Gravitational waves from asymmetric-mass binaries with spin and orbital precession

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Extreme-mass ratio inspirals (EMRIs)

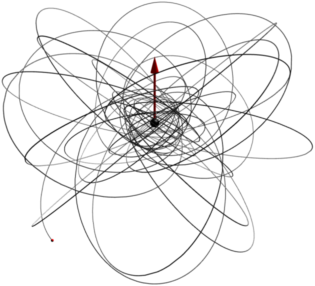
Primary mass M: $10^{5.5} - 10^{7} M_{\odot}$ Secondary mass μ: $1 - 100 M_{\odot}$

$\epsilon = \mu/\mathrm{M} \sim 10^{-4} - 10^{-7} \implies$ expand Einstein equations in ϵ

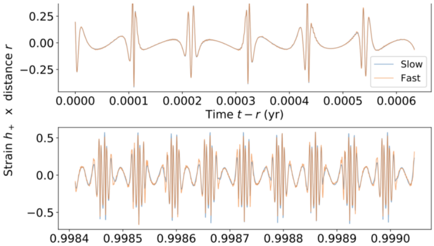
Some features:

mHz GWs $\implies$ targets for LISA

$\sim 1/\epsilon$ orbits in 1 year before inspiral in strong gravity regime

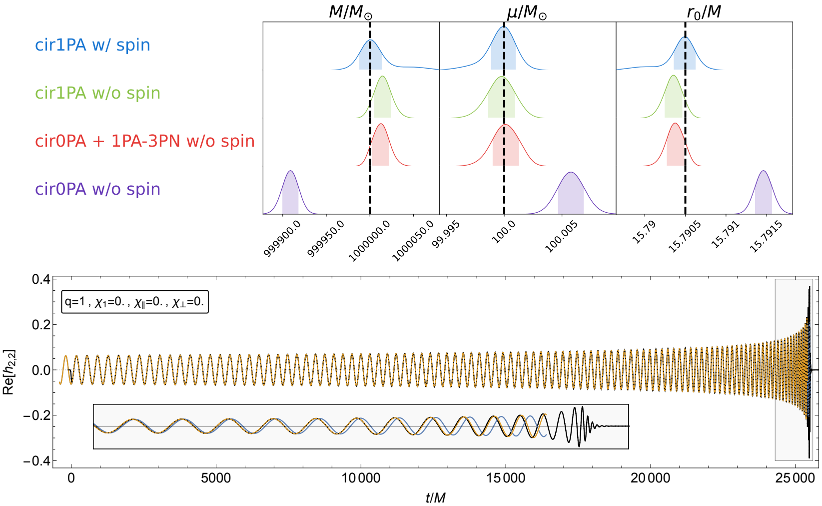


Credit: Maarten van de Meent



Credit: Niels Warburton

Why do we want to include the small body spin?



Top: Burke, Piovano+ (2023) Bottom: Mathews+, (2025)

But how big is the small body spin?

Spin magnitude $$S = \frac{1}{2} S^{\rho\sigma} S_{\rho\sigma}$$

In units μM

$$\frac{S}{\mu M} = \frac{S}{\mu^2}\epsilon = \chi\epsilon \qquad \text{with } \epsilon \ll 1$$

Earth: 740 Sun: 0.21 fastest white dwarf: 10

fastest pulsar 0.3 neutron star $|\chi| \leq 0.6$ Kerr BHs $|\chi| \leq 1$

only need linear terms $\epsilon\chi$ for EMRIs (and maybe most IMRIs)

Multi-Scale Expansion

Radiation reaction time $T_{RR} \gg T_{\text{orb}}$

$\Longrightarrow$ expand GW phase Φ_{GW} in two two-times scale expansion

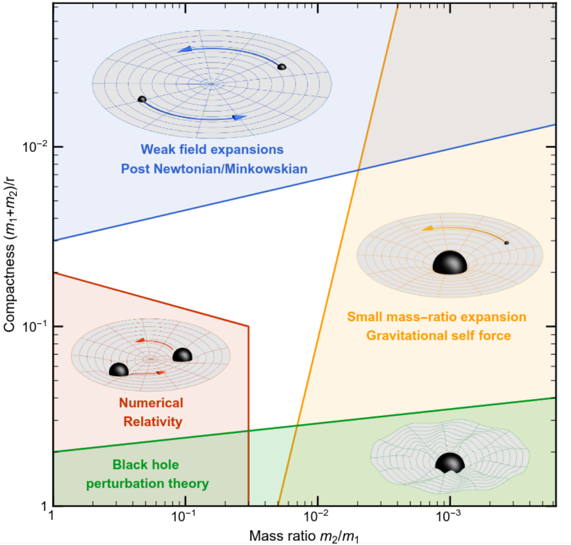
$$\Phi_{\text{GW}} = \underbrace{\epsilon^{-1}\mathcal{C}^{(0)}}_{\text{adiabatic}} + \underbrace{\epsilon^{-1/2}\mathcal{C}^{(1/2)}}_{\text{resonances}} + \underbrace{\epsilon^{0}\mathcal{C}^{(1)}}_{\text{post-1-adiabatic}} + \mathcal{O}(q)$$

- $\mathcal{C}^0$: dissipative 1SF
- $\mathcal{C}^1$: conservative 1SF , dissipative 2SF , secondary spin

$\Longrightarrow$ need first and second order metric perturbation

Hinderer&Flanagan (2008). See also the review by Pound&Wardell, (2021), Mathews&Pound (2025)

Approximation schemes to solve the Einstein equations



Credit: Maarten van de Meent.

Outline of the presentation

1. Chapter I
   Orbital Motion*
   (*radiation-reaction not included)
2. Chapter II
   Waveform From Extreme Asymmetrical-Mass Binaries
3. Chapter III
   Extreme ϵ Waveforms For Not-So-Extreme ϵ Binaries
4. End Credits

Chapter I
Orbital Motion*

(*radiation-reaction not included)

(linearized) MPD equations of motion - 2nd order form

EoM at order $\mathcal{O}(\epsilon\chi)$ in Kerr background

$$S^{\mu\nu} v_\nu^{\mathrm{g}} = 0 \quad \text{Tulczyjew-Dixon condition}$$

$$\chi = S/\mu^2 = \sqrt{\chi_\parallel^2 + \chi_\perp^2}$$

$$\begin{cases} \dfrac{D_{\mathrm{g}} v_{\mathrm{g}}^\mu}{\mathrm{d}\tau} = 0 \quad \text{geodesic equations} \\ \dfrac{D_{\mathrm{g}} v_{\mathrm{s}}^\mu}{\mathrm{d}\tau} = -\dfrac{1}{2} R^\mu{}_{\nu\rho\sigma} v_{\mathrm{g}}^\nu s^{\rho\sigma} \\ \dfrac{D_{\mathrm{g}} S^{\mu\rho}}{\mathrm{d}\tau} = 0 \end{cases}$$

Spin vector: $s^\mu = \chi_\perp \left(\tilde{e}^\mu{}_{(1)} \cos\psi_{\mathrm{p}} + \tilde{e}^\mu{}_{(2)} \sin\psi_{\mathrm{p}} \right) + \chi_\parallel e^\mu{}_{(3)}$

Precession angle: ψ_{p}

s^μ and ψ_{p} known in analytic form (Marck, 1983 ; van de Meent, 2020)

Symmetries to the rescue

Conserved quantities in Kerr background

- energy $E = E_{\rm g} + \epsilon\chi\delta E$
- angular momentum $L_z = L_{z{\rm g}} + \epsilon\chi\delta L_z$
- secondary spin χ
- Rüdinger constants (only at order $O(\epsilon\chi)$)
  - $Q = Q_{\rm g} + \epsilon\chi\delta Q$
  - $\chi_{\|}$

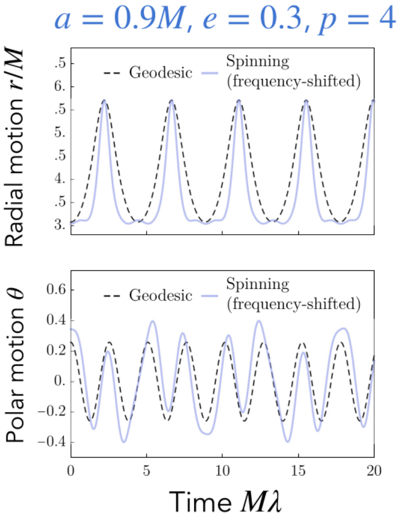
$\implies$ **integrability up to $\mathcal{O}(\epsilon\chi)$**

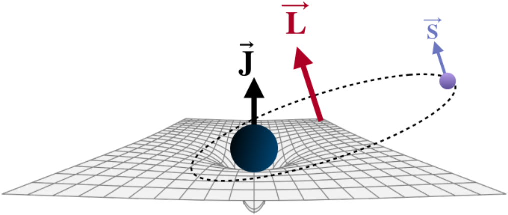
when the secondary is a BH, integrability up to $\mathcal{O}(\epsilon^2\chi^2)$

But how do I fix $\delta E, \delta L_z, \delta Q$? Different possible choices (or "**spin gauges**")

Rüdinger (1981,1983), Semeráck (1999,2007), Witznay (2019), Compère (2023), Ramond+ (2026) For spin gauges, see Piovano+ (2024) , Mathews&Pound (2025)

Solving the MPD equations - 2nd order form





Solving 2nd order MPD equations

$$\frac{D_{\mathrm{g}} v_{\mathrm{s}}^{\mu}}{\mathrm{d}\tau} = -\frac{1}{2} R^{\mu}{}_{\nu\rho\sigma} v_{\mathrm{g}}^{\nu} s^{\rho\sigma}$$

via Fourier series expansion.
Fixed turning points spin gauge

Drummond&Hughes (2022), Viktor+ (2023). Credit picture: Lisa Drummond

Solving the MPD equations - 1st order form

$z = \cos\theta$

$$\frac{\mathrm{d}\delta r}{\mathrm{d}\lambda} = \pm\frac{\delta R(r_{\rm g}, z_{\rm g}, \psi_p)}{2\sqrt{R_{\rm g}(r_{\rm g})}} \qquad \frac{\mathrm{d}\delta z}{\mathrm{d}\lambda} = \pm\frac{\delta Z(r_{\rm g}, z_{\rm g}, \psi_p)}{2\sqrt{Z_{\rm g}(z_{\rm g})}}$$

$$\frac{\mathrm{d}\delta t}{\mathrm{d}\lambda} = \delta T(r_{\rm g}, z_{\rm g}, \psi_p) \qquad \frac{\mathrm{d}\delta \phi}{\mathrm{d}\lambda} = \delta \Phi(r_{\rm g}, z_{\rm g}, \psi_p)$$

EoM **non-separable**

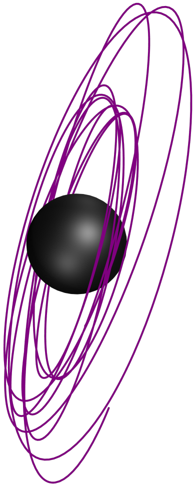
$\implies$ shifts turning points NON constants

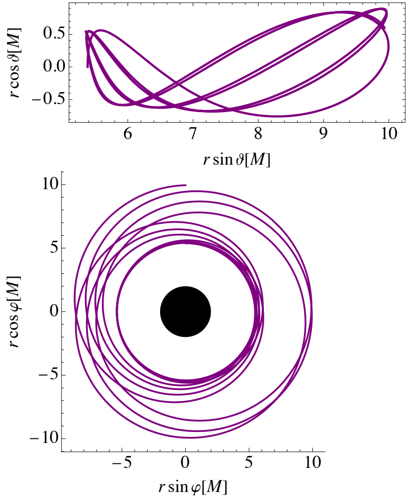
Two approaches to solve 1st order EoM

1. Fourier series expansions (numerical). Piovano+ (2024)
   Matches results Drummond&Hughes, (2022)
2. Virtual geodesics (analytical). Skuopy&Witzany (2nd paper 2024)
   Not known how to map virtual geodesic solutions to other methods

$$x^\mu = \tilde{x}^\mu(\tilde{C}) + q\chi\delta x^\mu(x^\mu_{\rm g}) \qquad x = t, r, z, \phi$$

Solutions in Schwarzschild spacetime





Analytic solutions in Schwarzschild spacetime.

Solutions by quadrature for static, spherical symmetric spacetime

Witzany & Piovano (2023)

Solutions almost equatorial orbits

$$z \simeq 0 + \delta z \implies \text{radial and polar motion decoupled.}$$

Orbital plane precession due to spin precession (Drummond&Hughes, 2022)

$$r = r_{\rm g} + \epsilon\chi_{\parallel}\delta r(r_{\rm g}) \qquad t = t_{\rm g}(r_{\rm g}) + \epsilon\chi_{\parallel}\delta t(r_{\rm g}) \qquad \phi = \phi_{\rm g}(r_{\rm g}) + \epsilon\chi_{\parallel}\delta\phi(r_{\rm g})$$

$$\delta z(r_{\rm g}) = -\chi_{\perp}\frac{\cos\psi_{\rm p}\sqrt{r_{\rm g}^2 + (L_{z\rm g} - aE_{\rm g})^2}}{r_{\rm g}(L_{z\rm g} - aE_{\rm g})}$$

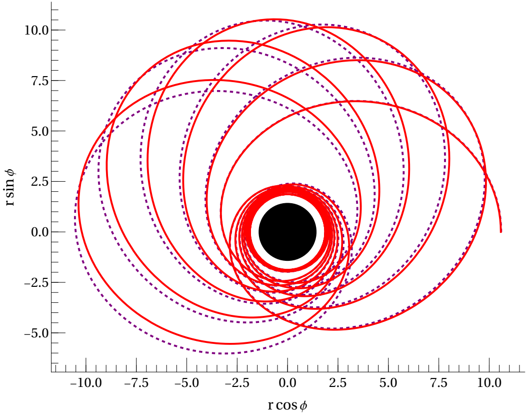
$\delta r(r_{\rm g}), \delta t(r_{\rm g}), \delta\phi(r_{\rm g})$ given as elliptic integrals or elementary functions (Piovano, 2025; work in progress)

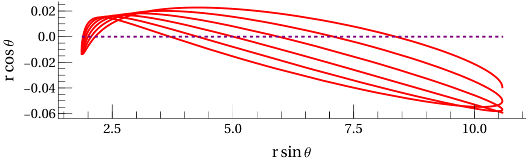
Problem:

$\delta r, \delta t, \delta\phi$ diverges at the geodesic separatrix in all spin gauges...

...except one: fixed eccentricity spin gauge

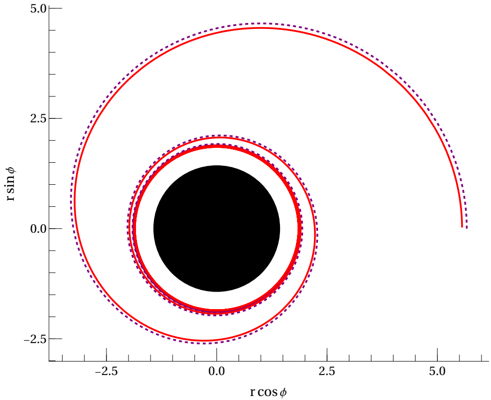
Nearly equatorial zoom-whirl orbit

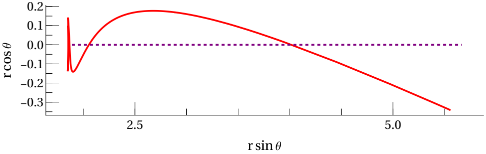




Spinning particle (red-line) and fiducial geodesic (purple, dashed line). Piovano, (2025)

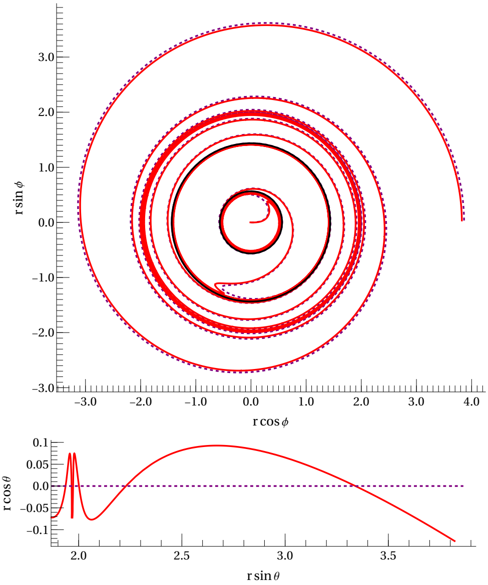
Nearly equatorial homoclinic orbit





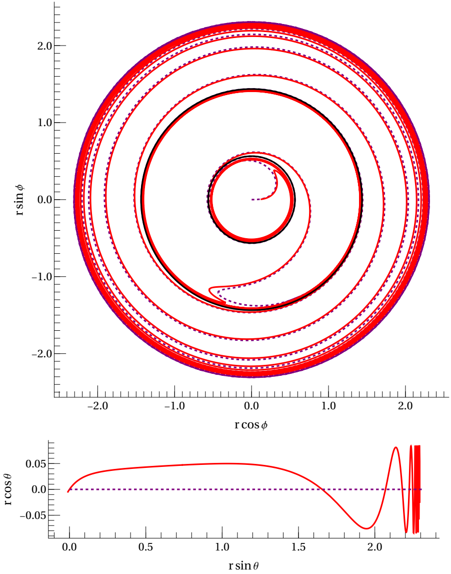
Spinning particle (red-line) and fiducial geodesic (purple, dashed line). Piovano, (2025)

Nearly equatorial homoclinic+critical plunge



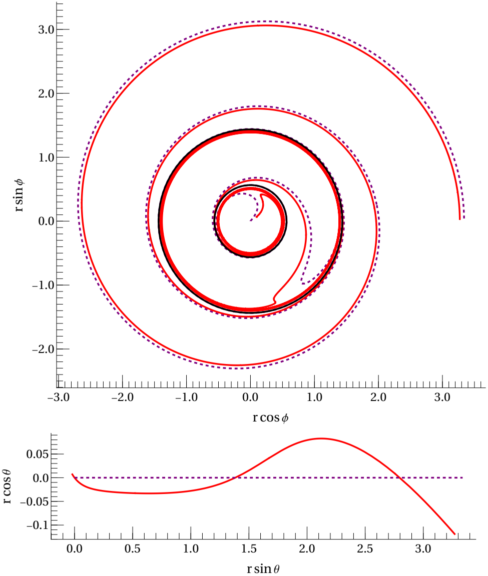
Spinning particle (red-line) and fiducial geodesic (purple, dashed line). Piovano, (soon)

Nearly equatorial ISCO plunge



Spinning particle (red-line) and fiducial geodesic (purple, dashed line). Piovano, (soon)

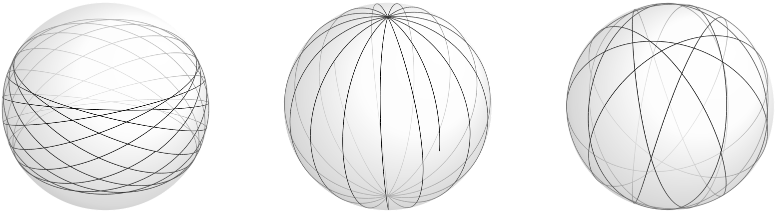
Nearly equatorial generic plunge



Spinning particle (red-line) and fiducial geodesic (purple, dashed line). Piovano, (soon)

Quasi-spherical inspirals in Kerr spacetime

- Boyer Linquist radius $r =$ const and inclination $x \neq \pm 1$.
  Precession orbital plane (Teo, 2021)
- "spherical evolves into spherical" (Kennefick&Ori, 1996)
- Adiabatic inspiral (Hughes, 2001), adiabatic+1SF (Lynch+, 2024)



Examples of spherical geodesic orbits (Credit: Teo, 2021)

Semi-analytic solutions for the orbits

$\delta r(\lambda)$ and frequency shifts known analytically
$\delta t(\lambda), \delta\phi(\lambda), \delta z(\lambda)$ known as Fourier series

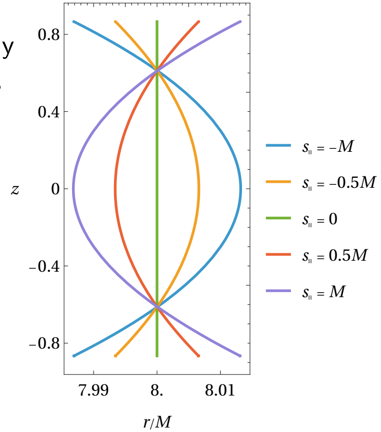
Problem:

$$\frac{\mathrm{d}\delta r}{\mathrm{d}\lambda} \neq 0 \text{ and } \langle \delta r \rangle \neq 0$$

for generic spin gauge.

We use Drummond - Hughes (DH) spin gauge:

$$\langle \delta r \rangle = \langle \delta z \rangle = 0$$

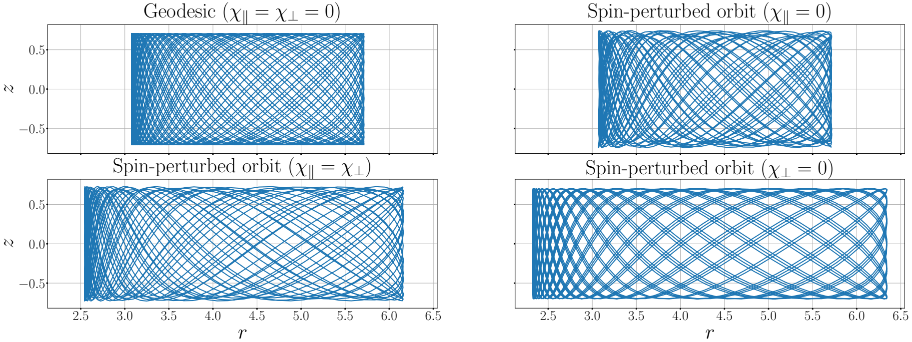


$a = 0.95,\ p_{\rm g} = 8,\ x_{\rm g} = 0.5$
$r = r_{\rm g} + q\chi_{\parallel}\delta r$

Skuopy, Piovano, Witznay (2025)

Spin flags! - fixed constants of motion

Solutions 1st order MPD equation generic orbits.



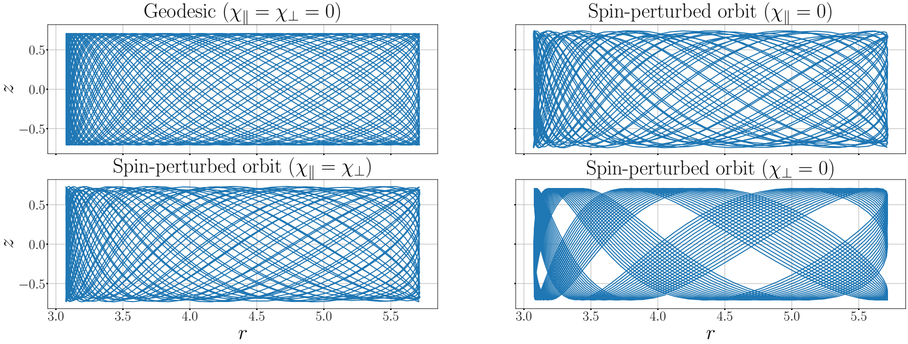
r vs z plots, $\epsilon = 1/20$

$\delta E = \delta L_z = \delta Q = 0$

Piovano+ (2024)

Spin flags! - fixed turning points (on average)

Solutions 1st order MPD equation generic orbits.



r vs z plots, $\epsilon = 1/20$

$\langle \delta(\text{turning points}) \rangle = 0$

Piovano+ (2024)

Chapter II
Waveform From Extreme Asymmetrical-Mass Binaries

Two-timescale expansion

- constants of motion $C_{\mathsf{A}} = (M, a, E, L_z, Q, \chi, \chi_\parallel)$
- phases $\varphi_{\mathsf{A}} = (\varphi_r, \varphi_\theta, \varphi_\phi, \varphi_\psi)$

$$\frac{\mathrm{d}\tilde{\varphi}_{\mathsf{A}}}{\mathrm{d}t} = \Omega_{\mathsf{A}}(C_{\mathsf{B}}) + \epsilon\chi_\parallel \delta\Omega_{\mathsf{A}}^{\mathsf{spin}}(C_{\mathsf{B}}) + \epsilon\delta\Omega_{\mathsf{A}}^{\mathsf{1SF}}(C_{\mathsf{B}})$$
$$\frac{\mathrm{d}C_{\mathsf{A}}}{\mathrm{d}t} = \epsilon\Big[\mathcal{F}_{\mathsf{A}(0)}(C_{\mathsf{B}}) + \epsilon\chi_\parallel \delta\mathcal{F}_{\mathsf{A}(1)}^{\mathsf{spin}}(C_{\mathsf{B}}) + \epsilon\delta\mathcal{F}_{\mathsf{A}(1)}^{\mathsf{2SF}}(C_{\mathsf{B}})\Big]$$

$$\frac{\mathrm{d}\chi}{\mathrm{d}t} = \frac{\mathrm{d}\chi_\parallel}{\mathrm{d}t} = 0 \quad \text{at 1PA}$$

$\chi_\perp$ effects purely oscillatory $\implies \langle \chi_\perp \text{ effects}\rangle = 0$

$\mathcal{F}_{\mathsf{A}(0)}(C_{\mathsf{B}})$ dissipative 1SF

$\delta\Omega_{\mathsf{A}}^{\mathsf{spin}}(C_{\mathsf{B}})$ conservative spin effects

$\delta\Omega_{\mathsf{A}}^{\mathsf{1SF}}(C_{\mathsf{B}})$ conservative 1SF

$\delta\mathcal{F}_{\mathsf{A}(1)}^{\mathsf{spin}}(C_{\mathsf{B}})$ dissipative spin effects

$\delta\mathcal{F}_{\mathsf{A}(1)}^{\mathsf{2SF}}(C_{\mathsf{B}})$ dissipative 2SF

How do we compute waveforms in practice?

Grid of $a, E, L_z, Q,.$

Adiabatic regime $T_{RR} \gg T_{\text{orb}} \implies$ we use balance laws

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -I^{E}_{\text{GW flux}}(J_{\text{A}}) \qquad \frac{\mathrm{d}L_z}{\mathrm{d}t} = -I^{L_z}_{\text{GW flux}}(J_{\text{A}})$$

- $\mathcal{F}_{\text{A}(0)}(C_{\text{B}})$, $\delta\Omega^{\text{spin}}_{\text{A}}(C_{\text{B}})$, $\delta\mathcal{F}^{\text{spin}}_{\text{A}(1)}(C_{\text{B}})$ asymptotic info is sufficient
- $\delta\Omega^{\text{1SF}}_{\text{A}}(C_{\text{B}})$, $\delta\mathcal{F}^{\text{2SF}}_{\text{A}(1)}(C_{\text{B}})$ require local info (self-force)

$$h = -\frac{2\mu}{r}\sum_{\ell m\vec{\kappa}}\left(\mathcal{A}^{\mathrm{g}}_{\ell m\vec{k}} + \epsilon\delta\mathcal{A}^{\mathrm{2SF}}_{\ell m\vec{k}} + \epsilon\chi_{\parallel}\delta\mathcal{A}^{\chi\parallel}_{\ell m\vec{k}} + \epsilon\chi_{\perp}\delta\mathcal{A}^{\chi\perp}_{\ell m\vec{k}}\right)e^{im\varphi}e^{-i(\omega^{\mathrm{g}}_{m\vec{\kappa}} + \epsilon\chi_{\parallel}\delta\omega_{m\vec{\kappa}})}$$

with $\vec{\kappa} = (n, k, j)$. $n, k \in \mathbb{Z}$, $j = -1, 0, 1$. $\omega_{m\vec{\kappa}} = m\Omega_{\phi} + k\Omega_z + r\Omega_r + j\Omega_p$.

Pound&Wardell (2021), Mathews&Pound (2025), "Waveform Modeling for the Laser Interferometer Space Antenna" (2023)

Methods to compute spin shifts to asymptotic fluxes

Modified version of Teukolsky formalism.

Spin effects

- corrections orbits $\delta t, \delta r, \delta z, \delta\phi$
- precession frequency Ω_p
- shift frequencies $\Omega_i = \Omega_{i\mathrm{g}} + \chi_{\parallel}\delta\Omega_i \quad i = r, z, \phi$
- stress energy tensor dipole particle: $T^{\mu\nu} = T^{\mu\nu\,\mathrm{p.p.}} + T^{\mu\nu\,\mathrm{dipole}}$

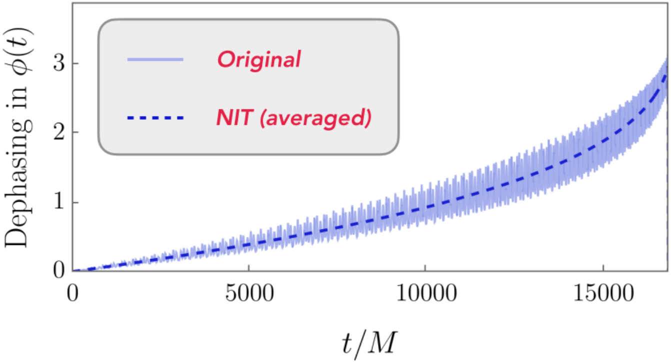
Codes to compute $\delta\mathcal{F}^{\mathrm{spin}}_{E(1)}(C_{\mathrm{B}})$, $\delta\mathcal{F}^{\mathrm{spin}}_{L_z(1)}(C_{\mathrm{B}})$

- 2nd order MPD (Viktor+,2023)
- 1st order MPD (Piovano+,2024)
  https://github.com/gabriel-andres-piovano/Spinning-Body-Hamilton-Jacobi
- shifted geodesic formalism (to appear soon).
  See Skuopy&Witzany (2nd paper 2024)

No code to compute $\delta\mathcal{F}^{\mathrm{spin}}_{Q(1)}(C_{\mathrm{B}})$ (yet)

Recipe for $\delta\mathcal{F}^{\mathrm{spin}}_{Q(1)}(C_{\mathrm{B}})$ now available. Grant (2024), Witznay+ (2024), Mathews&Pound (2025)

Osculating geodesic+NIT for inspiral spinning particle

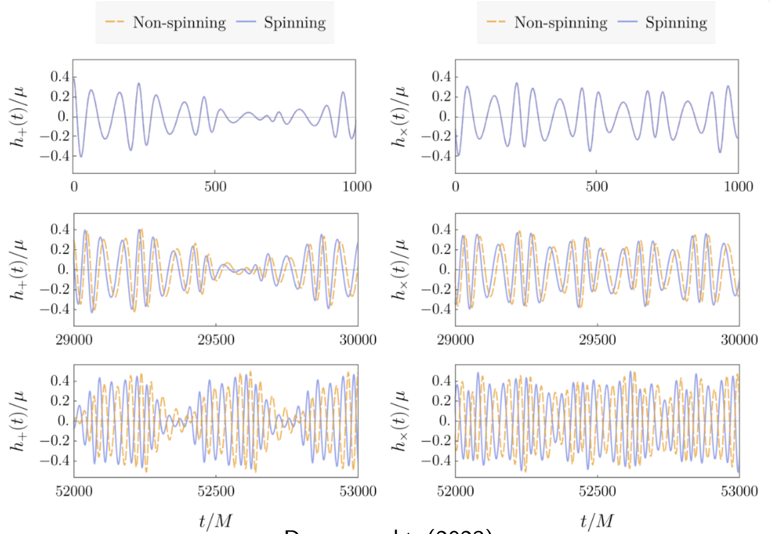


Drummond+ (2023)

Inspiral approximated via osculating geodesics method

Lynch+(2021), Lynch+(2023), Drummond+ (2023)

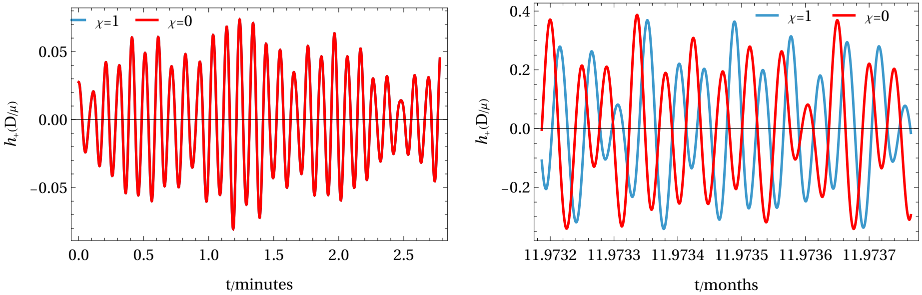
0PA+conservative spin effects waveform



Drummond+ (2023)

Spins (anti-) aligned, generic inspiral with $\mathcal{F}_{\mathrm{A}(0)}(C_{\mathrm{B}})$, $\delta\Omega_{\mathrm{A}}^{\mathrm{spin}}(C_{\mathrm{B}})$

Quasi-spherical inspiral



$(D/\mu)h_+(\mathrm{t})$ for spinning and non spinning body. $a = 0.95$, $x_f = 0.66$

Waveform model include $\mathcal{F}_{\mathrm{A}(0)}(C_{\mathrm{B}})$, $\delta\Omega^{\mathrm{spin}}_{\mathrm{A}}(C_{\mathrm{B}})$, $\delta\mathcal{F}^{\mathrm{spin}}_{\mathrm{A}(1)}(C_{\mathrm{B}})$

We use Drummond - Hughes (DH) spin gauge: $\langle \delta r\rangle = \langle \delta z\rangle = 0$

Skuopy, Piovano, Witznay (2025)

Does "spherical still evolve into spherical"?

Gold Observation (Quasi-spherical adiabatic inspiral spinning particle)

$$\dot{e} \sim e \quad \text{for } \; e \ll 1$$

1st argument. Evolution radial action J_r

In the DH gauge, spin correction radial action $\delta J_r \sim \mathcal{O}(e^2)$. Then $J_r = J_{r\mathrm{g}} + q\chi\delta J_r$

$$\frac{\mathrm{d}J_r}{\mathrm{d}t} \sim e \implies \dot{e} \sim e$$

2nd argument. Virtual Geodesic formalism

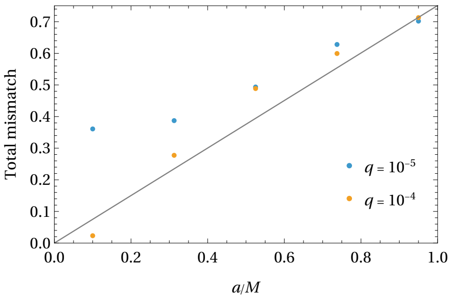
$$x^\mu = \tilde{x}^\mu(\tilde{C}) + q\chi\delta x^\mu(x^\mu_{\mathrm{g}}) \qquad x = t, r, z, \phi$$

Virtual Geodesic $\tilde{x}^\mu(\tilde{C})$ stays spherical $\implies$ $e \sim q\chi$ during inspiral

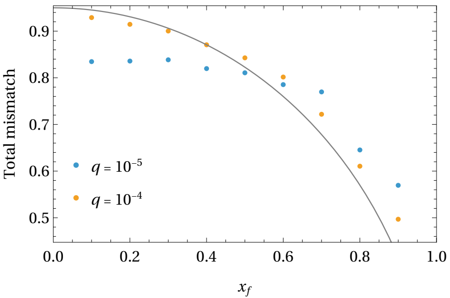
Skuopy, Piovano, Witznay (2025)

Is the small body spin detectable?

Mismatches waveforms for a spinning ($\chi_{\parallel} = 1$) and spinless secondary...



...vs primary spin. $x_f = 0.66$



...vs inclination. $a = 0.95$

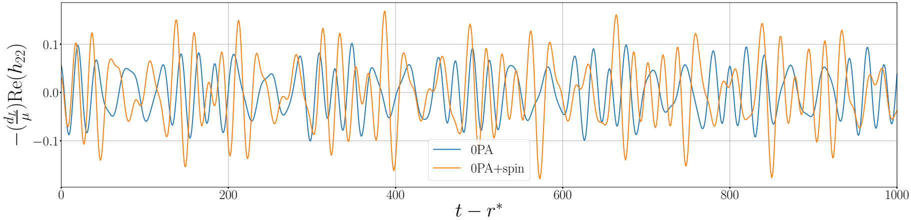
Gray line is $(a/M)\sqrt{1 - x_{\rm f}^2}$.

Lindlom criterion: two waveforms distinguishable if

$$\mathcal{M} > \frac{\#\ \text{parameters}}{2\text{SNR}^2} \simeq 1.5 \times 10^{-2} \qquad \text{for SNR} = 20$$

Skuopy, Piovano, Witznay (2025)

Waveform snapshots for spin precessing secondary



Piovano+ (2024)

0PA (blue curve) and 0PA+spin (orange curve). $\epsilon = 1/20$

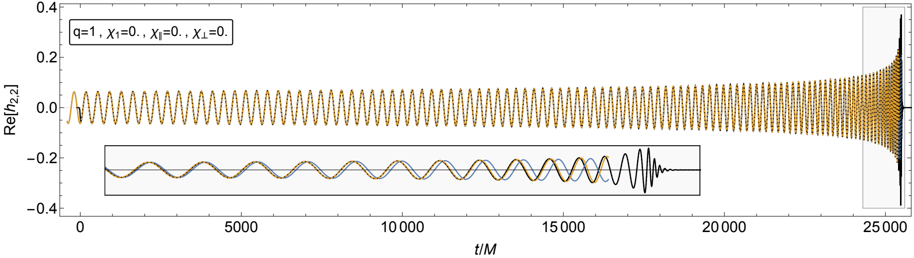
$$h = -\frac{2\mu}{r}\sum_{\ell m\vec{\kappa}}\left(\mathcal{A}^{\mathrm{g}}_{\ell m\vec{k}} + \epsilon\chi_{\parallel}\delta\mathcal{A}^{\chi\parallel}_{\ell m\vec{\kappa}} + \epsilon\chi_{\perp}\delta\mathcal{A}^{\chi\perp}_{\ell m\vec{\kappa}}\right)e^{im\varphi}e^{-i(\omega^{\mathrm{g}}_{m\vec{\kappa}}+\epsilon\chi_{\parallel}\delta\omega_{m\vec{\kappa}})}$$

with $\vec{\kappa} = (n, k, j)$. $n, k \in \mathbb{Z}$, $j = -1, 0, 1$. $\omega_{m\vec{\kappa}} = m\Omega_\phi + k\Omega_z + r\Omega_r + j\Omega_p$

$$\mathcal{A}^{\mathrm{g}}_{\ell mnkj} = \delta\mathcal{A}^{\chi\parallel}_{\ell mnkj} = 0 \text{ for } j \neq 0$$

Spin gauge $\delta E = \delta L_z = \delta Q = 0$

1PA self-force vs NR waveform



Credit: Mathews+, (2025)

Complete 1PA waveform (orange) vs vs 0PA (blue) vs NR simulation (black). Non-spinning binary

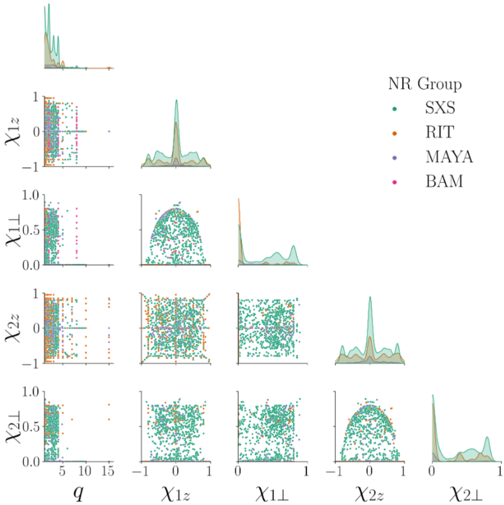
What we have

Waveform for circular orbits Schwarzschild spacetime (Soon in Kerr?!?)

$\mathcal{F}_{\mathrm{A}(0)}(\mathcal{C}_{\mathrm{B}})$, $\delta\Omega_{\mathrm{A}}^{\mathrm{spin}}(\mathcal{C}_{\mathrm{B}})$, $\delta\mathcal{F}_{\mathrm{A}(1)}^{\mathrm{spin}}(\mathcal{C}_{\mathrm{B}})$, $\delta\Omega_{\mathrm{A}}^{\mathrm{1SF}}(\mathcal{C}_{\mathrm{B}})$, $\delta\mathcal{F}_{\mathrm{A}(1)}^{\mathrm{2SF}}(\mathcal{C}_{\mathrm{B}})$

Chapter III
Extreme ϵ Waveforms For Not-So-Extreme ϵ Binaries

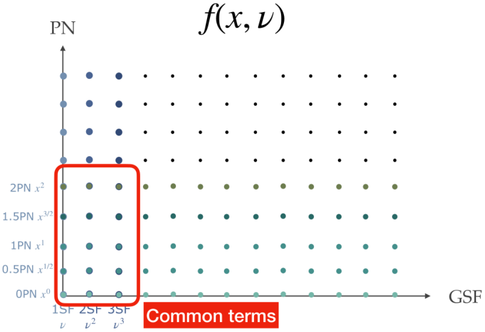
Motivation



Credit: LISA Waveform Working Group Whitepaper

- $q \lesssim 8$
  (few up to $q = 128$)
- challenging regions are sparsely covered
- high resolution needed for small q or high spins
- high SNR for new detectors
  $\Longrightarrow$ need longer signals
  $\Longrightarrow$ high resolutions
  $\Longrightarrow$ eccentricity?
  $\Longrightarrow$ spin precession?
- **SF waveforms may help cover parameters space efficiently**

Hybrid waveforms: self-force+PN



Credit: Löic Honnet

$$f(x,\nu) = \sum_{k=0}^{\infty}\sum_{n=0}^{\infty} f_{k(n)} x^{k/2}\nu^n$$

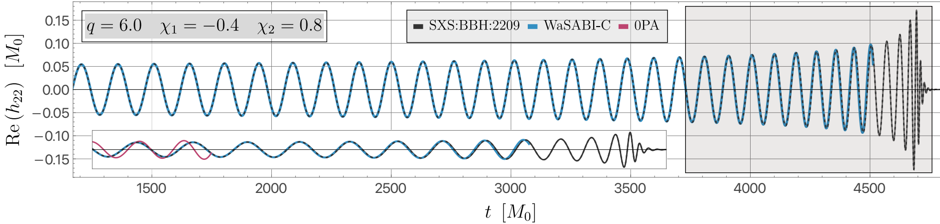
$$x = \omega^{2/3} \sim 1/r$$

$$\nu = m_1 m_2 / M_{\rm tot}^2 \sim \epsilon$$

- resummed PN info $\to$ high ν order (2SF, 3SF,...)
- expansion in ω SF info $\to$ high PN order

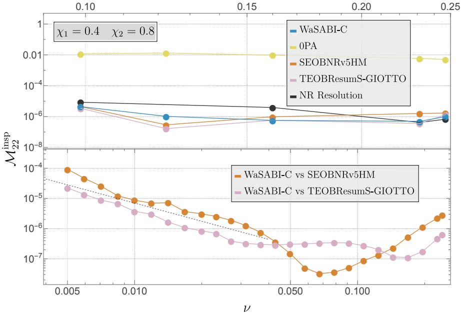
Evolution $M_{\rm tot}$, ν, ω , a from first-law and flux balance equations

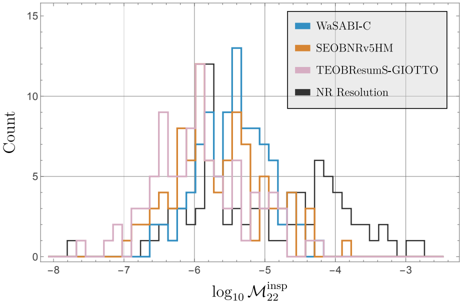
Hybrid vs NR waveforms



Honet,Mathews,Compère,Pound, Wardell, Piovano, van de Meent, Warburton (2025)
See also Mathews+, (2025); Honet+ (first paper, 2025)

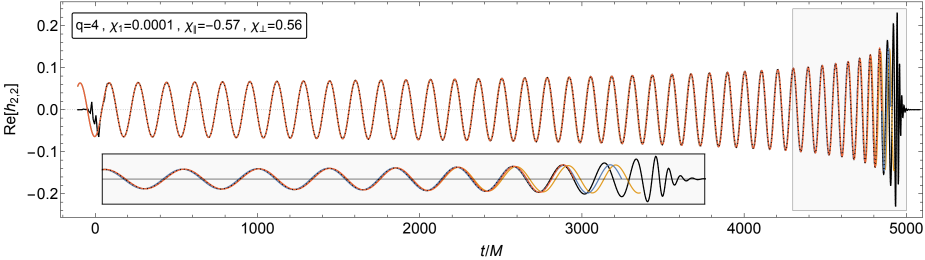
Hybrid vs NR waveforms





Honet,Mathews,Compère,Pound, Wardell, Piovano, van de Meent, Warburton, 2025

1PA Precessing waveform vs NR



Mathews+, (2025)

1PA model vs NR. Blue: 1PAT1e-a Orange: 1PAT1e-χ. Dashed, red: 1PAT1R

- 1PAT1e-χ: expand in ν at fixed M_{tot})
- 1PAT1e-a: expand in ν, fixed primary spin $a_1 = S_1/(m_1 M_{\text{tot}})$
- 1PAT1R: expand in ν but in

$$\frac{\mathrm{d}\Omega}{\mathrm{d}t} = \frac{\mathcal{N}}{\mathcal{D}}$$

only $\mathcal{N}$ and $\mathcal{D}$ are expanded

End Credits

Final notes and acknowledgments

G.A.P. acknowledges the support of the Win4Project grant ETLOG of the Walloon Region for the Einstein Telescope

- fancy doing perturbation theory? Check the BHPToolkit! https://bhptoolkit.org/
- for your tensor needs, check the Mathematica package "xAct": www.xact.es
- need an EMRI waveform pronto? Use FEW! https://github.com/BlackHolePerturbationToolkit/FastEMRIWaveforms
- wanna play with spin? Check https://github.com/gabriel-andres-piovano/Spinning-Body-Hamilton-Jacobi
- Feel free to contact me at gabriel.andres.piovano@ulb.be

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Thank you for you attention!