Quasi-local black hole horizons

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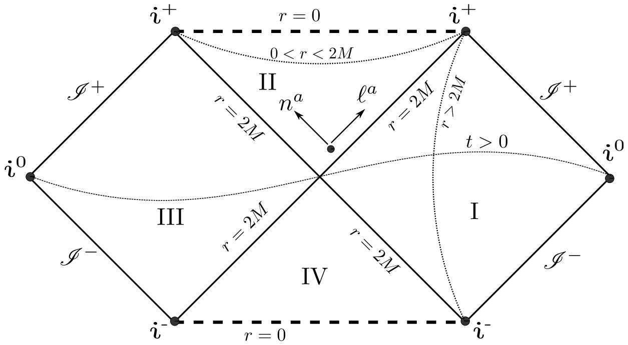
Outline

Event Horizons

- Black hole surfaces traditionally defined via event horizons



Event horizons

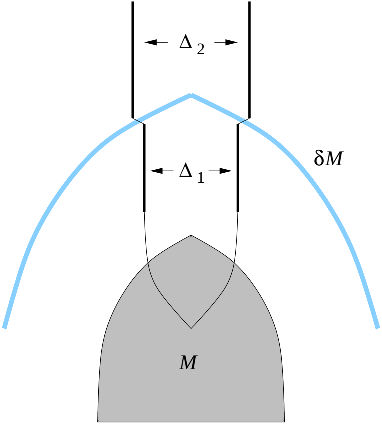
Event horizons have been used in key “classic” investigations in black hole physics

- The area increase theorem
- Black hole thermodynamics
- The black hole uniqueness theorems
- Black hole perturbation theory
- Topological censorship
- Astrophysics – all we need is Kerr + perturbations?

Is there any need to go beyond event horizons?

Event horizons are global and teleological

- Need to know the entire spacetime to locate event horizons (which is why numerical simulations do not rely on them)



Usual first law is not local

- Consider the "usual" first law

$$\delta M_{ADM} = \frac{\kappa\delta A}{8\pi G} + \Omega\delta J_{ADM}$$

- A, Ω, κ are defined at the horizon (though the Killing vector is normalized at ∞)
- M_{ADM} and J_{ADM} are defined at spatial infinity
- We would like a quasi-local version applicable to BHs in our universe

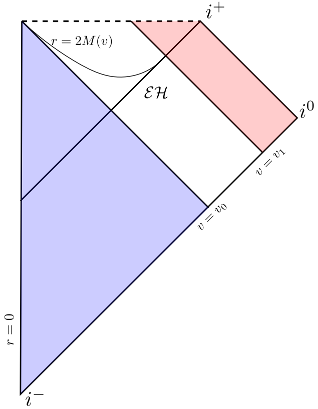
The Physical Process version of the second law

- Simple generalization of Schwarzschild for a varying mass function $M(v)$ with $\dot{M}(v) \geq 0$ and $\lim_{v\to\infty} M(v)$ finite
- Vaidya spacetime represents collapse of spherically symmetric null-dust

$$ds^2 = -\left(1 - \frac{2M(v)}{r}\right) dv^2 + 2dv\,dr + r^2 d\Omega^2 \,.$$

$$T_{ab} = \frac{\dot{M}(v)}{4\pi r^2} \nabla_a v \nabla_b v \,.$$

Event horizons can grow in flat space



- No general second law can be formulated for event horizons i.e. we cannot write ΔA as a sum of local fluxes
- This can only be done in perturbative settings + end state boundary condition (Hartle & Hawking (1972))

Quantum fluctuations can remove EHs

- Semiclassical calculations (Hajicek (1987)) show that quantum fluctuations near the singularity can completely remove the event horizon
- This is a semiclassical study of the Vaidya collapse model
- Shows that the Hawking radiation is associated with the apparent horizon, and not the event horizon

The singularity theorems

- There is one classic result which does not use event horizons directly
- The singularity theorem: the presence of a *closed trapped surface* (+energy condition) implies geodesic incompleteness in the future (Penrose (1965), Penrose)
- This paper also introduced closed trapped surfaces which are key to understanding black holes quasi-locally

Expansion, shear and twist

Take a spacelike 2-surface S with intrinsic metric q_{ab} and null normals ℓ^a, n^a

- The expansions are

$$\Theta_{(\ell)} = q^{ab}\nabla_a \ell_b \,, \quad \Theta_{(n)} = q^{ab}\nabla_a n_b \,.$$

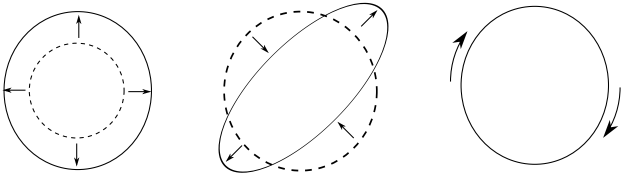
$$\to \Theta = \frac{1}{A}\frac{dA}{dt}$$

- Shear is the symmetric tracefree part:

$$\sigma^{(\ell)}_{ab} := q^c_{(a} q^d_{b)} \nabla_c \ell_d - \frac{1}{2}\Theta_{(\ell)} q_{ab}$$

- (...and twist is the anti-symmetric part)

Expansion, shear and twist

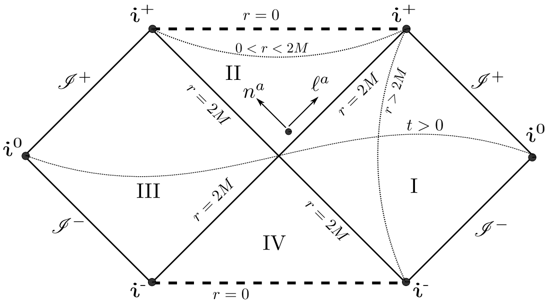


Definition of a trapped surface

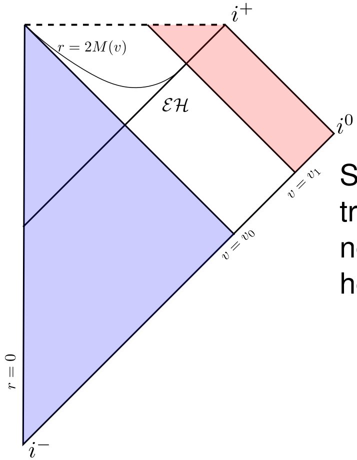
- *S*: closed spacelike spherical surface
- Outgoing and in going null normals: ℓ and n
- For a trapped surface, both sets of null rays are converging: $\Theta_{(\ell)} < 0$ and $\Theta_{(n)} < 0$
- Outer trapped surface: $\Theta_{(\ell)} < 0$, and no condition on $\Theta_{(n)}$
- Signature of black holes but *not* necessarily of strong gravitational field
- Marginally trapped surface: $\Theta_{(\ell)} = 0,\ \Theta_{(n)} < 0$
- Marginally outer trapped (MOTS): Only $\Theta_{(\ell)} = 0$
- Outermost MTS on a Cauchy surface is called the apparent horizon
- Boundary of trapped region on spatial slice is MTS – Hayward & Kriele (1997), Andersson & Metzger (2007)

The boundary of the trapped region

- Alternative definition of a black hole surface: boundary of spacetime region containing trapped surfaces
- In Schwarzschild trapped surfaces extend all the way up to the event horizon



Trapped surfaces in Vaidya



Spherically symmetric trapped surfaces do not extend up to event horizon

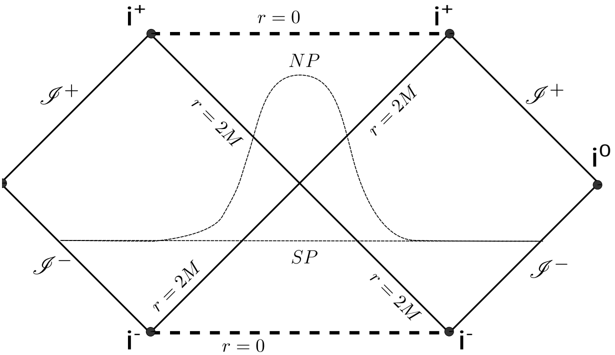
Eardley's conjecture

- Event horizons generally have positive expansion –> cross sections of the event horizon cannot be a MOTSs
- As we try to push a MOTS towards the event horizon, we cannot do it "uniformly" – and the limit will be singular
- The basic problem already exists in spherical symmetry (but with dynamics), and including angular momentum and other complications etc. are (probably) not qualitatively different
- Eardley was the first to recognize this in 1998 ("Black hole boundary conditions and coordinate conditions", Phys.Rev.D **57** (1998) 2299-2304)
- Eardley's conjecture: the event horizon is the boundary of the trapped spacetime region
- A MOTS needs to be deformed in long "needle" like shapes extending far to the future (or past) to approach the event horizon

Shapiro & Teukolsky and Wald & Iyer

- In 1991, Shapiro & Teukolsky claimed the existence of naked singularities in a numerical simulation of the Einstein-Vlasov system – they did not find any apparent horizons though they had indications of singularities
- Wald & Iyer pointed out that there exist Cauchy slices close to the singularity, even just in Schwarzschild, which do not contain any trapped surfaces

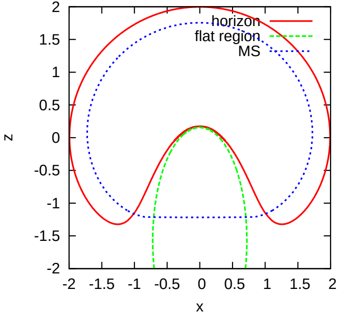
Shapiro & Teukolsky and Wald & Iyer



Shapiro & Teukolsky and Wald & Iyer

- Actually, numerical horizon finders would happily find an “apparent horizon” on these kinds of Cauchy surfaces (i.e. $\Theta_{(\ell)} = 0$).
- The “apparent horizon” would not satisfy $\Theta_{(n)} < 0$
- As far as we know, standard gauge choices in NR do not lead to such Cauchy surfaces
- Finally, as we shall see later, for BBH mergers we generally have MOTSs which do not satisfy $\Theta_{(n)} < 0$ everywhere (which are though entirely in the black hole region)

The trapping boundary



- In Vaidya we can push outer marginally trapped surfaces arbitrarily close to the EH (only $\Theta_{(\ell)} = 0$)
- Marginal surfaces can also extend into flat region (Schnetter & Krishnan, 2006)
- Analytic proof by Ben-Dov for Vaidya (2006)

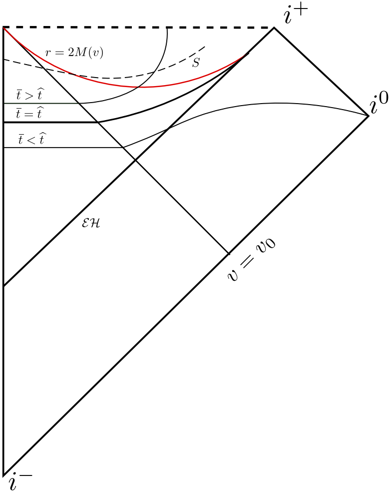
The trapping boundary

- Define "Kodama time" as

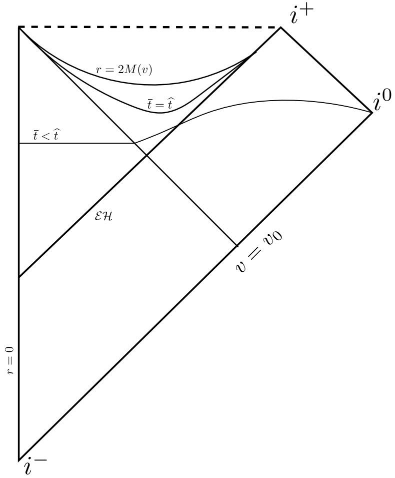
$$\xi_a = \nabla_a r - \left(1 - \frac{2M(v)}{r}\right)\nabla_a v\,.$$

- Boundary of trapped surfaces is $\widehat{\Sigma}$ and not the event horizon (Senovilla & Bengtsson (2010))
- The boundary may not always extend to the flat portion
- This is the case when $\lim_{v\to 0^+} M(v)/v < 1/8$

The trapping boundary

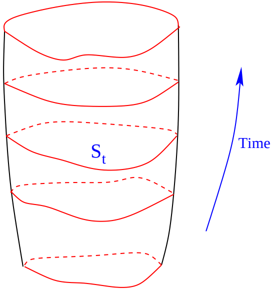


The trapping boundary



Marginally trapped tubes (MTTs)

- Marginally trapped tubes obtained by stacking up Marginally outer trapped surfaces (MOTS)



- Think of marginally trapped tube as the smooth time evolution of a MOTS
- Marginally trapped tube is defined as a 3-surface foliated by MOTSs
- This is a covariant spacetime definition even if we might use a 3+1 split to locate MOTSs in practice

Isolated, dynamical and trapping horizons

- All these are different kinds of MTTs ($\Theta_{(\ell)} = 0$)
- **Dynamical horizon**: spacelike MTT with $\Theta_{(n)} < 0$
- **Isolated horizon**: null MTT, no restriction on $\Theta_{(n)}$
- **Trapping horizon**: Null or spacelike, $\Theta_{(n)} < 0$, $\mathcal{L}_n\Theta_{(\ell)} < 0$
- **Timelike membrane**: timelike MTT

Some Review Articles:

- Ashtekar & Krishnan, Living Reviews (2004)
- Booth, Can. J. Phys. (2005)
- Gourgoulhon & Jaramillo, Physics Reports (2006)
- Ashtekar & Krishnan, Living Reviews (2025)

The equilibrium case

- The goal is to model a black hole in equilibrium in a universe that is otherwise dynamical
- Infinite dimensional set of solutions provided by characteristic initial value formulation (see later)
- This is an approximation in several cases of interest (inspiral, ringdown....)

Non-expanding horizons

A smooth 3-dimensional null surface Δ is said to be a *non-expanding horizon* if:

- Δ has topology $S^2 \times \mathbb{R}$, and if $\Pi : S^2 \times \mathbb{R} \to S^2$ is the natural projection, then $\Pi^{-1}(x)$ for any $x \in S^2$ are null curves on Δ.
- The expansion $\Theta_{(\ell)} := q^{ab}\nabla_a \ell_b$ of any null normal ℓ^a of Δ vanishes.
- The Einstein field equations hold at Δ, and the matter stress-energy tensor T_{ab} is such that for any future directed null-normal ℓ^a, $-T^a_b \ell^b$ is future causal.

Weakly exanding horizon

A weakly isolated horizon is a NEH Δ with a choice of null normal ℓ^a (defined up to constant positive rescalings) such that

$$\mathcal{L}_\ell \omega_a = 0$$

where ω_a is defined as

$$X^a \nabla_a \ell^b = X^a \omega_a \ell^b$$

for any X^a tangent to Δ

- Surface gravity is $\kappa_\ell = \ell^a \omega_a$ which is constant on Δ (zeroth law)
- Shear of ℓ^a vanishes $\rightarrow$ no fluxes across the horizon

The zeroth and first laws

(Ashtekar, Beetle & Lewandowski (2001, 2002), Ashtekar, Fairhurst & BK (2000), Booth (2001))

- The Weyl tensor is algebraically special on a NEH

$$\Psi_0 = \Psi_1 = 0$$

$$\Psi_2 \to \text{time independent}$$

$$\Psi_3, \Psi_4 \text{ determined by } \Psi_2$$

- Ψ_2 is thus gauge invariant
- The angular modes of Ψ_2 are then the BH multipole moments

The zeroth and first laws

- Turn now to angular momentum and the first law
- Let φ^a be an axial symmetry of the intrinsic horizon geometry
- Let ϕ^a be an axial vector in spacetime (not necessarily a symmetry) such that

$$\phi^a \to \varphi^a \quad \text{on} \quad \Delta$$

$$\phi^a \to \varphi^a_\infty \quad \text{at} \quad S^\infty$$

- The Hamiltonian which generates motions along ϕ^a on phase space consists of boundary terms

$$H^\phi = \oint_S \cdots + \oint_{S^\infty} \cdots$$

The zeroth and first laws

- The term at infinity is the ADM angular momentum
- Identify the horizon term with the BH angular momentum

$$J_\Delta^\varphi = -\frac{1}{8\pi G}\oint_S \omega_a \varphi^a \, d^2V$$

- (Used routinely in NR simulations)
- For the energy consider a time evolution vector field

$$t^a \to c\ell^a - \Omega\varphi^a \quad \text{on} \quad \Delta$$

$$\phi^a \to \tau^a \quad \text{at infinity}$$

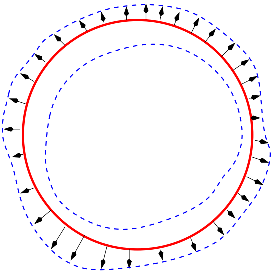
- Turns out that the corresponding Hamiltonian does not always exist

The zeroth and first laws

- The first law is the necessary and sufficient condition, i.e. if
$$dE_\Delta = \frac{1}{8\pi G}\kappa_{(c\ell)} dA + \Omega dJ_\Delta^{(\varphi)}$$
is an exact variation in phase space
- Thus we want c, Ω and the energy to be functions only of $(A, J_\Delta^{(\varphi)})$
- We can then consistently choose E_Δ, c, Ω to be the corresponding Kerr values as functions of $(A, J_\Delta^{(\varphi)})$
- This gives then the mass according to the usual Kerr formula

Evolution of MTSs in Time

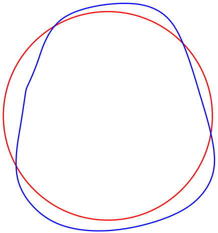
- MTT shown to exist if MTS is *strictly-stably-outermost*
    - Andersson, Mars & Simon (2005,2007)



- S is *strictly-stably-outermost* if it satisfies a stability condition
- Some outward deformation along $f\mathbf{r}$ makes S untrapped to linear order: $\delta_{f\mathbf{r}}\Theta_{(\ell)} > 0$ for $f \geq 0$

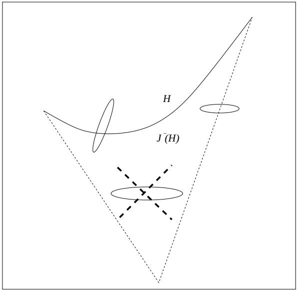
Properties of MTTs (Illustrative results)

- Key tool used in existence results is the elliptic stability operator: $\delta_{f\mathbf{r}}\Theta_{(\ell)} = L_{\mathbf{r}} f$
- The MTT in the neighborhood of a stable MTS is achronal
- It is spacelike if $T_{ab}\ell^a\ell^b$ is non-vanishing somewhere on S – DHs are the most relevant for physics
- The foliation of a DH is unique (Ashtekar & Galloway)
- DHs have generically $S^2 \times \mathbb{R}$ topology
- Strictly Stable MTSs are barriers for trapped and untrapped surfaces, i.e. this cannot happen:



Uniqueness results (Ashtekar & Galloway)

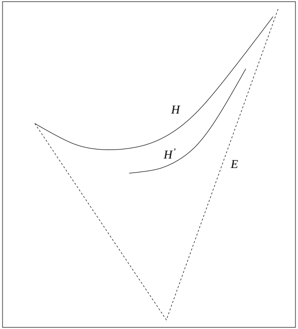
- How many DHs are there in a typical BH spacetime?



No closed MTS in $D^{-}(H) - H$: Ashtekar & Galloway 2005)

Uniqueness results

- Uniqueness result rules this out:



- But we can still have H' "threading" through H

References – Mostly with Brill-Lindquist

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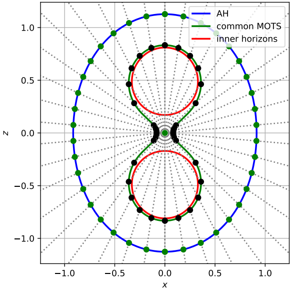
- N. Khera et al, Phys.Rev.Lett. **131** (2023) 23, 231401

Star shaped MOTSs

Usual assumption: MOTS must be star shaped

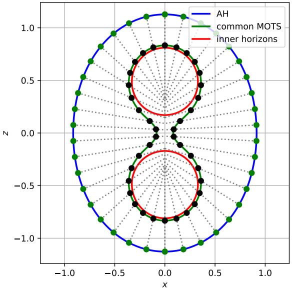
- MOTS is defined by $r = h(\theta, \phi)$



More general idea

Use a parameterized reference surface

- MOTS is then defined by $d = h(\theta, \phi)$
- Reference surface does not need to be star shaped



Results with Brill-Lindquist

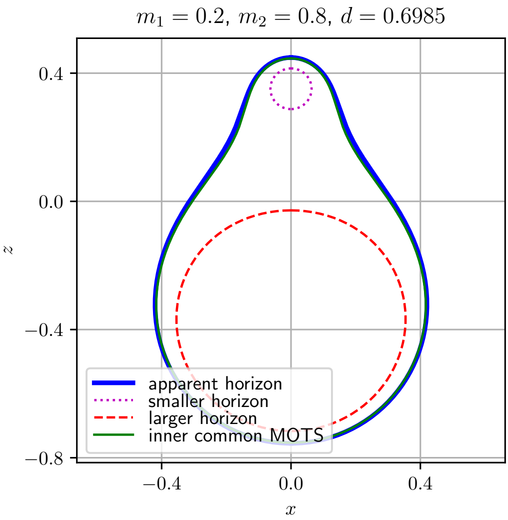
- First consider results with sequence of Brill-Lindquist initial data

Time symmetric data: non-spinning BHs with zero momentum and separation d

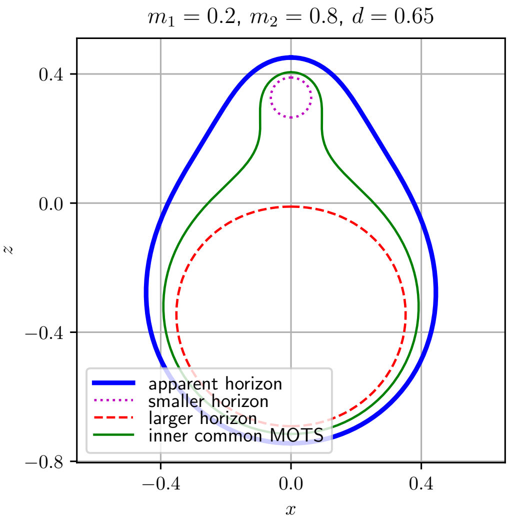
$$q_{ab} = \phi^4 \delta_{ab}$$

$$\phi = 1 + \frac{m_1}{2r_1} + \frac{m_2}{2r_2}$$

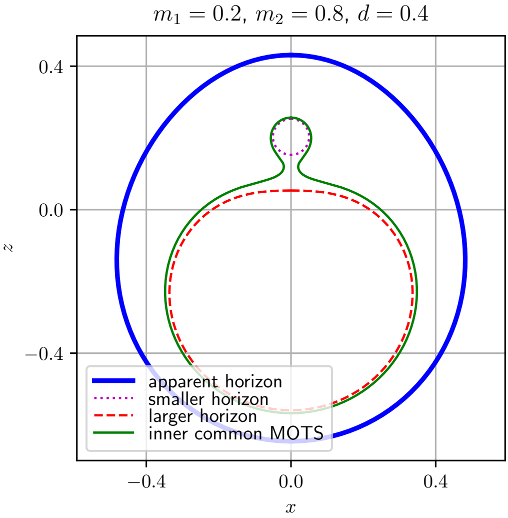
Results with Brill-Lindquist



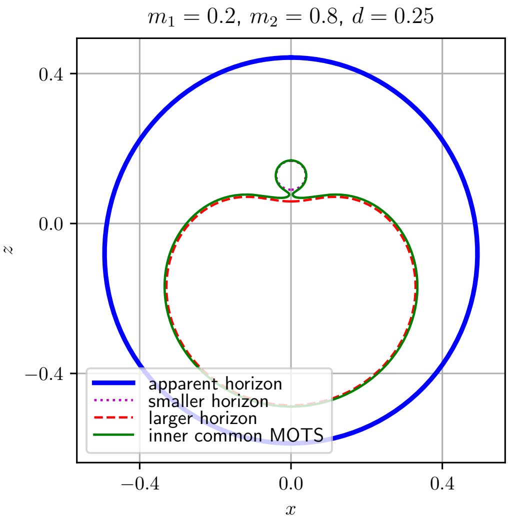
Results with Brill-Lindquist



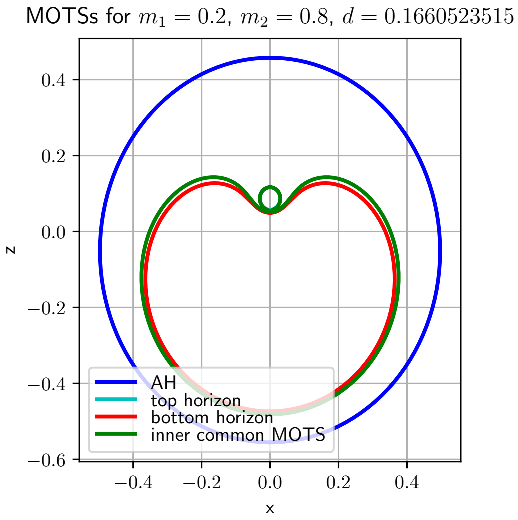
Results with Brill-Lindquist



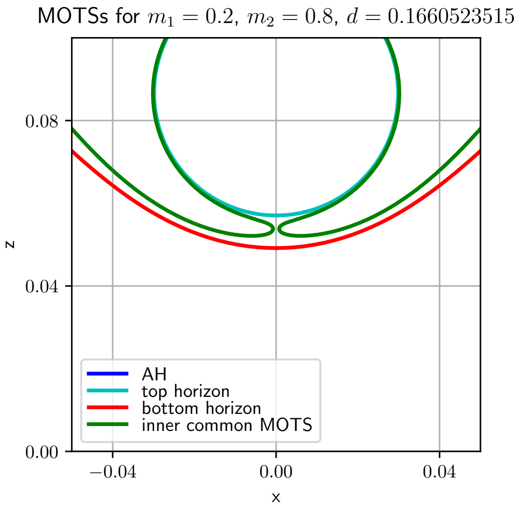
Results with Brill-Lindquist



Results with Brill-Lindquist



Results with Brill-Lindquist

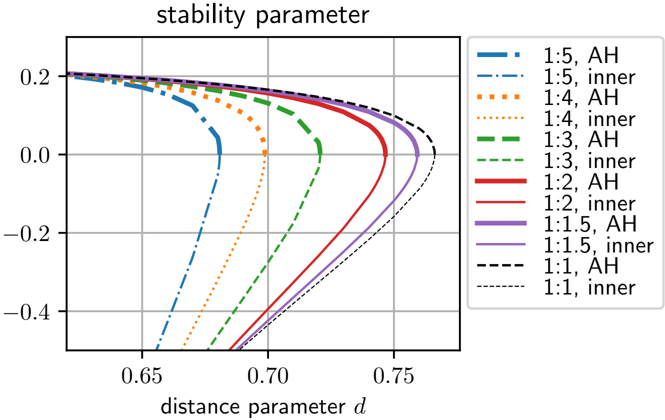


Properties of the 1:4 configuration

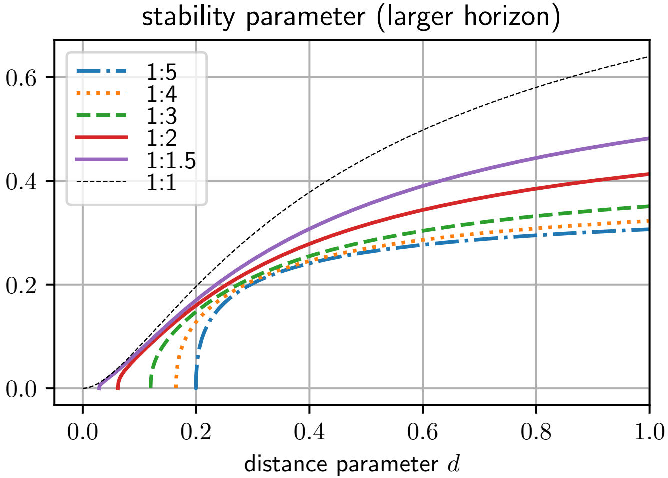
Large d	Only $\mathcal{S}_{large}$, $\mathcal{S}_{small}$ exist.
$d \approx 0.6987$	Common horizon appears and bifurcates into $\mathcal{S}_{AH}$ and $\mathcal{S}_{in}$.
$d \lesssim 0.6987$	$\mathcal{S}_{in}$ becomes increasingly distorted.
$d \lesssim 0.1660$	$\mathcal{S}_{in}$ no longer found.
$d \lesssim 0.1646$	$\mathcal{S}_{large}$ no longer found.
Small d	Only $\mathcal{S}_{AH}$ and $\mathcal{S}_{small}$ exist.

How can we explain this behavior?

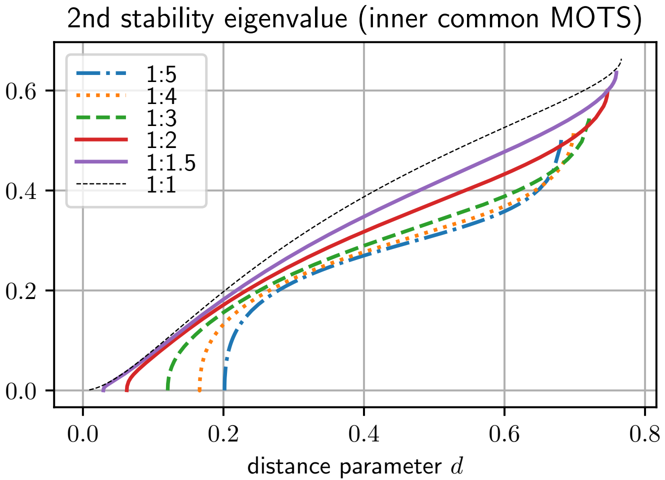
Stability of the outer horizons



Stability of the outer horizons



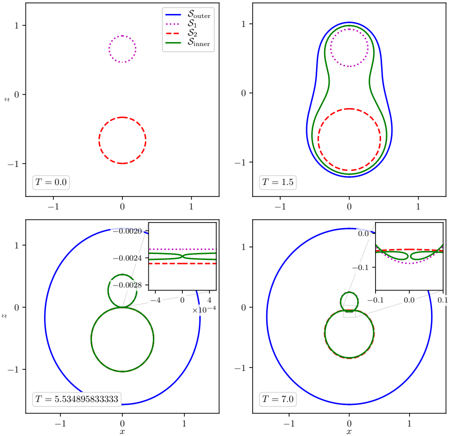
Stability of the outer horizons



Time evolution of Brill-Lindquist

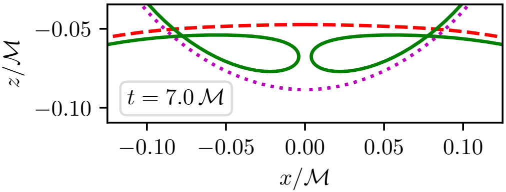
- After the study with initial data, we can apply the new horizon finder to time evolution
- Brill-Lindquist a good starting point – aim is to find sequence of MOTS connecting initial and final BHs

Time evolution of Brill-Lindquist

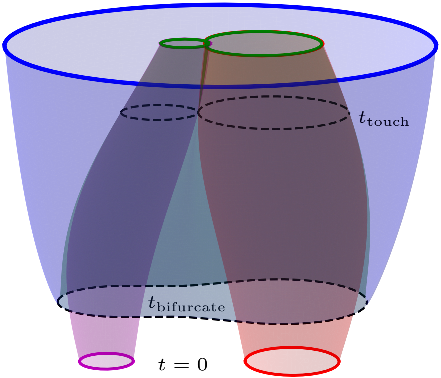


- Dramatic difference in behavior – the individual MOTSs now penetrate each other and merge with inner-common MOTS
- Self-intersections develop in inner-common MOTS immediately after merger

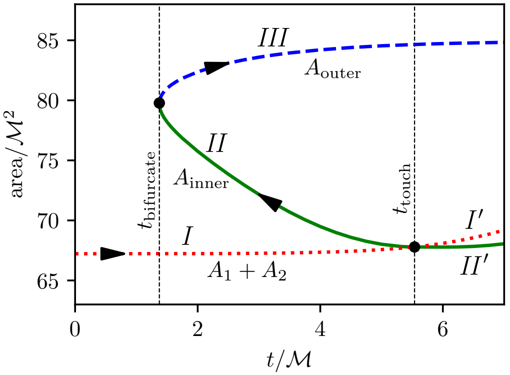
The self-intersecting MOTS



Analog of the "pair of pants picture"



The area increase law

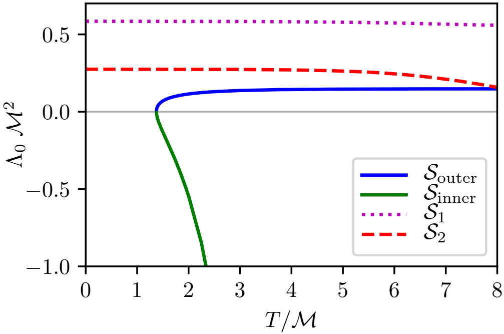


The stability spectrum and the merger

- Our numerical method allows us to calculate the spectrum highly accurately – operator is self-adjoint in the head-on collision of non-spinning black holes
- We need to watch out for vanishing eigenvalues – indicates a possible qualitative change in the geometry
- Start with the principal eigenvalue – the common horizon is born with $\Lambda_0 = 0$
- The outer horizon has always $\Lambda_0 > 0$, but the inner horizon is more interesting
- $\mathcal{S}_{inner}$ has $\Lambda_0 < 0$ but $\Lambda_1 > 0$

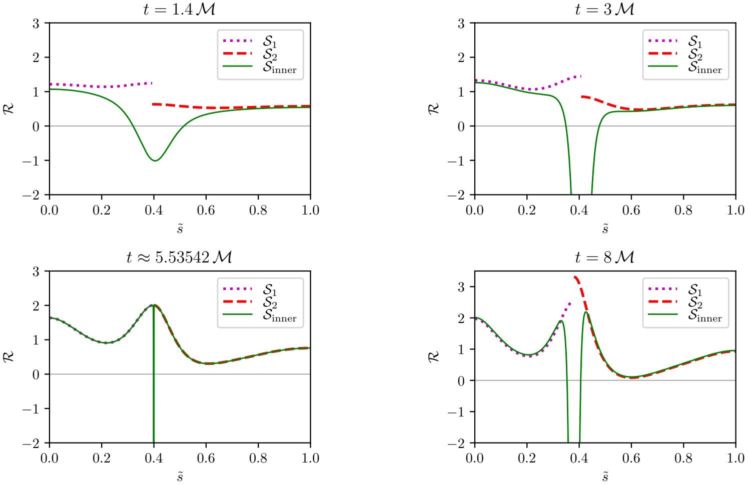
The stability spectrum and the merger

- Λ_0 becomes increasingly negative



Details of the cusp formation

(Vretinaris et al, In preparation)



- Allows us to track individual BH dynamics all the way through the merger

Binary black hole mergers

- In a BBH merger, we start from two BHs and end up with a single BH
- What happens to all of these inner horizons?
- Inricate sequence of bifurcations and "mergers" with more eigenvalues becoming successively unstable
- See talk by Ivan

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The characteristic initial value formulation

- Our approach is based instead on a characteristic initial value formulation for hyperbolic equations

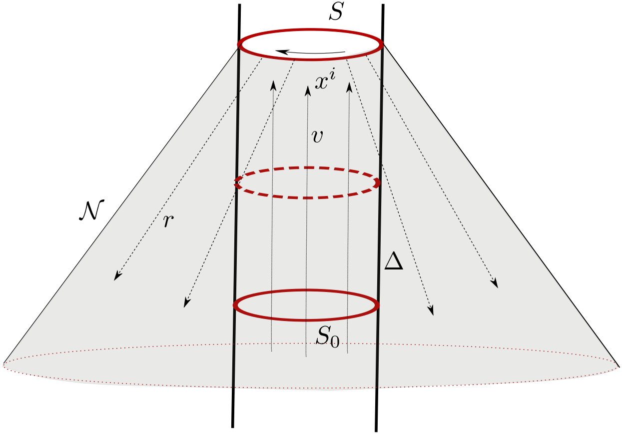
$$\sum_{J=1}^{N} A^a_{IJ}(x,\psi)\partial_a\psi_J + F_I(x,\psi) = 0\,.$$

$$A^a_{IJ}\xi_a > 0 \quad \text{for some } \xi_a$$

then equations are hyperbolic

- Standard initial value problem specifies ψ_I at $t = 0$ and solution is unique & guaranteed to exist locally
- Alterative: specify data on pair of intersecting null surfaces (Friedrich (1981), Friedrich & Rendall (2000))
- For us, the null surfaces are a weakly isolated horizon Δ and a transverse light cone $\mathcal{N}$

Near horizon coordinates



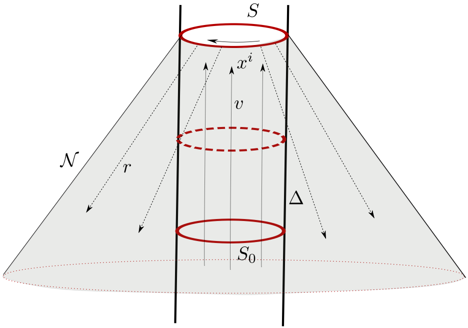
The Newman-Penrose formalism

- Tetrad formalism $(\ell, n, m, \bar{m})$ such that $\ell \cdot n = -1$, $m \cdot \bar{m} = 1$:

$$g_{ab} = -\ell_a n_b - n_a \ell_b + m_a \bar{m}_b + \bar{m}_a m_b \,.$$

- Spacetime connection encoded in 12 complex scalars (spin coefficients)
- Weyl tensor is broken up as 5 complex scalars Ψ_k $(k = 0, 1, \ldots, 4)$

Near horizon coordinates



- Start with coordinates x^i on S_0 and m^a
- Propagate on Δ as $\ell^a \nabla_a x^i = 0$, $\mathcal{L}_\ell m = 0$
- Propagate away from Δ using past null geodesics: $n^a \nabla_a \ell^b = 0$, $n^a \nabla_a m^b = 0$, $\ell^a \nabla_a x^i = 0$
- Affine parameter along $-n^a$ is r

The structure of the field equations

- Choose $n_a = -\nabla_a v$ and $n^a\nabla_a = -\partial_r$
- This implies

$$\ell^a\nabla_a := D = \frac{\partial}{\partial v} + U\frac{\partial}{\partial r} + X^i\frac{\partial}{\partial x^i}\,,$$

$$m^a\nabla_a := \delta = \Omega\frac{\partial}{\partial r} + \xi^i\frac{\partial}{\partial x^i}\,.$$

$$U, X^i, \Omega \sim \mathcal{O}(r)$$

- The variables are: U, X^i, Ω, 12 spin coefficients, 5 complex Weyl tensor components Ψ_k

The structure of the field equations

The field equations are of 3 types

- Evolution equations along v
- Purely angular equations
- Radial equations along r

Strategy for solving them:

- Propagate data on S_0 to all points of Δ consistent with angular equations.
- Proagate transverse to Δ using radial eauations
- Ensure consistency with non-radial equations away from Δ

The structure of the field equations

- In general, solution guaranteed only near Δ and global solution will exist only in certain cases

Free data:

- On horizon, we need a choice of ℓ^a, ω_a, geometry of S_0, shear and expansion of n^a
- On $\mathcal{N}$ we need Ψ_4 – The Bianchi identities do not specify radial derivative of Ψ_4
- We can follow the procedure iteratively in powers of r
- This results in solutions to the frame fields, Weyl tensor and spin coefficients as a power series in r

$$\Psi_k = \Psi_k^{(0)} + r\Psi_k^{(1)} + \frac{1}{2}r^2\Psi_k^{(2)} + \dots$$

$$g_{ab} = g_{ab}^{(0)} + g_{ab}^{(1)}r + \frac{1}{2}g_{ab}^{(2)}r^2 + \dots$$

Expansion of the Weyl tensor

Expressions for the Weyl tensor at first order:

$$\Psi_0 = \mathcal{O}(r^2)\,,$$

$$\Psi_1 = -r\eth\Psi_2^{(0)} + \mathcal{O}(r^2)\,,$$

$$\Psi_2^{(1)} = -(\eth + \bar{\pi}^{(0)})\Psi_3^{(0)} + 3\mu^{(0)}\Psi_2^{(0)}\,,$$

$$\Psi_3^{(1)} = -(\eth + 2\bar{\pi}^{(0)})\Psi_4^{(0)} + 4\mu^{(0)}\Psi_3^{(0)}$$

Expansion of the metric up to $\mathcal{O}(r^2)$

$$g_{ab}^{(0)} = 2\partial_{(a}r\partial_{b)}v + 2m_{(a}^{(0)}\bar{m}_{b)}^{(0)}\,,$$

$$\begin{aligned} g_{ab}^{(1)} &= -\Big(2\tilde{\kappa}\partial_{(a}v\partial_{b)}v + 4\pi^{(0)}m_{(a}^{(0)}\partial_{b)}v + 4\bar{\pi}^{(0)}\bar{m}_{(a}^{(0)}\partial_{b)}v \\ &+ 4\mu^{(0)}m_{(a}^{(0)}\bar{m}_{b)}^{(0)} + 2\lambda^{(0)}m_{(a}^{(0)}m_{b)}^{(0)} + 2\bar{\lambda}^{(0)}\bar{m}_{(a}^{(0)}\bar{m}_{b)}^{(0)}\Big)\,, \end{aligned}$$

Expansion of the metric

Ψ_4 appears only at the second order

$$
\begin{aligned}
g^{(2)}_{ab} &= 4\left(|\pi^{(0)}|^2 - \mathrm{Re}[\Psi^{(0)}_2]\right)\partial_{(a}v\partial_{b)}v \\
&+ 4\left(\mu^{(0)}\pi^{(0)} + \lambda^{(0)}\bar{\pi}^{(0)} - \Psi^{(0)}_3\right) m^{(0)}_{(a}\partial_{b)}v \\
&+ 4\left(\mu^{(0)}\bar{\pi}^{(0)} + \bar{\lambda}^{(0)}\pi^{(0)} - \bar{\Psi}^{(0)}_3\right) \bar{m}^{(0)}_{(a}\partial_{b)}v \\
&+ 4\left((\mu^{(0)})^2 + \left|\lambda^{(0)}\right|^2\right) m^{(0)}_{(a}\bar{m}^{(0)}_{b)} \\
&+ \left(4\mu^{(0)}\lambda^{(0)} - 2\Psi^{(0)}_4\right) m^{(0)}_{(a}m^{(0)}_{b)} + \left(4\mu^{(0)}\bar{\lambda}^{(0)} - 2\bar{\Psi}^{(0)}_4\right) \bar{m}
\end{aligned}
$$

Perturbative approach

- Stationary BHs can be reconstructed from horizon data alone
- Perturb horizon data + $\Psi_4 \rightarrow$ perturbed near horizon spacetime
- First order calculations are “straightforward”, but at second order we would need dynamical horizons (or “slowly evolving” horizons (Booth & Fairhurst (2004, 2007))
- Applications to tidal Love number calculations, ringdown,...
- (Talk by Beatrice)

Conclusion

- Trapped surfaces are not as ill-behaved as one might think
- Can prove useful mathematical results for them and can use them to study black holes
- The trapping boundary is understood in some simple cases, but the general case is still open.
- Applications in diverse areas from quantum gravity, GW Astronomy to numerical relativity
- Still left with the question: what is a dynamical black hole?
- Surface not unique but restrictions knowm
- Suggestion by Senovilla: core(s) of the trapped region?