Guiding center quantization of a quantum Hall analog of Hawking radiation

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From Black Holes to the Cosmos: Matt Visser's journey through space and time
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Guiding-centre Lagrangian and quasisymmetry (arXiv:2408.05919, J. Plasma Phys. **91** (2025) E46)
Guiding center quantization of a quantum Hall analog of Hawking radiation (Rodrigo Andrade e Silva & TJ, arXiv:2608.xxxxx)
Quantization of the guiding guiding center: do quantization and coarse graining commute? (RAeS & TJ, arXiv:2608.xxxxx)

black hole entropy in higher curvature theories

acoustic metrics

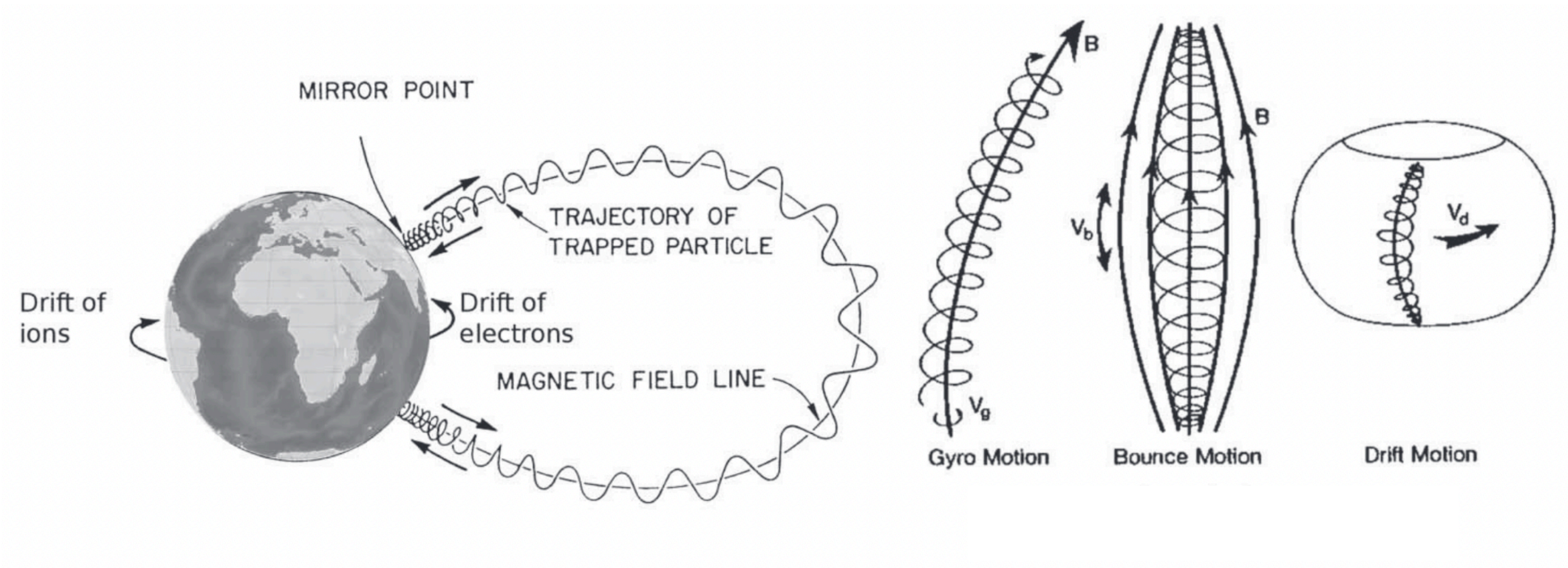
field theory with dispersion

BEC analogue spacetimes

Lorentz symmetry violation

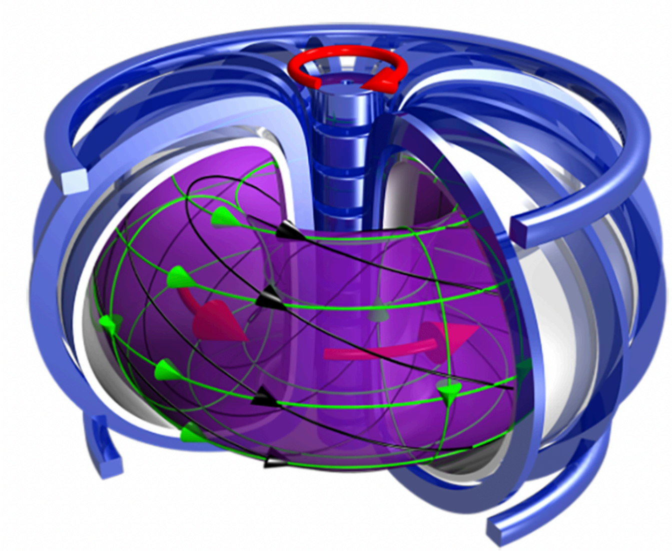
Hořava gravity

Guiding center approximation



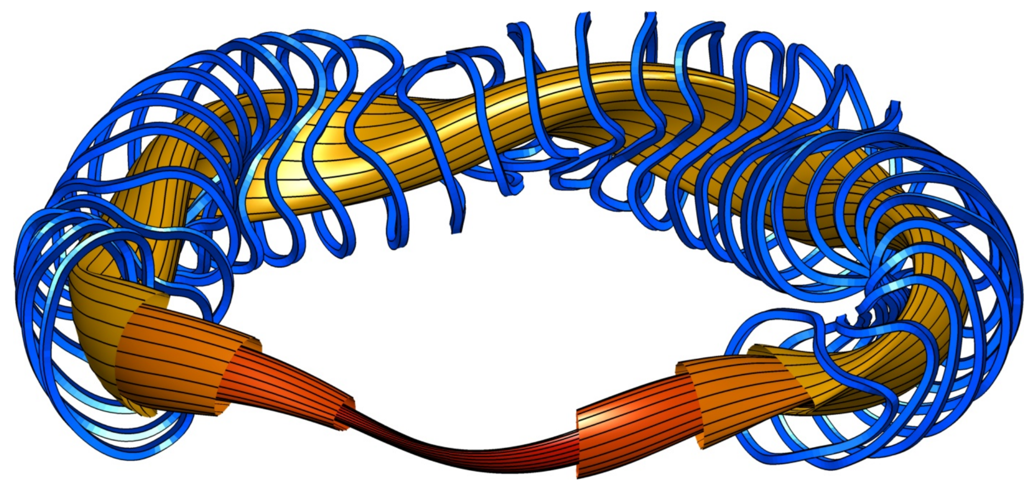
figures from M. Regi, IL NUOVO CIMENTO 39 C (2016) 285
(adapted from elsewhere)

Magnetic confinement of plasma for fusion



Tokamak

figure from IAEA



Stellarator

W7-X

Confinement to axisymmetric “flux surface” via twisting

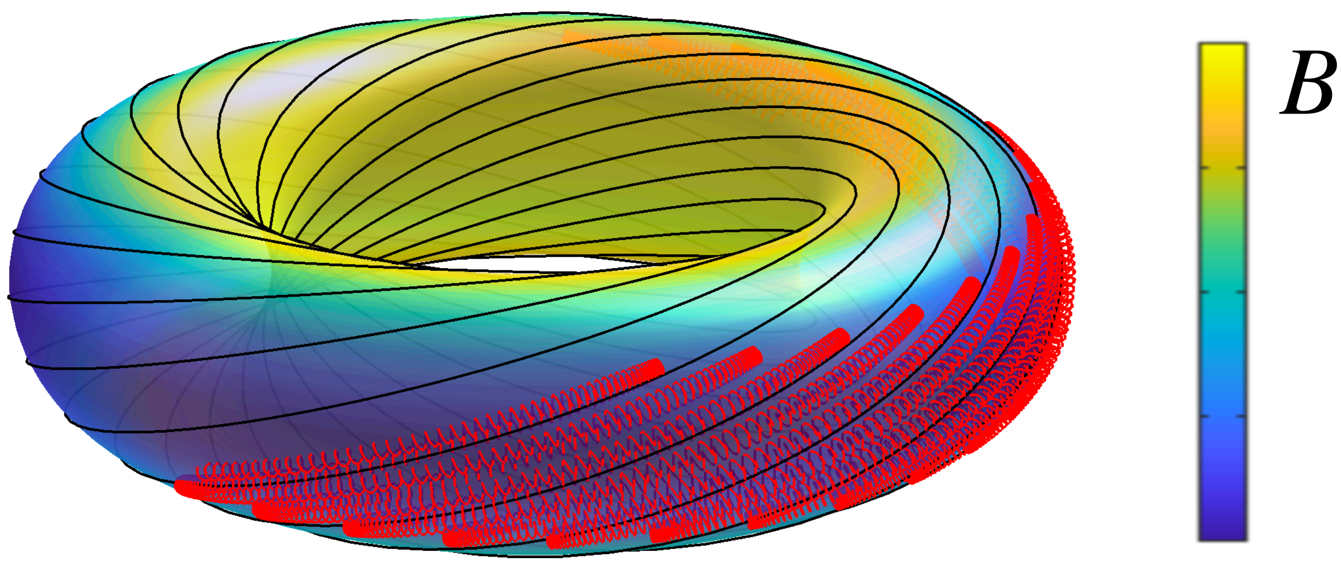


figure Matt Landreman
UMD Stellarator Group
terpconnect.umd.edu/~mattland/

Effective Lagrangian for guiding center motion

gyro radius small compared to other relevant lengths
gyro period short compared to other relevant times

"Microscopic Lagrangian" $$L = \frac{1}{2}m\mathrm{v}^2 + qA_i\mathrm{v}^i$$

velocity decomposition $$\mathbf{v} = \mathbf{v}_{\|} + \mathbf{v}_{\perp} = \mathbf{v}_{\|} + \mathbf{v}_{\text{gyro}} + \mathbf{v}_{\text{drift}}$$

kinetic energy decomposition $$\frac{1}{2}m\mathrm{v}^2 = \frac{1}{2}\mathrm{m}(\mathrm{v}_{\|}^2 + \underbrace{\mathrm{v}_{\text{gyro}}^2}_{\frac{2}{m}\mu B} + \underbrace{\mathrm{v}_{\text{drift}}^2}_{\text{neglect}} + \underbrace{2\mathbf{v}_{\text{gyro}}\cdot\mathbf{v}_{\text{drift}}}_{\text{averages to zero}})$$

Treat μB as an effective potential energy

μ: orbital magnetic moment
adiabatic invariant

Effective Lagrangian for guiding center motion

$$L^{\text{eff}} = \frac{1}{2}m(b_i v^i)^2 + qA_i v^i - \mu B$$

$b^i = B^i/B, \quad b_i = h_{ij}b^j, \quad h_{ij} =$ spatial metric

First order guiding center equations of motion

$$\boxed{\begin{aligned} m\dot{v}_{\|} &= -\mu b^i B_{,i} \\ v_{\perp}^j &= (qB)^{-1}\epsilon^{ij}\left(m v_{\|}^2 \kappa_i + \mu B_{,i}\right) \end{aligned}}$$

1. $v_{\perp}$ is first order in gradients of the background fields

2. add electrostatic and/or other potentials: $\mu B \rightarrow \mu B + \Phi$

Quantum theory of drift velocity? WHY ASK THAT?

1. **Curiosity**: No kinetic term, momenta 100% constrained.
   Does canonical quantization make sense? If so, what does it yield?

2. **Practicality**: Does it yield useful results with simplified calculations?

3. **Do quantization and coarse graining commute?**
   Lessons for quantizing a classical effective theory (e.g. GR)?

Quantum theory of drift velocity?

In **two spatial dimensions** the Lagrangian with an electrostatic potential is

$$L^{\text{eff}} = qA_i v^i - \mu B + q\Phi$$

The momentum is $p_i = qA_i$, which is a pair of primary constraints on the phase space.

The reduced phase space is the configuration space, with symplectic form

$$\omega = qF = qB\, dx \wedge dy \qquad (x, y \text{ area-adapted})$$

The quantized position coordinates are noncommutative.

The Hamiltonian is $$H = \mu B + q\Phi$$

Note: Theory has a background area element but no metric dependence.

Group theoretic quantization

Choose a Poisson algebra that generates a symplectic group that acts transitively on the phase space.

Include as many generators of symmetries of the classical system as possible.

Choose a unitary irreducible projective representation of the algebra such that
any classically complete sub-algebra is also irreducibly represented
&
any Casimir invariant of the classical algebra has the same value in the quantum representation

Closed isomagnetic contours

Assume flux function $2\pi\Phi$ has nondegenerate Hessian at "origin". Then

$$F = d\Phi \wedge d\theta$$ for smooth θ with period 2π

Algebra generators $\Phi, \quad X = \sqrt{2\Phi}\cos\theta, \quad Y = \sqrt{2\Phi}\sin\theta$

Φ generates area-preserving θ shifts

Poisson algebra

$$\{X, Y\} = -1$$
$$\{X, \Phi\} = -Y$$
$$\{Y, \Phi\} = X\,.$$

Quantum algebra

$$[\widehat{X}, \widehat{Y}] = -i\hbar$$
$$[\widehat{X}, \widehat{\Phi}] = -i\hbar\widehat{Y}$$
$$[\widehat{Y}, \widehat{\Phi}] = i\hbar\widehat{X}$$

Casimir matching $\widehat{\Phi} = \dfrac{\widehat{X}^2 + \widehat{Y}^2}{2}$

This is the Hamiltonian of a harmonic oscillator, so the flux is quantized like the energies of an oscillator — which means the allowed "radial" position of the particle is discretized
(although not larger than the coarse-graining scale)

This prediction of the quantized coarse-grained theory is spurious: the “microscopic” theory has no quantization of radial position.

A cautionary tale: a quantized effective theory may make spurious predictions even with no sign of internal breakdown.

Could this be relevant in the case of quantum gravity, e.g. with the area quantization in LQG?

If B is constant,

$$[\hat{x}, \hat{y}] = -i\ell_B^2, \qquad \ell_B = \sqrt{\hbar/qB}$$

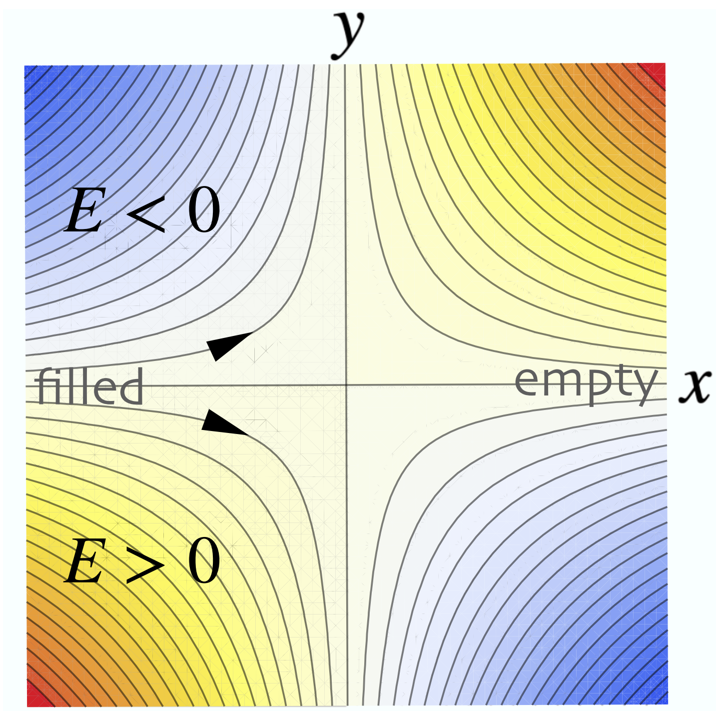
Phase space is the spatial plane, with noncommutativity scale the LLL orbit

The density of states matches that of one Landau level.
One recovers the quantum Hall resistivity with the filled states.

$$\rho_{xy} := \frac{E^y}{j^x} = \frac{2\pi\hbar}{q^2}$$

Michael Stone's model

CQG 30 (2013) 085003



$$\Phi = \lambda xy$$

$$v^x = -\frac{\lambda}{B}x \qquad v^y = \frac{\lambda}{B}y$$

Edge modes: chiral fermions in the metric

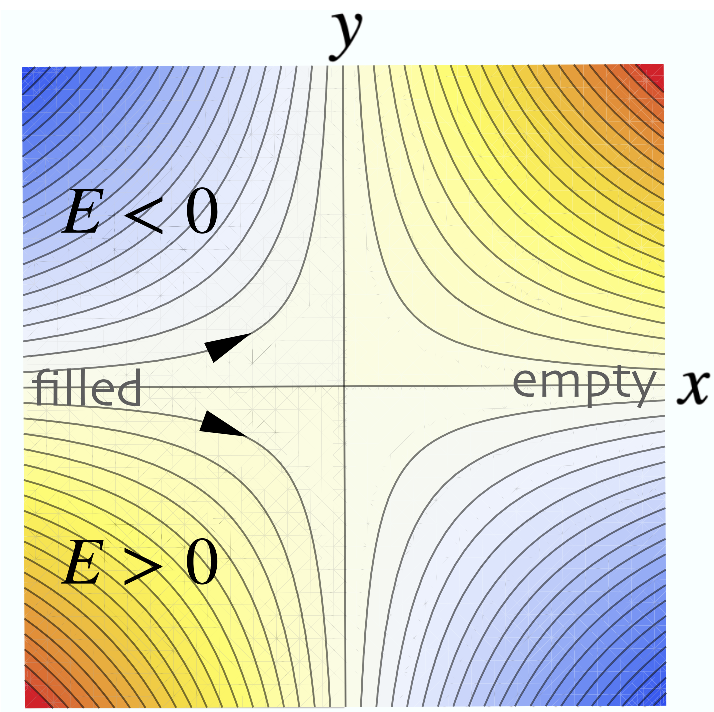
$$ds^2 = (v^y)^2 dt^2 - dy^2$$

EFT derived from edge current algebra.

M. Stone, Ann. Phys. **207**, 38 (1991)

The edge mode EFT has been derived starting from a bulk Chern-Simons action, with uniform electric field. It could be interesting to re-do this with Φ.

Michael Stone's model



$$ds^2 = (v^y)^2 dt^2 - dy^2$$

Prepare a state with charges entering from $x \to -\infty$.

Stone studied energy eigenfunctions $\psi_E(x, y)$, which involve parabolic cylinder functions. He compared

$$\psi_E(x = 0, y = \pm\infty)$$

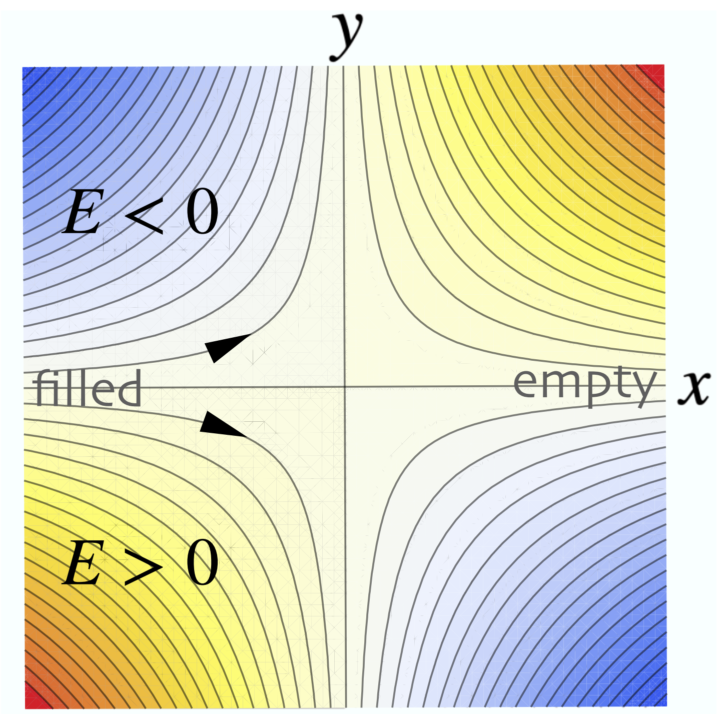
and showed that there is thermal emission of particles and holes at the Hawking temperature

$$T = \frac{\hbar\kappa}{2\pi}, \qquad \kappa = \frac{dv^y}{dy} = \frac{\lambda}{B}$$

Guiding center quantization

$$H = q\lambda xy \longrightarrow \hat{H} = \frac{q\lambda}{2}(\hat{x}\hat{y} + \hat{y}\hat{x})$$

Hamiltonian is a squeeze generator



$$ds^2 = (v^y)^2 dt^2 - dy^2$$

Heisenberg picture

$$\hat{x}(t) = e^{-\kappa t}\,\hat{x}(0), \qquad \hat{y}(t) = e^{\kappa t}\,\hat{y}(0)$$

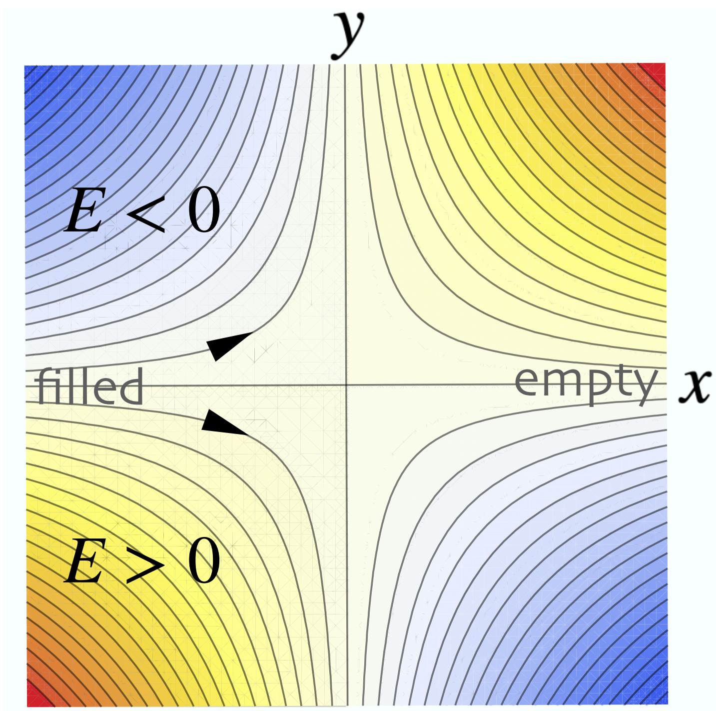
Schrödinger picture(s)

$$\hat{y} = \frac{i\hbar}{B}\frac{d}{dx}, \qquad \hat{x} = -\frac{i\hbar}{B}\frac{d}{dy}$$

General solution to time-dependent Schrödinger eqn

$$\phi(x,t) = f(x\,e^{\kappa t})\,e^{\kappa t/2}$$
$$\psi(y,t) = g(y\,e^{-\kappa t})\,e^{-\kappa t/2}$$

A charge initially confined to a half-plane never leaves that half-plane.

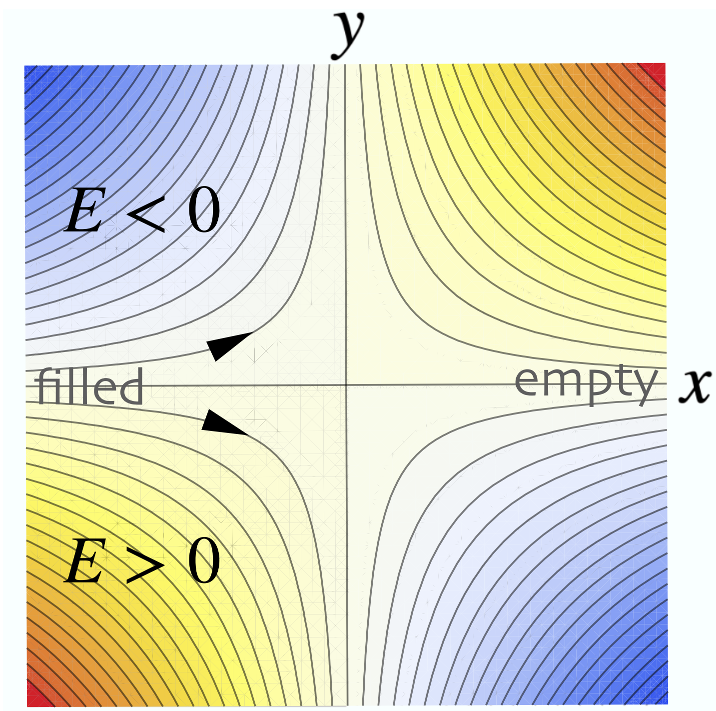


$$ds^2 = (v^y)^2 dt^2 - dy^2$$

A charge initially confined to a half-plane never leaves that half-plane …

HOWEVER, if $\phi(x) = 0$ for $x > 0$,
the Fourier transform is analytic in the lower half y plane,
so it must be supported at both $y > 0$ and $y < 0$.

$$\psi(y) \propto \int_{-\infty}^{0} dx\, e^{ixy/\ell_B^2}\, \phi(x)$$



$$ds^2 = (v^y)^2 dt^2 - dy^2$$

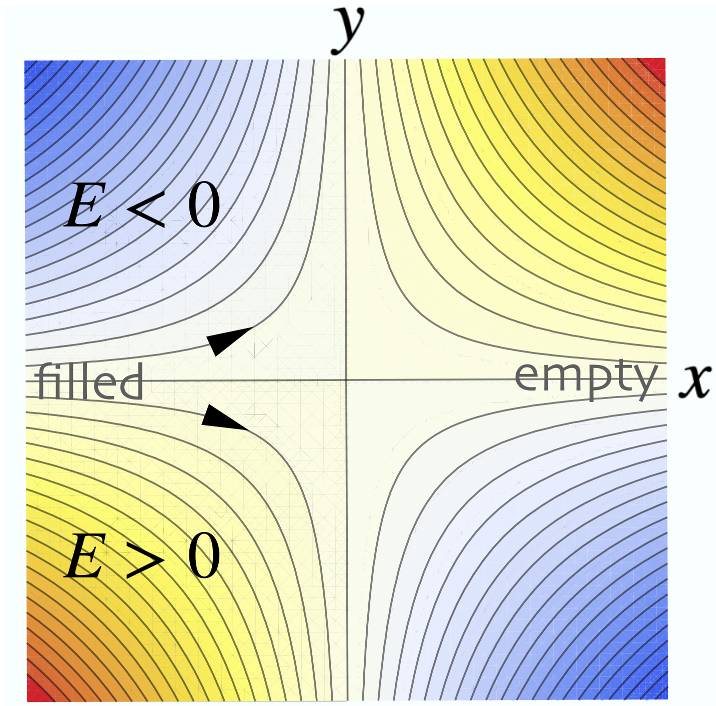
Energy eigenstates in the y representation

$$\psi_E(y) = \begin{cases} \psi_+ \, y^{-\frac{1}{2}+iE/\hbar\kappa} & y > 0 \\ \psi_- \, (-y)^{-\frac{1}{2}+iE/\hbar\kappa} & y < 0 \end{cases}$$

Analyticity in lower half plane implies $\psi_- = ie^{\pi E/\hbar\kappa}\psi_+$

Probability current

$$J^y = v^y |\psi(y)|^2 = \kappa y |\psi(y)|^2 = \kappa |\psi_+|^2 \begin{cases} 1 & y > 0 \\ -e^{E/T} & y < 0 \end{cases}$$



$$ds^2 = (v^y)^2 dt^2 - dy^2$$

Probability of emission to $y = +\infty$

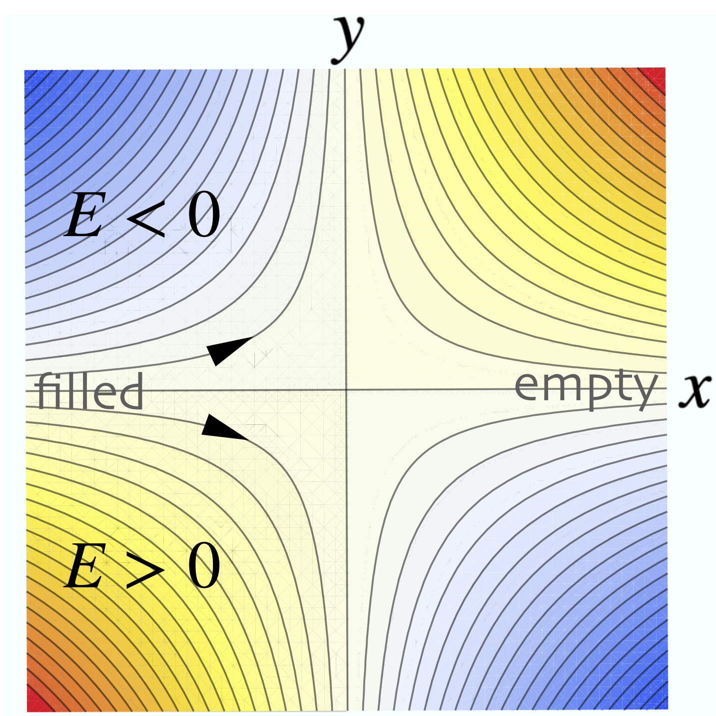
$$P_E = \frac{J_{+\infty}}{J_{+\infty} + (-J_{-\infty})} = \frac{1}{e^{E/T}+1}$$

Fermi-Dirac distribution

With incoming filled Fermi sea,
positive energy particles are emitted with probability p_E,
negative energy **holes** are emitted with probability $1 - p_E = p_{-E'}$
and the state is the usual two-mode squeezed state:

$$|\Omega\rangle \propto \exp\left[i \int_0^\infty dE\, e^{-E/2T}(a_-(E) a_+^\dagger(E) + a_-^\dagger(-E) a_+(-E)) \right] |0\rangle$$

Comments

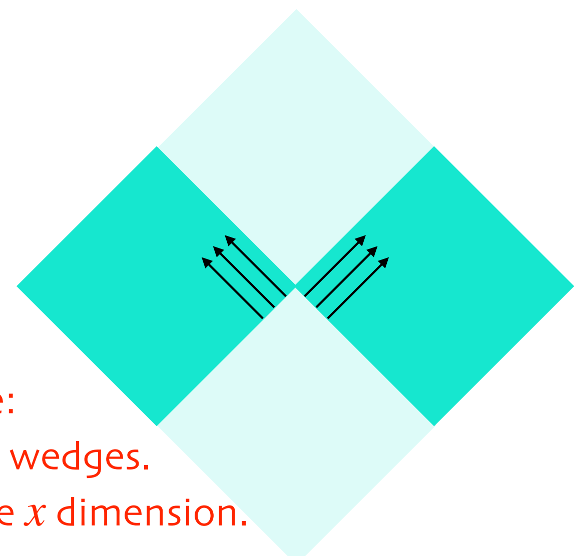


$$ds^2 = (v^y)^2 dt^2 - dy^2$$

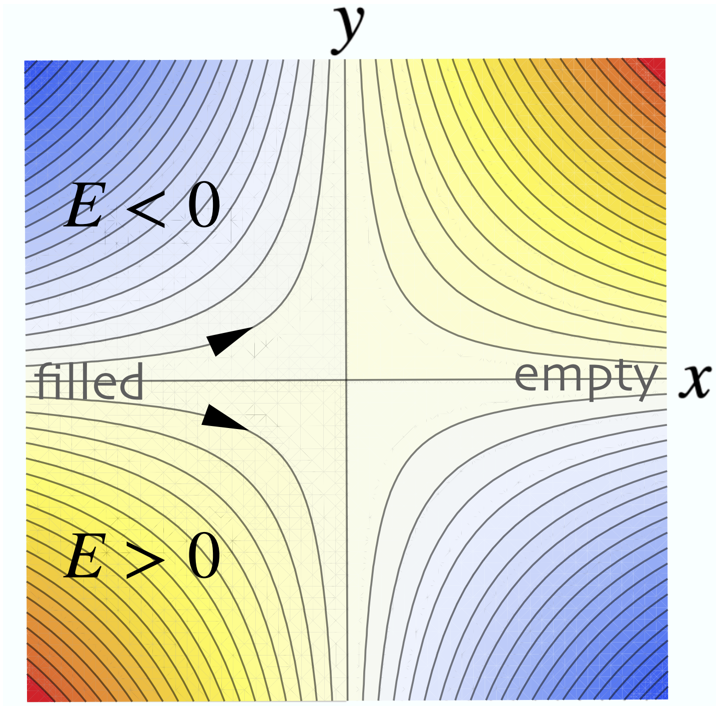
The effective spacetime is FLAT,
so this is the Unruh effect, not the Hawking effect.

BUT, there is outgoing "lab frame" radiation,
and no ingoing radiation, because:

a/ the lab time t is the boost angle in the effective metric
b/ the edge modes are chiral



The analog spacetime is incomplete:
it has only the left and right Rindler wedges.
The physical particles enter from the x dimension.



$$ds^2 = (v^y)^2 dt^2 - dy^2$$

Questions

1. Observability? See Stone (CQG) for comments.

2. Relation to Robinson-Wilczek chiral anomaly derivation of the Hawking effect?