Volume-law entanglement from the Hawking radiation of analogue black holes

"From Black Holes to the Cosmos: Matt Visser's journey through space and time"

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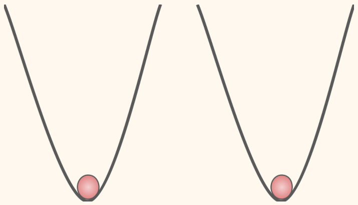
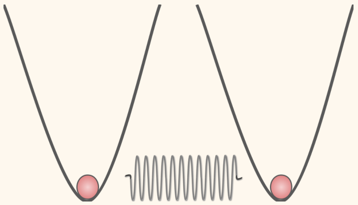
Based on arXiv:2604.02075 [gr-qc] with **S. Mahesh Chandran**

Quantum entanglement: Basics





- Subsystems are **separable**:
  $|\Psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$
- Described by **pure states**:
  $\rho_1 = |\psi_1\rangle \langle\psi_1|$, $\rho_2 = |\psi_2\rangle \langle\psi_2|$.

- Subsystems are **entangled**:
  $|\Psi\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle$
- Described by **mixed states**:
  $\rho_1 = \mathrm{Tr}_2 |\Psi\rangle \langle\Psi|$, $\rho_2 = \mathrm{Tr}_1 |\Psi\rangle \langle\Psi|$.

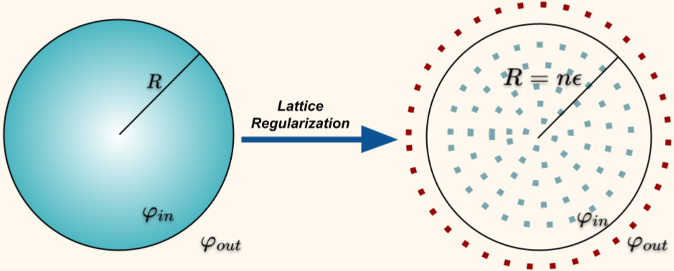
Quantifiable in terms of **entanglement entropy**: $S = -\,\mathrm{Tr}\,\rho_{1(2)} \ln \rho_{1(2)}$

Entanglement in QFT: Probing the vacuum structure

- **Vacuum correlators** of scalar field are **divergent** as $x_1 \to x_2$: [Unruh & Wald RPP '17]

$$\langle \Psi | \hat{\varphi}(x_1) \hat{\varphi}(x_2) | \Psi \rangle \propto \frac{1}{\sigma(x_1, x_2)} \quad ; \quad \sigma \equiv \text{Squared geodesic distance}$$

- **Vacuum entanglement entropy** (3+1D **flat** spacetime), regularized by UV-cutoff ϵ



$$S \sim c_0 \frac{R^2}{\epsilon^2} \quad \Leftrightarrow \quad S_{\mathrm{BH}} = \frac{A}{4}$$

Area law: For both vacuum entanglement and BH entropy! [Bombelli et al PRD '86, Srednicki PRL '93]

Hawking radiation and analogue experiments

- BHs: **area-law** scaling for entanglement entropy via "**brick-wall**" models:
  Vacuum strictly outside horizon; decoupled from interior (inaccessible) [Mukohyama PRD '98]

  $\Longrightarrow$ **However: Does not incorporate Hawking radiation!**

- **Analogue gravity** simulations of curved spacetime QFT:

  (i) Observation of **Hawking radiation** in BEC black-hole analogues

  [Steinhauer Nature Phys '16, de Nova et al Nature '19]

  (ii) Observation of **area-law** of mutual information in BEC flat-spacetime analogue

  [Tajik et al Nature Phys '23]

  $\Longrightarrow$ Experimental access to entanglement scaling of HR within sight!

  $\Longrightarrow$ Simple toy model for insights extensible to real BHs

How **vacuum structure** altered by **Hawking radiation**?

Observable signatures in analogue experiments?

Analogue black hole setup

- **Quasi-1D BEC** with flow velocity $\mathbf{v_0} = -v_0\hat{\imath}$ and local sound speed $c(x)$

- **Hydrodynamical regime**: Phonons propagate on **effective spacetime** (Painlevé-Gullstrand metric)

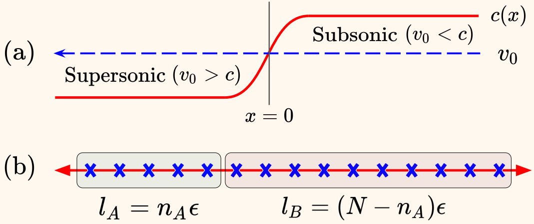
  $$\Box\varphi = 0; \quad ds^2 = -c^2dT^2 + (dx + v_0dT)^2$$

  Phase ($\theta = \theta_0 + \theta_1$) and density ($n = n_0 + n_1$) fluctuations conjugate:

  $$\varphi = \sqrt{\frac{nl_\perp^2}{c}}\theta_1 \quad ; \quad \pi = -\sqrt{\frac{cl_\perp^2}{n}}n_1$$

- BH horizon simulated via **transonic** $c(x)$

  Surface gravity $\kappa = \frac{dc}{dx}\Big|_{x=0}$ (Fig a)



(a) Sound speed $c(x)$ with horizon at $x = 0$

(b) Bipartition scheme for N lattice points sampled in $x \in \left[-\frac{L}{2}, \frac{L}{2}\right]$ with spacing ϵ

Gaussian quantum information

- **Gaussian quantum states** fully described by **covariance matrix** $\boldsymbol{\Sigma}$:

$$\boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{\Sigma}_{\varphi\varphi} & \boldsymbol{\Sigma}_{\varphi\pi} \\ \boldsymbol{\Sigma}_{\varphi\pi}^T & \boldsymbol{\Sigma}_{\pi\pi} \end{bmatrix}; \quad (\Sigma_{\varphi\varphi})_{ij} = \frac{\langle\{\hat{\varphi}_i, \hat{\varphi}_j\}\rangle}{2}, \ (\Sigma_{\varphi\pi})_{ij} = \frac{\epsilon\langle\{\hat{\varphi}_i, \hat{\pi}_j\}\rangle}{2}, \ (\Sigma_{\pi\pi})_{ij} = \frac{\epsilon^2\langle\{\hat{\pi}_i, \hat{\pi}_j\}\rangle}{2}$$

Commutation and uncertainty relations for quadratures $Q_i = \hat{\varphi}_i,\ Q_{i+N} = \epsilon\hat{\pi}_i$

$$[Q_i, Q_j] = i\Omega_{ij}; \qquad \boldsymbol{\Sigma} + \frac{i}{2}\boldsymbol{\Omega} \geq 0; \qquad \boldsymbol{\Omega} = \begin{bmatrix} O & \mathbb{I} \\ -\mathbb{I} & O \end{bmatrix} \text{ (symplectic matrix)}$$

- **PPT criterion**: If $\bar{\boldsymbol{\Sigma}}$ **partially transposed** covariance matrix w.r.t subsystem A

$\implies$ A and B **entangled** if $\bar{\boldsymbol{\Sigma}} + \frac{i}{2}\boldsymbol{\Omega} \geq 0$ violated

Logarithmic negativity $\Rightarrow$ additive entanglement measure for **PPT criterion**

$$\mathcal{E}_N = -\sum_j \ln\left[\min\left(1, 2\bar{\nu}_j\right)\right], \qquad \{\pm\bar{\nu}_j\} = \mathrm{eig}\left[\mathrm{i}\boldsymbol{\Omega}\bar{\boldsymbol{\Sigma}}\right] \text{ (symplectic eigenvalues)}$$

NB! Unlike entanglement entropy, $\mathcal{E}_N$ operational even when $A \cup B$ not pure!

Quantum state tomography: Experiment vs simulation

- **Tomography**: Experimentally reconstructing $\boldsymbol{\Sigma}$ via in-situ imaging of BEC

  $\Longrightarrow$ Recently demonstrated for flat analogue!

  [Tajik et al Nature Phys '23]

- To accurately simulate $\boldsymbol{\Sigma}$, impose:
  (i) **spatial resolution bandwidth** k_{UV}
  (ii) **coarse-graining length scale** ϵ

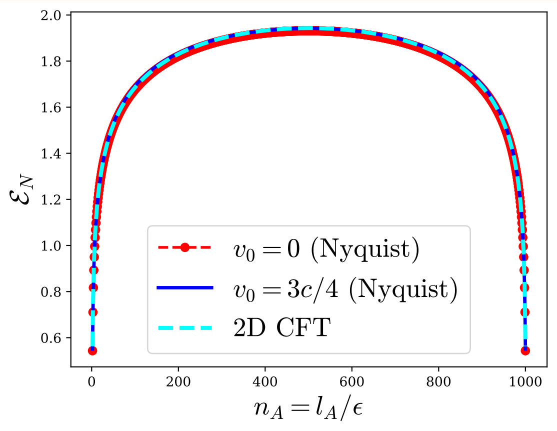
- Fix **Nyquist spacing** $\epsilon_{\mathrm{Nyq}} = \pi / k_{\mathrm{UV}}$, for max entanglement experimentally resolvable

  (NB! In general, $\epsilon_{\mathrm{Nyq}} \leq \epsilon \lesssim 2\epsilon_{\mathrm{Nyq}}$ suffices)

- **Vacuum scaling** simulation matches known

  UV-divergent result in 2D CFT: [Calabrese et al '12]

$$\mathcal{E}_N(l_A) = \frac{1}{4} \ln \left[\frac{l_A(L - l_A)}{L\epsilon} \right] + \text{const.}$$



Fixing initial and final phonon states

- **Initial state conformal Painlevé-Gullstrand vacuum** $(\hat{a}_u|0\rangle = \hat{a}_v|0\rangle = 0)$:

$$\hat{\varphi} = \int_0^\infty d\omega \left[\hat{a}_u(\omega) \frac{e^{-i\omega u}}{\sqrt{4\pi\omega}} + \hat{a}_v(\omega) \frac{e^{-i\omega v}}{\sqrt{4\pi\omega}} + h.c. \right]$$

- Switching on transonic flow **mimics gravitational collapse**

Real and analogue BHs:, Asymptotically **Unruh state** $(\hat{a}_K|U\rangle = \hat{a}_v|U\rangle = 0)$

$$\hat{\varphi} = \int_0^\infty d\omega_K \left[\hat{a}_K(\omega_K) \frac{e^{-i\omega_K U_K}}{\sqrt{4\pi\omega_K}} + h.c \right] + \int_0^\infty d\omega \left[\hat{a}_v(\omega) \frac{e^{-i\omega v}}{\sqrt{4\pi\omega}} + h.c \right]$$ [Unruh PRD '76, Candelas PRD '80]

defined **globally** (unlike "brick-wall") via Kruskal-Szekeres coordinates: [Anderson et al PRD '13]

$$U_K = \begin{cases} e^{-\kappa u}/\kappa & x < 0 \\ -e^{-\kappa u}/\kappa & x > 0 \end{cases} \quad ; \quad V_K = \begin{cases} e^{\kappa v}/\kappa & x < 0 \\ e^{\kappa v}/\kappa & x > 0 \end{cases}$$

- **Outgoing modes thermally distributed** $\implies$ spontaneous **Hawking radiation!**:

$$\langle U|\hat{a}_u^\dagger(\omega)\hat{a}_u(\omega')|U\rangle = N_\omega \delta(\omega - \omega'); \quad N_\omega = \frac{1}{e^{2\pi\omega/\kappa} - 1} \implies T_{\mathrm{H}} = \frac{\kappa}{2\pi}$$

Nyquist-spaced two-point correlators

- Decomposing **initial vacuum** correlators into **outgoing** (I) and **ingoing** (J) branches:

$$\mathbf{\Sigma}_{\varphi\varphi} = \mathbf{I}_{\varphi\varphi} + \mathbf{J}_{\varphi\varphi};$$

$$(I_{\varphi\varphi})_{jk} = \int_{\tilde{\omega}_{0u}}^{\tilde{\omega}_{1u}} \frac{d\tilde{\omega}}{4\pi\tilde{\omega}} \cos\left(\frac{\tilde{\omega}|j-k|}{1-\tilde{v}_0}\right) \qquad (J_{\varphi\varphi})_{jk} = \int_{\tilde{\omega}_{0v}}^{\tilde{\omega}_{1v}} \frac{d\tilde{\omega}}{4\pi\tilde{\omega}} \cos\left(\frac{\tilde{\omega}|j-k|}{1+\tilde{v}_0}\right)$$

- Correlators for final (**Unruh**) state decompose similarly:

$$(I_{\varphi\varphi})_{jk} = \int_{\tilde{\omega}_{0u}}^{\tilde{\omega}_{1u}} \frac{d\tilde{\omega}}{4\pi\tilde{\omega}} f_{\omega}(\tilde{\kappa}) \cos\left[\tilde{\omega}(\tilde{u}_j - \tilde{u}_k)\right] \qquad (J_{\varphi\varphi})_{jk} = \int_{\tilde{\omega}_{0v}}^{\tilde{\omega}_{1v}} \frac{d\tilde{\omega}}{4\pi\tilde{\omega}} \cos\left[\tilde{\omega}(\tilde{v}_j - \tilde{v}_k)\right]$$

$$f_{\omega}(\tilde{\kappa}) = \begin{cases} \coth\left(\frac{\pi\tilde{\omega}}{\tilde{\kappa}}\right) & \text{for L-L or R-R} \\ \operatorname{csch}\left(\frac{\pi\tilde{\omega}}{\tilde{\kappa}}\right) & \text{for L-R} \end{cases}; \quad \tilde{u}_j\epsilon = c_R u|_{x=j\epsilon}; \ \tilde{v}_j\epsilon = c_R v|_{x=j\epsilon}; \ \tilde{\kappa} = \frac{\kappa\epsilon}{c_R}$$

- **Nyquist spacing for UV cutoffs** incorporated via dispersion $\omega_{1u,1v}(k_{\mathrm{UV}} = \pi/\epsilon)$

Nonlocal correlations from Hawking radiation

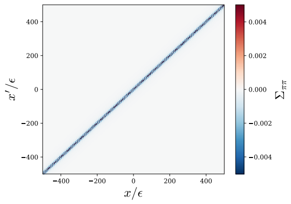
- **Initial vacuum** state features a **local correlation peak** along $x \sim x'$ (Fig a)
- **Unruh state**: Positive, **nonlocal peak** emerges in $\mathbf{I}_{\varphi\varphi}$ correlator, along **paths of Hawking quanta**:

$$\frac{x}{|v_0 - c_L|} + \frac{x'}{|v_0 - c_R|} = 0$$

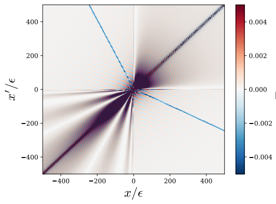
- Corresponds to **negative peak** in $\mathbf{I}_{\pi\pi}$

  $\implies$ outgoing Hawking modes with **opposite momenta** (Fig b)

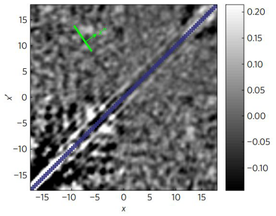
- Experimentally confirmed via **density-density correlations**! (Figs c,d)



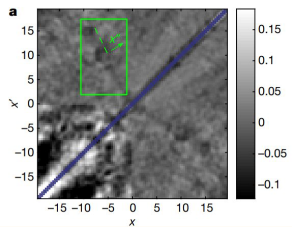
(a) Initial vacuum



(b) Unruh state



(c) [Steinhauer '16]



(d) [de Nova et al '19]

Entanglement scaling for Hawking radiation

- Numerical simulations:

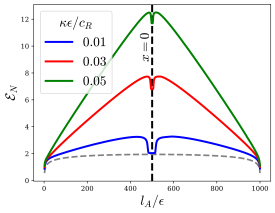
$$\mathcal{E}_N \sim \mathcal{E}_N^{(\mathrm{vac})} + \mathcal{E}_N^{(\mathrm{HR})}$$

- Hawking radiation piece **UV-finite**:

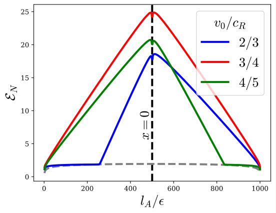
$$\mathcal{E}_N^{(\mathrm{HR})}(l_A) \sim \frac{\kappa}{8 v_{\mathrm{H}}^{L,R}} \max\left[0, \frac{l_h v_{\mathrm{H}}^{L,R}}{v_{\mathrm{H}}^{\max}} - |l_A - l_h|\right]$$

$v_{\mathrm{H}}^{L,R} = |v_0 - c_{L,R}|$
Hawking mode velocities in L/R regions

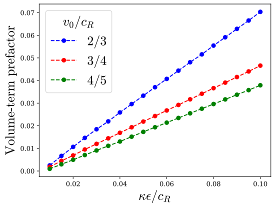
$v_{\mathrm{H}}^{\max} = \max[v_{\mathrm{H}}^{L}, v_{\mathrm{H}}^{R}]$ and $\quad l_h = N\epsilon/2$



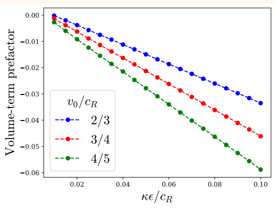
(a) $v_0/c_R = 3/4$



(b) $\kappa\epsilon/c_R = 0.1$



(c) $l_A < l_H$



(d) $l_A > l_H$

Volume-law = entanglement density $\times$ subsystem volume

- Unruh state expressible by **two-mode squeezed state** in each ω-sector:

$$|U\rangle^{(\omega)} = \frac{1}{\cosh r_\omega}\sum_n (\tanh r_\omega)^n |n\rangle_L^{(\omega)} |n\rangle_R^{(\omega)}\,; \quad r_\omega = \sinh^{-1}\sqrt{N_\omega}\,; \quad N_\omega = \frac{1}{e^{2\pi\omega/\kappa}-1}$$

- **Pairwise negativity**: Entanglement in each Hawking pair $\mathcal{E}_N^{(\omega)} = 2r_\omega$

- Local (in L or R region) **spatial density** for pairwise negativity:

$$\int_{k_{0u}}^{k_{1u}} \frac{dk}{2\pi}\mathcal{E}_N^{(\omega)} = \int_{\omega_{0u}}^{\omega_{1u}} \frac{d\omega \;\; \ln\left[\coth\left(\frac{\pi\omega}{2\kappa}\right)\right]}{2\pi v_{\mathrm{H}}^{L,R}} \sim \frac{\kappa}{8 v_{\mathrm{H}}^{L,R}}$$

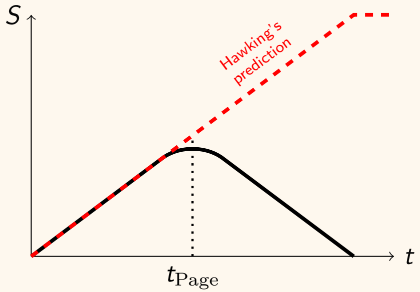
Spatial density of pairwise negativity in L/R regions
encodes both volume-law **prefactor** and its **UV-finiteness!**

Final comments: Page curve

- Subsystems with n_A and n_B qudits (dimension d)
- Average entanglement entropy (leading order) over random **pure states**: [Page PRL '93]

  $\langle S_A \rangle \sim (\ln d) \min[n_A, n_B] \implies$ **volume-law**

  $\implies$ turnover at $n_A = n_B$



- **Page curve**: **Unitarity benchmark**; crucial to black hole **information loss problem**
- Obtained **Page-curve**-like **volume-law scaling**

  $\mathcal{E}_N^{(\mathrm{HR})} \sim \frac{\kappa}{8 v_{\mathrm{H}}} \min\left[l_A, l_B\right]$ **from microscopic description**

  for early stages of evaporation (**Unruh state**) NB! No statistical averaging required!

  [Time dependence $\implies$ Future work]

Conclusions and Outlook

- **Conclusions:**

  Analogue BH: Simple extraction of **entanglement scaling** for Hawking radiation **experimentally verifiable**

- **Hawking radiation** $\Longrightarrow$ **UV-finite volume-law** emerges from entangled Hawking pairs

- For **real BHs** **supersedes** brick-wall **area-law** (obtained in absence of Hawking radiation) — **volume-law significant for smaller/hotter BHs!**

- **Outlook:**

  Extension to real BHs, dispersive effects, backreaction, late-stage evaporation

- Tomography + analogue BH setup $\Longrightarrow$ **Near-future experimental access!**

arXiv:2604.02075 [gr-qc] with **S. Mahesh Chandran**

Hawking Radiation with~~out~~ Black Hole Entropy

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Hawking radiation is a purely kinematic effect that is generic to Lorentzian geometries containing event horizons; it is independent of dynamics. On the other hand, the classical laws of black hole mechanics and the semiclassical laws of black hole thermodynamics are both inextricably linked with dynamics: Black hole entropy is proportional to area (plus corrections) if and only if the dynamics is Einstein-Hilbert (plus corrections). Hawking radiation can occur in physical situations in which the laws of black hole mechanics do not apply, and in physical situations in which the notion of black hole entropy does not even make any sense. [S0031-9007(98)05868-2]

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