Analog regular black holes and black hole mimickers in a Bose-Einstein condensate

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Intro and Motivation: Analog gravity with wave acoustics

- Take the velocity potential ϕ of an irrotational fluid flow $\mathbf{v} = \nabla\phi$
- Take perturbation $\delta\phi$ of the velocity potential, linearize Bernoulli and continuity, and write the resulting *massless* wave equation in the form:

$$\Delta\delta\phi \equiv \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\, g^{\mu\nu}\, \partial_\nu\delta\phi) = 0 \tag{1}$$

- The effective, "acoustic" metric used to write the wave eq. in diff.geom. form is:

$$ds^2_{\mathrm{AC}} = \frac{\rho}{c_s}[-(c_s^2 - \mathbf{v}^2)dt^2 - 2\mathbf{v}\cdot d\mathbf{x}dt + d\mathbf{x}^2] \tag{2}$$

- Waves in the fluid with speed c_s propagate effectively as if on a curved spacetime
- Test kinematic gravitational phenomena on such analog platforms: QNMs, SR, HR, *instabilities of inner horizon/light ring*

Intro and Motivation: Regular black holes and Black hole mimickers

- Singularities and Cauchy horizons as breakdown of predicatbility in GR - to be cured by QG?
- In the absence of QG, look for singularity-free phenomenological BH models within classical gravity:
  - regular black holes (RBHs)
  - black hole mimickers (BHMs)
- Nomenclature: Ultracompact objects (UCOs):
  UCOs $\approx$ (R)BHs + BHMs

Ultracompact objects (UCO) theory and phenomenology

- Simplest (static, spherically symmetric) phenomenological models:

$$ds^2 = -\left(1 - \frac{2m(r)}{r}\right) dt^2 + \left(1 - \frac{2m(r)}{r}\right)^{-1} dr^2 + d\Omega^2 \qquad (3)$$

- Example: Hayward model $m(r) = \frac{Mr^3}{r^3+2M\ell^2}$. Limiting values:

$\rightarrow$ $m(r) \rightarrow M$ as $r \rightarrow \infty$

$\rightarrow$ $m(r) \rightarrow \frac{r^3}{2\ell^2}$ as $r \rightarrow 0 \implies$ de-Sitter core (regular!)

- Horizon structure:

$\rightarrow$ for $0 \leq \ell < \ell_*$ inner and outer horizon $\implies$ RBH

$\rightarrow$ for $\ell > \ell_* = \frac{4M}{\sqrt{27}}$ no horizons $\implies$ BHM

Ultracompact objects' structure

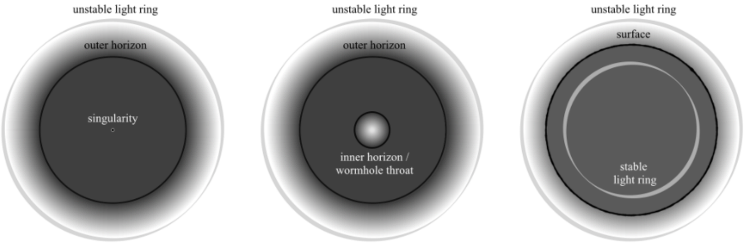

Figure: Towards a non-singular paradigm of BH physics: singular BH, regular BH, BH mimicker. Carballo-Rubio et al. 2025

Ultracompact objects' instabilities

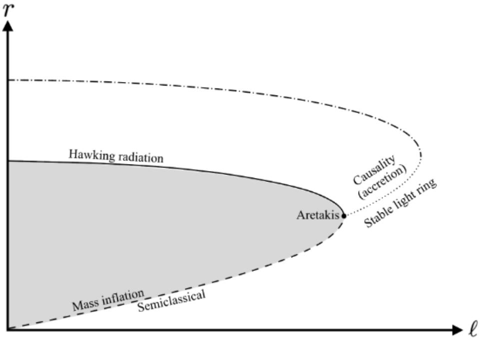

Figure: Schematic representation of instabilities for spherically symmetric UCO models. Carballo-Rubio et al. 2025

Realizing an UCO with BEC. Metric correspondence

- PG transformed gravitational metric

$$ds^2_{\mathrm{GR}} = -\left(1 - \frac{2m(r)}{r}\right) dt^2_{\mathrm{PG}} + 2\sqrt{\frac{2m(r)}{r}}\, dr\, dt_{\mathrm{PG}} + dr^2 + r^2 d\Omega^2 \,. \quad (4)$$

- Acoustic metric (up to conformal factor) in *radially flowing BEC*:

$$ds^2_{\mathrm{AC}} = -(c_s^2 - v_r^2)dt^2 - 2v_r drdt + dr^2 + dz^2 + r^2 d\phi^2 \qquad (5)$$

- "Acoustic" time transformation:

$$dt \to dt_{\mathrm{AC}} \equiv c_{\mathrm{s}}(r)dt \qquad (6)$$

gives

$$ds^2_{\mathrm{AC}} = -(1 - \frac{v_r^2}{c_s^2})dt^2_{\mathrm{AC}} - 2\frac{v_r}{c_s}drdt_{\mathrm{AC}} + dr^2 + dz^2 + r^2 d\phi^2 \qquad (7)$$

$\rightarrow$ Acoustic and gravitational metric become directly mappable

Realizing an UCO with BEC. Velocity profile

- Correspondence between condensate and gravitational quantities:

$$\frac{v_r(r)}{c_s(r)} = -\sqrt{\frac{2m(r)}{r}} \tag{8}$$

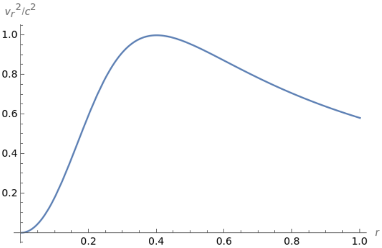

Figure: Velocity profile for realizing a Hayward UCO.

→ Ensures an analogy for the entire spacetime, not just an analog horizon or ergosphere structure!

→ Find controllable analogues of parameters ℓ and M which control the properties of the spacetime, including instabilities!

Realizing an UCO with BEC. Equations and quantities.

- "Correspondence" equation

$$\frac{1}{2}v_r^2 = \frac{m(r)}{r}c_s^2(r) \tag{9}$$

- Continuity equation

$$\frac{d}{dr}(\rho v_r r) = 0 \tag{10}$$

- Bernoulli equation

$$\frac{1}{2}v_r(r)^2 + U(r) + h(r) + Q(r) = 0 \tag{11}$$

$\rightarrow$ The *external potential* $U(r)$ gives us great experimental control
$\rightarrow$ The *enthalpy* $h(r) = \int_0^\rho \frac{d\rho}{\rho} c_s^2$ is under our control indirectly:
$\rightarrow$ The *interaction strength* $g(r)$ inside $c_s^2(r) = \frac{g(r)\rho(r)}{m_p^2}$ is controlled
...and with $g(r)$ we can ensure dominance of $h(r)$ and $U(r)$ over
$\rightarrow$ The *quantum pressure* $Q(r) = \frac{\hbar^2}{2m_p^2}\frac{1}{\sqrt{\rho}}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\sqrt{\rho}}{\partial r}\right)$ at *all* scales

Realizing an UCO with BEC. Calculations

- Problem: quantum pressure is dominant at scales below the *healing length* $\xi = \frac{\hbar}{\sqrt{2\rho(r)g(r)}}$, erasing any trace of the analog parameters

$\rightarrow$ To stay highly super-healing $\ell, M \gg \xi$, either make ℓ, M big, or engineer $g(r)$ with the Feshbach resonance to make ξ small

- $g(r) \propto r^{-2}$ easy to engineer with a two-coil magnetic trap

$\rightarrow$ If you then engineer, at $r \rightarrow 0$,

$$U(r) \sim -\frac{2Bg^{2/3}(r)l^{2/3}}{m_p^2 r^{4/3}} \propto -r^{-4/3}g^{2/3} \propto -r^{-8/3} \tag{12}$$

you satisfy the correspondence and the fluid equations self-consistently

$\rightarrow$ You have the analog of the central region of an UCO

$\rightarrow$ The proportionality constant in $U(r) \propto -r^{-8/3}$ serves as TUNABLE analog for ℓ

Realizing an UCO with BEC. Calculations

- Whereas, at $r \to \infty$:

$$U(r \to \infty) \to -\frac{Bg^{2/3}(r)}{m_p^2 M^{1/3} r^{1/3}} \propto -r^{-1/3} g^{2/3}(r) \propto -r^{-5/3} \qquad (13)$$

keeping $g \propto r^{-2}$ like in the central region

$\rightarrow$ You have the analog of the external region of an UCO

$\rightarrow$ The proportionality constant in $U(r) \propto -r^{-5/3}$ serves as TUNABLE analog for M

- NB: If we realize this regime *globally*, we obtain an analog for SS!

Outlook: modified dispersion relations

- To understand the instabilities in BEC with healing length ξ, predict how they behave with **modified dispersion relations**:

$$\omega^2 = c_s^2 k^2 (1 + 2k^2\xi^2) \tag{14}$$

- Expected observational signatures (w/o backreaction):

$\rightarrow$ BHM: Growing echo signal for inner light ring instability

$\rightarrow$ RBH: Excess over Hawking spectrum in the UV, concentrated in frequency bands set by κ_-, with a correlation structure set by round trip time between r_- and r_+

Outlook: Experimental realization in Heidelberg

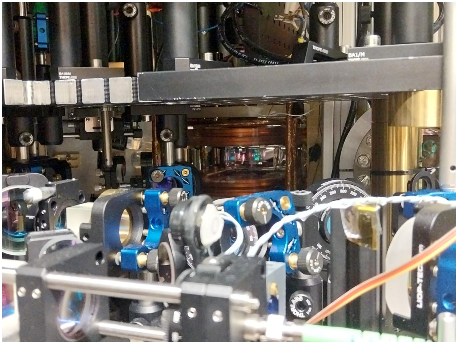

Figure: Photo of the condensate of ultracold potassium-39 atoms in the Atomic Quantum Systems Lab @ Heidelberg. Taken during my visit 4-8 May.

- Observe what happens experimentally, get hints on object post-instability (collaboration with Prof. Oberthaler & co)

Summary

- We have proposed the simulation in analog gravity of *entire regions* of a target gravitational spacetime by putting the sonic and the gravitational metrics in the same form
- This allows us to test for features of the spacetime that do not rely solely on a horizon or ergosphere structure - instabilities of IH or ILR!
- We have shown how to set up a BEC in a way that simulates both the central and the external region of our target UCO spacetime
- (Not in this talk) We have estimates of the expected instability timescales using experimental values from modern BEC experiments

Thank you for your attention!

- The work behind this talk will be on the arxiv in 1-2 weeks
- Check out our paper on the same construction for surface-gravity waves in fluids https://arxiv.org/pdf/2604.08671
- Q&A

- Extra slides

Extra slide: Blueshift instability in bouncing geometries

- Vary $m(r, t)$ with time in a way that turns on and off the trapping $\rightarrow$ bouncing geometry (relaxing the staticity assumption above)

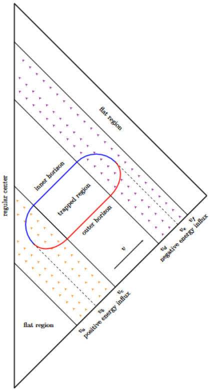

Figure: Penrose diagram of the everywhere regular bouncing geometry with outer and inner apparent horizons and trapping region forming and disappearing in finite time.