ASPECTS OF COSMOLOGICAL COUPLING

MASSIMILIANO RINALDI

Department of Physics - University of Trento
TIFPA - INFN

From Black Holes to the Cosmos:
Matt Visser's journey through space and time

Trieste, August 24th 2026

- Cosmological coupling (CC) is a proposal in GR that a global FLRW cosmology and local regions of relativistic stress within it are strongly coupled.
- The idea is not new: McVittie published an exact solution almost a century ago, which interpolates between a local black hole and large-scale expanding FLRW metric. His solution is, however, pathological.
- The idea revamped after some recent observational indications that some populations of BH masses grow as a^k (D. Farrah et al.). Eventually, if $\kappa = 3$ they dilute as a^{-3}, leading to an emergent DE without fine tuning (see Croker et al.)
- Proposed interpolating models beyond McVittie are generally non-singular (see e.g. Cadoni et al.)

- Cosmological coupling (CC) is a proposal in GR that a global FLRW cosmology and local regions of relativistic stress within it are strongly coupled.
- The idea is not new: McVittie published an exact solution almost a century ago, which interpolates between a local black hole and large-scale expanding FLRW metric. His solution is, however, pathological.
- The idea revamped after some recent observational indications that some populations of BH masses grow as a^k (D. Farrah et al.). Eventually, if $\kappa = 3$ they dilute as a^{-3}, leading to an emergent DE without fine tuning (see Croker et al.)
- Proposed interpolating models beyond McVittie are generally non-singular (see e.g. Cadoni et al.)

- Cosmological coupling (CC) is a proposal in GR that a global FLRW cosmology and local regions of relativistic stress within it are strongly coupled.
- The idea is not new: McVittie published an exact solution almost a century ago, which interpolates between a local black hole and large-scale expanding FLRW metric. His solution is, however, pathological.
- The idea revamped after some recent observational indications that some populations of BH masses grow as a^k (D. Farrah et al.). Eventually, if $\kappa = 3$ they dilute as a^{-3}, leading to an emergent DE without fine tuning (see Croker et al.)
- Proposed interpolating models beyond McVittie are generally non-singular (see e.g. Cadoni et al.)

- Cosmological coupling (CC) is a proposal in GR that a global FLRW cosmology and local regions of relativistic stress within it are strongly coupled.
- The idea is not new: McVittie published an exact solution almost a century ago, which interpolates between a local black hole and large-scale expanding FLRW metric. His solution is, however, pathological.
- The idea revamped after some recent observational indications that some populations of BH masses grow as a^k (D. Farrah et al.). Eventually, if $\kappa = 3$ they dilute as a^{-3}, leading to an emergent DE without fine tuning (see Croker et al.)
- Proposed interpolating models beyond McVittie are generally non-singular (see e.g. Cadoni et al.)

- The complexity of the equations describing solutions that interpolate between compact objects with FLRW universes can be circumvented by studying the general behaviour of event and apparent horizons (MR and V. Faraoni, gr-qc: 2407.14549 and gr-qc: 2608.21222)
- CC can have profound impacts in the evolution of primordial black holes (MR et al astro-ph 2607.13857).
- CC affecting inspiralling SMBH can leave an imprint in the data of Pulsar Timing Array (MR et al. gr-qc: 2409.01801)

- The complexity of the equations describing solutions that interpolate between compact objects with FLRW universes can be circumvented by studying the general behaviour of event and apparent horizons (MR and V. Faraoni, gr-qc: 2407.14549 and gr-qc: 2608.21222)
- CC can have profound impacts in the evolution of primordial black holes (MR et al astro-ph 2607.13857).
- CC affecting inspiralling SMBH can leave an imprint in the data of Pulsar Timing Array (MR et al. gr-qc: 2409.01801)

- The complexity of the equations describing solutions that interpolate between compact objects with FLRW universes can be circumvented by studying the general behaviour of event and apparent horizons (MR and V. Faraoni, gr-qc: 2407.14549 and gr-qc: 2608.21222)
- CC can have profound impacts in the evolution of primordial black holes (MR et al astro-ph 2607.13857).
- CC affecting inspiralling SMBH can leave an imprint in the data of Pulsar Timing Array (MR et al. gr-qc: 2409.01801)

1. Static horizons cannot exist in FLRW
2. How does an horizon evolve, then?
3. Some phenomenology of CC
4. Conclusions

Table of Contents

Static event horizons become null singularities

- The most general spherically symmetric and time-dependent metric can be written as

$$ds^2 = -T^2(t,r)dt^2 + a^2(t,r)\left(dr^2 + r^2 d\Omega^2\right)$$

The areal radius is defined as $R(t,r) = r\,a(t,r)$. The apparent horizon (AH) position is fixed by

$$\boxed{\nabla_\alpha R \nabla^\alpha R = 0}\,.$$

- Null radial geodesics have tangent $u^\alpha = (u^0, u^1, 0, 0)$ that satisfy the geodesic equation

$$\frac{du^\alpha}{d\lambda} + \Gamma^\alpha_{\mu\nu} u^\mu u^\nu = 0\,.$$

Static event horizons become null singularities

- The most general spherically symmetric and time-dependent metric can be written as

$$ds^2 = -T^2(t,r)dt^2 + a^2(t,r)\left(dr^2 + r^2 d\Omega^2\right)$$

The areal radius is defined as $R(t,r) = r\,a(t,r)$. The apparent horizon (AH) position is fixed by

$$\boxed{\nabla_\alpha R \nabla^\alpha R = 0}\,.$$

- Null radial geodesics have tangent $u^\alpha = (u^0, u^1, 0, 0)$ that satisfy the geodesic equation

$$\frac{du^\alpha}{d\lambda} + \Gamma^\alpha_{\mu\nu} u^\mu u^\nu = 0\,.$$

- Combining with normalisation conditions

$$u^\alpha u^\beta g_{\alpha\beta} = 0 \quad \text{and} \quad ds^2 = d\Omega^2 = 0$$

we find

$$\boxed{\frac{d}{d\lambda}\left(\frac{1}{u^0}\right) = \frac{\dot{T}}{T} \pm \frac{2T'}{a} + \frac{\dot{a}}{a}}, \quad \boxed{\frac{d}{d\lambda}\left(\frac{1}{u^1}\right) = \frac{T'}{T} \pm \frac{2\dot{a}}{T} + \frac{a'}{a}}$$

- In the near-horizon region the Schwarzschild metric can be written as the Rindler metric plus small corrections

$$\boxed{ds^2 = ds^2_{\text{Rindler}} + \ldots = -\frac{x^2}{4}dt^2 + 4M^2\left[dx^2 + d\Omega^2\right] + \ldots}$$

where $r = M(1+x)/2$ and $0 < x \ll 1$. The coordinate x measures an arbitrarily small distance from the horizon.

- Combining with normalisation conditions

$$u^\alpha u^\beta g_{\alpha\beta} = 0 \quad \text{and} \quad ds^2 = d\Omega^2 = 0$$

we find

$$\boxed{\frac{d}{d\lambda}\left(\frac{1}{u^0}\right) = \frac{\dot{T}}{T} \pm \frac{2T'}{a} + \frac{\dot{a}}{a}}, \quad \boxed{\frac{d}{d\lambda}\left(\frac{1}{u^1}\right) = \frac{T'}{T} \pm \frac{2\dot{a}}{T} + \frac{a'}{a}}$$

- In the near-horizon region the Schwarzschild metric can be written as the Rindler metric plus small corrections

$$\boxed{ds^2 = ds^2_{\text{Rindler}} + \ldots = -\frac{x^2}{4}dt^2 + 4M^2\left[dx^2 + d\Omega^2\right] + \ldots}$$

where $r = M(1+x)/2$ and $0 < x \ll 1$. The coordinate x measures an arbitrarily small distance from the horizon.

- The equations of motion for u_0 can then be solved

$$u^1 \simeq \pm \frac{x}{4M} u^0 \,, \quad u^0 = \frac{C}{\lambda - \lambda_0} > 0 \,,$$

the positivity condition being required to maintain the vector u future-directed. Thus we have two possibilities:

- $C > 0$, $\lambda > \lambda_0$, which implies

$$\lim_{\lambda \to \infty} u^0 = 0 \,,$$

that is "time stops" while $u^1 \to 0$ and also velocity vanishes.

- $C < 0$, $\lambda < \lambda_0$, which implies

$$\lim_{\lambda \to \lambda_0} u^0 = \infty \,,$$

so the null geodesics stops propagating at a finite λ_0.
Same results are obtained with timelike radial geodesics.

- The equations of motion for u_0 can then be solved

$$u^1 \simeq \pm \frac{x}{4M} u^0 \,, \quad u^0 = \frac{C}{\lambda - \lambda_0} > 0 \,,$$

the positivity condition being required to maintain the vector u future-directed. Thus we have two possibilities:

- $C > 0$, $\lambda > \lambda_0$, which implies

$$\lim_{\lambda \to \infty} u^0 = 0 \,,$$

that is "time stops" while $u^1 \to 0$ and also velocity vanishes.

- $C < 0$, $\lambda < \lambda_0$, which implies

$$\lim_{\lambda \to \lambda_0} u^0 = \infty \,,$$

so the null geodesics stops propagating at a finite λ_0.
Same results are obtained with timelike radial geodesics.

- The equations of motion for u_0 can then be solved

$$u^1 \simeq \pm \frac{x}{4M} u^0 \,, \quad u^0 = \frac{C}{\lambda - \lambda_0} > 0 \,,$$

the positivity condition being required to maintain the vector u future-directed. Thus we have two possibilities:

- $C > 0$, $\lambda > \lambda_0$, which implies

$$\lim_{\lambda \to \infty} u^0 = 0 \,,$$

that is "time stops" while $u^1 \to 0$ and also velocity vanishes.

- $C < 0$, $\lambda < \lambda_0$, which implies

$$\lim_{\lambda \to \lambda_0} u^0 = \infty \,,$$

so the null geodesics stops propagating at a finite λ_0.
Same results are obtained with timelike radial geodesics.

The static event horizon becomes singular

- We found that null geodesics end (cannot originate) on (from) the horizon (the same happens for timelike geodesics).
  This is a clear case of geodesic incompleteness, which usually leads to a singularity.
- A legitimate suspicion is that these results are artefacts of a bad coordinate choice. To clear the issue, we compute the Ricci scalar in the near-horizon limit By matching again with the Rindler near-horizon metric we find

$$\mathcal{R} \simeq \frac{1}{x}$$

  which blows up for $x \to 0$. Thus, forcing a horizon to be static in an expanding universe turns it into a naked singularity.

The static event horizon becomes singular

- We found that null geodesics end (cannot originate) on (from) the horizon (the same happens for timelike geodesics).
  This is a clear case of geodesic incompleteness, which usually leads to a singularity.
- A legitimate suspicion is that these results are artefacts of a bad coordinate choice. To clear the issue, we compute the Ricci scalar in the near-horizon limit By matching again with the Rindler near-horizon metric we find

$$\mathcal{R} \simeq \frac{1}{x}$$

  which blows up for $x \to 0$. Thus, forcing a horizon to be static in an expanding universe turns it into a naked singularity.

Table of Contents

- We consider again the metric

$$ds^2 = -T^2(t,r)dt^2 + a^2(t,r)\left(dr^2 + r^2 d\Omega^2\right)$$

- If the AH exists, its areal radius satisfies

$$\nabla^c R(t,r)\nabla_c R(t,r)\Big|_{\mathrm{AH}} = 0$$

so that

$$R_{\mathrm{AH}}(t) = a\left(t, r_{\mathrm{AH}}(t)\right) r_{\mathrm{AH}}(t)$$

- We find that

$$\frac{1}{R_{\mathrm{AH}}}\frac{dR_{\mathrm{AH}}}{dt} = \frac{1}{M}\frac{dM}{dt} = H_{\mathrm{AH}}\left(1 - \frac{a_{\mathrm{AH}}\dot{r}_{\mathrm{AH}}}{T_{\mathrm{AH}}}\right),$$

where M is the Misner-Sharp-Hernandez mass.

- We consider again the metric

$$ds^2 = -T^2(t,r)dt^2 + a^2(t,r)\left(dr^2 + r^2 d\Omega^2\right)$$

- If the AH exists, its areal radius satisfies

$$\nabla^c R(t,r)\nabla_c R(t,r)\Big|_{\mathrm{AH}} = 0$$

so that

$$R_{\mathrm{AH}}(t) = a\left(t, r_{\mathrm{AH}}(t)\right) r_{\mathrm{AH}}(t)$$

- We find that

$$\frac{1}{R_{\mathrm{AH}}}\frac{dR_{\mathrm{AH}}}{dt} = \frac{1}{M}\frac{dM}{dt} = H_{\mathrm{AH}}\left(1 - \frac{a_{\mathrm{AH}}\dot{r}_{\mathrm{AH}}}{T_{\mathrm{AH}}}\right),$$

where M is the Misner-Sharp-Hernandez mass.

- We consider again the metric

$$ds^2 = -T^2(t,r)dt^2 + a^2(t,r)\left(dr^2 + r^2 d\Omega^2\right)$$

- If the AH exists, its areal radius satisfies

$$\nabla^c R(t,r)\nabla_c R(t,r)\Big|_{\mathrm{AH}} = 0$$

so that

$$R_{\mathrm{AH}}(t) = a\left(t, r_{\mathrm{AH}}(t)\right) r_{\mathrm{AH}}(t)$$

- We find that

$$\frac{1}{R_{\mathrm{AH}}}\frac{dR_{\mathrm{AH}}}{dt} = \frac{1}{M}\frac{dM}{dt} = H_{\mathrm{AH}}\left(1 - \frac{a_{\mathrm{AH}}\dot{r}_{\mathrm{AH}}}{T_{\mathrm{AH}}}\right),$$

where M is the Misner-Sharp-Hernandez mass.

- The solution $R_{\rm AH} = \text{const}$ leads to a static, null event horizon, so it is unphysical. It is obtained when $\dot{r}_{\rm AH} = T_{\rm AH}/a_{\rm AH}$.
- When $\dot{r}_{\rm AH} = 0$ the AH is exactly comoving: then $M(t) = M_0 a(t)$ and

$$\frac{1}{M}\frac{dM}{dt} = \frac{1}{R_{\rm AH}}\frac{dR_{\rm AH}}{dt} = H_{\rm AH}$$

and the MSH mass shows a linear cosmological coupling.
- When the AH forms, it can be slower or faster than comoving. We found only one possible realistic, (qualitative) trajectory.

- The solution $R_{\rm AH} = \text{const}$ leads to a static, null event horizon, so it is unphysical. It is obtained when $\dot{r}_{\rm AH} = T_{\rm AH}/a_{\rm AH}$.
- When $\dot{r}_{\rm AH} = 0$ the AH is exactly comoving: then $M(t) = M_0 a(t)$ and

$$\frac{1}{M}\frac{dM}{dt} = \frac{1}{R_{\rm AH}}\frac{dR_{\rm AH}}{dt} = H_{\rm AH}$$

  and the MSH mass shows a linear cosmological coupling.
- When the AH forms, it can be slower or faster than comoving. We found only one possible realistic, (qualitative) trajectory.

- The solution $R_{\rm AH} = \text{const}$ leads to a static, null event horizon, so it is unphysical. It is obtained when $\dot{r}_{\rm AH} = T_{\rm AH}/a_{\rm AH}$.
- When $\dot{r}_{\rm AH} = 0$ the AH is exactly comoving: then $M(t) = M_0 a(t)$ and

$$\frac{1}{M}\frac{dM}{dt} = \frac{1}{R_{\rm AH}}\frac{dR_{\rm AH}}{dt} = H_{\rm AH}$$

  and the MSH mass shows a linear cosmological coupling.
- When the AH forms, it can be slower or faster than comoving. We found only one possible realistic, (qualitative) trajectory.

Possible evolutions of slower than comoving AH:

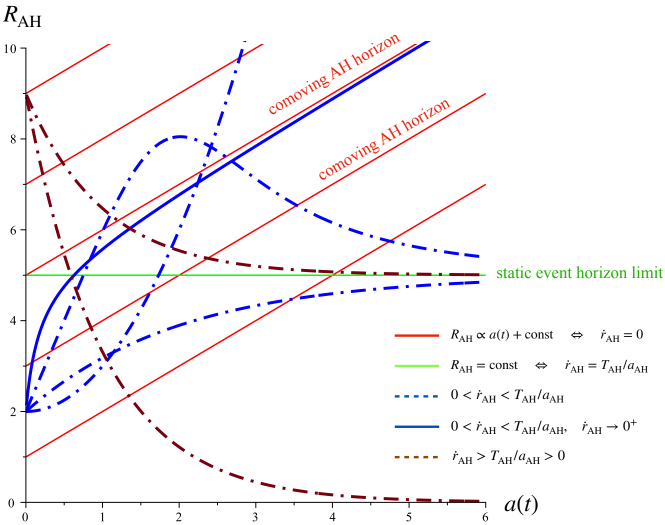


Table of Contents

Effects on Hawking evaporation and PBH as dark matter

Hawking evaporation and growth due to CC are competing processes.

- Assuming that CC switched on at some $z_{\rm CC}$ so that

$$M(z) = M\theta(z - z_{\rm CC}) + M\left(\frac{1 + z_{\rm CC}}{1 + z}\right)^k \theta(z_{\rm CC} - z)$$

  that is $M \propto a^k$ after some $z_{\rm CC}$ and assuming adiabaticity

- we can model the two effects as (MR et al astro-ph 2607.13857)

$$\frac{dM(t)}{dt} = \Gamma_{\rm CC}[M(t)] + \Gamma_{\rm CC}[M(t), t]$$

  where

$$\Gamma_{\rm CC} = -\frac{\mathcal{C}_{\rm HR}}{M(t)^2}, \qquad \Gamma_{\rm CC}[M(t), t] = kH(t)M(t)$$

Effects on Hawking evaporation and PBH as dark matter

Hawking evaporation and growth due to CC are competing processes.

- Assuming that CC switched on at some $z_{\rm CC}$ so that

$$M(z) = M\theta(z - z_{\rm CC}) + M \left(\frac{1 + z_{\rm CC}}{1 + z}\right)^k \theta(z_{\rm CC} - z)$$

  that is $M \propto a^k$ after some $z_{\rm CC}$ and assuming adiabaticity

- we can model the two effects as (MR et al astro-ph 2607.13857)

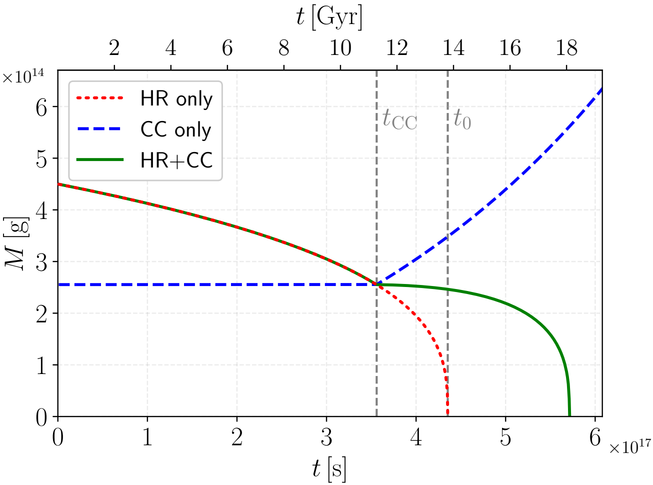
$$\frac{dM(t)}{dt} = \Gamma_{\rm CC}[M(t)] + \Gamma_{\rm CC}[M(t), t]$$

  where

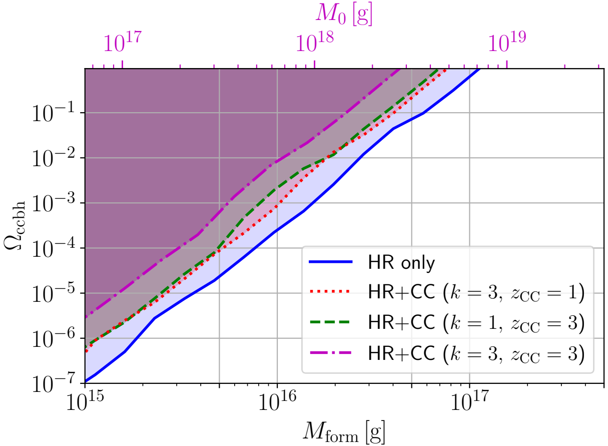
$$\Gamma_{\rm CC} = -\frac{\mathcal{C}_{\rm HR}}{M(t)^2}, \qquad \Gamma_{\rm CC}[M(t), t] = kH(t)M(t)$$

Effects on Hawking evaporation and PBH as dark matter

Evolution of a primordial BH with initial mass 4.5×10^{14} g:



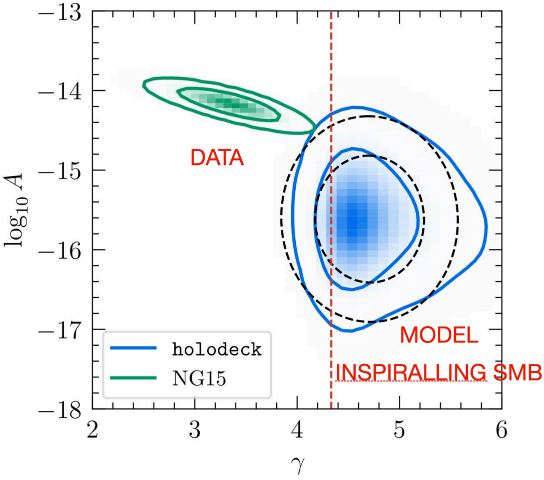
Effects on Hawking evaporation and PBH as dark matter



If dark matter is made of primordial black holes, cosmological coupling can alter the mass window.

Effects on Pulsar Timing Array

Pulsar Timing Array (PTA) experiments like NanoGrav are designed to measure the stochastic background of gravitational waves by means of pulsar signal distortions. If the background is generated by merging supermassive black holes there is a problem with data.



Effects on Pulsar Timing Array

- Cosmological coupling can leave an imprint on PTA data on the stochastic gravitational wave background (SGWB), when attributed to the merger of supermassive black holes. The standard amplitude is:

$$A^2 = \frac{4(G\mathcal{M})^{5/3}}{3\pi^{1/3}H_0}\int_{z_{\min}}^{z_{\max}} \frac{\dot{n}(z)dz}{(1+z)^{4/3}E(z)}$$

- Being the inspiraling phase very long, the mass growth can significantly alter the amplitude of SGWB through an increasing chirp mass $\mathcal{M}$. Assume Croker's growth model ($-3 < k < 3$)

$$M(z) = M_i\Theta(z - z_i) + M_i\left(\frac{1+z_i}{1+z}\right)^k\Theta(z_i - z)$$

Effects on Pulsar Timing Array

- Cosmological coupling can leave an imprint on PTA data on the stochastic gravitational wave background (SGWB), when attributed to the merger of supermassive black holes. The standard amplitude is:

$$A^2 = \frac{4(G\mathcal{M})^{5/3}}{3\pi^{1/3}H_0}\int_{z_{\min}}^{z_{\max}} \frac{\dot{n}(z)dz}{(1+z)^{4/3}E(z)}$$

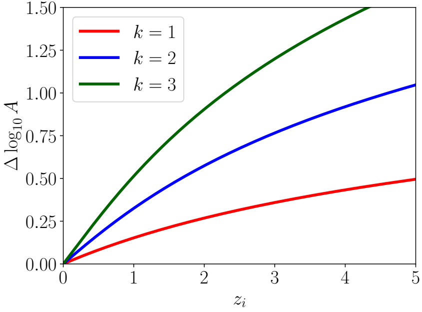
- Being the inspiraling phase very long, the mass growth can significantly alter the amplitude of SGWB through an increasing chirp mass $\mathcal{M}$. Assume Croker's growth model ($-3 < k < 3$)

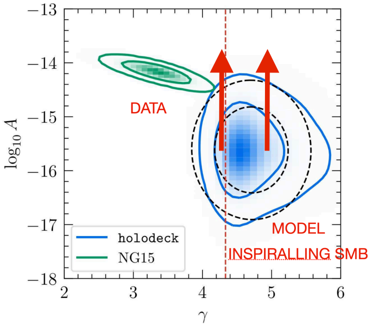
$$M(z) = M_i\Theta(z - z_i) + M_i\left(\frac{1+z_i}{1+z}\right)^k \Theta(z_i - z)$$

Effects on Pulsar Timing Array

The amplitude changes according to

$$A^2 = \frac{4(G\mathcal{M})^{5/3}}{3\pi^{1/3}H_0}(z_i+1)^{5k/3}\int_{z_{\rm min}}^{z_{\rm max}} dz\,\frac{\dot n(z)}{(1+z)^{(5k+4)/3}E(z)}$$





M. Calzà, F. Gianesello, M.R:, S. Vagnozzi, *Sci.Rep. 14 (2024) 31296*

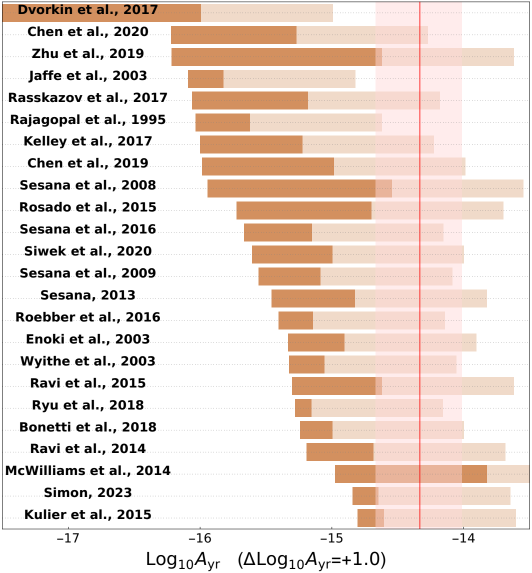


Table of Contents

Conclusions

- An exactly static black hole event horizon embedded in the time-dependent geometry generates a naked null singularity located at a finite areal radius. All radial null and timelike geodesics are future-inextensible at this static horizon leading to geodesic incompleteness, which define spacetime singularities (Hawking-Penrose theorems). This is confirmed by a divergent Ricci scalar.
- Scale separation is not sufficient: static horizon in expanding universes cause catastrophic effects no matter the magnitude of the expansion rate!
- The evolution of apparent horizons seems to be generic: in the long term they become comoving.
- CC has an impact in modelling DM as primordial black holes. It also affects PTA signals.

Conclusions

- An exactly static black hole event horizon embedded in the time-dependent geometry generates a naked null singularity located at a finite areal radius. All radial null and timelike geodesics are future-inextensible at this static horizon leading to geodesic incompleteness, which define spacetime singularities (Hawking-Penrose theorems). This is confirmed by a divergent Ricci scalar.
- Scale separation is not sufficient: static horizon in expanding universes cause catastrophic effects no matter the magnitude of the expansion rate!
- The evolution of apparent horizons seems to be generic: in the long term they become comoving.
- CC has an impact in modelling DM as primordial black holes. It also affects PTA signals.

Conclusions

- An exactly static black hole event horizon embedded in the time-dependent geometry generates a naked null singularity located at a finite areal radius. All radial null and timelike geodesics are future-inextensible at this static horizon leading to geodesic incompleteness, which define spacetime singularities (Hawking-Penrose theorems). This is confirmed by a divergent Ricci scalar.
- Scale separation is not sufficient: static horizon in expanding universes cause catastrophic effects no matter the magnitude of the expansion rate!
- The evolution of apparent horizons seems to be generic: in the long term they become comoving.
- CC has an impact in modelling DM as primordial black holes. It also affects PTA signals.

Conclusions

- An exactly static black hole event horizon embedded in the time-dependent geometry generates a naked null singularity located at a finite areal radius. All radial null and timelike geodesics are future-inextensible at this static horizon leading to geodesic incompleteness, which define spacetime singularities (Hawking-Penrose theorems). This is confirmed by a divergent Ricci scalar.
- Scale separation is not sufficient: static horizon in expanding universes cause catastrophic effects no matter the magnitude of the expansion rate!
- The evolution of apparent horizons seems to be generic: in the long term they become comoving.
- CC has an impact in modelling DM as primordial black holes. It also affects PTA signals.

THANK YOU FOR YOUR ATTENTION!