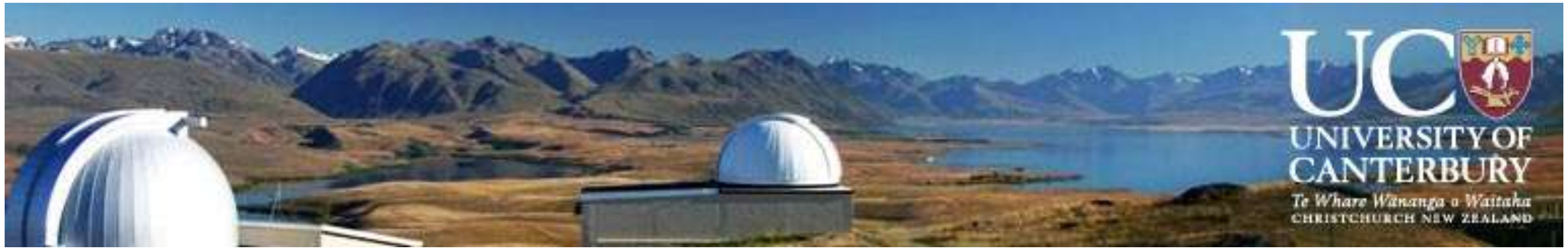
Parallel journeys in space, time and the timescape

David L Wiltshire
(University of Canterbury, Christchurch, NZ)



MARSDEN FUND
TE PŪTEA RANGAHAU A MARSDEN

ROYAL SOCIETY TE APĀRANGI

Outline

0. Origins
1. Timescape cosmology: key concepts, status summary
2. Supernovae evidence for foundational change to cosmological models
3. Exact solutions for differentially rotating galaxies in general relativity
4. The future: viewpoint on lessons learned

0. Origins

Matt's best cited contributions ...

iNSPIRE HEP | authors | Help

- book 3
- introductory 2
- review 2
- proceedings 1
- thesis 1

Collaborators

- Stefano Liberati 74
- Carlos Barcelo 31
- Silke E.Ch. Weinfurtner 27
- Petarapa Boonserm 25
- Prado Martin-Moruno 21
- Alex Simpson 21
- Raúl Carballo-Rubio 18
- Francesco Di Filippo 18
- Sebastian Schuster 15
- Carmen Molina-Paris 15

Show 90 more

Affiliations

- Victoria U., Wellington 238
- Washington U., St. Louis 93
- Los Alamos 11
- Southern California U. 10
- UC, Berkeley 4
- LBL, Berkeley 2
- Southern Denmark U., CP3-Origins 1
- USC, Los Angeles 1
- Victoria U., Dept. Math. Stat. 1

#1 Lorentzian wormholes: From Einstein to Hawking
Matt Visser (Washington U., St. Louis) (1995)
cite | claim | reference search | 1,522 citations

#2 Analogue gravity
Carlos Barcelo (IAA, Granada), Stefano Liberati (INFN, Trieste and SISSA, Trieste), Matt Visser (Victoria U., Wellington) (May, 2005)
Published in: *Living Rev.Rel.* 8 (2005) 12, *Living Rev.Rel.* 14 (2011) 3 • e-Print: gr-qc/0505065 [gr-qc]
pdf | links | DOI | cite | claim | reference search | 1,495 citations

#3 Acoustic black holes: Horizons, ergospheres, and Hawking radiation
Matt Visser (Washington U., St. Louis) (Dec, 1997)
Published in: *Class.Quant.Grav.* 15 (1998) 1767-1791 • e-Print: gr-qc/9712010 [gr-qc]
pdf | DOI | cite | claim | reference search | 723 citations

#4 Jerk and the cosmological equation of state
Matt Visser (Victoria U., Wellington) (Sep, 2003)
Published in: *Class.Quant.Grav.* 21 (2004) 2603-2616 • e-Print: gr-qc/0309109 [gr-qc]
pdf | DOI | cite | claim | reference search | 679 citations

#5 Traversable wormholes: Some simple examples
Matt Visser (Los Alamos) (Jan, 1989)
Published in: *Phys.Rev.D* 39 (1989) 3182-3184 • e-Print: 0809.0907 [gr-qc]
pdf | DOI | cite | claim | reference search | 661 citations

#6 An Exotic Class of Kaluza-Klein Models
Matt Visser (Southern California U.) (Jul, 1985)
Published in: *Phys.Lett.B* 159 (1985) 22-25 • e-Print: hep-th/9910093 [hep-th]
pdf | DOI | cite | claim | reference search | 603 citations

#7 Traversable wormholes with arbitrarily small energy condition violations
Matt Visser (Victoria U., Wellington), Sayan Kar (Indian Inst. Tech., Kharagpur), Naresh Dadhich (IUCAA, Pune) (Jan, 2003)
Published in: *Phys.Rev.Lett.* 90 (2003) 201102 • e-Print: gr-qc/0301003 [gr-qc]
pdf | DOI | cite | claim | reference search | 590 citations

#8 Stable gravastars: An Alternative to black holes?
Matt Visser (Victoria U., Wellington), David L. Wiltshire (Canterbury U.) (Oct, 2003)
Published in: *Class.Quant.Grav.* 21 (2004) 1135-1152 • e-Print: gr-qc/0310107 [gr-qc]
pdf | DOI | cite | claim | reference search | 524 citations

A beginning of parallel journeys
… amongst brane world pioneers

Volume 159B, number 1 PHYSICS LETTERS 12 September 1985

AN EXOTIC CLASS OF KALUZA–KLEIN MODELS

Matt VISSER
Physics Department, University of Southern California, Los Angeles, CA 90080-0484, USA

Received 22 April 1985

We discuss an exotic class of Kaluza–Klein models in which the internal space is neither compact nor even of finite volume. Rather than using the usual compact internal space we consider the case where particles are gravitationally trapped near a four-dimensional submanifold of the higher-dimensional spacetime. A specific model exhibiting this phenomenon is constructed in five dimensions.

Introduction. In Kaluza–Klein theories of the ordinary type spacetime is assumed to be of the form $M_4 \times K$; M_4 a four-dimensional manifold and K a compact manifold of "internal" coordinates. Classically particles are free to move around on K, thus K must be assumed to be very small. The alternative we wish to begin to explore in this note is to somehow arrange for an absence of translational invariance in the extra "internal" directions, and then use gravity to trap particles near a four-dimensional submanifold. Suggestions along similar lines (without gravity) have

tors and, therefore, four constants of the motion for particles following geodesics

$$E = -(P, \partial/\partial t)$$

$$= -P^A g_{AB}(\partial/\partial t)^B = P^0 e^{2\phi} ,$$

$$p^i = (P, \partial/\partial x^i)$$

$$= P^A g_{AB}(\partial/\partial x^i)^B = P^i \qquad (i = 1, 2, 3) .$$

A particle having a definite rest mass in the five-

Nuclear Physics B287 (1987) 717–742
North-Holland, Amsterdam

SPACETIME AS A MEMBRANE IN HIGHER DIMENSIONS

G.W. GIBBONS and D.L. WILTSHIRE

Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Silver Street, Cambridge CB3 9EW, England

Received 29 September 1986
(Revised 5 December 1986)

By means of a simple model we investigate the possibility that spacetime is a membrane embedded in higher dimensions. We present cosmological solutions of d-dimensional Einstein-Maxwell theory which compactify to two dimensions. These solutions are analytically continued to obtain dual solutions in which a $(d-2)$-dimensional Einstein spacetime "membrane" is embedded in d dimensions. The membrane solutions generalise Melvin's 4-dimensional flux tube solution. The flat membrane is shown to be classically stable. It is shown that there are zero mode solutions of the d-dimensional Dirac equation which are confined to a neighbourhood of the membrane and move within it like massless chiral $(d-2)$-dimensional fermions. An investigation of the spectrum of scalar perturbations shows that a well-defined mass gap between the zero modes and massive modes can be obtained if there is a positive cosmological term in $(d-2)$ dimensions or a negative cosmological term in d dimensions.

Two 20th century paradigm revolutions in one line each:

- Relativity: Space and Time are a relational structure between Things
- Quantum Mechanics: But what are Things?
  - When in doubt about observations pointing to some Thing, assume an invisible fluid with phenomenological properties leading to new experiments and its eventual detection. E.g.:
    Plumb pudding atoms → Rutherford scattering → Nuclear atom
    Genetic material → X-ray crystallography → DNA

    Or otherwise. E.g.: Phlogiston, Ether
  - ? Dark Matter, Dark Energy

INSTITUTE OF PHYSICS PUBLISHING CLASSICAL AND QUANTUM GRAVITY

Class. Quantum Grav. **21** (2004) 1135–1151 PII: S0264-9381(04)71221-6

Stable gravastars—an alternative to black holes?

? No!

Matt Visser[1] and David L Wiltshire[2]

[1] School of Mathematical and Computing Sciences, Victoria University of Wellington, New Zealand
[2] Department of Physics and Astronomy, University of Canterbury, Private Bag 4800, Christchurch, New Zealand

E-mail: matt.visser@vuw.ac.nz and d.wiltshire@phys.canterbury.ac.nz

Received 31 October 2003
Published 22 January 2004
Online at stacks.iop.org/CQG/21/1135 (DOI: 10.1088/0264-9381/21/4/027)

Abstract
The 'gravastar' picture developed by Mazur and Mottola is one of a very small number of serious challenges to our usual conception of a 'black hole'. In the gravastar picture there is effectively a phase transition at/near where the event horizon would have been expected to form, and the interior of what would have been the black hole is replaced by a segment of de Sitter space. While Mazur and Mottola were able to argue for the thermodynamic stability of their configuration, the question of dynamic stability against spherically symmetric perturbations of the matter or gravity fields remains somewhat obscure. In this paper we construct a model that shares the key features of the Mazur–Mottola scenario, and which is sufficiently simple for a full dynamical analysis. We find that there are *some* physically reasonable equations of state for the transition layer that lead to stability.

Class. Quantum Grav. **21** (2004) 2603–2615 PII: S0264-9381(04)76772-6

Jerk, snap and the cosmological equation of state

Matt Visser

School of Mathematical and Computing Sciences, Victoria University of Wellington,
PO Box 600, Wellington, New Zealand

E-mail: matt.visser@mcs.vuw.ac.nz

Received 27 February 2004
Published 26 April 2004
Online at stacks.iop.org/CQG/21/2603
DOI: 10.1088/0264-9381/21/11/006

Abstract
Taylor expanding the cosmological equation of state around the current epoch

$$p = p_0 + \kappa_0(\rho - \rho_0) + \frac{1}{2}\left.\frac{d^2 p}{d\rho^2}\right|_0 (\rho - \rho_0)^2 + O[(\rho - \rho_0)^3],$$

is the simplest model one can consider that does not make any *a priori* restrictions on the nature of the cosmological fluid. Most popular cosmological models attempt to be 'predictive', in the sense that once some *a priori* equation of state is chosen the Friedmann equations are used to determine the evolution of the FRW scale factor $a(t)$. In contrast, a 'retrodictive' approach might usefully take observational data concerning the scale factor, and use the Friedmann equations to *infer* an observed cosmological equation of state. In particular, the value and derivatives of the scale factor determined at the current epoch place constraints on the value and derivatives of the cosmological equation of state at the current epoch. Determining the first three Taylor coefficients of the equation of state at the current epoch requires a measurement of the deceleration, jerk and snap—the *second*, *third* and *fourth* derivatives of the scale factor with respect to time. Higher-order Taylor coefficients in the equation of state are related to higher-order time derivatives of the scale factor. Since the jerk and snap are rather difficult to measure, being related to the *third* and *fourth* terms in the Taylor series expansion of the Hubble law, it becomes clear why direct observational constraints on the cosmological equation of state are so relatively weak, and are likely to remain weak for the foreseeable future.

Discussion of reconstruction of effective fluid equation of state

$$P = w\,\rho$$

via cosmographic series

$$D = \frac{cz}{H_0}\left\{1 - \left[1 + \frac{q_0}{2}\right]z + \left[1 + q_0 + \frac{q_0^2}{2} - \frac{j_0}{6}\right]z^2 - \left[1 + \frac{3}{2}q_0(1+q_0) + \frac{5}{8}q_0^3 - \frac{1}{2}j_0 - \frac{5}{12}q_0 j_0 - \frac{s_0}{24}\right]z^3 + O(z^4)\right\}$$

$$d_L(z) = \frac{cz}{H_0}\left\{1 + \frac{1}{2}[1 - q_0]z - \frac{1}{6}\left[1 - q_0 - 3q_0^2 + j_0 + \frac{kc^2}{H_0^2 a_0^2}\right]z^2 + \frac{1}{24}\left[2 - 2q_0 - 15q_0^2 - 15q_0^3 + 5j_0 + 10q_0 j_0 + s_0 + \frac{2kc^2(1+3q_0)}{H_0^2 a_0^2}\right]z^3 + O(z^4)\right\}.$$

Buchert coarse-graining and the classical energy conditions

Matt Visser

School of Mathematics and Statistics, Victoria University of Wellington, Wellington 6140, New Zealand
E-mail: matt.visser@msor.vuw.ac.nz
http://www.victoria.ac.nz/sms

So-called "Buchert averaging" is actually a coarse-graining procedure, where fine detail is "smeared out" due to limited spatio-temporal resolution. For technical reasons, (to be explained herein), "averaging" is not really an appropriate term, and I shall consistently describe the process as a "coarse-graining". Because Einstein gravity is nonlinear the coarse-grained Einstein tensor is typically not equal to the Einstein tensor of the coarse-grained spacetime geometry. The discrepancy can be viewed as an "effective" stress-energy, and this "effective" stress-energy often violates the classical energy conditions. To keep otherwise messy technical issues firmly under control, I shall work with conformal-FLRW (CFLRW) cosmologies. These CFLRW-based models are particularly tractable, and are also particularly attractive observationally: the CMB is not distorted. In this CFLRW context one can prove some rigorous theorems regarding the interplay between Buchert coarse-graining, tracelessness of the effective stress-energy, and the classical energy conditions.

Keywords: Buchert averaging; coarse-graining; smearing; FLRW cosmology; CFLRW cosmology.

1. Introduction

The cosmological back-reaction problem continues to generate considerable (and sometimes heated) debate.[1–11] There is significant disagreement as to just how one should split the universe into "smooth background" plus "local perturbations", and yet more disagreement as to whether or not these perturbations remain small. I shall work within the framework of conformally-FLRW (CFLRW) cosmologies,[13] where the mathematics and physics are both firmly under control.[13] These CFLRW cosmologies are simply generic FLRW spacetimes distorted by a (possibly non-perturbatively large) conformal factor. Cosmographically[12] this is the unique non-perturbative distortion that can be applied to FLRW cosmologies without grossly modifying the CMB.[13] In counterpoint, when working in this CFLRW framework, Buchert's coarse-graining procedure simplifies tremendously. The contribution to the effective stress-energy due to coarse graining can be explicitly calculated. This coarse-graining contribution to the effective stress-energy need not be traceless, nor need it satisfy any of the classical energy conditions. (Further details are in preparation.[14])

Arxiv:1512.05729
Proc. 14th Marcel Grossmann Conference (2017); ref. [14] "in preparation"details

Back to first principles: questions to answer

- What is the origin of scale?
- Where is infinity? (i0 i- i+ …)?
- Why is there an average isotropic expansion law on (empirical) scales > 100/h Mpc when the Universe is lumpy (inhomogeneous) and anisotropic on smaller scales?
- Why is there a transition to “homogeneity”(empirically) around 70 – 120/h Mpc ?….
- History, 20th century: *assume* FLRW average on any scale
- Making history, 21st century: physics can explain surely?
- The time is now: we just needed data, a revolution is coming?!

1. Timescape cosmology: key concepts, status summary

Falsifiable predictions with answers by 2030:

FLRW model → Clarkson-Bassett-Lu test → ΛCDM

Or

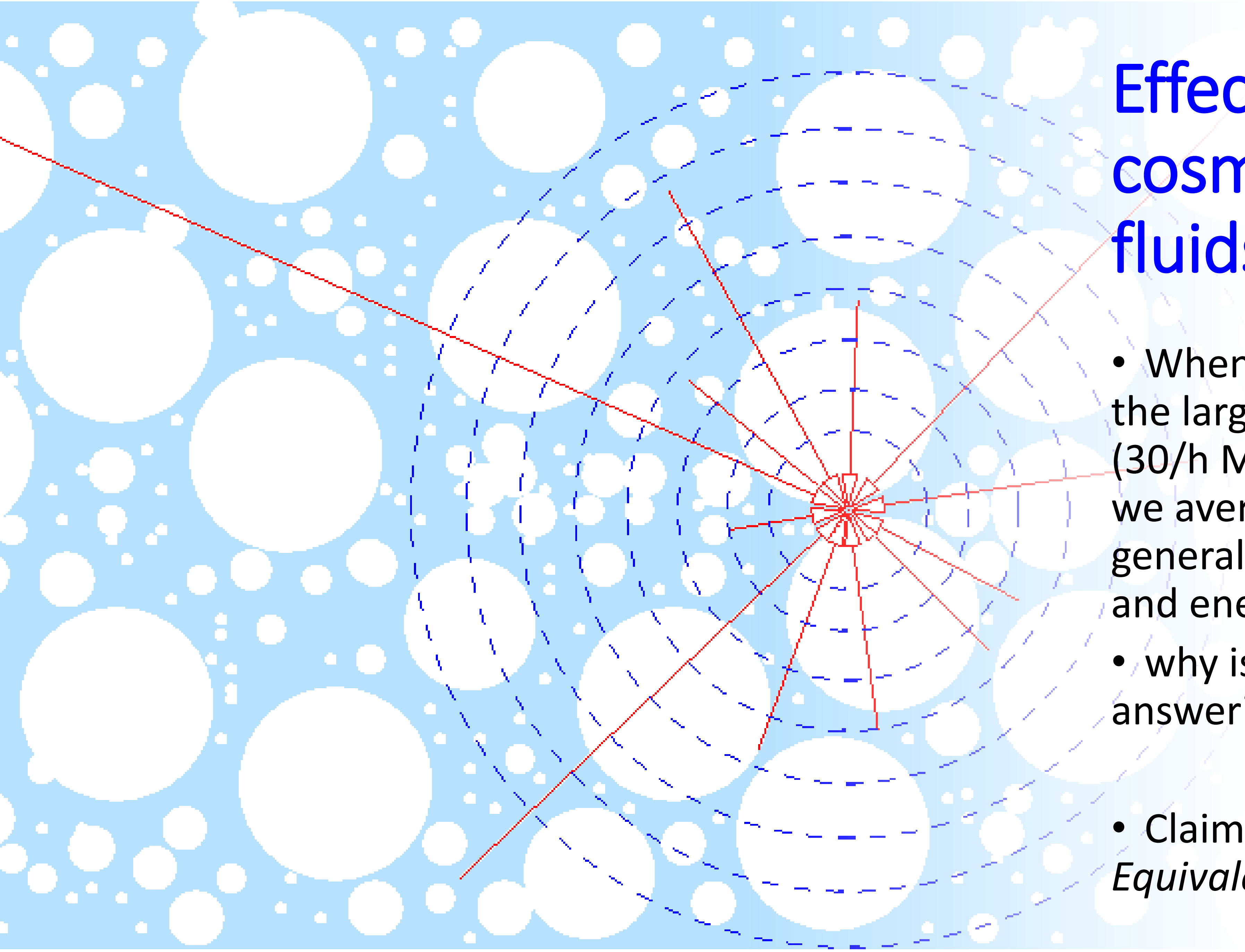
non-FLRW → Clarkson-Bassett-Lu test → Timescape

Takeaway moral: Gradients of kinetic energy of expansion in full GR are non-FLRW!

What is a cosmological "particle" ("dust")?

- In FLRW one takes observers "comoving with the dust"
- Traditionally galaxies were regarded as dust. However,
  - Galaxies, clusters not homogeneously distributed today
  - Dust particles should have (on average) invariant masses over the timescale of the problem
- Must coarse-grain over expanding fluid elements larger than the largest typical structures [voids of diameter $30\,h^{-1}$Mpc with $\delta_\rho \sim -0.95$ are $\gtrsim 40\%$ of $z=0$ universe]

$$\left.\begin{array}{c} g_{\mu\nu}^{\text{stellar}} \to g_{\mu\nu}^{\text{galaxy}} \to g_{\mu\nu}^{\text{cluster}} \to g_{\mu\nu}^{\text{wall}} \\ \vdots \\ \qquad\qquad\qquad\qquad\qquad\qquad\quad g_{\mu\nu}^{\text{void}} \end{array}\right\} \to g_{\mu\nu}^{\text{universe}}$$

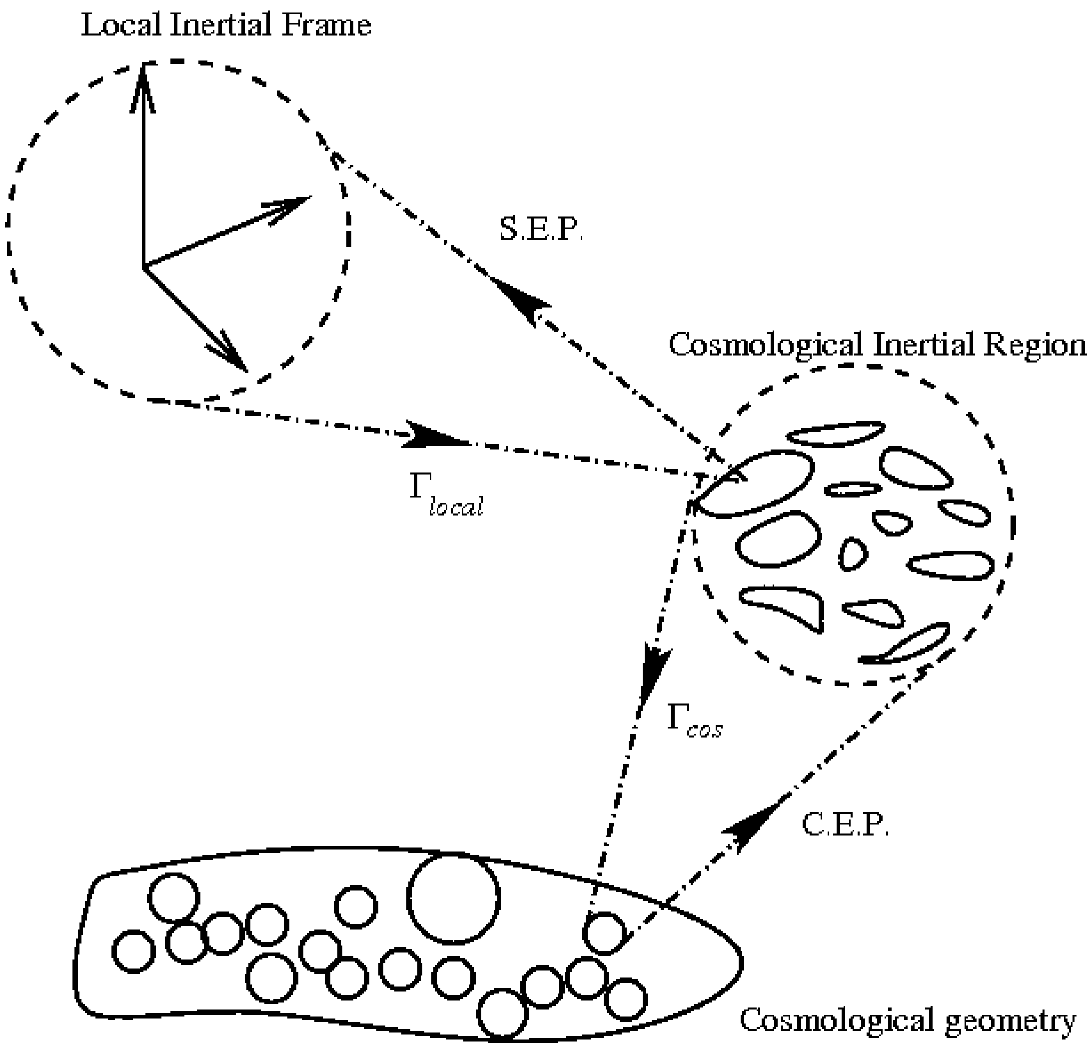


Effective cosmological fluids

- When coarse-graining over the largest *typical* structures (30/h Mpc diameter voids) we average over non-linear general relativistic matter and energy:
- why is there a simple answer?

- Claim: *Cosmological Equivalence Principle!*

Cosmological Equivalence Principle



Conceptual picture

Hubble parameter, involving first derivatives of the (statistical) metric, is to some extent a gauge parameter for connection of a (statistical) coarse grained geometry

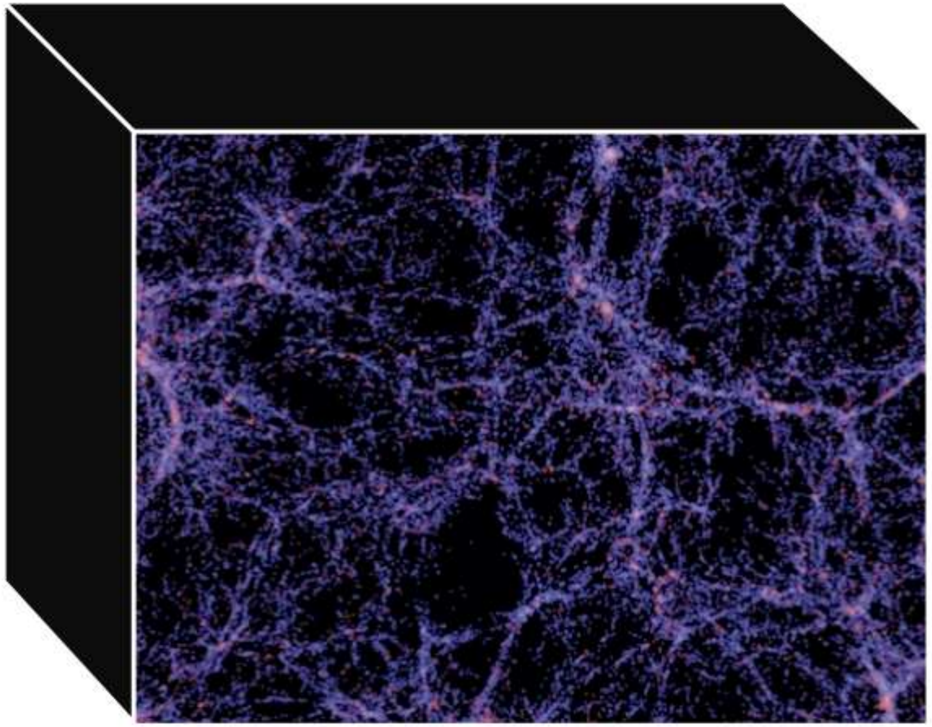
Averaging and backreaction

- *Fitting problem* (Ellis 1984):
  On what scale are Einstein's field equations valid?

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- In general $\langle G^{\mu}{}_{\nu}(g_{\alpha\beta})\rangle \neq G^{\mu}{}_{\nu}(\langle g_{\alpha\beta}\rangle)$
- *Weak backreaction*: *Assume* global average is an exact solution of Einstein's equations on large scale
- *Strong backreaction*: Fully nonlinear; assume alternative solution for homogeneity at last scattering
  - Einstein's equations are causal; no need for them on scales larger than light has time to propagate
  - Inflation becomes more a quantum geometry phenomenon, impact in present spacetime structure

Statistical Homogeneity Scale (SHS) average cell



- Need to consider relative position of observers over scales of tens of Mpc over which $\delta\rho/\rho \sim -1$.
- Gradients in spatial curvature and gravitational energy can lead to calibration differences between rulers & clocks of bound structures and volume average

Statistical Copernican Principle

- Retain Copernican Principle - we are at an average position *for observers in a galaxy*
- Observers in bound systems are not at a volume average position in freely expanding space
- By Copernican principle other average observers should see an isotropic CMB
- BUT *nothing in theory, principle nor observation demands that such observers measure the same mean CMB temperature nor the same angular scales in the CMB anisotropies*
- Average mass environment (galaxy) can differ significantly from volume–average environment (void)

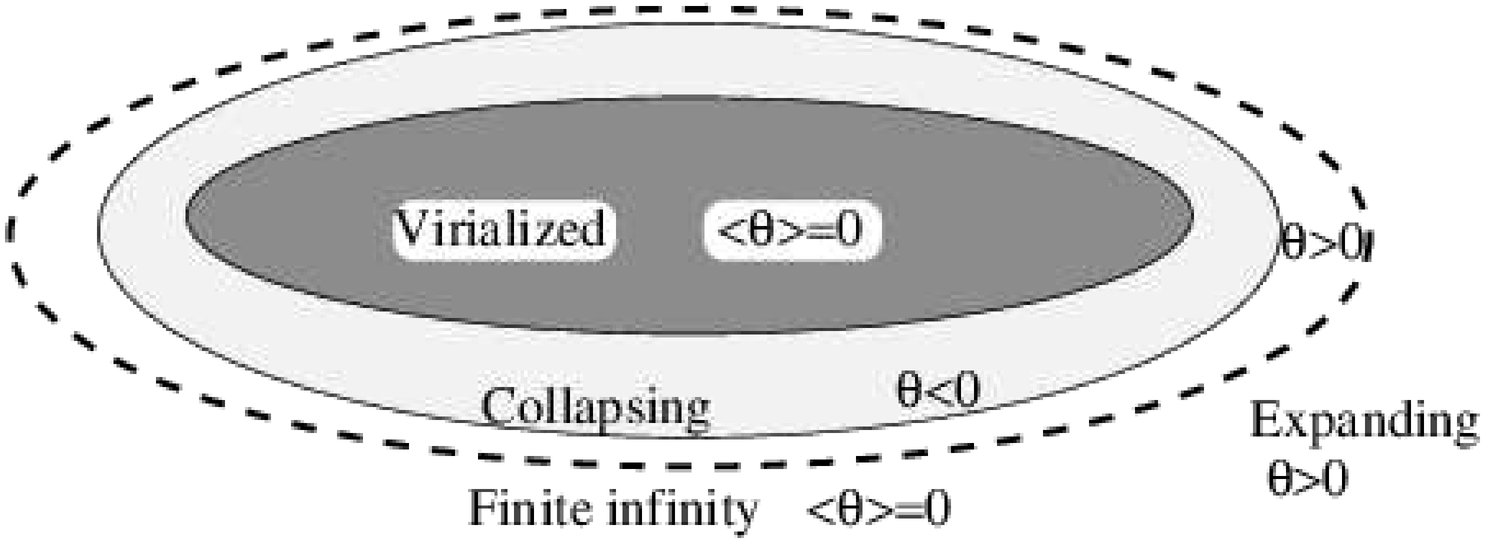
Cosmological Equivalence Principle

- *In cosmological averages it is always possible to choose a suitably defined spacetime region, the cosmological inertial region, on whose boundary average motions (timelike and null) can be described by geodesics in a geometry which is Minkowski up to some time-dependent conformal transformation,*

$$\mathrm{d}s^2_{\mathrm{CIR}} = a^2(\eta)\left[-\mathrm{d}\eta^2 + \mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\right] ,$$

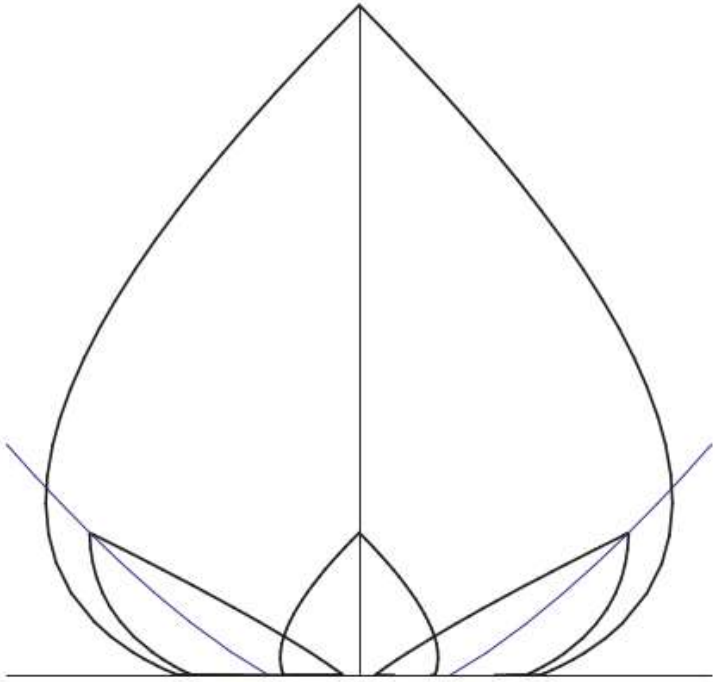
- Defines Cosmological Inertial Region (CIR) in which *regionally isotropic* volume expansion is equivalent to a velocity in special relativity
- Integrate on a bounding 2-sphere to define *"kinetic energy of expansion"*: globally it has gradients

Finite infinity



- Define *finite infinity*, "*fi*" as boundary to *connected* region within which *average expansion* vanishes $\langle\vartheta\rangle = 0$ and expansion is positive outside.
- Shape of *fi* boundary irrelevant (minimal surface generally): could typically contain a galaxy cluster.

Timescape: Physical interpretation (2007)



- Interpret solution of Buchert equations by radial null cone average

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \bar{a}^2(t)\,\mathrm{d}\bar{\eta}^2 + A(\bar{\eta},t)\,\mathrm{d}\Omega^2,$$

where $\int_0^{\bar{\eta}_{\mathcal{H}}} \mathrm{d}\bar{\eta}\, A(\bar{\eta},t) = \bar{a}^2(t)\mathcal{V}_{\mathrm{i}}(\bar{\eta}_{\mathcal{H}})/(4\pi)$.

- LTB metric but NOT an LTB solution

Timescape: Physical interpretation (2007)

- Conformally match radial null geodesics of spherical Buchert geometry to those of finite infinity geometry with *uniform quasilocal Hubble flow* condition $\mathrm{d}t = \bar{a}\,\mathrm{d}\bar{\eta}$ and $\mathrm{d}\tau_{\mathrm{w}} = a_{\mathrm{w}}\mathrm{d}\eta_{\mathrm{w}}$. But $\mathrm{d}t = \bar{\gamma}\mathrm{d}\tau_{\mathrm{w}}$ and $a_{\mathrm{w}} = {f_{\mathrm{wi}}}^{-1/3}\,(1 - f_{\mathrm{v}})\,\bar{a}$. Hence *on radial null geodesics*

$$\mathrm{d}\eta_{\mathrm{w}} = \frac{{f_{\mathrm{wi}}}^{1/3}\mathrm{d}\bar{\eta}}{\bar{\gamma}\,(1 - f_{\mathrm{v}})^{1/3}}$$

  Define η_{w} by integral of above on radial null-geodesics.

- Extend spatially flat wall geometry to dressed geometry

$$\mathrm{d}s^2 = -\mathrm{d}\tau_{\mathrm{w}}^2 + a^2(\tau_{\mathrm{w}})\left[\mathrm{d}\bar{\eta}^2 + r_{\mathrm{w}}^2(\bar{\eta},\tau_{\mathrm{w}})\,\mathrm{d}\Omega^2\right]$$

  where $r_{\mathrm{w}} \equiv \bar{\gamma}\,(1 - f_{\mathrm{v}})^{1/3}\,{f_{\mathrm{wi}}}^{-1/3}\eta_{\mathrm{w}}(\bar{\eta},\tau_{\mathrm{w}})$, $a = \bar{a}/\bar{\gamma}$.

Timescape: Physical interpretation (2007)

- N.B. The extension is NOT an isometry

$$\text{N.B.} \qquad \mathrm{d}s^2_{fi} = -\mathrm{d}\tau_{\mathrm{w}}^2 + a_{\mathrm{w}}{}^2(\tau_{\mathrm{w}})\left[\mathrm{d}\eta_{\mathrm{w}}^2 + \eta_{\mathrm{w}}^2\mathrm{d}\Omega^2\right]$$
$$\rightarrow \mathrm{d}s^2 = -\mathrm{d}\tau_{\mathrm{w}}^2 + a^2\left[\mathrm{d}\bar\eta^2 + r_{\mathrm{w}}^2(\bar\eta, \tau_{\mathrm{w}})\,\mathrm{d}\Omega^2\right]$$

- Extended metric is an effective "spherical Buchert geometry" adapted to wall rulers and clocks.
- Since $\mathrm{d}\bar\eta = \mathrm{d}t/\bar a = \bar\gamma\,\mathrm{d}\tau_{\mathrm{w}}/\bar a = \mathrm{d}\tau_{\mathrm{w}}/a$, this leads to *dressed parameters* which do not sum to 1, e.g.,

$$\Omega_M = \bar\gamma^3\bar\Omega_M\,.$$

- Dressed average Hubble parameter

$$H = \frac{1}{a}\frac{\mathrm{d}a}{\mathrm{d}\tau_{\mathrm{w}}} = \frac{1}{\bar a}\frac{\mathrm{d}\bar a}{\mathrm{d}\tau_{\mathrm{w}}} - \frac{1}{\bar\gamma}\frac{\mathrm{d}\bar\gamma}{\mathrm{d}\tau_{\mathrm{w}}}$$

Timescape: Physical interpretation (2007)

- H is greater than wall Hubble rate; smaller than void Hubble rate measured by wall (or any one set of) clocks

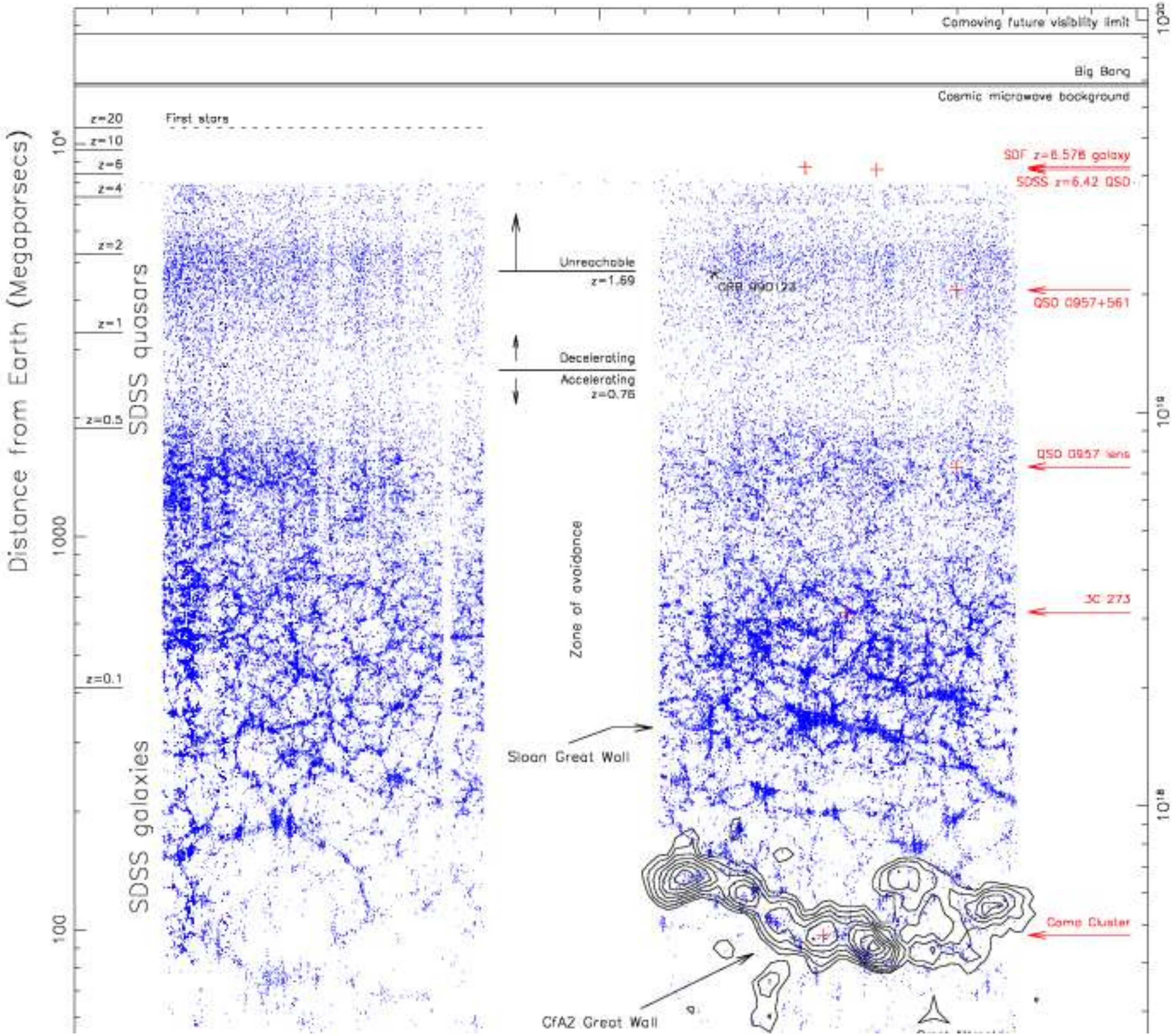
$$\bar{H}(t) = \frac{1}{\bar{a}}\frac{\mathrm{d}\bar{a}}{\mathrm{d}t} = \frac{1}{a_{\mathrm{v}}}\frac{\mathrm{d}a_{\mathrm{v}}}{\mathrm{d}\tau_{\mathrm{v}}} = \frac{1}{a_{\mathrm{w}}}\frac{\mathrm{d}a_{\mathrm{w}}}{\mathrm{d}\tau_{\mathrm{w}}} < H < \frac{1}{a_{\mathrm{v}}}\frac{\mathrm{d}a_{\mathrm{v}}}{\mathrm{d}\tau_{\mathrm{w}}}$$

- For tracker solution $H = (4{f_{\mathrm{v}}}^2 + f_{\mathrm{v}} + 4)/6t$
- Dressed average deceleration parameter

$$q = \frac{-1}{H^2 a^2}\frac{\mathrm{d}^2 a}{\mathrm{d}\tau_{\mathrm{w}}^2}$$

Can have $q < 0$ even though $\bar{q} = \frac{-1}{\bar{H}^2 \bar{a}^2}\frac{\mathrm{d}^2\bar{a}}{\mathrm{d}t^2} > 0$; difference of clocks important.

Cosmic coincidence problem



The timescape cosmology has always offered a solution to the : apparent cosmic *cosmic coincidence problem* acceleration starts when voids overtake filaments and walls in dominating volume averages

Cosmic coincidence? No problem

- Volume average observer sees no apparent cosmic acceleration

$$\bar{q} = \frac{2\,(1 - f_{\rm v})^2}{(2 + f_{\rm v})^2}\,.$$

As $t \to \infty$, $f_{\rm v} \to 1$ and $\bar{q} \to 0^+$.

- A wall observer registers apparent cosmic acceleration

$$q = \frac{-\,(1 - f_{\rm v})\,(8 f_{\rm v}{}^3 + 39 f_{\rm v}{}^2 - 12 f_{\rm v} - 8)}{\left(4 + f_{\rm v} + 4 f_{\rm v}{}^2\right)^2}\,,$$

Effective deceleration parameter starts at $q \sim \frac{1}{2}$, for small $f_{\rm v}$; changes sign when $f_{\rm v} = 0.5867\ldots$, and approaches $q \to 0^-$ at late times.

Physical Review Letters **99**, 251101 (2007)

First principles: redshift

$$
\begin{aligned}
1 + z_{\rm obs} &= \frac{-\mathbf{U}_{\rm em}\cdot\mathbf{k}}{-\mathbf{U}_{\rm obs}\cdot\mathbf{k}} = \frac{\lambda_{\rm obs}}{\lambda_{\rm em}} = \frac{E_{\rm em}}{E_{\rm obs}} \\
&\neq \sqrt{\frac{c+v}{c-v}} \simeq 1 + \frac{v}{c} + \mathcal{O}\left(\frac{v^2}{c^2}\right) \\
&= \frac{\mathbf{U}_0\cdot\mathbf{k}}{\mathbf{U}_{\rm obs}\cdot\mathbf{k}}\,\frac{\mathbf{U}_1\cdot\mathbf{k}}{\mathbf{U}_0\cdot\mathbf{k}}\,\frac{\mathbf{U}_{\rm em}\cdot\mathbf{k}}{\mathbf{U}_1\cdot\mathbf{k}} \\
&= (1+z_{\rm pec,0})\,\frac{g_{\mu\nu}U_1^\mu k^\nu\big|_{\mathbf{x}_1}}{g_{\alpha\beta}U_0^\alpha k^\beta\big|_{\mathbf{x}_0}}\,(1+z_{\rm pec,1})
\end{aligned}
$$

Adapted to ideal observers at rest $\mathrm{d}x^i = 0$, define

$$
(1 + z_{\rm cos}) \equiv \frac{g_{\mu\nu}U_1^\mu k^\nu\big|_{\mathbf{x}_1}}{g_{\alpha\beta}U_0^\alpha k^\beta\big|_{\mathbf{x}_0}} = \frac{g_{tt}U_1^t k^t\big|_{\mathbf{x}_1}}{g_{tt}U_0^t k^t\big|_{\mathbf{x}_0}} \tag{1}
$$

First principles: redshift

For models with average isotropic expansion

$$
\begin{aligned}
(1+z_{\rm cos}) &\equiv (1+z_{\phi,0})\,(1+\bar z)\,(1+z_{\phi,1}) \\
\text{Interpret}\quad 1+z_{\phi,0} &= 1/(-g_{tt}U_0^t)\big|_{\mathbf{x}_0} = 1/\sqrt{-g_{tt}(\mathbf{x}_0)} \\
1+z_{\phi,1} &= (-g_{tt}U_1^t)\big|_{\mathbf{x}_1} = \sqrt{-g_{tt}(\mathbf{x}_1)} \\
1+\bar z &= k^t(\mathbf{x}_1)/k^t(\mathbf{x}_0)
\end{aligned}
$$

as "gravitational redshifts" versus "background expansion"

Notes
- FLRW: $1+\bar z = a(t_0)/a(t_1)$
- Standard cosmology: $z_{\phi,0},\ z_{\phi,1} \sim 10^{-5}$
- Λ-Szekeres: $z_{\phi,0},\ z_{\phi,1} \sim 10^{-3}$
- Timescape: Null cone conformal frame degeneracy
- Bondi-Metzner-Sachs group relevant generally

First principles: distances

Observational definition $\quad d_L \equiv \sqrt{\frac{\mathcal{L}}{4\pi\mathcal{F}}}$

Etherington relation $\quad d_A = \frac{D}{1+z} = \frac{d_L}{(1+z)^2}$

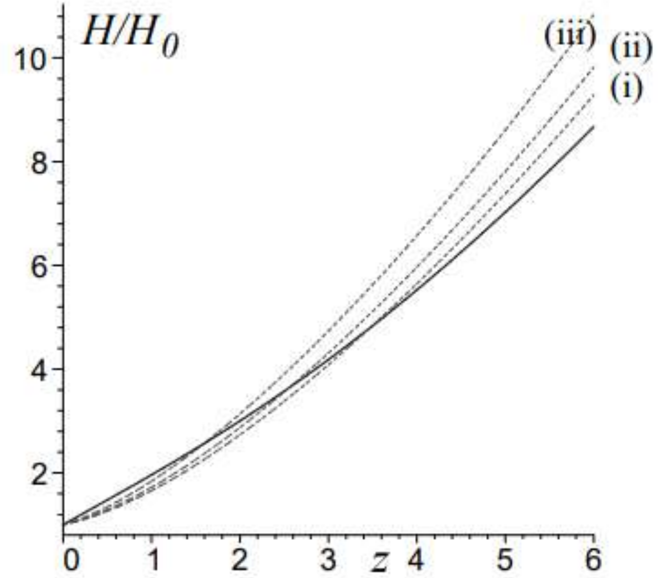
- d_L based on Euclidean $\mathcal{F} = \mathcal{L}/(4\pi {d_L}^2)$
- When non-Euclidean relation to null geodesics differs
- $d_L = (1+z)^2 d_A$ quite general, applies here.
- FLRW $D = \frac{c}{H_0\sqrt{|\Omega_{k0}|}}\,\mathrm{sinn}\left(\sqrt{|\Omega_{k0}|}\int_{1/(1+z)}^{1}\frac{\mathrm{d}y}{\sqrt{\Omega_{R0}+\Omega_{M0}y+\Omega_{k0}y^2+\Omega_{\Lambda 0}y^4}}\right)$

  $\mathrm{sinn}(x) = \{\sinh(x),\ \Omega_{k0}>0;\ x,\ \Omega_{k0}=0;\ \sin(x),\ \Omega_{k0}<0\}$
- Timescape $D = c\,(1+z)t^{2/3}\int_t^{t_0}\frac{2\,\mathrm{d}t'}{(2+f_v(t'))(t')^{2/3}} = c\,(1+z)t^{2/3}(\mathcal{T}(t_0)-\mathcal{T}(t))$

  $\mathcal{T}(t) = 2t^{1/3} + \frac{b^{1/3}}{6}\ln\left(\frac{(t^{1/3}+b^{1/3})^2}{t^{2/3}-b^{1/3}t^{1/3}+b^{2/3}}\right) + \frac{b^{1/3}}{\sqrt{3}}\tan^{-1}\left(\frac{2t^{1/3}-b^{1/3}}{\sqrt{3}\,b^{1/3}}\right)$

  $b = \frac{2(1-f_{v0})(2+f_{v0})}{9f_{v0}\bar{H}_0}, \qquad \text{bare } \bar{H}_0 = \frac{2(2+f_{v0})H_0}{(4{f_{v0}}^2+f_{v0}+4)}$

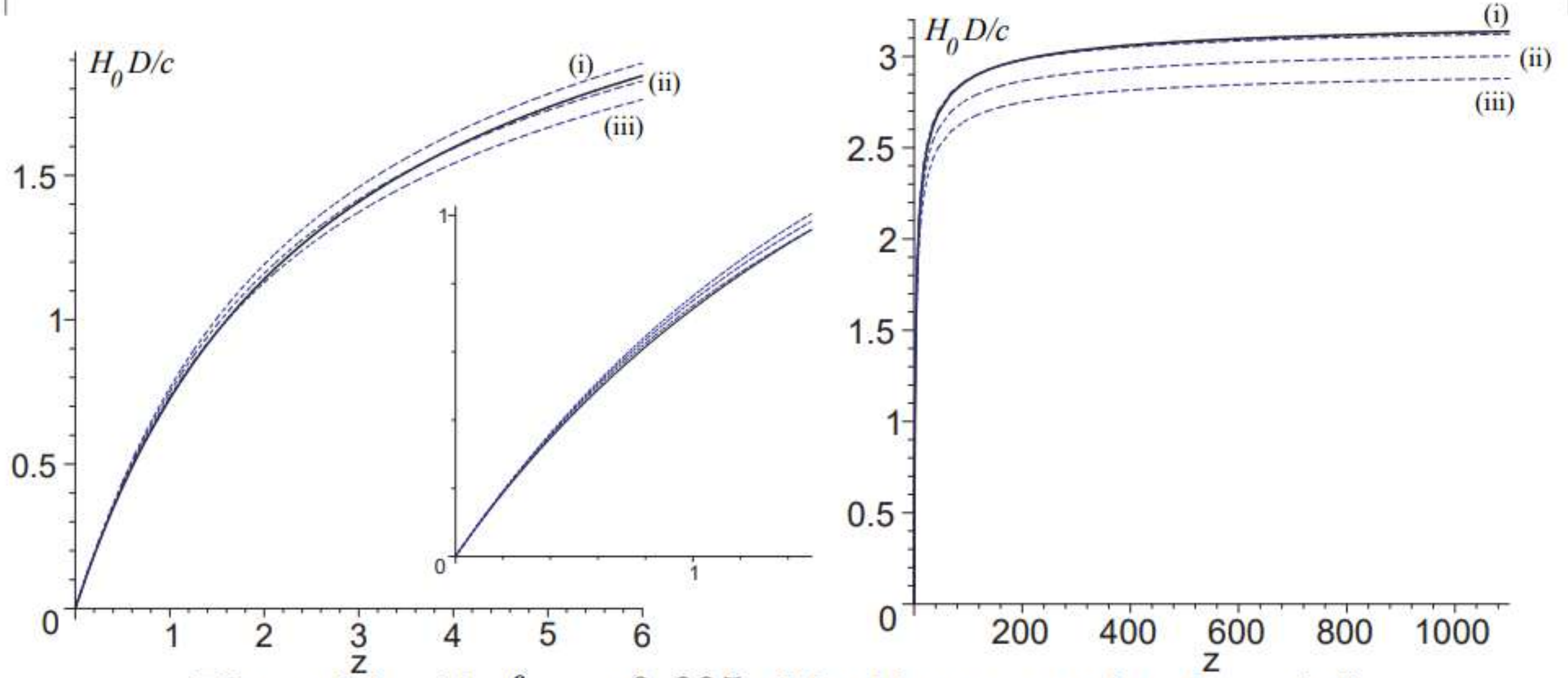
Average expansion: $H(z)/H_0$



$H(z)/H_0$ for $f_{v0} = 0.762$ (solid line) is compared to three spatially flat ΛCDM models: **(i)** $(\Omega_{M0}, \Omega_{\Lambda0}) = (0.249, 0.751)$; **(ii)** $(\Omega_{M0}, \Omega_{\Lambda0}) = (0.279, 0.721)$ **(iii)** $(\Omega_{M0}, \Omega_{\Lambda0}) = (0.34, 0.66)$;.

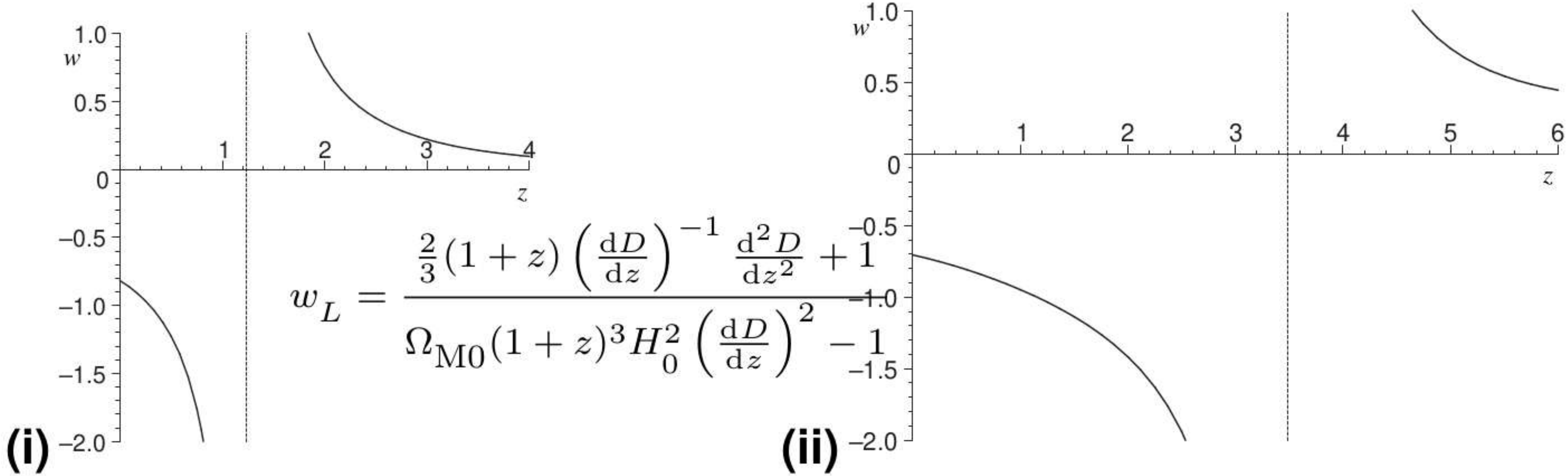
- Function $H(z)/H_0$ displays quite different characteristics
- For $0 < z \lesssim 1.7$, $H(z)/H_0$ is larger for TS model, but value of H_0 assumed also affects $H(z)$ numerical value

Dressed "comoving distance" $D(z)$



TS model, with $f_{v0} = 0.695$, **(black)** compared to 3 spatially flat ΛCDM models (blue): **(i)** $\Omega_{M0} = 0.3175$ (best-fit ΛCDM model to Planck); (ii) $\Omega_{M0} = 0.35$; **(iii)** $\Omega_{M0} = 0.388$.

Effective “dark energy” equation of state

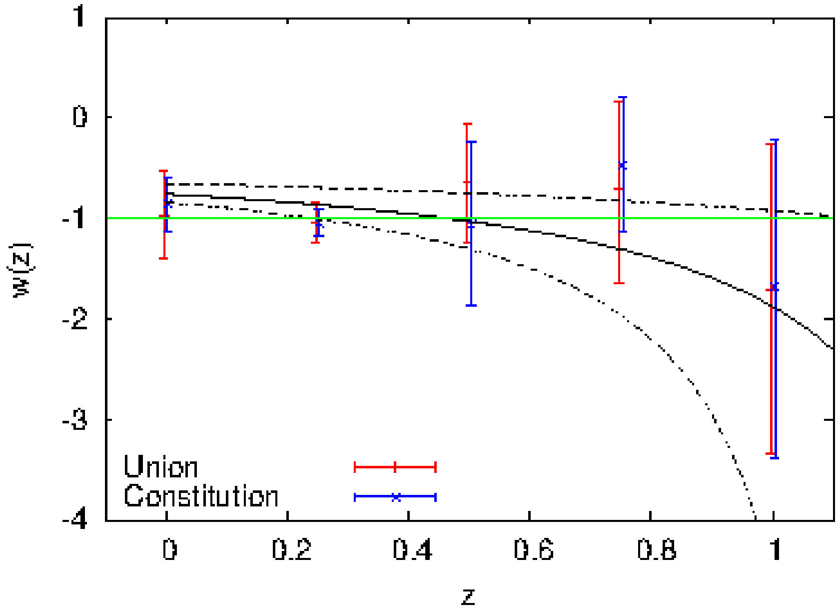


$$w_L = \frac{\frac{2}{3}(1+z)\left(\frac{\mathrm{d}D}{\mathrm{d}z}\right)^{-1}\frac{\mathrm{d}^2D}{\mathrm{d}z^2}+1}{\Omega_{\mathrm{M0}}(1+z)^3H_0^2\left(\frac{\mathrm{d}D}{\mathrm{d}z}\right)^2-1}$$

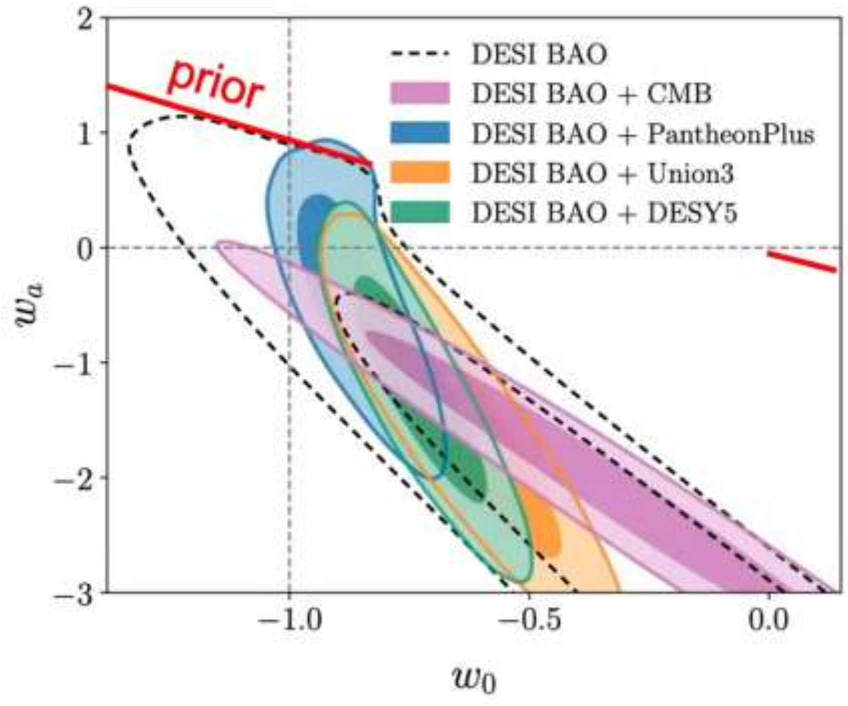
A formal “dark energy equation of state” $w_L(z)$ for the TS model, with $f_{\mathrm{v0}} = 0.695$, calculated directly from $r_w(z)$: **(i)** $\Omega_{\mathrm{M0}} = 0.41$; **(ii)** $\Omega_{\mathrm{M0}} = 0.3175$.

Description by a “dark energy equation of state” makes no fundamental sense when there’s no physics behind it; but average value w $\simeq -1$ for z < 0.7 makes empirical sense.

Effective evolving "dark energy"



Physical Review D ***80***, 123512 (2009), Figure 4



DESI 2024 results, now further tightened

DESI BAOs find evidence for "dark energy" evolving in a manner consistent with timescape – defining a precision test will require recalibration of the sound speed in the primordial plasma, the dark matter/baryonic matter ratio

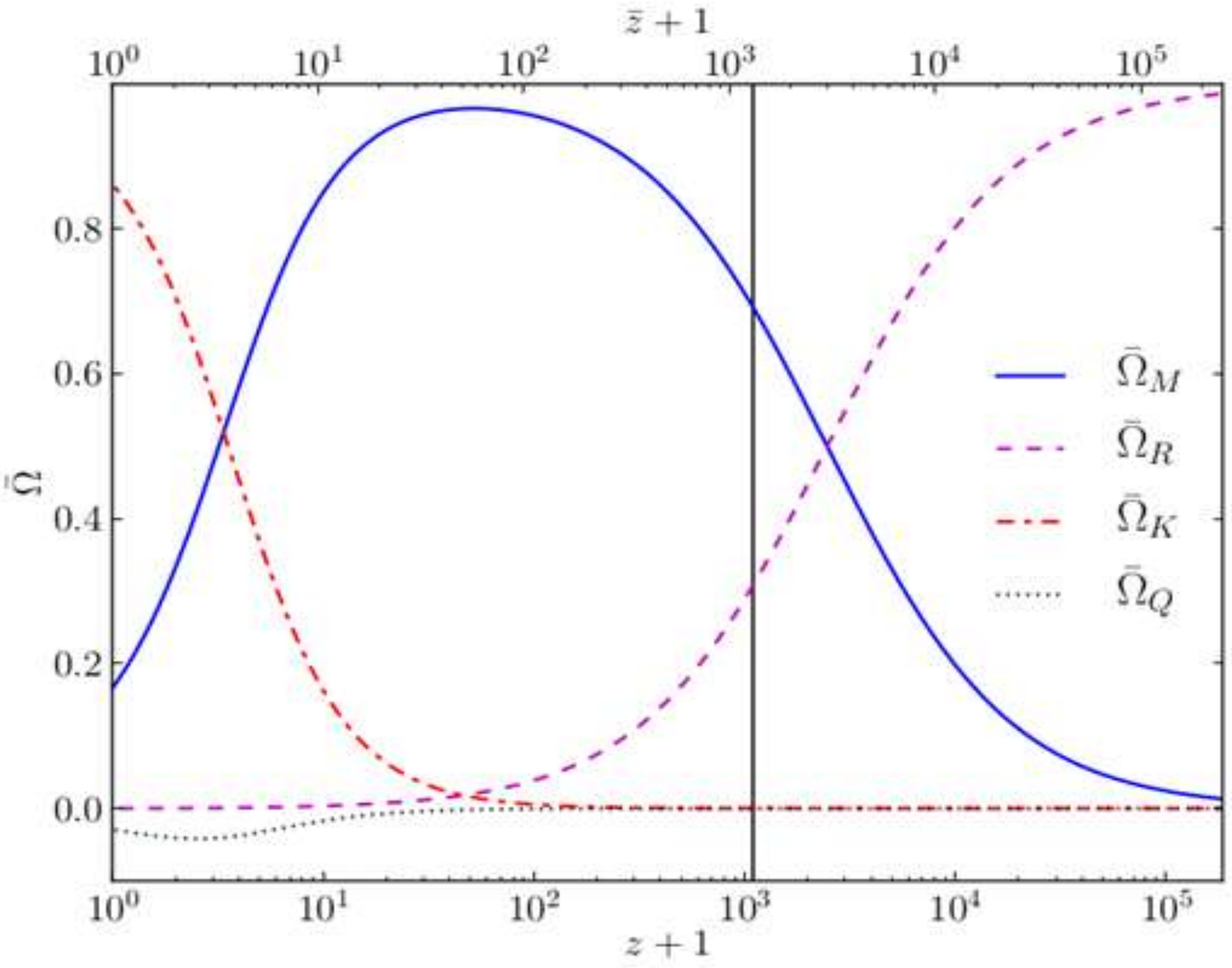
Timescape Planck CMB Constrained Sum Rule

$$\bar{\Omega}_{\mathrm{M}} + \bar{\Omega}_{\mathrm{R}} + \bar{\Omega}_{\mathrm{K}} + \bar{\Omega}_{\mathcal{Q}} = 1$$

$$0.167 + {\sim}10^{-5} + 0.862 - 0.0293 = 1$$

Classical & Quantum Gravity Review D ***30***, 175006 (2013)

The role of non-rigid spatial curvature



NOT FLRW

$$\bar{\Omega}_K \neq -k/(\bar{a}^2\bar{H}^2)$$

TIMESCAPE

$$\bar{\Omega}_K \propto f_v^{1/3}/(\bar{a}^2\bar{H}^2)$$

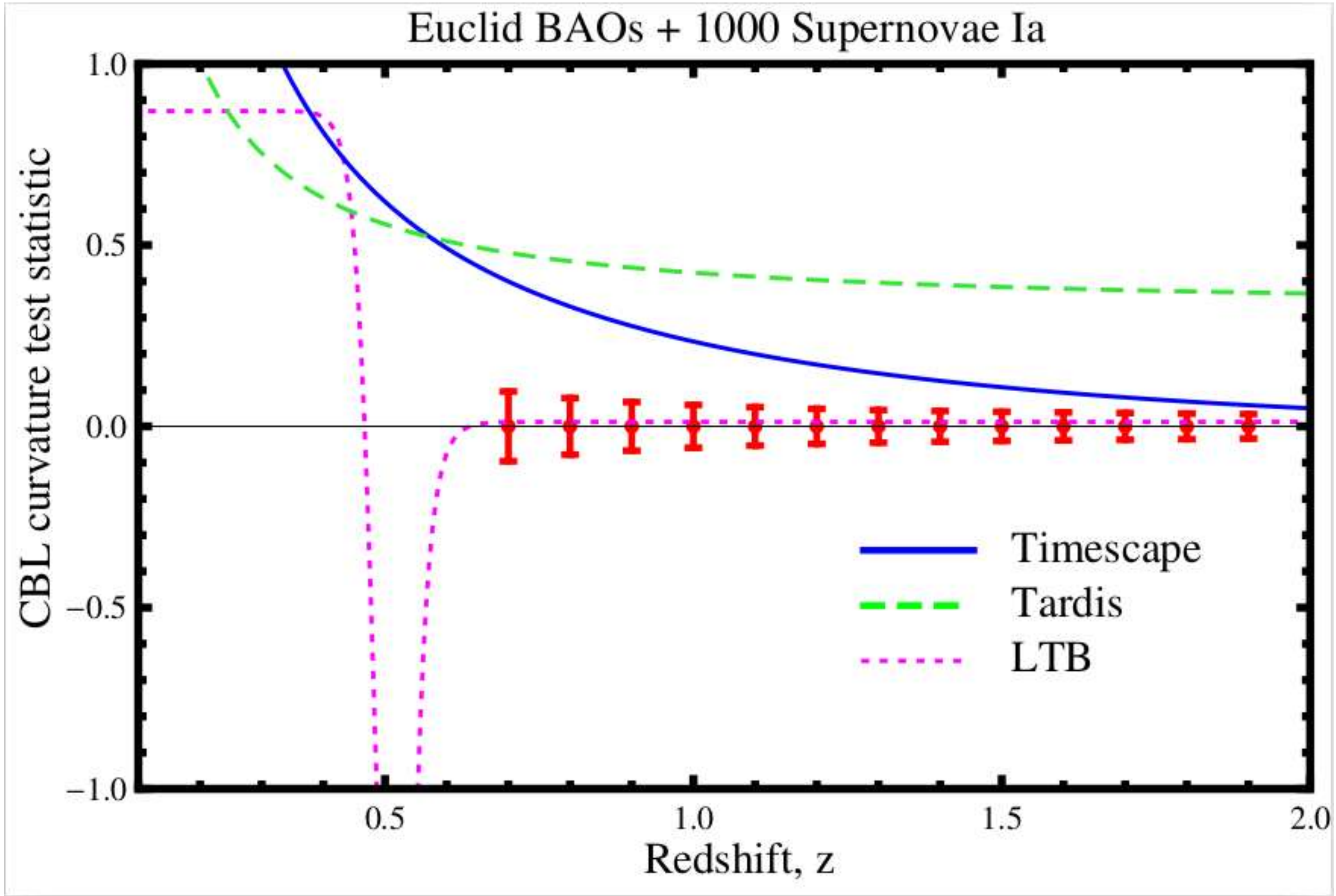
Classical & Quantum Gravity Review D ***30***, 175006 (2013)

Other key parameters – fit to Planck CMB data

Age of Universe – Observer dependent		Units Gyr (billion years)
Age of universe: wall/galaxy observer	τ_{w0}	14.2 ± 0.5
Age of universe: volume average observer	t_0	17.5 ± 0.6
Age of universe: void centre observer	τ_{v0}	21.3 ± 0.8

Hubble "constant"		Units (km/s/Mpc)
Bare (within filament, galaxy observer)	$\bar{H}_0$	50.1 ± 1.7
Dressed	H_0	61.7 ± 3.0
Dressed FLRW equivalent	H_{FLRW0}	67.4 ± 1.3
Void as seen by galaxy observer	H_{v0}	75.2 ± 2.6

Euclid will test FLRW & timescape

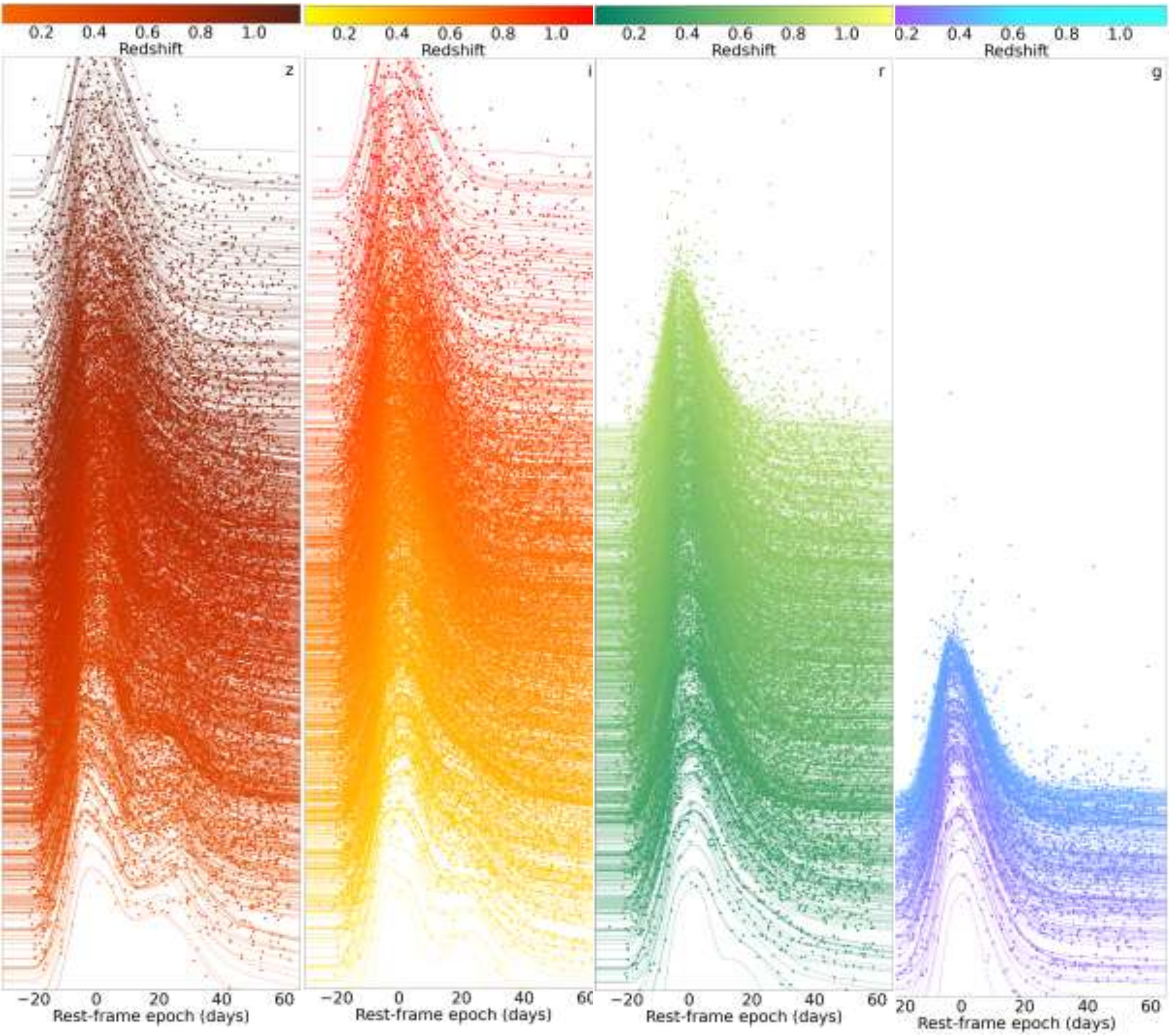


Images:
Clarkson-Bassett-Lu test: projection for Euclid, D Sapone, E Majerotto, S Nesseris, Phys Rev D 90 023021 (2014)

2. Supernovae evidence for foundational change to cosmological models, MNRAS Letters **537**, L55 (2025); A Seifert, ZG Lane, M Galoppo, R Ridden-Harper, DLW

+ Cosmological foundations revisited with Pantheon+ MNRAS **536**, 1752 (2025); ZG Lane, A Seifert, R Ridden-Harper, DLW

Pantheon+ Revisited (MNRAS and MNRAS Letters 2025)



Tripp relation is based on empirical supernova light curve parameters:

$$\mu = m - M = m_B^* - M_B + \alpha x_1 - \beta c,$$

but is related to cosmological models which predict a *bolometric* luminosity distance d_L:

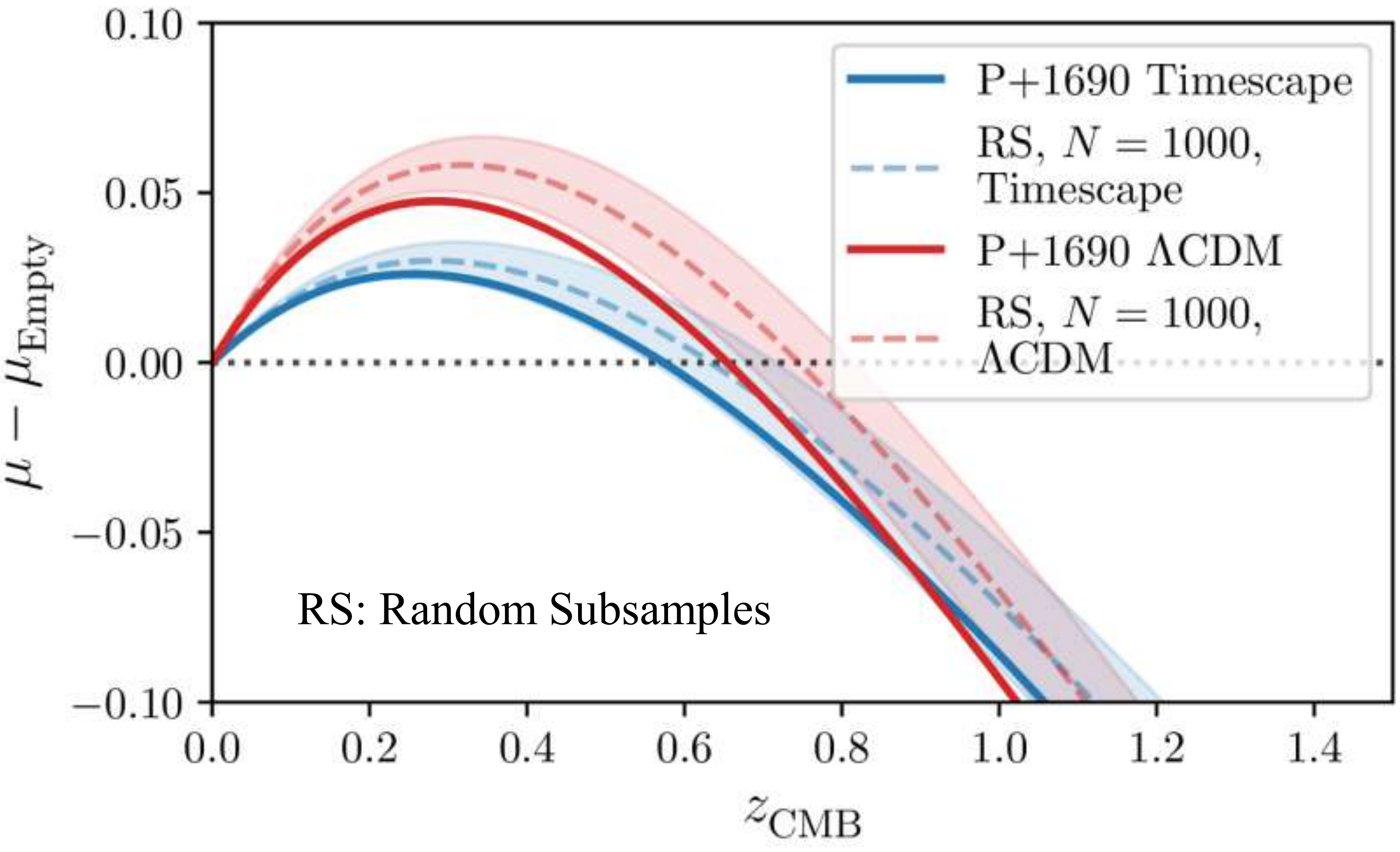
$$\mu \equiv 25 + 5 \log_{10}\left(\frac{d_L}{\mathrm{Mpc}}\right),$$

Observed distance modulus serves as a correction model

$$\Delta\mu_B \equiv (m - M) - (m_B^* - M_B) = \alpha x_1 - \beta c,$$

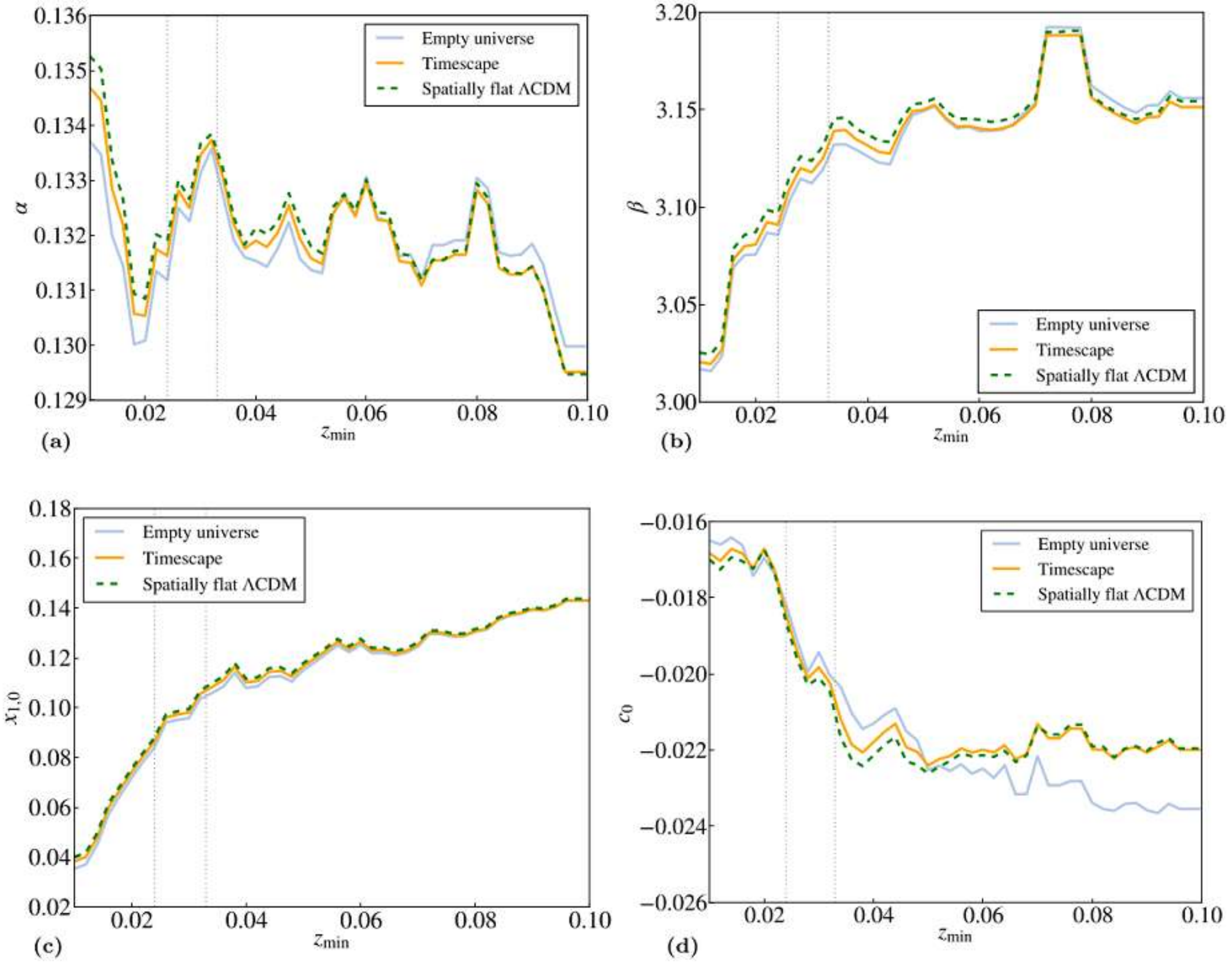
But $\Delta(M - M_B)$ degenerate with ΔH_0 !

Pantheon+ Revisited (arXiv:2311.01438)



Residual Distance Moduli ΛCDM / Timescape

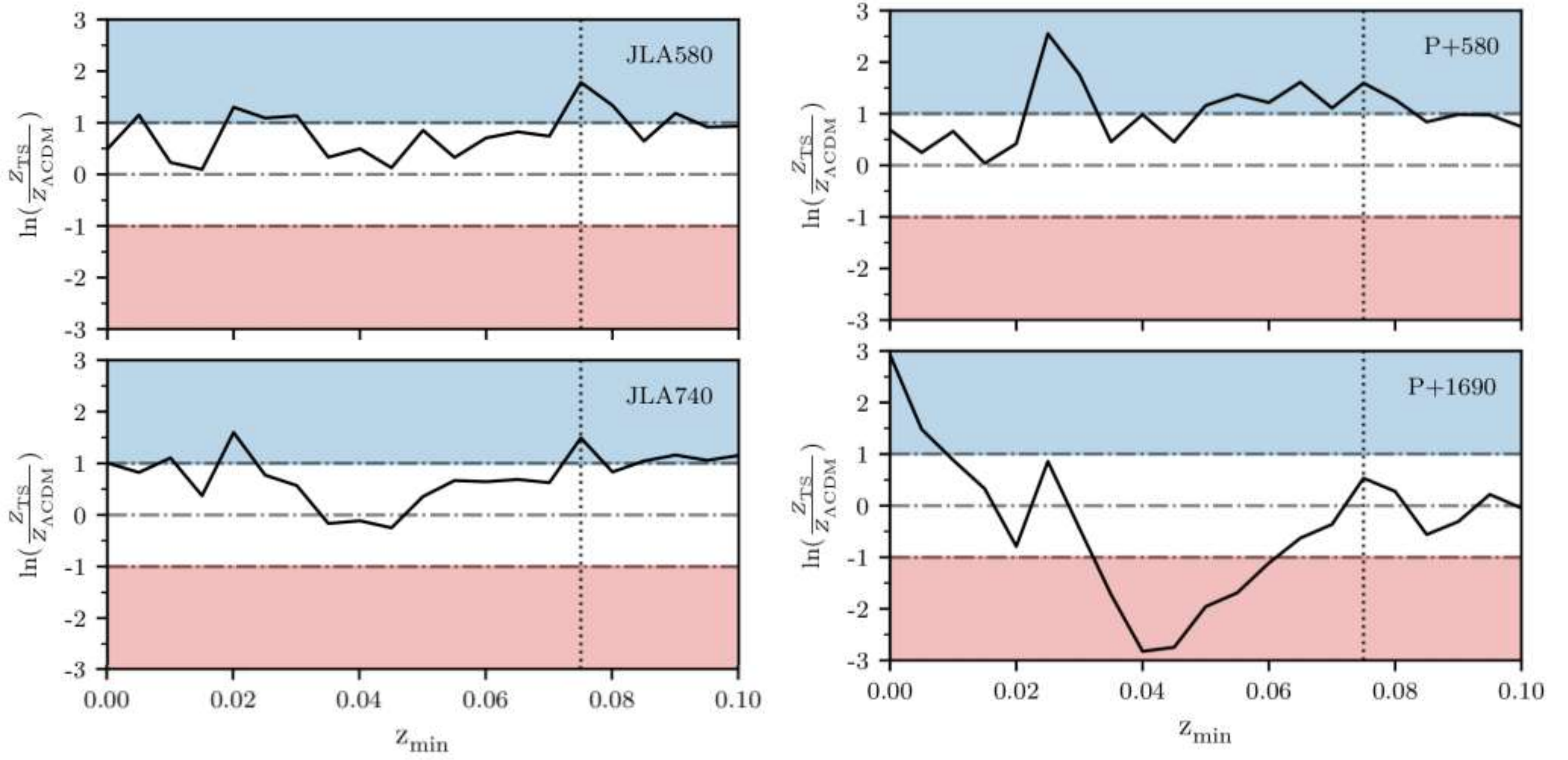
Dam, Heinesen + DLW, JLA sample



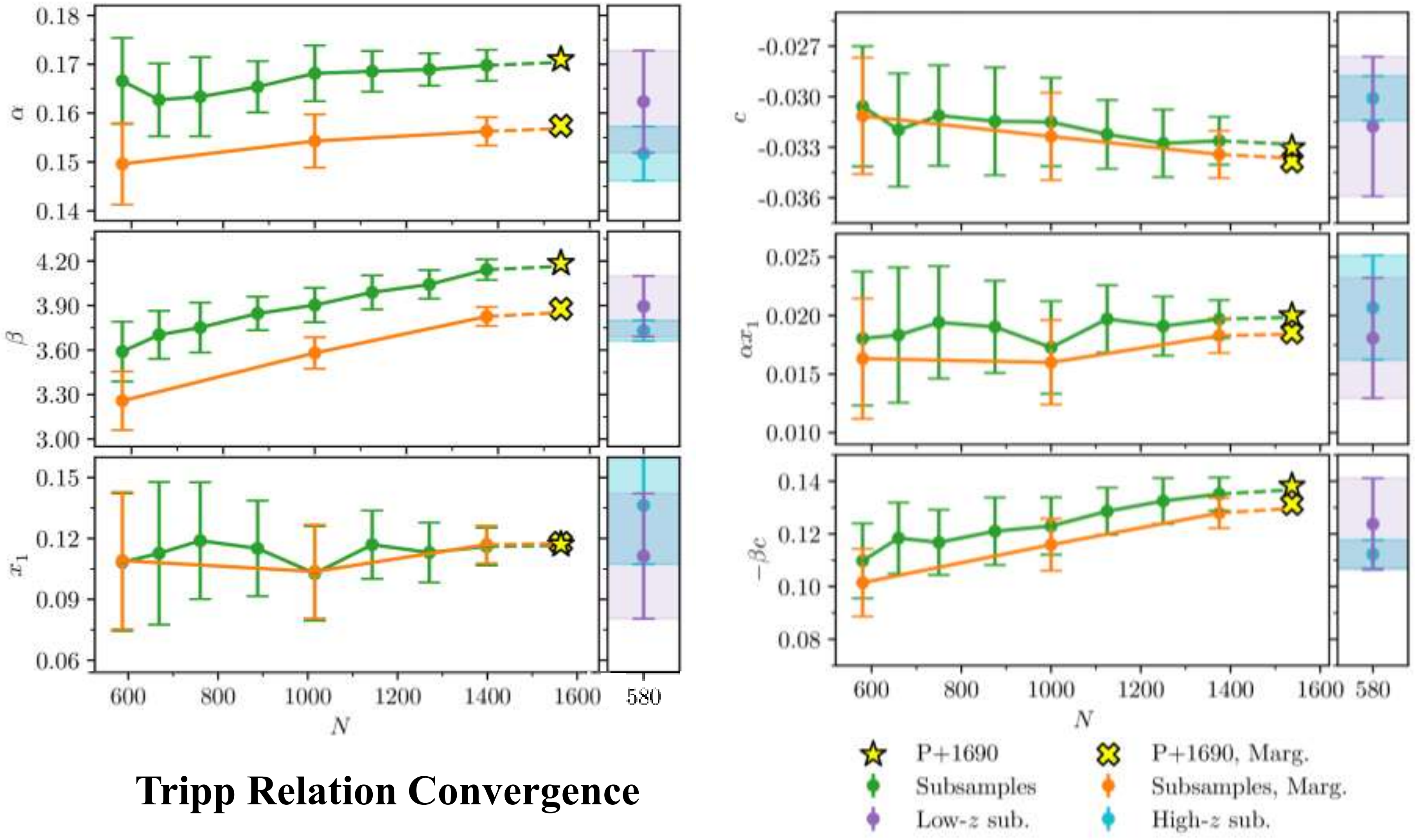
MNRAS **472**, 875 (2017)

- 740 Sne Ia
- Work in heliocentric frame
- Remove data below minimum $z_{\rm min}$ and refit as $z_{\rm min}$ varied, reanalyse
- Assume independent Gaussians for colour c, stretch x_1
- Marginal preference for timescape

Hubble tension revisited



Pantheon+ Revisited (arXiv:2311.01438)

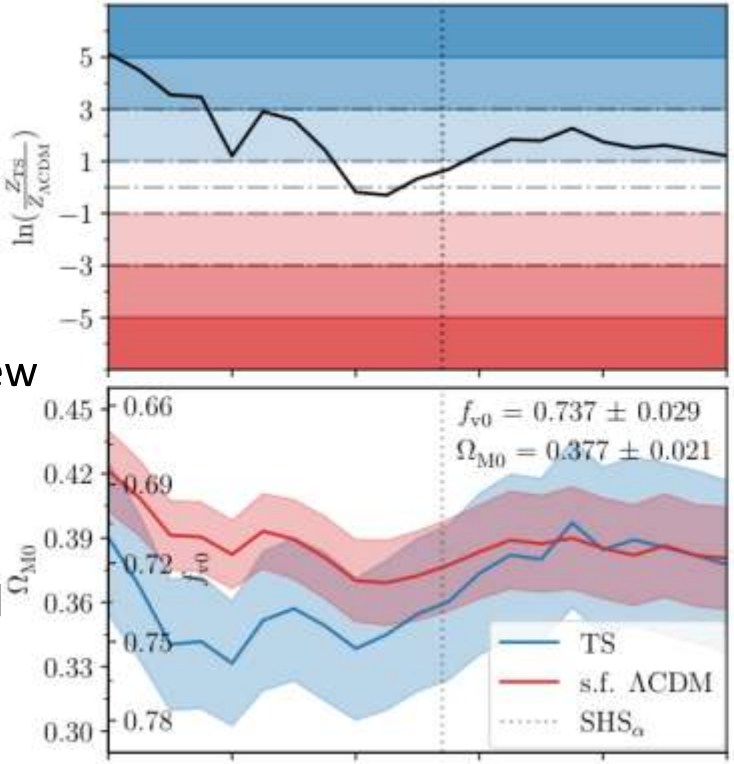


Tripp Relation Convergence

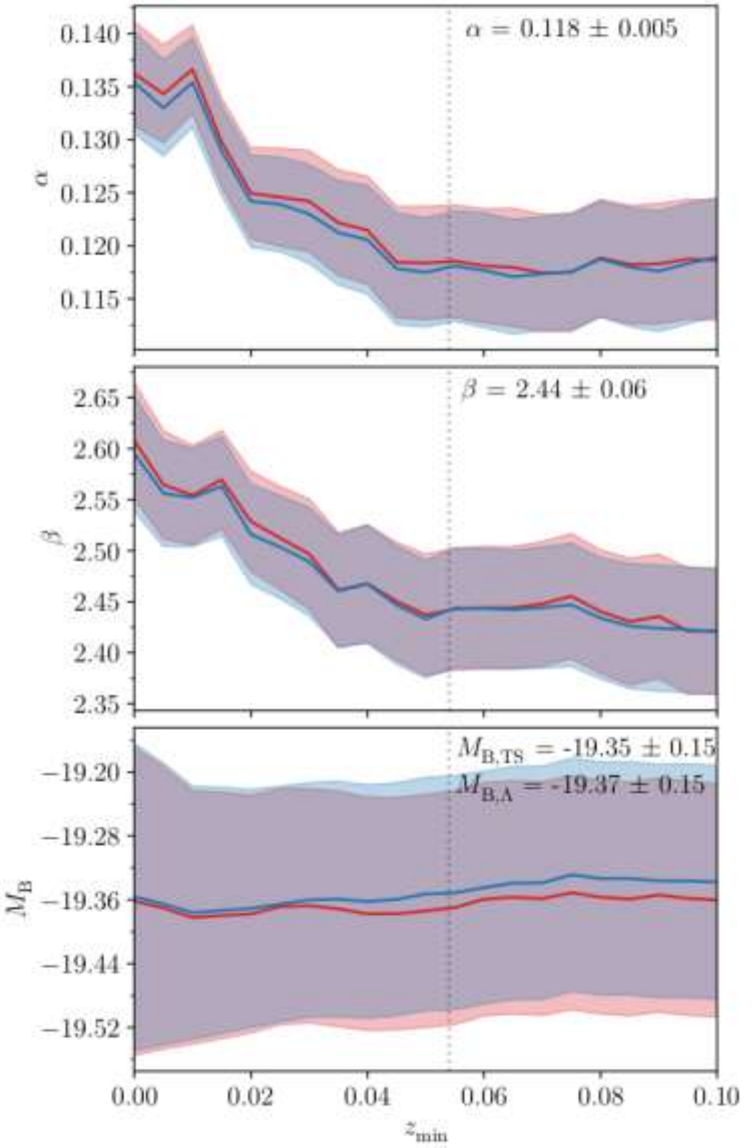
Hubble tension gone

Following a suggestion of the referee of arXiv:2311.01438, with new methodology:

- Timescape fits better than LCDM with >99% confidence, ln B > 5
- Parameters agree quantitatively with new DESI results that disfavour LCDM
- The parameters agree with Planck CMB geometric scales, & strong gravitational lensing distance ratio fits
- The parameters are consistent with numerical relativity simulations, MJ Williams et al, MNRAS 536, 2645 (2025)



New results:
A. Seifert et al
MNRAS Letters



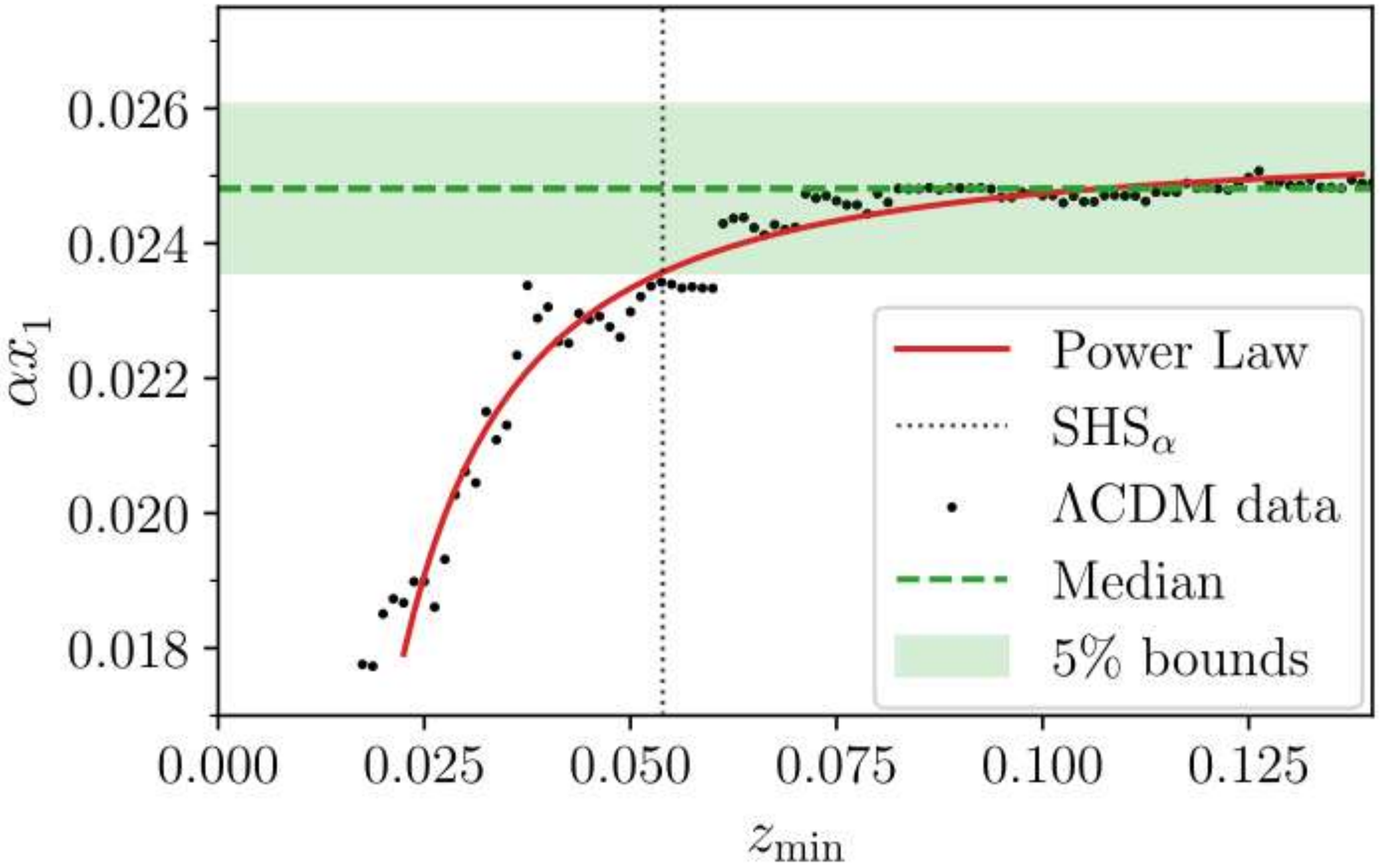


Figure 3. The convergence of the αx_1 light-curve parameter for the spatially flat ΛCDM model across various redshift cuts, where x_1 is the median value from the distribution. A power-law model has been fit to the data, and the shaded band represents within 5 per cent of the median value within the range $0.1 \leq z_{\rm min} < 0.14$ indicating when the model converges. The vertical dotted line represents the SHS_α found at $z_{\rm min} = 0.054^{+0.007}_{-0.012}$. The power-law uncertainty is smaller than the plotted line.

MNRAS Letters **537**, L55 (2025)

- Product αx_1
- Power law fit
  $-az^{-n}+C$

Convergence defines empirical statistical homogeneity scale SHS α

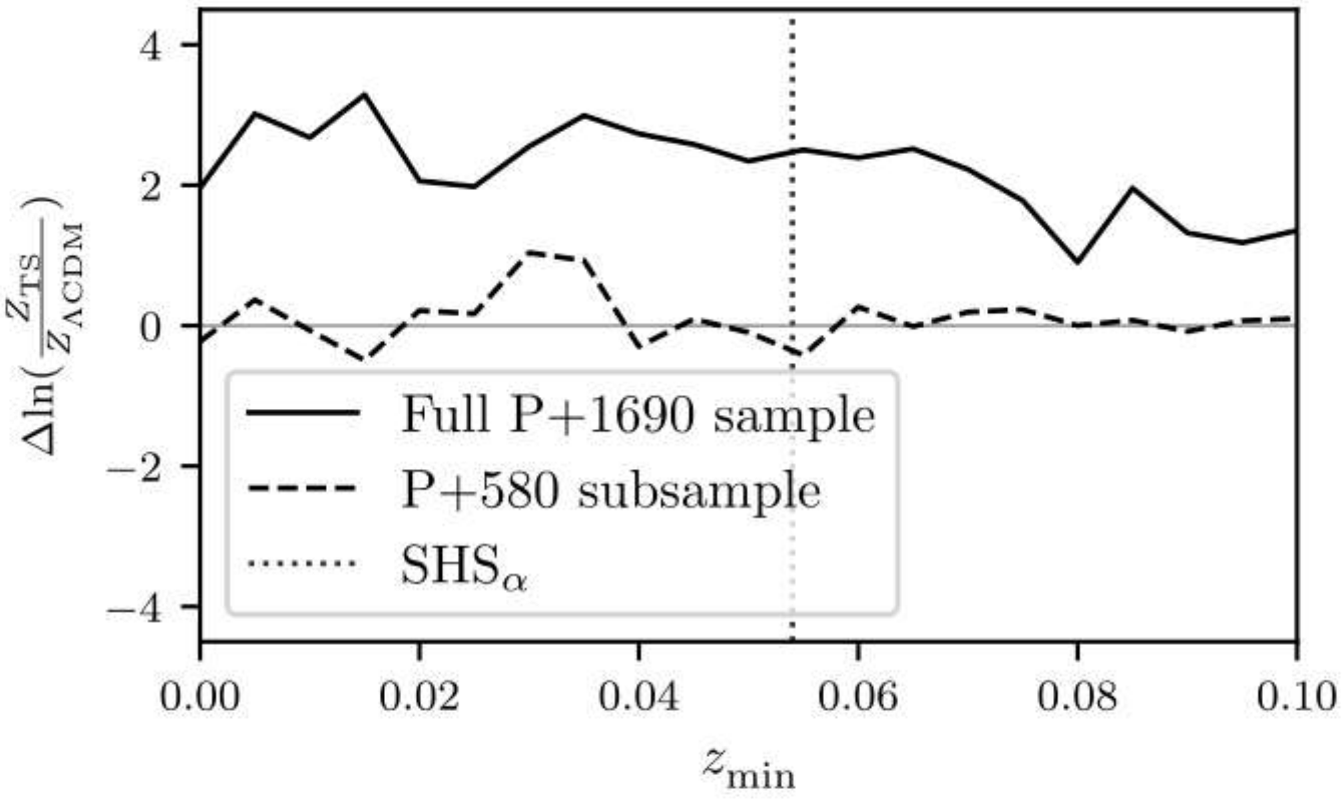
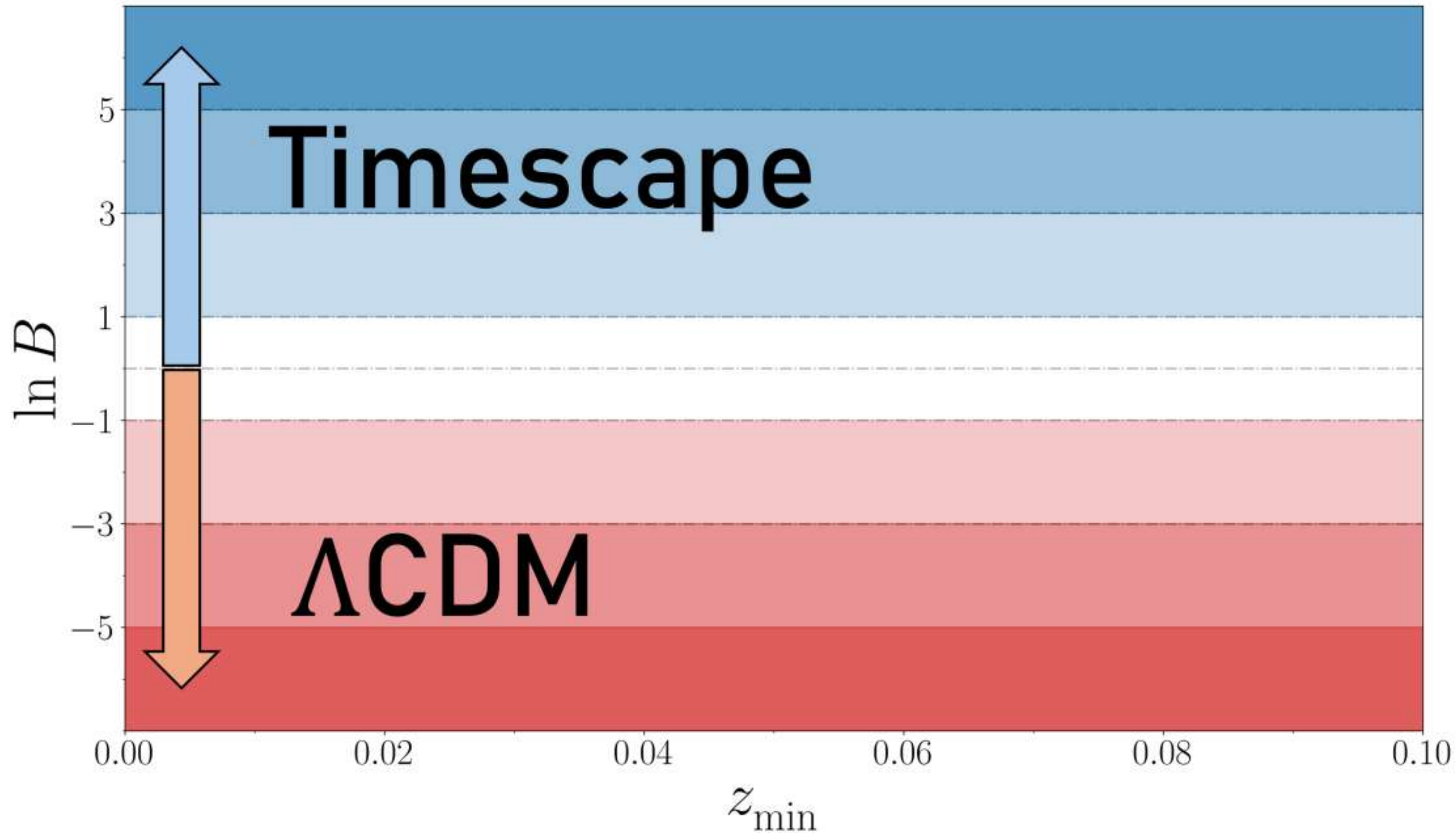
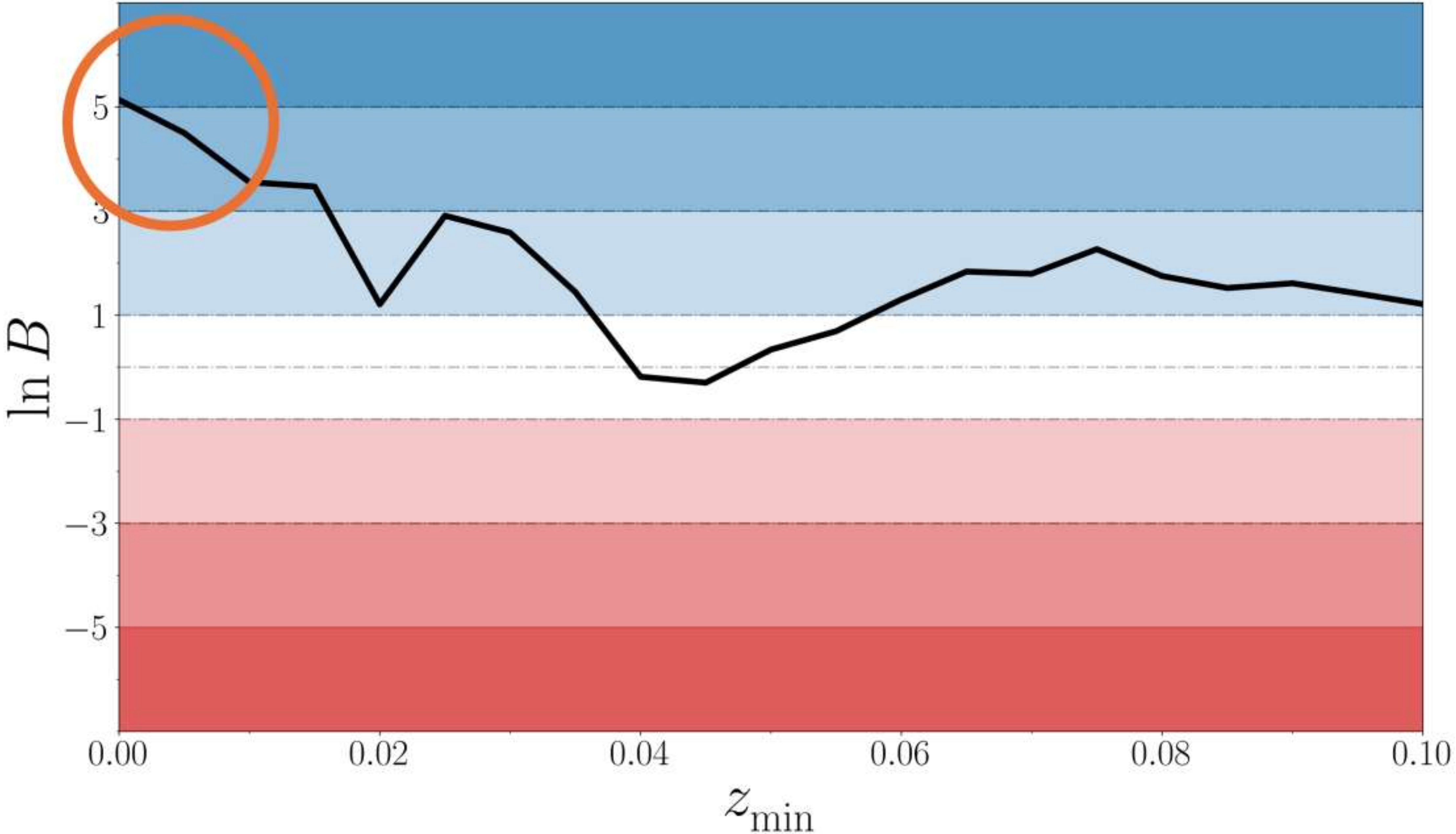
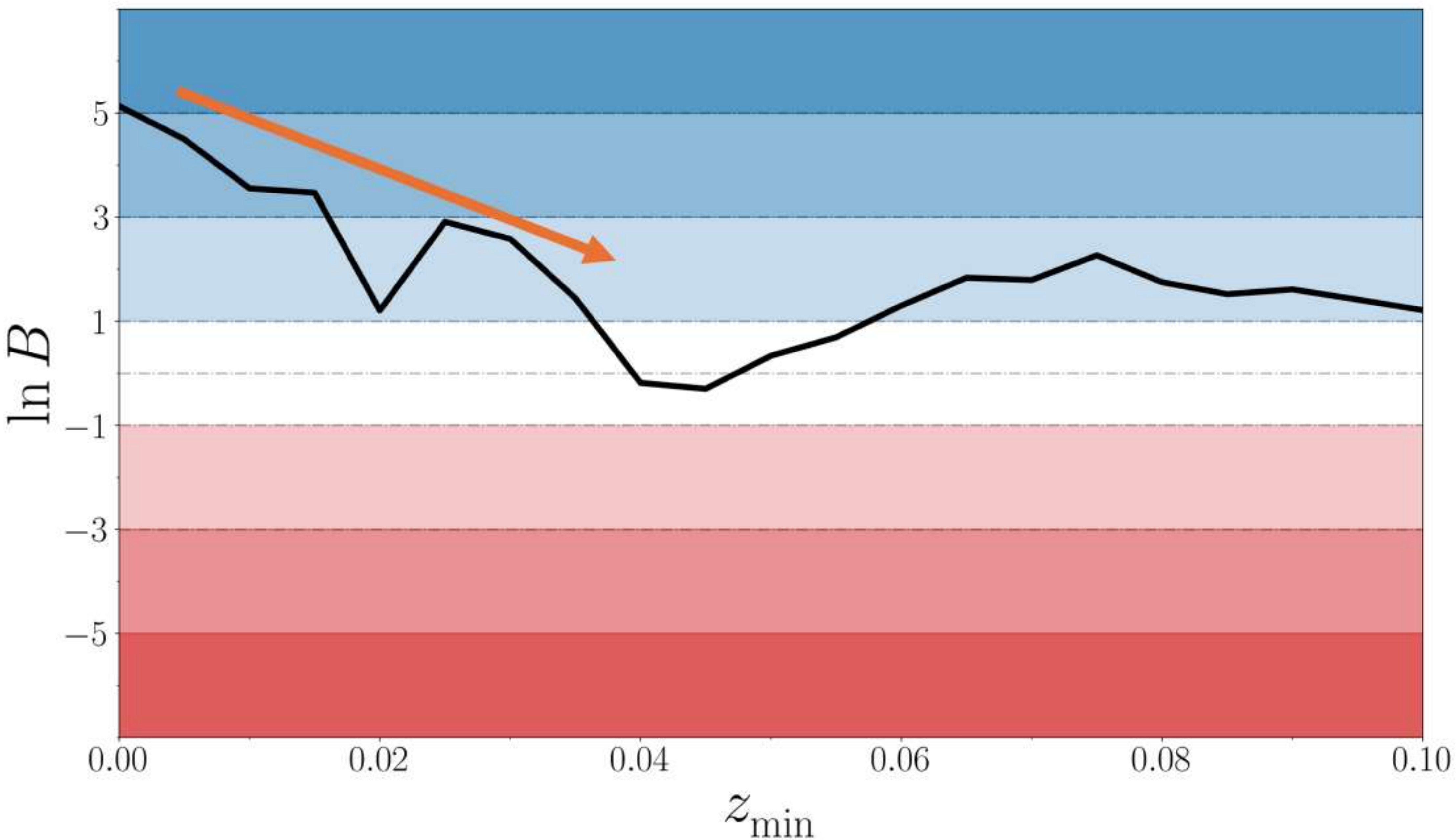


Figure 2. The difference in the Bayes factors for the full P+1690 sample and the P+580 subsample between Lane et al. (2024) and our results. For the subsample, the results from the new analysis presented here align very well with the results by Lane et al. (2024), while for the full sample the new analysis greatly increases the preference in favour of timescape.

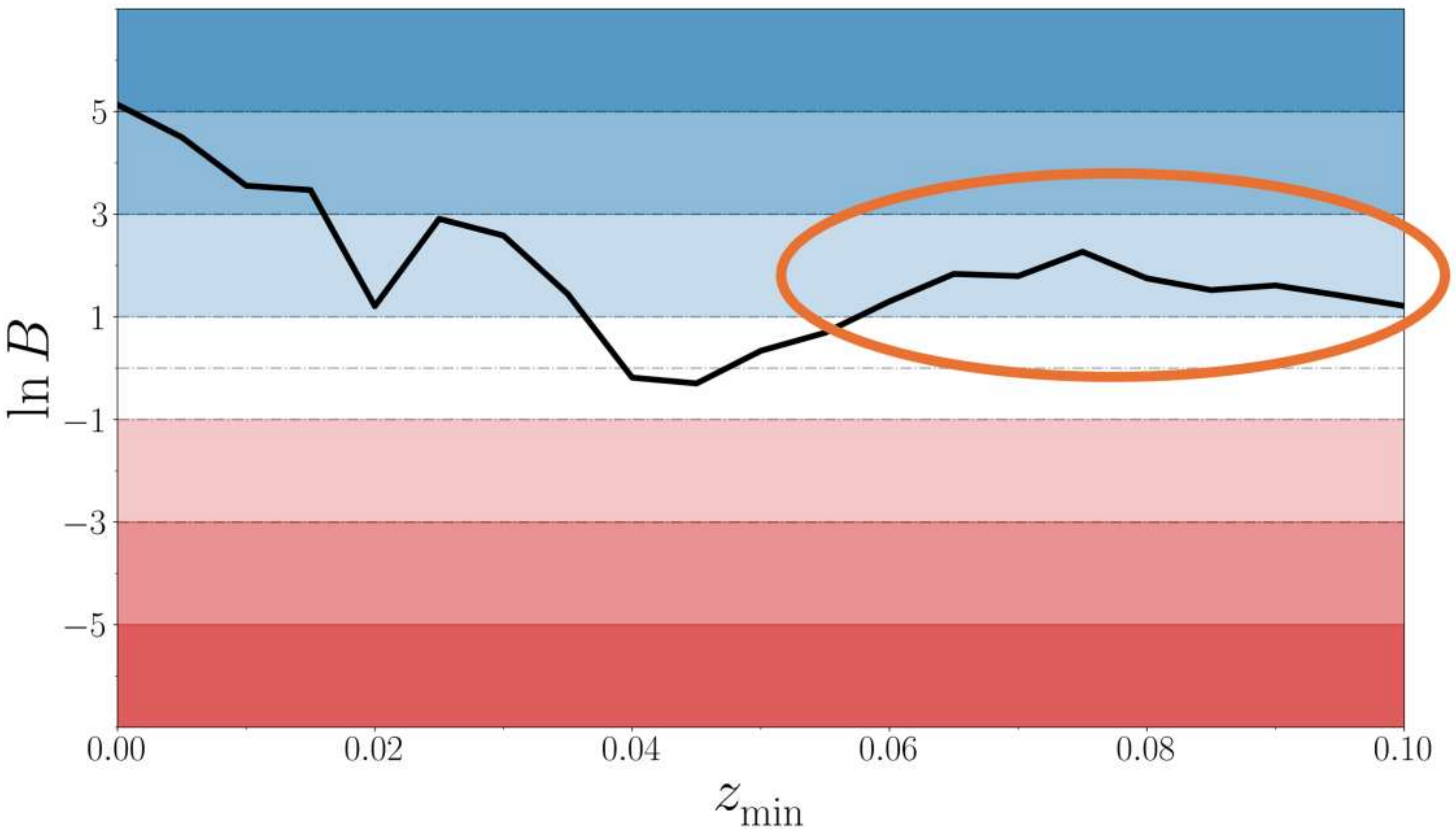

Timescape
ΛCDM
5
3
1
−1
−3
−5
ln B
0.00
0.02
0.04
0.06
0.08
0.10
$z_{\rm min}$



5
3
1
−1
−3
−5
$\ln B$
0.00
0.02
0.04
0.06
0.08
0.10
z_{min}



5
3
1
−1
−3
−5
ln B
0.00
0.02
0.04
0.06
0.08
0.10
z_min



5
3
1
−1
−3
−5
ln B
0.00
0.02
0.04
0.06
0.08
0.10
z_min

DES (Camilleri et al, MNRAS 533 (2024) 2615)

Timescape	$\Omega_{\rm m}^{\S}$	f_{v0}
DES-SN5YR$_{\rm cut}$		
Timescape*	$0.292^{+0.043}_{-0.051}$	$0.791^{+0.039}_{-0.034}$
Flat-ΛCDM	$0.362^{+0.019}_{-0.018}$	-
DES-SN5YR$_{\rm cut}$ + BAO-$\theta_{*\perp}$		
Timescape*	$0.446^{+0.010}_{-0.009}$	$0.665^{+0.008}_{-0.009}$
Flat-ΛCDM	$0.332^{+0.011}_{-0.010}$	-

- These results use zmin = 0.33

	DES-SN5YR$_{\rm cut}$				DES-SN5YR$_{\rm cut}$ + BAO-$\theta_{*\perp}$		
Flat-ΛCDM	0.0	0.0	1616	Flat-ΛCDM	0.0	0.0	1624
Timescape	−1.7	−1.72	1612	Timescape	6.3	6.17	1637

- DES BAO does not yet use direct BAO extraction from raw data [cf A Heinesen et al JCAP 03 (2019) 003]
- To fit timescape expansion history, sound speed in early universe must be recalibrated with different dark matter density (and potential new gravitational physics if "dark matter" is non-particulate)

New formalism for cosmological perturbation theory without a global background

Cosmological perturbations on an averaged background

Marco Galoppo[a,*] **and Pierre Mourier**[a]

[a] *School of Physical & Chemical Sciences, University of Canterbury, Private Bag 4800, Christchurch 8140, New Zealand*

E-mail: Marco.Galoppo@canterbury.ac.nz, Pierre.Mourier@canterbury.ac.nz

ABSTRACT: In relativistic cosmology, the formation of nonlinear inhomogeneities can induce non-negligible backreaction on late-time expansion. Among the important consequences for precision cosmology is the potential impact on the linear growth of large-scale structures. We address this impact by combining covariant spatial averaging with covariant and gauge-invariant perturbation theory. We focus on irrotational dust model spacetimes. The effects of backreaction and nontrivial dynamical curvature on the average cosmological dynamics are formulated as the addition of an effective perfect fluid with pressure. We then introduce an effective background driven by both the averaged dust density and the emergent effective fluid, and derive the general evolution equations for linear perturbations of this system. The residual freedom in this framework amounts to specifying the properties of the effective-fluid perturbations as a closure condition. We analyse two physically motivated choices for this condition. In addition, we clarify the conditions under which the coupling between linear structure growth and perturbations of the effective fluid can be neglected. Finally, we apply this formalism to four examples of averaged cosmological models from the literature, three of which — intended as effective full descriptions of the largest scales — have been shown to provide a good fit to observational data. Our results highlight the importance of backreaction effects in shaping linear structure growth in such models. Neglecting these effects may thus lead to biased predictions for the development of large structures, even when the models provide a good description of the general background observables.

KEYWORDS: cosmic web, Cosmological perturbation theory in GR and beyond, gravity, baryon acoustic oscillations

ARXIV EPRINT: 2511.17160

JCAP02(2026)066

M Galoppo
P Mourier
JCAP
02(2026)066

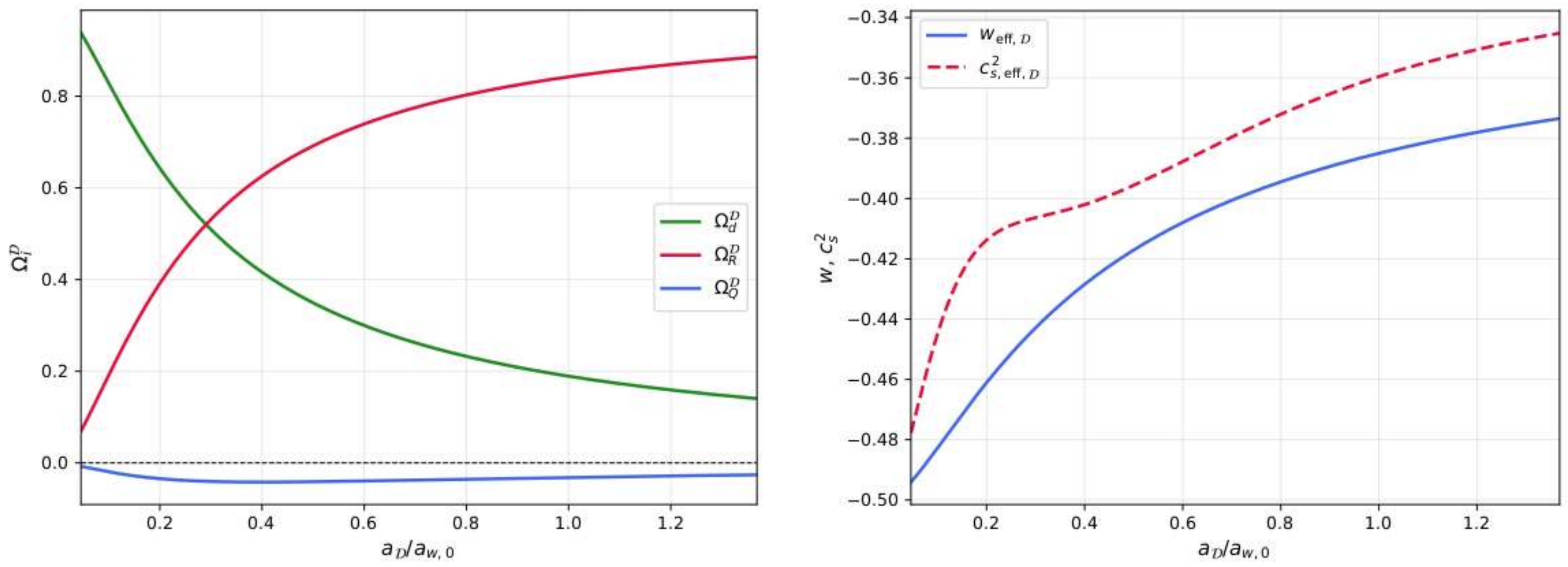


Figure 1. The Ω variables (left panel) and the effective fluid EoS parameter and squared sound speed (right panel) in the timescape model, as functions of the rescaled volume-average scale factor $a_{\mathcal{D}}/a_{w,0}$, with $f_{v,0} = 0.737$, and $H_0 = 73.03\,\mathrm{km/(s \cdot Mpc)}$. The model has no cosmological constant.

- Meszaros approximation using covariant gauge-invariant variables (Ellis & Bruni 1989)

  Evolution of dust perturbations $\Delta^{(\mathrm{d})}$:

$$\ddot{\Delta}^{(d)} + \frac{2}{3}\Theta\dot{\Delta}^{(d)} - 4\pi G\rho^{(d)}\Delta^{(d)} \simeq 0$$

  Θ = background expansion scalar driven by multi fluid components, and $\rho^{(d)}$ (which reduces to its background value $\langle\varrho\rangle_{\mathrm{D}}$) evolves according to the full background scale factor.

Summary thus far: Extraordinary Claims?

- The cosmological constant is effectively
  i. a ***counter term*** (analogous to QFT) constant balancing the all sky monopole average expansion of:
  ii. the ***kinetic energy*** $\Omega_k(z, \vartheta, \varphi)$ of negatively curved voids dominating present epoch
- Essential ingredients (Timescape model)
  i. CMB power spectrum consistent initial conditions;
  ii. backreaction of inhomogeneities – non-FLRW average expansion (Einstein equations, better than Friedmann);
  iii. ***quasilocal uniform Hubble expansion*** condition – also accounts for Hubble tension

3. Exact solutions for differentially rotating galaxies in general relativity
M. Galoppo & DLW, arXiv:2406.15134

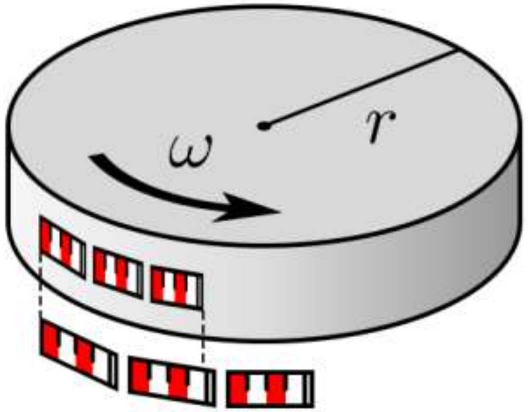
Quasilocal Newtonian limit of general relativity and galactic dynamics
M. Galoppo, F. Re & DLW, arXiv:2408.00358
Class. Quantum Grav. **42** (2025) 135004

Quasilocal angular momentum conservation

The observed Universe

- Is not empty
- Is not globally rotating
- Quasilocal angular momentum, J, and energy must be defined & coupled in a consistent manner so that, $J \to 0$, at a finite distance from isolated rotating galaxies in the background cosmological model
- Realistic solutions to Einstein's Field Equations (EFE) involve differential rotation and nonrelativistic frame-dragging

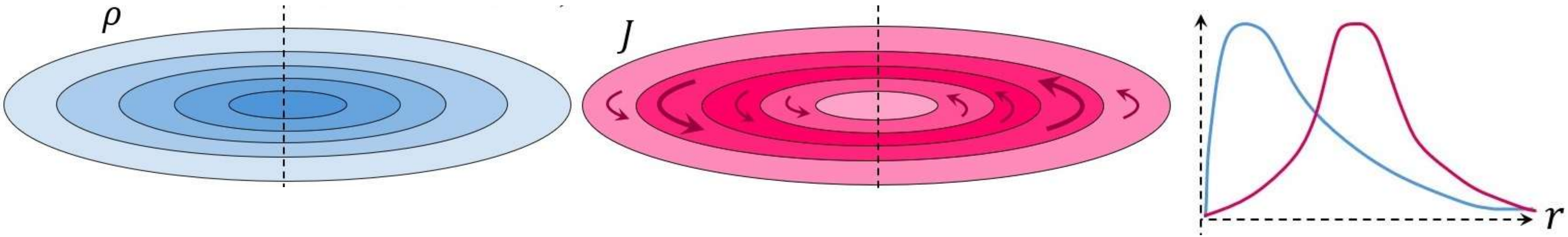
Non-Euclidean kinematics: analogy



- Ehrenfest paradox / Born rigidity 1909
  - Global Minkowski observer, $\mathcal{C}_0 = 2\pi R$
  - Centre disk observer, $\mathcal{C}_{\mathrm{in}} = 2\pi\gamma R < \mathcal{C}_0$
  - Rotational kinetic energy "produces" non-Euclidean geometry even in special relativity

Now replace by concentric differentially rotating disks; add EFE

Differential rotation: frame-dragging



As angular momentum increases from centre outward:

- each fluid element is dragged radially out and in rotation direction
- as star/gas density drops, angular momentum drops from its peak, rings dragged radially inward
- Inward dragging flattens rotation curve still where only gas, but eventually v of diffuse particles rises as they are swept in, at putative rotosurface v → c

Non-commutativity of averaging for weak field perturbations

1. Assume small metric perturbations, i.e., $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/c^2$ with $h_{\mu\nu}/c^2 \ll 1$;
2. Deduce the Einstein Field Equations;
3. Find Newton, i.e.,
4. Obtain Newtonian gravity:

$$\Delta\Phi = 4\pi G\rho_M + \mathcal{O}\left(v^2/c^2\right)$$

$\neq$

1. Write the general metric which respects the symmetries of the system;
2. Deduce the EFE;
3. Take the nonrelativistic limit, i.e., expand to the lowest order in v/c;
4. Obtain first-order correction to Newton:

$$\Delta\Phi + \ldots = 4\pi G\rho_M + \mathcal{O}\left(v^2/c^2\right)$$

Conventional approach

Our approach

At leading order coincides with Ehlers' Newton-Cartan limit

- First physically relevant non-conventional example

Exact solution for differentially rotating disc galaxies

- Building blocks: generalized Lewis–Papapetrou–Weyl metric & dust + pressure energy-momentum tensor. Astrophysically hold for timescales of $10^7 \lesssim t \lesssim 10^9$ yr

Metric & EMT

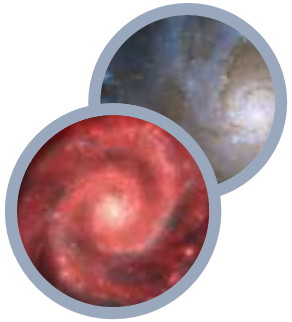
$$ds^2 = -c^2 e^{2\boxed{\Phi(r,z)}/c^2}(\mathrm{d}t + \boxed{A(r,z)}\,\mathrm{d}\phi)^2 + e^{-2\Phi(r,z)/c^2}\left[\boxed{W(r,z)}^2\,\mathrm{d}\phi^2 + e^{2\boxed{k(r,z)}/c^2}(\mathrm{d}r^2 + \mathrm{d}z^2)\right]$$

$$T^{\mu\nu} = (\boxed{\rho_M(r,z)} + \boxed{p(r,z)}/c^2)\,U^\mu U^\nu + p(r,z)g^{\mu\nu}$$

$$U^\mu \partial_\mu = (-H(r,z))^{-1/2}(\partial_t + \boxed{\Omega(r,z)}\,\partial_\phi)$$

$$H = -e^{2\Phi/c^2}(1 + A\,\Omega)^2 + e^{-2\Phi/c^2}W^2\,\Omega^2/c^2$$

- We can now move on to calculate the EFE and the equations of motion (EOM).



- We then create a mock rotation curve for a Milky Way-like galaxy following the Newtonian + DM paradigm and assuming an isothermal halo, i.e. :

$$\rho_{DM}(R) = \frac{\rho_{DM0}}{1+(R/R_{DM})^2}\,.$$

To solve the quasilocal equations we define

$$\Delta\Phi_D = \frac{||\vec{\nabla}Ł_D||^2}{2r^2}\,.$$

We can obtain and solve the equations

$$\vec{\nabla}Ł_D \cdot \left[\vec{\nabla}Ł_D - 2\vec{\nabla}(rv_K)\right] + 2r\left(v_K^2 - v_B^2\right)_{,r} = 0\,.$$

Its solution allows us to check the dragging velocity magnitude: it must be nonrelativistic.

EXACT SOLUTIONS: Having checked the self-consistent form of *H(η)* in the low energy limit, now adopt global ansatz $H(\eta) = -1 + \epsilon\,\eta^2/(bc)^2$

Exact solution: more details

$$H = -e^{2\Phi/c^2}(1 + A\,\Omega)^2 + e^{-2\Phi/c^2} r^2\,\Omega^2/c^2$$

$$\mathrm{d}\Omega = c^2\,\mathrm{d}H/(2\eta)$$

Now choose an H(η(r,z)) and a Ψ such that

$$\Psi = \eta + c^2 \frac{r^2}{2} \int \frac{H'}{H} \frac{d\eta}{\eta} - \frac{1}{2} \int \frac{H'}{H} \eta\, d\eta\,.$$

which is a solution of the homogeneous Grad-Shafranov equation $\hat{\Delta} = 0$:

$$\hat{\Delta}\Psi := \Psi_{,rr} - \frac{1}{r}\Psi_{,r} + \Psi_{,zz} = 0,$$

General solution

$$\Psi(r,z) = \mathcal{C}_0 + r \sum_{n=0}^{\infty} \mathcal{A}_n I_1(a_n r) \cos(a_n z) + r \sum_{m=0}^{\infty} \mathcal{B}_m K_1(b_m r) \cos(b_m z)$$

Exact solution: more details

$$\mathrm{d}\Omega = c^2\,\mathrm{d}H/(2\eta)\,. \tag{5}$$

In view of $\boxed{5}$ we call H the *differential rotation state function*, since a choice of H fixes Ω. The norms of the two Killing vectors, $\xi^\mu\partial_\mu = \partial_t$ and $\zeta^\mu\partial_\mu = \partial_\phi$, and their inner product are then respectively given by

$$\xi^\mu\xi_\mu = -c^2 e^{2\Phi/c^2} = c^2\,\frac{\left(H - \eta\,\Omega/c^2\right)^2 - (r\,\Omega/c)^2}{H}\,, \tag{6}$$

$$\zeta^\mu\zeta_\mu = r^2 e^{-2\Phi/c^2} - c^2 A^2 e^{2\Phi/c^2} = \frac{r^2 - (\eta/c)^2}{(-H)}\,, \tag{7}$$

$$\xi^\mu\zeta_\mu = -c^2 A e^{2\Phi/c^2} = \eta - \frac{(\eta/c)^2 - r^2}{H}\,\Omega\,. \tag{8}$$

Exact solution: two classes

$$\eta(r,z) = b_\epsilon c \,\mathrm{tann}\left(\frac{b_\epsilon \Psi(r,z)}{b_\epsilon^2 c - \epsilon r^2 c}\right), \tag{30}$$

where

$$\mathrm{tann}(x) = \begin{cases} \tanh(x), & \epsilon = +1 \\ x, & \epsilon = 0 \\ \tan(x), & \epsilon = -1 \end{cases}. \tag{31}$$

Moreover, by $(\boxed{5})$ it follows that

$$\Omega = \Omega_0 + \epsilon \frac{\eta}{b_\epsilon^2}. \tag{32}$$

Furthermore, to ensure zero rotation on the symmetry axis we choose $\Omega_0 = 0$. We thus find

$$v_K = \epsilon \frac{rc}{b_\epsilon} \,\mathrm{tann}\left(\frac{b_\epsilon \Psi(r,z)}{b_\epsilon^2 c - \epsilon r^2 c}\right), \tag{33}$$

and

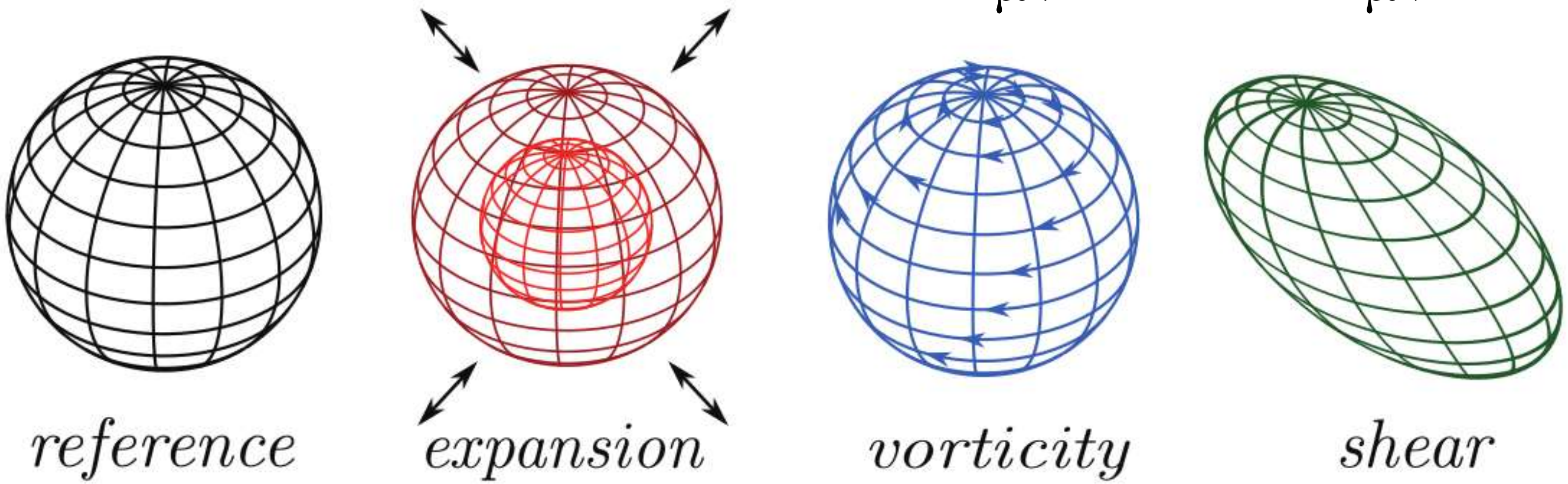
$$v_D = c\left(\frac{r}{b_\epsilon}\epsilon - \frac{b_\epsilon}{r}\right) \,\mathrm{tann}\left(\frac{b_\epsilon \Psi(r,z)}{b_\epsilon^2 c - \epsilon r^2 c}\right). \tag{34}$$

Differential rotation is crucial mathematically since the term $\epsilon r^2 c$ regularizes potential divergences at infinity. In particular, we choose solutions to the homogeneous Grad-Shafranov equation which are regular about the z-axis and diverge as

$\Psi \sim r^\alpha,\ \alpha < 2$ as $r \to \infty$

Raychaudhuri equation: geodesic deviation

θ $\quad\omega_{\mu\nu}$ $\quad\sigma_{\mu\nu}$



$$\theta = \nabla_\mu U^\mu$$

$$\omega_{\mu\nu} = U_{[\mu;\nu]}$$

$$\sigma_{\mu\nu} = U_{(\mu;\nu)} \square \frac{1}{3}\theta h_{\mu\nu}$$

$$\dot{\Theta} \equiv U^\nu \nabla_\nu \Theta = -\frac{1}{3}\Theta^2 + \omega^2 - \sigma^2 - R_{\mu\nu}U^\mu U^\nu$$

For stationary, axisymmetric solutions $\Theta = 0, \ \dot{\Theta} = 0,$

$$\omega^2 - \sigma^2 = 4\pi G(\rho + 3P/c^2)$$

Raychaudhuri equation: shear, vorticity, “density”

$$\sigma^2 = \frac{e^{2\Xi} c^4 r^2 (\eta_{,r}^2 + \eta_{,z}^2)\, \ell^2(\eta)}{8\eta^2}$$

$$\omega^2 = \frac{e^{2\Xi} (\eta_{,r}^2 + \eta_{,z}^2)\, \Delta^2(\eta)}{8r^2}$$

where $\Xi(r,z) = [\Phi(r,z) - k(r,z)]/c^2$

$$R_{\mu\nu} U^\mu U^\nu = 4\pi\, G\rho$$

$$8\pi G\rho = \frac{\eta_{,r}^2 + \eta_{,z}^2}{4\eta^2 r^2} \left[\eta^2 \Delta^2(\eta) - c^4 r^4 \ell(\eta)^2\right] e^{2\Xi},$$

where $\ell(\eta) = H'/H$ and $\Delta(\eta) = 2 - \eta\,\ell(\eta)$,

Rotosurface $r \to b_+$ at finite z: new finite infinity?!

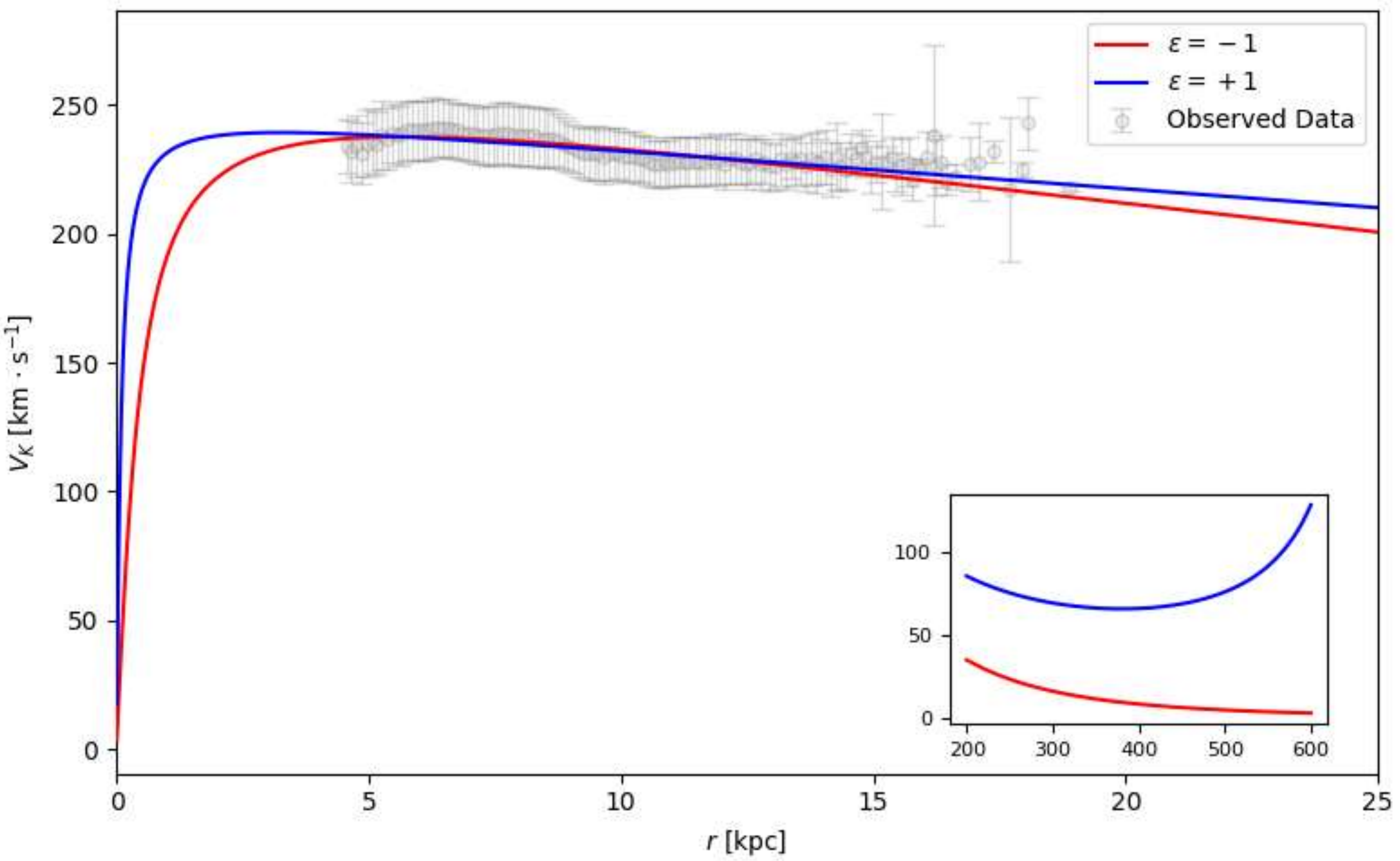
Killing vector norms diverge like at an horizon BUT *pointwise*

$$\omega^2 = \sigma^2 = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} \simeq \frac{e^{2\Xi}\Psi^2}{8|b_+ - r|^4} \to 0,$$

since Ψ is finite and $e^{2\Xi} \sim e^{-\Psi^2/(bc^2|b_+ - r|)}$ as $r \to b_+$

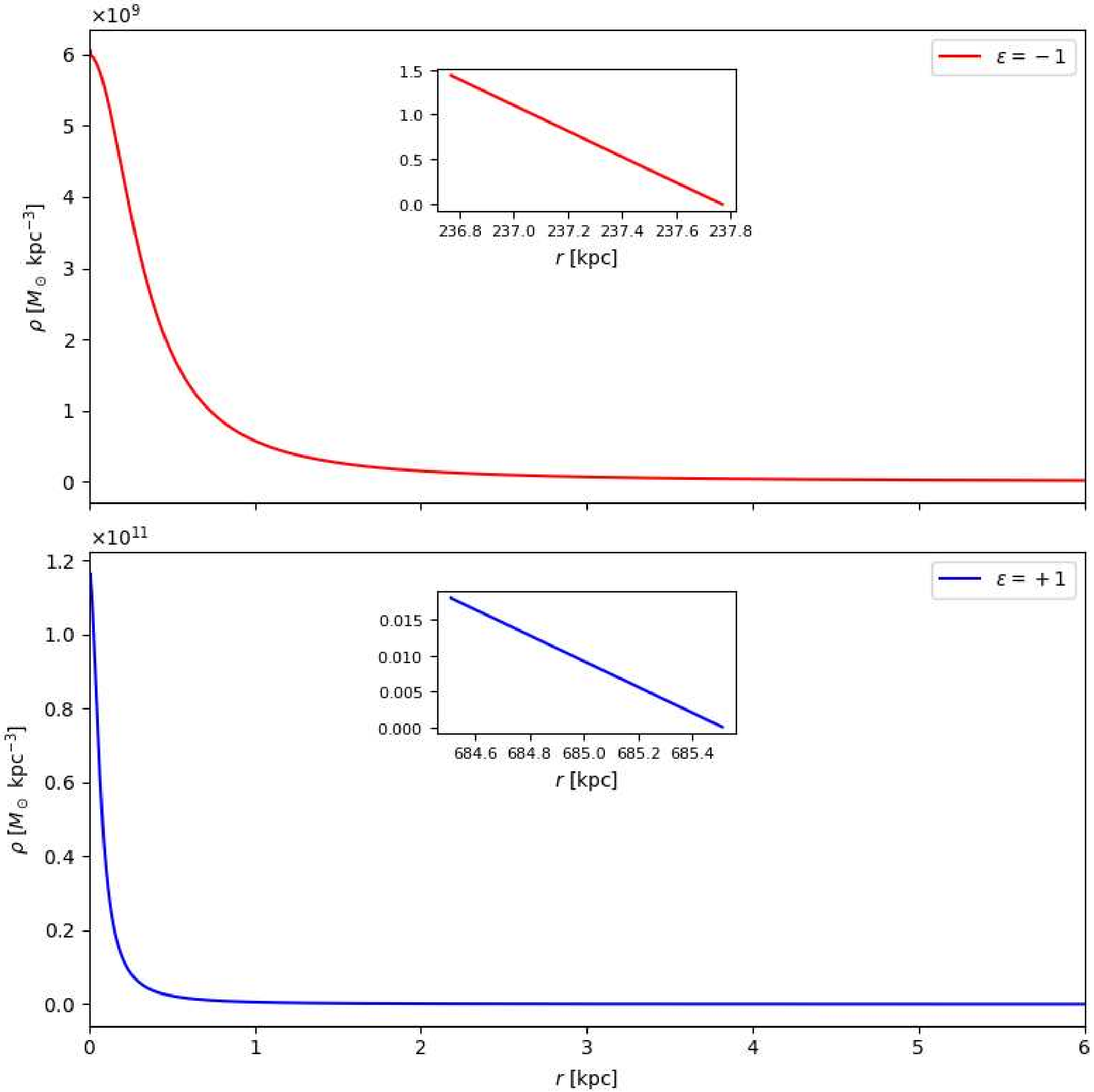
- On rotosurface need to rotate with v → c to have zero angular momentum
- Rotosurface has characteristic of shock front
- Milky Way model, v_K increases beyond 500 kpc, v_K→ c at 700 kpc
- *Astrophysical consequence*? Synchrotron radiation source contributing to galactic plane foregrounds, modelled empirically for CMB anisotropy map construction?

Example rotation curves fit to Milky Way



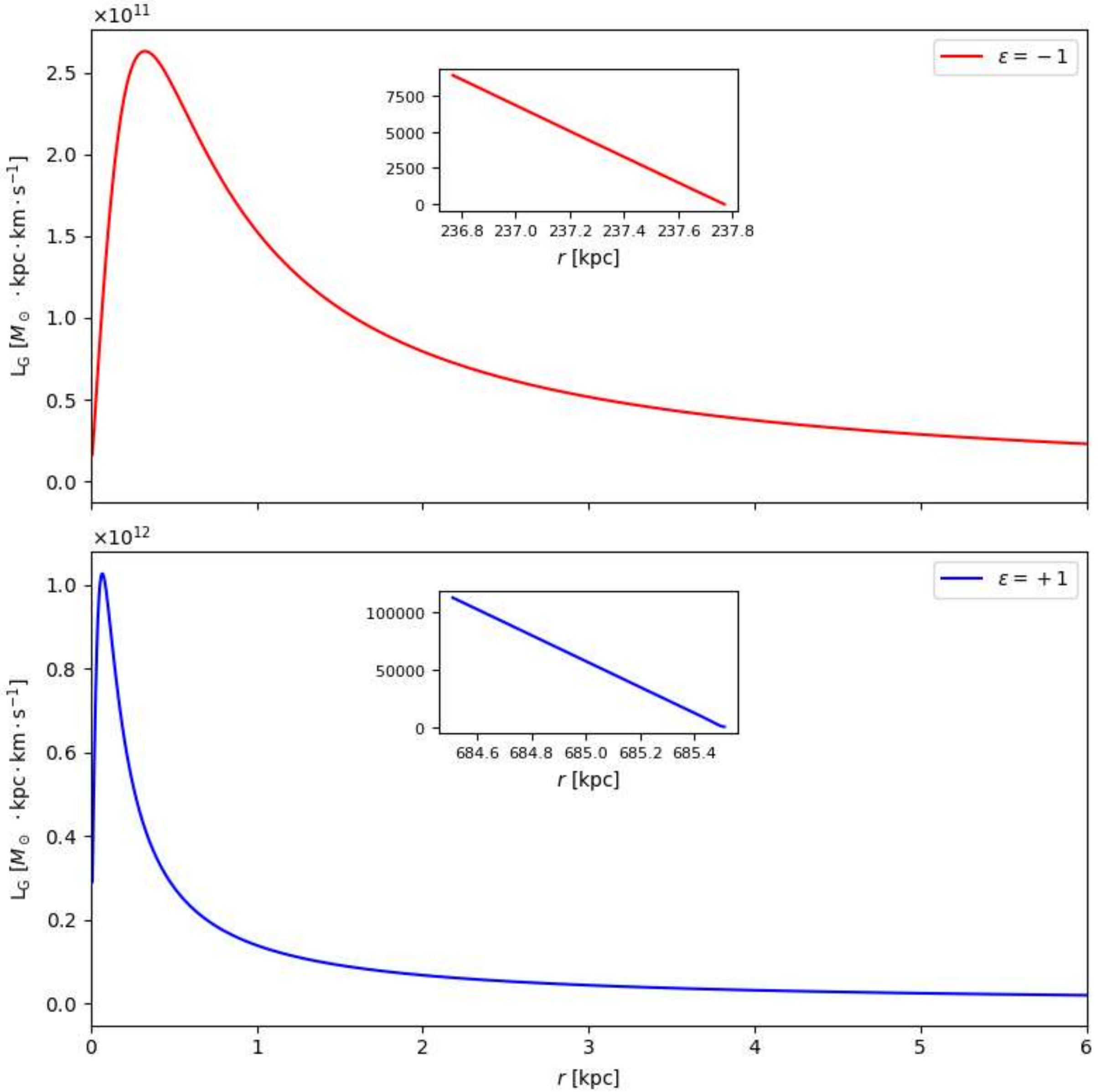
Rotation velocity curves for both differentially rotating solution classes fit to GAIA-DR3 data sample “ALL” of Beordo et al. (2024) (superposed). Insets show the approach r → b− and r → b+.

Radial effective dust density distributions



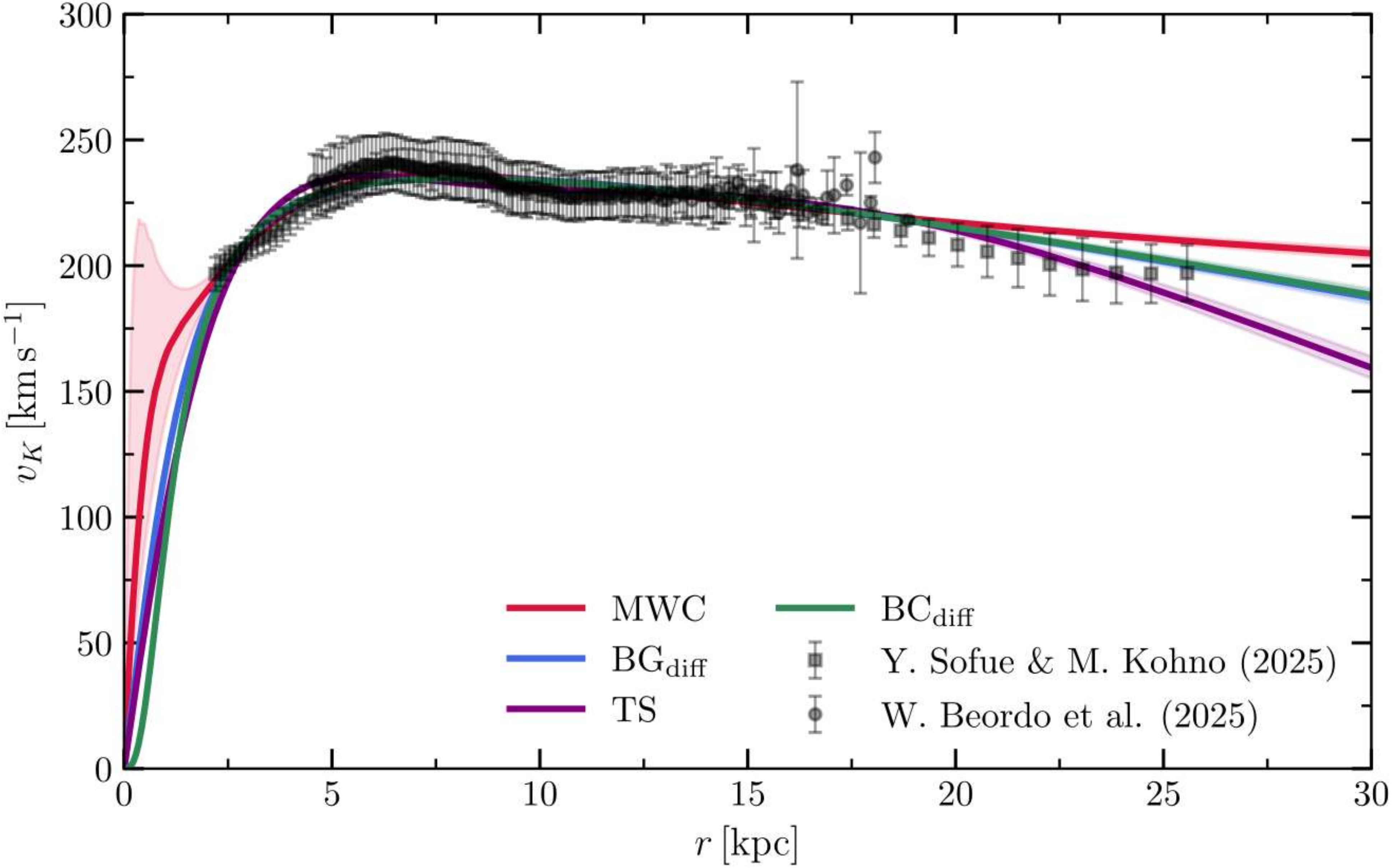
Radial effective dust density distribution for both $\epsilon = -1$ and $\epsilon = +1$ example spacetimes above. Insets show the approach $r \to b-$ and $r \to b+$.

Radial angular momentum density distributions



Angular momentum density distribution measured by the stationary observer, i.e., $Ł_G := r\rho v_K$ for both $\epsilon = -1$ and $\epsilon = +1$ example spacetimes above. Insets show the approach $r \to b-$ and $r \to b+$.

Example rotation curves fit to Milky Way



Rotation velocity curves for ϵ =-1 classes fit to GAIA-DR3 + additional data, comparing Newtonian Milky Way Classical (MWC) with 3 differentially rotating relativistic galaxies.

Akaike & Bayes Info Criteria Comparisons

Classical (MWC) Thick and thin discs + dark matter bulge: 10 free parameters

$$\Psi_{\mathrm{BC_{diff}}}(r,z) = -\Psi_0 \frac{r^2}{\left[r^2 + (a+z)^2\right]^{3/2}},$$

$$\Psi_{\mathrm{BG_{diff}}}(r,z) = -\Psi_0 \left[\frac{z+r_0}{\sqrt{r^2+(z+r_0)^2}} - \frac{z-r_0}{\sqrt{r^2+(z-r_0)^2}}\right],$$

$$\Psi_{\mathrm{TS}}(r,z) = -\Psi_0 \left[f \cos(s_1 z) r K_1(s_1 r) + (1-f)\cos(s_2 z) r K_1(s_2 r)\right].$$

BC Bonnor Cloud: 3 free parameters

BG Balasin-Grumiller: 5 free parameters

TS Two Scales (for exponential decay of disc in r,z directions): 6 free parameters

Model	$\bar{\chi}^2$	ΔAIC	ΔBIC
MWC	0.30	0.00	0.00
$\mathrm{BG_{diff}}$	0.30	−10.63	−33.40
$\mathrm{BC_{diff}}$	0.31	−8.96	−31.72
TS	0.35	0.68	−15.58

TABLE I. Model-comparison diagnostics for the Milky Way rotation-curve fits. Here, ΔAIC and ΔBIC are computed with respect to the Newtonian Milky Way benchmark.

Rotosurface: physics or mathematical science fiction?

Killing vector norms diverge like at an horizon BUT *pointwise*

$$\omega^2 = \sigma^2 = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} \simeq \frac{e^{2\Xi}\Psi^2}{8|b_+ - r|^4} \to 0,$$

since Ψ is finite and $e^{2\Xi} \sim e^{-\Psi^2/(bc^2|b_+ - r|)}$ as $r \to b_+$

- **However,** Komar integrals for mass & angular momentum diverge
- BUT Komar for 2 Killing vectors is adapted to **empty** asymptotically flat universe
- Ehrenfest paradox/Born rigidity should be a warning. Can we define alternative quasilocal integrals?
- **MATT: PLEASE HELP!!!**

4. The future: viewpoint on lessons learned

Conjecture

- There are **two** essential layers of shell-cross singularities requiring redefinition of effective regional fluid elements
- 1. Rotational kinetic energy and geometry intertwined: ROTOSURFACE for galaxies within groups and clusters of galaxies
- 2. ”Thermal” velocity dispersion kinetic energy and geometry intertwined: TILTSURFACE – Finite Infinity of Timescape

IOP Publishing Classical and Quantum Gravity

Class. Quantum Grav. 34 (2017) 193001 (27pp) https://doi.org/10.1088/1361-6382/aa8535

Topical Review

Dimension and dimensional reduction in quantum gravity

S Carlip

Department of Physics, University of California, Davis, CA 95616, United States of America

E-mail: carlip@physics.ucdavis.edu

Received 18 October 2016, revised 1 August 2017
Accepted for publication 9 August 2017
Published 4 September 2017

CrossMark

Abstract

A number of very different approaches to quantum gravity contain a common thread, a hint that spacetime at very short distances becomes effectively two dimensional. I review this evidence, starting with a discussion of the physical meaning of 'dimension' and concluding with some speculative ideas of what dimensional reduction might mean for physics.

3. Evidence for dimensional reduction

- 3.1 High temperature string theory
- 3.2 Causal dynamical triangulations
- 3.3 Asymptotic safety
- 3.4 Causal set theory
- 3.5 Loop quantum gravity
- 3.6 The short distance Wheeler-DeWitt equation
- 3.7 Modified dispersion relations and noncommutative geometry
- 3.8 Minimum length
- 3.9 Modified gravity
- 3.10 Spacetime foam
- 3.11 Multifractional geometry

Personal viewpoint of next steps to Quantum Gravity:

- Space-Time Geometry is emergent, coupled directly to particle genesis
- Very early universe: when the average relationship between things is lightlike there is no passage of time – light carries no clock

 - Dimensional reduction to a 2 + 2 phase space encodes its geometry

 - Penrose's Weyl curvature hypothesis, Conformal Cyclic Cosmology is almost correct. But "cyclic" is extraneous: there is no global external time.

Personal viewpoint of next steps to Quantum Gravity:

- Origin of scale, particle rest mass & spatial dimensions > 2 are entwined

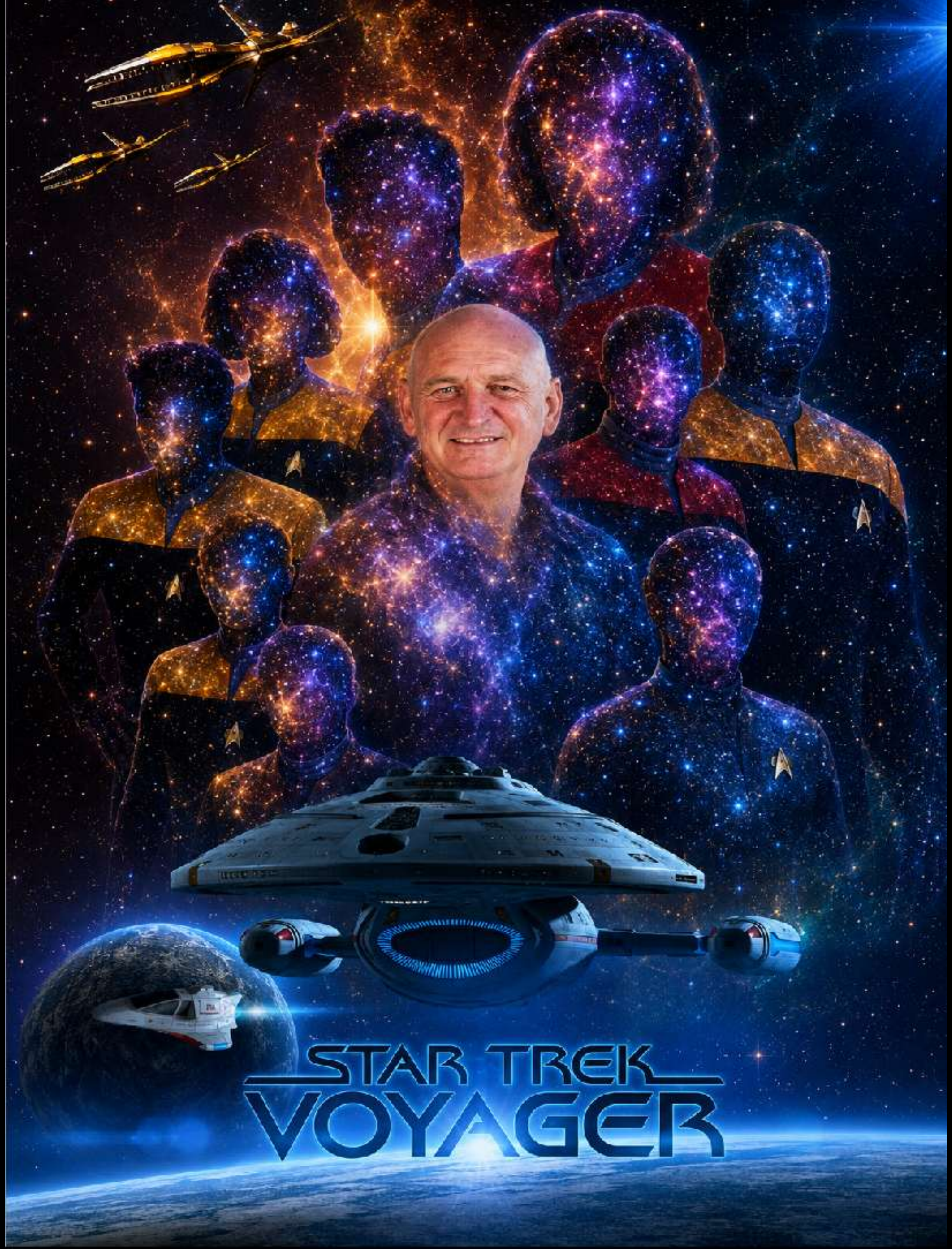
 - A geometrical description of the Higgs mechanism is key to understanding Kaluza-Klein, superstring & brane phenomenology

 - Extra spatial dimensions > 3 represent internal degrees of freedom of particles with rest-mass m≠0:

 D=1+3+1, U(1), $\{e^{\pm}, \mu^{\pm}, \tau^{\pm}\}$;

 D=1+3+2, SU(2), $\{(p^{+}, n), (p^{-}, n), \ldots\}$; …

- Theoretical Physicists will be busy still for decades ahead!



For those of you who have followed Matt through a wormhole into the Delta Quadrant and are looking for a way back down to Earth … there may be a way.

Exciting decades on the road to quantum gravity lie ahead!