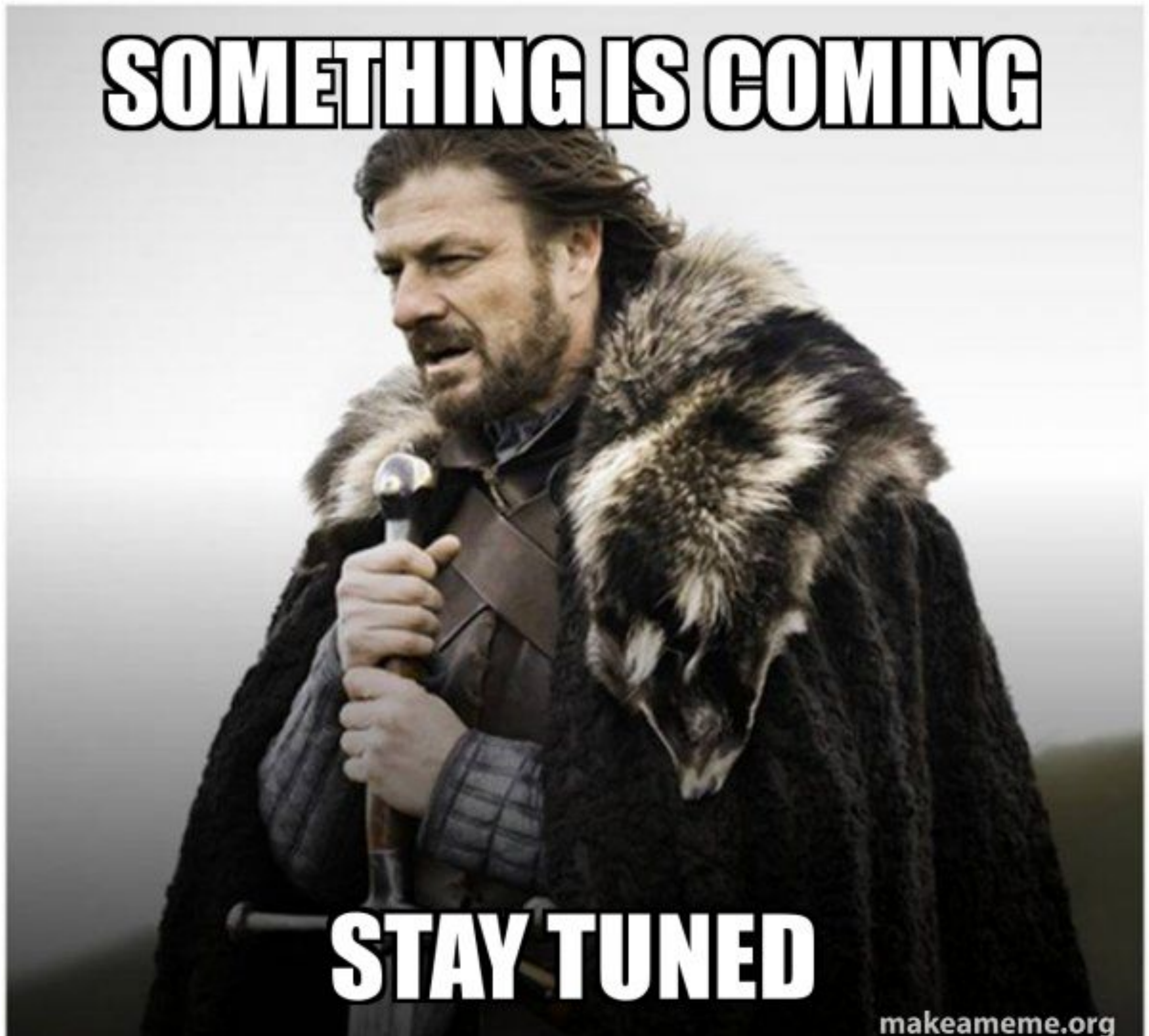
Preliminary: relative difference $\sigma_{l=2}$ EH vs MOTS



Preliminary: event horizon geometry



For QNM comparison…



Conclusion

- ★ First explicit analytic construction of black hole tomography
- ★ QNMs describe horizon geometry nicely
- ★ Comparison with black hole perturbation theory ongoing
- ★ Observations of quadratic modes very likely with ET & LISA, so the future is bright!