

Matt Visser:

Spacetime Sculptor

Thin shells, wormhole throats, exotic matter,
and black-bounces

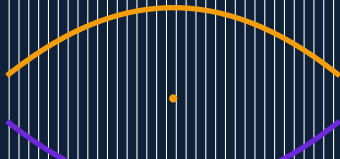
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Aug 24–28, 2026 SISSA (Miramare Campus)

From Black Holes to the Cosmos:

Matt Visser's journey through space and time



Talk outline:

Part I: historical and geometrical motivation

Flamm, Einstein–Rosen, Wheeler, Misner, and Morris–Thorne: from ER-bridges and topology to the flaring-out condition, Raychaudhuri defocusing, and exotic matter in GR.

Part II: thin-shell formalism as the technical backbone

We then build the Israel–Lanczos machinery: cut-and-paste geometry, induced metric, extrinsic curvature, surface stresses, conservation identity, equation of motion, and linearized stability.

Part III: Matt Visser's programme

With the formalism in place, the later slides become a chronological tour: surgical Schwarzschild wormholes; Poisson–Visser stability; throat theorems; integrated exoticity; scalar and brane variants; generic shells; gravastars; and black-bounces.

1. The Flamm–Einstein–Rosen bridge

Historical starting point

Wormhole physics can be tentatively traced back to Flamm in 1916, whose aim was to render the conclusions of the Schwarzschild solution in a clearer geometrical manner.

Einstein and Rosen, in 1935, were attempting to build a geometrical model of a physical elementary “particle” that would be finite and singularity-free. Their discussion was phrased in terms of a bridge across a double-sheeted physical space.

Einstein–Rosen, *Phys. Rev.* 1935

“These solutions involve the mathematical representation of physical space by a space of two identical sheets, a particle being represented by a ‘bridge’ connecting these sheets.”

ER discussed two types of bridges, neutral and quasicharged. These constructions can be generalized, but one must remember that, at the time, coordinate singularities and physical singularities had not yet been cleanly separated: to many physicists the event horizon was the singularity.

1.1. The neutral Einstein–Rosen bridge

The neutral Einstein–Rosen bridge is the observation that a suitable coordinate change seems to make the Schwarzschild coordinate singularity disappear. In modern language, ER found that certain coordinate systems naturally cover only two asymptotically flat regions of the maximally extended Schwarzschild spacetime.

Start from the ordinary Schwarzschild geometry,

$$ds^2 = - \left(1 - \frac{2M}{r} \right) dt^2 + \frac{dr^2}{1 - 2M/r} + r^2 d\Omega^2.$$

Introduce

$$u^2 = r - 2M, \quad r = 2M + u^2, \quad -\infty < u < +\infty.$$

What the coordinate change does

This coordinate transformation discards the region containing the curvature singularity and double-covers the Schwarzschild exterior.

The neutral bridge in Einstein–Rosen coordinates

With $u^2 = r - 2M$, the Schwarzschild metric becomes

$$ds^2 = -\frac{u^2}{2M + u^2} dt^2 + 4(2M + u^2) du^2 + (2M + u^2)^2 d\Omega^2.$$

The region near $u = 0$ is interpreted as a bridge connecting the asymptotically flat region near $u = +\infty$ with the asymptotically flat region near $u = -\infty$.

To justify the word “bridge”, consider a spherical surface at constant u :

$$A(u) = 4\pi(2M + u^2)^2, \quad A(0) = 4\pi(2M)^2.$$

The area has a minimum at $u = 0$. The narrowest part of the geometry is the throat, while the nearby region is the bridge, or in modern terminology a wormhole.

Neutral ER bridge: the modern interpretation

The net result is that the neutral Einstein–Rosen bridge, also called the Schwarzschild wormhole, is identical to a part of the maximally extended Schwarzschild geometry.

Non-traversability

The throat pinches off before an observer can traverse it. A minimal-area surface is therefore not sufficient for a traversable wormhole.

Conceptual distinction

The Einstein–Rosen bridge is not a traversable wormhole. It is a horizon-based bridge associated with the Schwarzschild black-hole geometry.

1.2. The quasicharged Einstein–Rosen bridge

ER also considered a charged-type construction. Start from the Reissner–Nordström form,

$$ds^2 = - \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} \right) dt^2 + \frac{dr^2}{1 - 2M/r + Q^2/r^2} + r^2 d\Omega^2.$$

To make the bridge construction work, ER found that a completely ad hoc mutilation of the theory was necessary: one has to reverse the sign of the electromagnetic stress-energy tensor, which implies a negative energy density (with $Q^2 \rightarrow -\varepsilon^2$).

Considering the massless case and the coordinate change

$$u^2 = r^2 - \varepsilon^2,$$

one obtains a very peculiar geometry: a massless quasicharged object, with negative energy density, a horizon at $r = \varepsilon$, $u = 0$. This was the object ER wished to interpret as an “electron”.

1.3. The generalized Einstein–Rosen bridge

Following Matt, consider an arbitrary static, spherically symmetric geometry with an event horizon,

$$ds^2 = -e^{2\Phi(r)} \left(1 - \frac{b(r)}{r} \right) dt^2 + \frac{dr^2}{1 - b(r)/r} + r^2 d\Omega^2.$$

The horizon occurs at $r = r_H$, defined by

$$b(r_H) = r_H.$$

Introduce the coordinate change

$$u^2 = r - r_H, \quad r = r_H + u^2.$$

The region near $u = 0$ is then the bridge connecting the two asymptotically flat exterior regions.

Generalized ER bridge: near-horizon form

In the coordinate u , the metric is

$$ds^2 = -e^{2\Phi(r_H+u^2)} \left(1 - \frac{b(r_H+u^2)}{r_H+u^2} \right) dt^2 \\ + \frac{4u^2 du^2}{1 - b(r_H+u^2)/(r_H+u^2)} + (r_H+u^2)^2 d\Omega^2.$$

Near the bridge/horizon, $r \simeq r_H$ and $u \simeq 0$, so the metric is qualitatively analogous to the neutral and quasicharged bridges.

Important

The key ingredient of the ER bridge construction is the existence of an event horizon. The ER bridge is a coordinate artefact arising from choosing a coordinate patch that double-covers the asymptotically flat region exterior to the black-hole event horizon.

2. Geons and spacetime foam

After the pioneering work of Einstein and Rosen in 1935, the field lay dormant for roughly twenty years.

Interest in such systems was rekindled in 1955 by John Wheeler, who was becoming interested in topological issues in general relativity. In his paper on geons, Wheeler described a metric that is nearly flat overall except in two widely separated regions where double connectedness becomes visible.

Wheeler's geon picture

"One can consider a metric which on the whole is nearly flat except in two widely separated regions, where a double connectedness comes into evidence..."

This marks the shift from the coordinate bridge of ER to the topological analysis of handles (tunnels), and field lines threading the geometry.

Geons: gravitational–electromagnetic entities

The word “geon” was coined by Wheeler to denote a gravitational–electromagnetic entity. Geons were hypothesized to be unstable but long-lived solutions of the coupled Einstein–Maxwell equations; in modern language, a geon is best viewed as a hypothetical unstable gravitational–electromagnetic quasisoliton.

Wheeler’s concept also suggested a model of nonzero charge with everywhere zero charge density:

Charge without charge

A divergence-free electromagnetic disturbance around one tunnel mouth sends lines of force into space and appears to have a charge. An equal number of lines must enter through the tunnel, so the other mouth manifests an equal and opposite charge.

Two routes then became available: classical dynamics of tunnel configurations, assuming their possible existence; and quantum gravitational processes that might give rise to such configurations, leading to Wheeler’s concept of spacetime foam.

First route: classical physics as geometry

Misner and Wheeler, in “Classical physics as geometry” (Annals of Physics, 1957), presented a tour de force in which the Riemannian geometry of manifolds with nontrivial topology was investigated with the ambitious goal of explaining all of physics.

This was one of the earliest uses of abstract topology, homology, cohomology, and differential geometry in physics. Their viewpoint is best summarized by the phrase

“Physics is geometry”.

More specifically, the already-unified classical theory was meant to describe, in terms of empty curved space:

- gravitation without gravitation;
- electromagnetism without electromagnetism;
- charge without charge;
- mass without mass.

Around the mouth of the wormhole, electromagnetic energy gives mass to this region of space.

The word “wormhole” enters physics

The Misner–Wheeler paper is the first paper that introduced the term “wormhole” to the scientific community:

Misner–Wheeler, *Annals Phys.* 1957

“There is a net flux of lines of force through what topologists would call a handle of the multiply-connected space and what physicists might perhaps be excused for more vividly terming a wormhole.”

Ultimately, the aim was to use source-free Maxwell equations coupled to Einstein gravity, seasoned with nontrivial topology, to build models for classical electrical charges and other particle-like entities in classical physics.

It is now known that this classical conception of “charge without charge” cannot work as originally conceived: classically, the tunnels collapse to form black holes, and the otherwise interesting topology is hidden behind event horizons.

Second route: spacetime foam

Wheeler emphasized the overwhelming importance of the Planck scale in gravitational physics. At distances below the Planck length, quantum fluctuations are so large that linearized theory breaks down irretrievably, and the quantum physics of full nonlinear Einstein gravity must be faced.

Wheeler's ocean analogy

“On the atomic scale the metric appears flat, as does the ocean to an aviator far above. The closer the approach, the greater the degree of irregularity. Finally, at distances of order ℓ_P , the fluctuations in a typical metric component $g_{\mu\nu}$ become of the same order as the metric components themselves.”

Once metric fluctuations become nonlinear and strongly interacting, they may “break” in a manner analogous to ocean waves. This suggests a foamlike structure in which, as Wheeler put it, “space resonates between one foamlike structure and another”.

Thus, spacetime foam refers to Wheeler's suggestion that the geometry and topology of space might be constantly fluctuating, possibly involving topology change.

3. Interregnum and Renaissance

A long pause in Lorentzian wormhole physics

There was a roughly thirty-year gap between Wheeler's and Misner's 1955–1957 work and the 1988 Morris–Thorne renaissance of wormhole physics.

Much was accomplished during this period, but relatively little effort was devoted to Lorentzian wormholes as traversable spacetime geometries. Considerable attention instead went into understanding Wheeler's geon idea and the spacetime-foam picture.

During this interregnum, Lorentzian wormholes were mostly treated as curiosities and relegated to the background. Important isolated developments nevertheless appeared, including Homer Ellis' drainhole construction and Kirill Bronnikov's tunnel-like solutions in the early 1970s.

The 1988 turning point

Only in 1988, with the Morris–Thorne analysis, did a full-fledged renaissance of wormhole physics begin - a renaissance that is still in full flight.

Morris–Thorne: reverse engineering Einstein's equations

The static, spherically symmetric traversable wormhole ansatz is

$$ds^2 = -e^{2\Phi(r)}dt^2 + \frac{dr^2}{1 - b(r)/r} + r^2d\Omega^2.$$

The two functions have sharply different roles:

$\Phi(r)$: redshift; finite if no horizon forms, $b(r)$: shape; $b(r_0) = r_0$.

Reverse philosophy

Choose the geometry first; then use the Einstein equations to infer the matter source.

Morris–Thorne field equations

For an orthonormal stress-energy tensor

$$T_{\hat{\mu}\hat{\nu}} = \text{diag}(\rho, p_r, p_t, p_t),$$

the Einstein field equations

$$G_{\hat{\mu}\hat{\nu}} = 8\pi T_{\hat{\mu}\hat{\nu}}$$

give the two components relevant for the radial geometry:

$$\begin{aligned} 8\pi\rho(r) &= \frac{b'(r)}{r^2}, \\ 8\pi p_r(r) &= -\frac{b(r)}{r^3} + 2\left(1 - \frac{b(r)}{r}\right) \frac{\Phi'(r)}{r}. \end{aligned}$$

Geometry determines matter

The shape function $b(r)$ fixes the energy density directly, while both $b(r)$ and the redshift function $\Phi(r)$ determine the radial pressure required to support the geometry.

Energy conditions: the null energy condition

For any null vector $k^{\hat{\mu}}$, the null energy condition (NEC) requires

$$T_{\hat{\mu}\hat{\nu}} k^{\hat{\mu}} k^{\hat{\nu}} \geq 0,$$

which, for radial and transverse null directions, gives

$$\boxed{\rho + p_r \geq 0}, \quad \boxed{\rho + p_t \geq 0}.$$

The remaining standard pointwise conditions are

$$\text{WEC:} \quad \rho \geq 0, \quad \rho + p_r \geq 0, \quad \rho + p_t \geq 0,$$

$$\text{SEC:} \quad \rho + p_r \geq 0, \quad \rho + p_t \geq 0, \quad \rho + p_r + 2p_t \geq 0,$$

$$\text{DEC:} \quad \rho \geq 0, \quad \rho \geq |p_r|, \quad \rho \geq |p_t|.$$

For the wormhole throat

The key feature is the radial NEC combination: $\rho + p_r$.

Flaring-out condition and NEC violation at the throat

At fixed t and $\theta = \pi/2$, the embedding geometry satisfies

$$\frac{dz}{dr} = \pm \left(\frac{r}{b(r)} - 1 \right)^{-1/2}, \quad \frac{d^2r}{dz^2} = \frac{b - rb'}{2b^2} > 0.$$

Thus, at the throat,

$$b(r_0) = r_0, \quad b'(r_0) < 1.$$

From the field equations,

$$8\pi(\rho + p_r) = \frac{rb' - b}{r^3} + 2 \left(1 - \frac{b}{r} \right) \frac{\Phi'}{r}.$$

Evaluating at $r = r_0$,

$$(\rho + p_r)|_{r_0} = \frac{b'(r_0) - 1}{8\pi r_0^2} < 0.$$

Flaring-out condition:

In GR, the flaring-out condition at a traversable wormhole throat violates the radial NEC locally.

Raychaudhuri focusing and the price of defocusing

For a hypersurface-orthogonal null congruence with tangent k^a ,

$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - \sigma_{ab}\sigma^{ab} - R_{ab}k^ak^b.$$

At a traversable throat, the null congruence must defocus:

$$R_{ab}k^ak^b < 0.$$

In Einstein gravity,

$$R_{ab}k^ak^b = 8\pi T_{ab}k^ak^b, \quad \Rightarrow \quad T_{ab}k^ak^b < 0.$$

Conceptual bridge

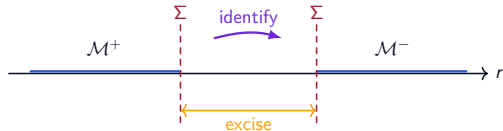
The embedding calculation and the null-congruence calculation tell the same story in two languages.

Why thin shells? From the flaring-out condition to localized exoticity

A thin-shell wormhole replaces an extended exotic fluid by a distributional surface layer on a timelike hypersurface Σ :

$$T^a_b = T^a_b|_+ \Theta(\ell) + T^a_b|_- \Theta(-\ell) + S^a_b \delta(\ell).$$

- choose two regular exterior regions;
- excise unwanted interiors;
- identify the two boundaries;
- compute the surface stress tensor required by the junction.



Core advantage

Exoticity is no longer hidden in a bulk ansatz: it is measured directly by S^i_j on Σ .

Cut-and-paste construction: the geometrical data

Take two static, spherically symmetric manifolds $\mathcal{M}_\pm$,

$$ds_\pm^2 = -e^{2\Phi_\pm(r)} \left(1 - \frac{b_\pm(r)}{r} \right) dt_\pm^2 + \frac{dr^2}{1 - b_\pm(r)/r} + r^2 d\Omega^2.$$

The shell is the timelike hypersurface

$$\Sigma : \quad r = a(\tau),$$

with intrinsic coordinates $\xi^i = (\tau, \theta, \phi)$ and induced metric

$$ds_\Sigma^2 = h_{ij} d\xi^i d\xi^j = -d\tau^2 + a^2(\tau) d\Omega^2.$$

First junction condition

The induced metric must be the same from both sides: $[h_{ij}] = 0$. This fixes the common area radius $a(\tau)$.

Shell kinematics: velocity and unit normal

The shell is located at $r = a(\tau)$, with

$$x_{\pm}^a = (t_{\pm}(\tau), a(\tau), \theta, \phi), \quad U_{\pm}^a = (\dot{t}_{\pm}, \dot{a}, 0, 0), \quad U_{\pm}^a U_a^{\pm} = -1.$$

Define

$$f_{\pm}(a) = 1 - \frac{b_{\pm}(a)}{a}, \quad F_{\pm}(a, \dot{a}) = f_{\pm}(a) + \dot{a}^2.$$

The unit normal satisfies

$$n_a^{\pm} U_{\pm}^a = 0, \quad n_{\pm}^a n_a^{\pm} = +1,$$

and, choosing it to point outward into each retained bulk region,

$$n_a^{\pm} = \pm \left(-e^{\Phi_{\pm}(a)} \dot{a}, \frac{\sqrt{F_{\pm}}}{f_{\pm}(a)}, 0, 0 \right).$$

Extrinsic curvature

The second fundamental form is

$$K_{ij}^{\pm} = -n_a^{\pm} \left(\frac{\partial^2 x_{\pm}^a}{\partial \xi^i \partial \xi^j} + \Gamma^a_{bc} \frac{\partial x_{\pm}^b}{\partial \xi^i} \frac{\partial x_{\pm}^c}{\partial \xi^j} \right).$$

For the generic spherical metrics above,

$$K^{\theta}_{\theta\pm} = K^{\phi}_{\phi\pm} = \pm \frac{\sqrt{F_{\pm}}}{a},$$

$$K^{\tau}_{\tau\pm} = \pm \left[\frac{\ddot{a} + \{b_{\pm}(a) - ab'_{\pm}(a)\}/(2a^2)}{\sqrt{F_{\pm}}} + \Phi'_{\pm}(a)\sqrt{F_{\pm}} \right].$$

Interpretation

K^{θ}_{θ} measures the bending of the spherical area radius; K^{τ}_{τ} contains the acceleration and the redshift-gradient force.

Israel–Lanczos equation

Let

$$[K^i_j] = K^i_{j+} - K^i_{j-}, \quad [K] = [K^i_i].$$

The surface stress tensor is

$$S^i_j = -\frac{1}{8\pi} ([K^i_j] - \delta^i_j [K]).$$

For spherical symmetry,

$$S^i_j = \text{diag}(-\sigma, p, p),$$

so the junction conditions reduce to

$$\sigma = -\frac{1}{4\pi a} \left(\sqrt{F_+} + \sqrt{F_-} \right),$$

$$p = \frac{1}{8\pi} \left[\frac{\sqrt{F_+} + \sqrt{F_-}}{a} + K^\tau_{\tau+} - K^\tau_{\tau-} \right].$$

Immediate consequence: negative surface energy

For a symmetric construction with identical sides,

$$F_+ = F_- = F, \quad \sigma = -\frac{1}{2\pi a} \sqrt{F}.$$

For a static Schwarzschild thin shell, $f(a) = 1 - 2M/a$,

$$\sigma_0 = -\frac{1}{2\pi a_0} \sqrt{1 - \frac{2M}{a_0}}, \quad p_0 = \frac{1}{4\pi a_0} \frac{1 - M/a_0}{\sqrt{1 - 2M/a_0}}.$$

The surface NEC is

$$\sigma_0 + p_0 = \frac{1}{4\pi a_0} \frac{3M/a_0 - 1}{\sqrt{1 - 2M/a_0}},$$

while the surface energy density itself satisfies $\sigma_0 < 0$ for every $a_0 > 2M$.

Negative energy density

The WEC is violated on the shell for all $a_0 > 2M$. The shell dictates exactly where the exotic matter resides; the sign of $\sigma_0 + p_0$ depends on the shell radius.

Conservation identity on the shell

The contracted Gauss–Codazzi relation gives the surface conservation law

$$D_i S^i_j = -[T_{ab} e^a_j n^b],$$

where D_i is the covariant derivative compatible with h_{ij} , and $e^a_j = \partial x^a / \partial \xi^j$.

For spherical symmetry,

$$\frac{d(\sigma A)}{d\tau} + p \frac{dA}{d\tau} = A \Xi \dot{a}, \quad A = 4\pi a^2.$$

For the general redshift functions,

$$\Xi = \frac{1}{4\pi a} \left[\Phi'_+(a) \sqrt{F_+} + \Phi'_-(a) \sqrt{F_-} \right].$$

Transparency condition

If $\Xi = 0$, then

$$\sigma' = -\frac{2}{a}(\sigma + p).$$

Equation of motion: the potential formulation

Define the surface mass and the symmetric/asymmetric bulk combinations

$$m_s(a) = 4\pi a^2 \sigma(a), \quad \bar{b}(a) = \frac{b_+(a) + b_-(a)}{2}, \quad \Delta(a) = \frac{b_+(a) - b_-(a)}{2}.$$

Solving the surface-density equation for $\dot{a}^2$ gives

$$\boxed{\frac{1}{2}\dot{a}^2 + V(a) = 0,}$$

with

$$\boxed{V(a) = \frac{1}{2} \left[1 - \frac{\bar{b}(a)}{a} - \left(\frac{m_s(a)}{2a} \right)^2 - \left(\frac{\Delta(a)}{m_s(a)} \right)^2 \right].}$$

Separation of roles

$\bar{b}$ and Δ encode the bulk geometry; $m_s(a)$ encodes the shell microphysics.

Equilibrium and linearized stability

A static thin-shell wormhole at $a = a_0$ satisfies

$$V(a_0) = 0, \quad V'(a_0) = 0.$$

Perturbing $a(\tau) = a_0 + \eta(\tau)$ gives

$$\frac{1}{2}\dot{\eta}^2 + \frac{1}{2}V''(a_0)\eta^2 + O(\eta^3) = 0.$$

Therefore

$$V''(a_0) > 0 \iff \text{linearized stability.}$$

In the transparent case, the equation of state enters through

$$\beta_0^2 = \left. \frac{\partial p}{\partial \sigma} \right|_{a_0}, \quad \sigma' = -\frac{2}{a}(\sigma + p),$$

so the stability problem becomes a sharp inequality on the material response of the shell.

Thin-shell formalism: what later slides will use

The complete computational chain

$$(\mathcal{M}_+, \mathcal{M}_-) \longrightarrow (h_{ij}, n_{\pm}^a, K_{ij}^{\pm}) \longrightarrow S^i_j = \text{diag}(-\sigma, p, p) \longrightarrow m_s(a), \Xi(a), V(a) \longrightarrow V''(a_0) > 0.$$

1989–1995

Surgical Schwarzschild wormholes and Poisson–Visser stability are direct applications of the symmetric, transparent version.

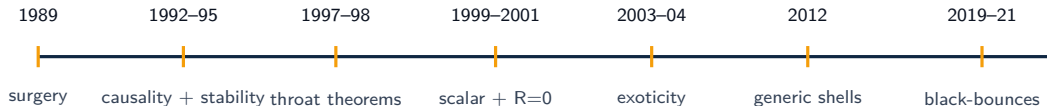
2012 onward

Generic shells, gravastars, and black-bounce shells use the asymmetric potential and the flux term.

Transition

The rest of the talk is best read as successive ways of choosing the two bulks and the shell equation of state.

Chronological roadmap of the Visser programme



1989–1995

Surgery, quantum shells, lensing, time machines, and linearized stability.

1997–2004

Roman rings, generic throat theorems, scalar/brane routes, and exoticity quantifiers.

2012–2021

Generic shells, gravastars, regular cores, Vaidya bounces, and black-bounces.

1989: polyhedral wormholes and localized exoticity

Visser constructed non-spherical traversable wormholes by joining two copies of Minkowski spacetime across the boundary of a compact region:

flat bulk + polyhedral throat $\implies$ localized exoticity.

- the flat faces carry no stress-energy;
- in the sharp-polyhedron limit, the corner contribution vanishes;
- exotic stress remains concentrated along the edges.

Key result

A traveller crossing through a flat face encounters neither exotic matter nor tidal forces. For an edge of bending angle ϕ ,

$$\mu = T = -\frac{\phi}{4\pi G},$$

so convex edges carry negative tension.

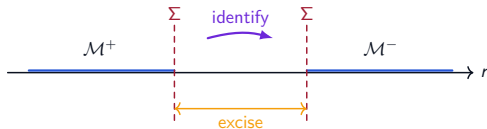
M. Visser, Phys. Rev. D 39, 3182(R) (1989).

1989: surgical Schwarzschild wormholes

Take two Schwarzschild exterior regions,

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2d\Omega^2, \quad f(r) = 1 - \frac{2M}{r}.$$

Excise $r < a$, with $a > 2M$, and identify the two timelike boundaries $r = a$.



Main result

The surgery removes the horizons and singular interiors, producing a traversable wormhole between two asymptotically flat regions. All non-vacuum stress-energy is concentrated on the throat, where the Israel junction conditions determine the required exotic surface layer. The construction is simple enough to permit a dynamical-stability analysis.

1995: Poisson–Visser linearized stability

For a Schwarzschild thin-shell wormhole, the radial dynamics reduces to

$$\dot{a}^2 + V(a) = 0, \quad V(a) = 1 - \frac{2M}{a} - [2\pi a \sigma(a)]^2.$$

The shell conservation law,

$$\sigma' = -\frac{2}{a}(\sigma + p), \quad \beta^2 \equiv \frac{\partial p}{\partial \sigma},$$

relates the potential directly to the local equation of state.

Linearizing about a static throat $a = a_0$,

$$V(a_0) = V'(a_0) = 0,$$

$$V''(a_0) > 0 \iff \text{stable.}$$

Key idea

The stability of the geometry is converted into a constraint on the material response of the exotic shell, encoded by β_0^2 .

Poisson–Visser: explicit stability criterion

For the Schwarzschild throat, $V''(a_0) > 0$ gives

$$\beta_0^2 \left(1 - \frac{3M}{a_0}\right) < -\frac{1 - 3M/a_0 + 3(M/a_0)^2}{2(1 - 2M/a_0)}.$$

Equivalently, the stability boundary is

$$\beta_{\text{crit}}^2(a_0) = -\frac{1 - 3M/a_0 + 3(M/a_0)^2}{2(1 - 2M/a_0)(1 - 3M/a_0)}.$$

$$a_0 > 3M$$

$$\beta_0^2 < \beta_{\text{crit}}^2 < 0.$$

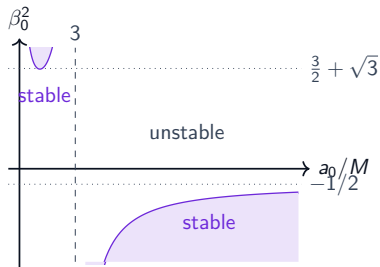
Large throats therefore require a negative equation-of-state slope.

$$2M < a_0 < 3M$$

$$\beta_0^2 > \beta_{\text{crit}}^2 > 0.$$

Stability requires a sufficiently large positive response.

Poisson–Visser: the stability map



Physical interpretation

If one naively imposes the conventional sound-speed range

$$0 < \beta_0^2 \leq 1,$$

no Schwarzschild thin-shell wormhole is linearly stable.

However, for exotic matter, β_0^2 need not represent a physical sound speed.

1992–1997: from wormhole to time machine

A traversable wormhole becomes a chronology problem once the mouths acquire a time shift. If the exterior separation is L , a closed timelike curve is available when

$$\Delta T > L \quad (c = 1).$$

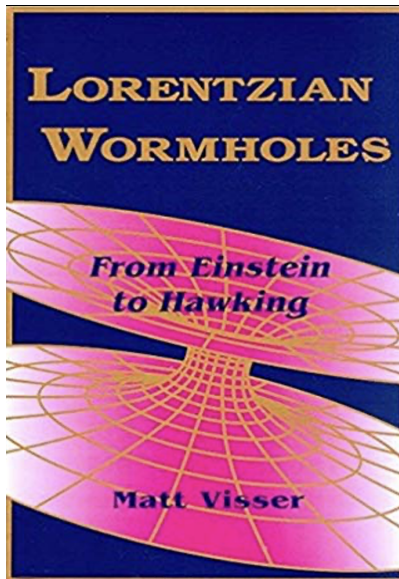
A Roman ring distributes the effect over several wormholes:

$$\sum_i \Delta T_i > \sum_i L_i.$$

Chronology-protection question

Can vacuum polarization, disruption, or gravitational back-reaction prevent the formation of macroscopic chronology violation?

M. Visser, Phys. Rev. D 47, 554 (1993); Phys. Rev. D 55, 5212 (1997).



1997–1998: generic throat theorems

Hochberg and Visser generalized the notion of a wormhole throat beyond spherical symmetry.

For a static wormhole, the throat is a minimal-area two-surface,

$$\text{tr } K = 0, \quad \partial_n(\text{tr } K) \leq 0,$$

with strict inequality corresponding to flaring out.

For dynamical wormholes, the throat is characterized by a marginal null surface,

$$\theta_{\pm} = 0, \quad \mathcal{L}_{k_{\pm}} \theta_{\pm} > 0.$$

Main result

Raychaudhuri's equation then implies

$$T_{ab} k^a k^b < 0$$

at a traversable wormhole throat in general relativity.

2000: brane surgery

In higher-dimensional settings, the same junction logic applies. A negative-tension brane has

$$S^i_j = -\lambda \delta^i_j, \quad \lambda < 0.$$

The Israel condition in $(n+1)$ dimensions keeps the same structural form:

$$[K^i_j] - \delta^i_j [K] = -8\pi G_{n+1} S^i_j.$$

At a glance

Brane surgery is thin-shell thinking lifted into a higher-dimensional arena: the sign and magnitude of the brane tension sculpt the global geometry.

C. Barceló and M. Visser, Nucl. Phys. B 584, 415 (2000).

2003–2004: exoticity as a quantitative resource

Visser, Kar, and Dadhich separated local NEC violation from the integrated amount of exotic matter. For a Morris–Thorne wormhole, one may use the volume-integral quantifier

$$\Omega_{\text{ANEC}} = \int (\rho + p_r) dV.$$

A more explicit integration-by-parts form, including both asymptotic regions, is

$$\oint (\rho + p_r) dV = - \int_{r_0}^{\infty} (1 - b') \ln \left[\frac{e^{2\Phi}}{1 - b/r} \right] dr.$$

Precise meaning

The local NEC violation at the throat does not vanish. What can be made arbitrarily small is the integrated amount of exotic matter.

M. Visser, S. Kar, and N. Dadhich, Phys. Rev. Lett. 90, 201102 (2003); S. Kar, N. Dadhich, and M. Visser, Pramana 63, 859 (2004).

2004: fundamental limitations on warp-drive spacetimes

Lobo and Visser analysed the Alcubierre and Natário warp drives, including the weak-field regime $v \ll 1$.

Linearized gravity allows a finite-mass spaceship inside the bubble. For bubble radius R and wall thickness Δ ,

$$M_{\text{warp}} \sim -\frac{v^2 R^2}{\Delta},$$

so requiring the negative warp energy not to dominate gives

$$\boxed{\frac{v^2 R^2}{\Delta} \lesssim M_{\text{ship}}}.$$

Main conclusion

Energy-condition violations persist even for arbitrarily small velocities, and the resulting negative-energy requirements impose severe bounds on physically useful warp-drive configurations.

F. S. N. Lobo and M. Visser, Class. Quantum Grav. 21, 5871 (2004).

(Nominated as one of the 2004 and 2005 Highlights of Class.Quant.Gravity)

2012: generic dynamic thin-shell wormholes

Montelongo García, Lobo, and Visser considered two arbitrary spherical bulks,

$$ds_{\pm}^2 = -e^{2\Phi_{\pm}(r)} \left(1 - \frac{b_{\pm}(r)}{r} \right) dt_{\pm}^2 + \frac{dr^2}{1 - b_{\pm}(r)/r} + r^2 d\Omega^2,$$

joined at $r = a(\tau)$.

The surface density is universal:

$$\sigma = -\frac{1}{4\pi a} \left(\sqrt{F_+} + \sqrt{F_-} \right), \quad F_{\pm} = 1 - \frac{b_{\pm}(a)}{a} + \dot{a}^2.$$

Unification

Symmetric and asymmetric thin-shell wormholes become special cases of one master formalism.

N. Montelongo García, F. S. N. Lobo, and M. Visser, Phys. Rev. D 86, 044026 (2012).

2012: master potential for asymmetric shells

Define

$$m_s(a) = 4\pi a^2 \sigma(a), \quad \bar{b}(a) = \frac{b_+(a) + b_-(a)}{2}, \quad \Delta(a) = \frac{b_+(a) - b_-(a)}{2}.$$

The dynamics takes the compact form

$$\frac{1}{2} \dot{a}^2 + V(a) = 0,$$

$$V(a) = \frac{1}{2} \left[1 - \frac{\bar{b}(a)}{a} - \left(\frac{m_s(a)}{2a} \right)^2 - \left(\frac{\Delta(a)}{m_s(a)} \right)^2 \right].$$

Structural elegance

$\bar{b}$ is the average bulk geometry, Δ is the asymmetry, and m_s is the shell microphysics.

2012: external forces and generic stability

The shell conservation identity becomes

$$\frac{d(\sigma A)}{d\tau} + p \frac{dA}{d\tau} = A \Xi \dot{a}, \quad A = 4\pi a^2,$$

with

$$\Xi = \frac{1}{4\pi a} \left[\Phi'_+(a) \sqrt{F_+} + \Phi'_-(a) \sqrt{F_-} \right].$$

At equilibrium,

$$V(a_0) = 0, \quad V'(a_0) = 0, \quad V''(a_0) > 0,$$

yielding inequalities involving $m_s''(a_0)$, $\Xi'(a_0)$, and derivatives of $b_{\pm}$, $\Phi_{\pm}$.

General lesson

Stability is equivalent to selecting suitable surface material properties for the chosen pair of bulk geometries.

2012: generic thin-shell gravastars

The same machinery applies to gravastars:

$$\mathcal{M}_{\text{int}} \cup_{\Sigma} \mathcal{M}_{\text{ext}}, \quad S^i_j = \text{diag}(-\sigma, p, p), \quad \frac{1}{2}\dot{a}^2 + V(a) = 0.$$

Wormhole

Two exterior regions are joined across a throat. The shell is a portal between asymptotic domains.

Gravastar

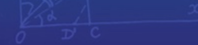
An interior compact region is joined to an exterior. The shell is a horizon-avoidance layer.

P. Martín-Moruno, N. Montelongo García, F. S. N. Lobo, and M. Visser, JCAP 03, 034 (2012).

Fundamental Theories of Physics 189

Francisco S.N. Lobo *Editor*

Wormholes, Warp Drives and Energy Conditions



Proper length of the identical bar

$$l = \frac{PP'}{OC} = \frac{P'P'}{OC}$$

M. Visser showed that:



2019: black-bounce to traversable wormhole

The Simpson–Visser metric is

$$ds^2 = - \left(1 - \frac{2m}{\sqrt{r^2 + a^2}} \right) dt^2 + \frac{dr^2}{1 - 2m/\sqrt{r^2 + a^2}} + (r^2 + a^2) d\Omega^2.$$

The area radius never vanishes:

$$R(r) = \sqrt{r^2 + a^2} \geq a, \quad r_H^2 = (2m)^2 - a^2.$$

At a glance

The parameter a replaces the singular center by a nonzero-area bounce surface.

A. Simpson and M. Visser, JCAP 02, 042 (2019).

Black-bounce taxonomy: one parameter, three causal regimes

$a = 0 \implies$ Schwarzschild,

$0 < a < 2m \implies$ regular black-bounce black hole,

$a = 2m \implies$ null bounce,

$a > 2m \implies$ traversable wormhole.

The message:

Black holes, regular black holes, and traversable wormholes are not isolated curiosities here: they are neighbouring sectors in a single sculptable parameter space.

2019–2021: dynamic bounces, shells, and regular cores

Vaidya black-bounces promote the mass to a null-time function,

$$m \rightarrow m(w), \quad ds^2 = - \left(1 - \frac{2m(w)}{\sqrt{r^2 + a^2}} \right) dw^2 \mp 2dwdr + (r^2 + a^2)d\Omega^2.$$

Dynamic thin-shell black-bounce wormholes keep the same mechanical structure,

$$\frac{1}{2} \dot{a}_\Sigma^2 + V(a_\Sigma) = 0,$$

with V now controlled by the black-bounce parameters on both sides.

Continuity of method

The later geometries change the bulk input, but not the thin-shell logic: junction data still determine $\sigma, p, V(a)$, and stability still means $V''(a_0) > 0$.

A. Simpson, P. Martín-Moruno, and M. Visser, CQG 36, 145007 (2019). F. S. N. Lobo, A. Simpson, and M. Visser, PRD 101, 124035 (2020).

Regular cores: changing the input function

Regular-core models replace the Schwarzschild mass by a profile

$$M(r) = m\mathcal{F}(r), \quad \mathcal{F}(0) = 0, \quad \mathcal{F}(\infty) = 1,$$

so that

$$f(r) = 1 - \frac{2M(r)}{r}$$

feeds into the same shell equations.

Geometric effect

The core removes the curvature singularity and changes the near-center causal structure.

Shell effect

The functions $f(a)$ and $f'(a)$ shift the required surface stresses and stability windows.

T. Berry, F. S. N. Lobo, A. Simpson, and M. Visser, Phys. Rev. D 102, 064054 (2020).

For a conference in Matt's honour: Scientific perspective

Methodological contributions

- geometric questions formulated in terms of precise, testable conditions;
- energy-condition violations quantified through local and integrated measures;
- stability analysed through explicit perturbative criteria;
- diverse compact-object and wormhole geometries treated within a common mathematical framework.

As a scientist: original, rigorous, incisive, and versatile.

As a person: generous, warm, witty, supportive, and remarkably unpretentious.

Broader impact

Matt Visser's work helped establish a systematic framework for analysing wormholes and related compact geometries in terms of their causal structure, energy-condition requirements, matter content, and stability. This methodology has subsequently been applied to gravastars, regular black holes, black-bounces, and a broad range of modified-gravity configurations.

Mentorship and scientific legacy – post 2002 (in NZ)

Postdoctoral researchers

Joey Medved (Canada)

Ana Alonso-Serrano (Spain)

Prado Martín-Moruno (Spain)

Current students

Kellie Jobe (NZ)

Christopher Simmonds (NZ)

PhD students

Silke Weinfurter (Germany)

Petarpa Boonserm (Thailand)

Céline Cattoën (France)

Gabriel Abreu (Venezuela)

Jozef Skakala (Slovakia)

Sebastian Schuster (Germany)

Valentina Baccetti (Italy)

Jessica Santiago (Brazil)

Del Rajan (NZ)

Rudeep Gaur (NZ)

Alex Simpson (NZ)

Joshua Baines (NZ)

MSc students

Damien Martin (NZ)

Petarpa Boonserm (Thailand)

Tristan Faber (Germany)

Bethan Cropp (NZ)

Alex van Brunt (NZ)

Finnian Gray (NZ)

Del Rajan (NZ)

Thomas Berry (NZ)

Aden Jowsey (NZ)

Joe Wilson (NZ)

A scientific legacy carried forward through generations of students, postdocs,
and long-term collaborators across the world.

Beyond the blackboard: Lisbon 2017 (“Cabo da Roca” and “Boca do Inferno”)



Cabo da Roca and the Atlantic edge of Europe.

Beyond the blackboard: New Zealand – 2019



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**Thank you
for your attention**