Meeting Matt in the Middle

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With
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TOP

General Relativity
Standard Model (QFT)

....

QFT in curved spacetime
Black hole thermodynamics
Semiclassical Gravity
Cosmology
Modified gravity
Modified causal ladder
Nontrivial spacetime topology

Matt's Domain:
Get creative with spacetime

MIDDLE

....

BOTTOM

Quantum Gravity

Plan

- The Feynmanian heuristic for the path integral: what recovery of continuum spacetime looks like in quantum gravity
- The causal set approach to quantum gravity
- The action of a causal set
- Some conjectures about the action of manifold-like causal sets.
- Example: can spacetime have timelike boundaries (boundaries of space)?

Recovering classical physics, approximately, from the path integral

(Feynman *et al.*, 1965):

> *"Suppose that for all paths, [the action] S is very large compared to $\hbar$. One path contributes a certain amplitude. For a nearby path, the phase is quite different, because with an enormous S even a small change in S means a completely different phase - because $\hbar$ is so tiny. So nearby paths will normally cancel their effects out in taking the sum - except for one region, and that is when a path and a nearby path all give the same phase in the first approximation (more precisely, the same action within $\hbar$). Only those paths will be the important ones."*

- This is a heuristic for how classical physics is recovered from quantum physics: the paths that dominate the sum are not themselves the classical trajectory but **they are all approximated by it**.
- We predict the classical trajectory up to quantum fluctuations i.e. the **common** properties of the paths that contribute.
- This heuristic explains why there is an action principle for classical mechanics.

Path Integral Quantum Gravity

$$“Z = \sum_{M \in \mathcal{M}} \int_{g \in \mathcal{G}} [dg]\, e^{iS[g]}”$$

is a statement of intent (Renate Loll), a hope for a gravitational sum-over-histories (SOH).

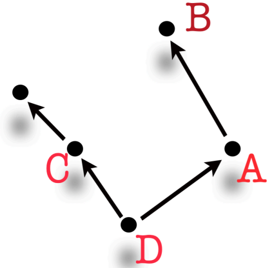
- SOH quantum foundations is work in progress (Jim Hartle, Rafael Sorkin).
- In the meantime, we can use the Feynman heuristic.
- For GR, say, to emerge from path integral quantum gravity à la Feynman we need:
  Suppression of class of all “bad” histories
  Domination of histories approximated by GR solutions
- (One aspect of) the cosmological constant problem in quantum gravity from the path integral perspective is that “bad histories threaten to dominate”.

I will use causal set quantum gravity to illustrate these issues.

3 pillars of Causal Set approach to quantum gravity

1. **Discreteness:** physical, fundamental atomicity of spacetime at the Planck scale.

2. **Order:** a more primitive organising principle for physics even than space and time: the spacetime causal order--of all continuum spacetime structure--survives in the deep theory.



Causal set = discrete order = transitive directed acyclic **graph**

$$\boxed{10^{240}}$$

3. **Path integral:**

Finite!

$$\text{“}\, Z(V) = \sum_M \int \mathcal{D}g \; e^{iS[g]} \longrightarrow Z(N) = \sum_{\text{causal sets of cardinality N}} e^{iS(C)} \,\text{”}$$

The Discrete-Continuum Correspondence for causal sets

$(C, \prec)$ recovers (M, g) if C **faithfully embeds** at Planckian density $\rho = l^{-d}$ in (M, g). In other words, there exists an embedding $C \hookrightarrow M$ such that

1. *Number* $\approx$ *Vol* in fundamental units in sufficiently nice regions;
2. The causal set order $\prec$ is the causal order of the embedded points in M;
3. (M, g) has no structure on the discreteness scale.

Given any (distinguishing) (M, g) we can produce faithfully embedded causal sets by a Poisson process of "sprinkling" at Planckian density $\rho = l^{-d}$

It is a conjecture (the Hauptvermutung of causal set theory) that this correspondence holds water. There is good evidence for it, including

Kronheimer-Penrose-Hawking-Malament theorem

If you know the causal order and the volume measure of a distinguishing Lorentzian geometry ($d \geq 3$) you know the whole geometry (M, g).

The fundamental gravitational path integral

The gravitational path integral is fundamentally a sum over causal sets:

$$Z(N) = \sum_{\text{causets of cardinality N}} e^{iS[C]} \tag{1}$$

For each nice Lorentzian geometry of volume approximately N in fundamental units, there are many causets in this sum that correspond to it.
But, most causets are not manifold-like.
So we need non-manifold-like causal sets to be suppressed in the sum, and then:

$$Z(N) = \sum_{\text{causets}} e^{iS[C]} \quad \longrightarrow \quad Z(V) = \sum_{M} \int \mathcal{D}g \, e^{iS[g]} \tag{2}$$

What $S[C]$ can do this job?

Causal set actions (Dionigi Benincasa, FD & L Glaser)

There is a family of bilocal actions $S^{(d)}[C]$ for a finite causal set C, one action for each spacetime dimension d.

The 4d BDG action is $$S^{(4)}[C] \;\equiv\; \zeta_4\Big[N - N_1 + 9N_2 - 16N_3 + 8N_4\Big]$$

where

- N is the cardinality of C and N_m is the number of *order intervals* of cardinality $m+1$ in C.
- ζ_4 is a dimensionless number of order 1
- in the continuum regime $\zeta_4 = -\frac{4}{\sqrt{6}}(\frac{l}{l_p})^2$ where l is the fundamental length and $l_p = \sqrt{8\pi G_N}$ is the (cosmologists') Planck length.
- *Sprinkling* into a 4-dimensional spacetime (M, g) at density $\rho = l^{-4}$ turns this causet action into a random variable, $\mathbf{S}_l^{(4)}(g)$
- How is the random variable related to the continuum Einstein Hilbert action of $S_{EH}[g]$?

Behaviour of Causal set actions I: towards a solution of the Cosmological Constant problem (taming the entropy)

The action has to be special. It must suppress the bad histories which dominate entropically in the SOH. The number of causal sets of cardinality N grows like $2^{N^2/4}$ and the vast majority of those are the Kleitman-Rothschild 3-layer orders.

- Remarkably, one can show (Loomis & Carlip, 2018; Carlip *et al.*, 2023)

$$\lim_{N\to\infty} \sum_{\text{KR orders}} e^{i\alpha S[C]} = 0$$

  and the limit is approached super-exponentially fast, for any choice of BDG action (for a range of possible α).

- k-layer orders are also suppressed, for any fixed k.
- So a large class of bad histories (the most numerous) are suppressed in the SOH
- Much more to do

Behaviour of Causal set actions II: Sprinklings

Conjecture 1: for $d = 4$, (M, g) globally hyperbolic and fixed, as $l \to 0$:

$$\lim_{l\to 0}\langle \boldsymbol{S}_l^{(4)}[g]\rangle = \frac{1}{l_p^2}\int_M d^4x\sqrt{-g}\,\frac{R}{2} \;+\; \frac{1}{l_p^2}\int_J d^2x\,\coth(\Theta(x))$$

where J is the "joint", the intersection of the past and future boundaries and Θ is the Lorentzian angle between the past and future boundaries in the co-dimension 2 plane normal to both.

- Verified for flat causal diamonds in any dimension: limit of mean action is the joint term which is just its (co-dimension 2) volume (Buck *et al.*, 2015).
- Verified for some examples of conformally flat spacetimes and in causal intervals in Riemann normal neighbourhoods to first order in curvature (FD; Machet & Wang)
- Evidence from deSitter simulations (D. Benincasa, D, Rideout unpublished)
- Evidence for joint term from examples and simulations in flat space (FD, Lloyd-Jones & Liu)

Behaviour of Causal set actions III

Conjecture 2: for (M, g) with **timelike** boundary Σ, as $l \to 0$:

$$\lim_{l \to 0} \langle \boldsymbol{S}_l^{(4)}[g] \rangle \, l = \frac{a}{l_p^2} \int_\Sigma d^{d-1}x \sqrt{-h}$$

where a is a constant of order 1. We have numerical evidence from flat space examples of this conjecture in lower dimensions, but not yet for $d = 4$ (FD, Lloyd-Jones& Liu).

This timelike boundary term $\langle \boldsymbol{S}_l^{(4)}(g) \rangle = O(l^{-1})$ and dominates over any EH or joint term which tend to constants in the limit.

Why do boundaries of space not exist?

Assuming the conjectures hold:

- Use action $d = 4$. Suppose all the non-manifoldlike causal sets are suppressed (à la Carlip et al)
- All with dimension $\neq 4$ will be suppressed (a small change in C will change $S[C]$ by order 1).
- Comparing causal sets with and without boundary, of realistic cosmological scales:

$$\frac{1}{\hbar}S(C_{edge}) \sim 10^{180}\,, \quad \frac{1}{\hbar}S(C_{no\ edge}) \sim 10^{120}$$

  The large ratio of the magnitudes of their actions indicates that the measure of the neighbourhood of C_{edge} over which the action is approximately constant is much smaller than that of $C_{no\ edge}$, leading to the suppression of M_{edge} in the gravitational path integral.
- For globally hyperbolic $d = 4$ causal sets, the action is close to the EH action* and a Feynmanian argument implies suppression of non-solutions compared to solutions.

*Fluctuations of the action around the mean can't be ignored.

More discussion points

- Fluctuations of the action around the mean can't be ignored.
- Fluctuations can be tamed by introducing a smearing mesoscale in the definition of the action.
- There's no Gibbons-Hawking-York boundary term
- The BDG action is real and has no imaginary contribution even when there's topology change in the approximating manifold. Is analytic continuation of the parameters of the action possible/needed?
- Spacetimes with closed causal loops are absent from the path integral and so cannot be recovered by causal set theory (sad face emoji).

Thank you for listening

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Causal set scalar curvature (D. Benincasa, FD & L. Glaser)

1-parameter family of scalar curvature estimators. C is a *finite* causet and l is the fundamental length.

The 2d Ricci scalar is $$R^{(2)}[C,x] \equiv \frac{4}{l^2}\Big[1 - 2N_1(x) + 4N_2(x) - 2N_3(x)\Big]$$

where $N_m(x)$ is the number of exclusive *order intervals* of cardinality $m-1$ in C with x as future endpoint.

The 4d Ricci scalar is $$R^{(4)}[C,x] \equiv \frac{8}{l^2\sqrt{6}}\Big[1 - N_1(x) + 9N_2(x) - 16N_3(x) + 8N_4(x$$

where l is the fundamental (discreteness) length.

d'Alembertian of scalar field in $\mathbb{M}^2$ (Sorkin, 2006)

Let C be a sprinkled causal set in $\mathbb{M}^2$ and let $\phi : \mathbb{M}^2 \to \mathbb{R}$ be a scalar field of compact support. ϕ can be restricted to C. The d'Alembertian of a scalar field on a lattice at x would be a difference of values of the field at x, its nearest-neighbours and next-nn's.

Ansatz:

$$B_2\phi(x) \;\equiv\; l^{-2}\left[A_0\phi(x) + A_1\sum_{y=nn}\phi(y) + A_2\sum_{y=nnn}\phi(y) + A_3\sum_{y=nnnn}\phi(y)\right]$$

Sprinkling makes this a random variable that depends on the density ρ. Demand that

$$\lim_{\rho\to\infty}\langle B_2\phi(x)\rangle = \Box\phi(x)$$

$$B_2\phi(x) \;\equiv\; l^{-2}\left[-2\phi(x) + 4\sum_{y=nn}\phi(y) - 8\sum_{y=nnn}\phi(y) + 4\sum_{y=nnnn}\phi(y)\right]$$

The **fluctuations** grow with density. Can fix that by smearing over a fixed physical "nonlocality scale". There's numerical evidence for

Claim

The fluctuations of the smeared $B_2\phi$ decrease like $N^{-1/2}$

What about $B_2\phi$ for sprinkling in curved 2D spacetime?

Claim

$$\lim_{\rho\to\infty}\langle B_d\phi(x)\rangle = \left[\Box - \frac{1}{2}R(x)\right]\phi(x)$$

- There is a B_d for every spacetime dimension and if one *assumes* that the continuum limit is local, then it is easy to calculate the coefficient $-\frac{1}{2}$ of R is dimension independent (Glaser, 2014).
- Status: can prove result for B_4 in a 4D Riemann Normal Neighbourhood (Belenchia *et al.*, 2016). And in 2D flat space (Sorkin, unpublished notes)
- Applying B_d to the constant field $= -2$ then gives a causet function, $\mathfrak{R}_d(x)$, whose mean over sprinklings of a d-dimensional spacetime should tend to $R(x)$ in the continuum limit.
- There are no analytical results on the variance but fluctuations are huge in simulations and we need to work with the smeared versions of $\mathfrak{R}_d(x)$ that involve a fixed smearing scale (Benincasa 2010). Numerical results in 4D: s.d. $\sim N^{-1/2}$.