THE NEWFOUNDLAND CENTRED WORLD

his is an AZIMUTHAL EQUIDISTANT PROJECTION centred on
t. John's, Newfoundland. Only distances and directions
neasured along straight lines radiating from the centre are true.
ll straight lines passing through St. John's are great circles.
eformation of earth surface area increases outward from the
entre and measurements taken other than through the centre
re inaccurate.

SCALE along any straight line through the centre

1000 0 2000 4000
Kilometres

1000 0 1000 2000 3000 4000
Miles

Black Hole Evolutions:

Lessons from bifurcation theory

arXiv:2605.30458 + in progress

Matt's Journey Through Space and Time
(Trieste, 2026)

Ivan Booth
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Mary Chiamaka Okpala
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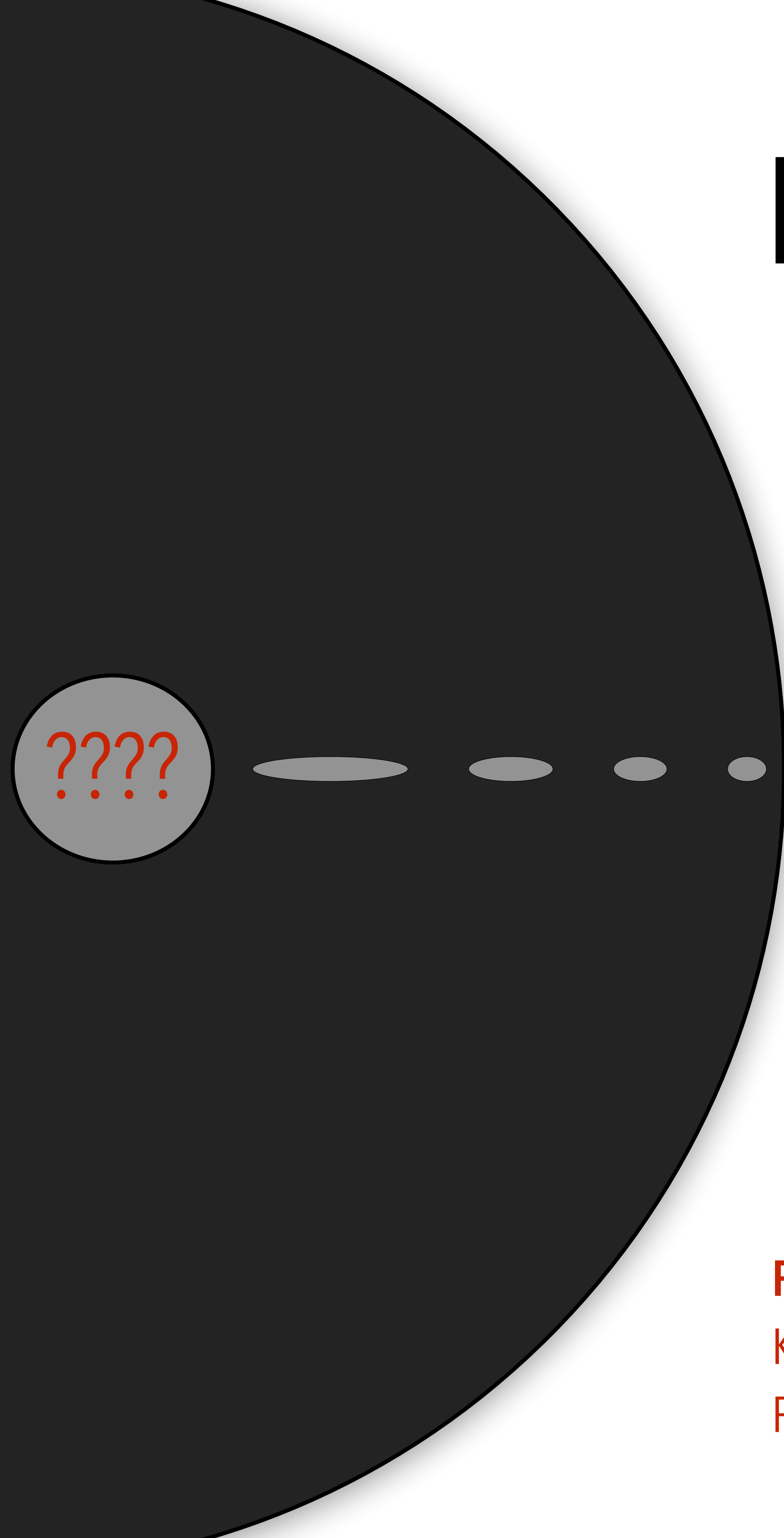

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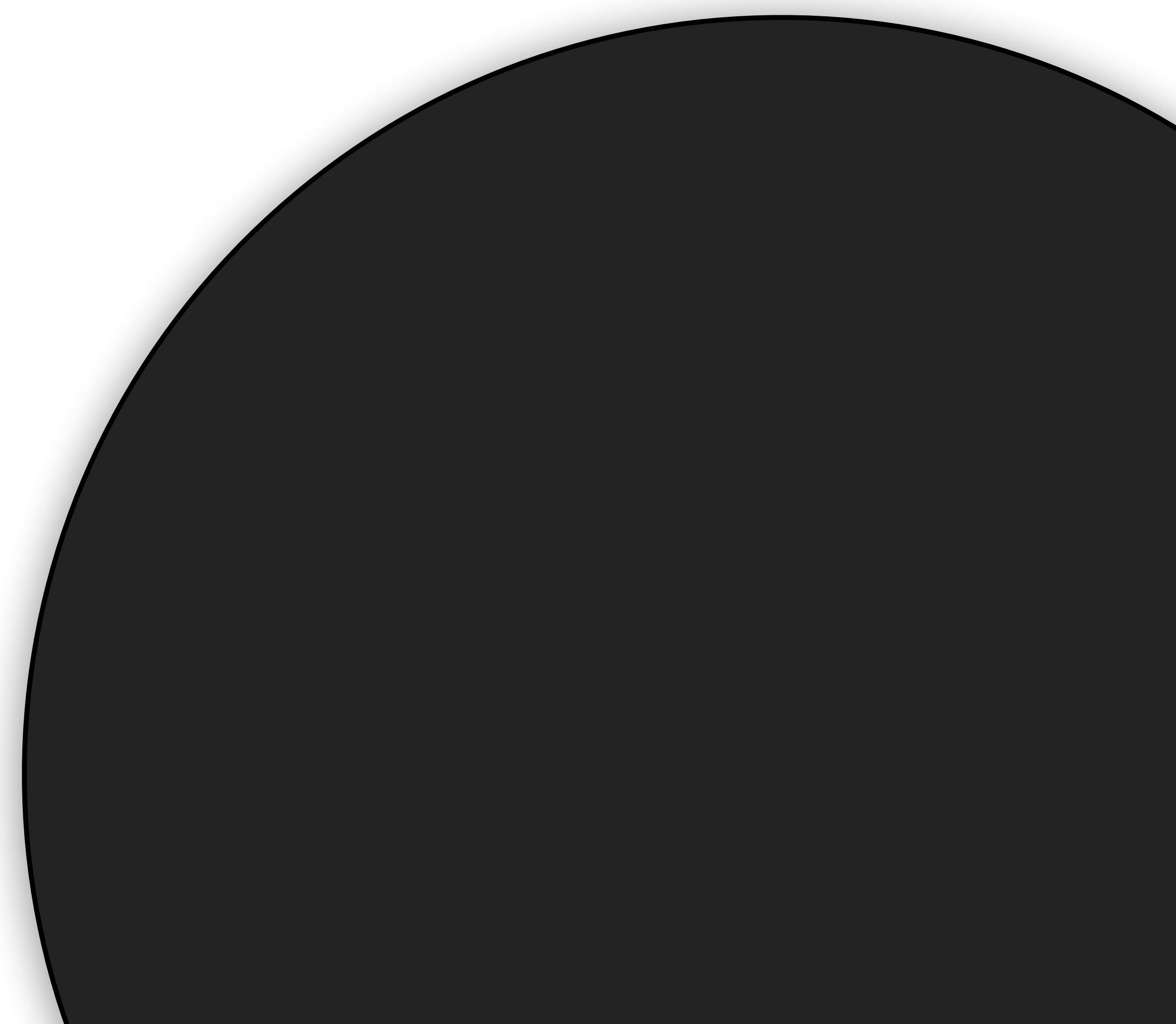
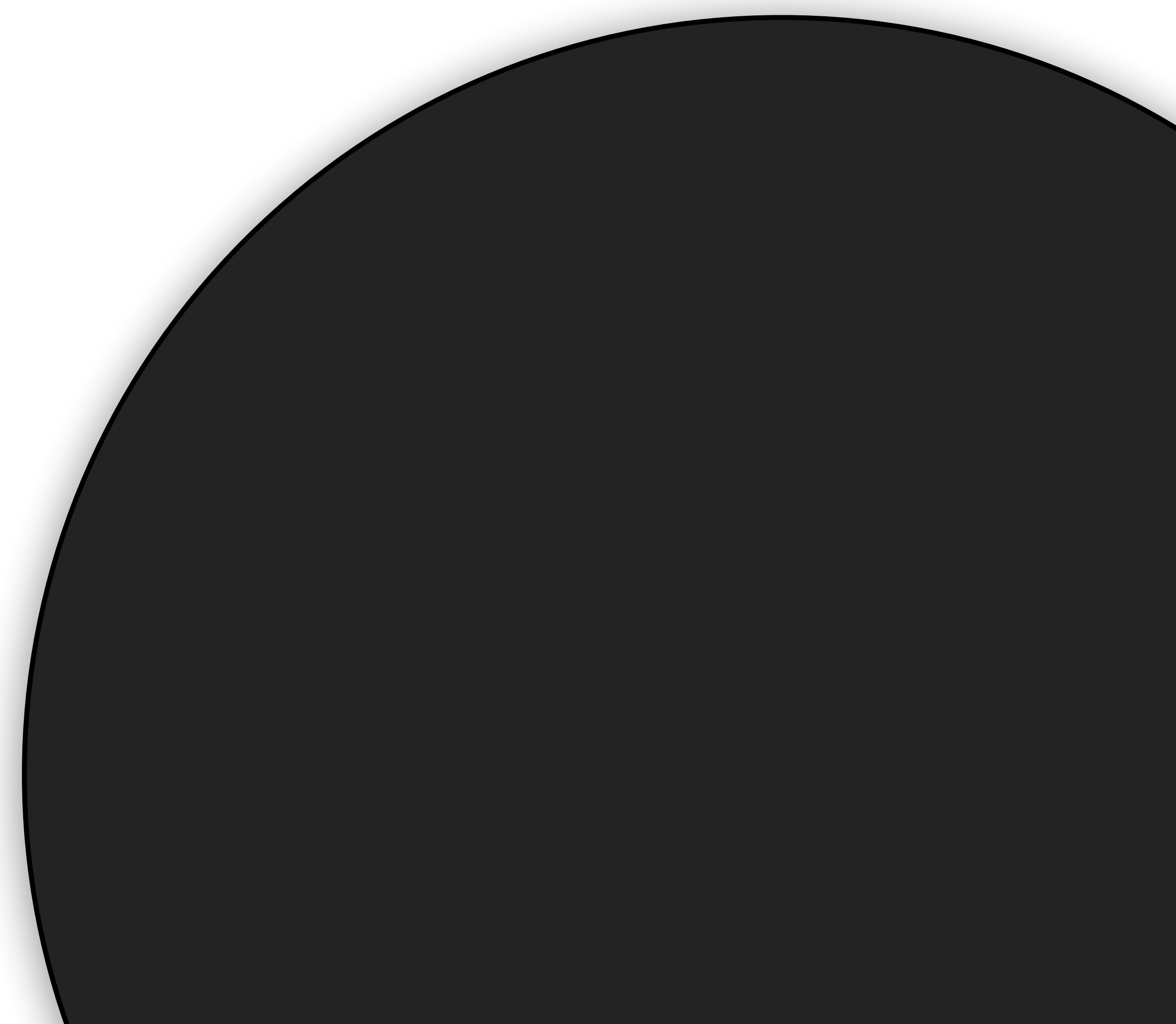
Physics Questions



Physics Questions

- During a merger what happens to the original black holes?



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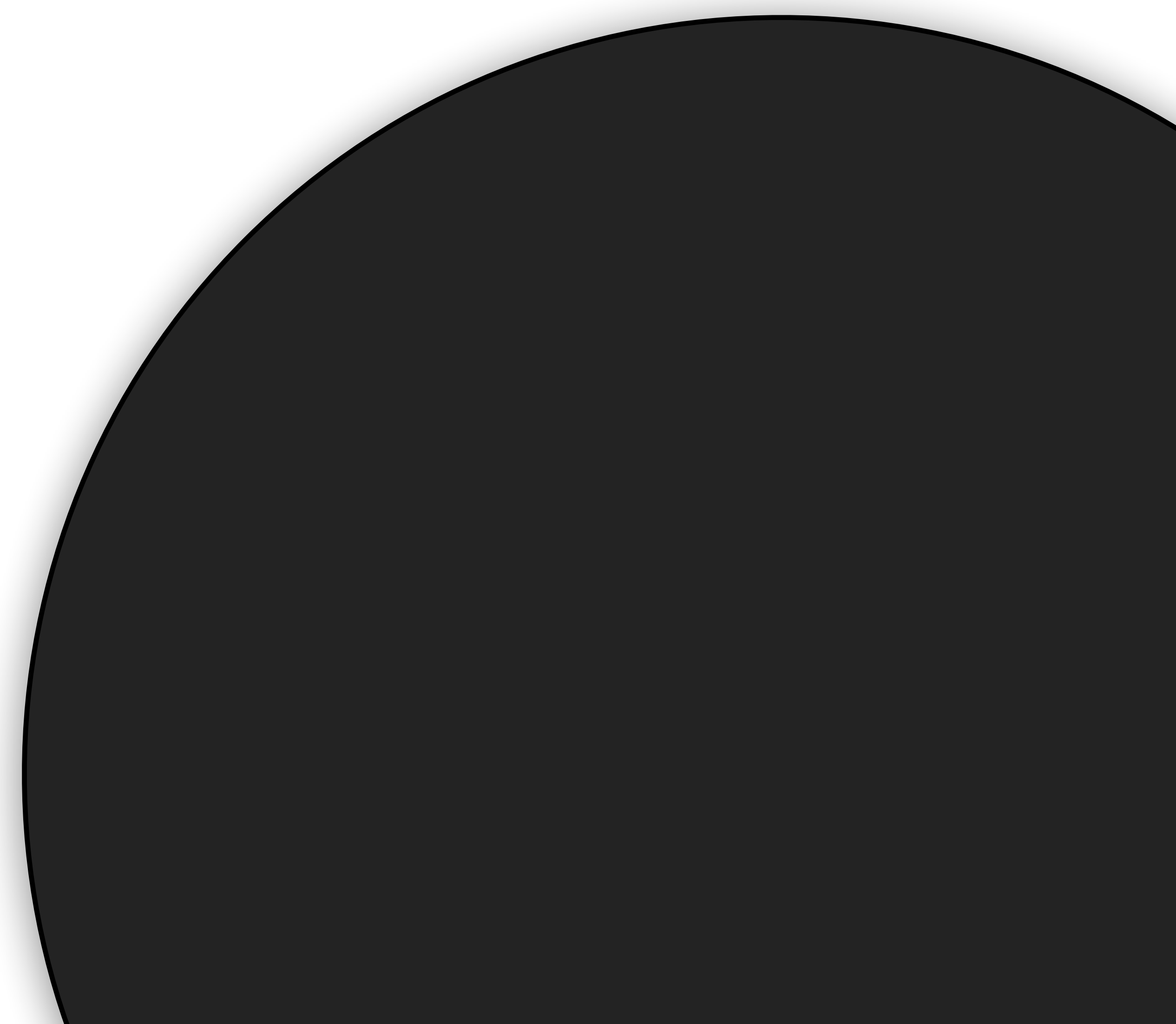
- What happens inside the final black after the new apparent horizon forms?



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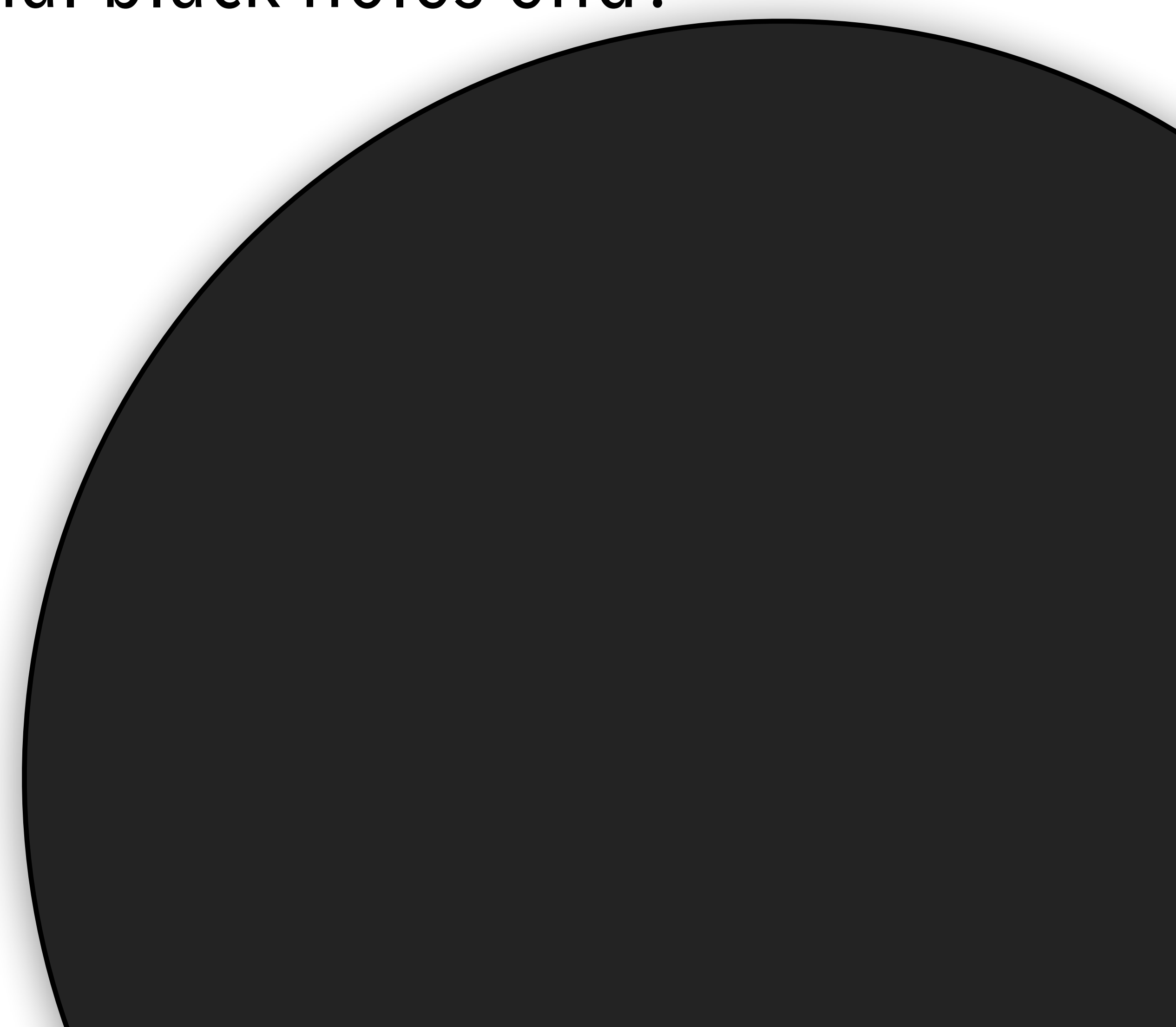

Physics Questions

- During a merger what happens to the original black holes?
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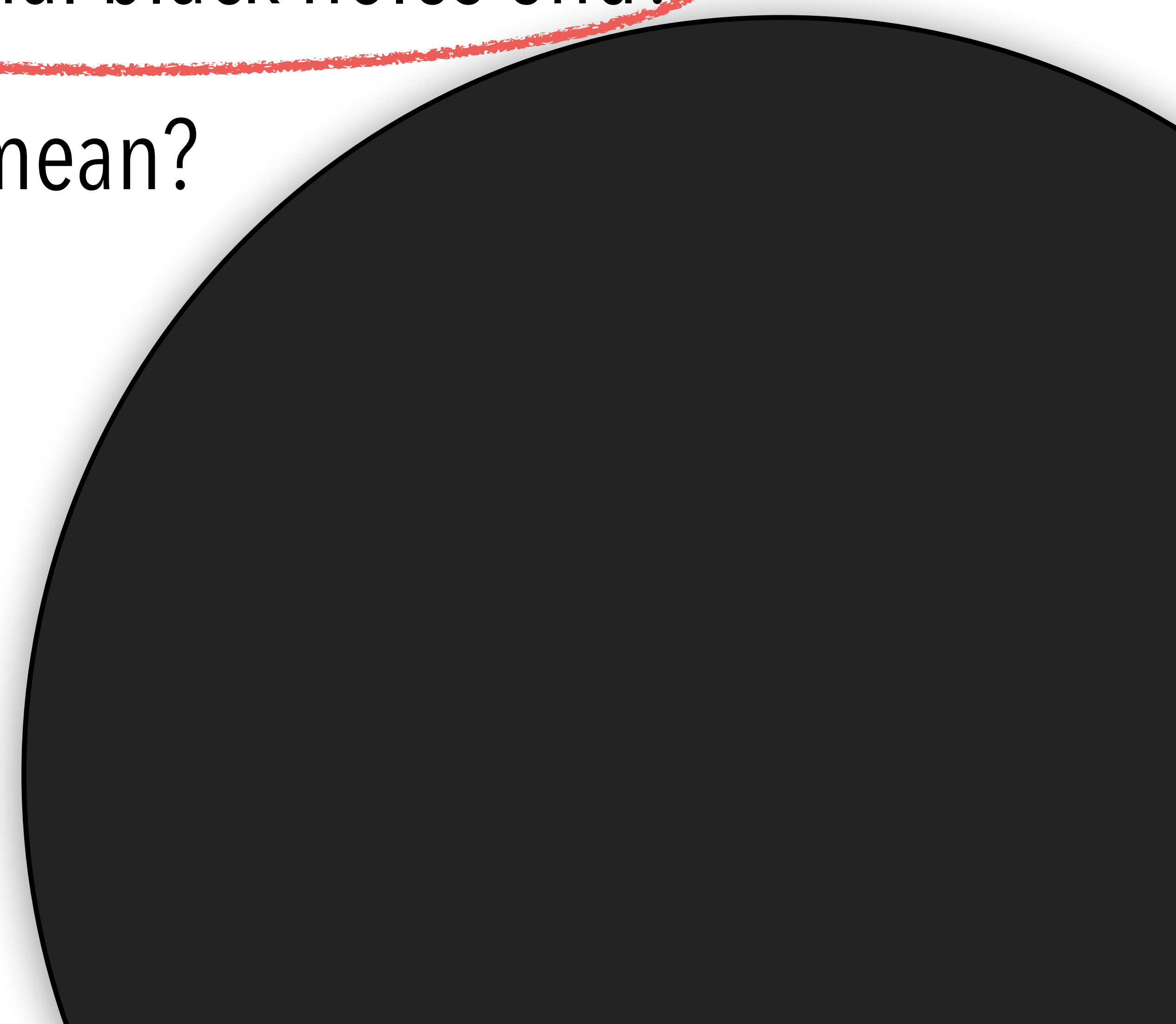
- More specifically : Do the original black holes end?
- What does this question even mean?



How do black holes end?

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black holes

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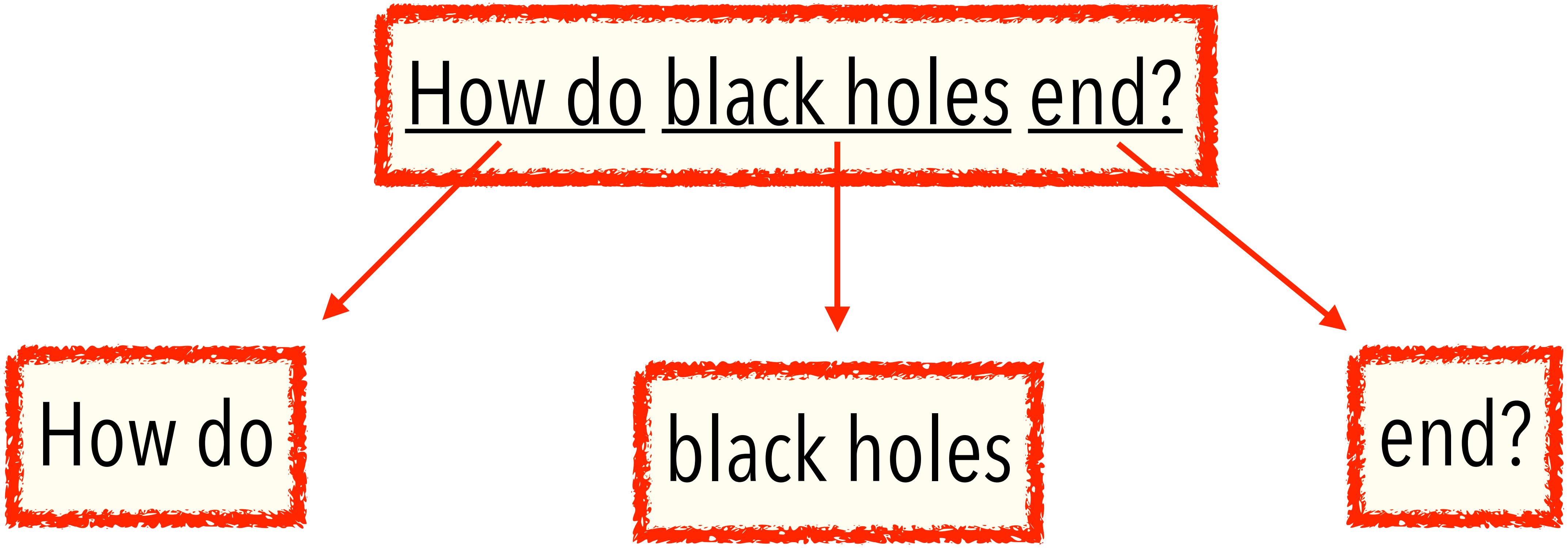

Should be:

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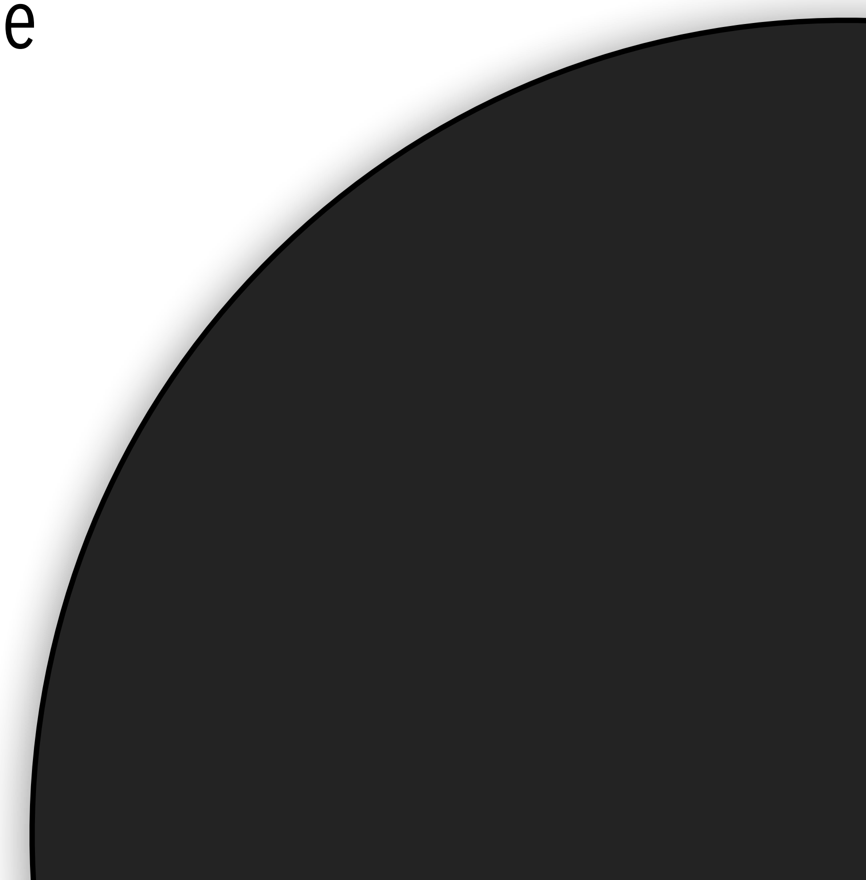
Should be:

i. gravitationally bound region from which light can't escape



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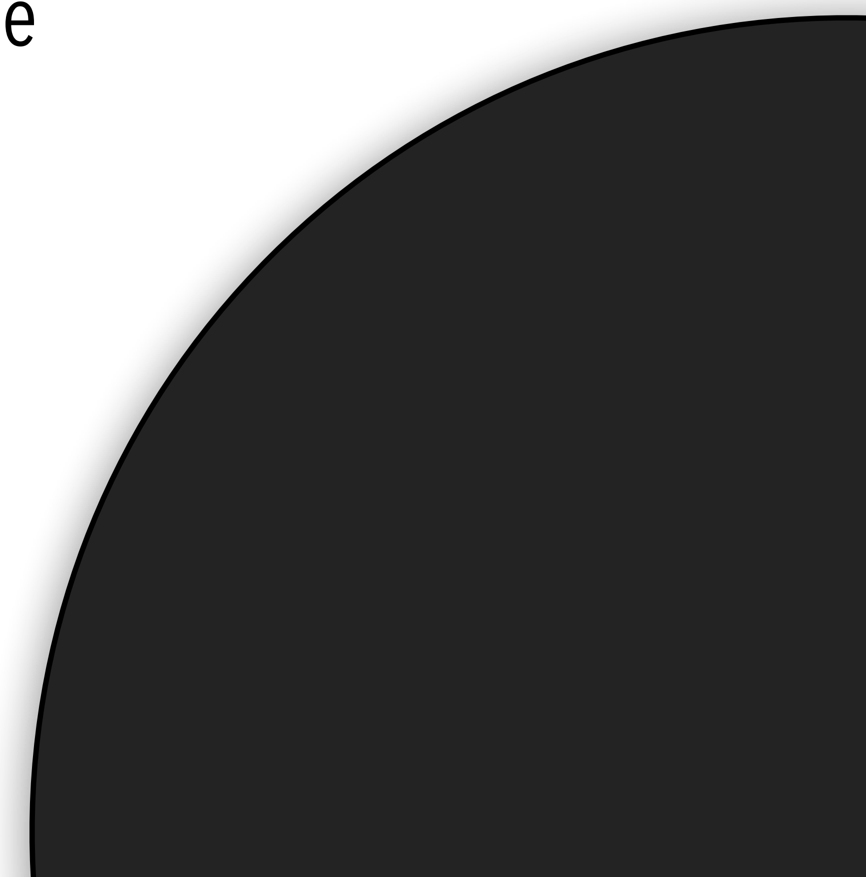

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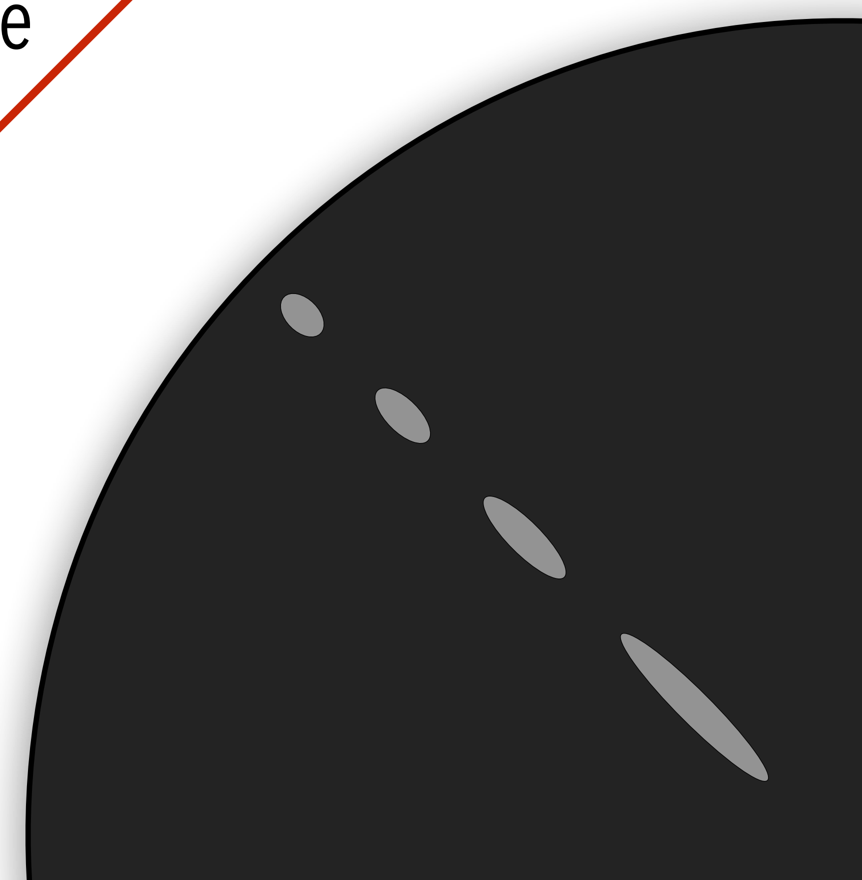

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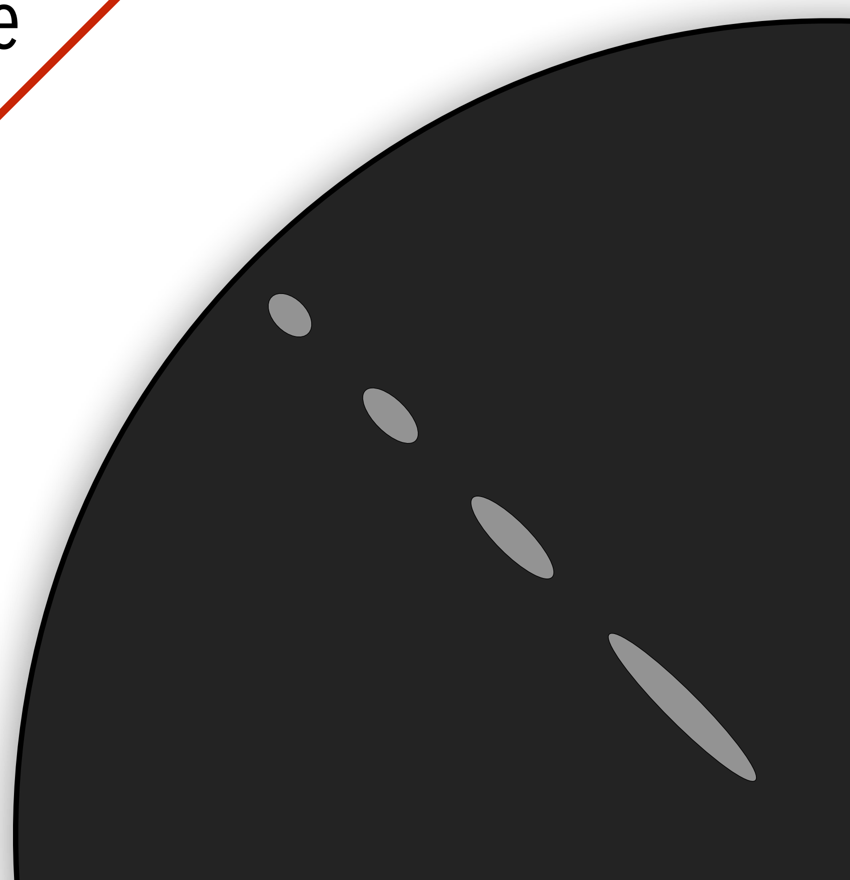
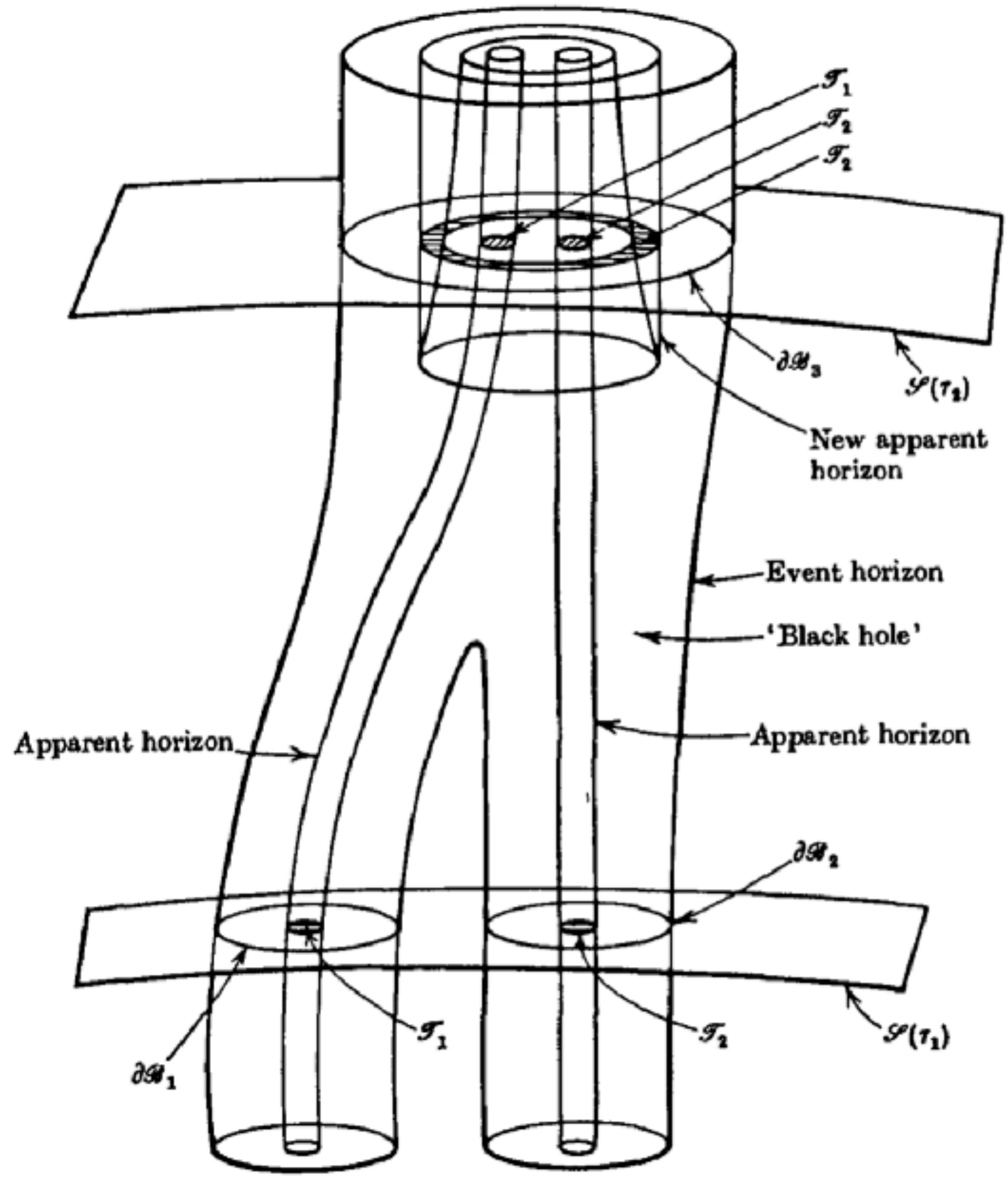
``Dissolved''



Black hole mergers and event horizons



Hawking-Ellis 1973

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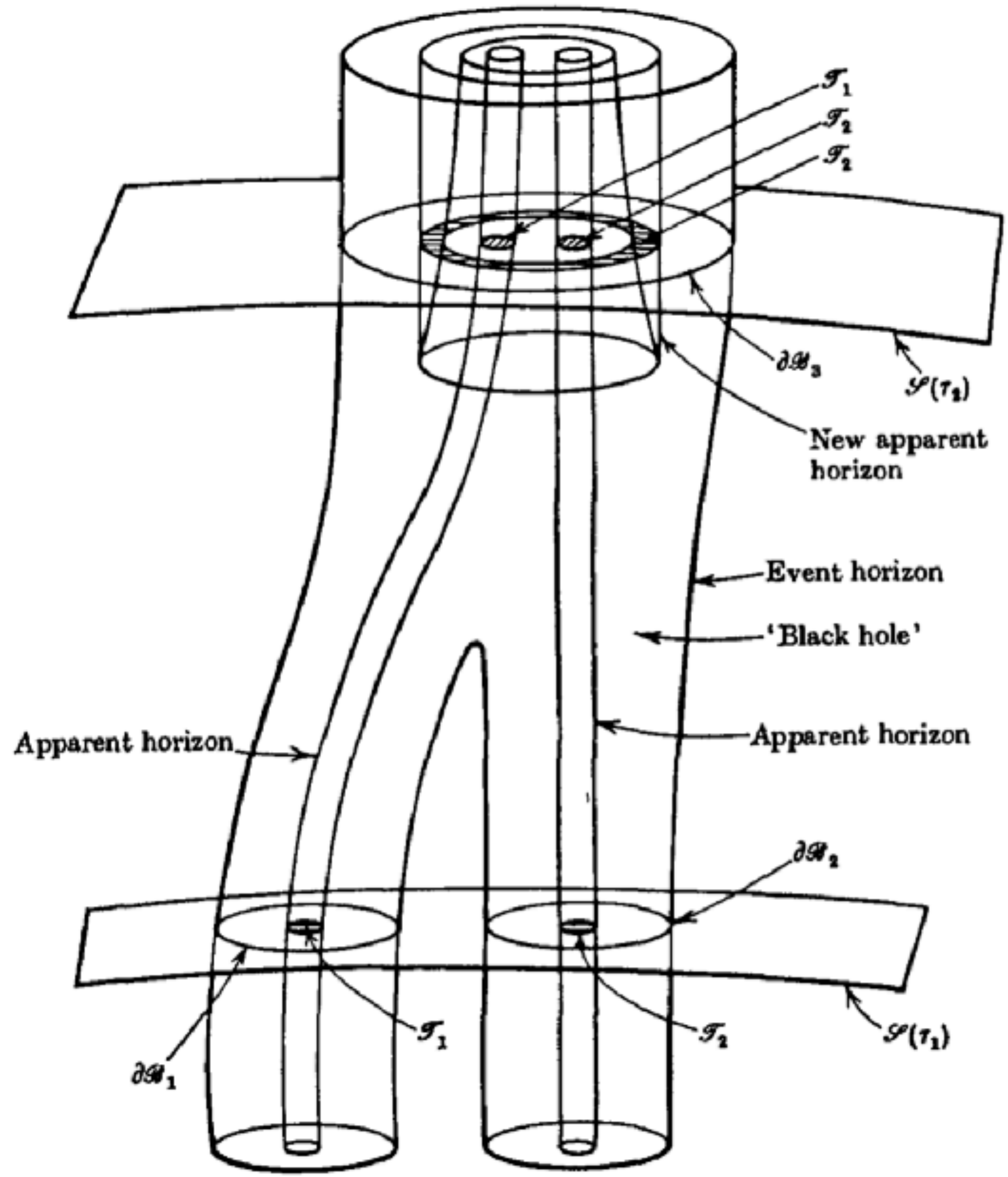

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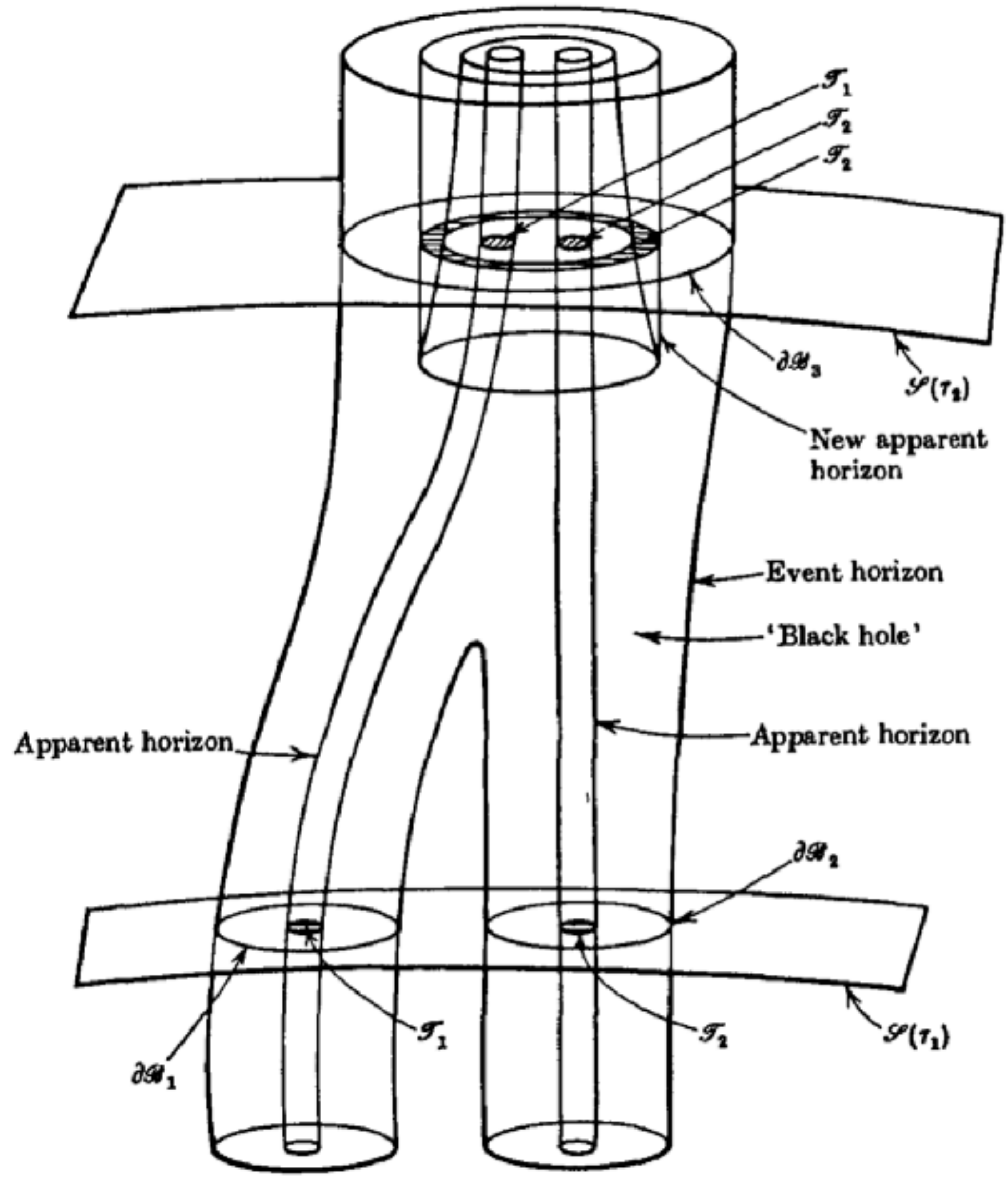

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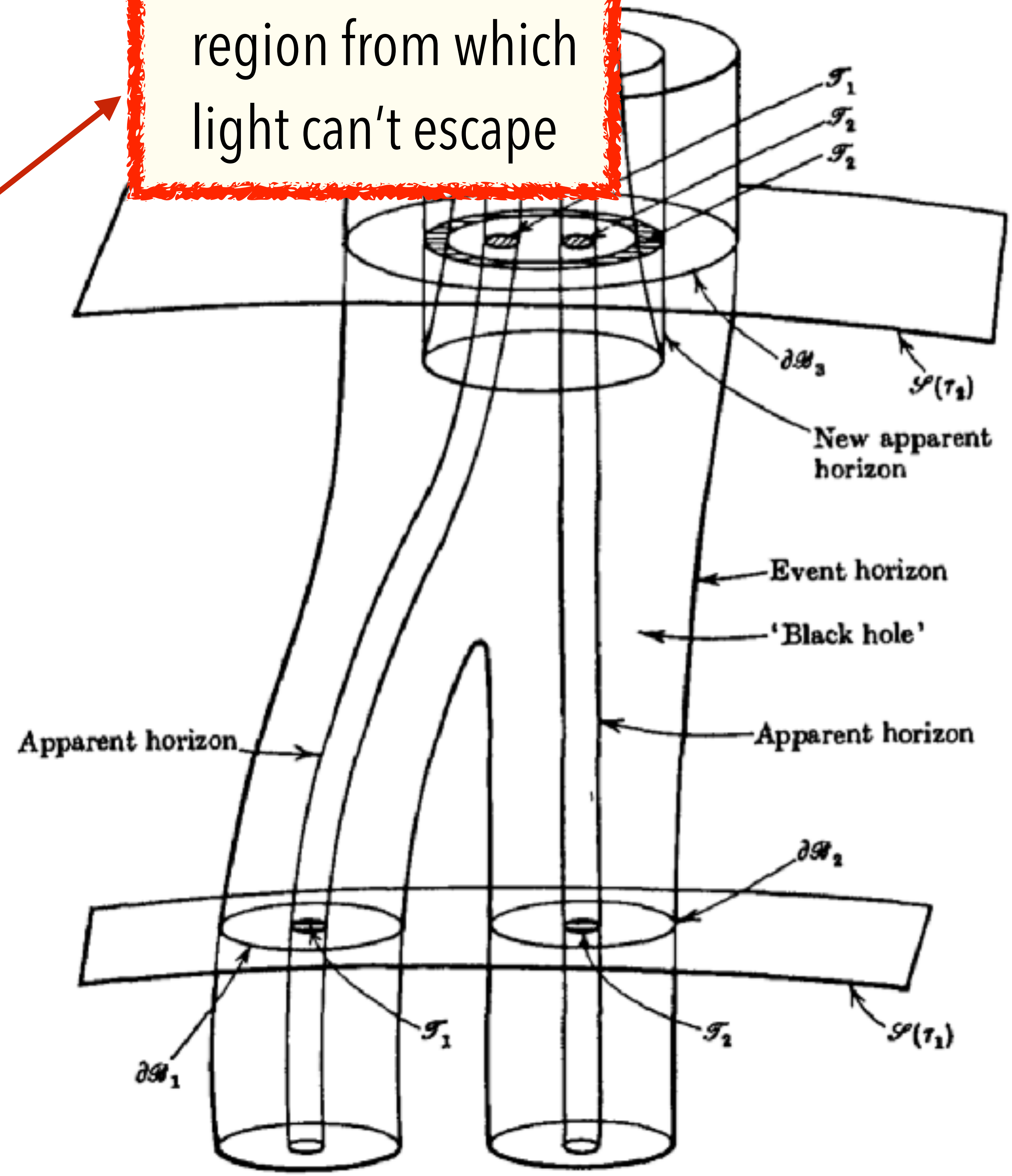

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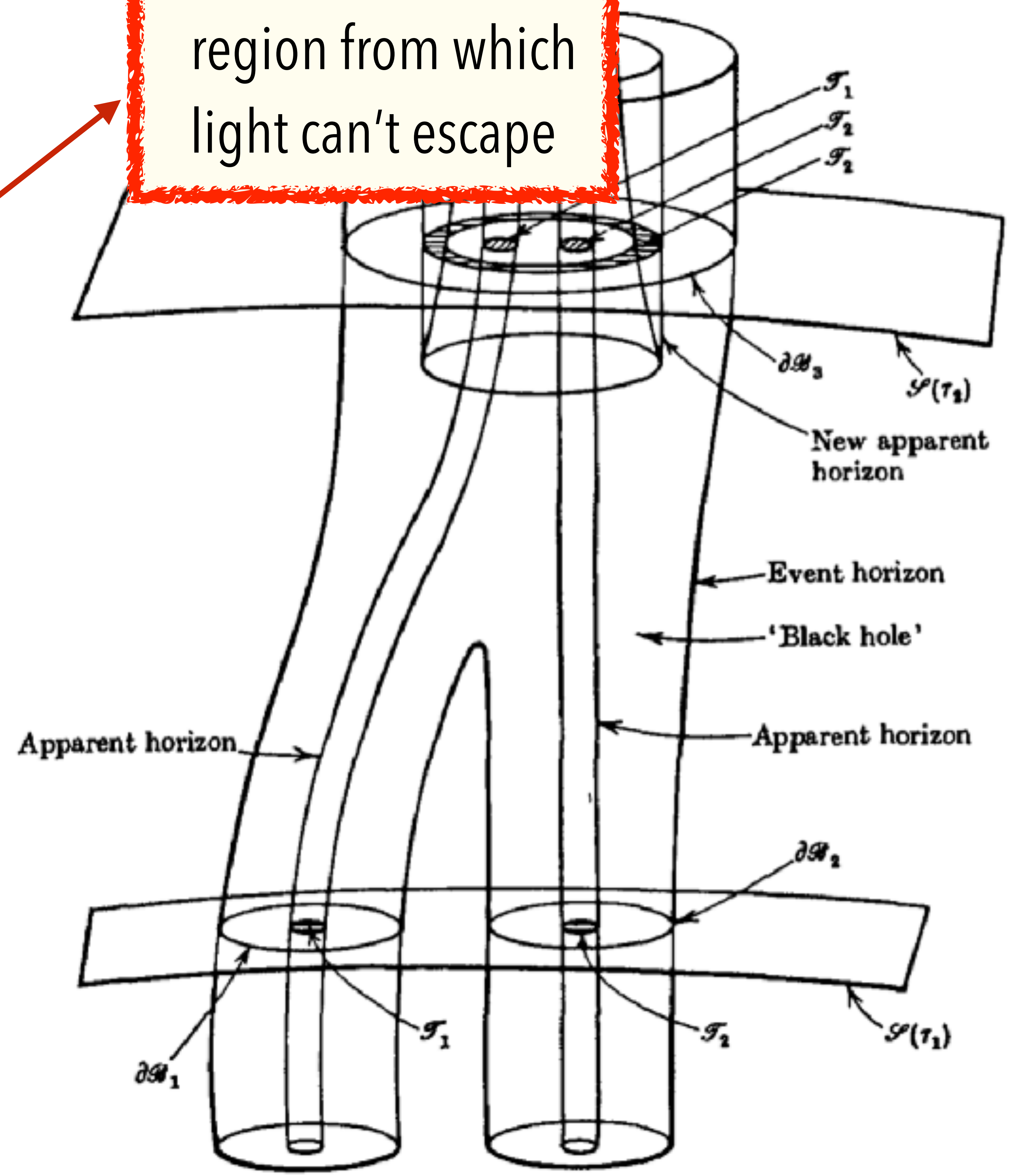

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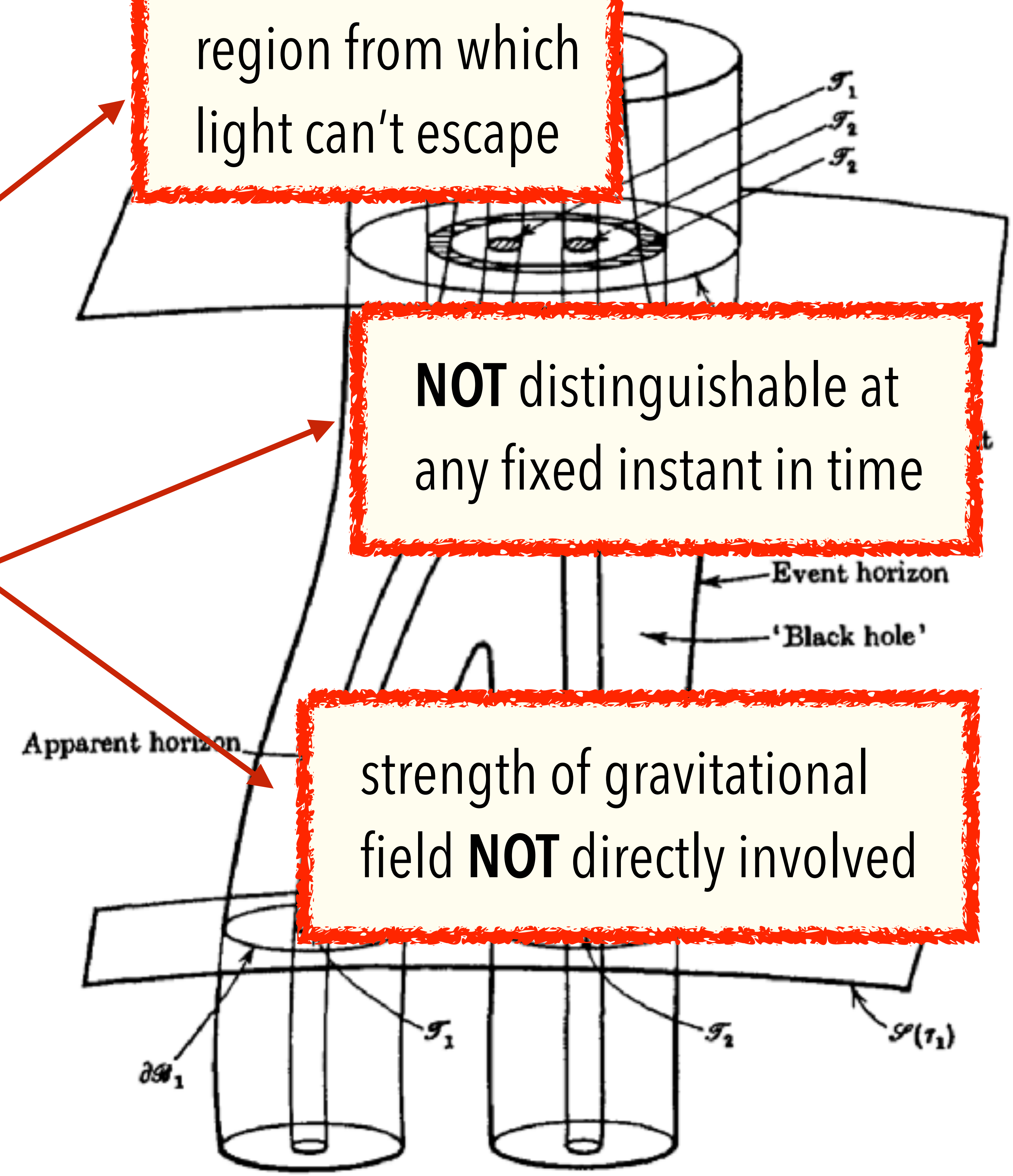

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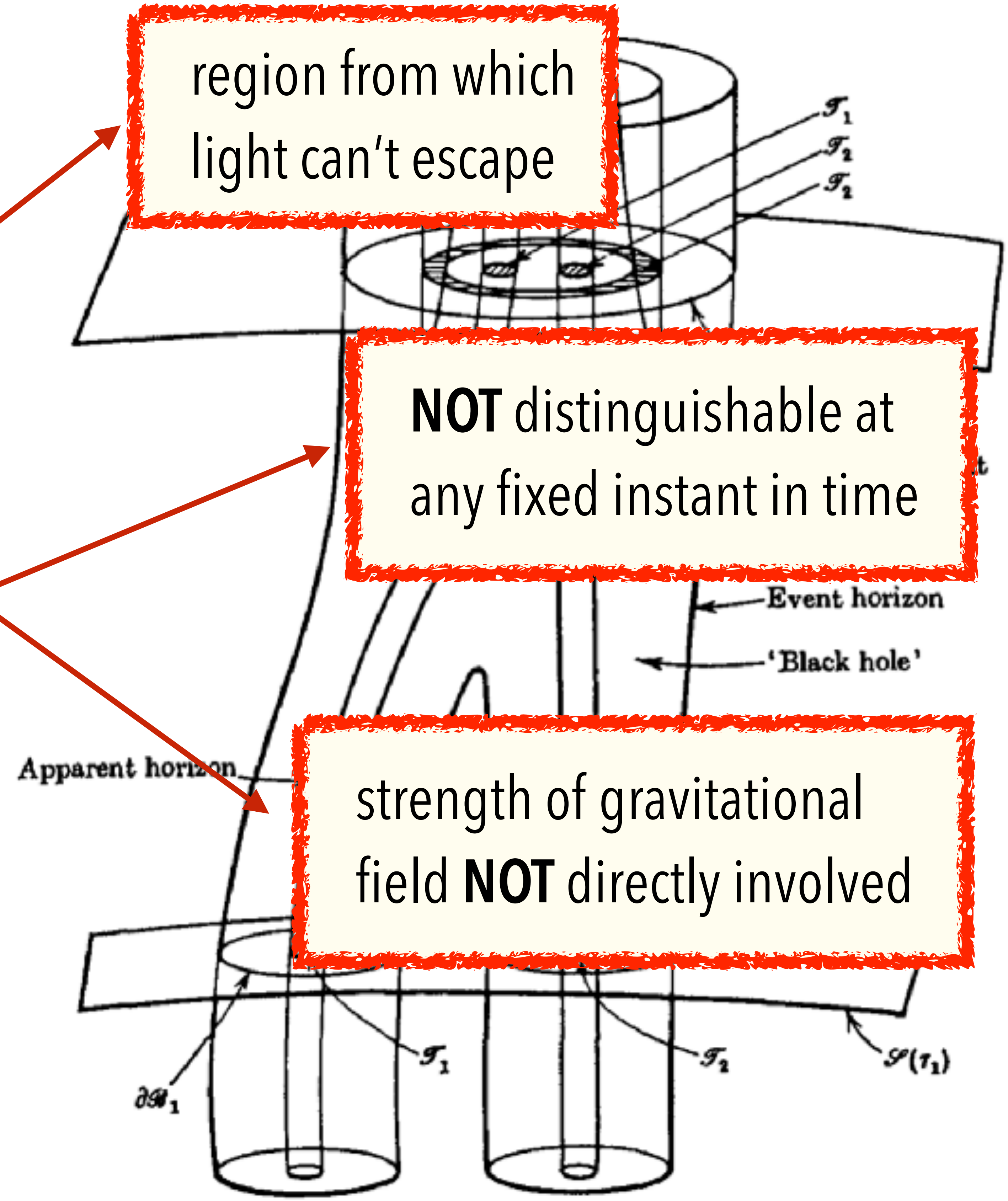

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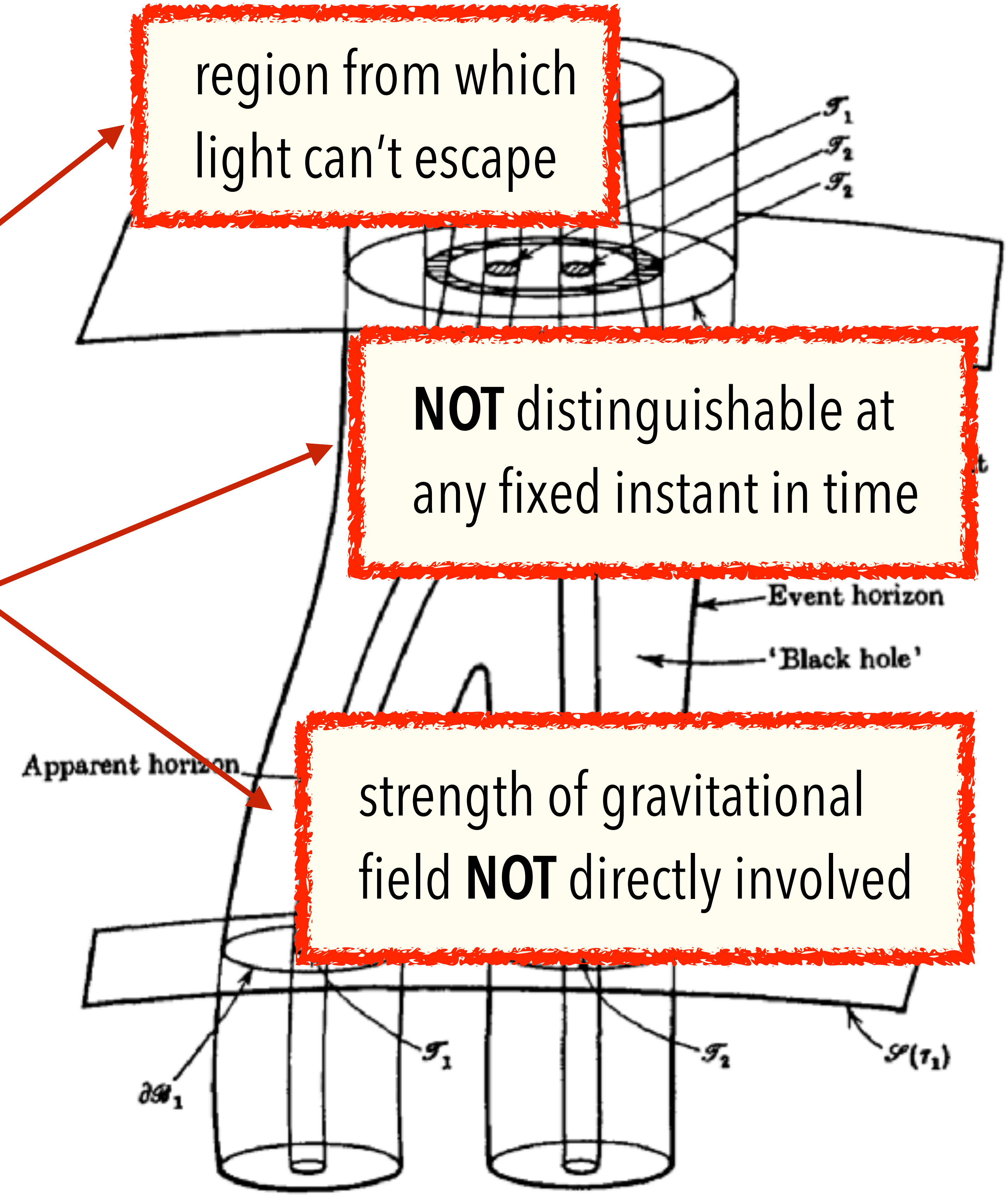

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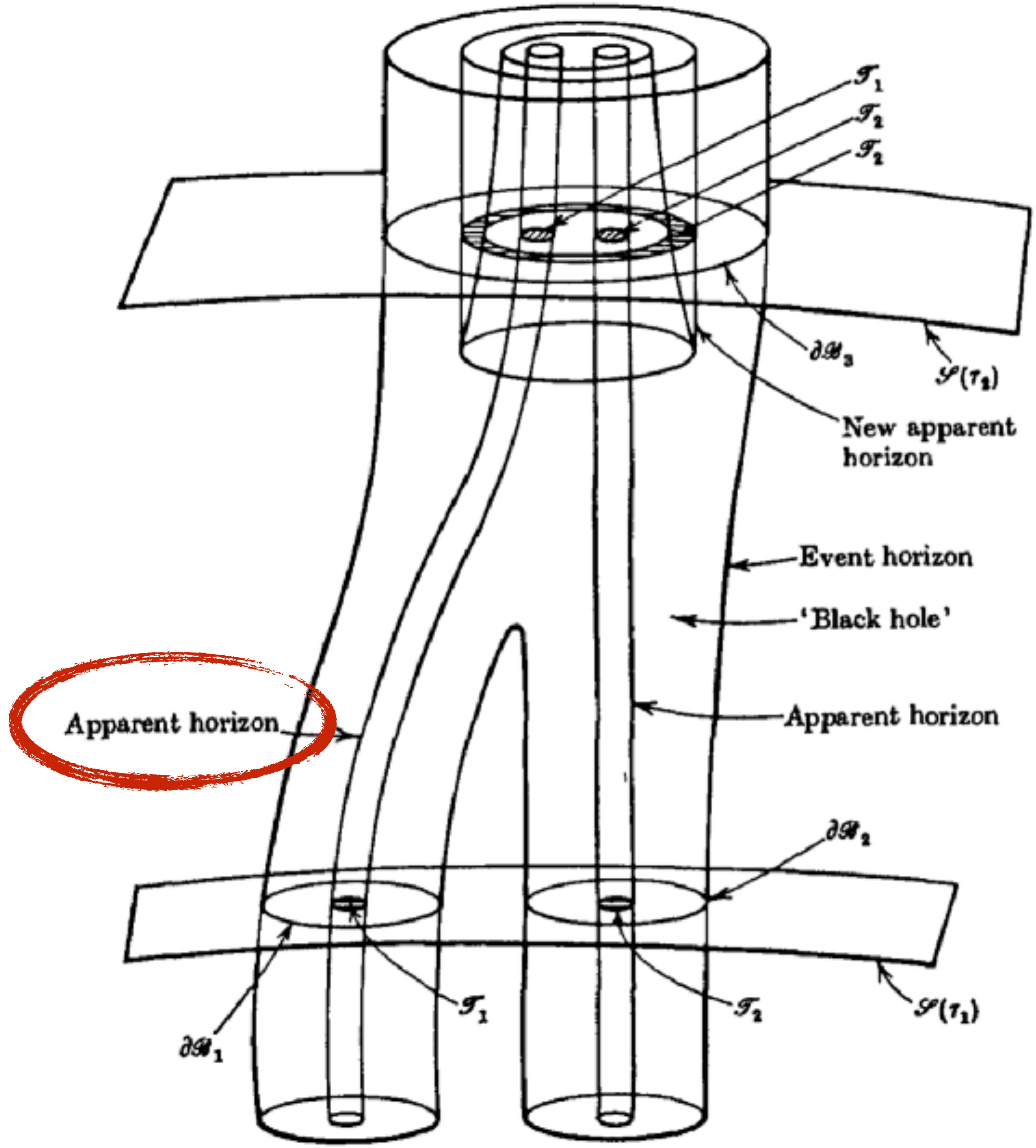

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Black hole mergers and event horizons

final fate of original AHs??

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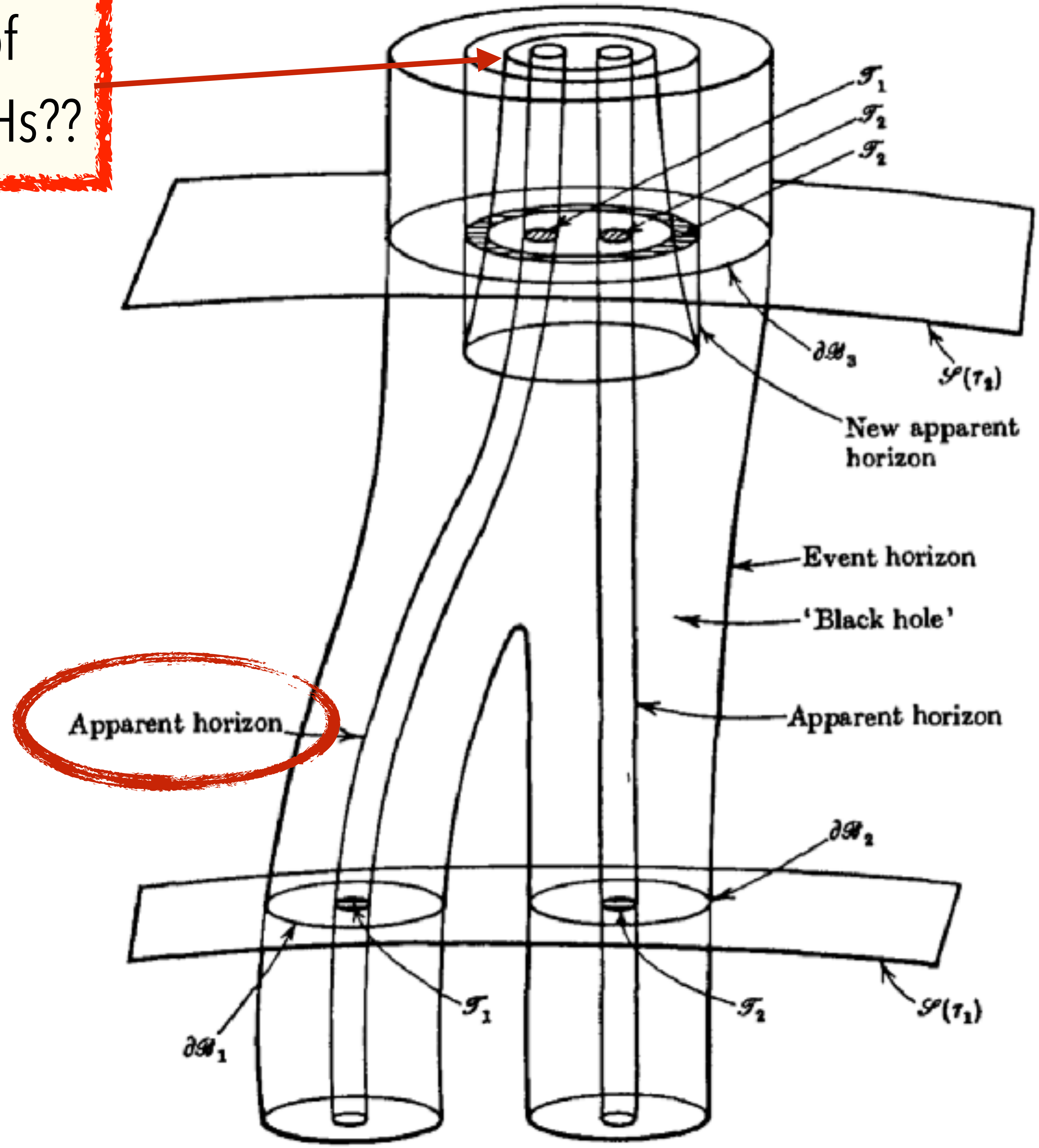
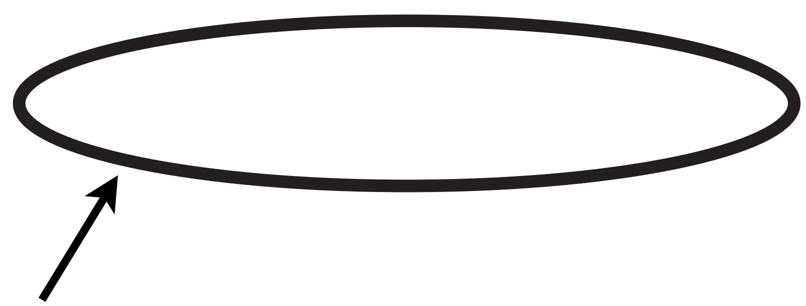


Hawking-Ellis 1973

How do you know if you are inside a black hole?



S: spacelike two-surface bounding a three-dimensional region

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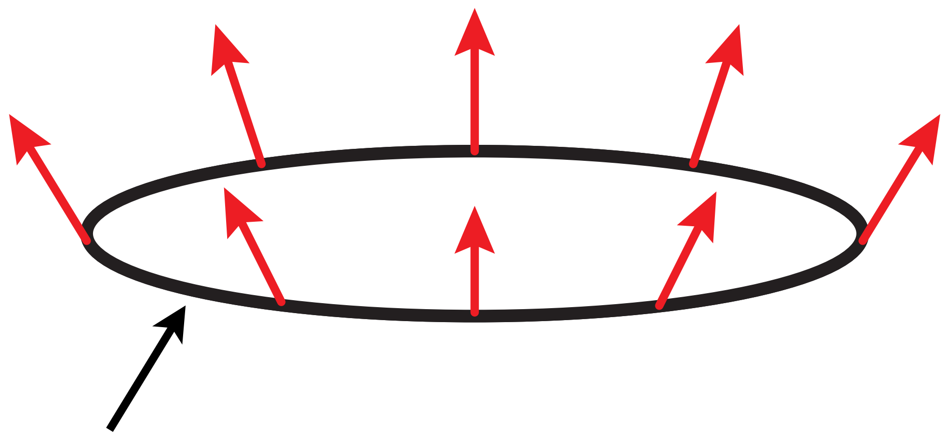

ℓ_+ - outward null normal

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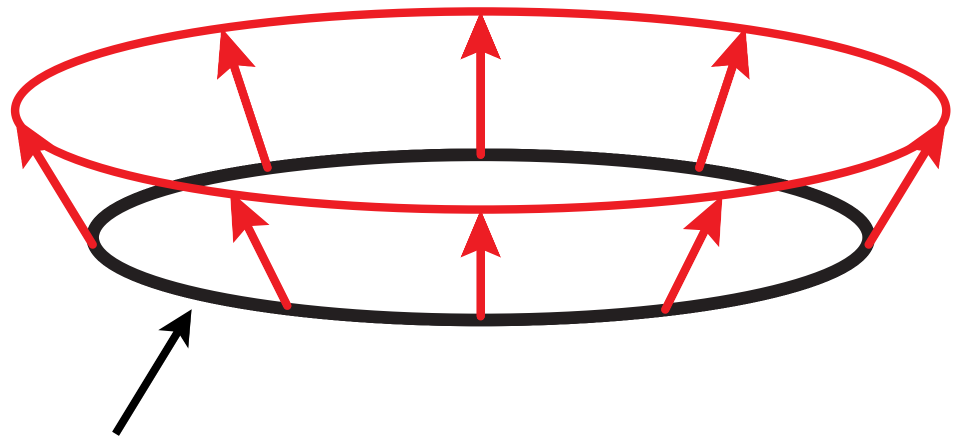

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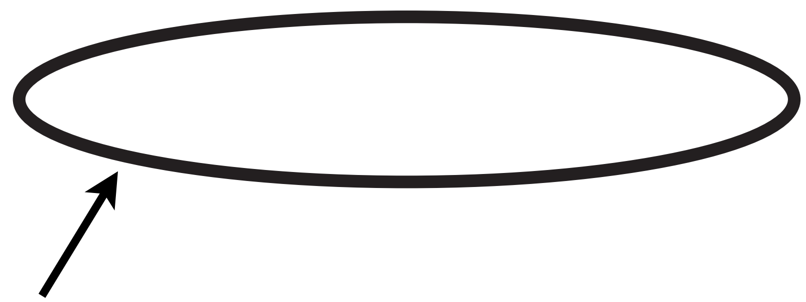

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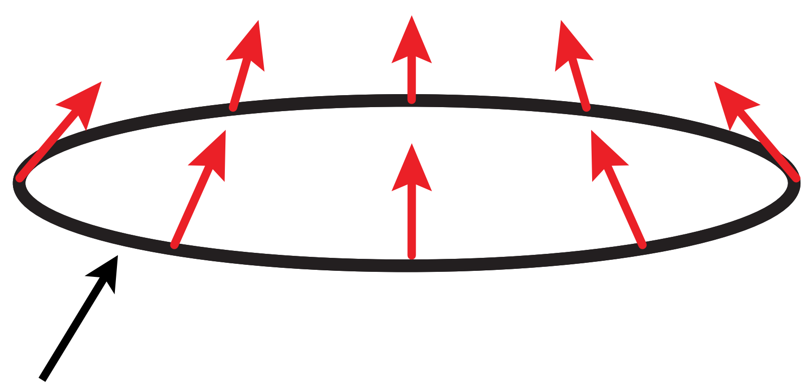

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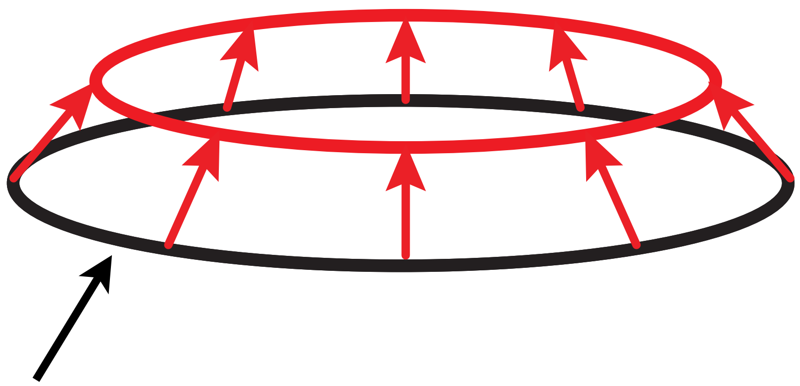

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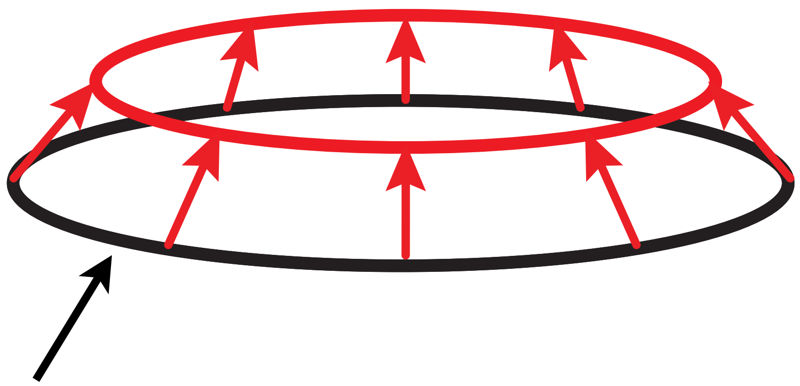

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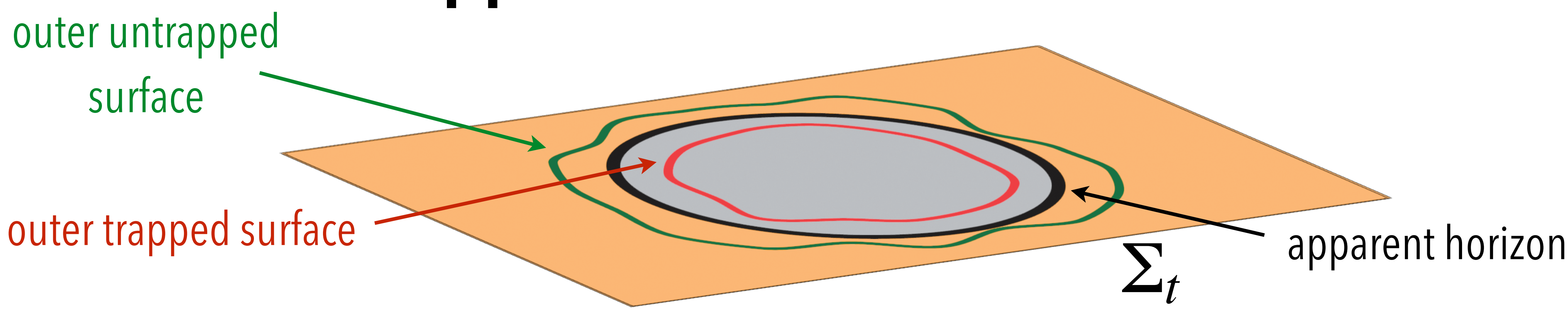
- Outer trapped surfaces cannot send signals to infinity (Penrose PRL 65)

Light cannot escape from an outer trapped surface

Apparent horizons and MOTS



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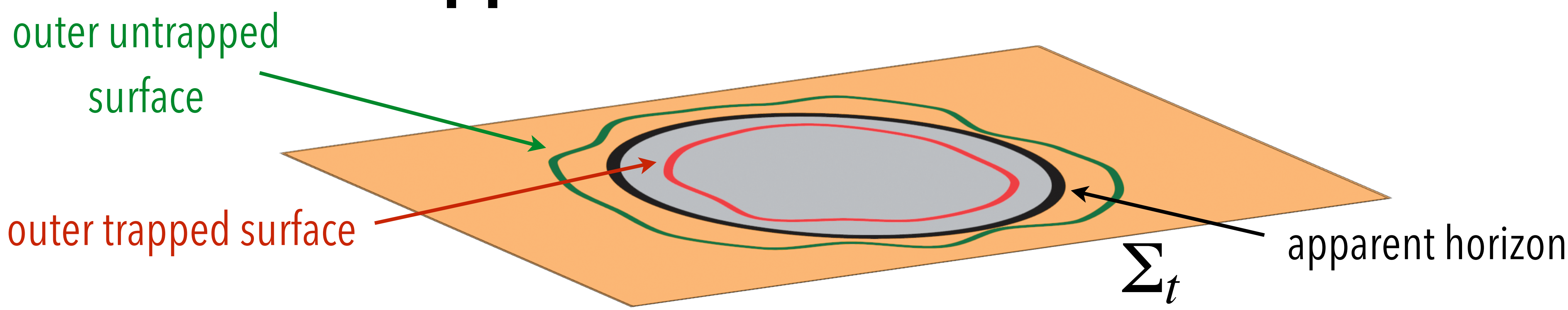

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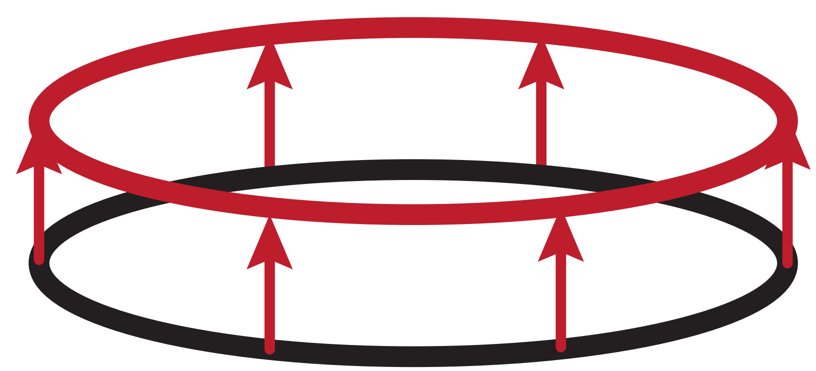

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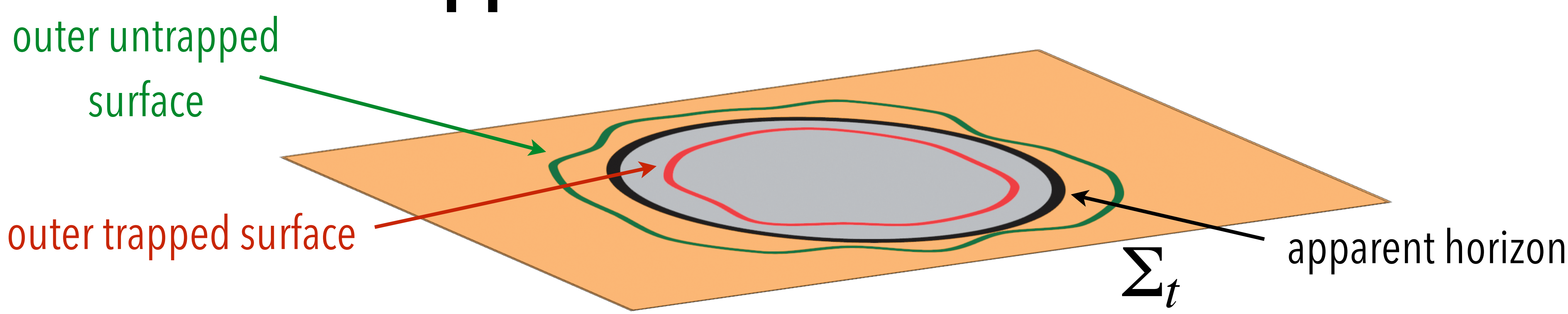

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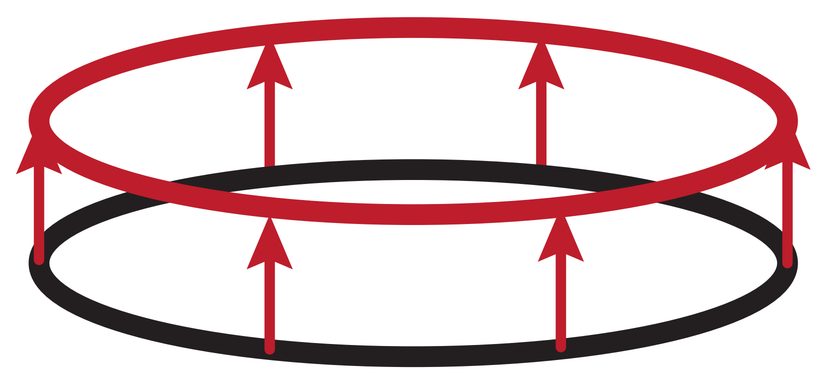

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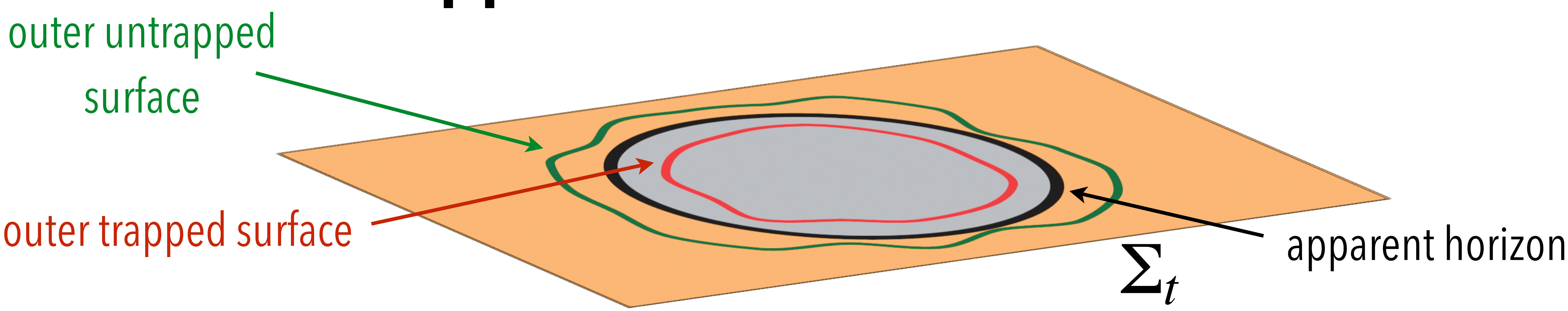

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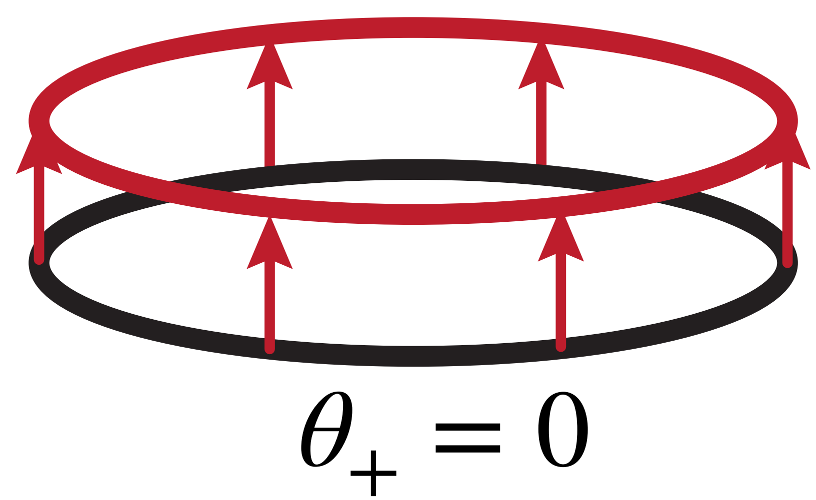

Schwarzschild, RN, Kerr event horizons are MOTS and apparent horizons

RN and Kerr inner horizons as well as cosmo horizons are MOTS

A local characterization of apparent horizons

A MOTS S in a time slice Σ_t is a (local) boundary and ***strictly stably outermost*** if:

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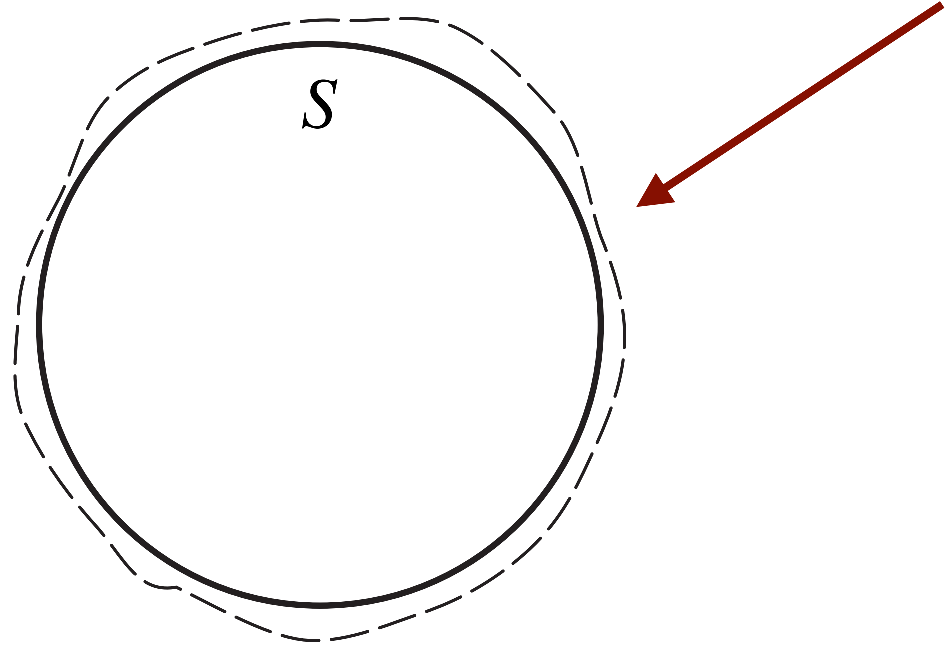

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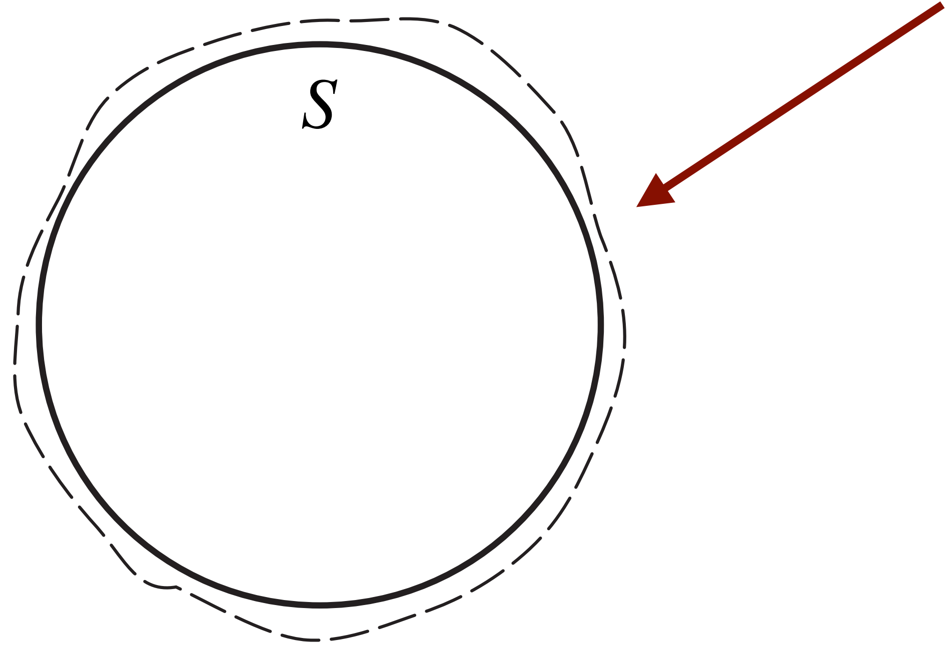

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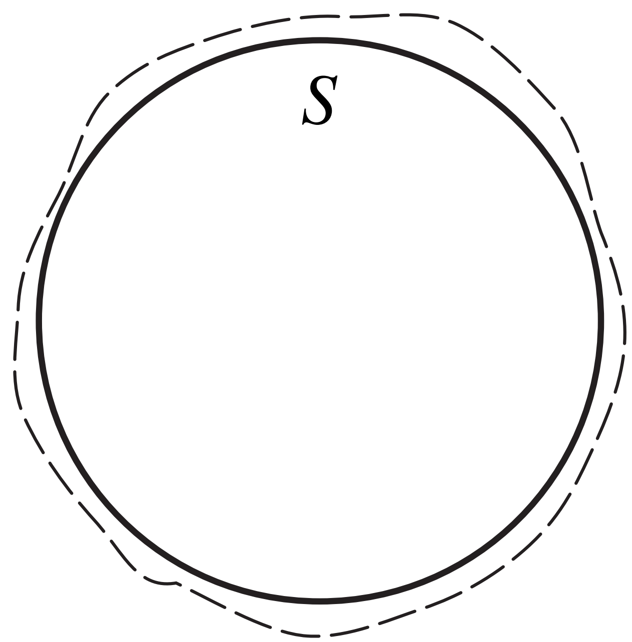

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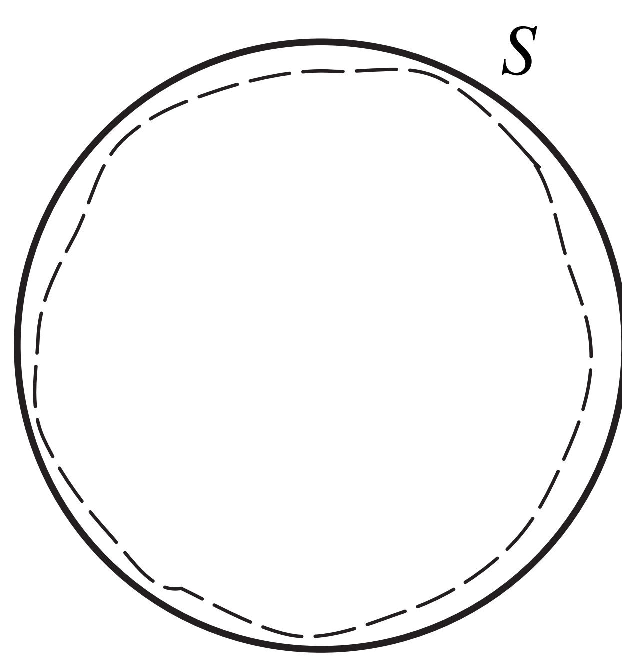

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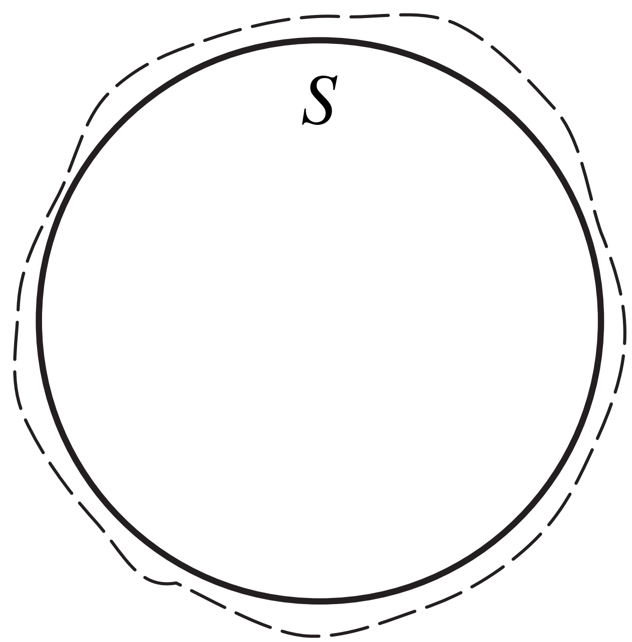

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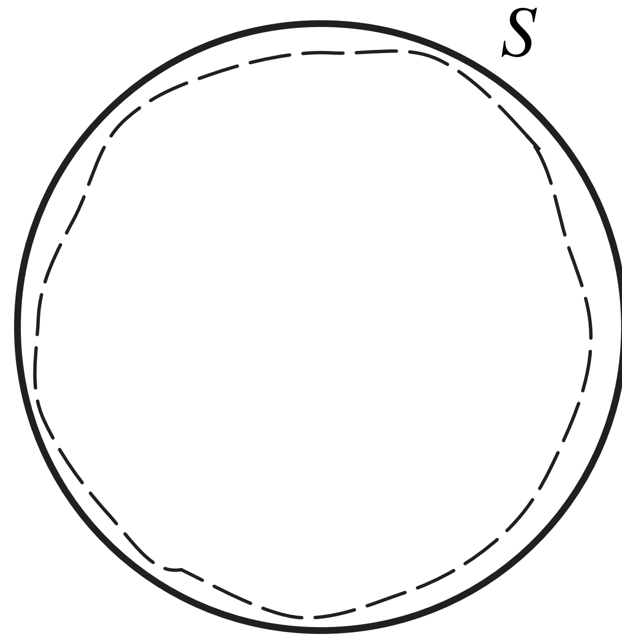
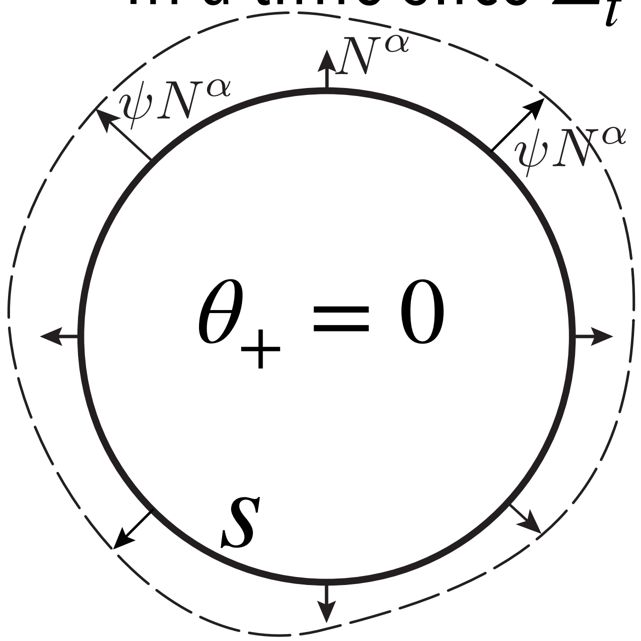


Newman 1987
Hayward 1994
Andersson, Mars, Simon 2005
Pook-Kolb, Hennigar, Booth 2021

3D Stability Operator for (non-rotating) MOTS

- In a time slice Σ_t with N unit normal to S and future outward null $l_+ = u + N$:



variation of expansion · stability operator · 2D surface Laplacian · Gauss curvature · square of null shear · matter term

$$\delta_{\psi N}\theta_+ = L_S\psi = -\mathscr{D}^2\psi + \left(\mathscr{K} - 2\|\sigma_+\|^2 - G_{+-}\right)\psi$$

(Andersson, Mars, Simon, 2005, PRL 111102)

(Andersson, Mars, Simon, Adv.Theor.Math.Phys. 12 (2008) 4, 853-888)

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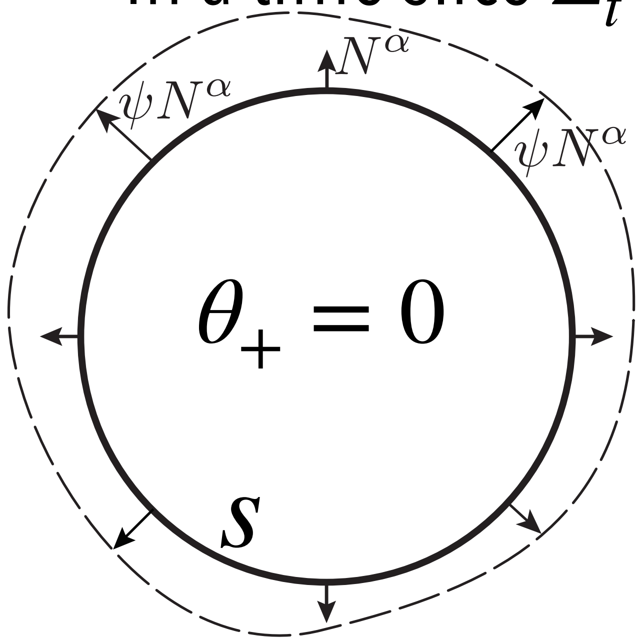

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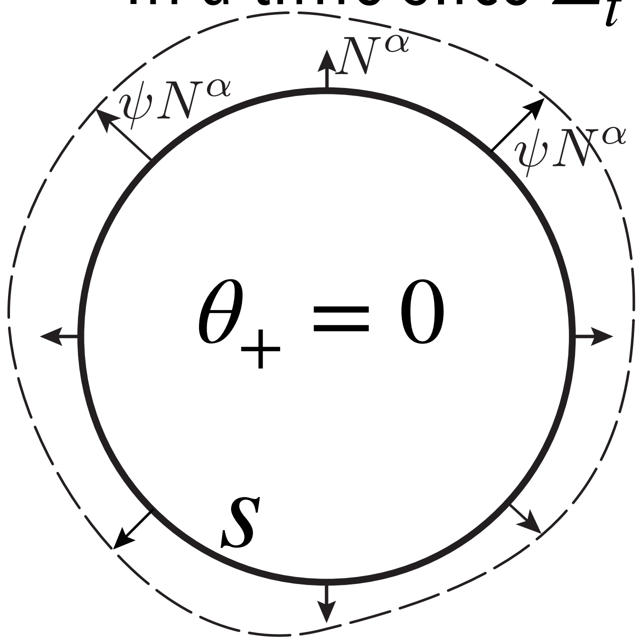

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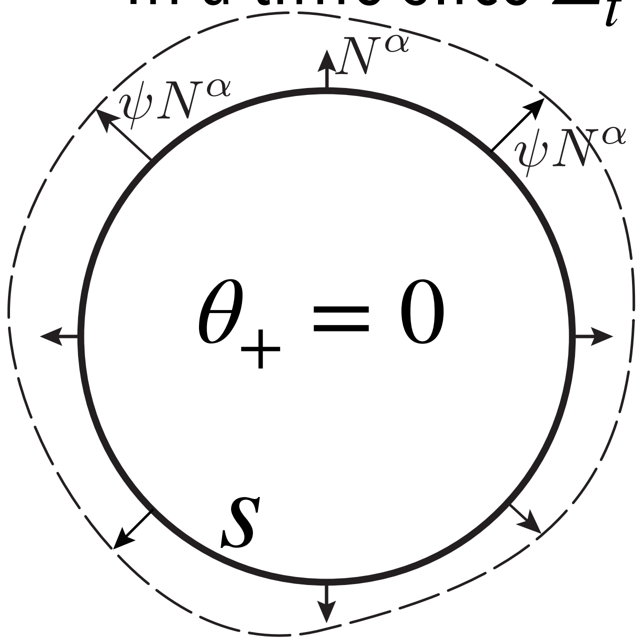

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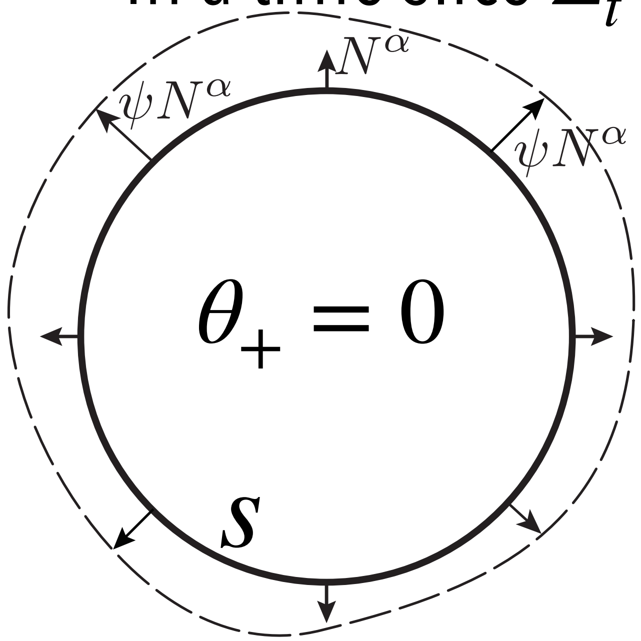
- There is a smallest **principal eigenvalue** λ_o. All others are larger.

3D Stability Operator for (non-rotating) MOTS

- In a time slice Σ_t with N unit normal to S and future outward null $l_+ = u + N$:



variation of expansion | stability operator | 2D surface Laplacian | Gauss curvature | square of null shear | matter term

$$\delta_{\psi N}\theta_+ = L_S\psi = -\mathscr{D}^2\psi + \left(\mathscr{K} - 2\|\sigma_+\|^2 - G_{+-}\right)\psi$$

(Andersson, Mars, Simon, 2005, PRL 111102)

(Andersson, Mars, Simon, Adv.Theor.Math.Phys. 12 (2008) 4, 853-888)

- $L_S\psi = \delta_{\psi N}\theta_+$ is a self-adjoint linear elliptic operator on ψ
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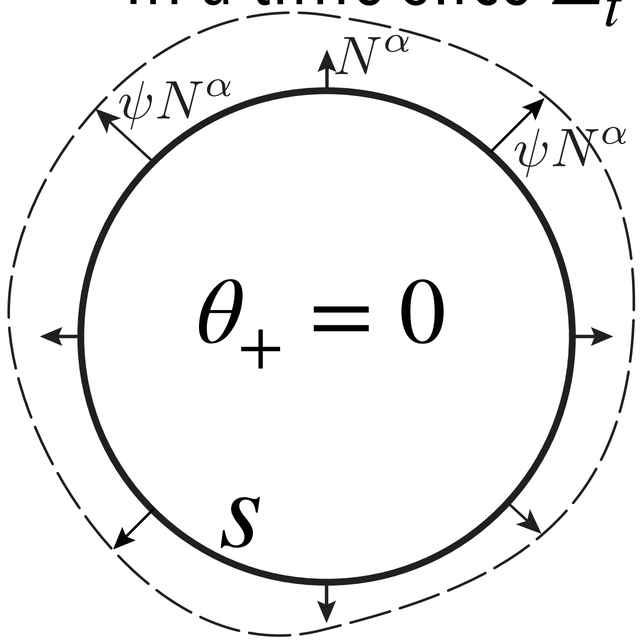

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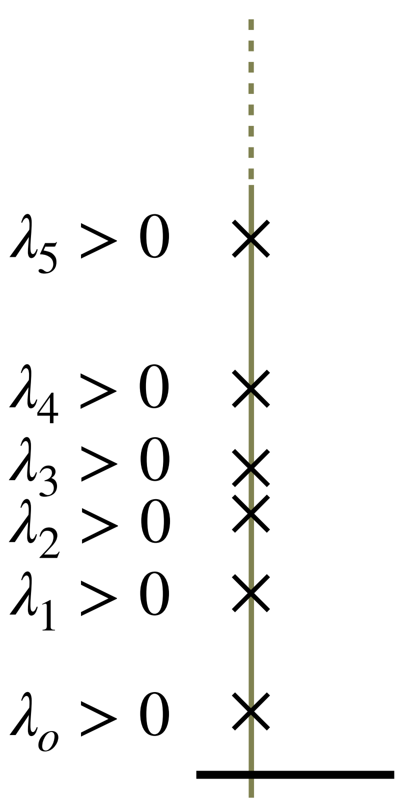

Strictly stable

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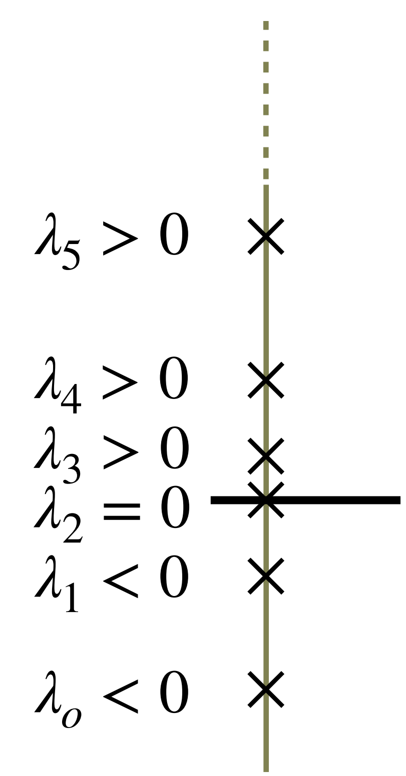
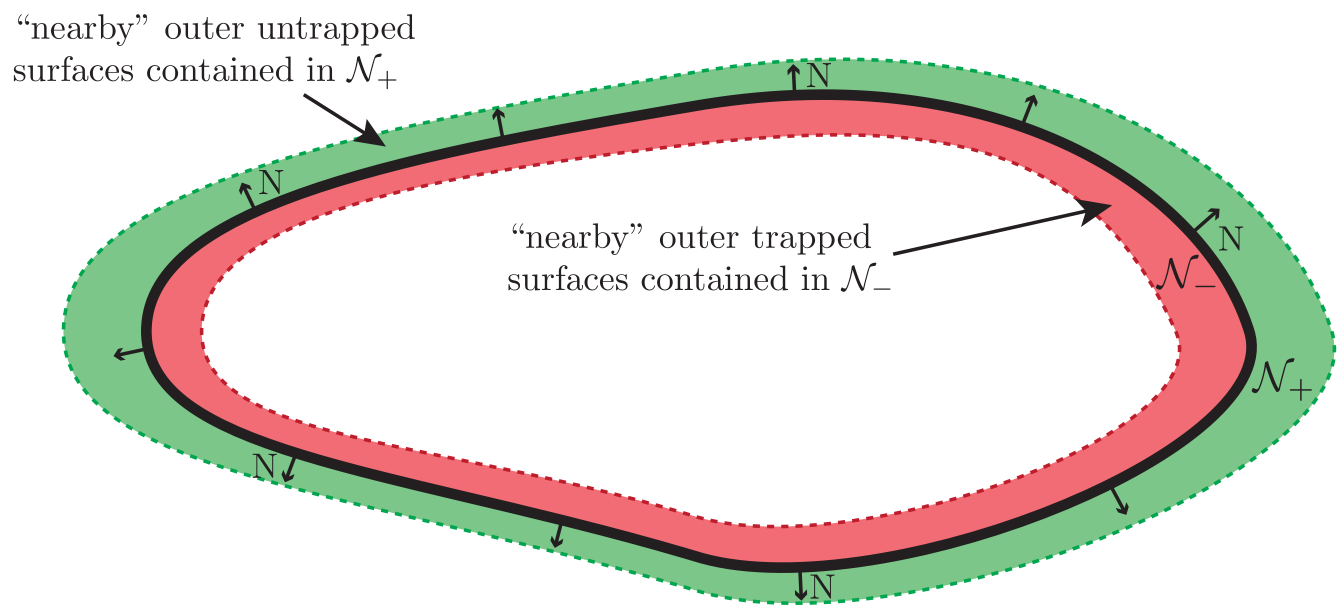
- There is a smallest **principal eigenvalue** λ_o. All others are larger.
- Principal eigenfunction $\psi_o > 0$
- Strictly stable: all eigenvalues positive

  Unstable: one or more negative eigenvalues



Unstable

Stability: MOTSs as local boundaries



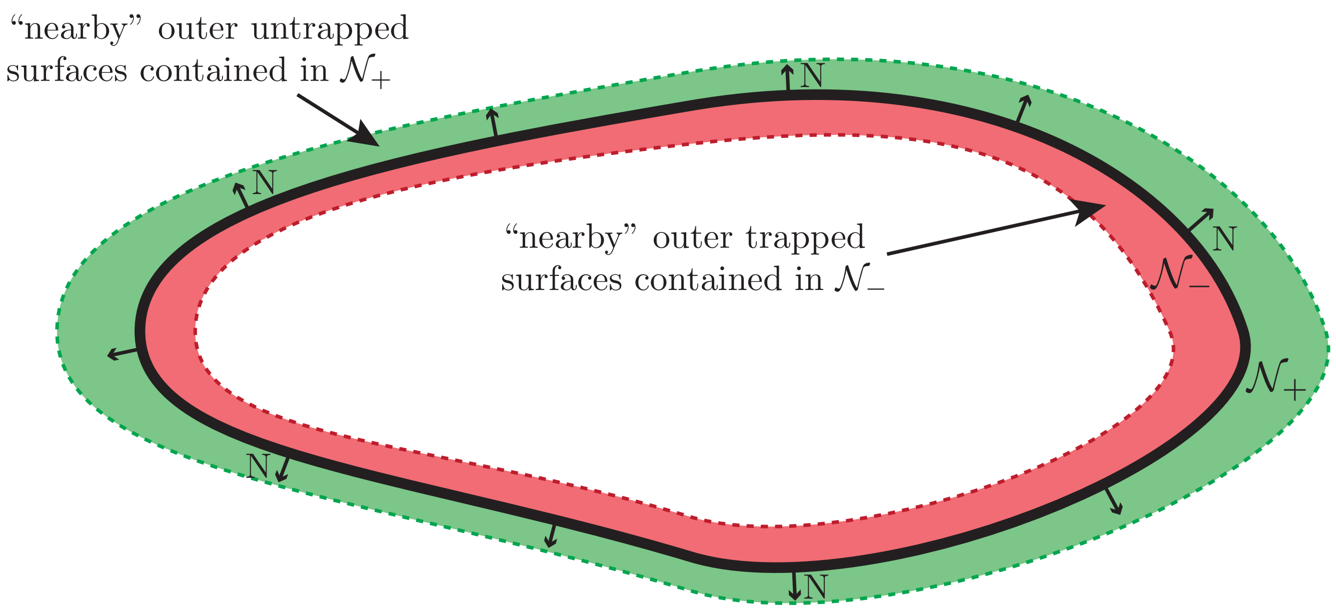
Strictly stable ($\lambda_o > 0$)

$\Rightarrow$ outer untrapped outside,

outer trapped inside

Local apparent horizon

Stability: MOTSs as local boundaries



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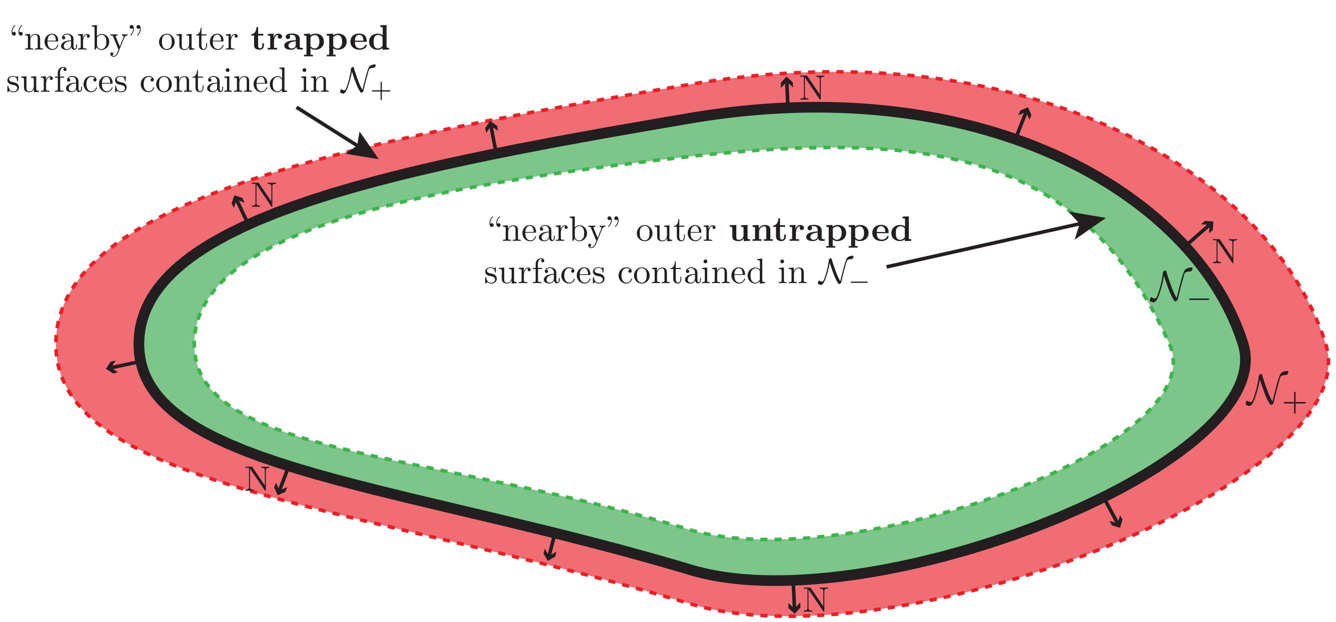

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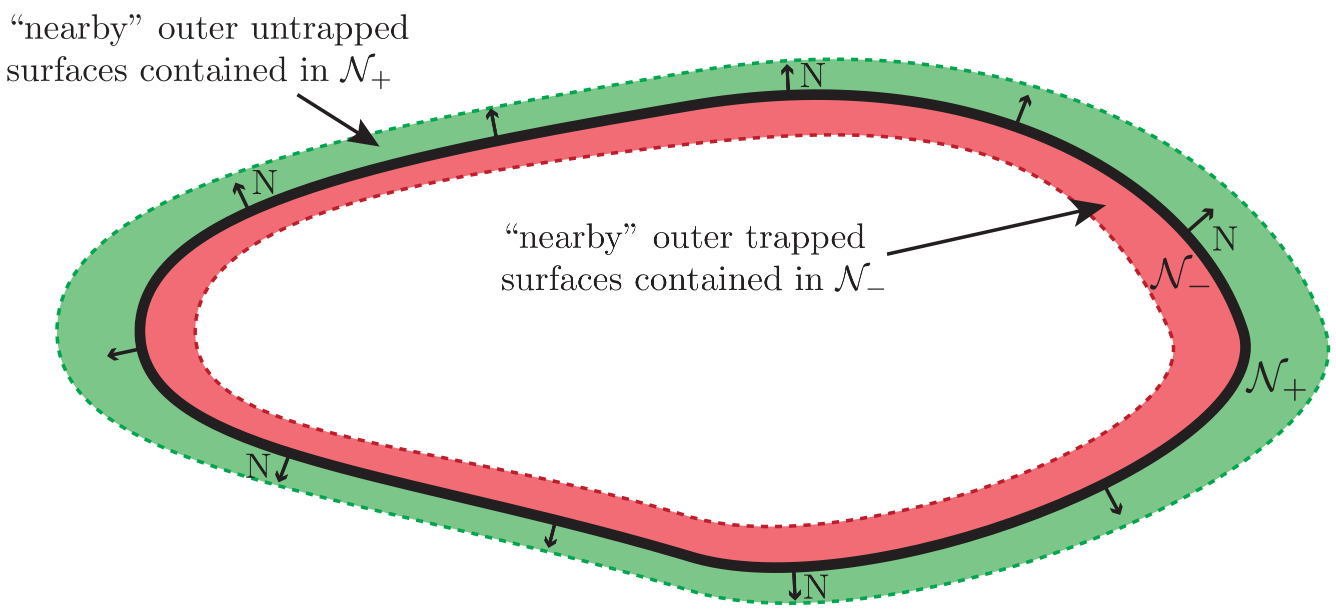
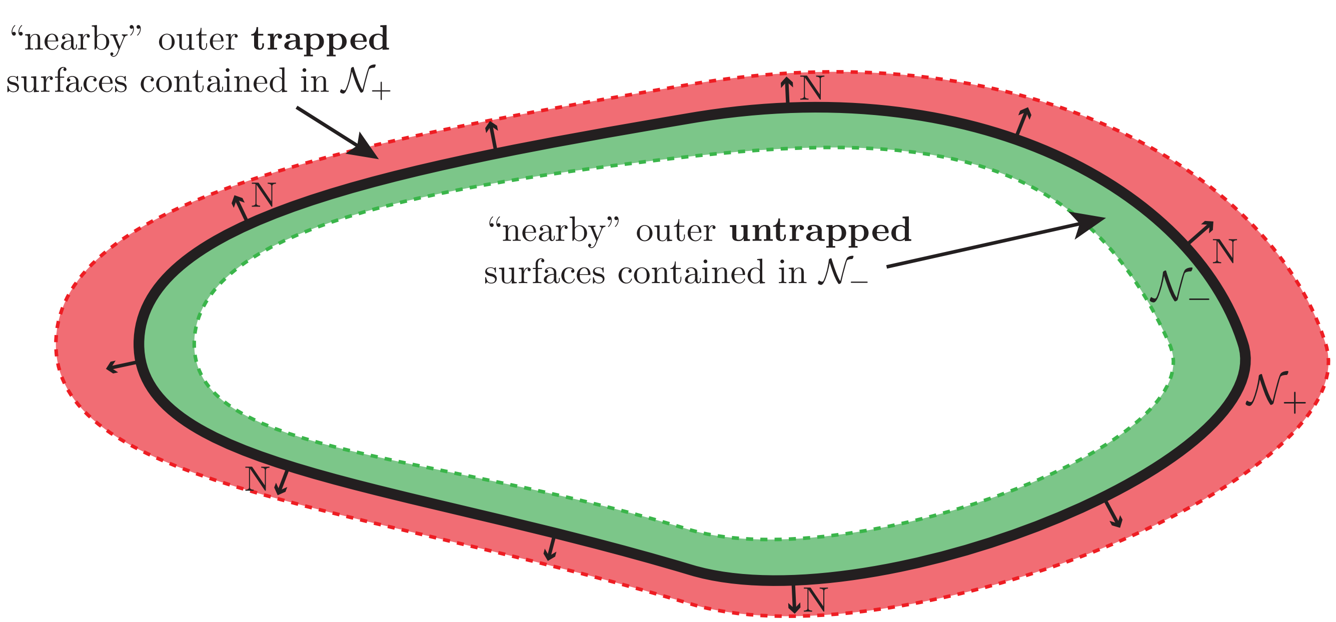


Strictly stable MOTS separates trapped from untrapped regions (distinguishable)



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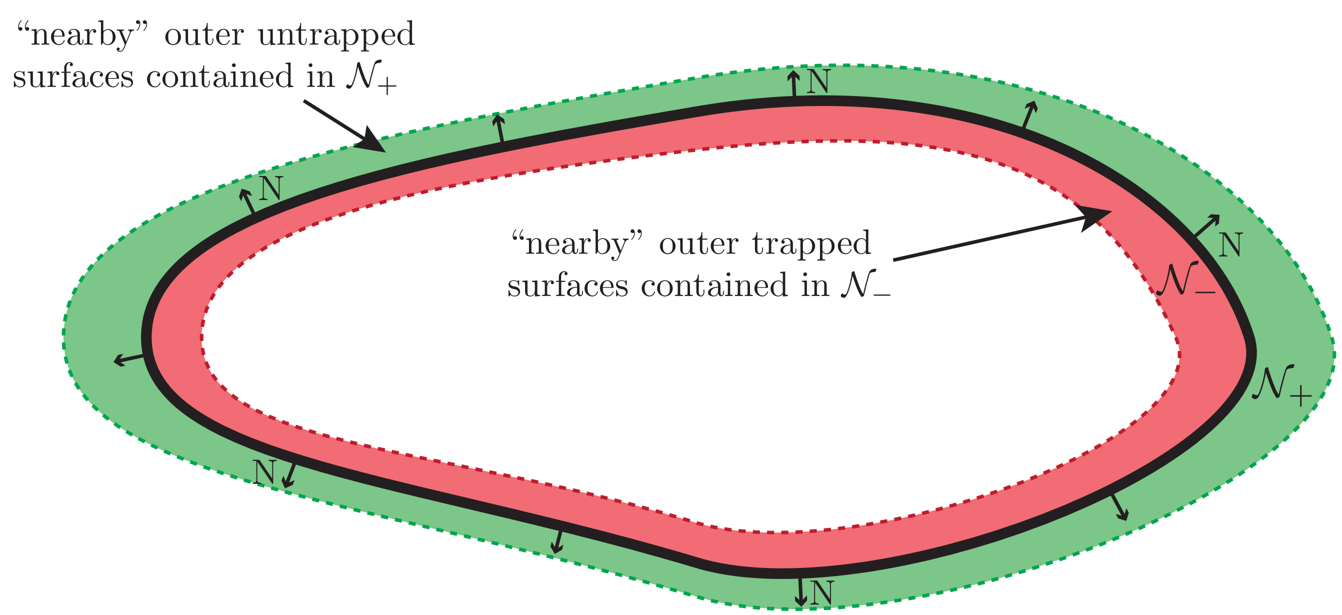

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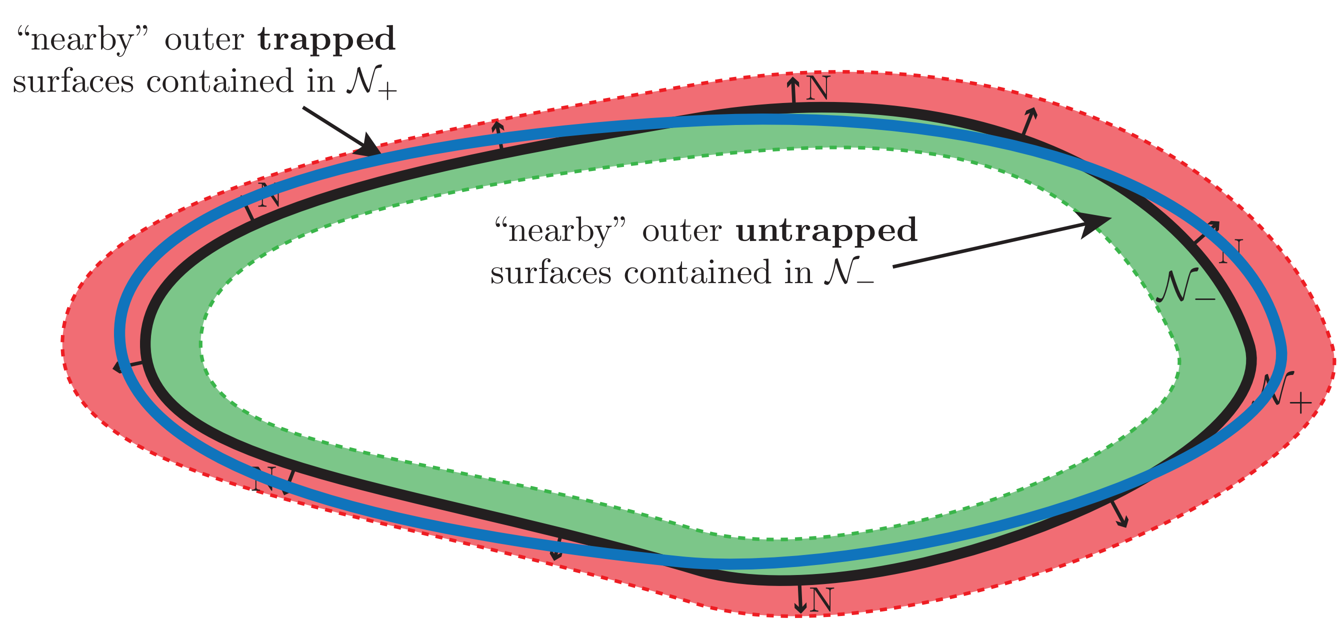


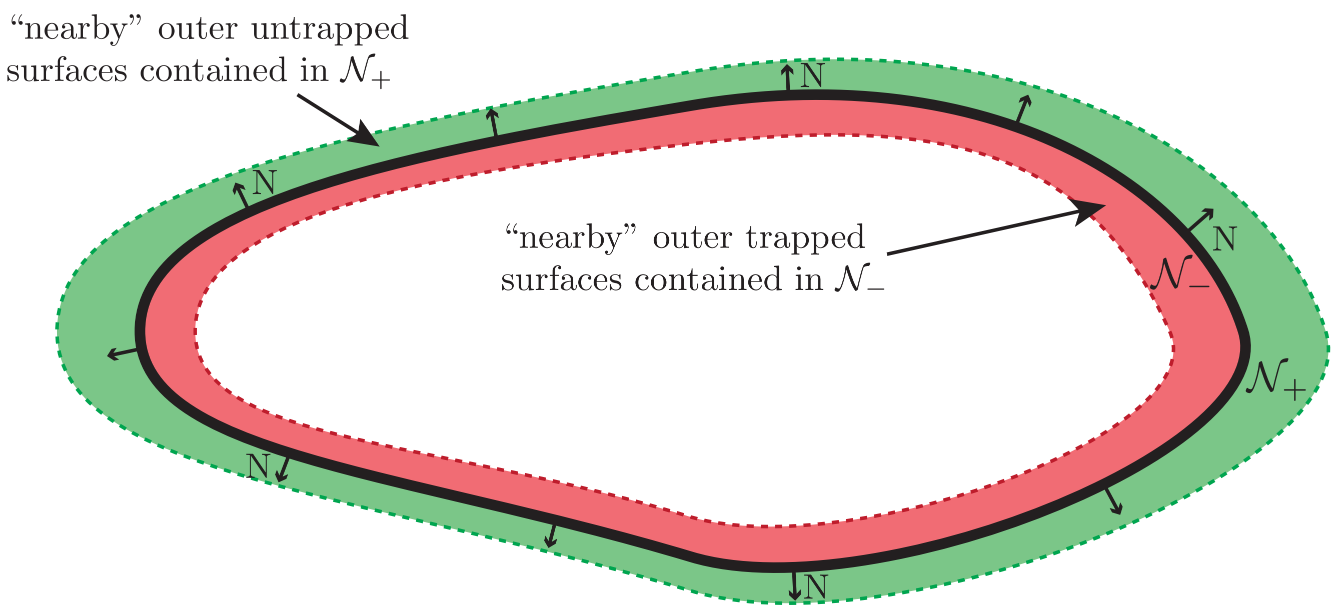
Strictly unstable ($\lambda_o < 0$)

$\Rightarrow$ outer trapped outside, outer untrapped inside

$\Rightarrow$ trapped and untrapped can also **straddle** MOTS

Local inner horizon

Stability: MOTSs as local boundaries



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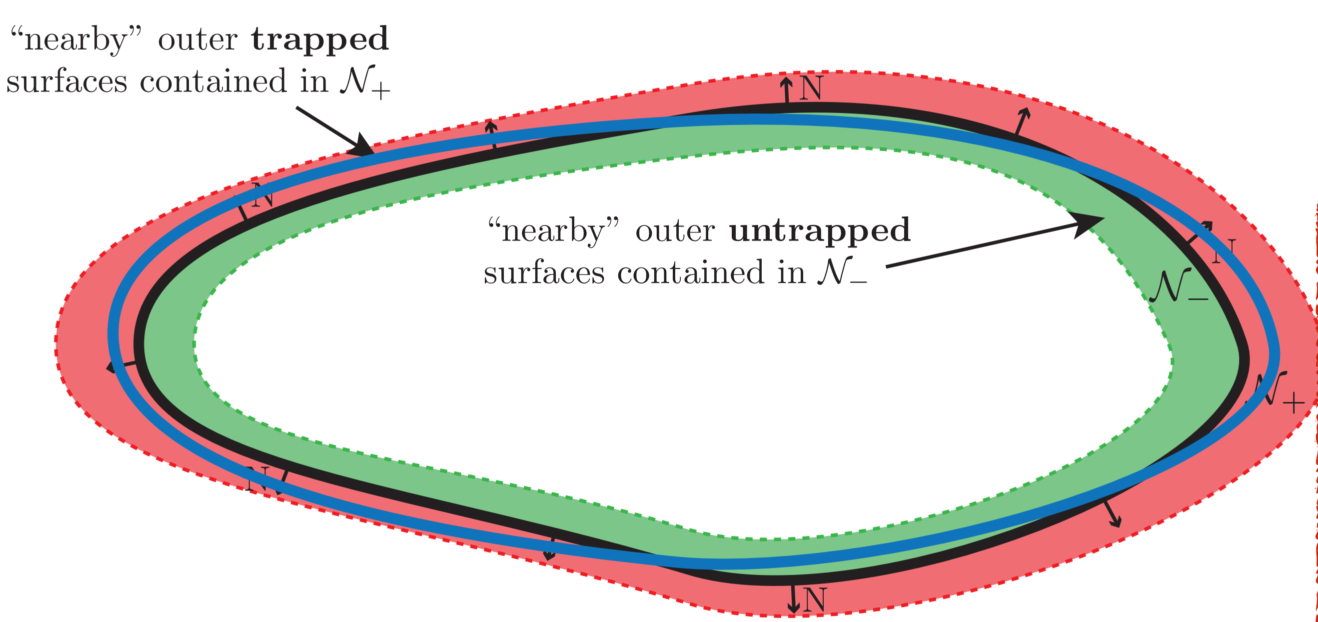

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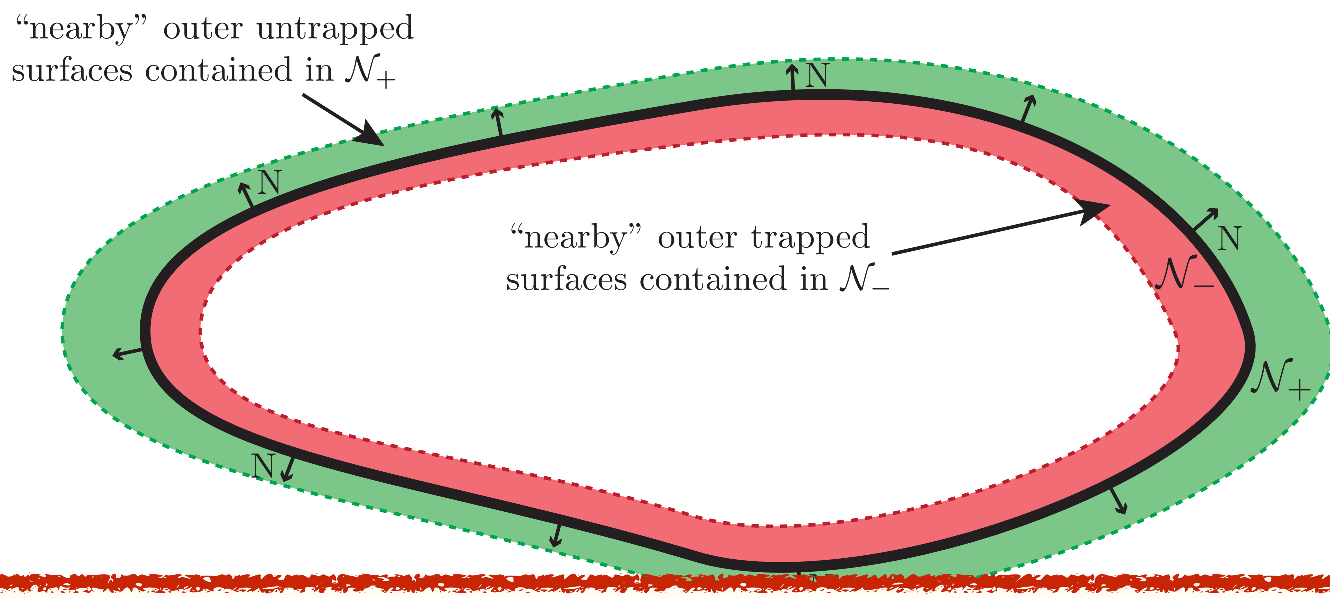
Unstable MOTS doesn't fully separate trapped from untrapped regions(distinguishable)

Stability: MOTSs as local boundaries



Strictly stable MOTS separates trapped from untrapped regions (distinguishable)

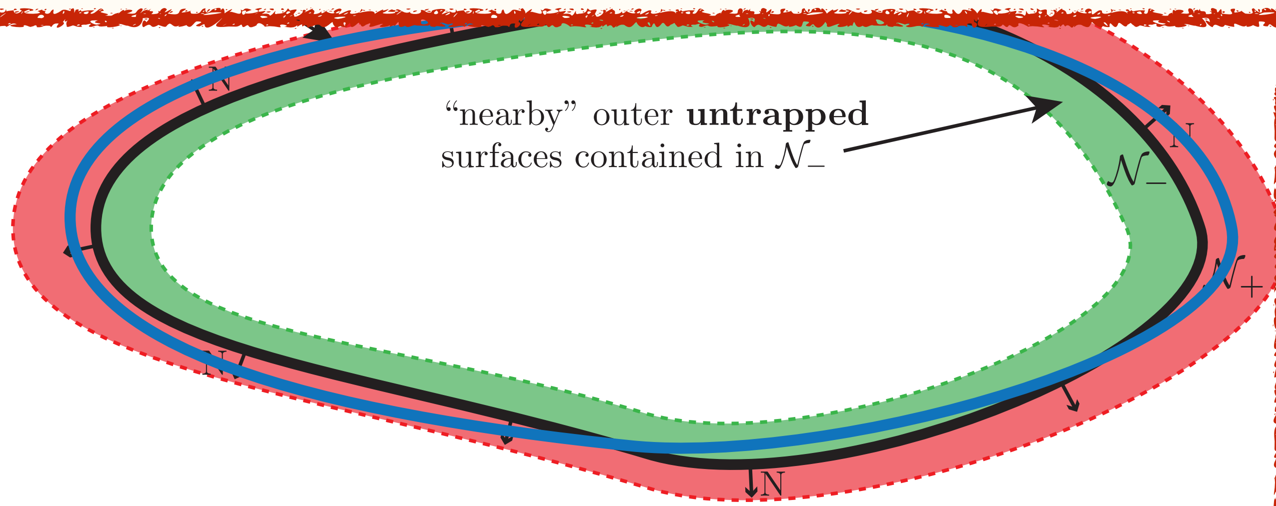
Canonical Example: Reissner-Nordström

Outer horizon $r_{\mathrm{OH}} = m + \sqrt{m^2 - q^2}$: strictly stable

Inner horizon $r_{\mathrm{IH}} = m - \sqrt{m^2 - q^2}$: strictly unstable

stable ($\lambda_o < 0$)

$\Rightarrow$ outer trapped outside,



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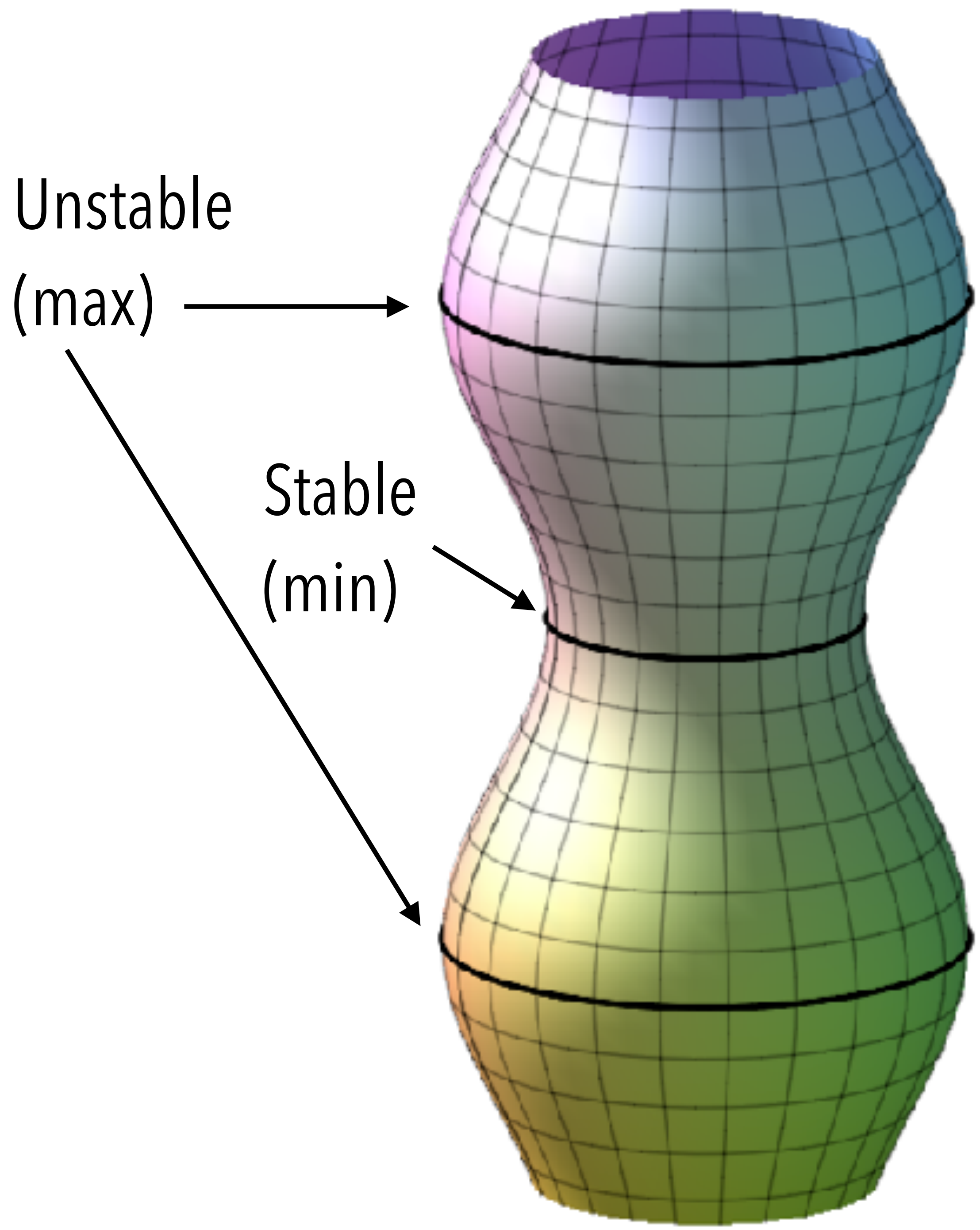
An aside…

What does "stable" mean?

This is *geometric stability* in the sense of minimal surfaces/geodesics.



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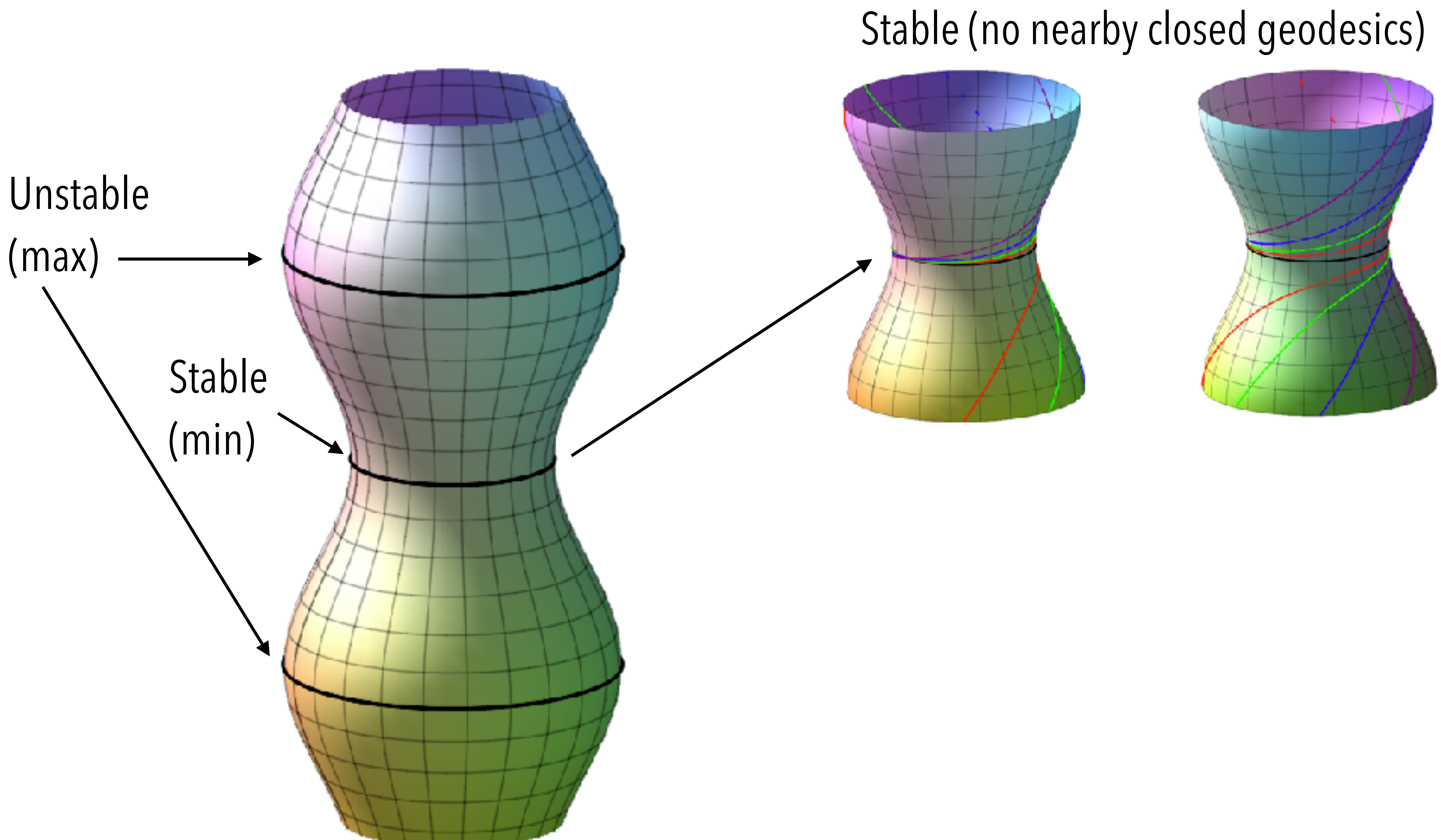

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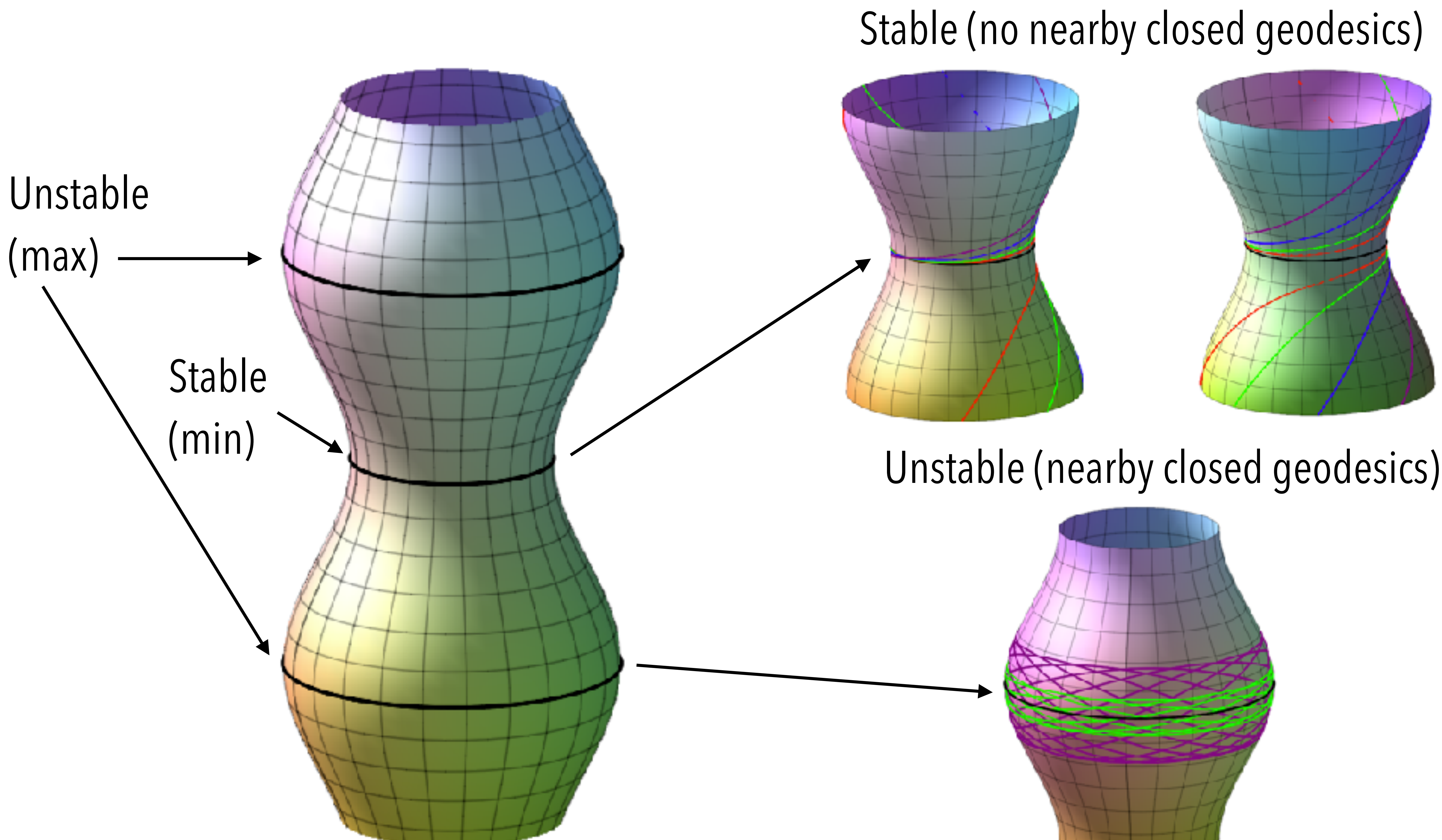

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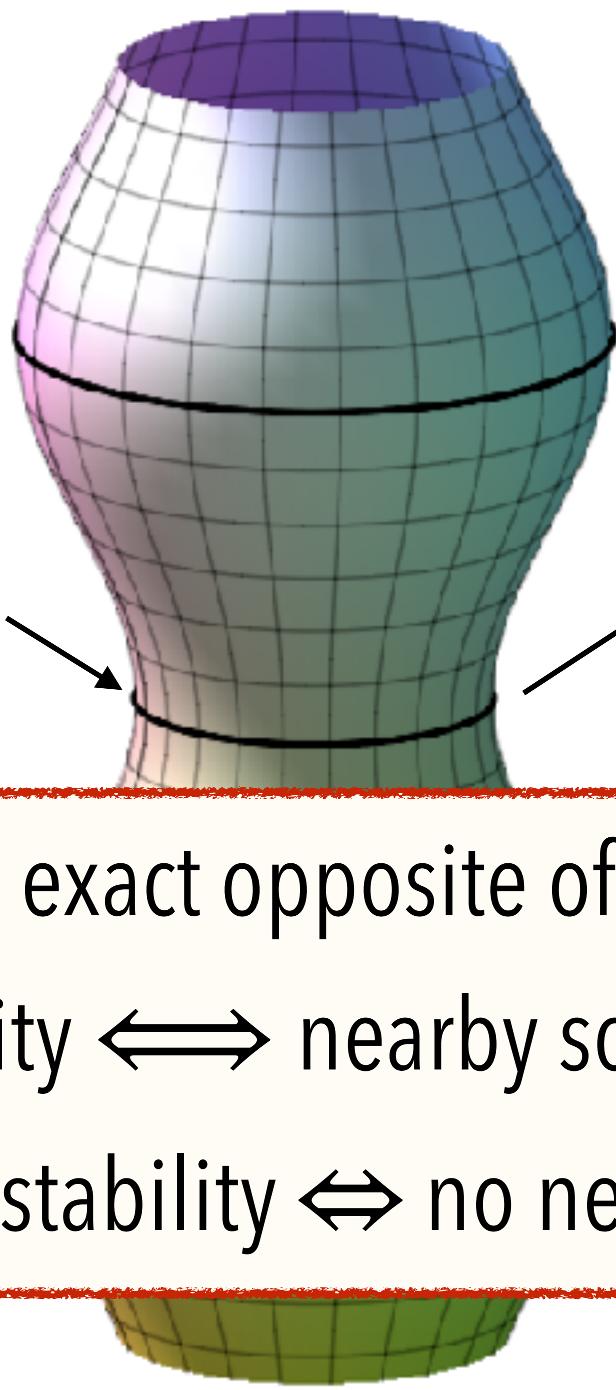
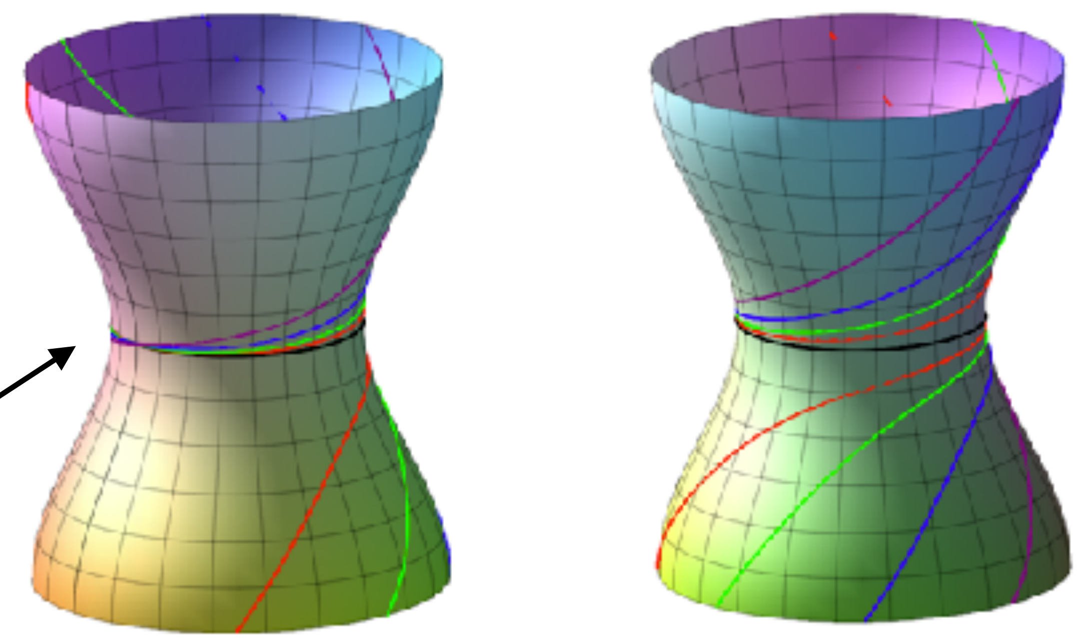
Unstable (max)

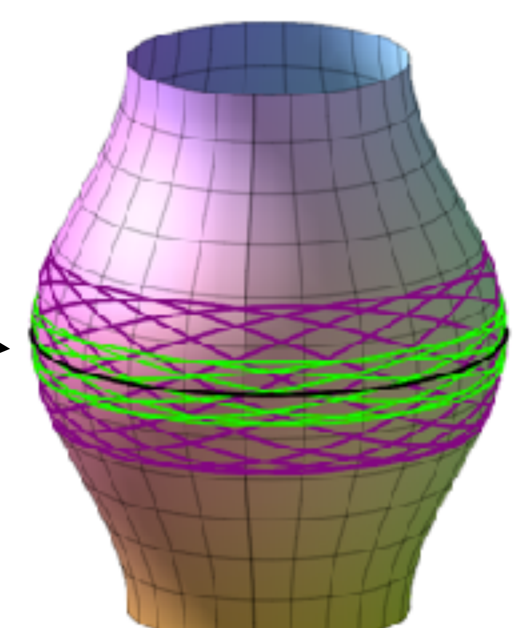
Stable (min)



Stable (no nearby closed geodesics)



Unstable (nearby closed geodesics)



Almost the exact opposite of ODE stability!

ODE stability $\Longleftrightarrow$ nearby solutions

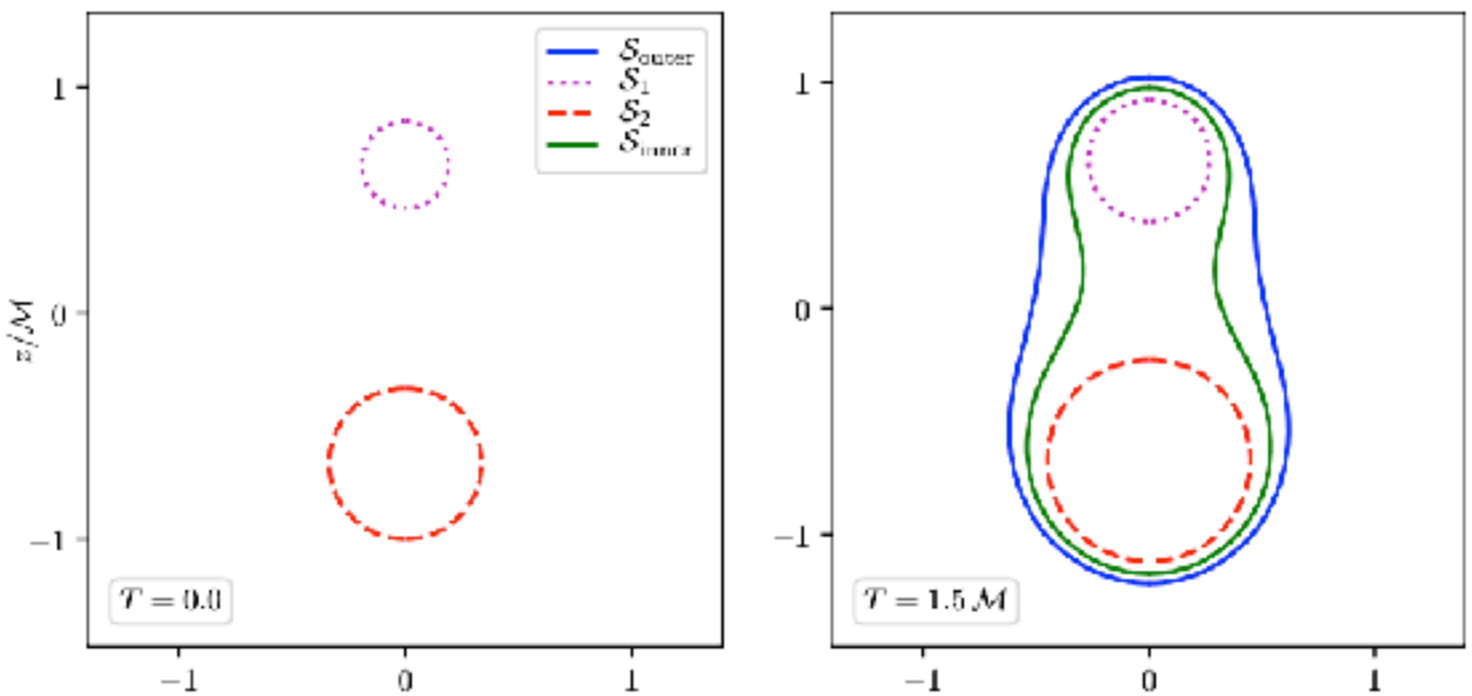
geometric stability $\Leftrightarrow$ no nearby solutions

MOTSs can have "exotic" properties

Self-intersecting marginally outer trapped surfaces

Daniel Pook-Kolb,[1,2] Ofek Birnholtz,[3] Badri Krishnan,[1,2] and Erik Schnetter[4,5,6]

Phys. Rev. D 100, 084044 (2019)



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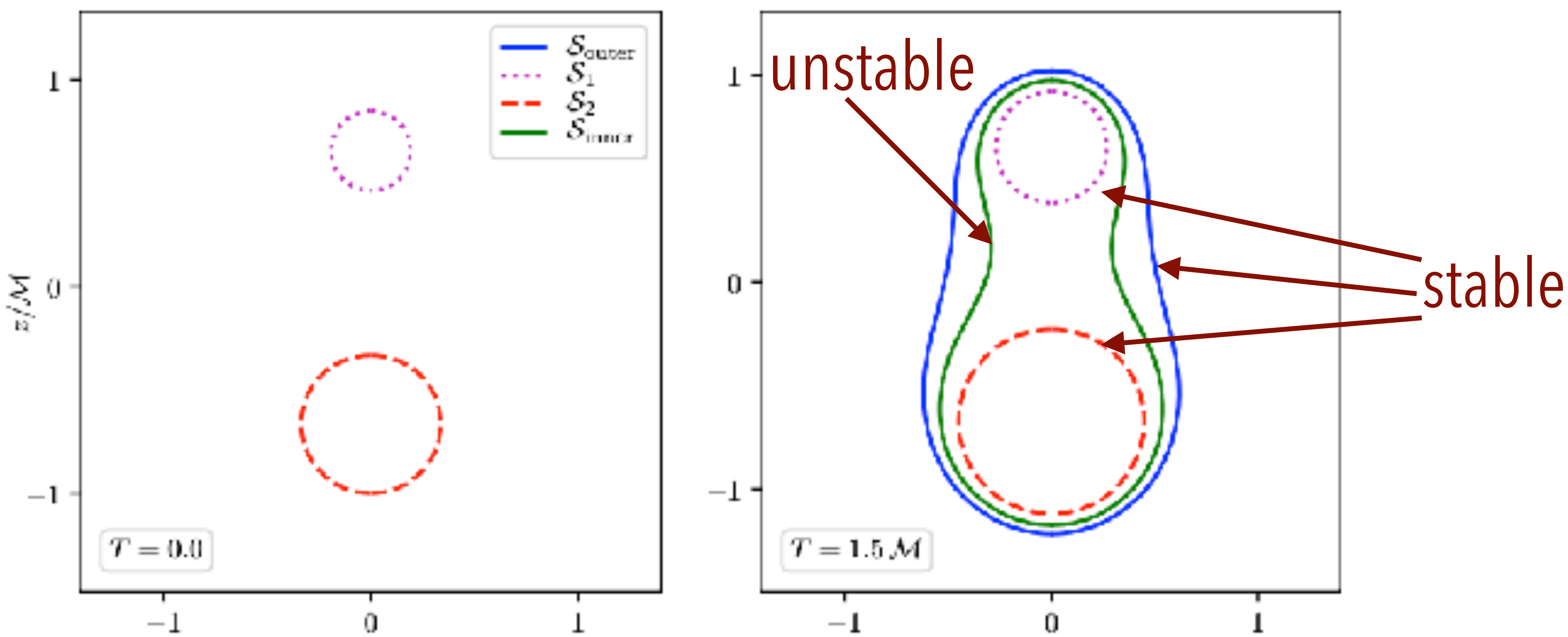

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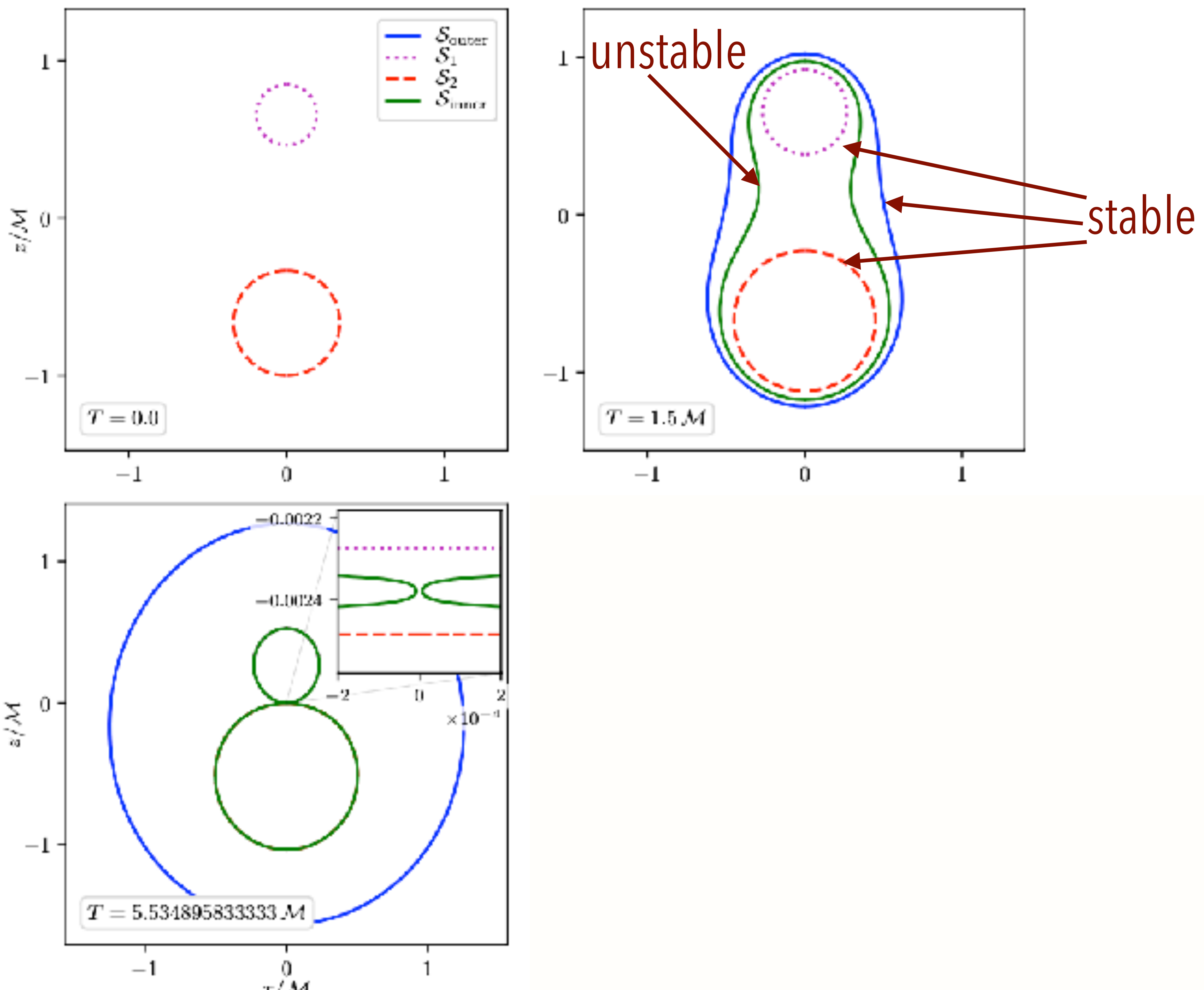

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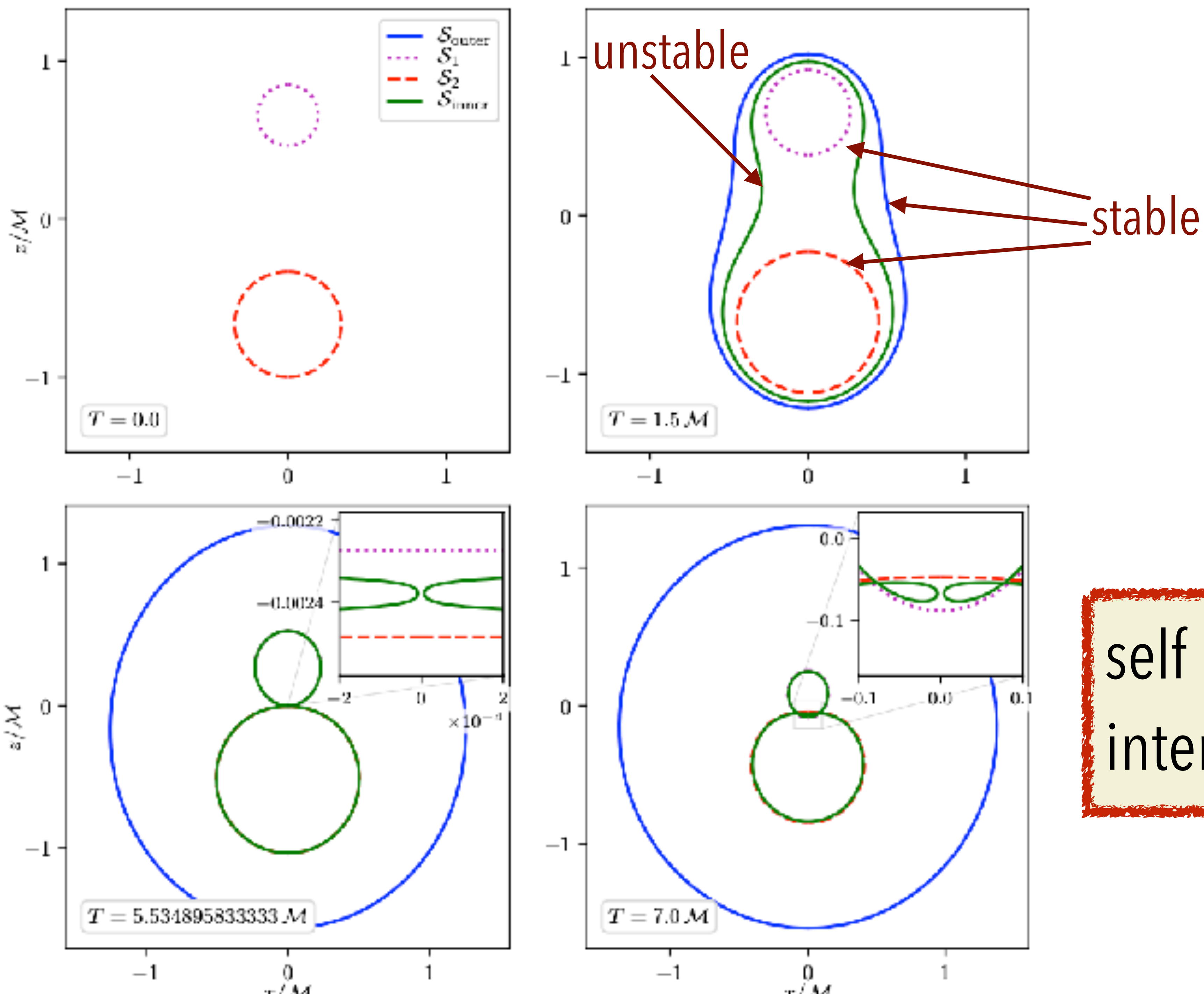
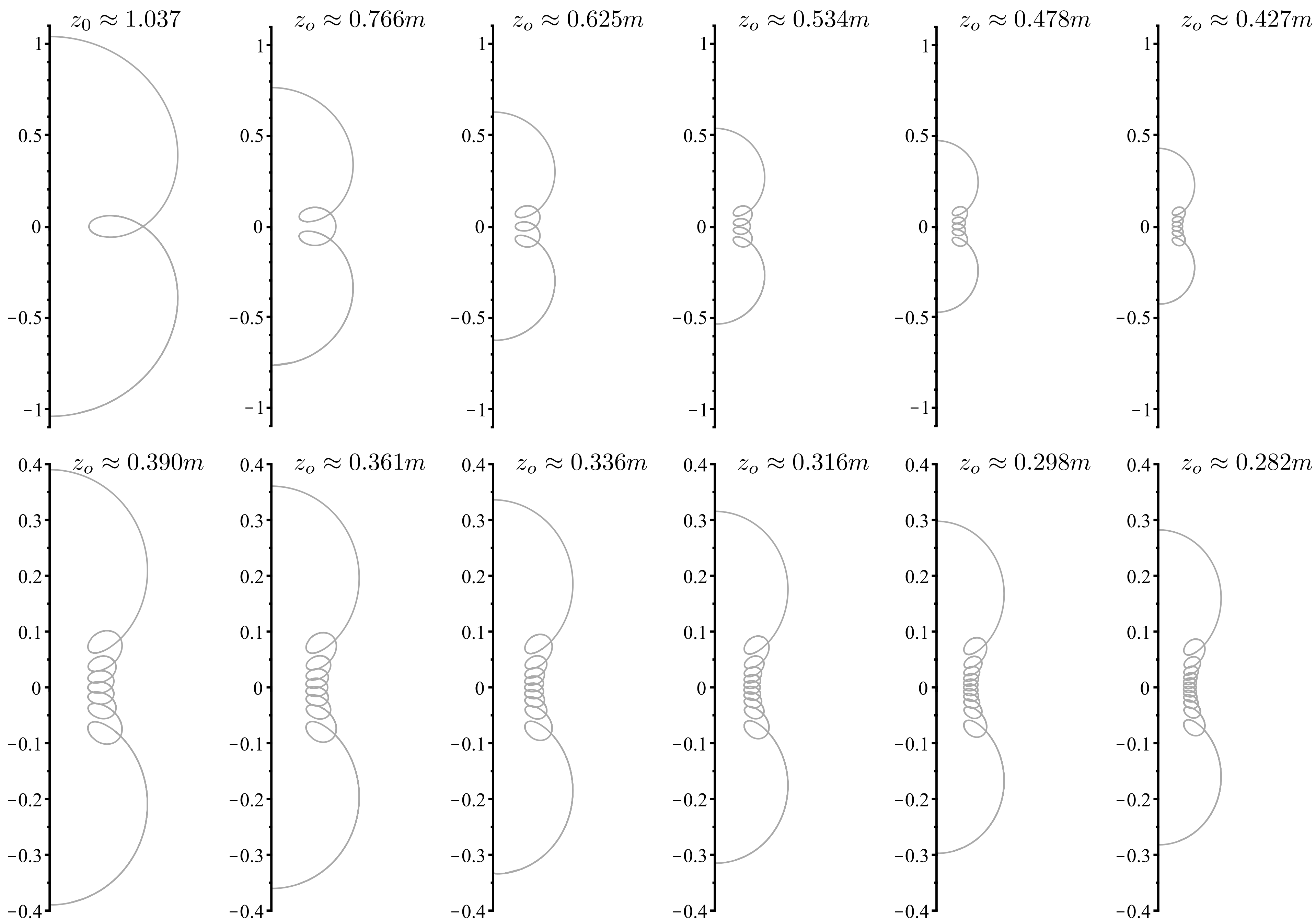


self intersections!

Most MOTS are not horizons/boundaries…

Schwarzschild



IB, R.Hennigar, S.Mondal (PRD 2020)

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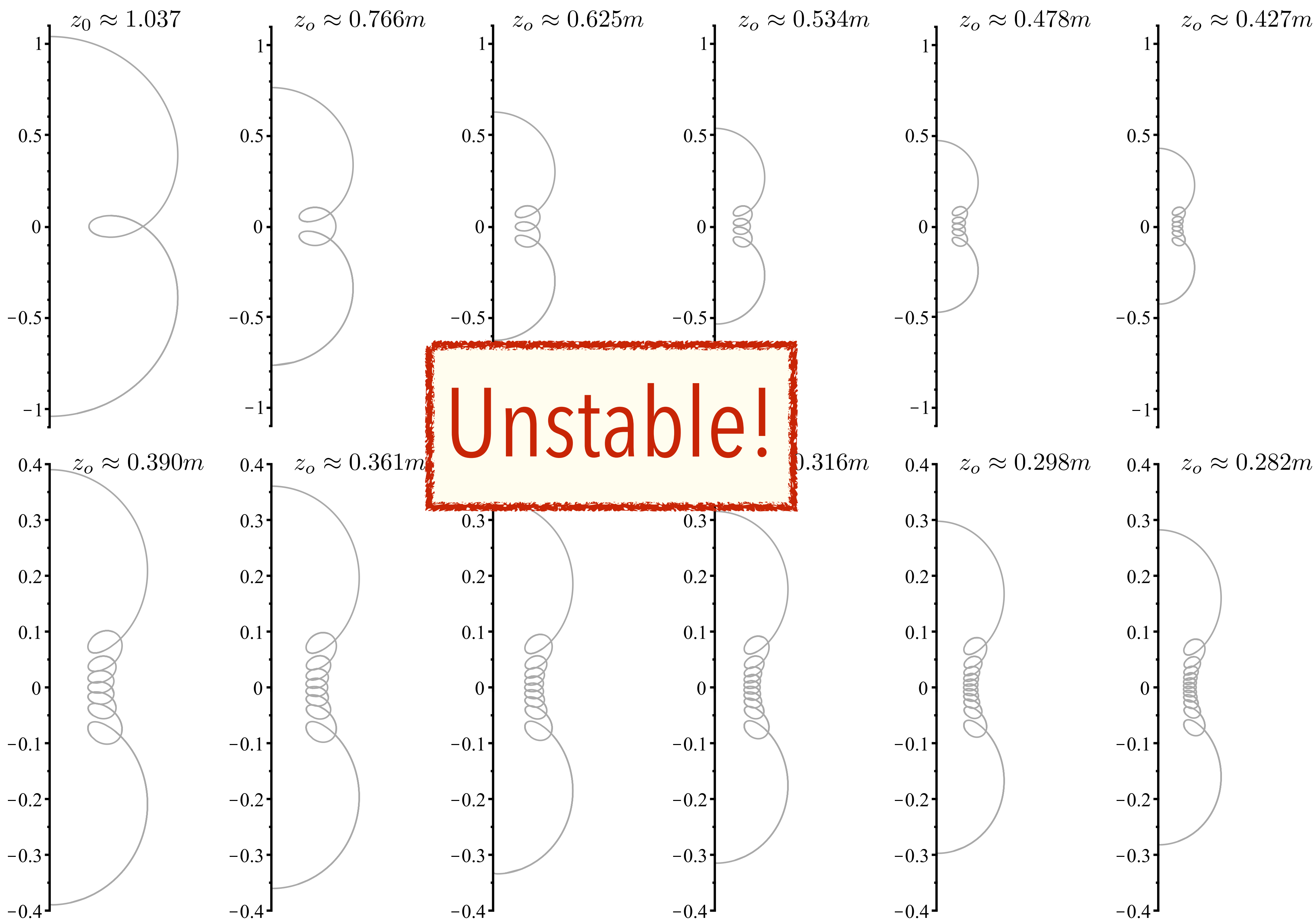
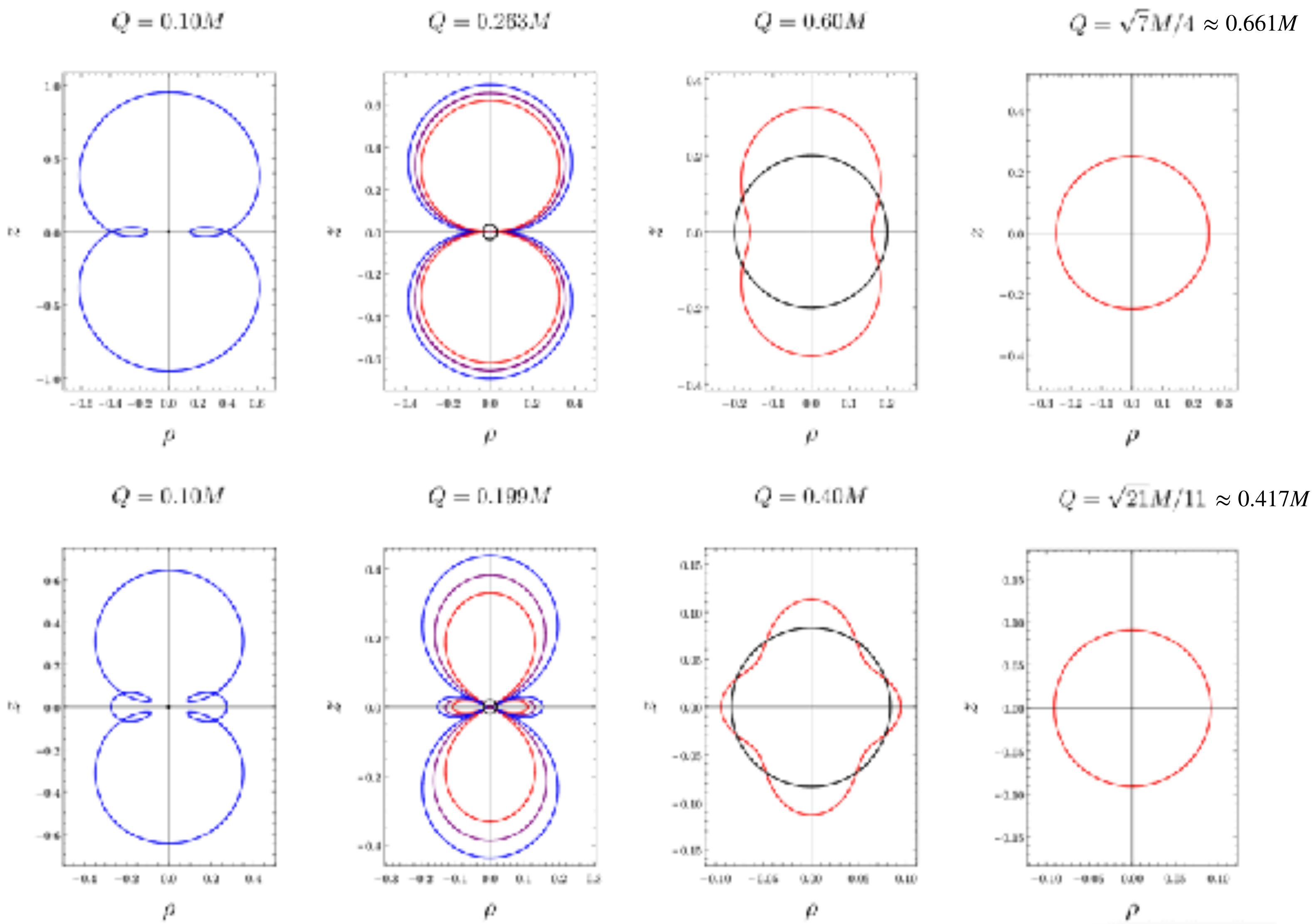


Unstable!

IB, R.Hennigar, S.Mondal (PRD 2020)

Reissner-Nordström (around inner horizon)



R.Hennigar, B.Chan/Sievers, L.Newhook, IB PRD 2022

Existence is robust across coordinate systems and solutions
(B.Sievers, R.Hennigar, H.Kunduri, S.Muth, L.Newhook, IB)

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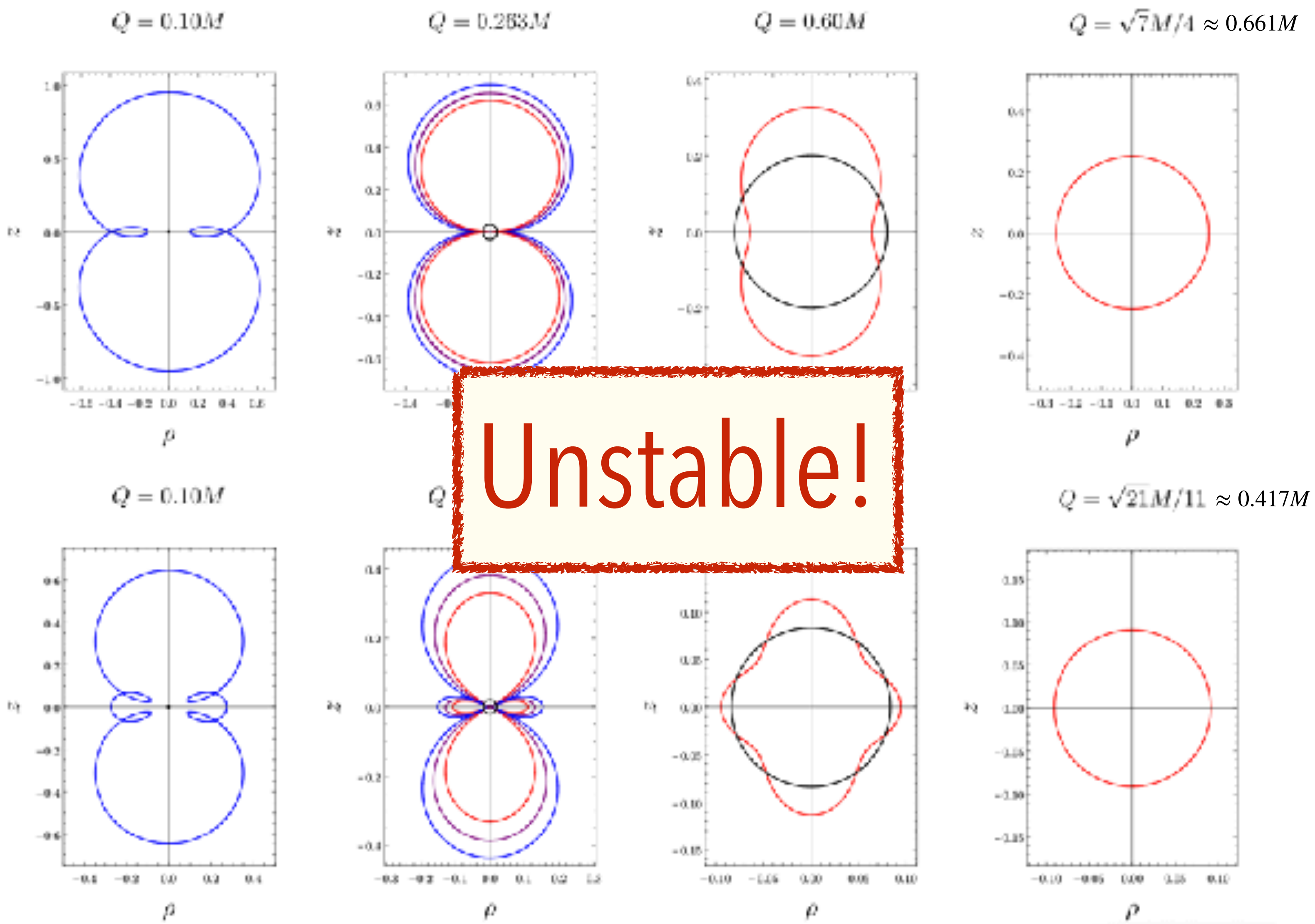

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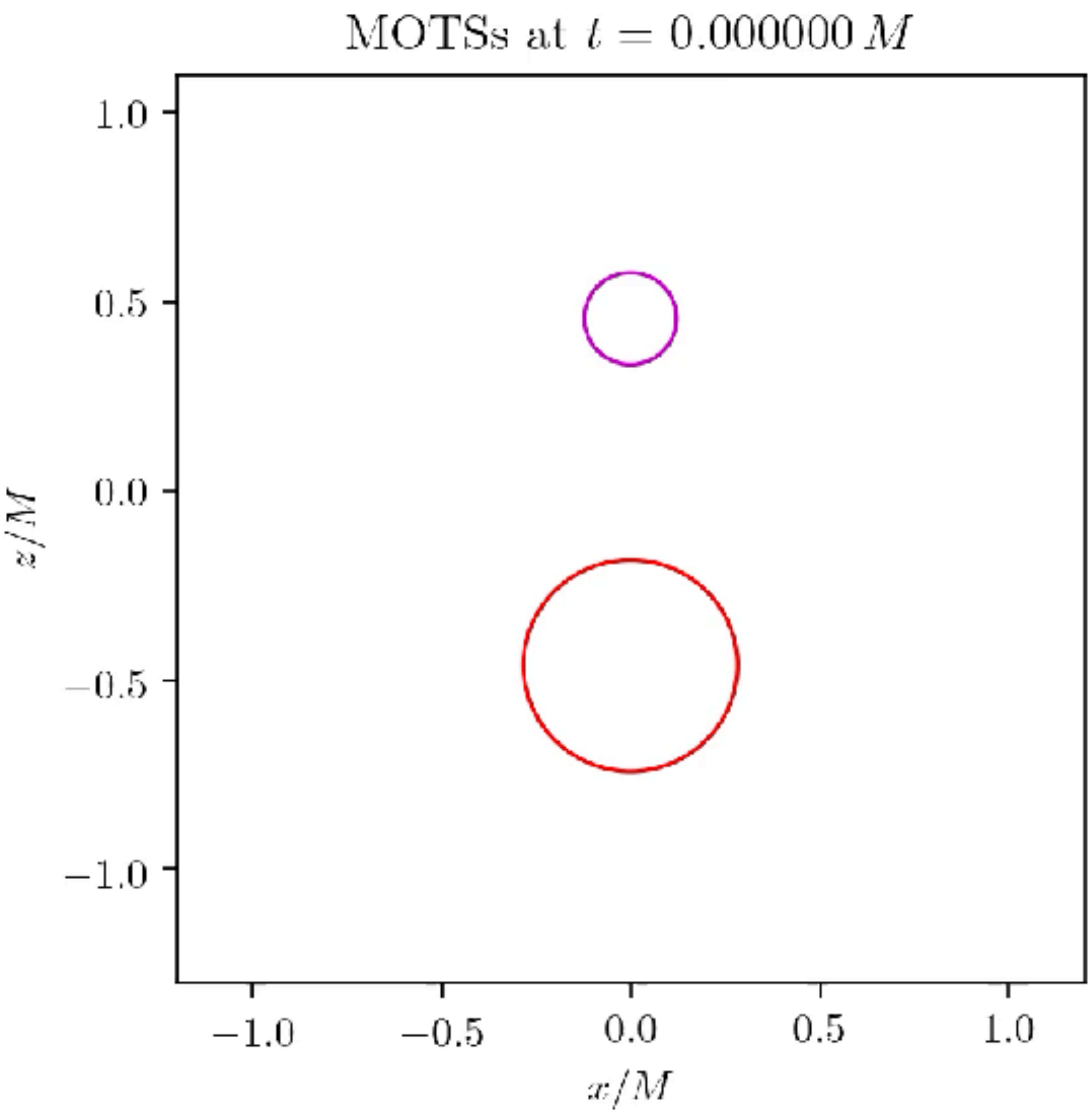
Mergers are complicated…

Pook-Kolb, Hennigar, Booth PRL 2021 (Summary)
Pook-Kolb, Hennigar, Booth PRD 2021 (Theory)
Booth, Hennigar, Pook-Kolb PRD 2021 (Numerical Results)



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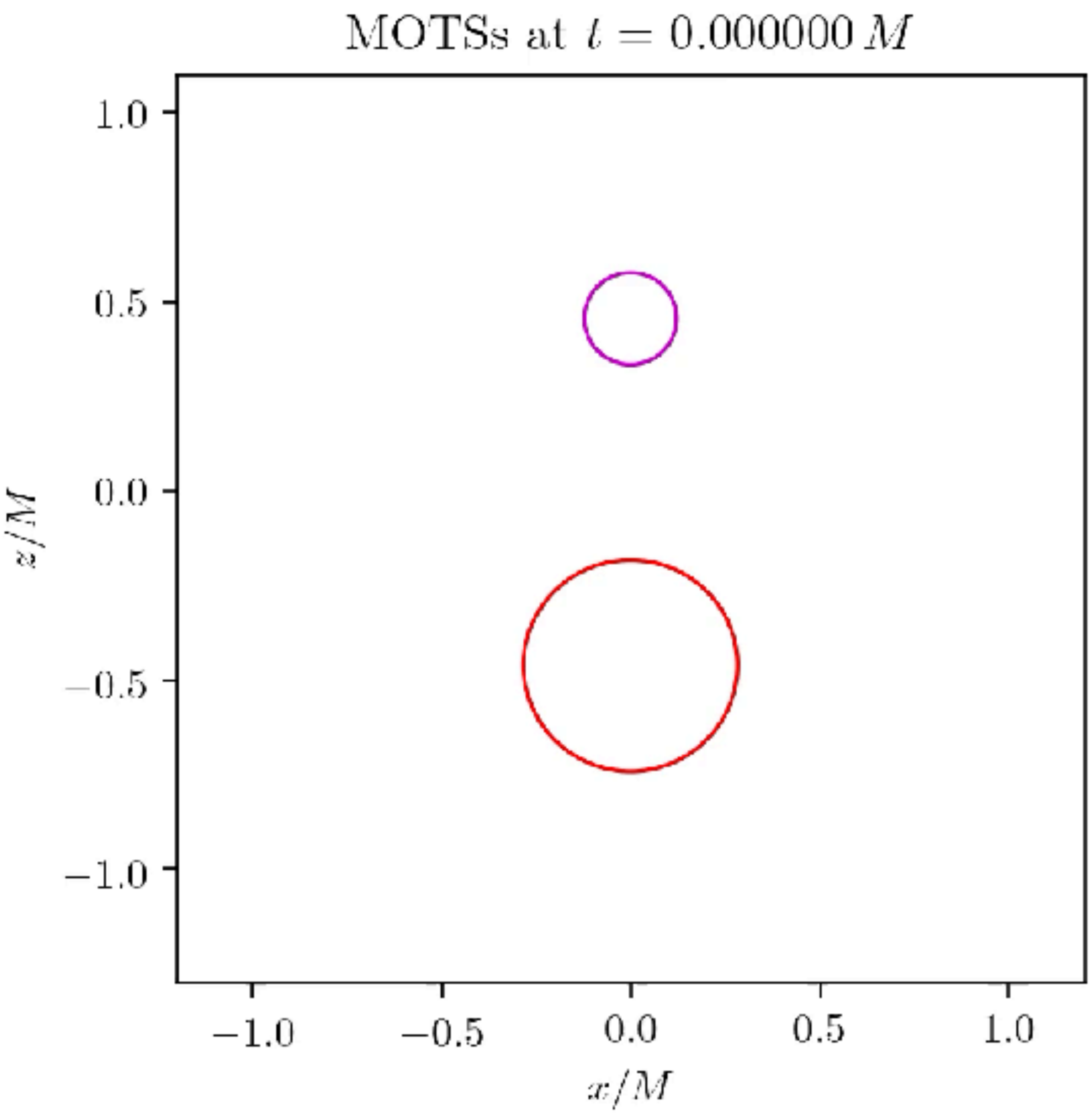
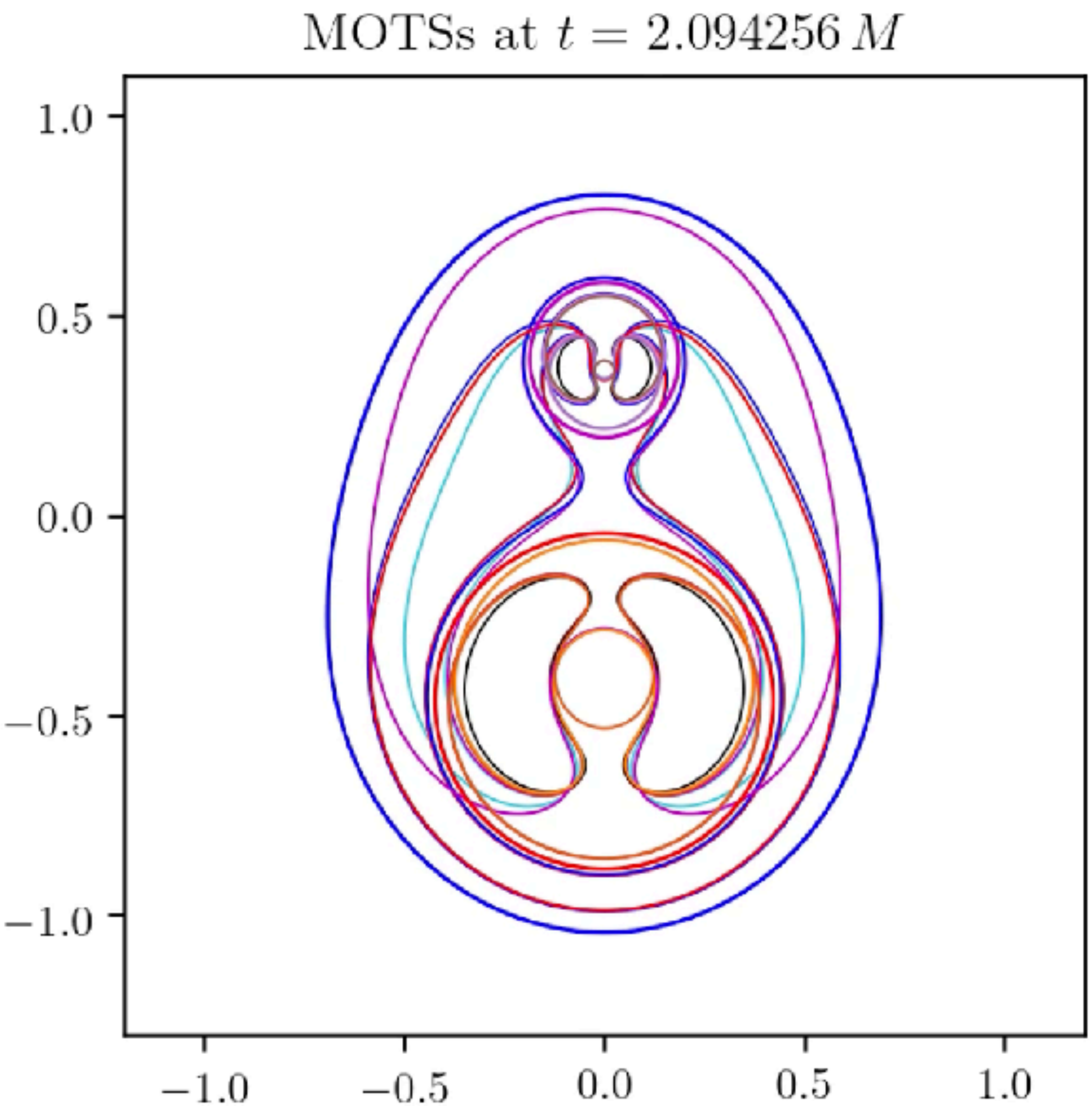
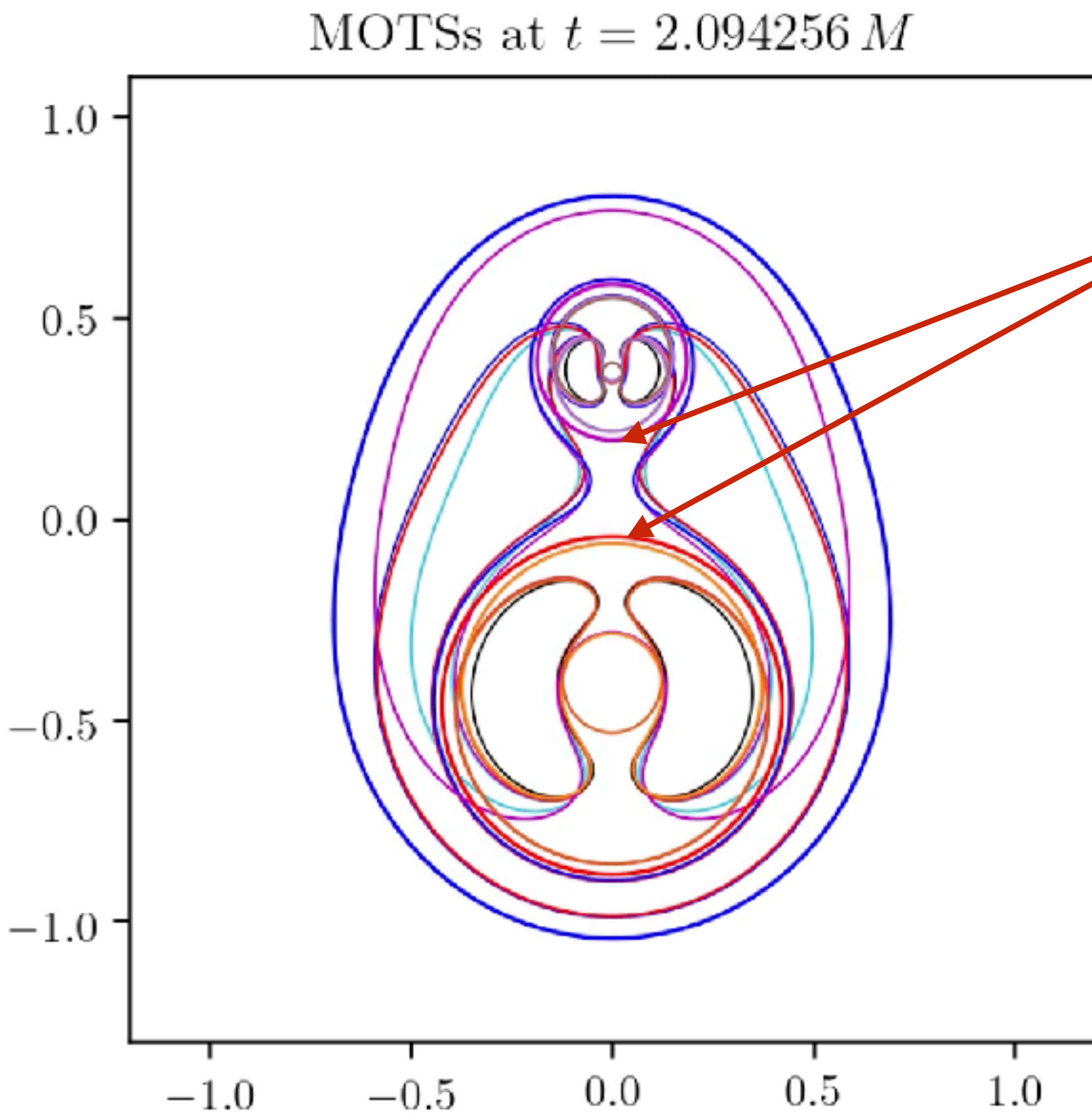


During a merger



The original and final apparent horizons are stable.

During a merger



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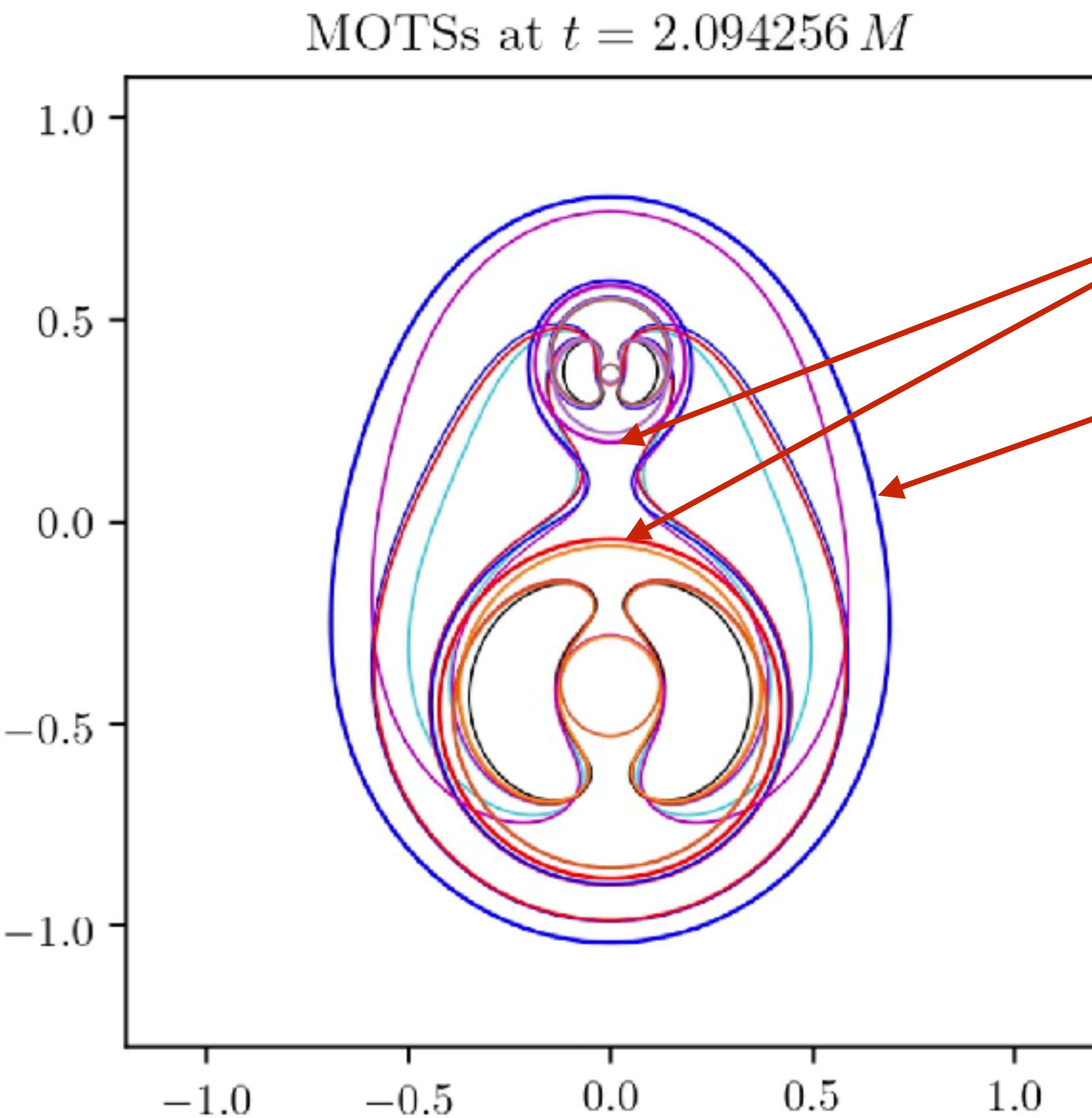

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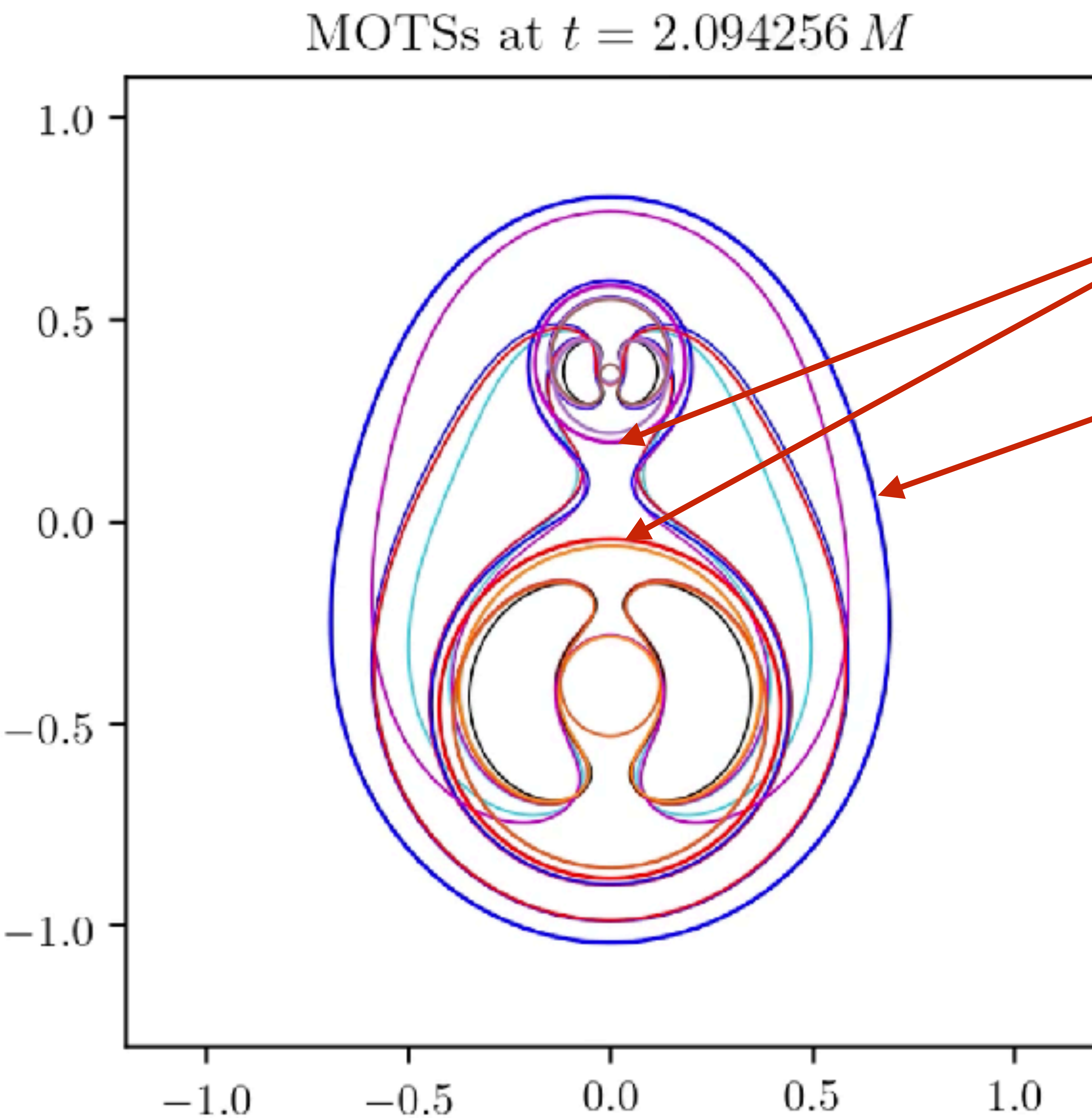

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All other MOTS are unstable.

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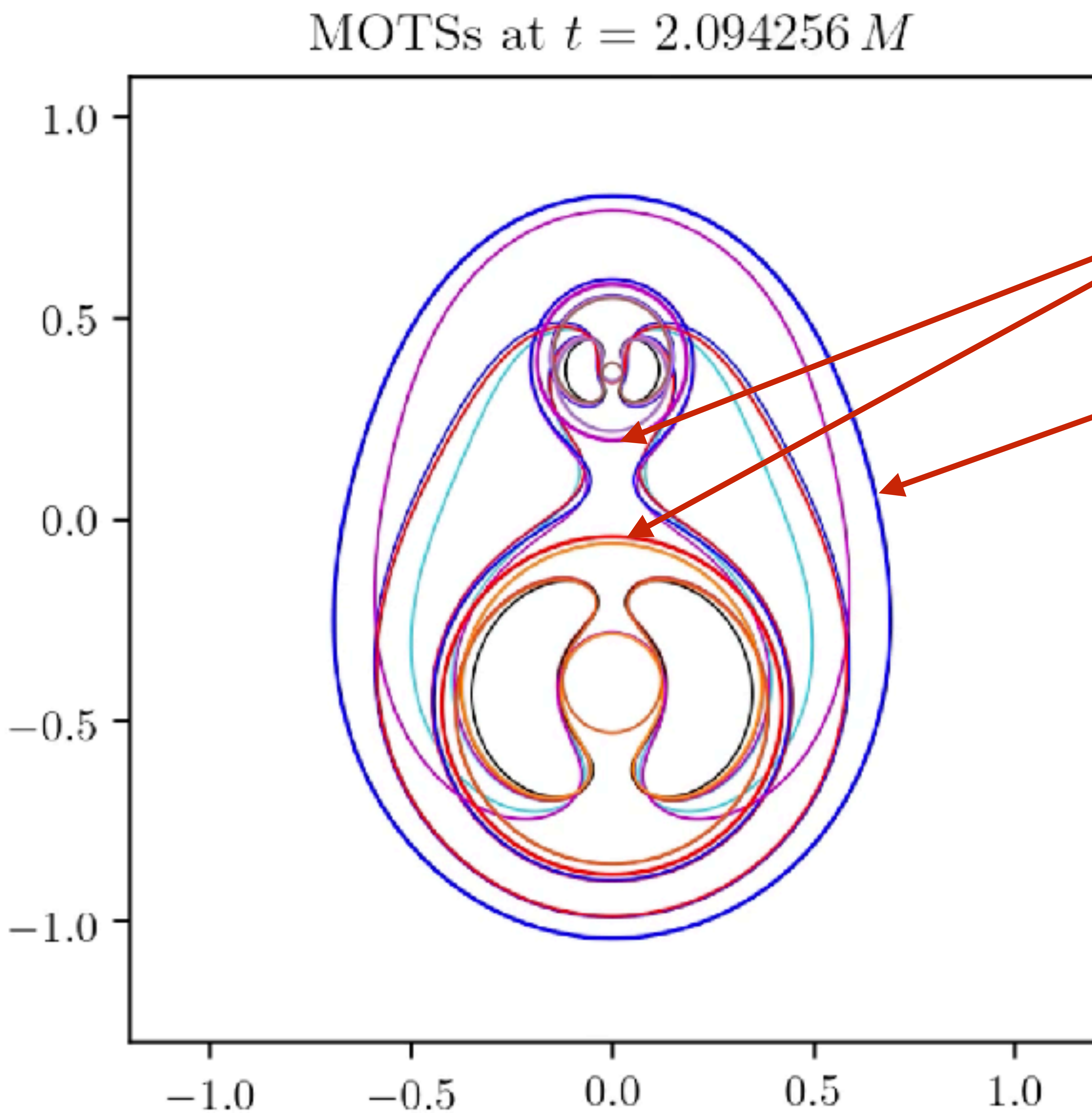

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All other MOTS are unstable.

Most MOTS do not bound trapped regions

Stability Operator and Evolution/Deformation

Problem:

Stability Operator and Evolution/Deformation

Problem: δ_α = one-parameter deformation of the geometry (h_{ij}, K_{ij}) of a slice Σ_t

Stability Operator and Evolution/Deformation

examples: time evolution, q in RN, Λ is dS, anything else

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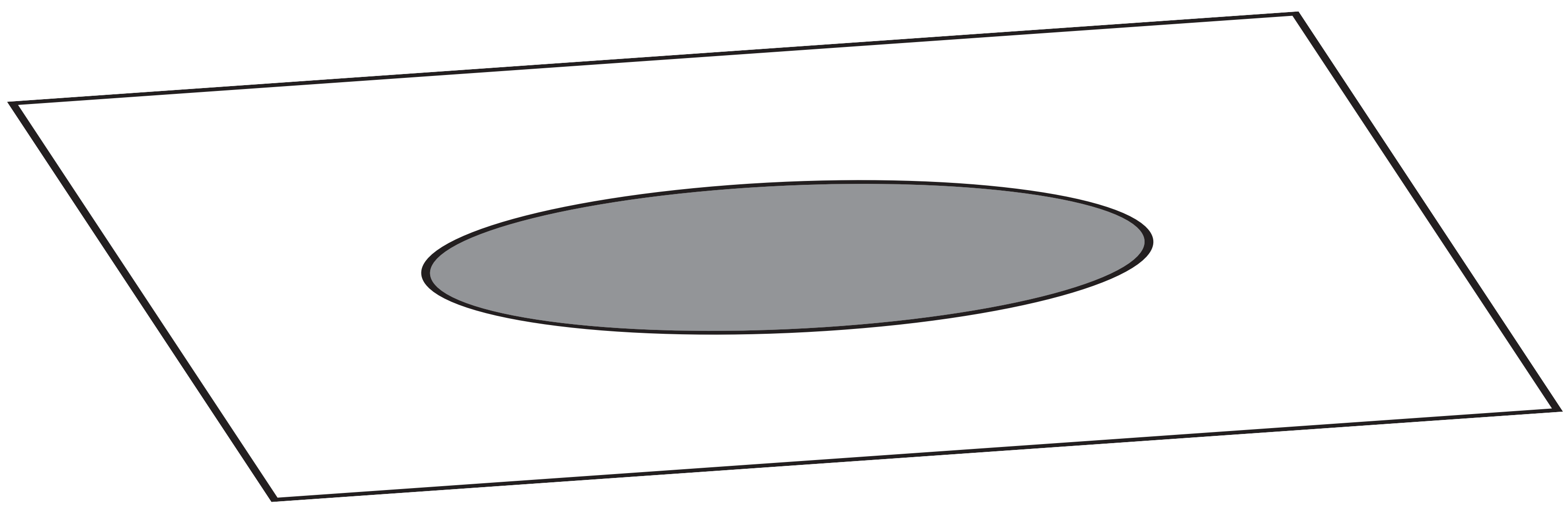

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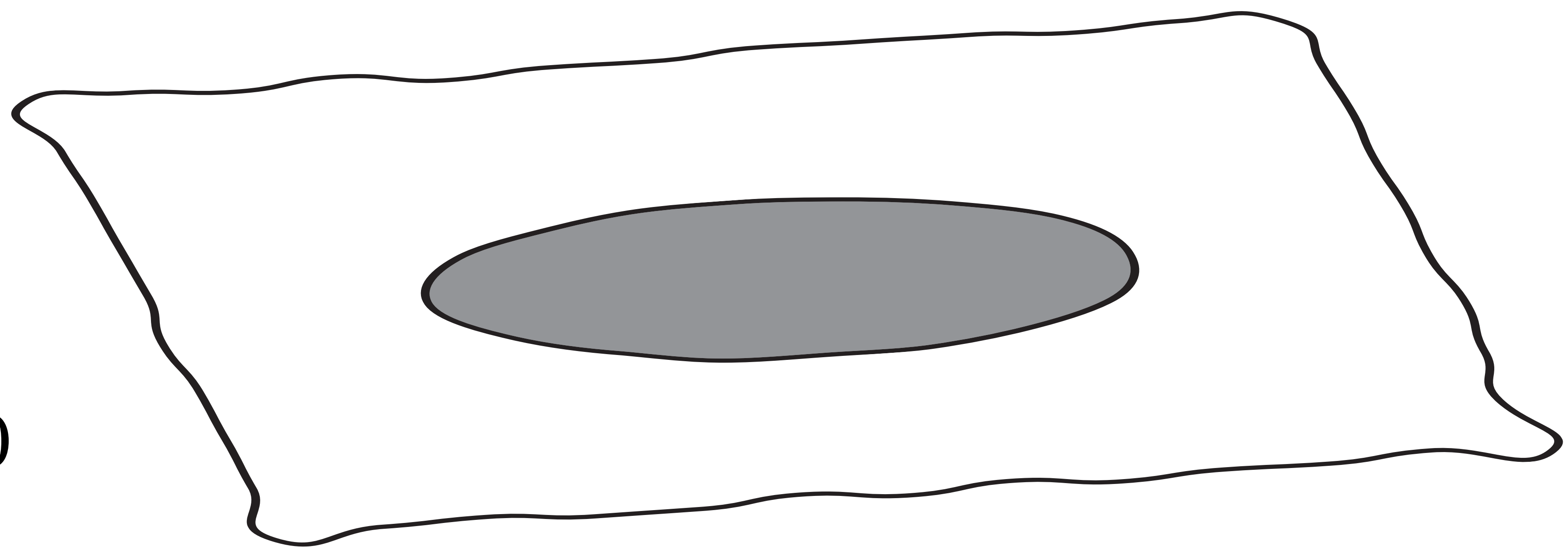
distorted

(h_{ij}, K_{ij})

$\theta_+\Big|_S \neq 0$



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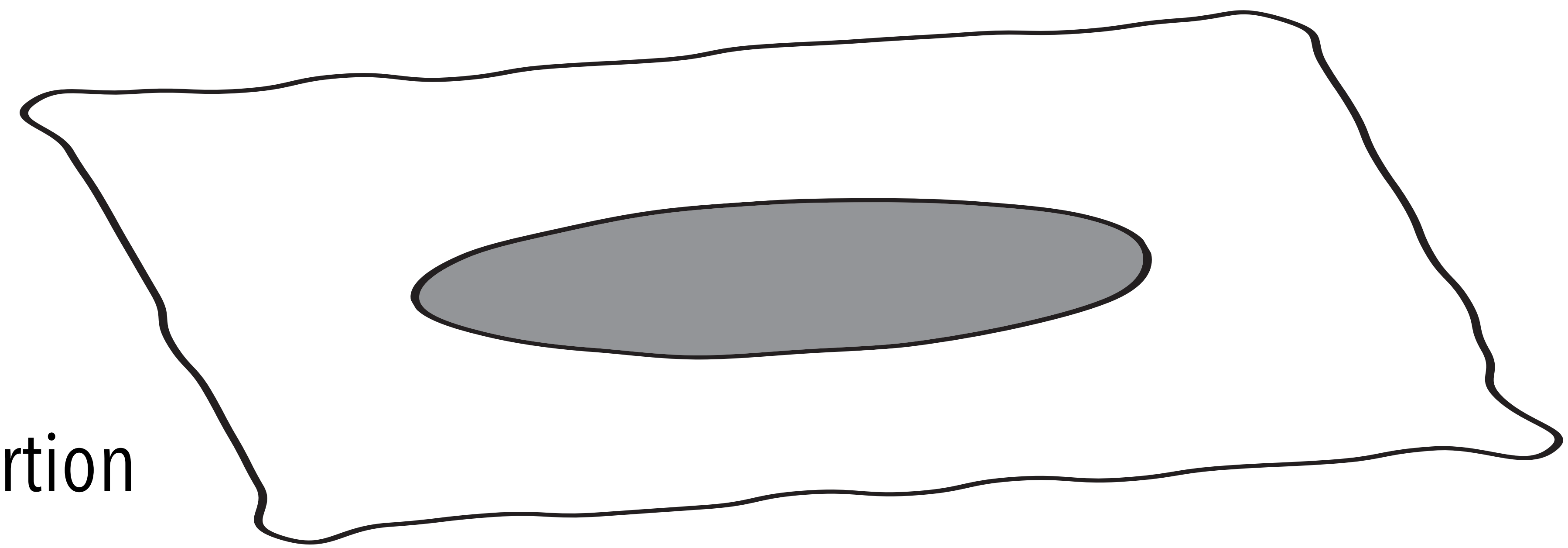

small distortion

of $S \rightarrow \theta_+ = 0$

Stability Operator and Evolution/Deformation

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total change in expansion

change due to (h, K) deformation

Solution: Look for a ψ for which: $0 = \delta\theta_+ = \delta_\alpha\theta_+ + \delta_{\psi N}\theta_+$

change from S perturbation

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$$\Longleftrightarrow 0 = \delta_\alpha\theta_+ + L_S\psi$$

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ψ is unique if L_S is invertible $\iff$ L_S has no vanishing eigenvalues

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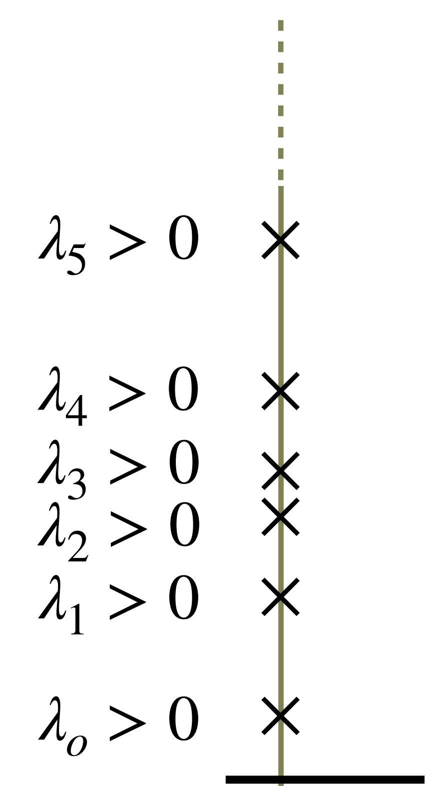
Strict stability ($\lambda_o > 0$) is sufficient but not necessary for uniqueness



Second application of stability operator

Stability Operator and Evolution/Deformation

examples: time evolution, q in RN, Λ is dS, anything else

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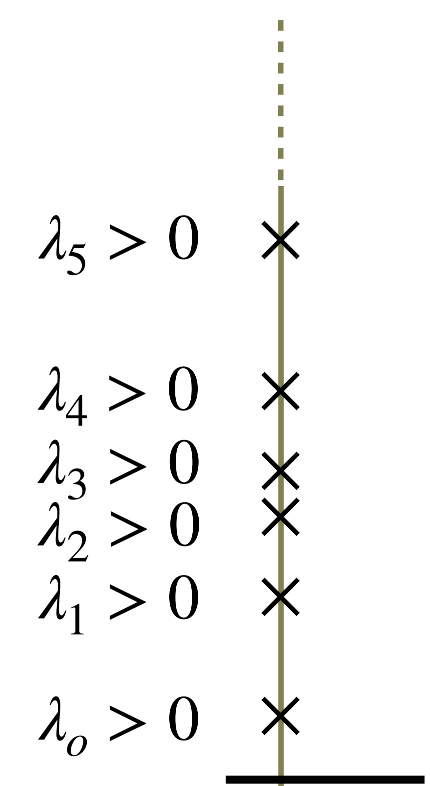
Strict stability ($\lambda_o > 0$) is sufficient but not necessary for uniqueness

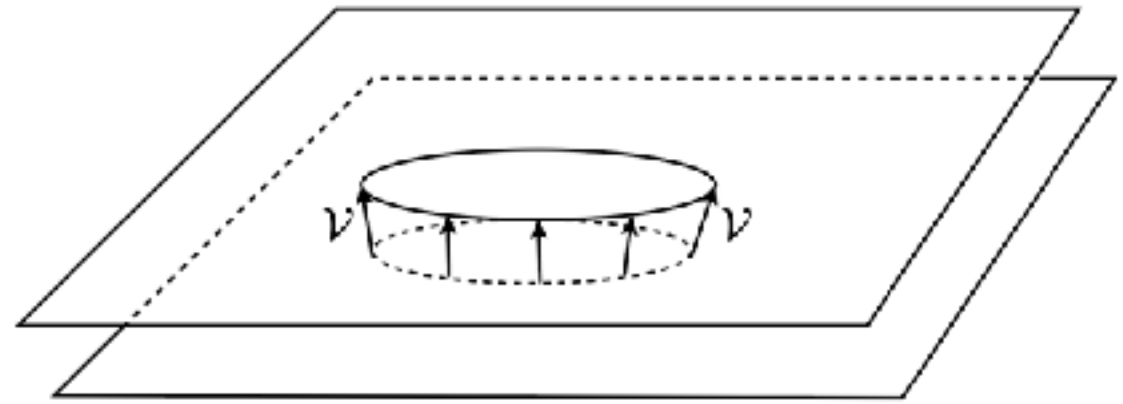
Non-uniqueness: Possible when an eigenvalue vanishes.



Example: time evolution

- Time evolution of a MOTS:

$$0 = \delta_t \theta_+ + \delta_{\psi N} \theta_+$$
$$\iff \psi = - L^{-1}(l_+, N)[\delta_t \theta_+]$$



$$\mathscr{V} = t + \psi N$$

Andersson Mars Simon, PRL 05

Example: time evolution

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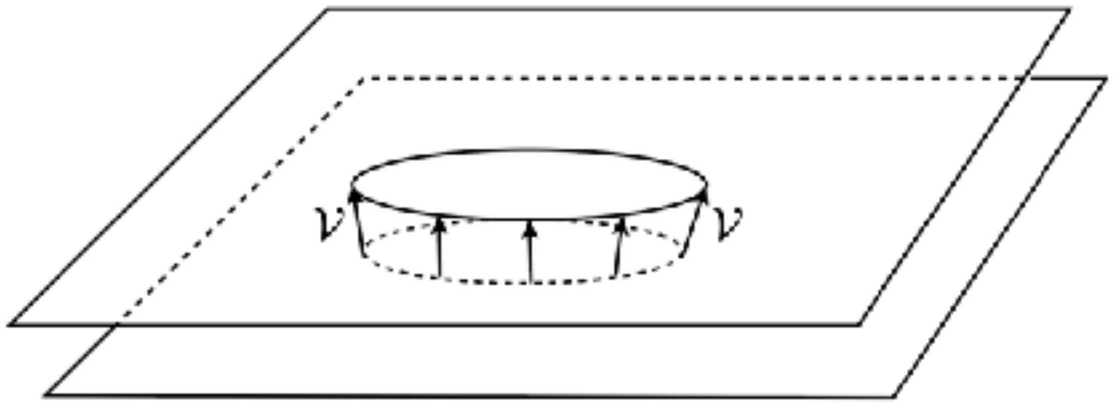
$$\Longleftrightarrow \psi = - L^{-1}(l_+, N)[\delta_t \theta_+]$$



$\mathscr{V} = t + \psi N$

- Unique if there are no vanishing eigenvalues

Andersson Mars Simon, PRL 05

Example: time evolution

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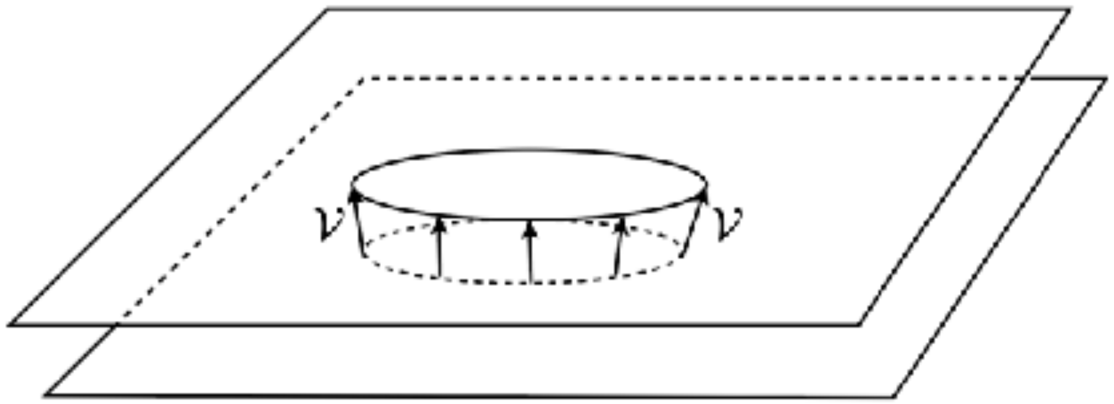

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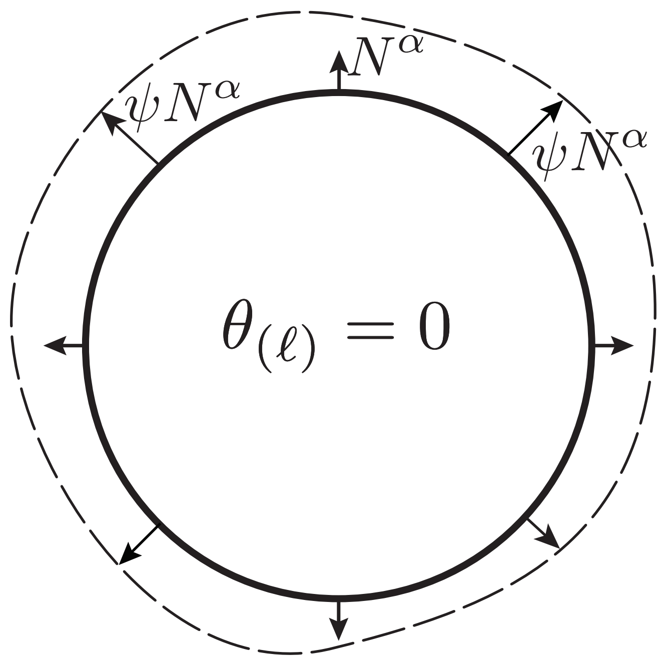

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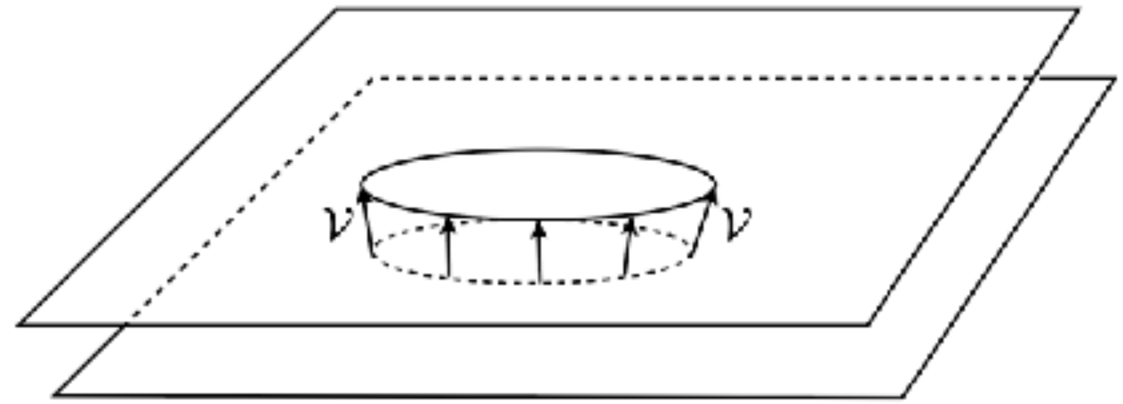

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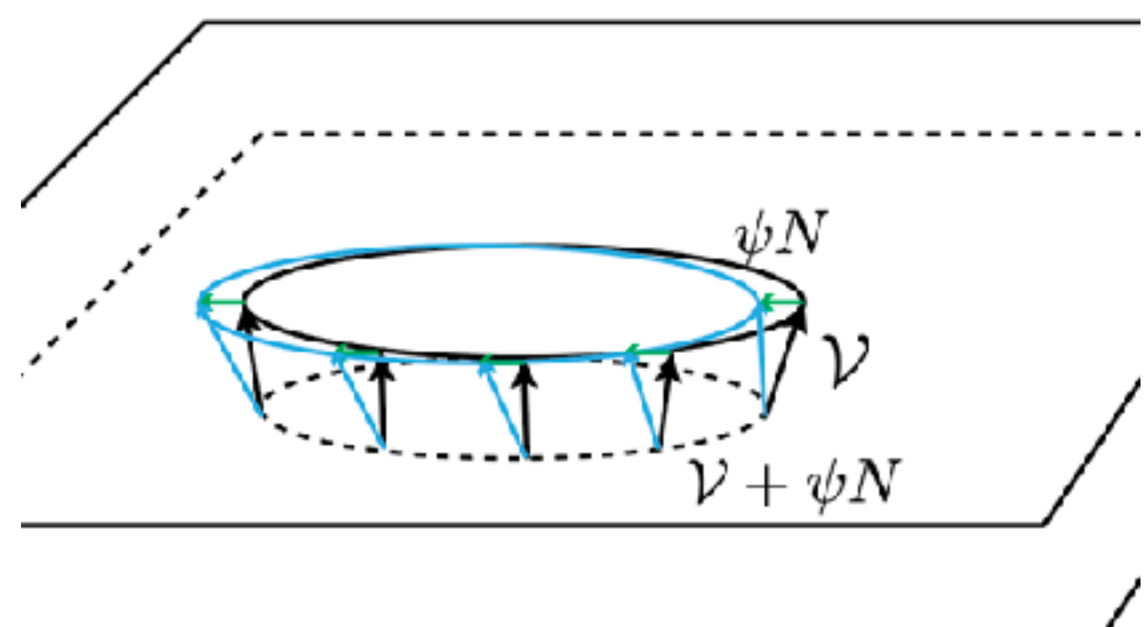
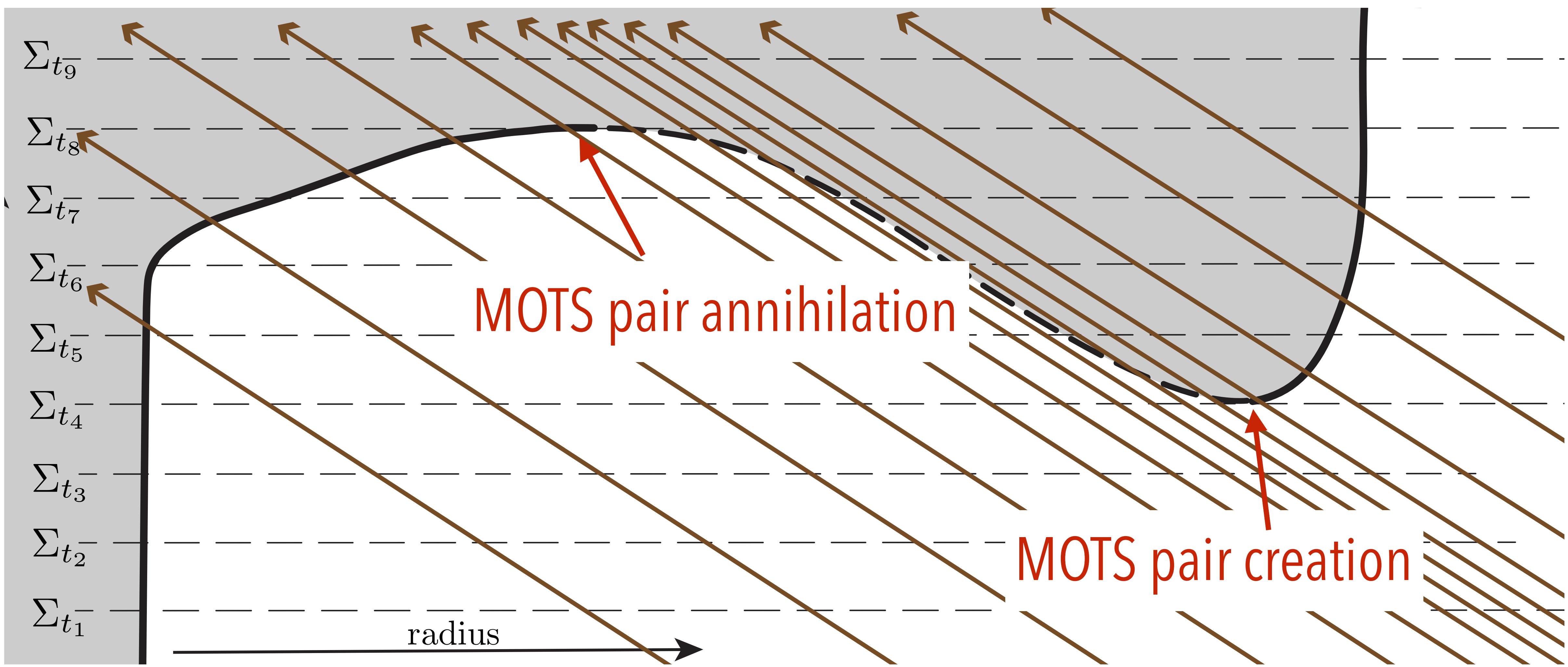
- Then: $\delta_{(\mathscr{V}+\psi_0 N)}\theta_+ = 0$ so: $\mathscr{V} + \psi_0 N$ is an equally good "time-evolution" vector



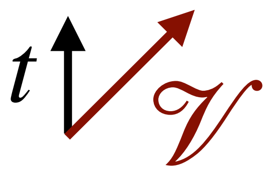
Andersson Mars Simon, PRL 05

MOTS time evolution - an example

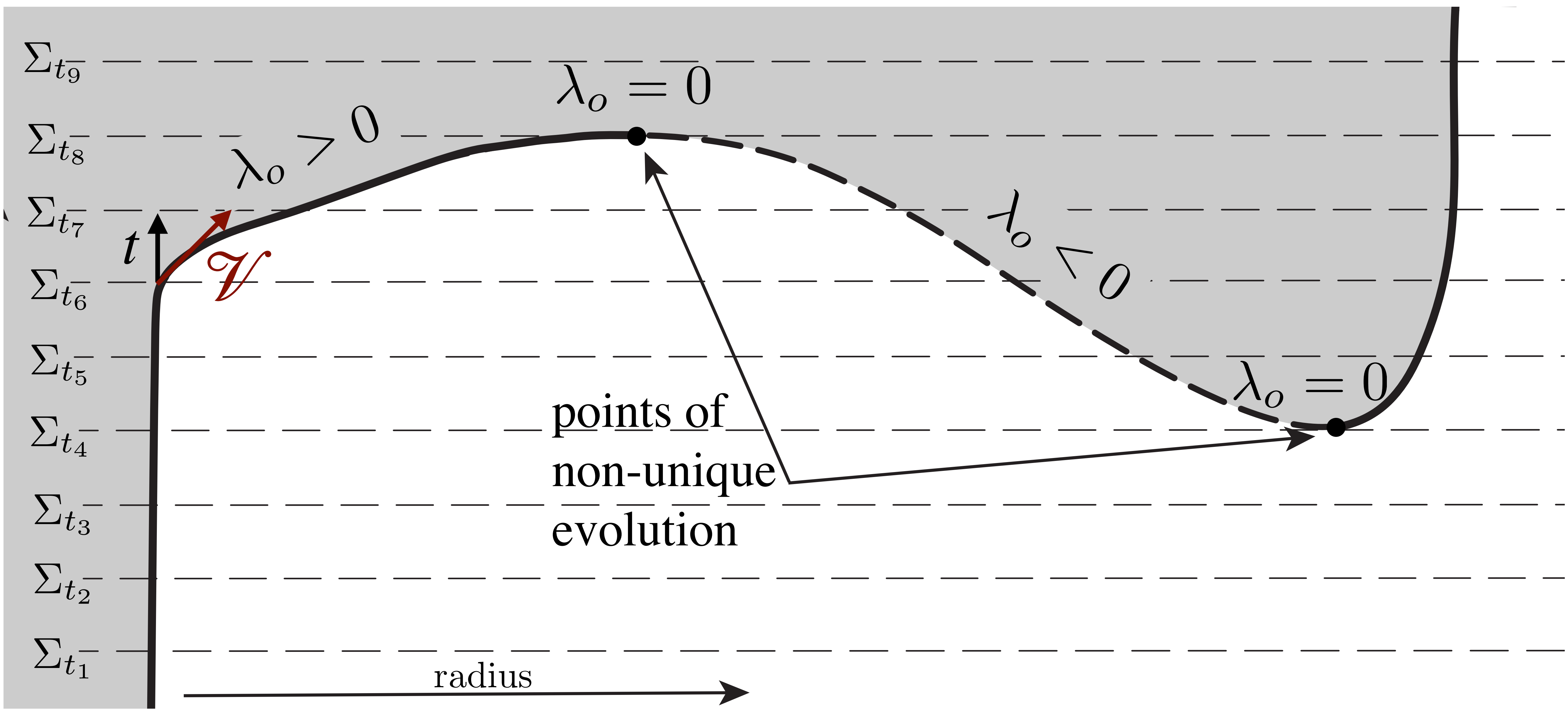


Booth, Brits, Gonzalez, Van Den Broeck
CQG 2006

MOTS evolution and the stability operator



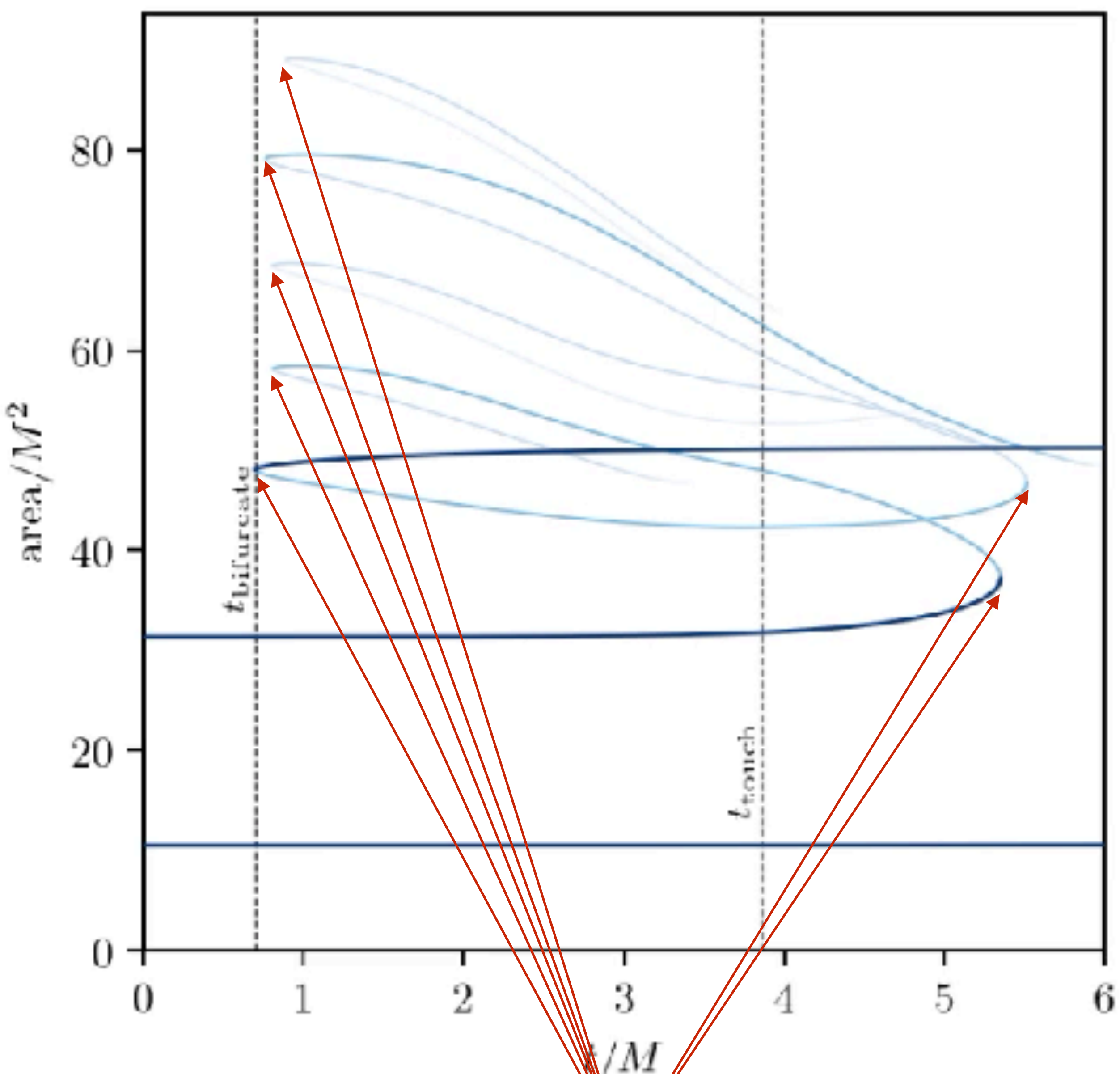
MOTS evolution and the stability operator



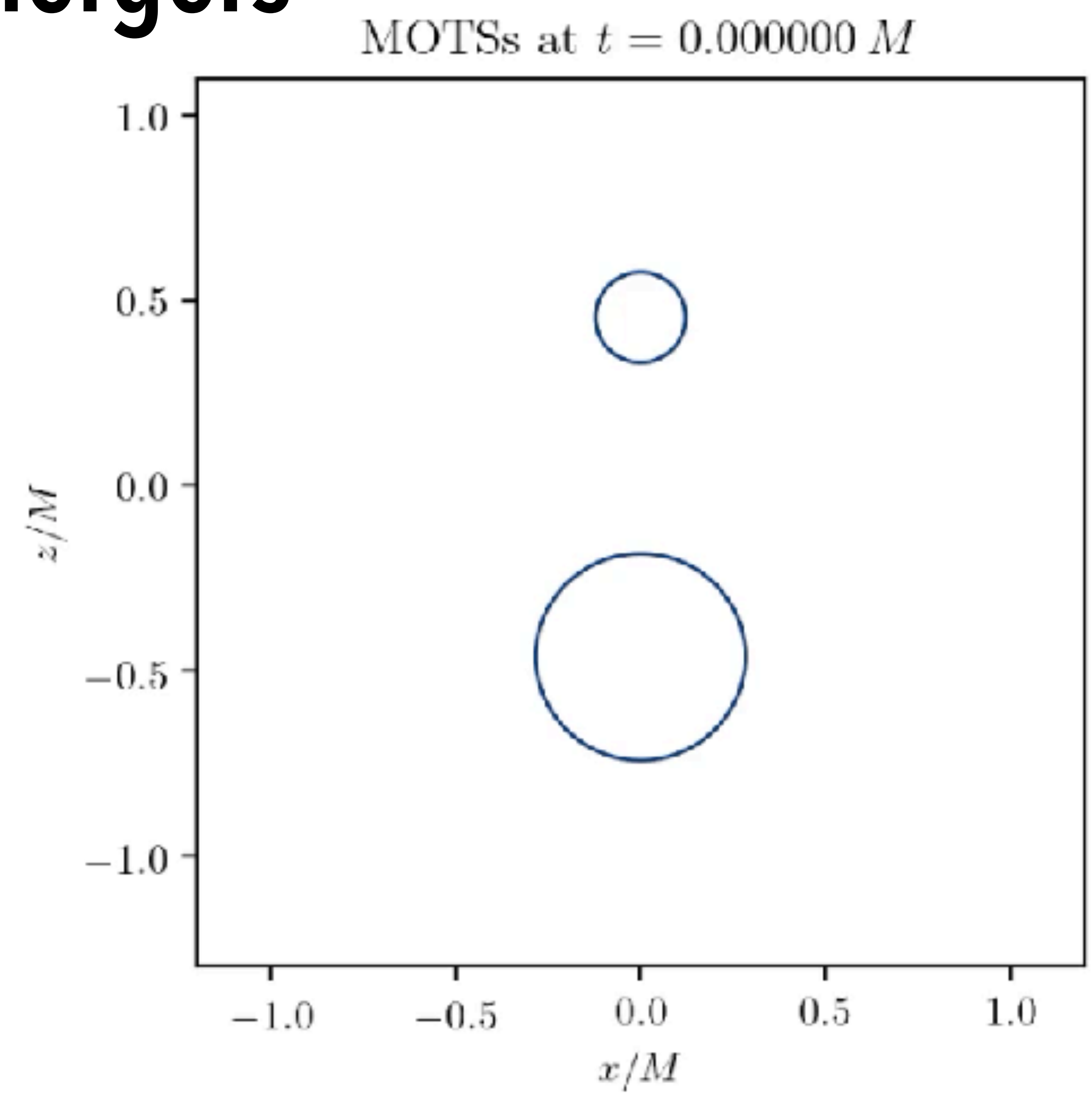
Numerical experiments:
horizon dynamics

Pook-Kolb, Hennigar, Booth PRL 2021 (Summary)
Pook-Kolb, Hennigar, Booth PRD 2021 (Theory)
Booth, Hennigar, Pook-Kolb PRD 2021 (Numerical Results)

MOTS during black hole mergers



L_Σ has a vanishing eigenvalue at creation and annihilation points



Black = stable MOTS = apparent horizons

Blue = unstable MOTS

Fading = lost for numerical reasons

(Pook-Kolb, Booth, Hennigar 2021)

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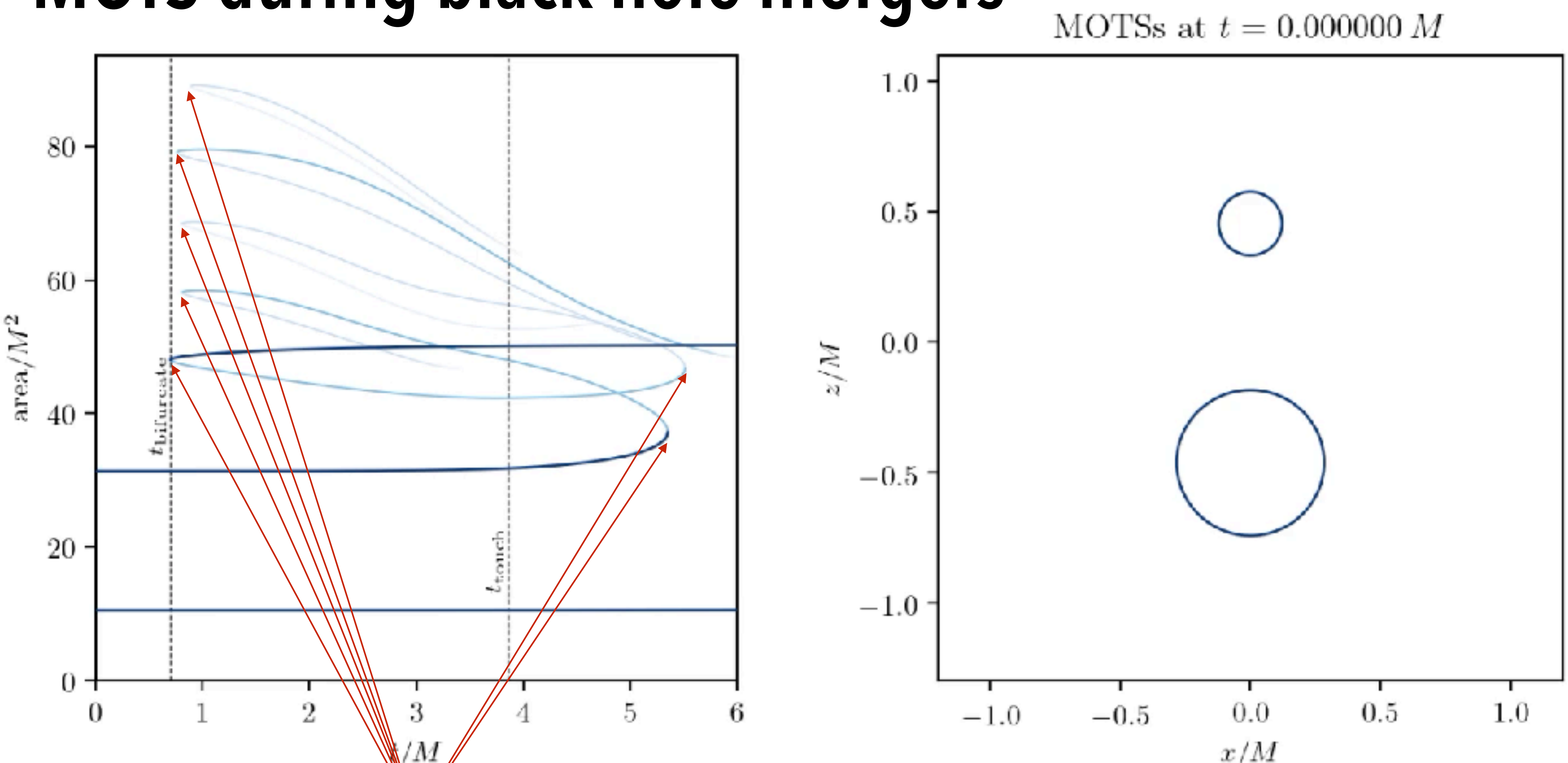

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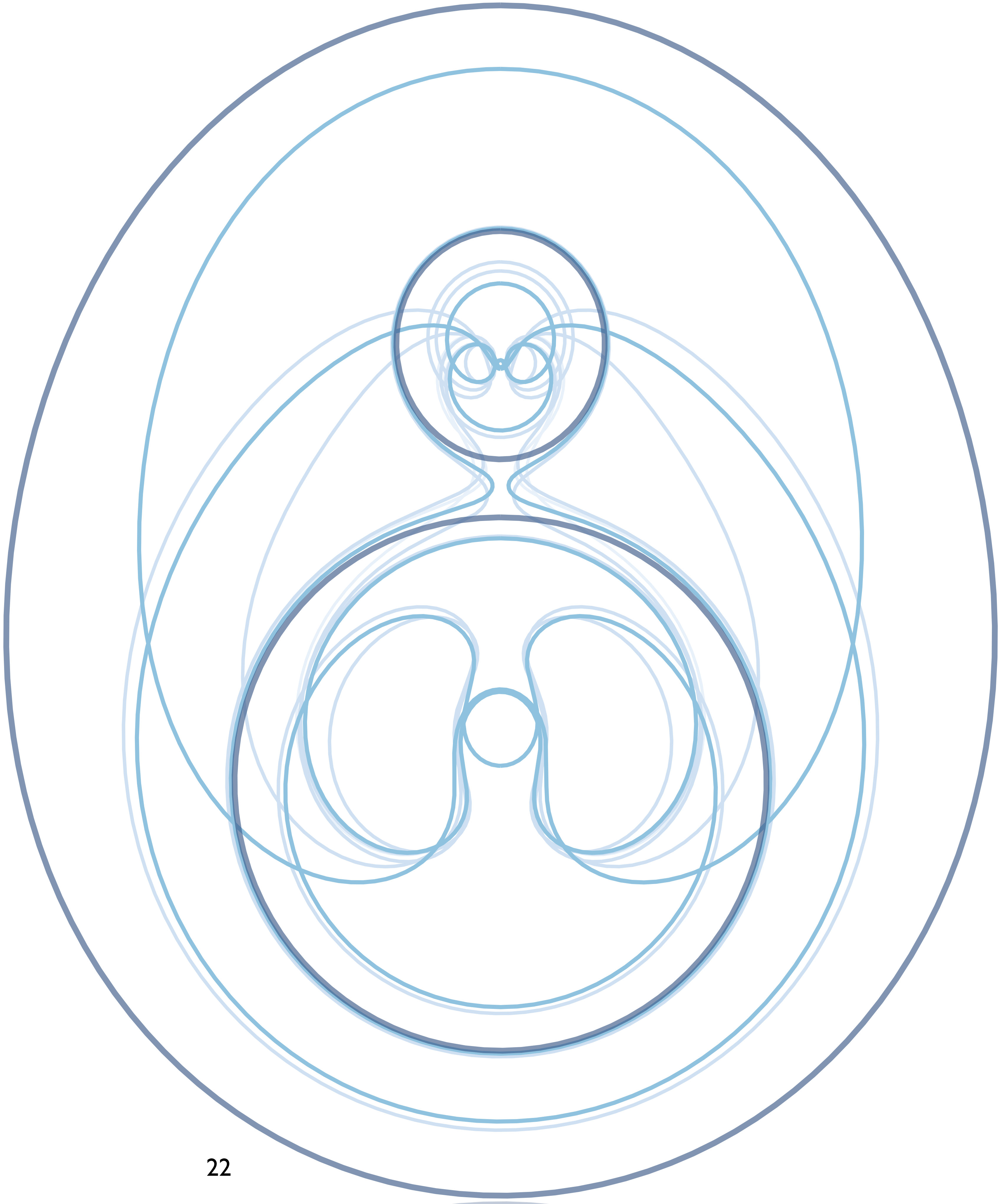

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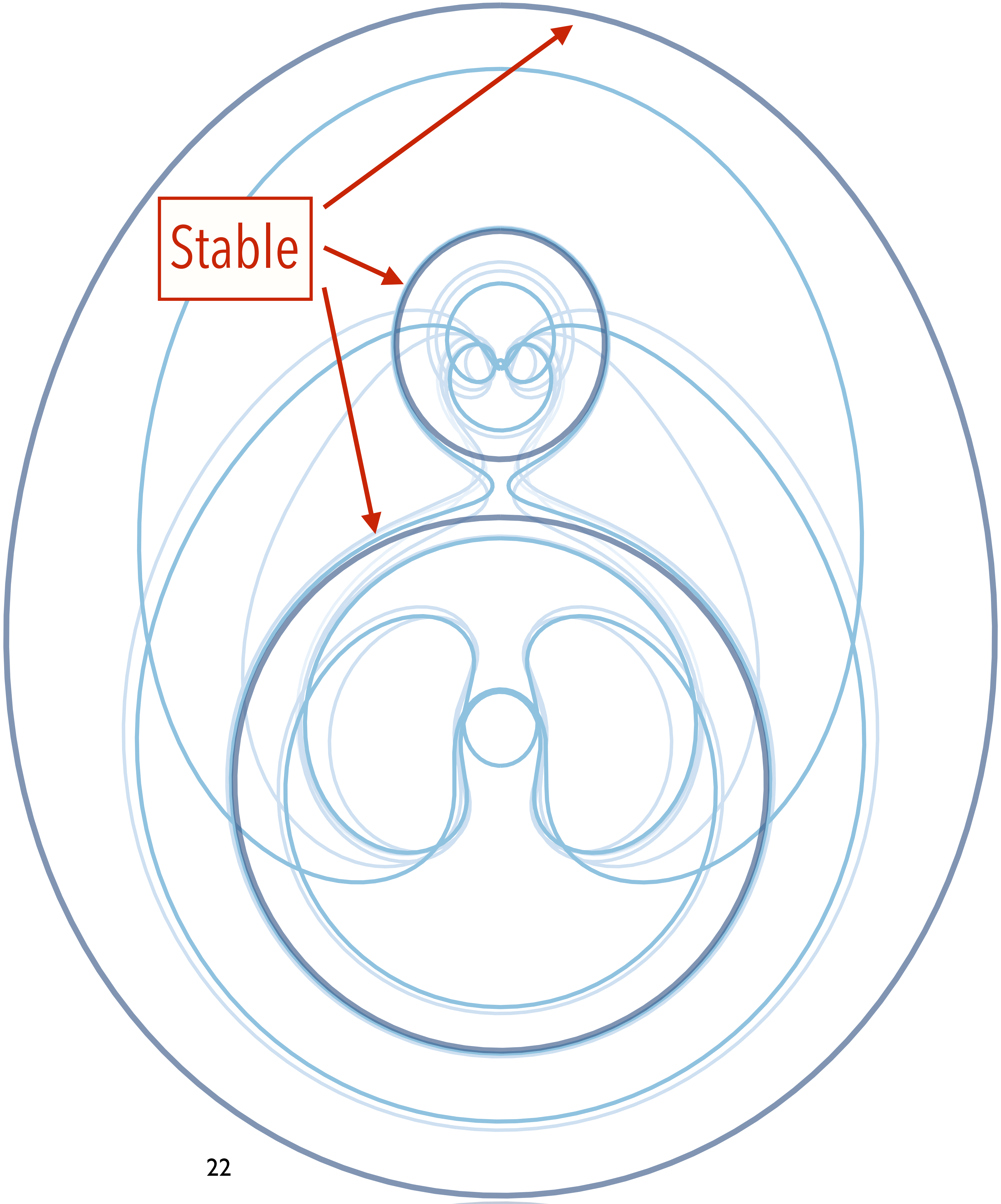
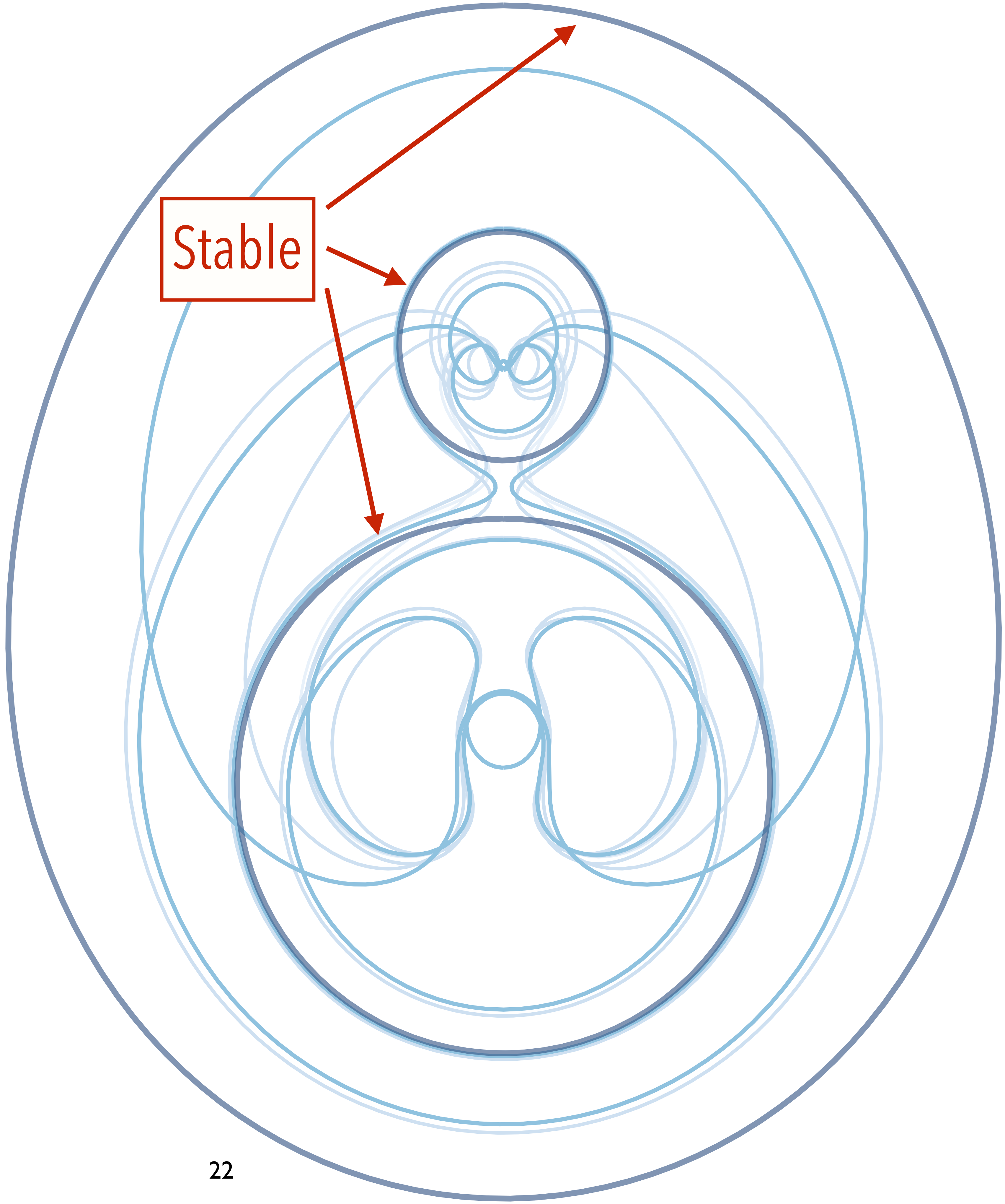


Binary Mergers

- Only the initial and final outer AHs are stable



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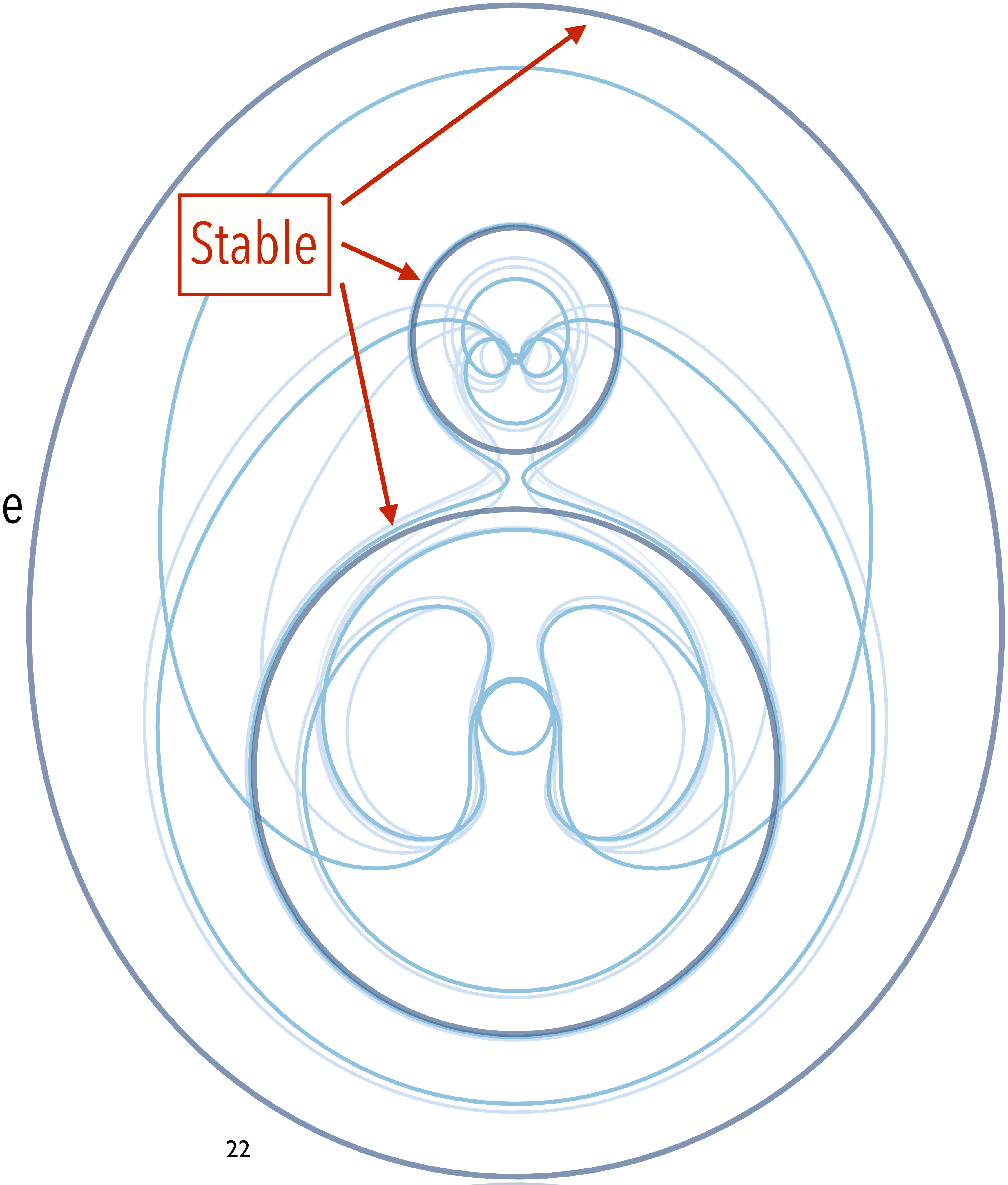

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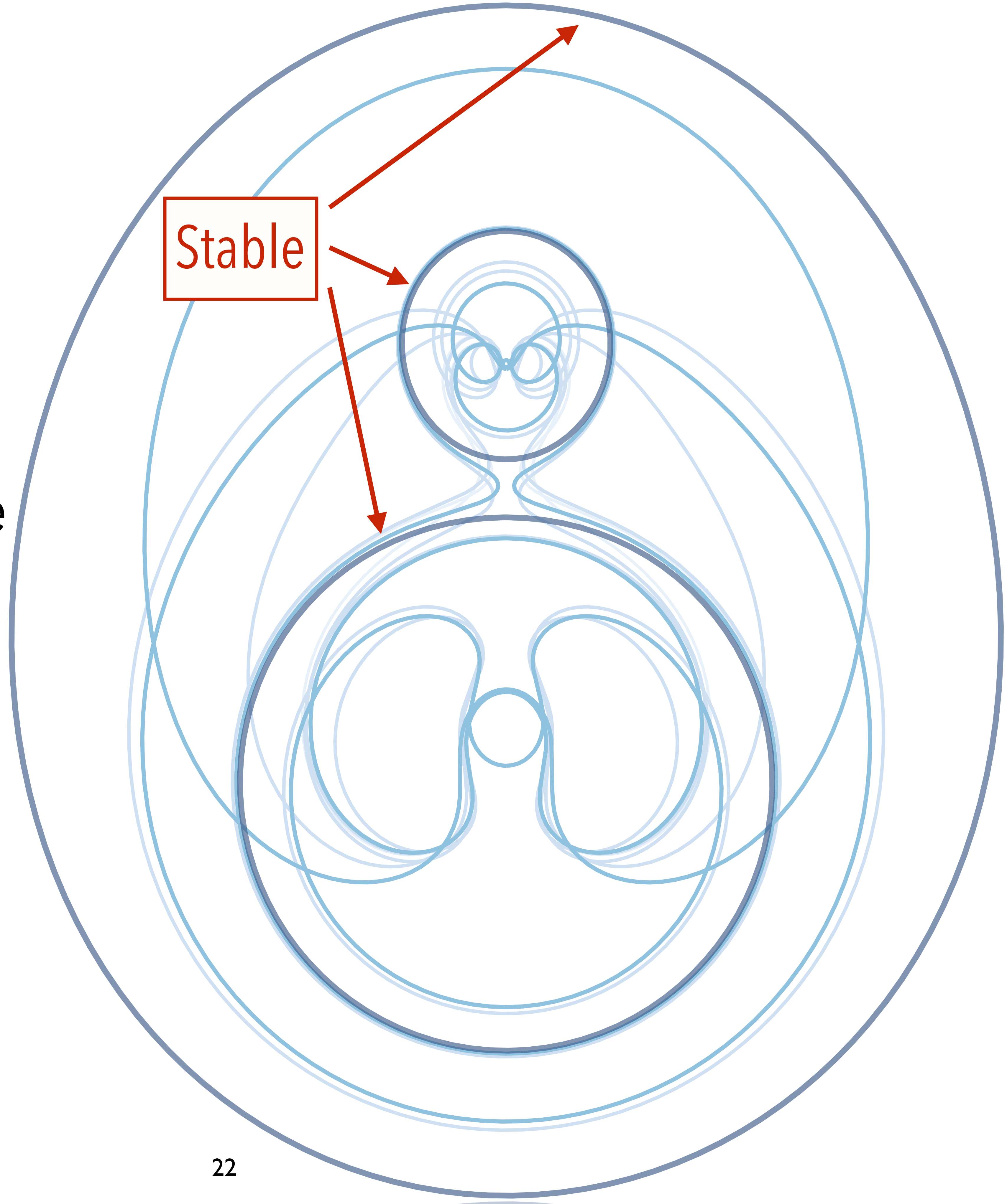

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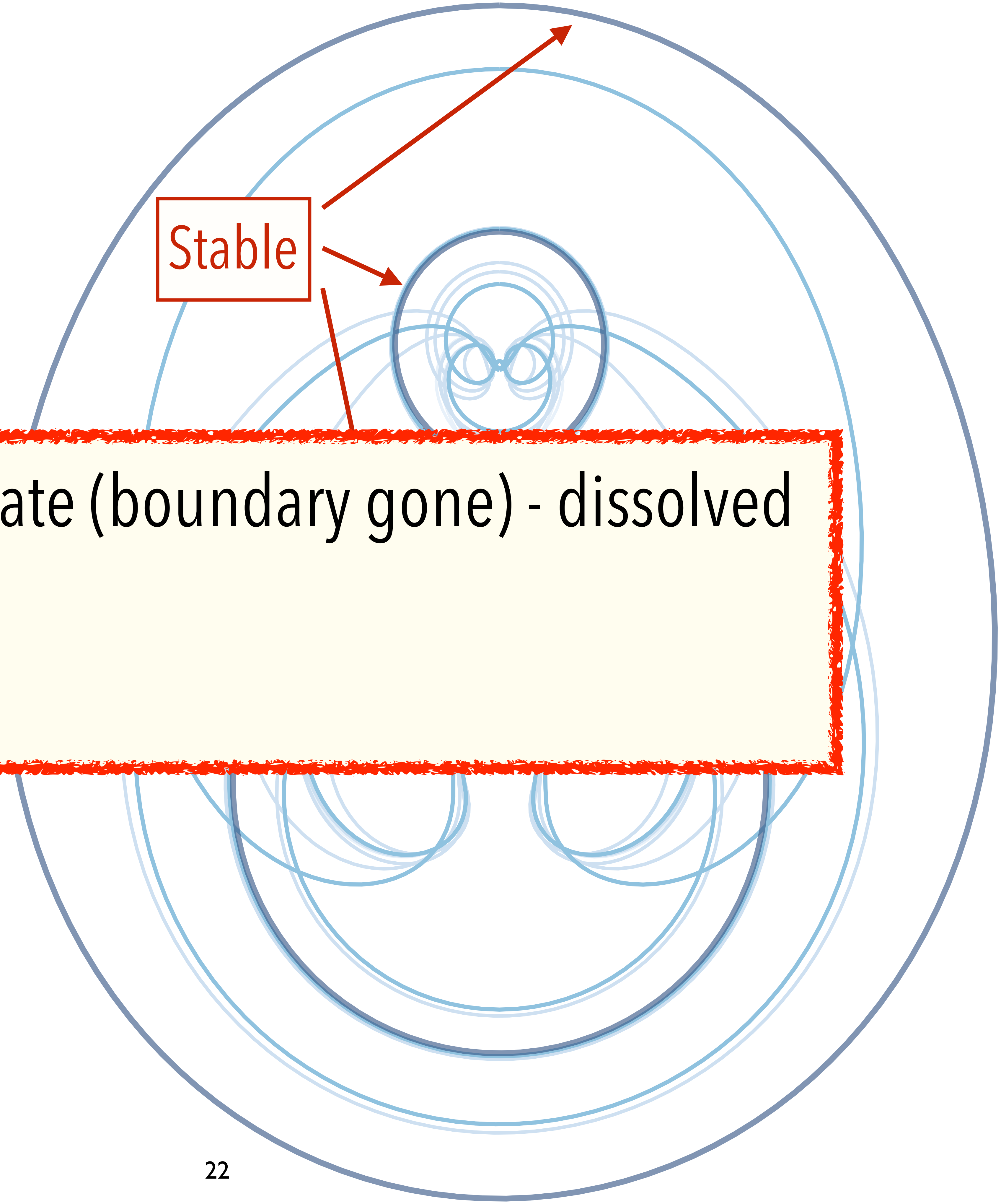

• MOTS can annihilate (boundary gone) - dissolved

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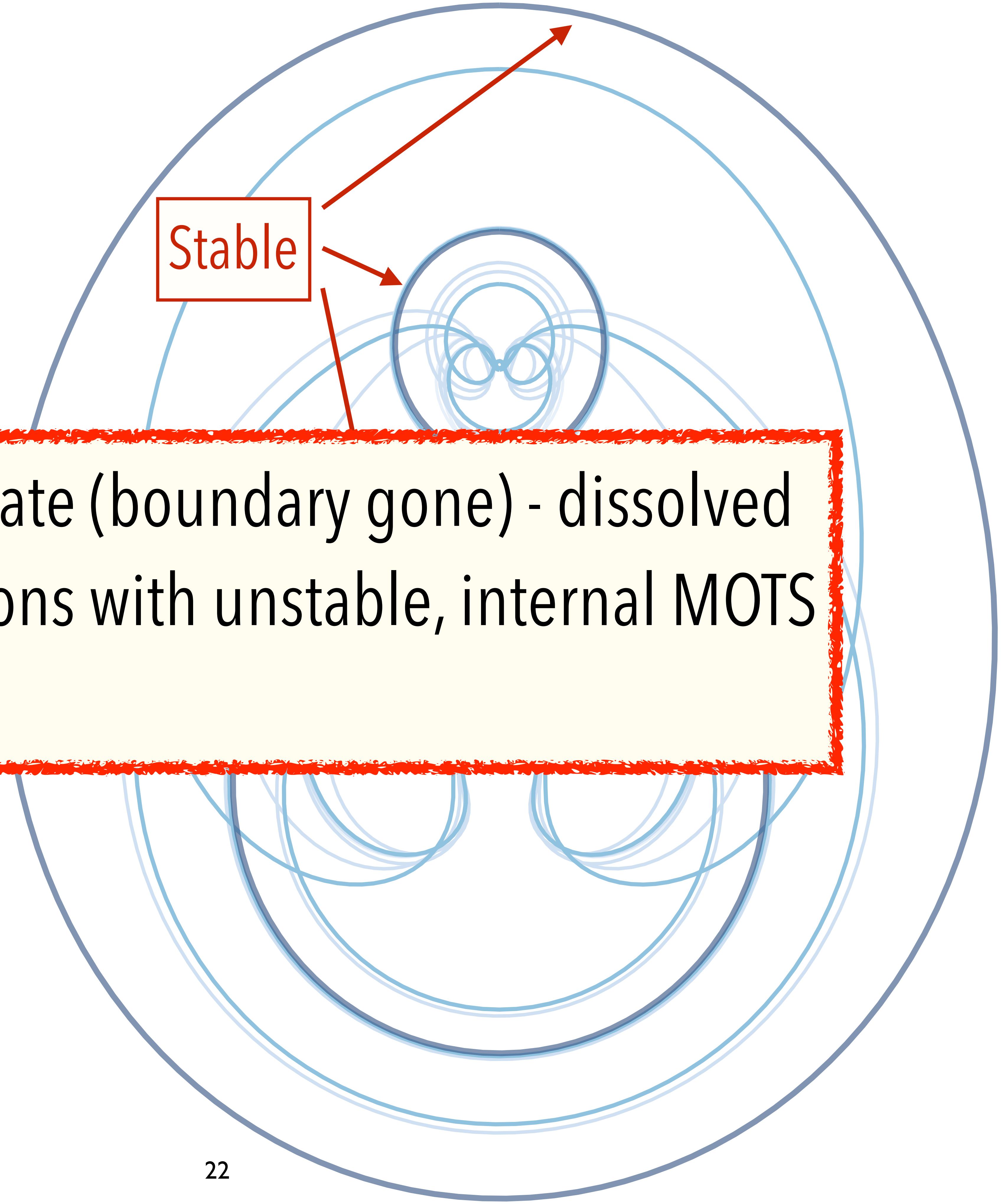
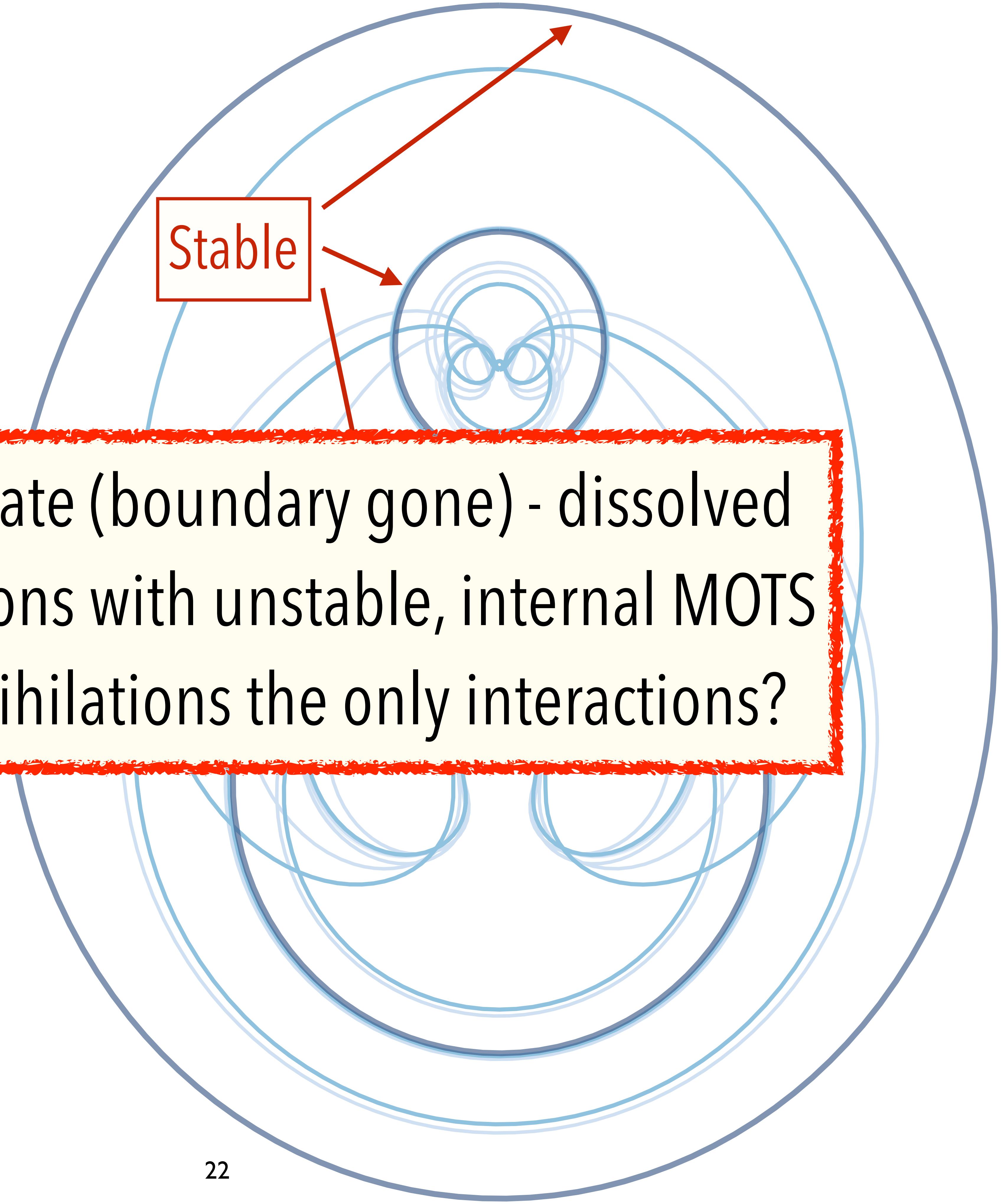


Binary Mergers

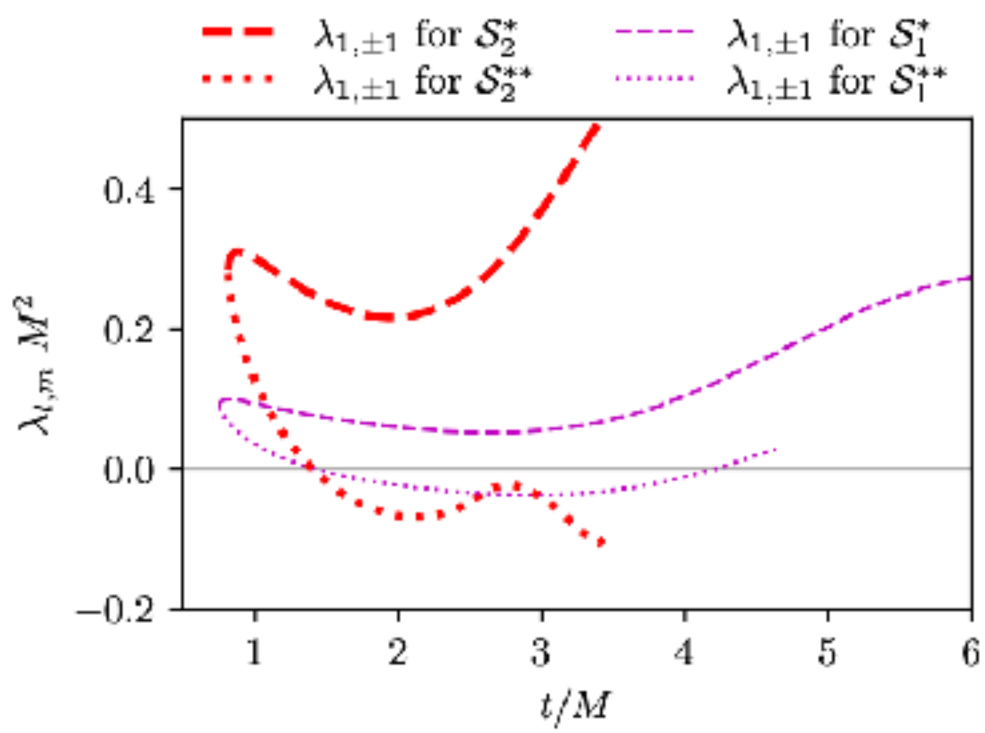
- Only the initial and final outer AHs are stable
- MOTS can annihilate (boundary gone) - dissolved
- Involves interactions with unstable, internal MOTS
- Are creations/annihilations the only interactions?
- Eigenvalues vanish at creation/annihilation events



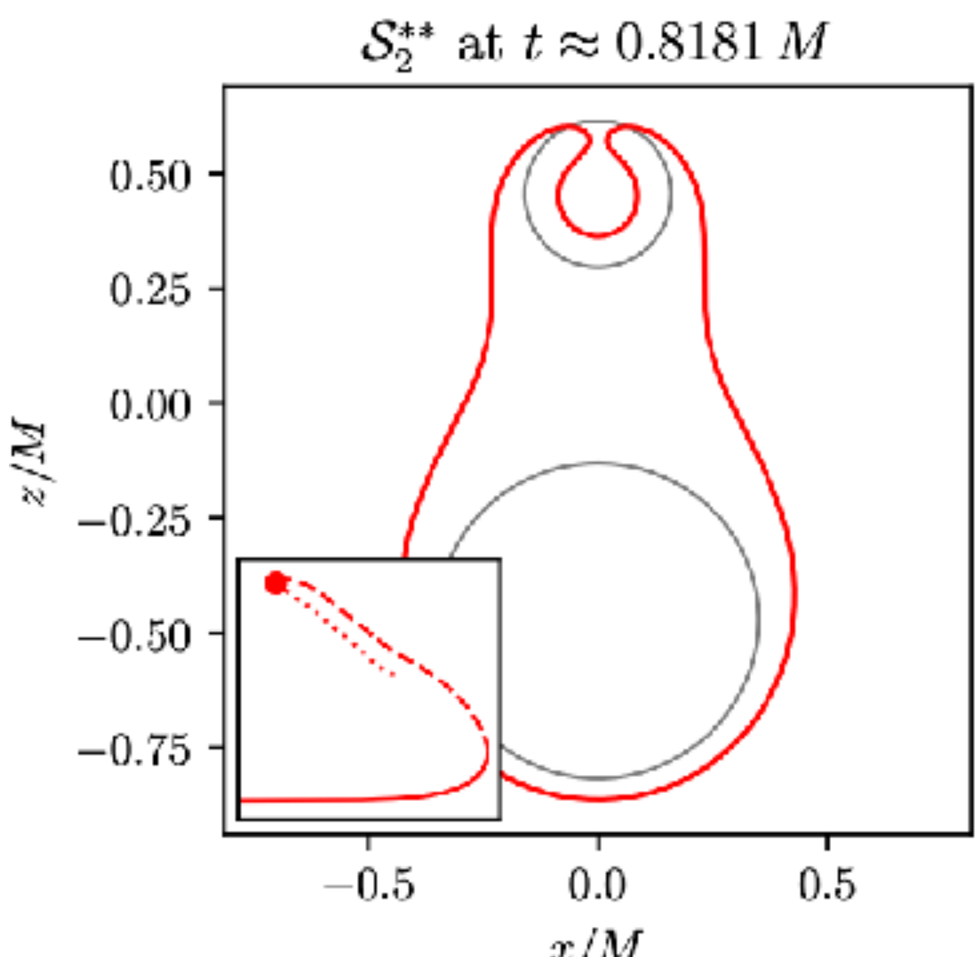
Clue: Vanishing eigenvalues, smooth evolution

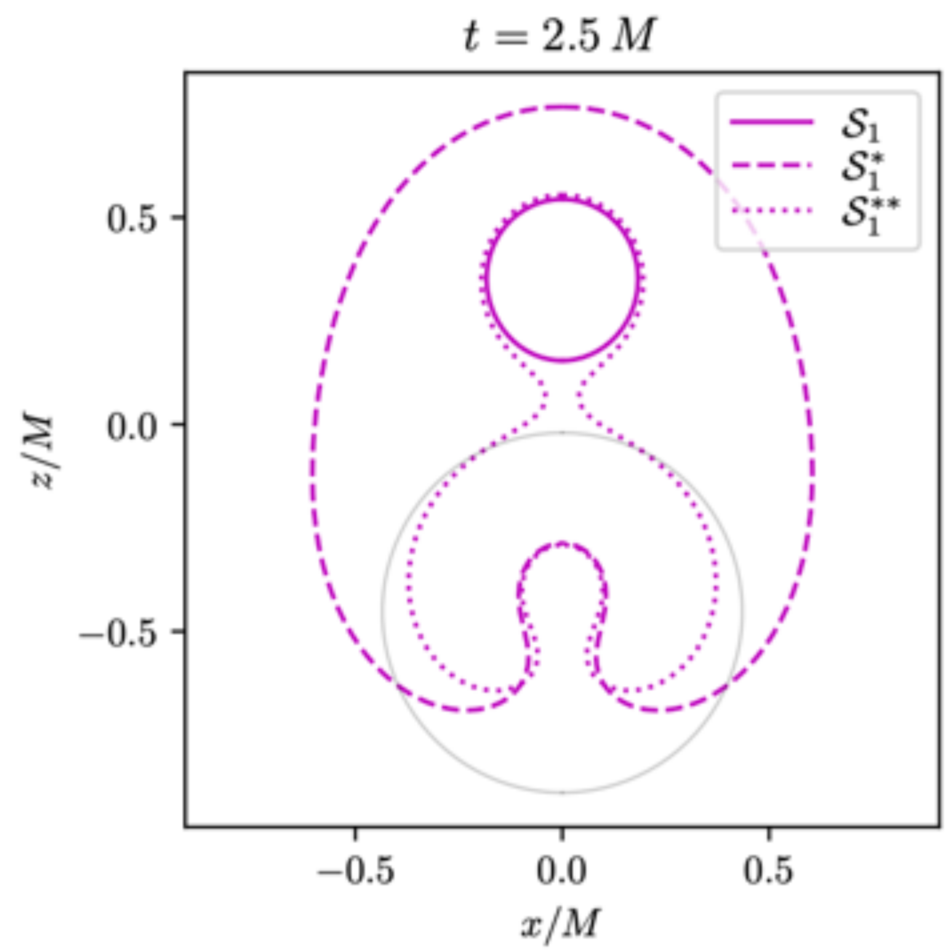
spacetime deformation

Pook-Kolb, Booth, Hennigar, PRD 2021 “Ultimate fate…II..”



Non-axisymmetric eigenvalues





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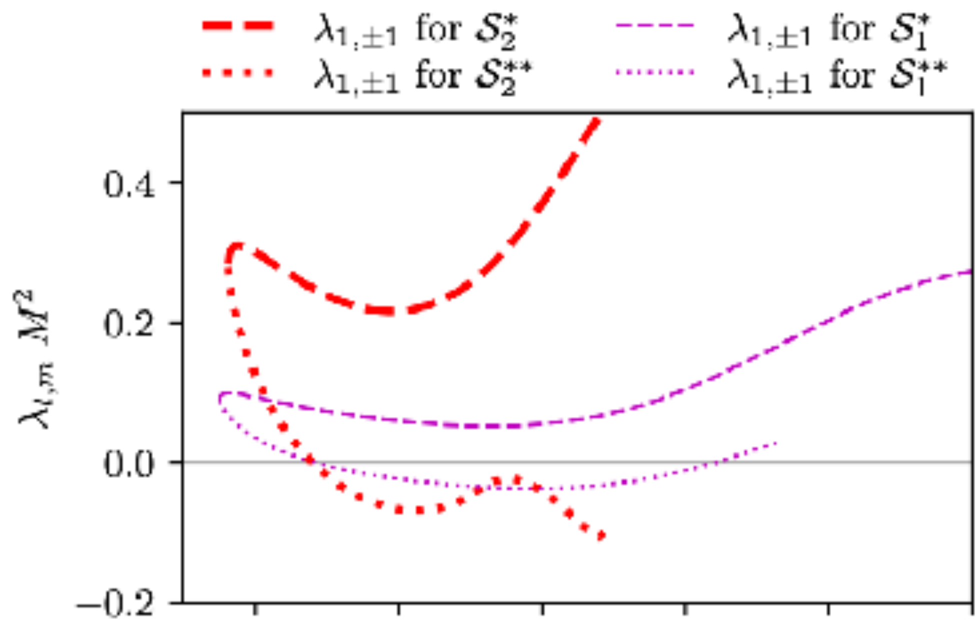
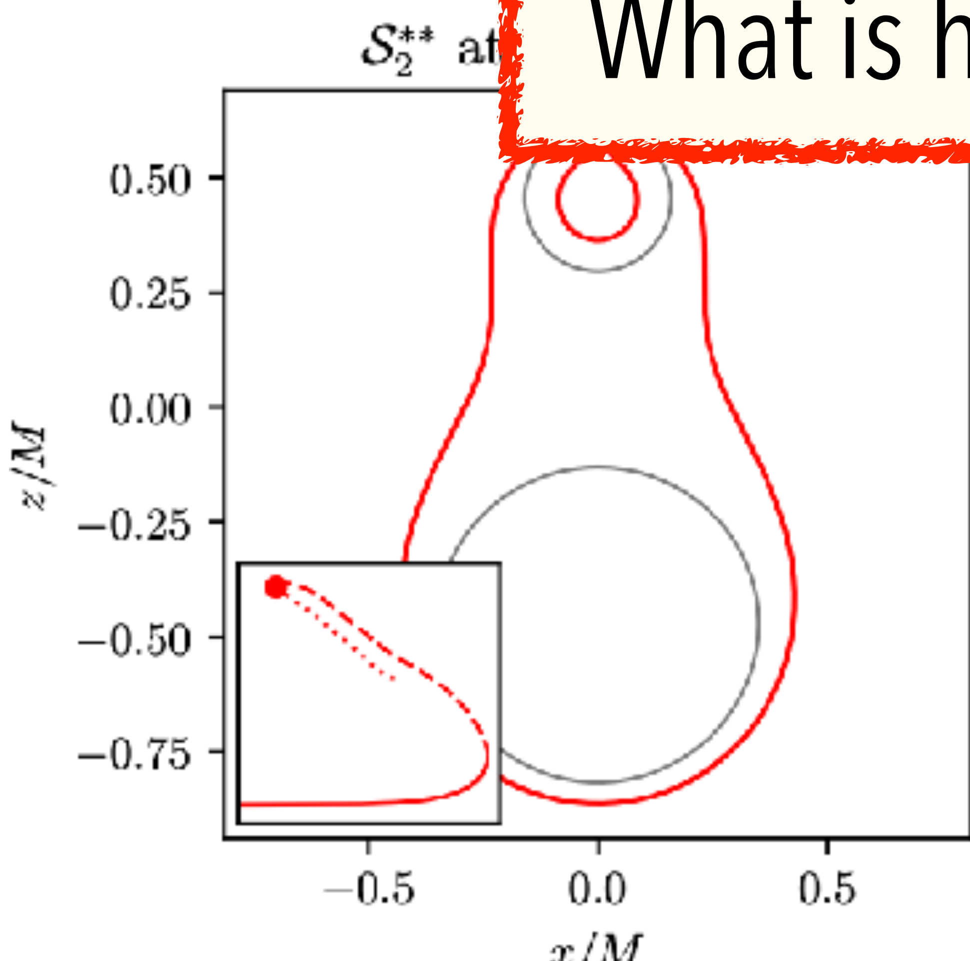
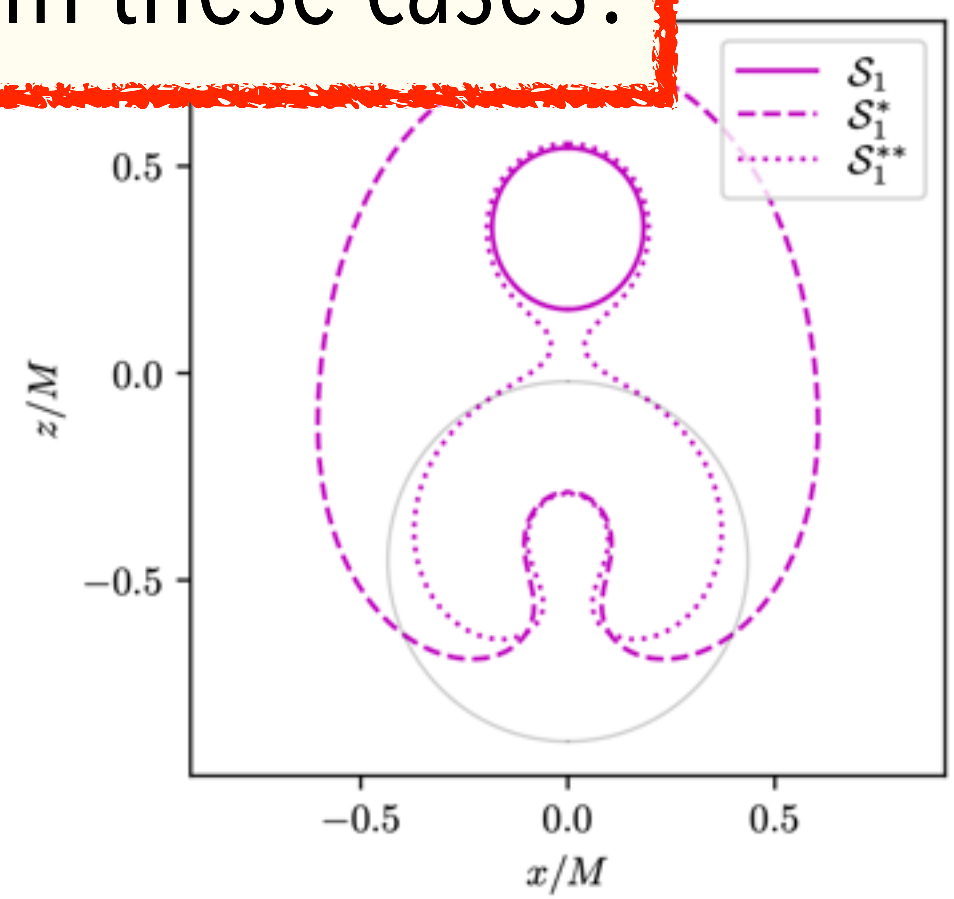


Non-axisymmetric eigenvalues

What is happening in these cases?





Recall bifurcation theory for dynamical systems…

Fixed points for differential equations

$$\dot{x} = \alpha + x^2 \qquad \dot{x} = x(\alpha + x) \qquad \dot{x} = x\left(\alpha + x^2\right)$$

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$x_o = \pm\sqrt{|\alpha|}$

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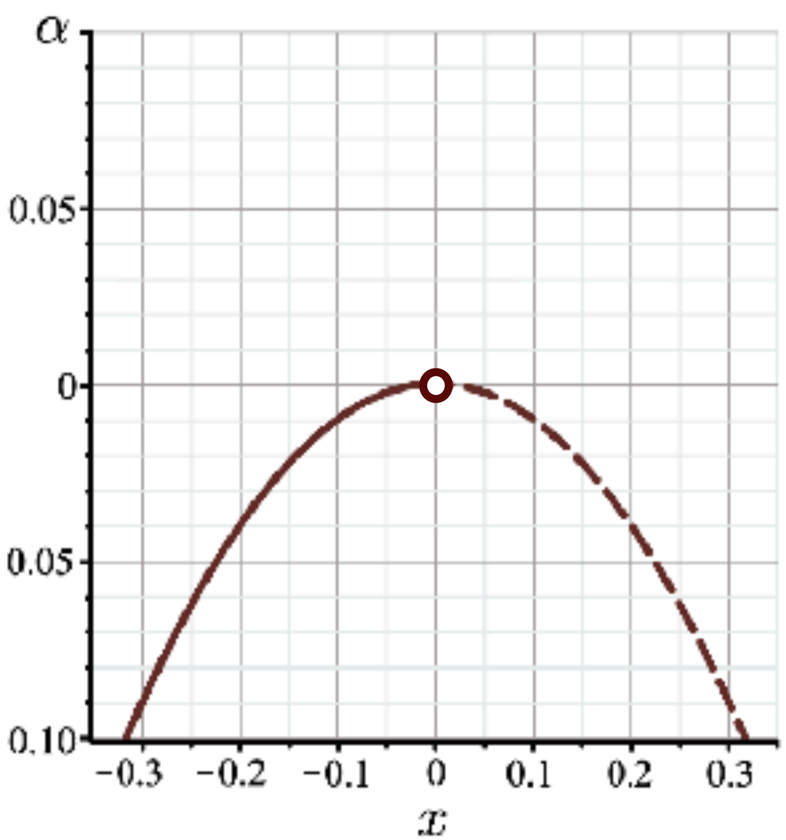
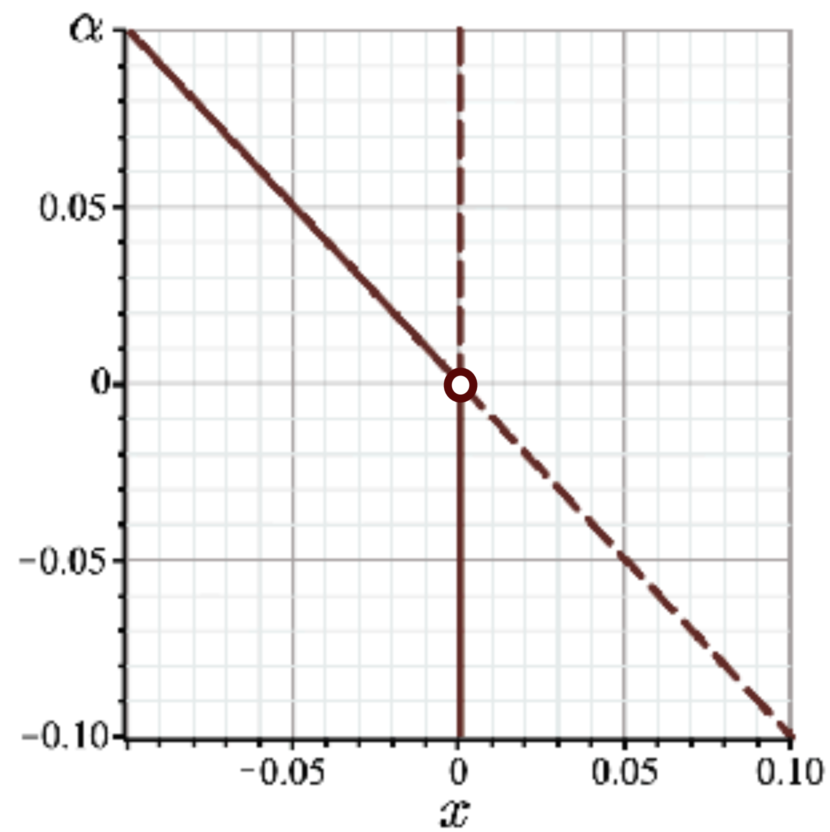
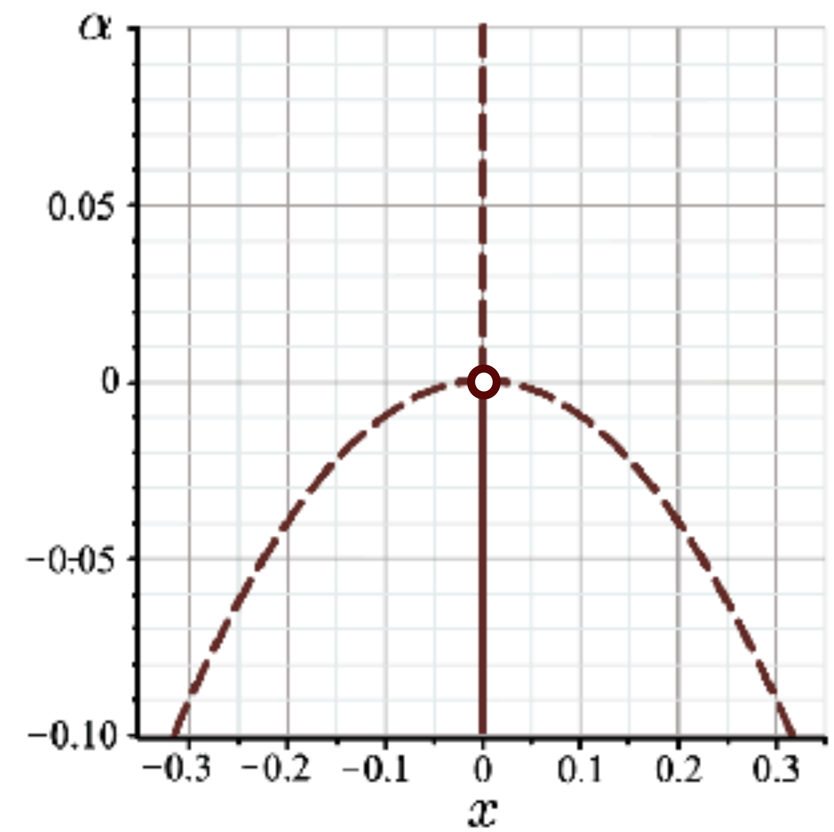


a) Saddle-node



b) Transcritical



c) Pitchfork

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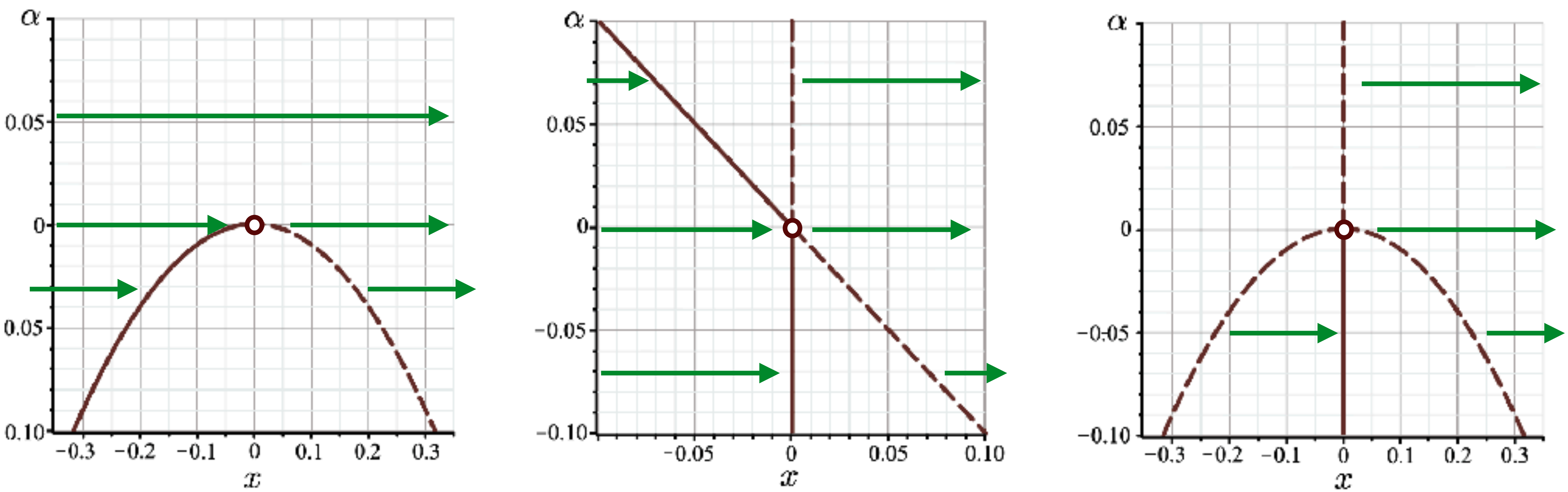

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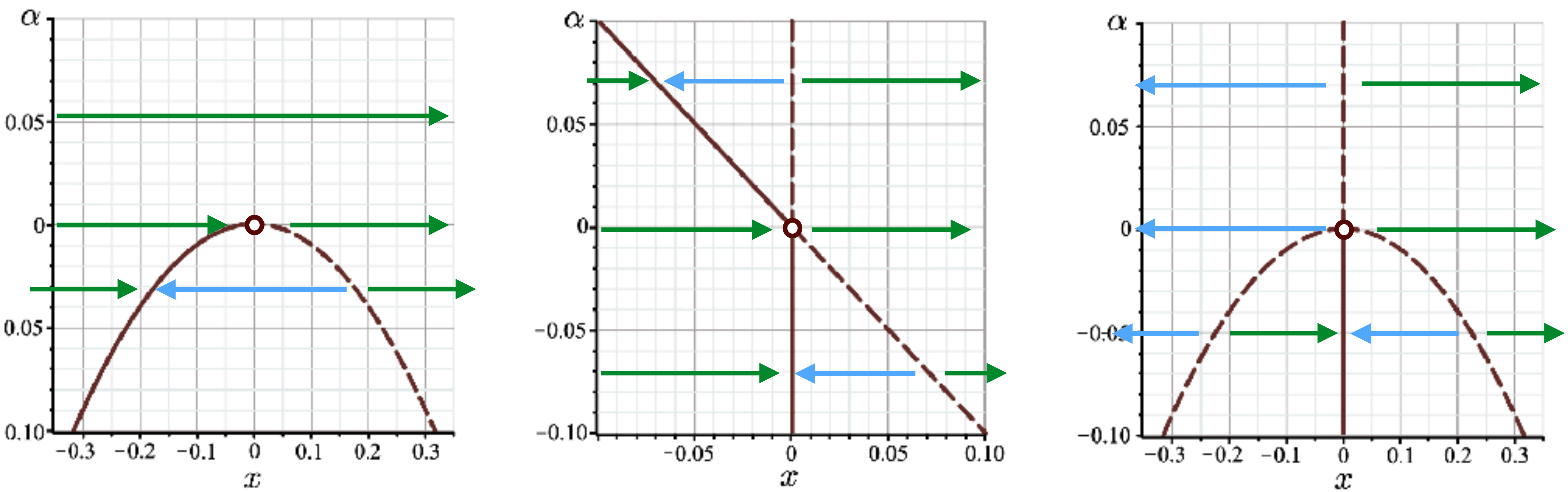

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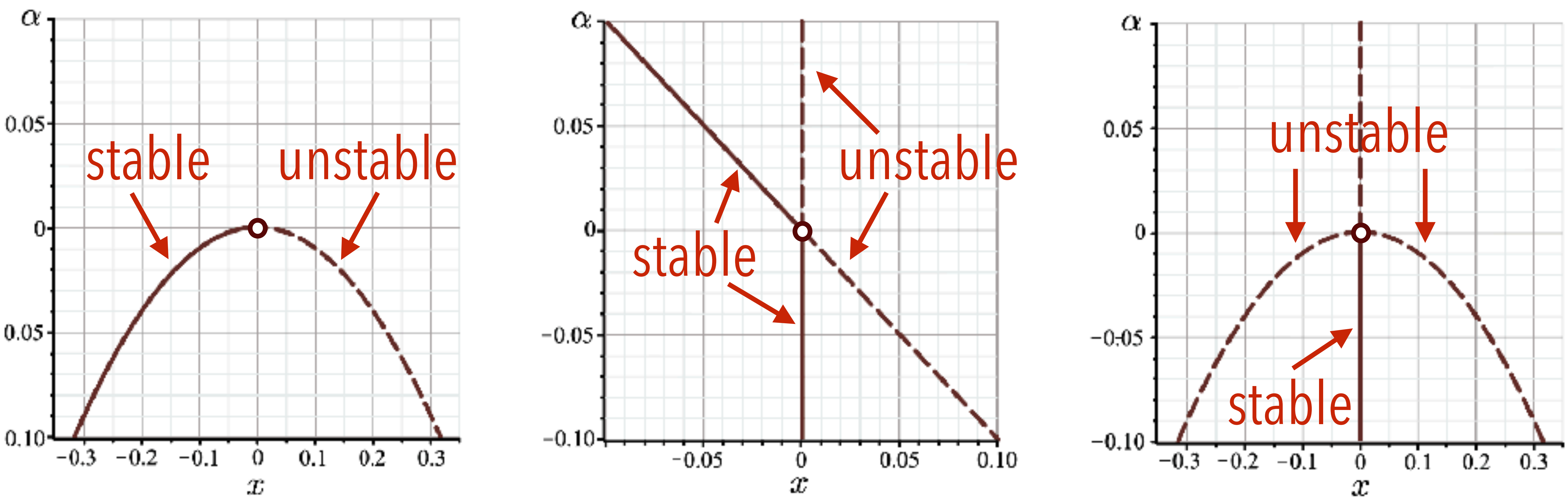

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And now: bifurcation theory for MOTSs - fix notation

<u>ODEs</u> <u>MOTSs</u>

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	<u>ODEs</u>	<u>MOTSs</u>
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opposite name conventions!!!

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"Direction"of bifurcation	eigenvector v_ν	eigenfunction ψ_ν

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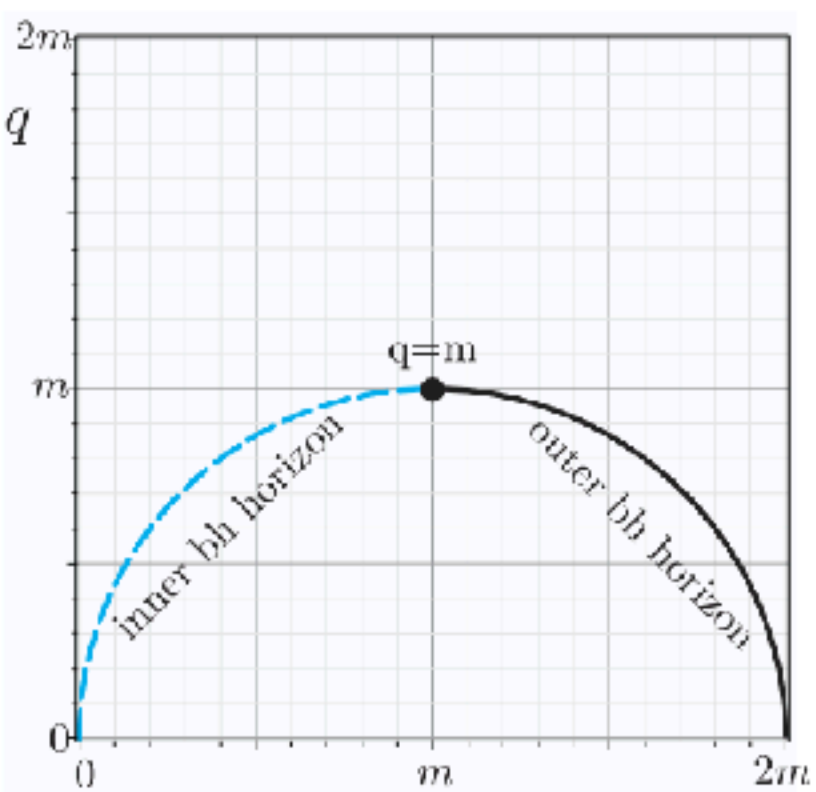
Reissner-Nordström-deSitter

$$ds^2 = -F\,dv^2 + 2dvdr + r^2 d\Omega^2$$

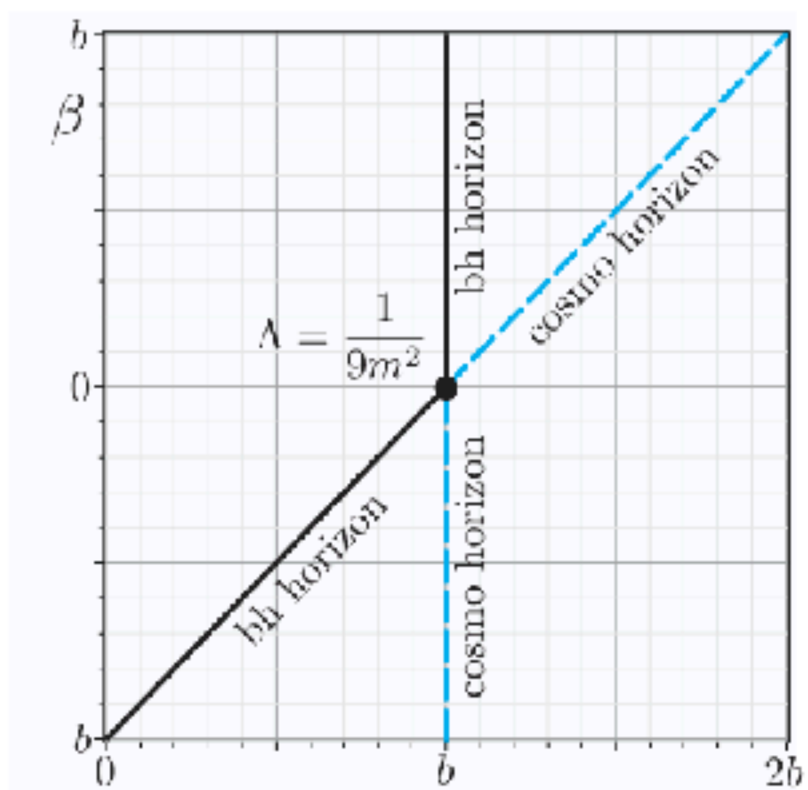
$$F = -\frac{\Lambda}{3}r^2 + 1 - \frac{2m}{r} + \frac{q^2}{r^2} \quad \text{for functions} \quad \Lambda(\alpha), m(\alpha), q(\alpha)$$

MOTSs bifurcations are **exactly** ODE bifurcations (modulo naming) with

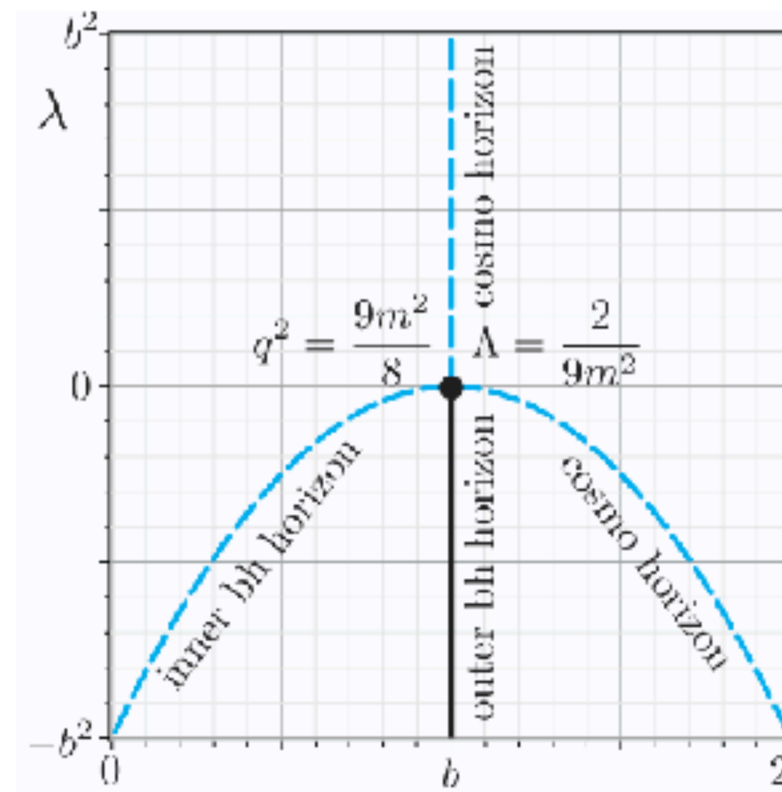
$$\dot{r} = \frac{dr}{dv} = f = \frac{F}{2} \text{ (radial null geodesics)}$$



a) Reissner-Nordström: saddle-node



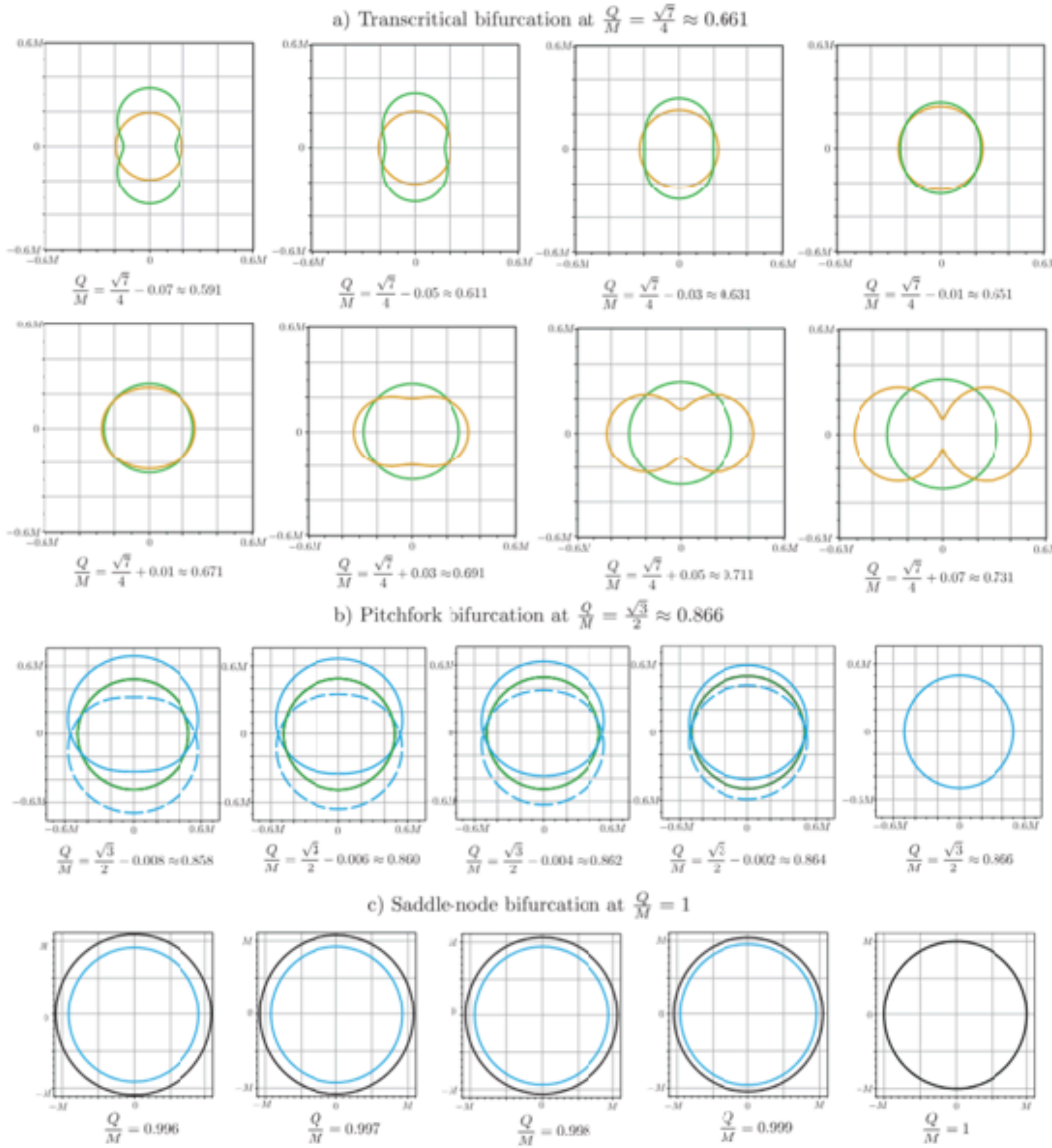
b) Schwarzschild-deSitter: transcritical



c) Reissner-Nordström-deSitter: pitchfork

Axi-symmetric bifurcations from spherical symmetry

Back to Reissner-Nordström: order emerges



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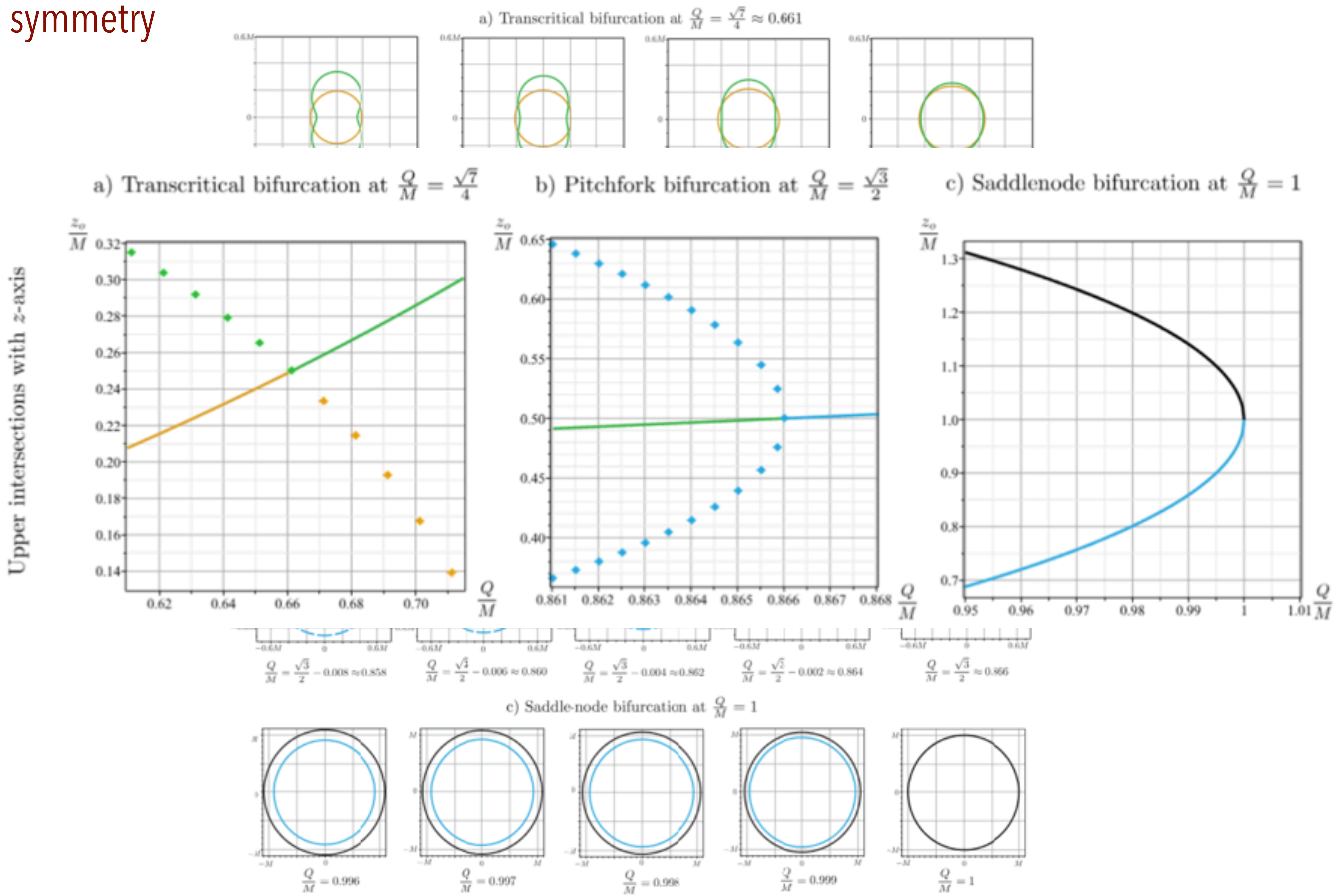

Weyl-Schwarzschild

- All static, axisymmetric metrics can be written in the form

$$ds^2 = -e^U dt^2 + e^{-2U+2V}\left(dz^2 + d\rho^2\right) + e^{-2U}\rho^2 d\phi^2$$

where $U = U(\rho, z)$ and $V = V(\rho, z)$.

- These are vacuum solutions if U is a solution of the flat space Laplace equation:

$$\frac{\partial^2 U}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial U}{\partial \rho} + \frac{\partial^2 U}{\partial z^2} = 0$$

and $\dfrac{\partial V}{\partial \rho} = \rho\left(\left[\dfrac{\partial U}{\partial \rho}\right]^2 + \left[\dfrac{\partial U}{\partial z}\right]^2\right) \;,\; \dfrac{\partial V}{\partial z} = 2\rho\dfrac{\partial U}{\partial \rho}\dfrac{\partial U}{\partial z}$.

- U solutions can be classified in the usual way: monopole, dipole, quadrupole, octopole etc.

Weyl-distorted Schwarzschild

$$ds^2 = -e^{2U}\left(1-\frac{2m}{r}\right) + e^{-2U+2V}\left(\frac{dr^2}{1-\frac{2m}{r}} + r^2 d\theta^2\right) + e^{-2U} r^2 \sin^2\theta d\phi^2$$

- If we demand no conical singularities on the horizon and deform from infinity, then the metric on the horizon at r=2m is:

$$dS^2 = 4m^2 e^{-2U}\left(e^{4U-4u_o} d\theta^2 + \sin^2\theta d\phi^2\right)$$

where $U(2m,\theta) = \sum_{i=0}^{\infty} \alpha_i P_i(\cos\theta)$ and $V(2m,\theta) = 2U(2m,\theta) - 2u_o$

with $U(2m,0) = U(2m,\pi) = u_o\ ,\ \sum_{k=1}^{\infty} \alpha_{2k-1} = 0\ ,\ \sum_{k=1}^{\infty} \alpha_{2k} = u_o$

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Focus on quadrupole case

Weyl-distorted Schwazschild II

T.Pilkington, A.Melanson,
J. Fitzgerald, IB
CQG 28:125018,2011

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- Quadrupole solutions give a one-parameter (α) family of distortions

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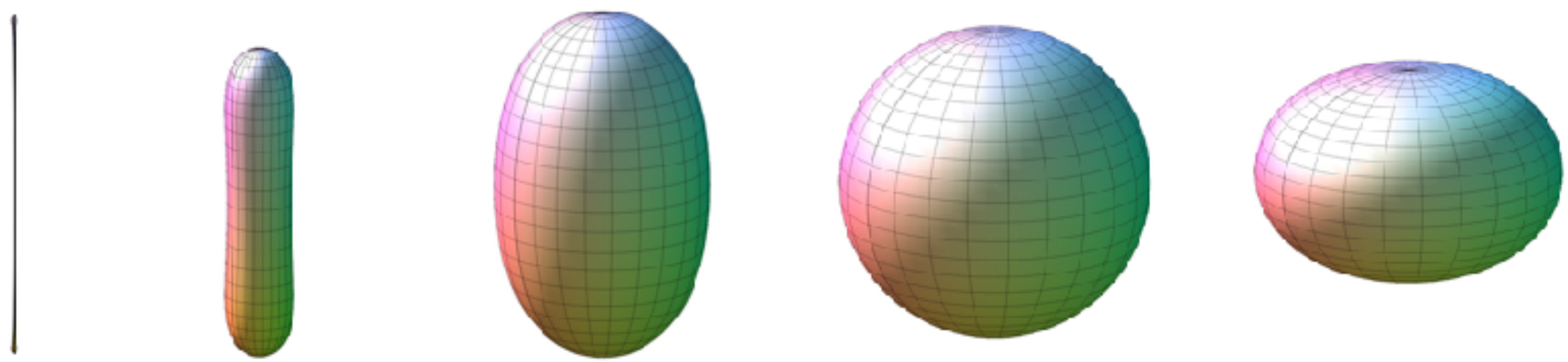
- $r = 2m$ bends but it doesn't break!



Not embeddable

$\alpha = -3.2$ $\alpha = -0.8$ $\alpha = -0.2$ $\alpha = 0$ $\alpha = 0.124$ $\alpha \geq 0.125$

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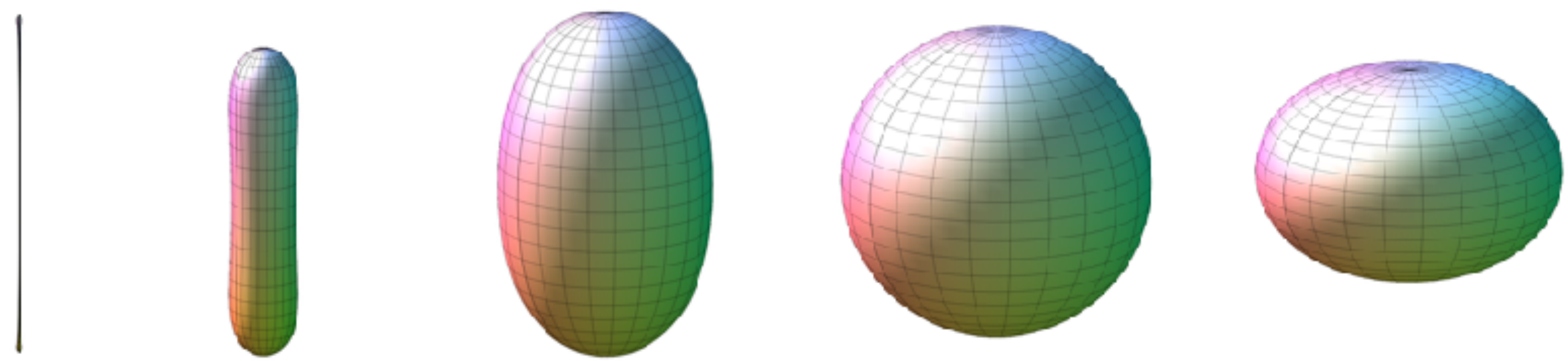

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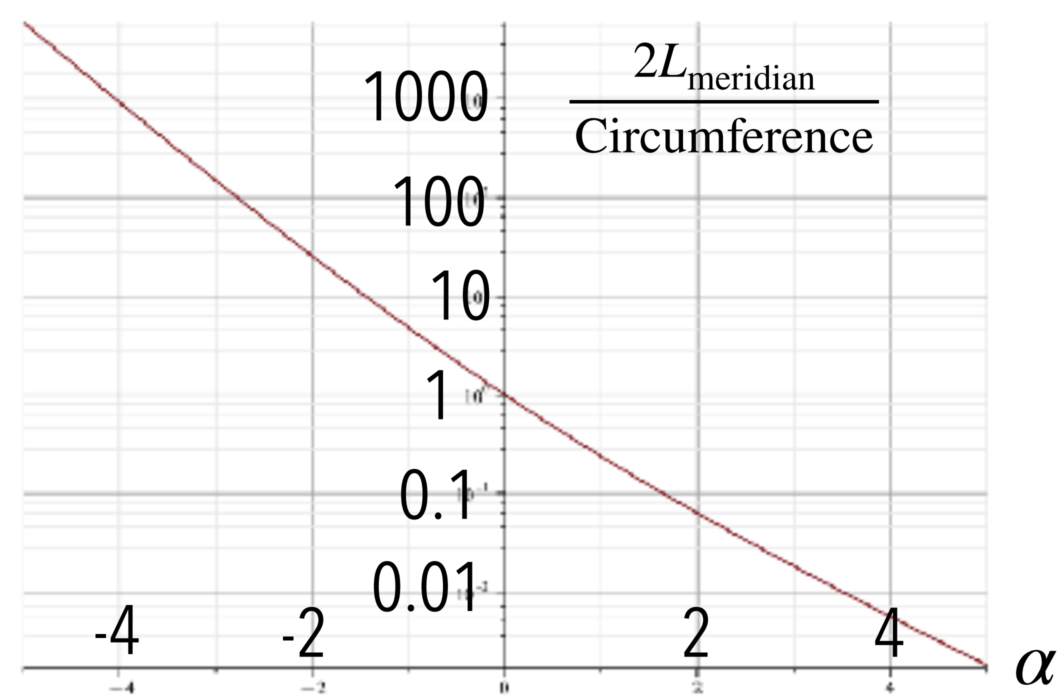
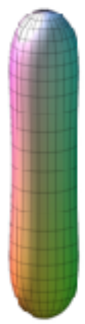
$\alpha = -3.2$ $\alpha = -0.8$ $\alpha = -0.2$ $\alpha = 0$ $\alpha = 0.124$ $\alpha \geq 0.125$

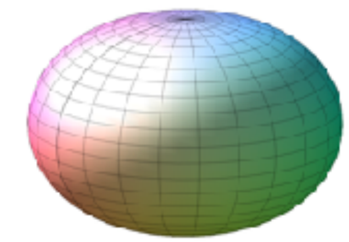
Area$=16\pi^2 m^2 e^{-2\alpha}$

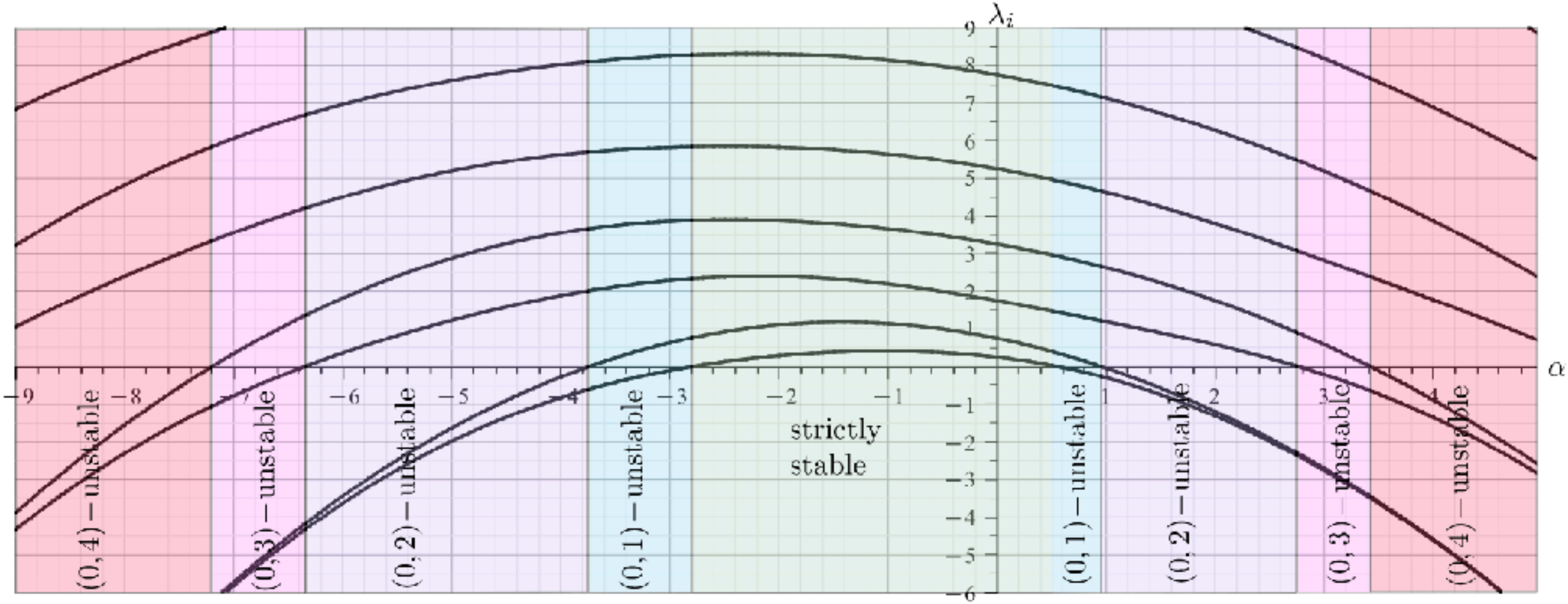
Circumference $= 4\pi m$





Eigenvalues of quadrupole-distorted Schwarzschild





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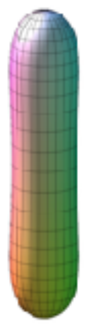

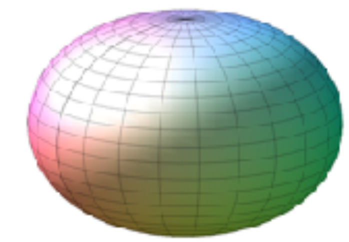

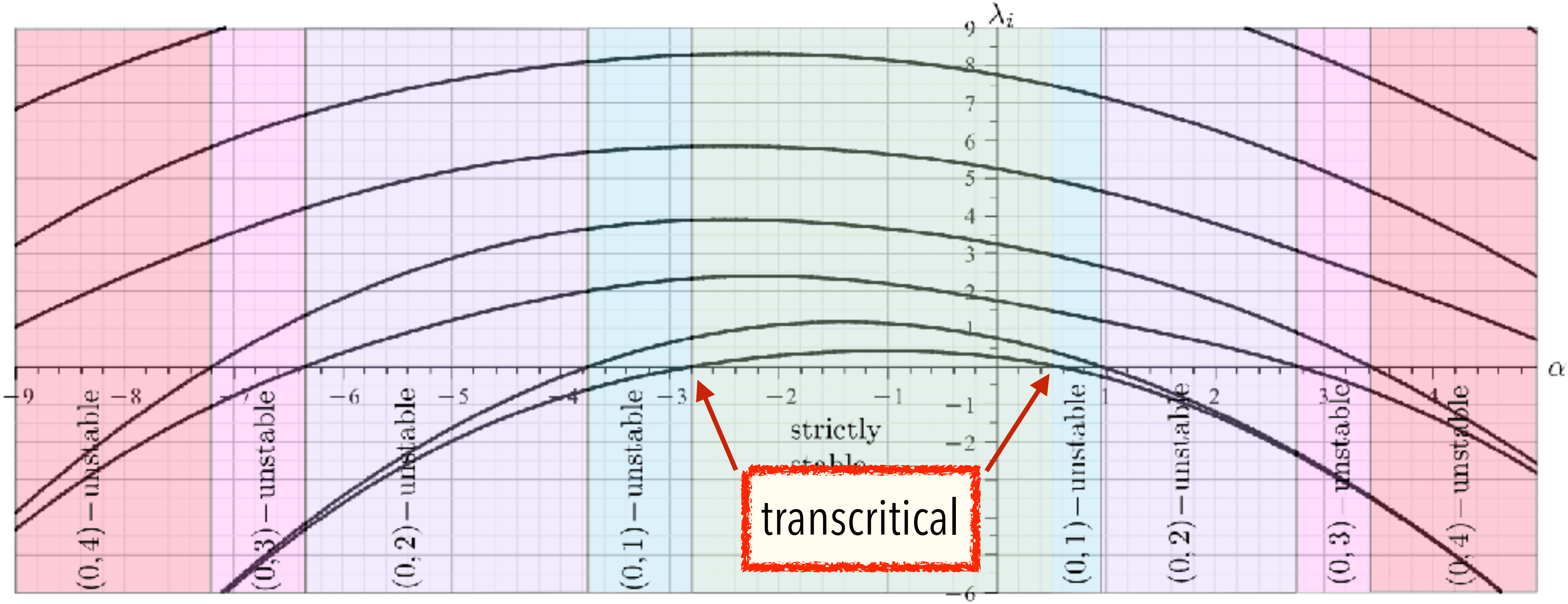

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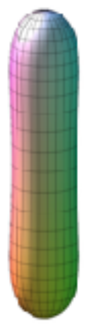

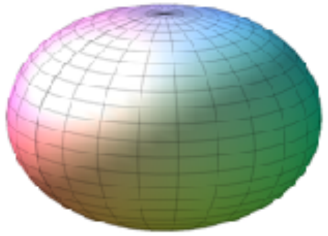

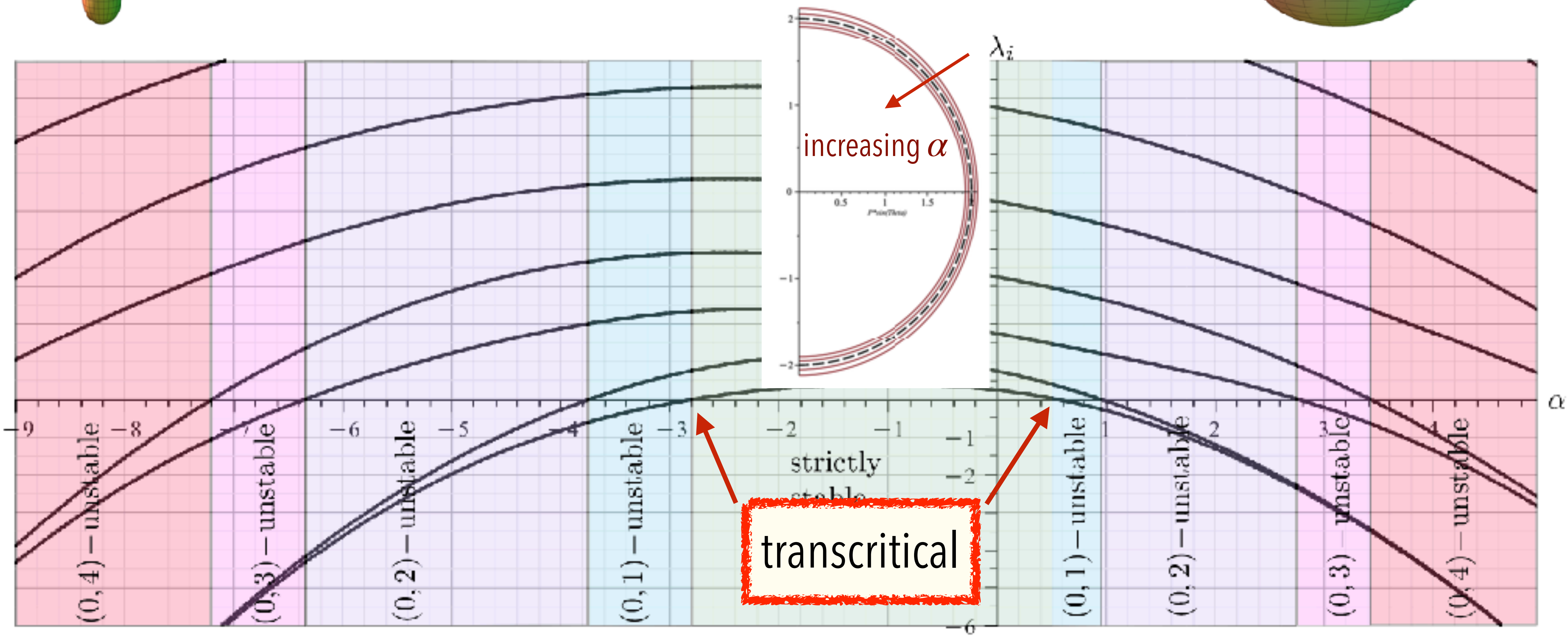

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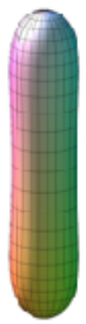

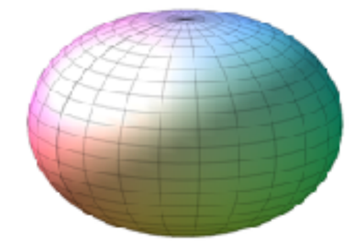

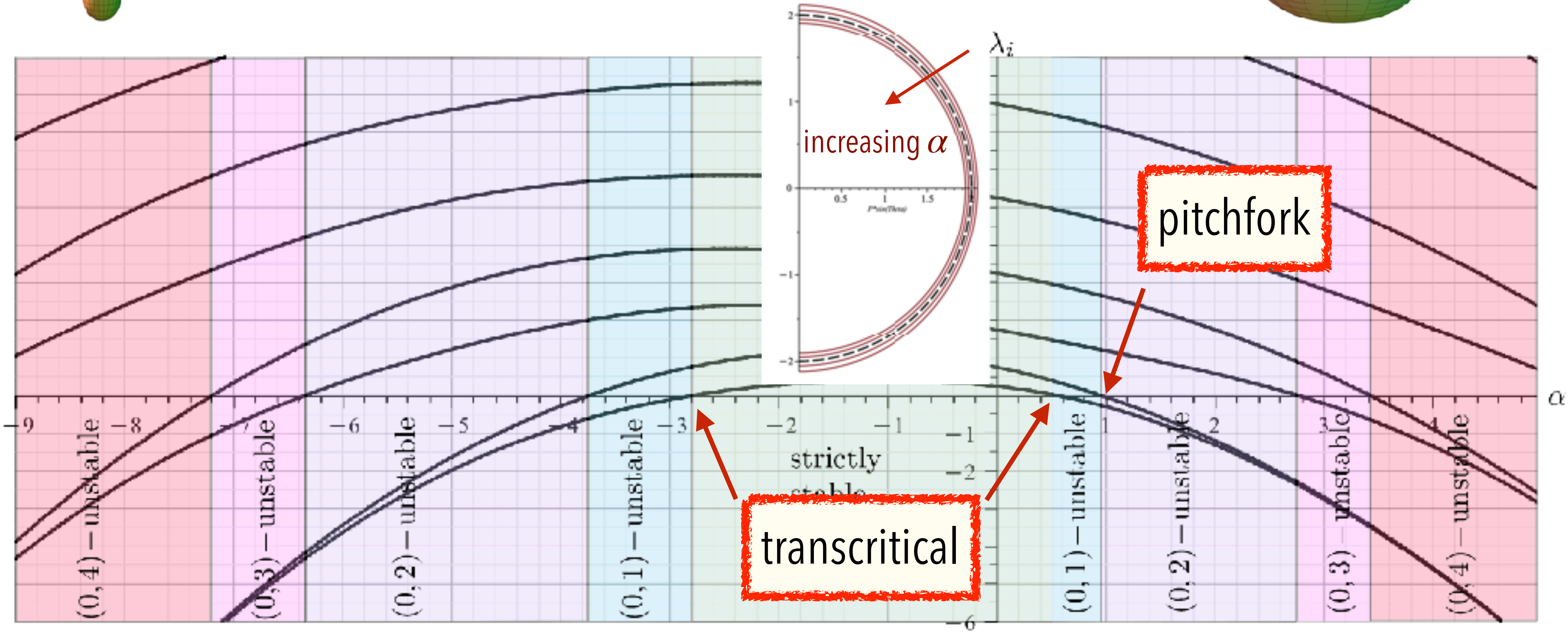

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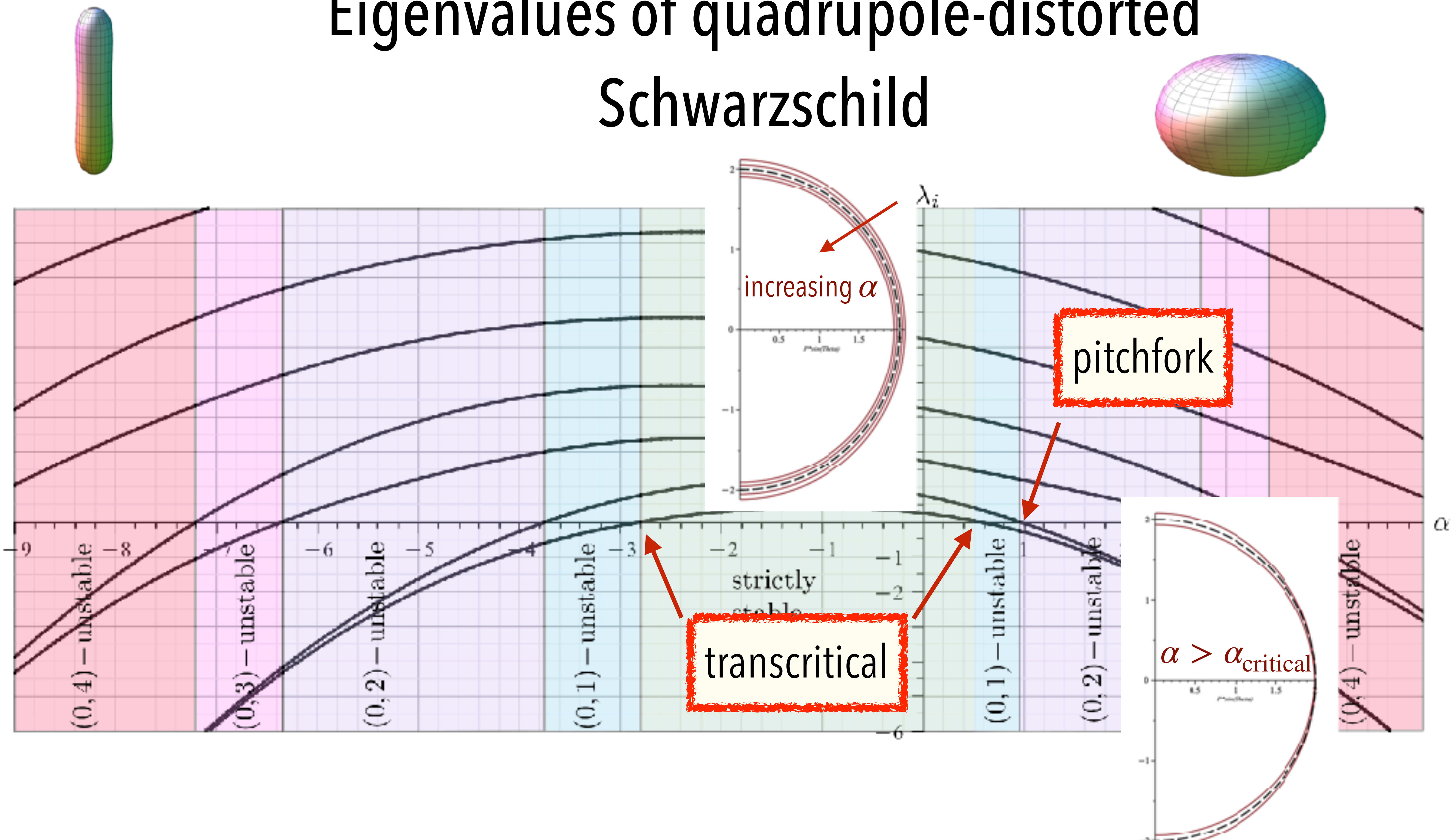
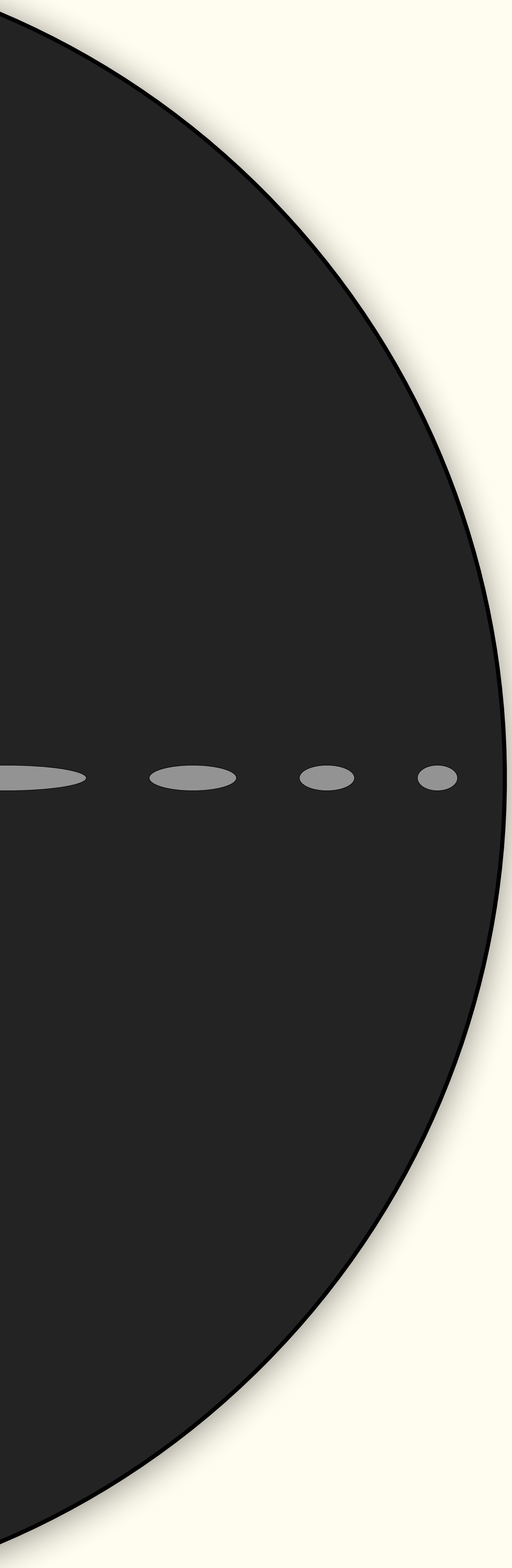


Take aways



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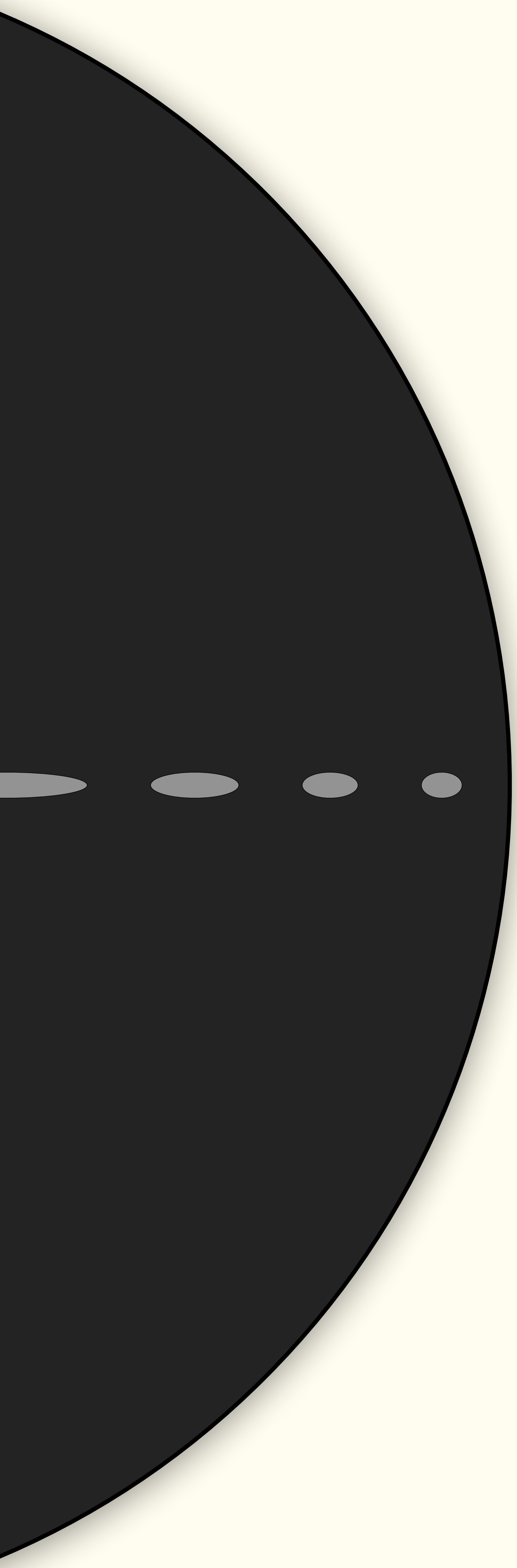
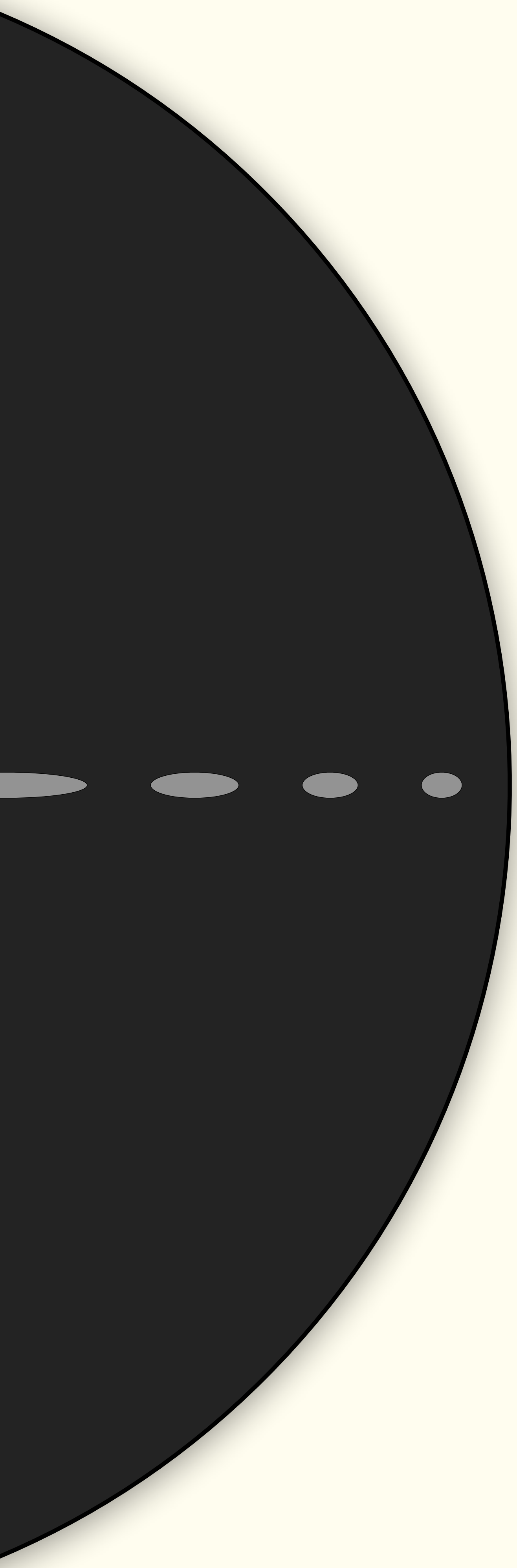


- Most MOTS are not boundaries of trapped regions: they simply identify regions of (very) strong gravity



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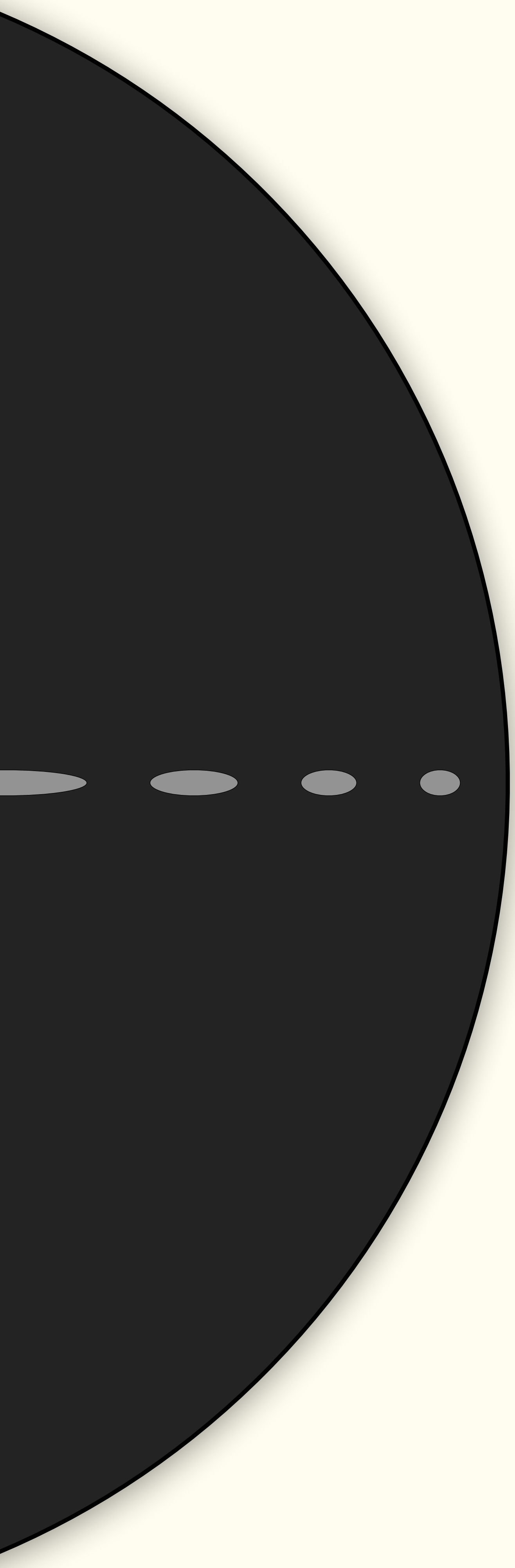

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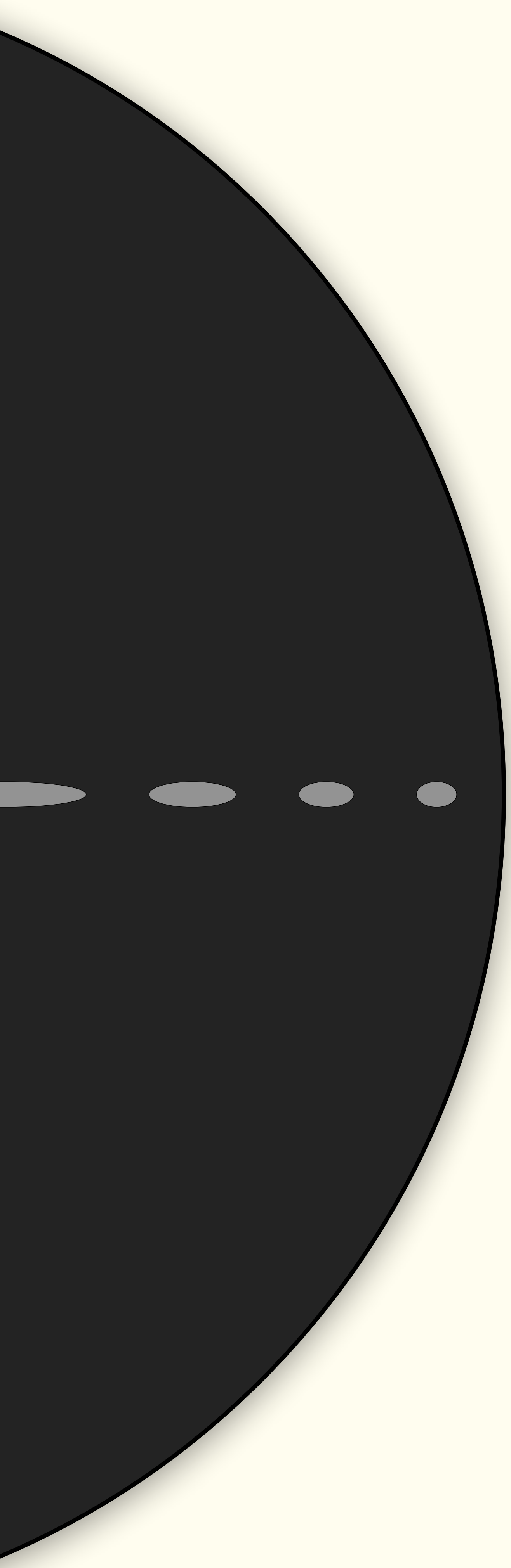

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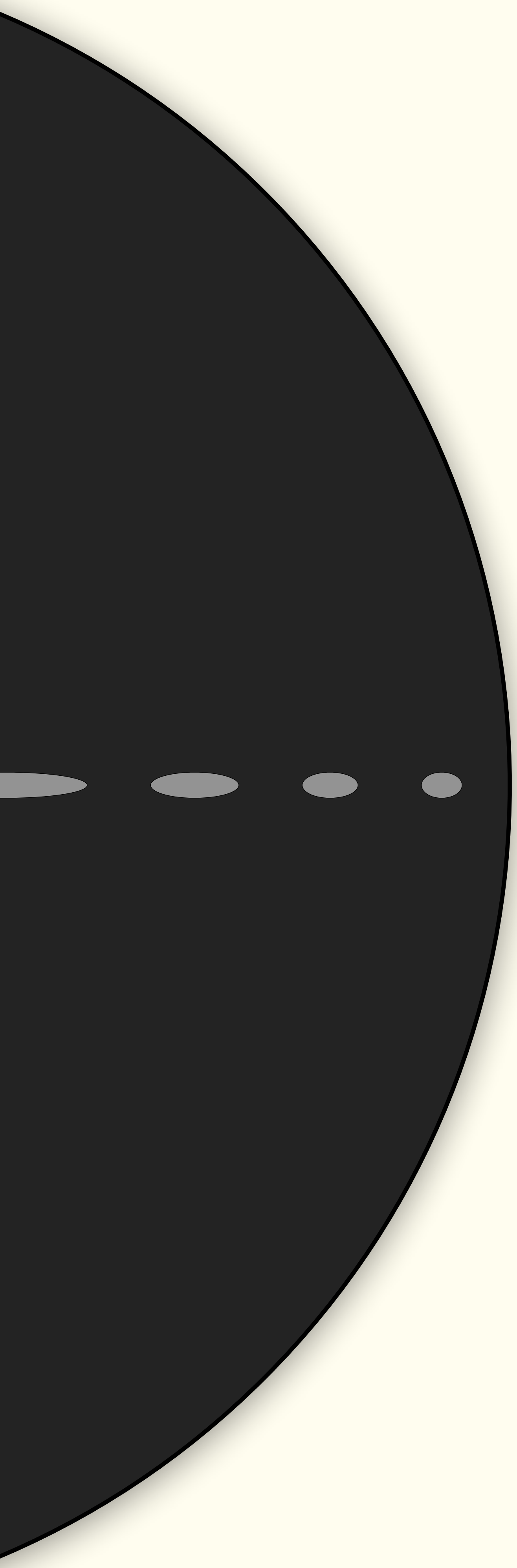

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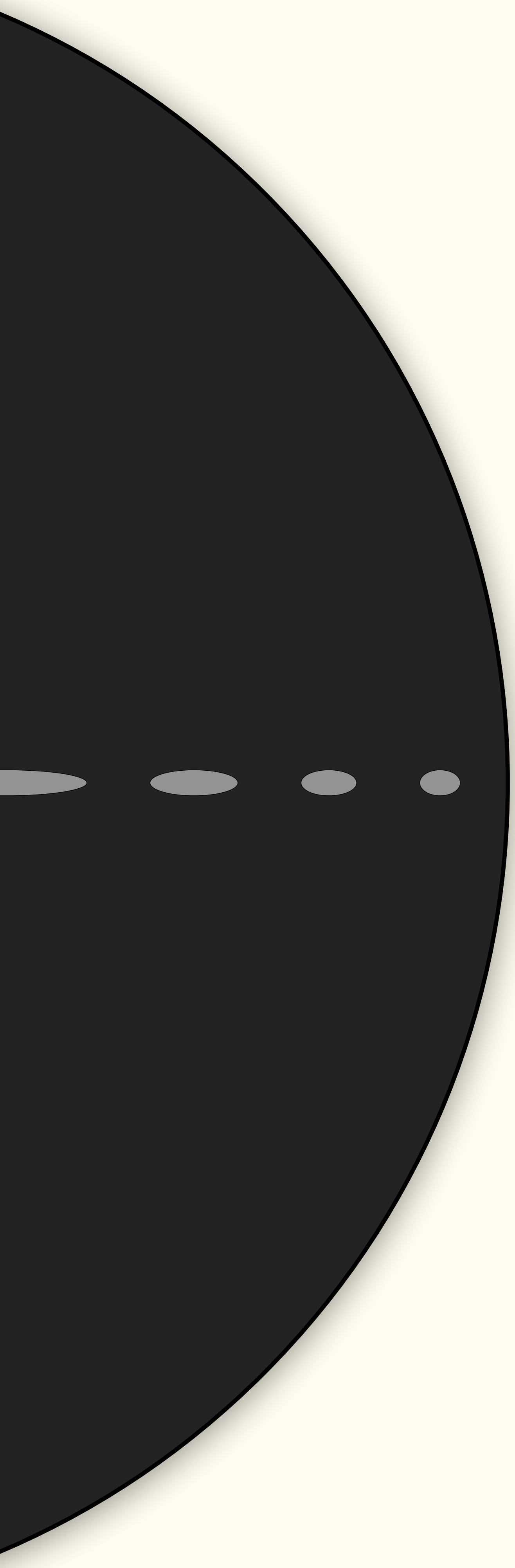

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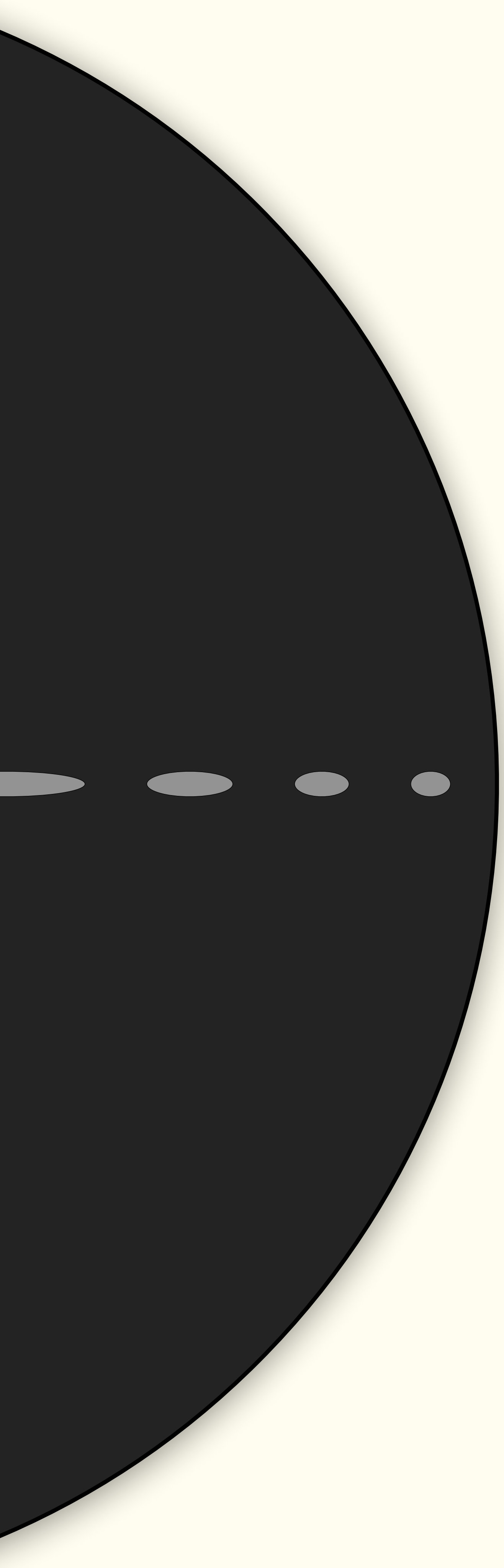

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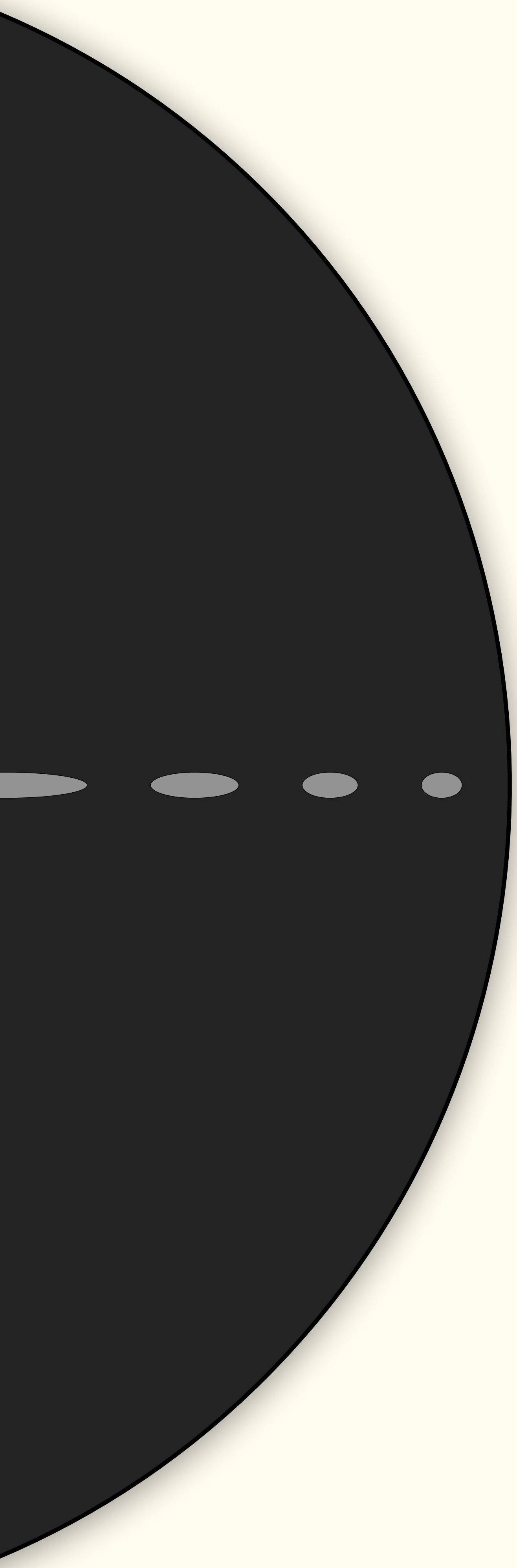

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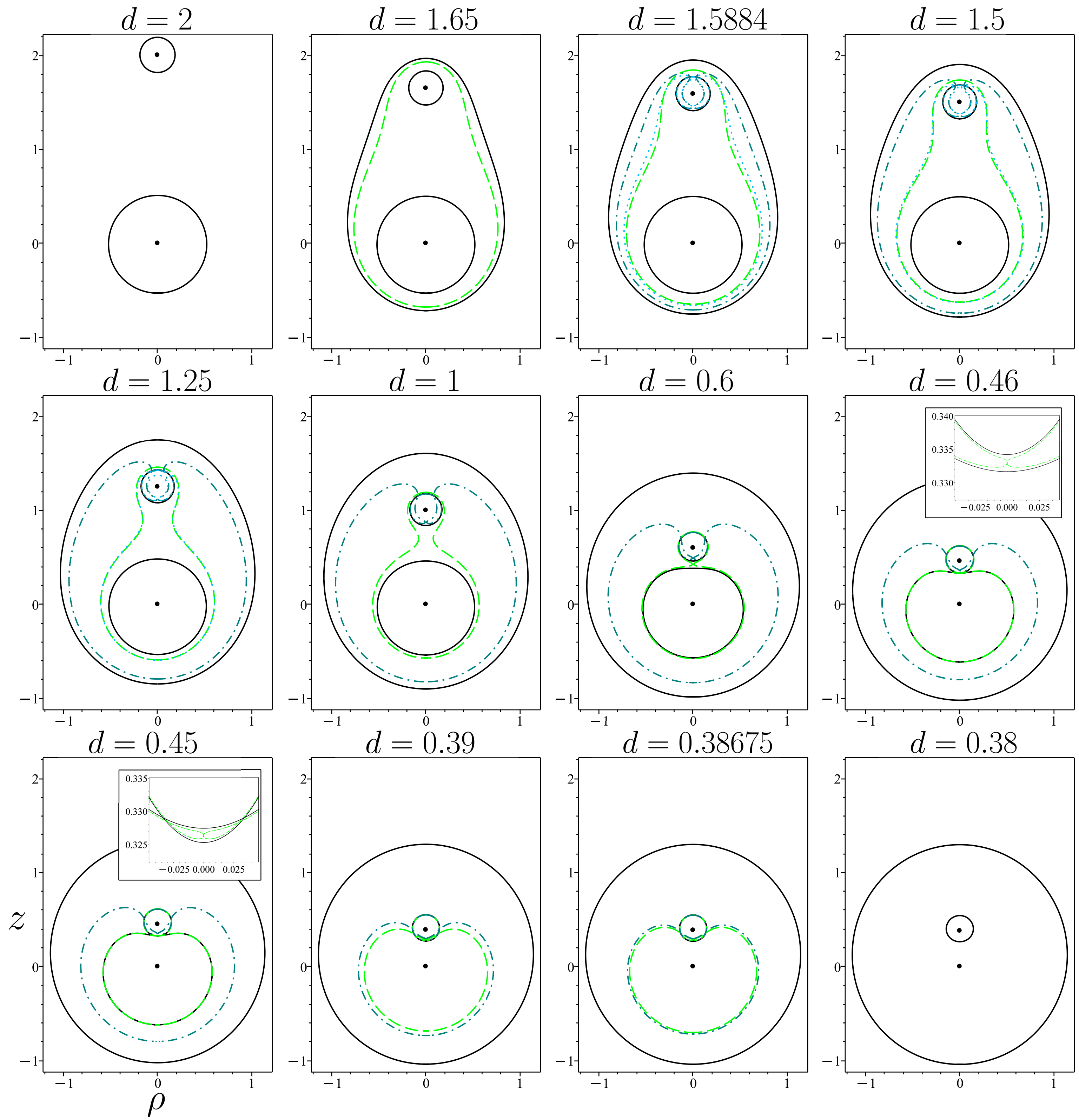
- Are their further restrictions for physical processes??

BLdS "crossing the cosmological horizon"



F. Elizondo Lopez, IB (in progress)

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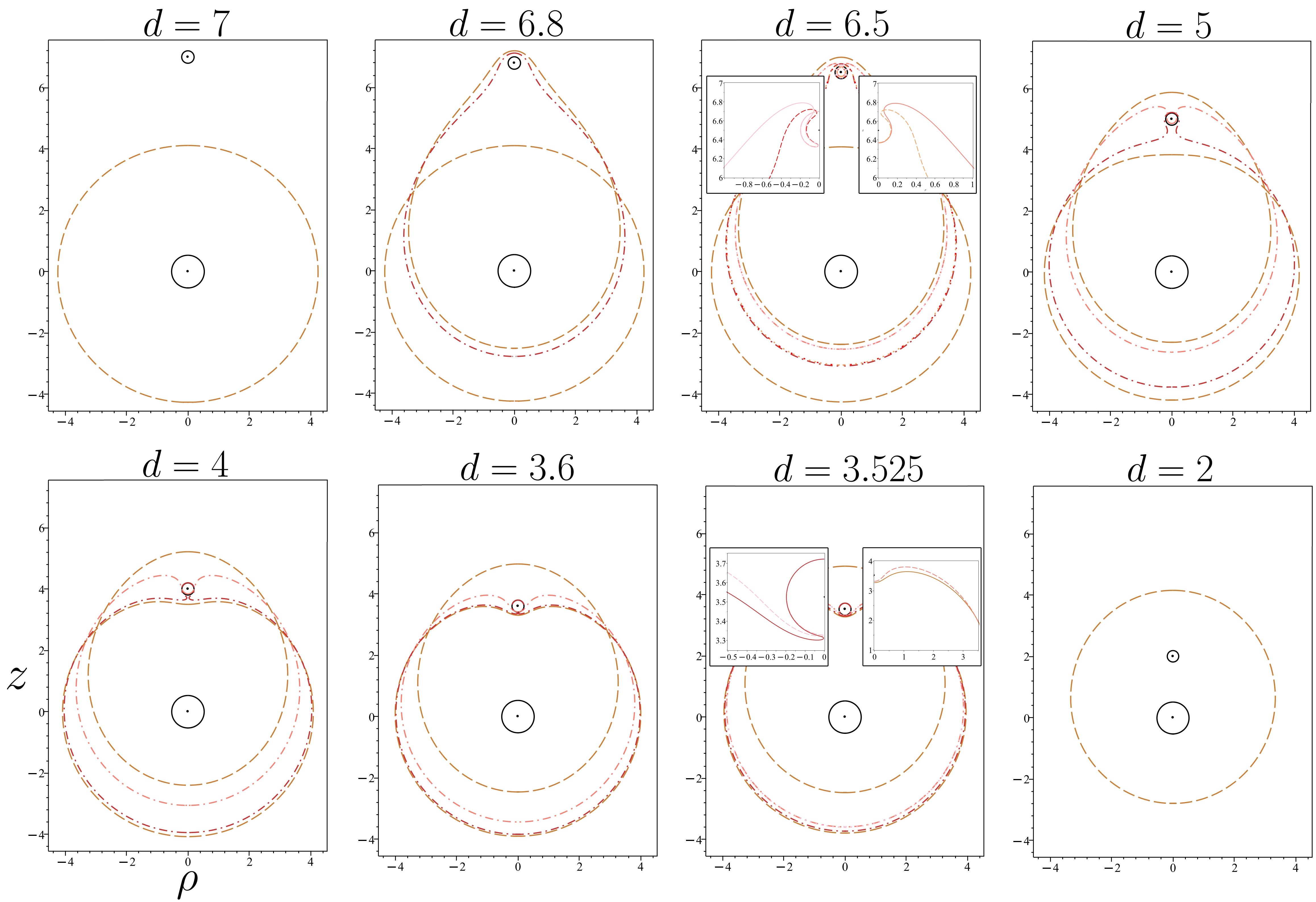

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