BREAKING SYMMETRIES *MA NON TROPPO*: TRANSVERSE DIFFEOMORPHISMS, INTERACTING DARK FLUIDS, AND UNIFIED DARK SECTOR.

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FROM BLACK HOLES TO THE COSMOS:
MATT VISSER'S JOURNEY THROUGH SPACE AND TIME
24-28 AUGUST 2026

UNIVERSITAS COMPLVTENSIS
FACULTAD DE CIENCIAS FÍSICAS
IPARCOS
MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES
Cofinanciado por la Unión Europea
AGENCIA ESTATAL DE INVESTIGACIÓN

OUTLINE

- Breaking symmetries *ma non troppo.*
- The TDiff scalar-field theory.
- Interacting dark sector from Diff breaking.
- The silent unified dark sector.
- Conclusions.

BREAKING SYMMETRIES

MA NON TROPPO

THE STANDARD PARADIGM

General Relativity + dark components is our best description of gravitational phenomena.

- **Principle of General Covariance**: "*the laws of physics retain the same form under arbitrary coordinate transformations*" -> **Diffeomorphism (Diff) invariance.**
  - Diff symmetry is the gauge symmetry of GR.
  - It fixes the matter couplings to gravity and the number of propagating d.o.f.

- The fundamental nature of the dark sector is poorly understood.
  - The standard model of cosmology is the **ΛCDM model**.
  - The ΛCDM model is very successful from a phenomenological point of view.

WHY WOULD ONE GO BEYOND THE STANDARD PARADIGM?

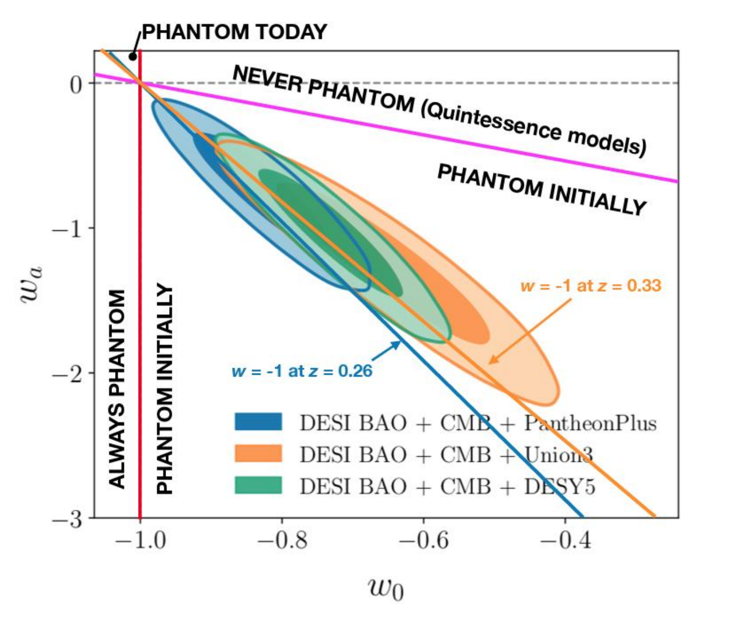
- Why is Λ so tiny, but still not zero? Do we need to find a reason? Carroll, arXiv: 0004075
  - **H0 tension** when assuming ΛCDM ($\sim 4\sigma$).

    Riess, Casertano, Yuan, Macri and Scolnic, arXiv: 1903.07603

    The tension may be alleviated considering:

    ➢ Phantom models ($w = p/\rho < -1$), dark sector interactions,…

    Valentino, Melchiorri, arXiv: 1704.08342. Valentino, Mukherjee, Sen, arXiv: 2005.12587.



- Recent data may indicate the need of **evolving dark energy**.
  - ➢ Weak preference against ΛCDM $\sim 3.2\sigma$ (updated from $\sim 4\sigma$).
  - ➢ If CPL parametrization, $w = w_0 + w_a(1-a)$

Karim et al. arXiv: 2503.14738. Popovic et al., arXiv:2511.07517
Figure: Cortês y Liddle, arXiv 2404.08056

We should examine the foundational pillars of the standard model.

WHY AND HOW DOES ONE BREAK DIFF INVARIANCE?

For the massless spin 2 case, classical stability just requires that the theory is invariant under **transverse diffeomorphisms** (TDiff)

$$\boxed{\hat{x}^\mu = x^\mu + \xi^\mu(x)} \quad \text{with} \quad \boxed{\partial_\mu \xi^\mu = 0}$$

Van der Bij, Van Dam and Ng, Physica A 116 (1982).

- ➢ There is one gauge freedom less than in general Diff theories.

Álvarez, Blas, Garriga, Verdaguer, hep-th/0606019.

This symmetry can be enhanced in one of two ways:

- Full diffeomorphisms (Diff) invariance is impossed: this is **General Relativity**.
- Weyl symmetry is required (WTDiff invariance): this is equivalent to **Unimodular Gravity**.
  - ➢ Trace-free Einstein equations. Non-dynamical field: $g = |\det(g_{\mu\nu})| = 1$ is fixed.
  - ➢ Λ emerges as a constant of integration, instead of being a parameter in the Lagrangian.

Einstein, (1919). Unruh, PRD 40, 1048 (1989).

TDIFF INVARIANT MATTER

TDiff theories:

- Physics laws are invariant only under coordinates transformations such that $J = \det\left(\frac{\partial \hat{x}^{\mu}}{\partial x^{\nu}}\right) = 1.$
- Tensor densities become tensors.

Alvarez and Faedo, arXiv: 0702184

In this talk Diff invariance breaks to TDiff **through the matter sector**:

$$\Rightarrow \quad d^4x\sqrt{g} \to d^4x\, f(g)$$

- The "weight of energy" is modified.

Álvarez, Faedo, and López-Villarejo, arXiv :0904.3298

- Einstein-Hilbert action for gravity: $\boxed{S = S_{EH} + S_m} \Rightarrow \boxed{G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}}$
- Even if Diff is broken, Bianchi identities still hold:
  - The **total SET** is conserved **on solutions of the Einstein equations**.
  - We will ultimately get a **constraint** for g.

$$\boxed{T_{\mu\nu} = \frac{2}{\sqrt{|g|}}\frac{\delta S_m}{\delta g^{\mu\nu}}}$$

Maroto, arXiv:2301.05713. Jaramillo Garrido, Maroto and PMM, arXiv: 2307.14861 [gr-qc]

THE TDIFF SCALAR FIELD THEORY

TDIFF SCALAR FIELD THEORY: *A SIMPLE EXAMPLE*

$$S_m = \int d^4x \left\{ \frac{f_k(g)}{2} g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi - \cancel{f_v(g) V(\psi)} \right\}$$

Shift-symmetric field or dominant kinetic limit.

- ψ field equation: $\partial_\mu \left(f_k \partial^\mu \psi \right) = 0$

- The SET of Einstein equations takes the simple form $T_{\mu\nu} = (\rho + p) u_\mu u_\nu - p g_{\mu\nu}$ $\qquad u^\alpha \equiv \dfrac{\partial^\alpha \psi}{\sqrt{(\partial\psi)^2}}$

This is a **perfect fluid** when u^α is timelike.

$$\rho = \frac{(\partial\psi)^2}{\sqrt{g}} \left(f_k - g f_k' \right), \quad p = \frac{(\partial\psi)^2}{\sqrt{g}} g f_k'.$$

$$w = \frac{p}{\rho} = \frac{g f_k'}{f_k - g f_k'}.$$

It's a function of g only!

- SET conservation + field equation imply a constraint on g: $\qquad (2F - 1) \dfrac{g}{f_k} = C_g(x) \delta V^2$

$F \equiv \dfrac{g f_k'}{f_k}$ $\qquad \theta = \nabla_\mu u^\mu = u^\mu \partial_\mu \ln \delta V$ $\qquad u^\mu \partial_\mu C_g(x) = 0$

Jaramillo Garrido, Maroto and PMM, arXiv: 2307.14861.

Cosmology:

- $b(\tau)$ is not a gauge function.

$$ds^2 = b^2(\tau) d\tau^2 - a^2(\tau) d\vec{x}^2 \Rightarrow g \propto a^{\frac{6}{1-\alpha}} \qquad b \propto a^{\frac{3\alpha}{1-\alpha}}$$

Maroto, arXiv:2301.05713

TDIFF SCALAR FIELD THEORY: *A SIMPLE EXAMPLE*

Combining the constraint on g with the field equation for $\alpha \neq 1/2$

$$\rho = \frac{c_\rho}{(w-1)\sqrt{g}} \; .$$

It's a function of g only!

Fluid perturbations are adiabatic for this simple case: $\delta p = c_s^2\, \delta\rho$

- Constant equation of state models: $f_k(g) = C|g|^{\hookleftarrow}$

$$w = \frac{\hookleftarrow}{1-\hookleftarrow}$$

$$c_s^2 = w$$

Diff invariant case	$\alpha = 1/2$	$w_\phi = 1$
TDiff dark matter	$\alpha = 0$	$w_\phi = 0$
Accelerated expansion	$\alpha < -1/2$	$w_\phi < -1/3$

Jaramillo Garrido, Maroto and PMM, arXiv: 2307.14861.

SYMMETRY RESTORATION I

Diff symmetry can be restored by introducing additional Stueckelberg-like fields.

- Each term of the TDiff action has the form

$$S_{\mathrm{TDiff}}[g_{\mu\nu},\Psi]=\int d^4x\, f(g)\,\mathcal{L}\left(g_{\mu\nu},\Psi,\partial_\mu\Psi\right)$$

TDiff frame $\bar{\mu}=1$

- Let's consider the Diff action

$$S_{\mathrm{Diff}}[g_{\mu\nu},\Psi,\bar{\mu}]=\int d^4x\,\sqrt{g}\,\underbrace{\left[\frac{\bar{\mu}}{\sqrt{g}}\,f(g/\bar{\mu}^2)\right]}_{H(\bar{\mu}/\sqrt{g})\,\equiv\,\frac{\bar{\mu}}{\sqrt{g}}\,f(g/\bar{\mu}^2)}\,\mathcal{L}\left(g_{\mu\nu},\Psi,\partial_\mu\Psi\right)$$

where we have introduce the scalar density $\bar{\mu}$, so that $\bar{\mu}/\sqrt{g}$ is a Diff scalar.

- How is the scalar density related to the new field?

Henneaux and Teitelboim, Phys. Lett. B 222, 195 (1989): we build the scalar density with a vector field $\bar{\mu}=\partial_\mu(\sqrt{g}A^\mu)$

$$\frac{\bar{\mu}}{\sqrt{g}}=\nabla_\mu A^\mu=Y \qquad H(Y)\equiv Y\,f(Y^{-2})$$

$$\boxed{S_{\mathrm{Diff}}[g_{\mu\nu},\psi,T^\mu]=S_{\mathrm{EH}}+\int d^4x\,\sqrt{g}\,[H_k(Y)X-H_v(Y)V]}$$

$$X\equiv\frac{1}{2}g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi$$

Other options are possible.

Jaramillo, Maroto, PMM arXiv: 2402.17422

GRAVITY IN THE DIFF FRAME (*GENERAL MODELS*)

$$S_{\text{Diff}}[\psi, g_{\mu\nu}, A^\mu] = S_{EH} + \int d^4x\sqrt{g}\,[H_k(Y)X - H_v(Y)V(\psi)]$$

- Einstein equations: $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ with $$T_{\mu\nu} = H_k(Y)\partial_\mu\psi\partial_\nu\psi - [H_k(Y)X - H_v(Y)V]\,g_{\mu\nu} + Y\,[H_k'(Y)X - H_v'(Y)V]\,g_{\mu\nu}$$
- ψ field equation: $\nabla_\nu\,[H_k(Y)g^{\mu\nu}\nabla_\mu\psi] + H_v(Y)V'(\psi) = 0\,.$
- A^μ field equation: $\partial_\nu\,[H_k'(Y)X - H_v'(Y)V] = 0\,.$ **TDiff constraint** $\Rightarrow$ $\boxed{H_k'X - H_v'V = -\frac{c_\rho}{2}}$

c_ρ is an integration constant.

$Y = Y(X, \psi)$ — K-essence theories

$X = X(\psi)$ — Mimetic theories

TDiff dictionary

$$Y \to 1/\sqrt{g}$$

$$H(Y) = Yf(Y^{-2}) \to f(g)/\sqrt{g}$$

Jaramillo, Maroto, PMM, arXiv: 2402.17422

Beltrán, Borislavov, Jaramillo, Maroto, PMM, arXiv: 2509.07715

THE TDIFF FLUID

- If u^μ is timelike, the SET takes the perfect fluid form

$$\rho = H_k X + H_v V - \frac{c_\rho}{2} Y$$
$$p = H_k X - H_v V + \frac{c_\rho}{2} Y$$

$$w = \frac{H_k X - H_v V + Y c_\rho/2}{H_k X + H_v V - Y c_\rho/2}$$

  - ➢ It cannot cross the phantom line: $\rho + p = 2H_k X \geq 0$

- Fluid perturbations

Using the constraint: $X = X(Y, \psi)$ ⇒ $\rho = \rho(\psi, Y)$, $p = p(\psi, Y)$ ⇒

$$\begin{cases} \delta\rho = \left.\dfrac{\partial\rho}{\partial Y}\right|_\psi \delta Y + \left.\dfrac{\partial\rho}{\partial\psi}\right|_Y \delta\psi \\ \delta p = \left.\dfrac{\partial p}{\partial Y}\right|_\psi \delta Y + \left.\dfrac{\partial p}{\partial\psi}\right|_Y \delta\psi \end{cases}$$

Shift-symmetric case is recovered for $X = X(\psi)$.

➢ We join $\delta\rho$ and δp expressions to write

$$\delta p = c_s^2\, \delta\rho + \alpha\, \delta\psi$$

$$c_s^2 \equiv \left.\frac{\partial p/\partial Y}{\partial \rho/\partial Y}\right|_\psi \qquad \alpha \equiv (-c_s^2) \left.\frac{\partial \rho}{\partial \psi}\right|_Y + \left.\frac{\partial p}{\partial \psi}\right|_Y$$

Effective speed of sound Non-adiabatic coefficient.

$$c_s^2 = \frac{A(Y,\psi)}{A(Y,\psi) - B(Y,\psi)}$$

$$A(Y,\psi) \equiv H_k \left[V\left(H_k'' H_v' - H_k' H_v''\right) - \frac{c_\rho}{2} H_k'' \right]$$
$$B(Y,\psi) \equiv 2(H_k')^2 \left(H_v' V - \frac{c_\rho}{2} \right)$$

➢ Stable perturbations: $c_s^2 \geq 0$ $\dfrac{B(Y,\psi)}{A(Y,\psi)} < 1$

➢ Avoid superluminalities: $c_s^2 \leq 1$ $\dfrac{B(Y,\psi)}{A(Y,\psi)} \leq 0$

Jaramillo, Maroto, PMM, arXiv: 2402.17422

SYMMETRY RESTORATION II

$$S_{\rm COV}[g_{\mu\nu},\psi,A^\mu]=\int \mathrm{d}^4x\,\sqrt{g}\Big[H_k(Y)X-H_v(Y)V(\psi)\Big]$$

Since the vector-field appears only through a divergence, it may be replaced by a second scalar field, taking some subtleties into account.

Blas, Shaposhnikov, Zenhausern, arXiv: 1104.1292.

- Let's consider a Lagrange multipler μ to enforce $\phi=\nabla_\alpha A^\alpha=Y$

$$S=\int \mathrm{d}^4x\sqrt{g}\left[H_k(\phi)X-H_v(\phi)V(\psi)+\mu\left(\nabla_\alpha A^\alpha-\phi\right)\right]$$

➢ Variation with respect to A^α leads to $\nabla_\alpha\mu=0$. So, up to a boundary term, we have

$$\Rightarrow\quad S[\psi,\phi]=\int \mathrm{d}^4x\sqrt{g}\left[H_k(\phi)X-H_v(\phi)V(\psi)-\mu_0\phi\right]\quad\Rightarrow$$

$$\begin{aligned}\delta\psi:&\quad \nabla_\alpha\left[H_k(\phi)\nabla^\alpha\psi\right]+H_v(\phi)V'(\psi)=0,\\ \delta\phi:&\quad H_k'(\phi)X-H_v'(\phi)V(\psi)=\mu_0,\end{aligned}$$

➢ Parametrized family of actions, one for each value of μ_0.

➢ The new field ϕ is a spectator field.

➢ μ_0 is a global degree of freedom.

$\Rightarrow$ $\mu_0=-c_\rho/2$ is a parameter in the Lagrangian, instead of a constant of integration.

De Cruz, Martoto,arXiv: 2504.02541.
De Cruz, Jaramillo, Maroto, PMM, 2607.19494.

INTERACTING DARK SECTOR FROM DIFF BREAKING

MULTI-FIELD TDIFF THEORY

Let's now consider 2 TDiff scalar fields **but no interaction term**: $S = S_{EH} + S_{mat}$

$$S_{mat} = \int d^4x[f_{K1}(g)X_1 - f_{V1}V_1(\phi_1) + f_{K2}(g)X_2 - f_{V2}V_2(\phi_2)]$$

$$S_{mat} = \int d^4x\sqrt{g}\,[H_{K1}(Y)X_1 - H_{V1}(Y)V_1(\phi_1) + H_{K2}(Y)X_2 - H_{V2}(Y)V_2(\phi_2)]$$

- The SET can be written as: $T_{\mu\nu} = T^{(1)}_{\mu\nu} + T^{(2)}_{\mu\nu}$ $\quad T^{(i)}_{\mu\nu} = (\rho_i + p_i)u_\mu u_\nu - p_i g_{\mu\nu}$ $\quad \rho_i(\phi_i, Y), p_i(\phi_i, Y)$

- Due to Bianchi identities: $\nabla_\mu T^{\mu\nu} = \nabla_\mu T^{(1)\mu\nu} + \nabla_\mu T^{(2)\mu\nu} = 0$ This does not imply the conservation of the individual SET.

An **effective interaction between the fields** is induced as a consequence of the symmetry breaking.

- **Cosmology** in the Diff framework: $ds^2 = a(\eta)^2(d\eta^2 - d\mathbf{x}^2)$

$$\dot\rho_1 + 3\frac{\dot a}{a}[\rho_1 + p_1] = Q,$$
$$\dot\rho_2 + 3\frac{\dot a}{a}[\rho_2 + p_2] = -Q,$$

Maroto, PMM, Tessainer, arXiv: 2409.11991, 2507.16616, 2605.18424

MULTI-FIELD TDIFF THEORY: *SHIFT-SYMMETRIC CASE*

$$S_{\rm mat} = \int d^4x \sqrt{g}[H_1(Y)X_1 + H_2(Y)X_2]$$

Shift-symmetric field or dominant kinetic limit.

- No interaction potential: 1 EoM for each scalar-field that is independent of the other field:

$$X_i = \frac{C_{\phi_i}^2}{2H_i(Y)^2 a^6}$$

- The A^μ-equation, or TDiff constraint, couples the scalar fields:

$$H_1'(Y)X_1 + H_2'(Y)X_2 = C_g$$

$$H_1'(Y)\frac{C_{\phi_1}^2}{2H_1(Y)^2} + H_2'(Y)\frac{C_{\phi_2}^2}{2H_2(Y)^2} = C_g a^6.$$

- ➢ The form of $Y(a)$ is determined by the dynamics of both fields, and it affects the dynamics of the fields.
- ➢ $Y \to 1/\sqrt{g}$, so this is provides us with $g(a)$ and, therefore, $b(a)$ in the TDiff frame.

Maroto, PMM, Tessainer, arXiv: 2409.11991, 2605.18424

SHIFT-SYMMETRIC CASE WITH CONSTANT W

$$H_i(Y) = \lambda_i Y^{\alpha_i}$$

$$w_i = \frac{1-\alpha_i}{1+\alpha_i}$$

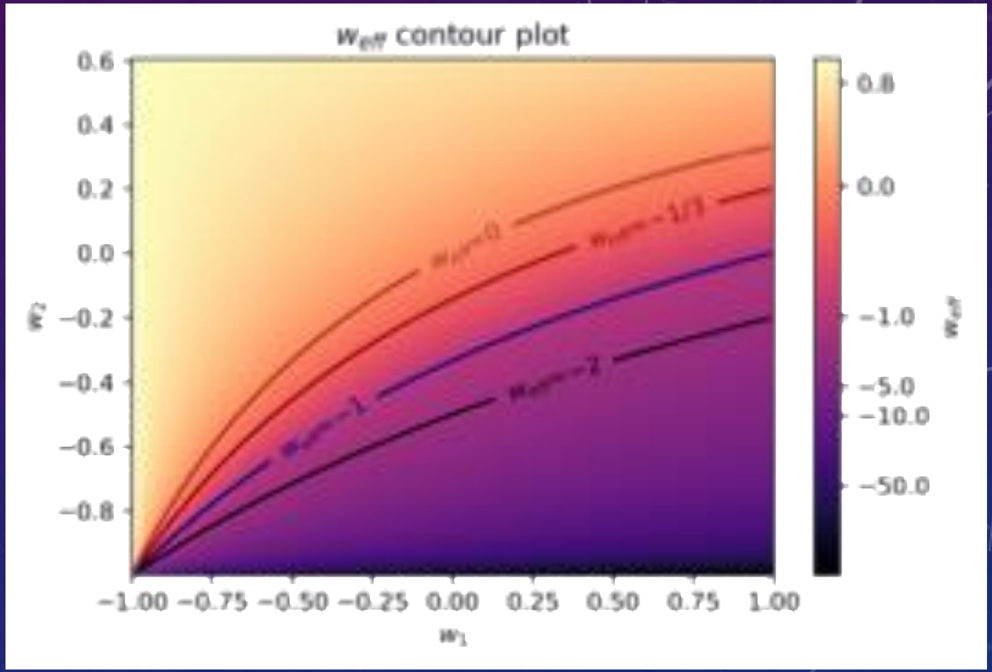
But, due to the interaction, the TDiff fluids can decay differently than in the Diff case.

- Single-field domination regimes:

  when ϕ_1 dominates, the asymptotic behaviour is

$$\rho_1(a) \propto a^{-3(1+w_1)} \quad \text{and} \quad \rho_2(a) \propto a^{-3(1+w_{\rm eff})}$$

> ➢ Even if $w_i \geq -1$, one fluid can present an **effective phantom behaviour** during the domination of the other.

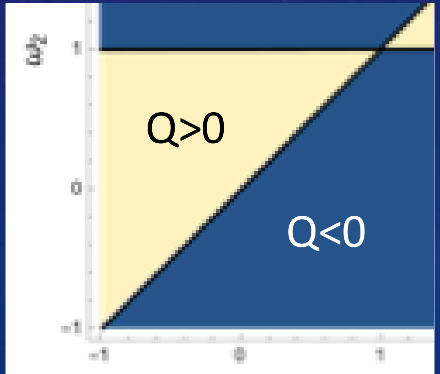


- Energy exchange: interacting kernel Q

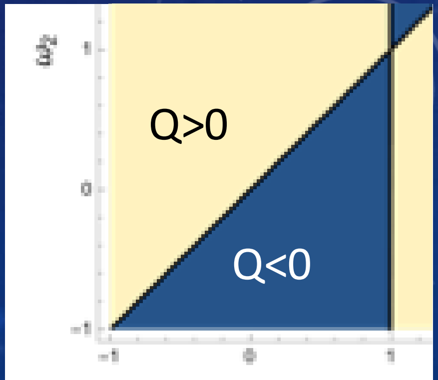
$$\dot{\rho}_1 + 3\frac{\dot{a}}{a}[\rho_1 + p_1] = Q,$$
$$\dot{\rho}_2 + 3\frac{\dot{a}}{a}[\rho_2 + p_2] = -Q.$$

- $w_1 < w_2 < 1 \Rightarrow Q > 0$: fluid 2 loses energy during all the evolution.
- $w_2 < w_1 < 1 \Rightarrow Q < 0$: vice versa

The fluid with the larger w always loses energy.



ϕ_1 dominates



ϕ_2 dominates

Maroto, PMM, Tessainer, arXiv: 2409.11991

MULTI-FIELD TDIFF THEORY

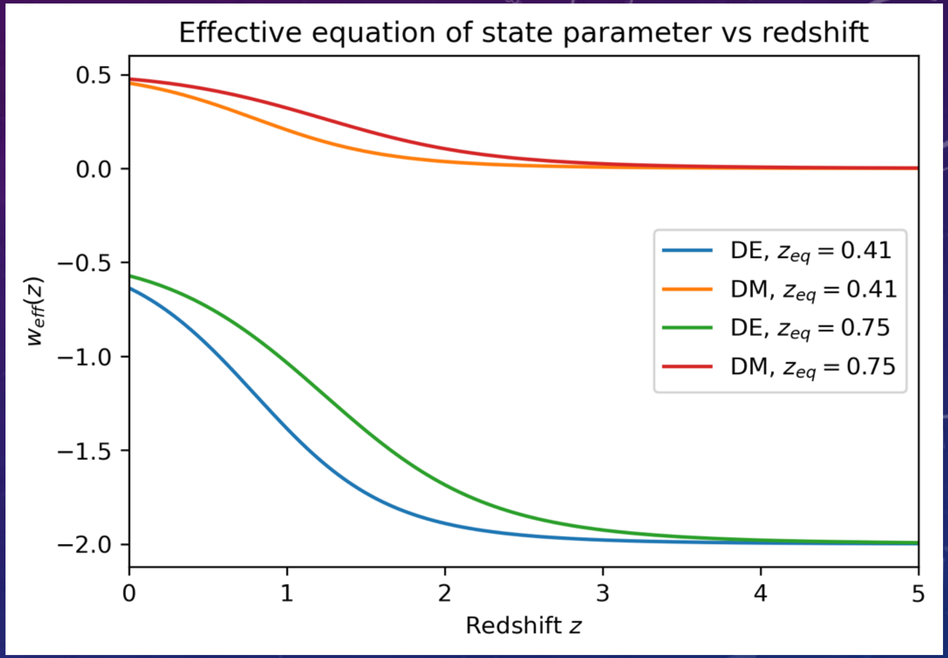
- Analytical model:

  Shift-symmetric model with power-law coupling functions, such that $w_1 = 0$ and $w_2 = -1/2$.

  - ➢ Dark matter gives energy to dark energy.
  - ➢ Phantom behaviour as a result of the energy exchange.

  Maroto, PMM, Tessainer, arXiv: 2409.11991

  - ➢ **However**, it can be argued that perturbations will become unstable at some point under ϕ_2 domination.



- Complete analysis of cosmological perturbations for 2 TDiff fields is presented in Maroto, PMM, Tessainer, arXiv: 2605.18424
  - ➢ Shift-symmetric power-law models
    - ➢ In **DM-DE models** with viable background evolution c_s^2 is too large before the instability.
    - ➢ Dark radiation-dark matter models may be of interest.
  - ➢ One can consider more complex coupling functions and/or go beyond the shift-symmetric case.

THE SILENT UNIFIED DARK SECTOR

DARK DEGENERACY

Cosmological observations are not sensitive to the individual components of the dark sector but rather to the total SET.

Wasserman, arXiv: 0203137.
Kunz, arXiv: 0702615.

- Dark energy and dark matter (e.g. ΛCDM).
- Interacting dark matter and dark energy.
- Unified dark sector.

They can reproduce an effective phantom behaviour if we artificially decompose them as non-interacting dark energy and dark matter.

Unified dark sector models describe the total dark sector as a ***single fluid***.

But…

- Consistent simple models require extremely small dark sector sound speed, $|c_s^2| \lesssim 10^{-5}$.
- Avoiding structural instabilities requires either a non-trivial fluid or fine-tuning.
- Mimetic models allow for unification, in some sense forced by a Lagrange multiplier.

Sandvik, Tegmark, Zaldarriaga, Waga, arXiv: 0212114.

Lim, Sawicki, Vikman,1003.5751.

TDIFF FLUIDS WITH A VANISHING SPEED OF SOUND

Let's take a single TDiff scalar field theory.

$$S_{\rm TDiff}[g_{\mu\nu},\psi] = \int d^4x\left[f_k(g)X - f_v(g)V(\psi)\right] \;\Rightarrow\; S[\psi,\phi] = \int d^4x\sqrt{g}\left[H_k(\phi)X - H_v(\phi)V(\psi) - \mu_0\phi\right]$$

- Perturbations take the form $\delta p = c_s^2\delta\rho + \alpha\delta\psi$ where the effective speed of sound is $c_s^2 = \dfrac{A}{A-B}$

➢ We restrict our focus those model characterized by $c_s^2 = 0$: $A = H_k\left[V\left(H_k''H_v' - H_k'H_v''\right) + \mu_0 H_k''\right] = 0$

$$\Rightarrow\; S_{c_s^2=0}[\psi,\varphi] = \int d^4x\sqrt{g}\left\{X - V(\psi) + \varphi\left[X - \beta V(\psi) - \gamma\right]\right\}$$

- β and γ are constants.
- $\phi \to \varphi$

⇒ If $\beta = 1$ and/or $V(\psi) = V_0$, we have **adiabatic fluids** ($\alpha = 0$).

⇒ Diff breaking realices in a natural way the idea behind mimetic models.

De Cruz, Jaramillo, Maroto, PMM, 2607.19494.

TDIFF FLUIDS WITH A VANISHING SPEED OF SOUND

$$S[\psi,\phi] = \int \mathrm{d}^4x\sqrt{g}\left[H_k(\phi)X - H_v(\phi)V(\psi) - \mu_0\phi\right]$$

$$\boxed{c_s^2 = 0} \Rightarrow V(\psi)\,[H_k'(\phi)]^2\,\frac{\mathrm{d}}{\mathrm{d}\phi}\left(\frac{H_v'(\phi)}{H_k'(\phi)}\right) = \mu_0 H_k''(\phi)$$

- Solution 1: $H_k(\phi) = k_1\phi + k_2$
- Solution 2: $H_k''(\phi) \neq 0$ and $\mu_0 = 0$

$\Rightarrow$ $V(\psi) = 0$ or $H_v(\phi) = c_1 H_k(\phi) + c_2$

Coupling functions are linearly related.

- Solution 3: $H_k''(\phi) \neq 0$ and $\mu_0 \neq 0$ $\Rightarrow$ $V(\psi) = V_0 \neq 0$ and $H_v(\phi) = c_1 H_k(\phi) + c_2 - \frac{\mu_0}{V_0}\phi$

- $\varphi = \frac{H_k(\phi)}{\sigma^2} - 1$
- $\psi \to \frac{\psi}{\sigma}, \qquad V(\psi) \to \frac{V(\psi) - \mu_0(\sigma^2 - k_2)/k_1}{c_1\sigma^2 + c_2}$

$$\beta = \frac{c_1\sigma^2}{c_1\sigma^2 + c_2}, \qquad \gamma = \frac{\mu_0\sigma^2(c_1k_2 + c_2)}{k_1(c_1\sigma^2 + c_2)}$$

$$\delta\psi: \quad \nabla_\alpha\left[(1+\varphi)\nabla^\alpha\psi\right] + (1+\beta\varphi)V'(\psi) = 0$$

$$\delta\varphi: \quad X = \beta V(\psi) + \gamma.$$

Now there is **no wave-like** equation.

$$\Rightarrow S_{c_s^2=0}[\psi,\varphi] = \int \mathrm{d}^4x\sqrt{g}\left\{X - V(\psi) + \varphi\left[X - \beta V(\psi) - \gamma\right]\right\}$$

φ emerges as a Lagrange multiplier.

De Cruz, Jaramillo, Maroto, PMM, 2607.19494.

THE UNIFIED FLUID AS AN INTERACTING DARK SECTOR

We have 2 parameters models, with SET: $T_{\mu\nu} = (1+\varphi)\nabla_\mu\psi\nabla_\nu\psi - \left[(\beta-1)V(\psi)+\gamma\right]g_{\mu\nu}$

We want to see if it can describe the dark sector of the Universe.

- This is a perfect fluid, which can be decomposed as $T^{(\mathrm{D})}_{\mu\nu} = T^{(c)}_{\mu\nu} + T^{(\lambda)}_{\mu\nu}$
  - It can be interpreted as cold dark matter ($w_c = 0$) and vacuumm energy ($w_\lambda = -1$).
  - **ΛCDM is recovered** for $\beta = 1$ and/or $V(\psi)$ =constant**!**

$$\rho_{\mathrm{D}} = \underbrace{2(1+\varphi)\left[\beta V(\psi)+\gamma\right]}_{\equiv\rho_c} + \underbrace{(1-\beta)V(\psi)-\gamma}_{\equiv\rho_\lambda}$$

$$p_{\mathrm{D}} = \underbrace{(\beta-1)V(\psi)+\gamma}_{\equiv p_\lambda}.$$

- **Interacting kernel Q^ν**

$$Q^\nu = \nabla_\mu T^{\mu\nu}_{(c)} = -\nabla_\mu T^{\mu\nu}_{(\lambda)}$$

  - Energy-momentum exchange: $Q^\nu = \nabla^\nu p_\lambda = (\beta-1)V'(\psi)\nabla^\nu\psi$
  - Proyection $Q = u^\nu Q_\nu$, energy exchange: $Q = (\beta-1)V'(\psi)\sqrt{2X}$

➡ Energy flows from (to) the vacuum energy to (from) the dust if $(\beta-1)V'(\psi) > 0$ (<0).

De Cruz, Jaramillo, Maroto, PMM, 2607.19494.

De Cruz, Jaramillo, Maroto, PMM, 2607.19494.

COSMOLOGICAL DINAYMICS

- Cosmological background $\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\mathrm{d}\mathbf{x}^2$
  - Friedmann equation $H^2 = \frac{\kappa^2}{3}(\rho_\mathrm{D} + \rho_\mathrm{B} + \rho_\mathrm{R})$ where we take into account the unified fluid (D), Diff barionic matter (B) and radiation (R).
  - Constraint/φ-field equation $X = \frac{1}{2}\dot{\psi}^2 = \beta V(\psi) + \gamma$ it simplifies for the $c_s^2 = 0$ models.
  - Conservation equation of D: $\dot{\rho}_\mathrm{D} + 3H(\rho_\mathrm{D} + p_\mathrm{D}) = 0$ $\Rightarrow$ $\dot{\rho}_c + 3H\rho_c = -\dot{\rho}_\lambda$

- Cosmological perturbations $\mathrm{d}s^2 = a^2(\eta)\left[(1+2\Phi)\,\mathrm{d}\eta^2 - (1-2\Psi)\,\mathrm{d}\mathbf{x}^2\right]$
  - Perturbations for a non-interacting perfect fluid $\delta = \delta\rho/\rho_0$, $\theta = -k^2 v$ and $c_a^2 = p_0'/\rho_0'$

$$\delta' = -(1+w)(\theta - 3\Psi') + 3\mathcal{H}w\delta + 9\mathcal{H}^2 c_a^2(1+w)\frac{\theta}{k^2}$$

$$\theta' = -\mathcal{H}\theta + k^2\Phi.$$

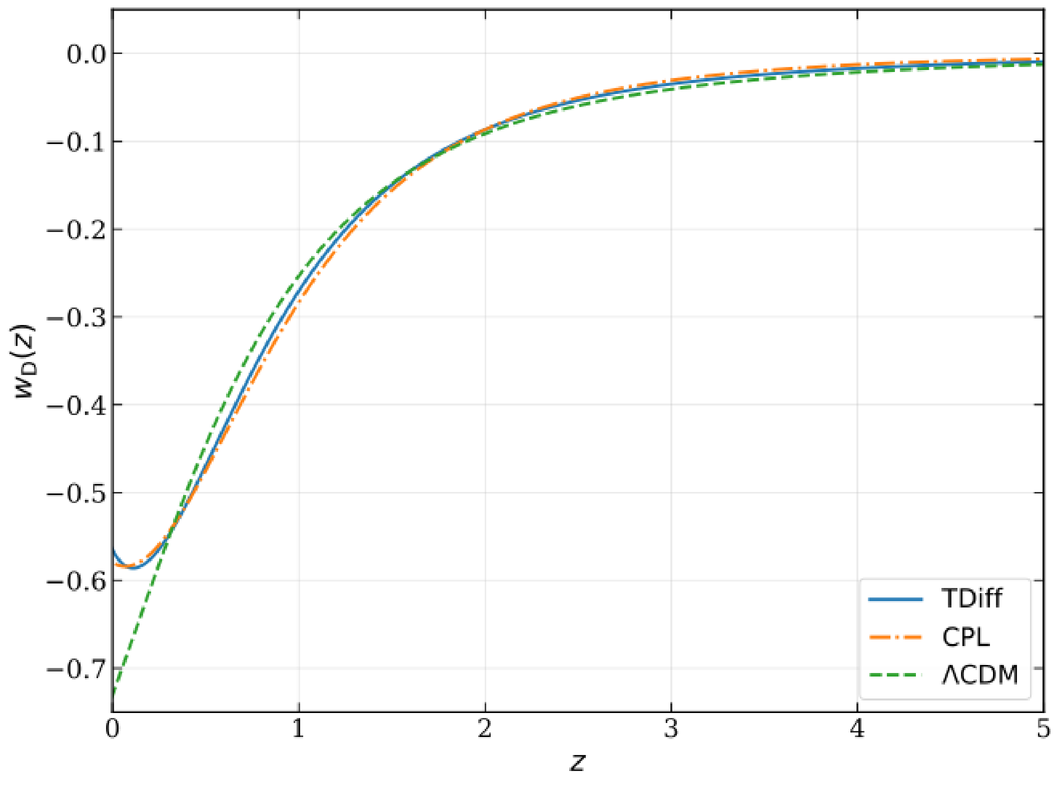
$$\boxed{c_s^2 = 0}$$

Due to their non-adiabaticity, $\delta p = -3\mathcal{H}c_a^2\rho_0(1+w)\frac{\theta}{k^2}$, we have a term proportional to $c_a^2\theta/k^2$.

De Cruz, Jaramillo, Maroto, PMM, 2607.19494.

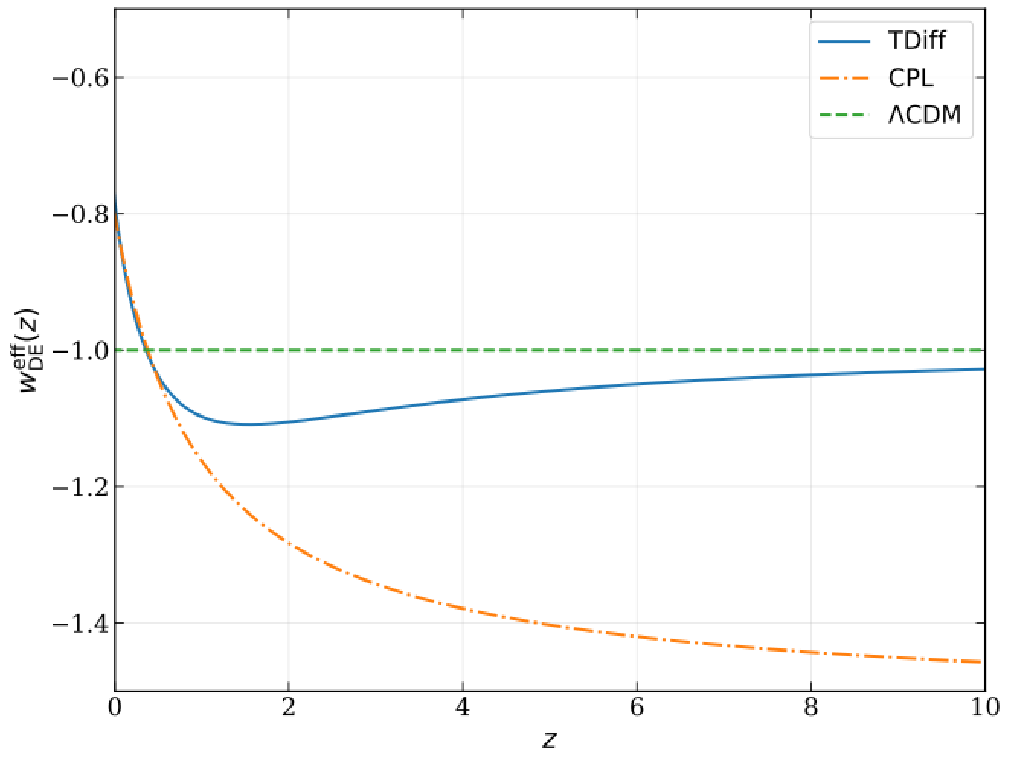
EXAMPLE: $V(\psi) = \frac{1}{2}\xi\psi^2 + V_0$ and $\beta = 0$

- 3 free parameters: (m, γ, ψ_0) ➡ Same as CPL: (Ω_c, w_0, w_a)



Equation of state parameter for the (total) dark sector: $w_D = \dfrac{-\rho_\Lambda}{\rho_c + \rho_\Lambda}$

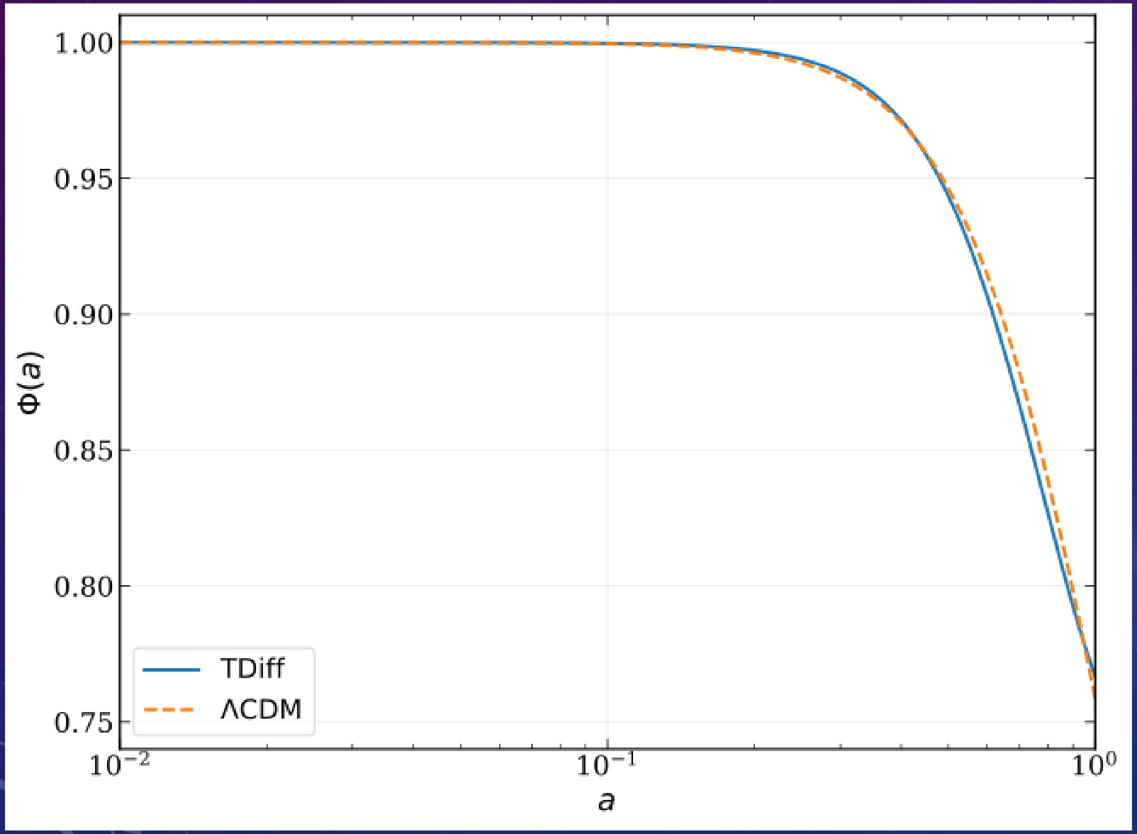
- Forcing a decomposition of our unified fluid as non-interacting cold dark matter and dark energy, with $w_{DE}^{eff}(z)$.å

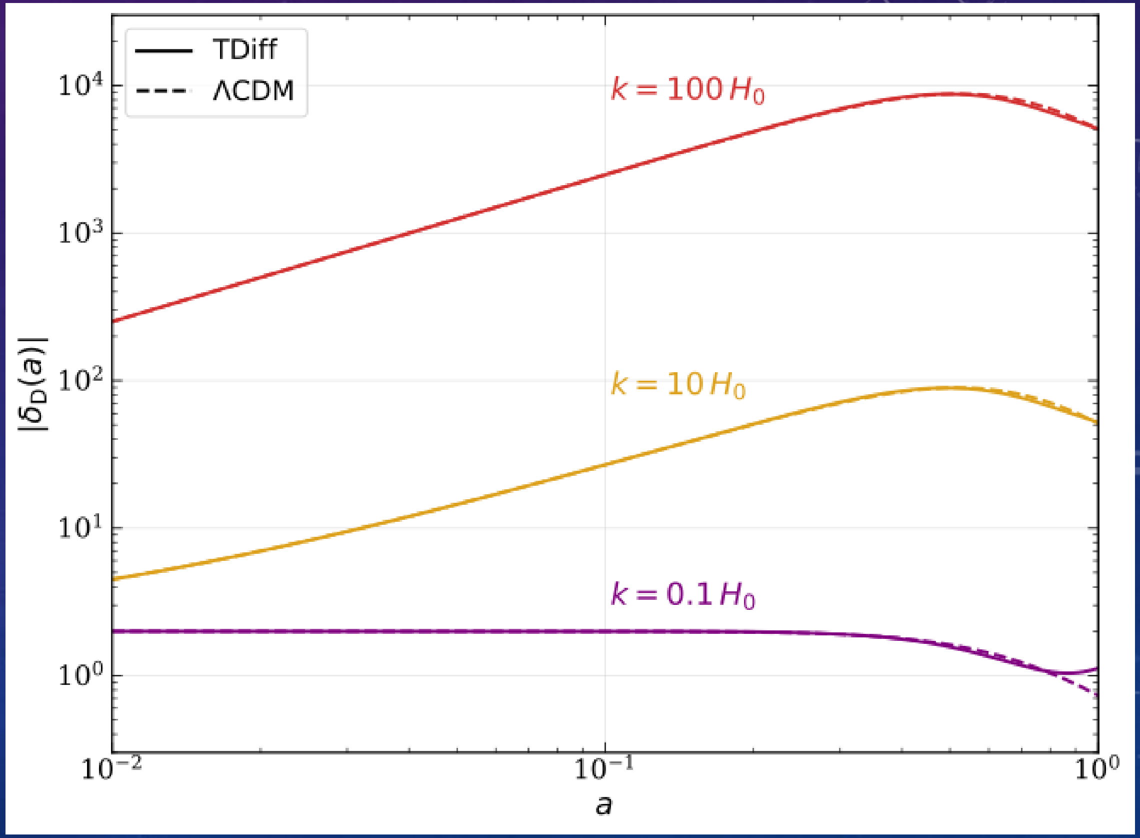


De Cruz, Jaramillo, Maroto, PMM, 2607.19494.

EXAMPLE: $V(\psi) = \frac{1}{2}\xi\psi^2 + V_0$ and $\beta = 0$

Simplified treatment (dark-sector-only) analysis $$\Phi'' + 3\mathcal{H}\left(1 + c_a^2\right)\Phi' + 3\mathcal{H}^2\left(c_a^2 - w\right)\Phi = 0$$





De Cruz, Jaramillo, Maroto, PMM, 2607.19494.

CONCLUSIONS

- By breaking Diff invariance down to TDiff in the matter sector, we can maintain **minimal coupling to gravity** while obtaining a wide range of simple, interesting models.

- Shift-symmetric single-field models are adiabatic.
  - By choosing different coupling functions one obtains a variety of distinct models.
  - A **dark matter model** emerges as a particular compelling scenario.

- **Restoring symmetry** yields a complementary framework to analyze the model's gravitational properties.

- In multi-field TDiff scenarios, only the total energy-momentum tensor is conserved.
  - **Symmetry breaking induces effective interactions** among the individual components.

- We have presented a class of unified dark sector models based on a single TDiff scalar field with $c_s^2 = 0$: ***the silent unified dark sector***.

  - Broken Diff symmetry naturally implements the constraints imposed in **mimetic models.**

  - **Adiabatic models** are equivalent to **ΛCDM**.

  - The dark sector equation of state avoids crossing the phantom divide.

  - We **can reproduce CPL** cosmic evolution with a fundamental model.

  - These models show **great promise** in addressing recent observational data.