Stability of thin shells: From Matt to now

Eric Poisson, University of Guelph

From black holes to the cosmos: Matt Visser's journey through space and time, Trieste, August 2026

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BRIEF REPORTS

Brief Reports are accounts of completed research which do not warrant regular articles or the priority handling given to Rapid Communications; however, the same standards of scientific quality apply. (Addenda are included in Brief Reports.) A Brief Report may be no longer than four printed pages and must be accompanied by an abstract.

Stability of a shell around a black hole

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We study the mechanical stability of a static, infinitely thin, spherically symmetric massive shell surrounding a classical Schwarzschild black hole. The shell is taken to have a non-negative surface energy density, and a speed of sound not greater than the speed of light. We show that the shell is stable against radial perturbations only outside a critical radius which is always larger than the radius of the circular photon orbit. The surface energy density of a stable shell is always larger than twice the surface pressure, and thus satisfies the dominant energy condition by a wide margin. We briefly discuss the effects of Hawking radiation in view of a path-integral approach to black-hole thermodynamics developed by York and collaborators. Our results suggest that a macroscopic thermal equilibrium situation associated with the canonical ensemble in this approach may not be realizable with a thin matter shell in Lorentzian spacetime.

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Thin-shell wormholes: Linearization stability

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The class of spherically symmetric thin-shell wormholes provides a particularly elegant collection of exemplars for the study of traversable Lorentzian wormholes. In the present paper we consider linearized (spherically symmetric) perturbations around some assumed static solution of the Einstein field equations. This permits us to relate stability issues to the (linearized) equation of state of the exotic matter which is located at the wormhole throat.

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Self-gravitating thin shells are dynamically unstable on all angular scales

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We establish the dynamical instability of a static, spherically symmetric, and infinitesimally thin shell in general relativity. The shell is made up of a perfect fluid with a barotropic equation of state, and it produces a Schwarzschild spacetime in its exterior and a Minkowski spacetime in its interior. We introduce a linear perturbation of the matter and spacetime, decompose it in spherical harmonics, and compute the shell's spectrum of quasinormal modes. We reveal the existence of two modes with a purely imaginary frequency, one negative (which describes stable oscillations), the other positive (which describes an exponential growth); these modes occur for all sampled values of the shell's compactness and adiabatic index, and all sampled values of the multipolar order $\ell \geq 2$, in the even-parity sector of the perturbation. All other quasinormal modes describe damped oscillations and do not contribute to the instability. This study complements a recent analysis by Yang, Bonga, and Pen [Phys. Rev. Lett. **130**, 011402 (2023)], which also concluded in a dynamical instability, but was limited by an eikonal approximation to small angular scales ($\ell \gg 1$); our treatment applies to all angular scales. The eigenvalue problem for the mode frequencies is

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Dynamical Instability of Self-Gravitating Membranes

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We show that a generic relativistic membrane with in-plane pressure and surface density having the same sign is unstable with respect to a series of warping mode instabilities with high wave numbers. We also examine the criteria of instability for commonly studied exotic compact objects with membranes, such as gravastars, anti–de Sitter bubbles, and thin-shell wormholes. For example, a gravastar which satisfies the weak energy condition turns out to be dynamically unstable. A thin-layer black hole mimicker is stable only if it has positive pressure and negative surface density (such as a wormhole), or vice versa.

Instability of thin shells: Summary

- Yang, Bonga, Pan (2023)
  Self-gravitating thin shells are dynamically unstable.
  In the eikonal regime ($l \gg 1$), there always exist an unstable mode.
  Black-hole mimickers that feature a thin shell are unstable.
  Thin-shell wormholes are unstable.

- Pitre, Schneider, Poisson (2026)
  Dynamical instability persists on **all angular scales** ($l \geq 2$).
  Shell's interior is taken to be vacuum (Minkowski).

Instability of thin shells: Preliminary remarks

An important ingredient in a stability analysis is the **adiabatic index**

$$\Gamma := \left(\frac{\partial \ln p}{\partial \ln \sigma}\right)_s$$

p: surface pressure

σ: rest-mass density

μ: energy density

The speed of sound is given by $v^2_{\text{sound}} = \left(\frac{\partial p}{\partial \mu}\right)_s = \Gamma \frac{p}{\mu + p}$

Radial stability requires $\Gamma \geq \frac{3}{2} + (M/R) + \frac{7}{4}(M/R)^2 + \cdots$

Nonradial instability: Unstable mode for each l and any value of Γ.

Newtonian analysis: Setup

- The shell is a closed 2D surface embedded in 3D flat space.
- We use intrinsic coordinates $\vartheta^A = (\vartheta, \varphi)$ on the shell.
- These are **Lagrangian coordinates**: a fluid element moves with $\vartheta^A =$ constant, and keeps its label during a perturbation.
- The shell possesses:

$$\text{mass density} : \sigma$$

$$\text{surface pressure} : p(\sigma)$$

$$\text{velocity vector} : \boldsymbol{v} = v_n\, \boldsymbol{n} + v^A\, \boldsymbol{e}_A$$

$$\text{induced metric} : h_{AB}$$

$$\text{extrinsic curvature} : K_{AB}$$

Newtonian analysis: Governing equations

- Mass conservation: $\partial_t \sigma + \sigma\left(v_n K + D_A v^A\right) = 0$

- Momentum conservation:

$$\sigma\left(\partial_t v_n + v^A \partial_A v_n - K_{AB} v^A v^B\right) = \sigma\langle g_n \rangle + pK$$
$$\sigma\left(\partial_t v_A - v_n \partial_A v_n - v_B D_A v^B\right) = \sigma g_A - \partial_A p$$

- Gravity:

$$\boldsymbol{g} = g_n\, \boldsymbol{n} + g^A\, \boldsymbol{e}_A = \boldsymbol{\nabla} U = \text{gravitational acceleration}$$
$$\nabla^2 U = 0, \qquad [U] = 0, \qquad [g_n] = -4\pi G\sigma$$

Newtonian analysis: Equilibrium configuration

- The unperturbed configuration is static and spherically symmetric.

- The governing equations imply

$$\sigma = \frac{M}{4\pi R^2}, \qquad p = \frac{GM}{16\pi R^3}$$

$$M = \frac{4}{\pi G^2}\frac{p^2}{\sigma^3}, \qquad R = \frac{1}{\pi G}\frac{p}{\sigma^2}$$

- This describes a sequence of equilibria parametrized by σ.

Newtonian analysis: Stability

- The perturbation is described by a **Lagrangian displacement vector**, which takes a fluid element from its unperturbed position to its new position.

$$\boldsymbol{\xi} = \xi_n \, \boldsymbol{n} + \xi^A \, \boldsymbol{e}_A$$

$$\xi_n = h^{\ell m}(t) \, Y^{\ell m}(\vartheta^A), \qquad \xi_A = j^{\ell m}(t) \, \partial_A Y^{\ell m}(\vartheta^A)$$

- All other perturbation variables are obtained from $\boldsymbol{\xi}$.
- The governing equations reduce to the matrix system

$$\ddot{\mathsf{h}} + \mathsf{A}\mathsf{h} = 0 \qquad \mathsf{h} = \begin{pmatrix} h^{\ell m} \\ j^{\ell m} \end{pmatrix}$$

Newtonian analysis: Stability

- Diagonalization of A produces the shell's eigenfrequencies and eigenvectors (normal modes).

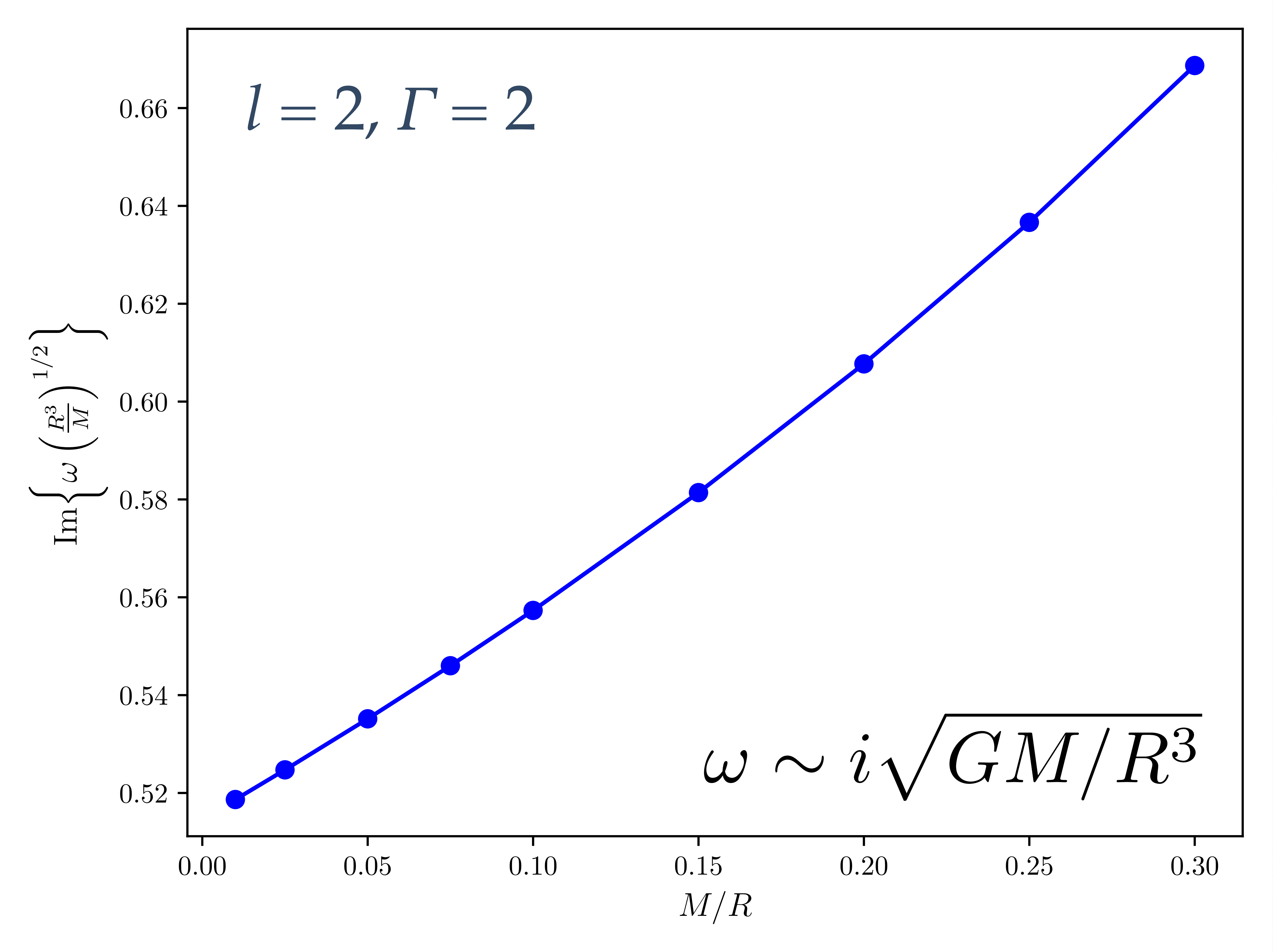
- For each l and any value of Γ, we find that

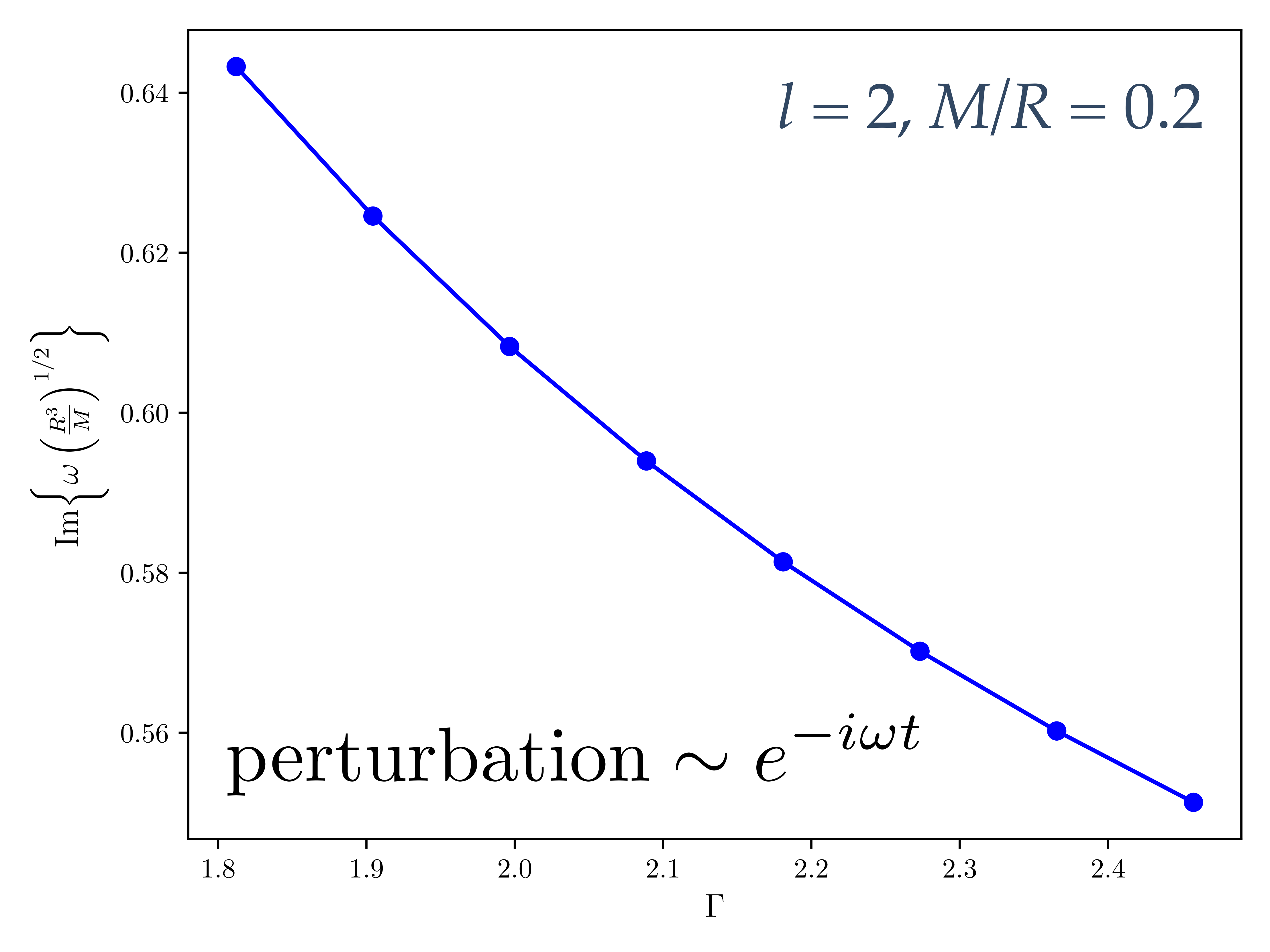
$$\begin{aligned} \omega_+^2 &> 0 \quad \rightarrow \quad \text{stable mode} \\ \omega_-^2 &< 0 \quad \rightarrow \quad \text{unstable mode} \end{aligned} \qquad \omega_\pm^2 = \frac{GM}{R^3} f_\pm(\ell, \Gamma)$$

- Conclusion: **the shell is dynamically unstable**.

General relativity: Matter modes

- The relativistic shell also has a mode with an imaginary frequency.

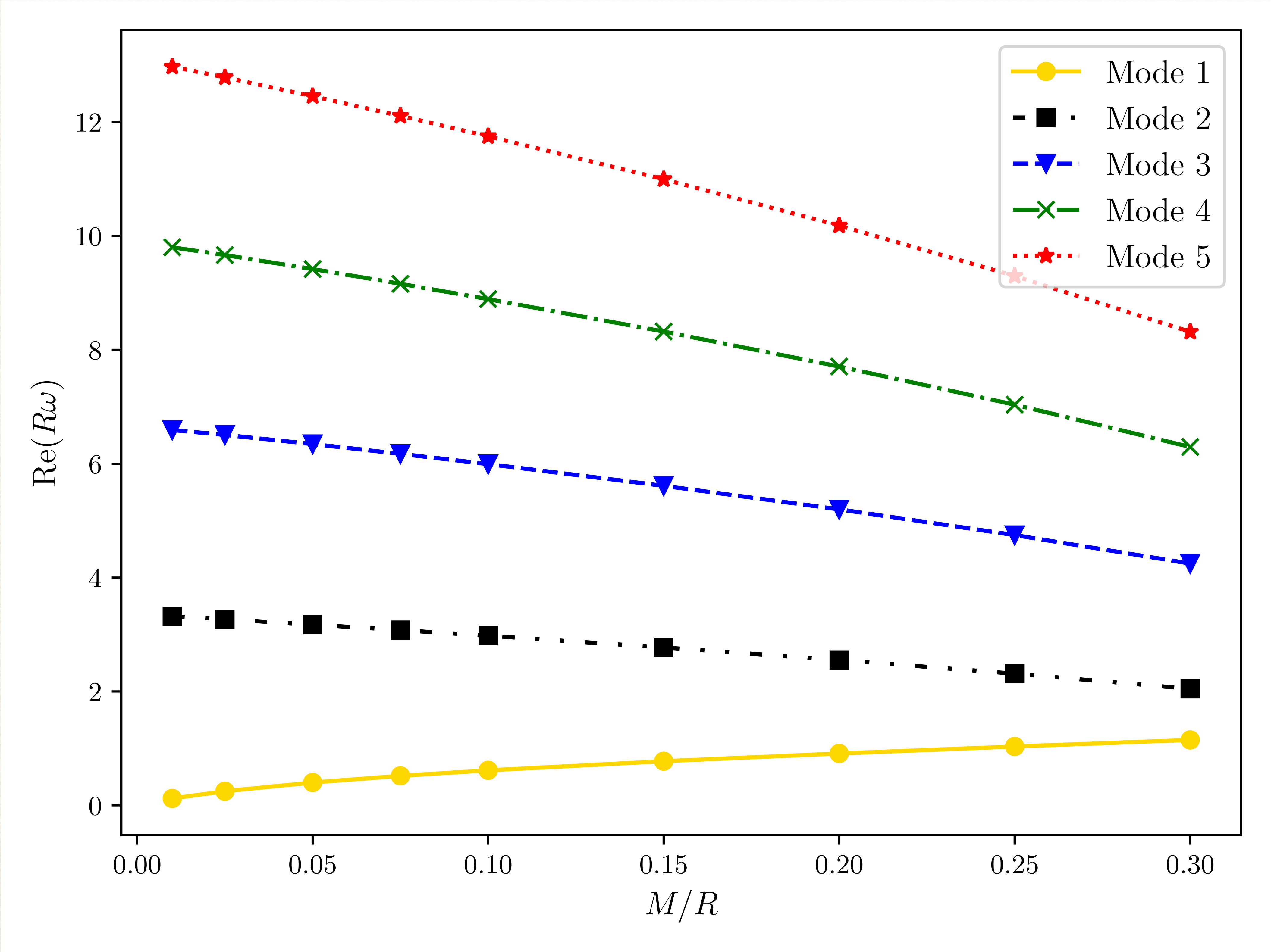


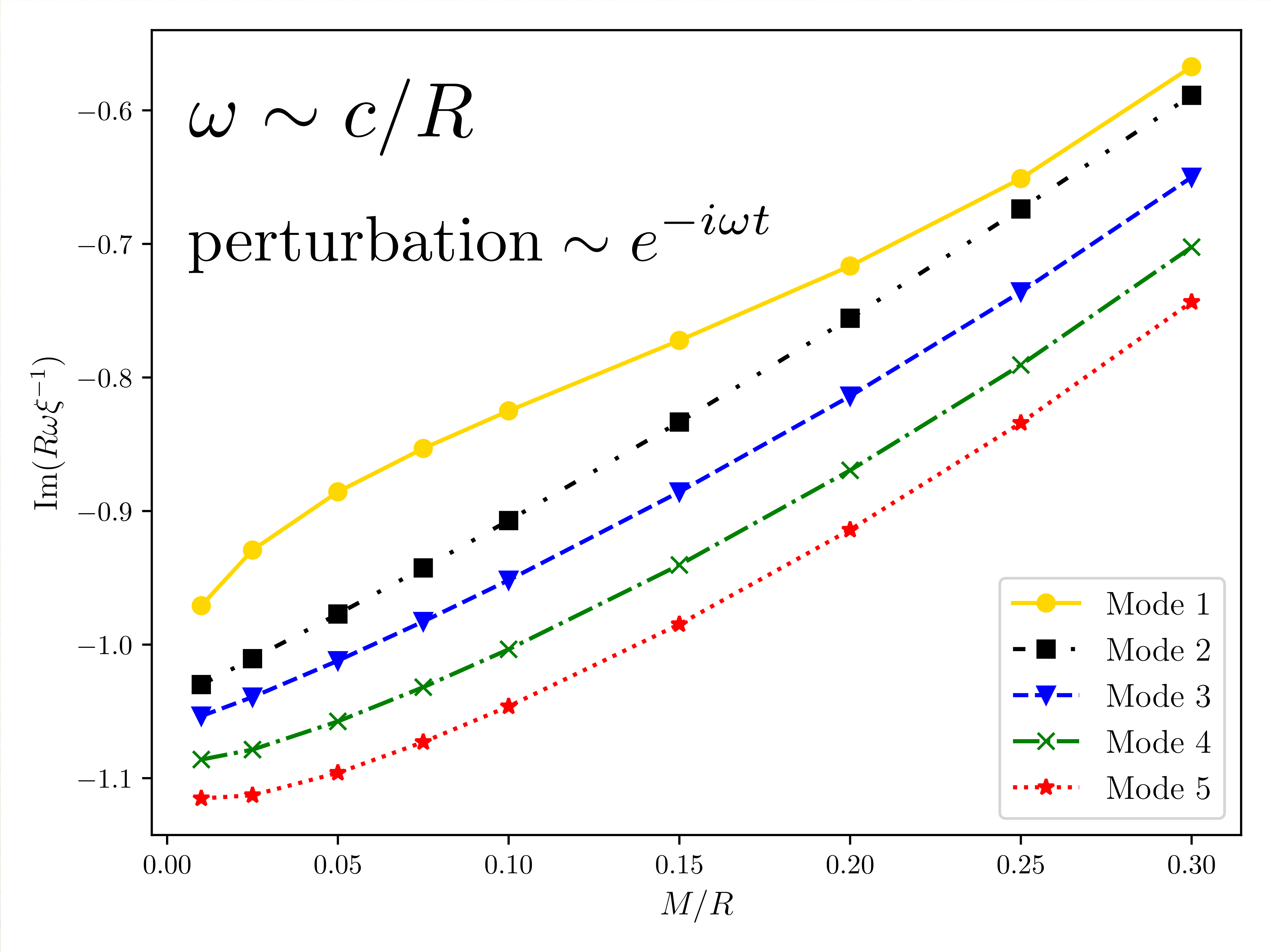


- The relativistic shell also is **dynamically unstable**.

General relativity: Wave modes

- In addition, the relativistic shell has wave modes; stable; complex frequencies with a negative imaginary part.





Conclusion

- A self-gravitating thin shell is unconditionally unstable to nonradial perturbations (any l, any Γ, any M/R).
- This was established for a vacuum interior.
- Interior matter might succeed in stabilizing the shell.
- Yang, Bonga, and Pan (2023) show that this does not happen for a de Sitter interior (gravastar).
- Conjecture: it will not happen for any interior matter.
- A black-hole mimicker that features a thin shell at its surface is dynamically unstable.

Thanks for your attention!