Semiclassical Black Hole $\to$ White Hole Transition: An Analytic Treatment

Massimiliano Spadafora

with Julio Arrechea and Stefano Liberati

SISSA · Astroparticle and gravitational physics (APG) group

From Black Holes to the Cosmos: Matt Visser's journey through space and time

The problem

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Possibility of non trivial solutions (Hawking process, White-holes, Wormholes...).

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Inner horizons

Generic whenever there is charge, rotation, or a regular core.

Mass inflation and semiclassical-instability.

Trapped and anti-trapped regions: the language of the talk

Null expansions $\theta_\pm = \frac{1}{A}\frac{dA}{d\lambda_\pm} \xrightarrow{\text{sph. sym.}} \frac{2}{r}\frac{dr}{d\lambda_\pm}$

Region	θ_+	θ_-
Untrapped	>0	<0
Trapped	<0	<0
Anti-trapped	>0	>0

- Trapped $\Leftrightarrow$ BH interior.
- Anti-trapped $\Leftrightarrow$ WH interior.
- In *dynamical* spacetimes: quasi-local **trapping horizons** ($\theta_\pm = 0$), not event horizons.

Horizon	θ_+	θ_-	$\mathcal{L}_n\theta$
FOTH (outer, BH)	0	<0	$\mathcal{L}_n\theta_+ < 0$
FITH (inner, BH)	0	<0	$\mathcal{L}_n\theta_+ > 0$
POTH (outer, WH)	>0	0	$\mathcal{L}_n\theta_- < 0$
PITH (inner, WH)	>0	0	$\mathcal{L}_n\theta_- > 0$

Future horizons (FOTH, FITH) $\leftrightarrow$ BH;
past horizons (POTH, PITH) $\leftrightarrow$ WH.

How a region disappears – and how a new one is born

$$\frac{d\theta_\pm}{d\lambda_\pm} = -\tfrac{1}{2}\theta_\pm^2 - 8\pi\, T_{ab}k^a k^b \qquad \text{(Raychaudhuri)}$$

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Classical matter obeys the **NEC**: $T_{ab}k^a k^b \geq 0$. The RSET generically *violates* it.

Role for each flux

$$d\theta_+/d\lambda_+ \supset -8\pi\langle T_{--}\rangle, \quad d\theta_-/d\lambda_- \supset -8\pi\langle T_{++}\rangle$$

Starting from an **untrapped** region ($\theta_+ > 0,\ \theta_- < 0$):

- $\langle T_{--}\rangle > 0 \Rightarrow \theta_+ \downarrow \to 0$: **trapped region**.
- $\langle T_{--}\rangle < 0 \Rightarrow \theta_+ \uparrow$: no trapping.
- $\langle T_{++}\rangle < 0 \Rightarrow \theta_- \uparrow \to 0$: **anti-trapped region**.
- $\langle T_{++}\rangle > 0 \Rightarrow \theta_- \downarrow$: no anti-trapping.

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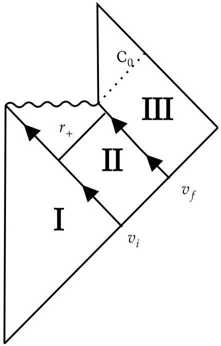
2. Birth of a new region. If $\langle T_{++}\rangle$ violates the NEC, θ_- can be driven through zero: **an anti-trapped region may form** inside the trapped one.

Nothing here is specific to a given metric: only the *signs* of the two fluxes matter.

Evaporation with an inner horizon is inside-out

Standard picture (Schwarzschild): Hawking radiation slowly shrinks the *outer* horizon; lifetime $t_{\text{evap}} \sim \mathcal{O}(M^3)$.



Evanescent Schwarzschild: one horizon r_+, no inner horizon, Hawking process.

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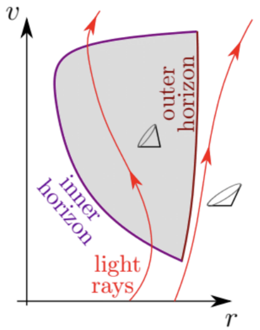
With an inner horizon r_- (charge, rotation, regular core):

- Ingoing flux $\propto -\kappa_h^2$ on *both* horizons.
- $\kappa_+ > 0$: FOTH contracts. $\kappa_- < 0$: FITH expands.

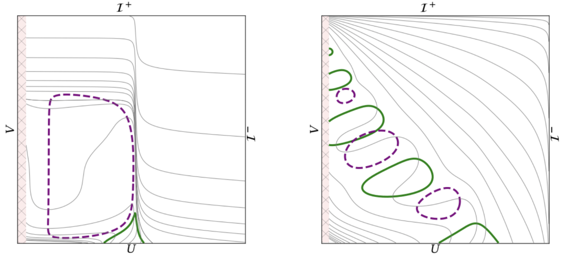
The key message

The trapped region is eaten **from the inside out**, and disappears in *finite* time when the two horizons merge. Not the Hawking process: faster, inner-horizon driven, **no extremal remnant**.



Semiclassical evolution of the trapped region: inner horizon out, outer horizon in [1].

What the numerical simulations show



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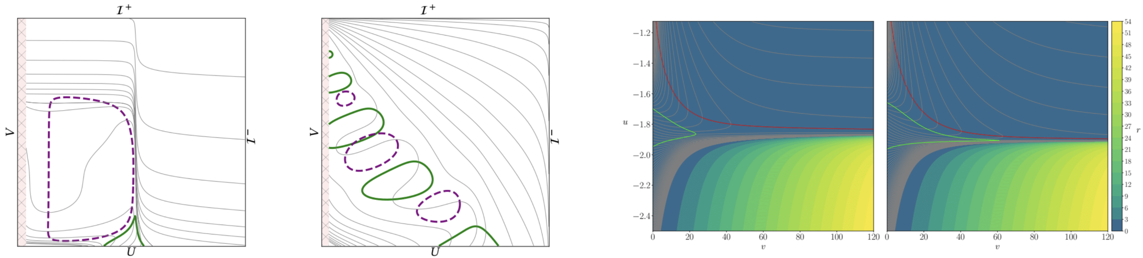

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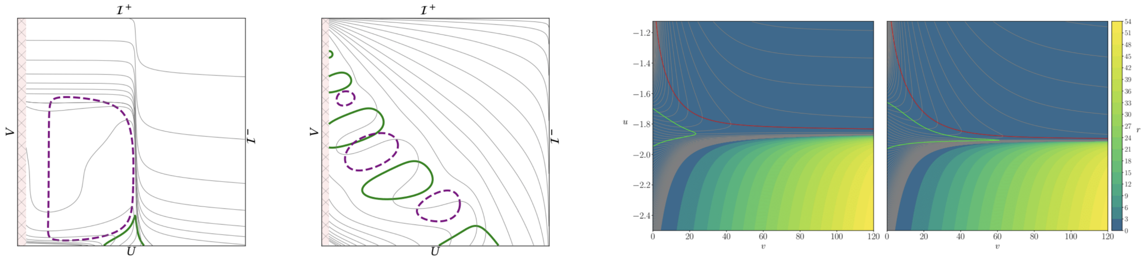

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Independent codes, different models, same qualitative story. Why?

Aim and main result

Central claim of this work

The $|in\rangle$ vacuum **generically** evaporates the trapped region *and* dynamically generates an anti-trapped region, in any BH with an inner and an outer horizon. Both statements follow from a **fully analytic** computation.

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Questions we can answer analytically:

- Which mechanism creates the WH region?
- Why does it evaporate for regular BHs but not for Reissner–Nordström?
- *Universal* or *model-dependent*?
- Self-propelled cascade?

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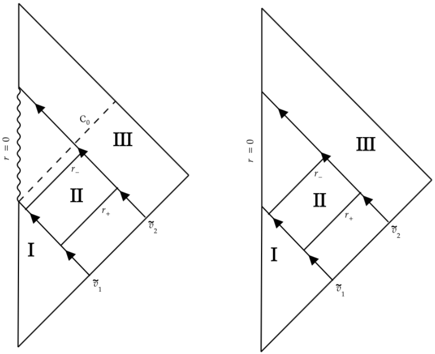
Procedure:

- Select a spacetime.
- Compute the RSET on it.
- Put it in the Raychaudhuri and see how the expansions evolve (insight on the backreaction).

Toy model: evanescent black hole

Two ingoing null shells ($+M$ then $-M$) at $\tilde{v}_1, \tilde{v}_2$:

Region	Metric	Content
I ($v < \tilde{v}_1$)	$-du_1\,dv$	flat, $\lvert in\rangle$ vacuum
II ($\tilde{v}_1 < v < \tilde{v}_2$)	$-f(r)\,du_2\,dv$	trapped region
III ($v > \tilde{v}_2$)	$-du_3\,dv$	post-evaporation



Backgrounds:

- Reissner–Nordström: $f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$
- Bardeen: $f = 1 - \frac{2M}{r}\left(\frac{r}{\sqrt{r^2+\ell^2}}\right)^3$
- Minkowski core, AdS-core Bardeen

All share $r_- < r_+$, with $\kappa_+ > 0$, $\kappa_- < 0$.

Framework

$$G_{\mu\nu} = 8\pi\left(T^{\mathrm{eff}}_{\mu\nu} + \hbar\,\langle\hat{T}_{\mu\nu}\rangle_{\mathrm{ren}}\right) \qquad \text{with} \qquad \boxed{\langle\hat{T}_{\mu\nu}\rangle_{4D} \simeq \frac{\langle\hat{T}_{ab}\rangle_{2D}}{4\pi r^2}\,\delta^a_\mu\delta^b_\nu}$$

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Polyakov (s-wave) approximation

Massless scalar, keep only $\ell = 0$, neglect backscattering (harmless near horizons, $V_\ell \to 0$).

The 2D RSET contains at most **second** derivatives of $f(r) \Rightarrow$ backreaction becomes tractable.

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The $|in\rangle$ state

Vacuum w.r.t. the Minkowski null coordinates of the far past: *no particles before collapse*.

Crossing a null shell, the state-dependent part of the RSET is fixed by a **Schwarzian derivative** of the matching map:

$$\langle in|\,\hat{T}_{ab}\,|in\rangle = \langle B|\,\hat{T}_{ab}\,|B\rangle - \tfrac{1}{24\pi}\{u_1, u_i\}\,\delta_{au_i}\delta_{bu_i}$$

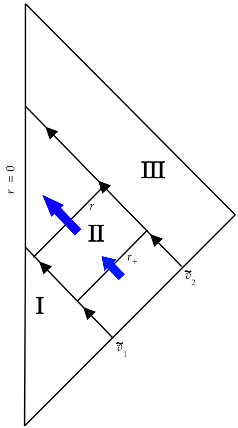
Regularity on future (past) horizons requires $\langle T_{uu}\rangle \sim f^2$ ($\langle T_{vv}\rangle \sim f^2$) – checked in all cases. [details in backup]

Ingoing flux: universal, and it drives the inside-out evaporation

On any trapping horizon

$$\boxed{\langle T_{vv}\rangle^{2D}_{\mathrm{phys}}\Big|_{r_h} = -\frac{\kappa_h^2}{48\pi}}$$

- **Universal negative sign**, for every $f(r)$.
- Depends *only* on the surface gravity.
- Constant in v; vanishes after evaporation.
- $\Rightarrow$ FOTH $\downarrow$, FITH $\uparrow$: the trapped region shrinks **from both sides**, faster from the inside.

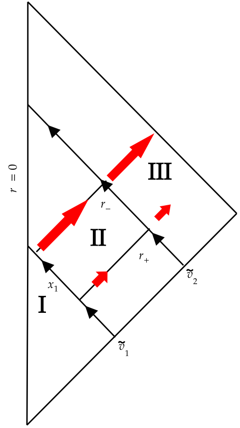


Outgoing flux: exponentially amplified at the inner horizon

On any trapping horizon

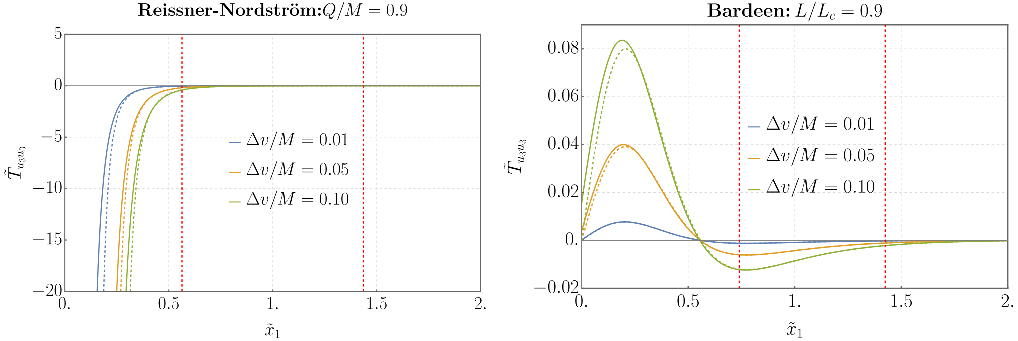
$$\langle T_{uu}\rangle^{2D}_{\rm phys}\Big|_{r_h} \propto \frac{f^{(3)}(r_h)}{\kappa_h}\left[1 - e^{-2\kappa_h(v-\tilde{v}_1)}\right]$$

- **Non-universal sign**: set by $f^{(3)}(r_h)$, i.e. by the *gradient of the curvature*.
- FOTH ($\kappa_h > 0$): saturates to a constant.
- **FITH ($\kappa_h < 0$):** $\langle T_{uu}\rangle \sim e^{2|\kappa_-|\Delta v}$



Terminology: this is a *semiclassical inner-horizon (blueshift) amplification* of the vacuum flux – distinct from mass inflation, which is a classical, nonlinear effect of infalling matter.

The amplified profile: RN vs regular black holes



Universal vs model-dependent

RN (left): outgoing flux always negative, diverging at the Cauchy horizon. **Bardeen** (right): negative, then *positive* and larger. The amplification is **universal**; the *sign structure* of the seed is set by the core, i.e. by $f^{(3)}(r)$ – and it decides the fate of the WH region.

Backreaction: feeding the fluxes into Raychaudhuri

Outgoing expansion θ_+ (sourced by T_{vv})

$$\left.\frac{d\theta_+}{dv}\right|_{r_h} \simeq -\frac{8\pi}{r_h^2}\langle T_{vv}^{(2D)}\rangle = \frac{\kappa_h^2}{6r_h^2} > 0$$

$\Rightarrow$ horizons merge at a finite $v_{\rm extr}$: the trapped region closes. (We take $\tilde{v}_2 \equiv v_{\rm extr}$.)

Ingoing expansion θ_- (sourced by T_{uu})

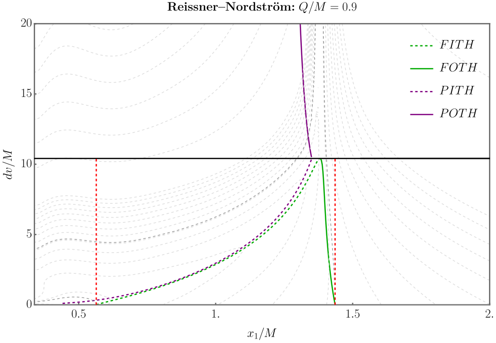
$$\frac{d\theta_-}{dx_1} = \frac{f(r)}{f(x_1)}\left[\theta_-^2 + \frac{16\pi}{r^2}\frac{\langle T_{u_2u_2}^{(2D)}\rangle}{f(r)^2}\right]$$

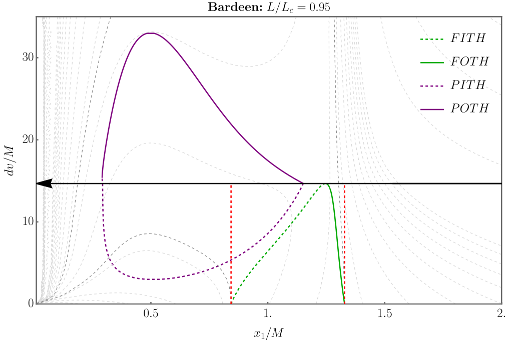
Start from $\theta_- = -1/r$. A *negative* T_{uu} pushes θ_- up through zero $\Rightarrow$ **anti-trapped region**.

A *positive* T_{uu} pushes it back $\Rightarrow$ the WH region evaporates.

Transverse fluxes kill a region; parallel fluxes create the next one.

The anti-trapped region: unbounded (RN) vs finite-lived (regular)

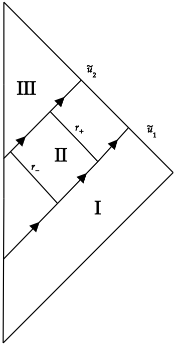




Conjecture

RN (left): the anti-trapped region grows without bound. **Regular cores** (right): PITH and POTH merge, **finite WH lifetime**. Any BH with inner *and* outer horizons generates an anti-trapped region; **regularity of the core** is what lets the WH evaporate in turn.

The white hole: same computation, roles exchanged



Two *outgoing* shells; region II is anti-trapped. Everything follows from $v \leftrightarrow -u$.

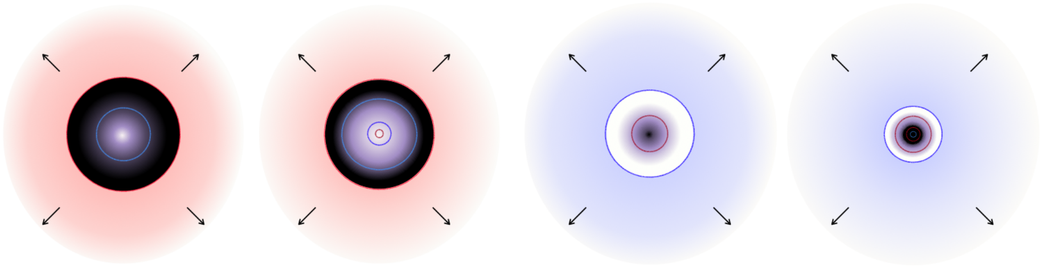
BH / WH dictionary

	Evanescent BH	Evanescent WH
Evaporating flux	$T_{vv} = -\kappa_h^2/48\pi$	$T_{uu} = -\kappa_h^2/48\pi$
Amplified flux	$T_{uu} \sim e^{2\lvert\kappa_-\rvert\Delta v}$	$T_{vv} \sim e^{2\kappa_+\Delta u}$
Amplifier	inner horizon r_-	outer horizon r_+
Triggers	WH formation	new BH formation

The exponentially amplified *ingoing* flux of the WH, if positive and large enough, drives θ_+ through zero: a **new trapped region** is seeded.

(Semiclassical counterpart of the classical Eardley instability of WH exteriors.)

A self-propelled cascade



Each region radiates while seeding the next one $\Rightarrow$ the cascade is **naturally damped**. Plausible end-state: a horizon-free (bouncing) geometry, *with no genuine quantum-gravity dynamics invoked*.

Conclusions, limitations, next steps

What we have shown

- Fully **analytic** $|in\rangle$ RSET for evanescent BHs and WHs.
- Transverse fluxes $\Rightarrow$ inside-out evaporation; parallel fluxes, exponentially amplified, $\Rightarrow$ birth of the next region.
- Amplification is **universal**; sign structure (hence WH lifetime) is **model-dependent**.
- Reproduces the numerics of [2, 3] with no fitting.

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Next:

- Self-consistent evolution (RSET in Vaidya spacetime).
- Beyond Polyakov: greybody factors, higher multipoles.
- Quantitative cascade lifetimes; Kerr.

Thank you!

References I

C. Barceló, V. Boyanov, R. Carballo-Rubio, L. J. Garay, *Classical mass inflation versus semiclassical inner horizon inflation*, Phys. Rev. D **106** (2022) 124006, arXiv:2203.13539.

J. Barenboim, A.V. Frolov, G. Kunstatter, *Evaporation of regular black holes in 2D dilaton gravity*, 2025.

V. Boyanov, D. Hilditch, A. Semião, *Semiclassical evolution of a dynamically formed spherical black hole with an inner horizon*, 2025.

F. Di Filippo, *Radiating Black Holes in General Relativity Need Not Be Singular*, Phys. Rev. Lett. **136** (2026) 161402.

W.A. Hiscock, *Models of Evaporating Black Holes*, Phys. Rev. D **23** (1981) 2813.

A. Fabbri, J. Navarro-Salas, *Modeling Black Hole Evaporation*, World Scientific (2005).

Backup slides

Phenomenology: timescales and observables

Lifetime of the trapped region

	Hawking	inner-horizon driven
scaling	$\mathcal{O}(M^3)$	$\mathcal{O}(M^n)$, $n \gtrsim 2$
$M = M_\odot$	$\sim 10^{63}$ yr	$\sim 10^{25}$ yr
explodes today	$M \sim 10^{12}$ kg	$M \sim 10^{23}$ kg

- Still *far* longer than the age of the Universe for astrophysical masses: **longevity is safe**.
- But $\mathcal{O}(M^2) \ll$ Page time: the trapped region opens **before** the Page time [4].
- The PBH mass window for bursts shifts by ~ 11 orders of magnitude.

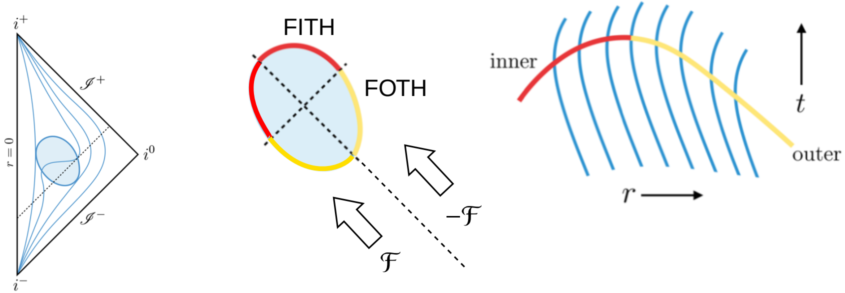
Potential observables

- At the end of each phase the stored energy is released as a **highly collimated pulse** at $u_3 = \tilde{v}_2 - 2r_h$.
- The cascade sends an alternating sequence of *negative and positive* fluxes to $\mathscr{I}^+$, with sign structure fixed by $f^{(3)}(r)$, i.e. by the **core**.

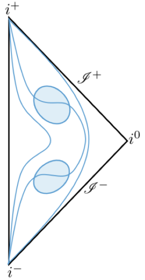
Take-home

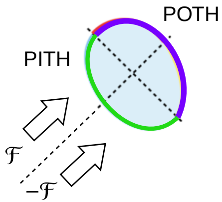
The structure of the core is **no longer unobservable**: it imprints itself on the radiation reaching infinity.

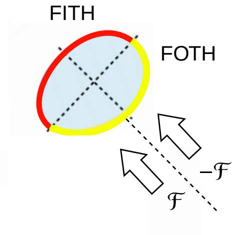
Backup: trapped region in detail



Backup: bouncing geometry in detail



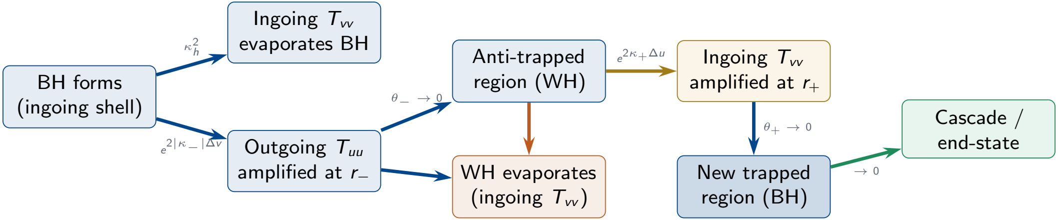




Spacelike vs timelike horizons

A trapping horizon is spacelike (timelike) when the energy flux crossing it is positive (negative) — i.e. when the NEC is satisfied (violated).

Backup: the cascade, step by step



Backup: the 2D Polyakov RSET

In conformal coordinates $ds^2 = -e^{2\sigma} dx^+ dx^-$:

$$\langle \Psi | \hat{T}_{\pm\pm} | \Psi \rangle^{2D} = \underbrace{-\frac{1}{12\pi}\left[(\partial_\pm \sigma)^2 - \partial_\pm^2 \sigma\right]}_{\text{geometric term } \Theta_{\pm\pm}(\sigma)} \underbrace{- \frac{t_\pm(x^\pm)}{12\pi}}_{\text{state-dependent term}}$$

Geometric part

Determined solely by the conformal factor σ. Corresponds to the zero-temperature **Boulware vacuum**: $t_\pm = 0$.

State-dependent part

$t_\pm(x^\pm)$ are freely specifiable — they encode the choice of vacuum (Boulware, Unruh, Hartle–Hawking, $|in\rangle$, ...).

Backup: two-dimensional RSET, explicit form

In $ds^2 = -e^{2\rho(u,v)}\, du\, dv$:

$$\langle \hat{T}_{uu} \rangle = \underbrace{-\frac{1}{12\pi}\left(\partial_u^2\rho - (\partial_u\rho)^2\right)}_{\text{geometric}} \underbrace{-\frac{t_u(u)}{12\pi}}_{\text{state}}$$

$$\langle \hat{T}_{vv} \rangle = \underbrace{-\frac{1}{12\pi}\left(\partial_v^2\rho - (\partial_v\rho)^2\right)}_{\text{geometric}} \underbrace{-\frac{t_v(v)}{12\pi}}_{\text{state}}$$

$$\langle \hat{T}_{uv} \rangle = \underbrace{-\frac{1}{12\pi}\partial_u\partial_v\rho}_{\text{geometric}} \propto R^{2D}$$

Note: $ds^2 = -f\, dt^2 + dr^2/f = -f\, du\, dv$ only for **static** spacetimes.

Choice of the state

- Boulware $\to t_u = t_v = 0$
- General vacuum (quantization w.r.t. (u_i, v_i)):

$$t_{u_i}(u) = \{u_i, u\}, \quad t_{v_i}(v) = \{v_i, v\}$$

with $\{f, x\} = \frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2$

Backup: changing the quantum state

Under a change of null coordinates $\tilde{x}_{\tilde{a}} \to x_a$:

$$\langle x_\pm | \hat{T}_{\tilde{a}\tilde{b}} | x_\pm \rangle = \Theta_{\tilde{a}\tilde{b}}(\tilde{\sigma}) - \frac{1}{24\pi} \{x_a, \tilde{x}_{\tilde{a}}\} \, \delta_{ab} \, \delta_{\tilde{a}\tilde{b}}$$

How to read this

- $\Theta_{\tilde{a}\tilde{b}}$: geometric (Boulware) part, fixed by the metric.
- $\{x_a, \tilde{x}_{\tilde{a}}\}$: Schwarzian of the coordinate map — encodes the **state**.

State dictionary

State	x_+	x_-
Boulware	u	v
Unruh	$U = -e^{-\kappa u}$	v
HH	$U = -e^{-\kappa u}$	$V = e^{\kappa v}$
$\lvert in \rangle$	u_1	v

Finiteness in the ingoing (outgoing) Eddington–Finkelstein frame requires:

Future (BH) horizons:

$$\langle T_{uu}\rangle^{\mathrm{BH}}_{\mathrm{phys}} = \frac{\langle\Psi|\hat{T}_{uu}|\Psi\rangle}{f^2} < \infty,$$

$$\langle T_{uv}\rangle^{\mathrm{BH}}_{\mathrm{phys}} = \frac{\langle\Psi|\hat{T}_{uv}|\Psi\rangle}{f} < \infty,$$

$$\langle T_{vv}\rangle^{\mathrm{BH}}_{\mathrm{phys}} = \langle\Psi|\hat{T}_{vv}|\Psi\rangle < \infty.$$

Past (WH) horizons:

$$\langle T_{uu}\rangle^{\mathrm{WH}}_{\mathrm{phys}} = \langle\Psi|\hat{T}_{uu}|\Psi\rangle < \infty,$$

$$\langle T_{uv}\rangle^{\mathrm{WH}}_{\mathrm{phys}} = \frac{\langle\Psi|\hat{T}_{uv}|\Psi\rangle}{f} < \infty,$$

$$\langle T_{vv}\rangle^{\mathrm{WH}}_{\mathrm{phys}} = \frac{\langle\Psi|\hat{T}_{vv}|\Psi\rangle}{f^2} < \infty.$$

Free-falling observer

$$\rho^{\mathrm{f.f.}}_{\mathrm{obs}} = (1+\alpha)^2(T_{uu})^{\mathrm{BH}}_{\mathrm{phys}} + 2(T_{uv})^{\mathrm{BH}}_{\mathrm{phys}} + \tfrac{(1-\alpha)^2}{f^2}(T_{vv})^{\mathrm{BH}}_{\mathrm{phys}}, \qquad \alpha = \sqrt{1-f(r)}.$$

Backup: strategy

1. **Construct a toy spacetime** using *static* metrics glued along null shells.

   Why static metrics?

   Exact analytic RSETs are available only in static backgrounds. The shell model still captures the causal structure of collapse and evaporation.

2. **Compute the RSET** in the $|in\rangle$ vacuum with the 2D Polyakov approximation.
3. **Analyse the backreaction**: feed $\langle \hat{T}_{\mu\nu} \rangle$ into Raychaudhuri and track $\theta_{\pm}$.

Backup: Israel matching and the $|in\rangle$ vacuum

Matching conditions

$$\frac{du_2}{du_1} = \frac{1}{f(x_1)}, \qquad \frac{du_2}{du_3} = \frac{1}{f(x_2)}$$

with x_1, x_2 the shell-crossing radii of a given outgoing ray.

$|in\rangle$ vacuum

$t_{u_1}(u_1) = 0$, $t_v(v) = 0$: no particles in the far past.

Consequence

Crossing a shell, the state part transforms via Schwarzians:

$$t_{u_2} = \{u_1, u_2\}, \qquad t_{u_3} = \{u_1, u_3\}$$

$$\{u_1, u_2\} = -\tfrac{1}{8}B(x_1),\ \{u_1, u_3\} = -\tfrac{1}{8}\frac{B(x_1)-B(x_2)}{f(x_2)^2}$$

Backup: RSET of the evanescent BH, region by region

Region II (inside the BH):

$$\langle T_{vv}\rangle^{\rm BH}_{\rm phys} = -\frac{B(r)}{192\pi}$$

$$\langle T_{u_2u_2}\rangle^{\rm BH}_{\rm phys} = \frac{B(x_1) - B(r)}{192\pi\, f(r)^2}$$

$$\langle T_{u_2v}\rangle^{\rm BH}_{\rm phys} = \frac{f''(r)}{96\pi}$$

Region III (post-evaporation):

$$\langle T_{u_3u_3}\rangle^{\rm BH}_{\rm phys} = \frac{B(x_1) - B(x_2)}{192\pi\, f(x_2)^2}$$

$$\langle T_{vv}\rangle^{\rm BH}_{\rm phys} = 0$$

The function $B(r)$

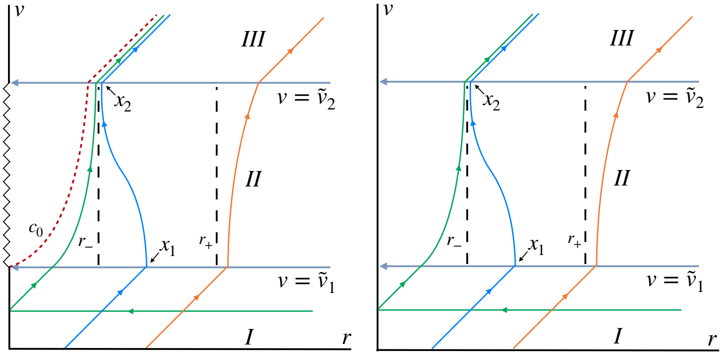
$$B(r) = f'(r)^2 - 2f(r)f''(r)$$

On a non-degenerate horizon $B(r_h) = 4\kappa_h^2$.

$\langle T_{vv}\rangle = 0$ in region III: no ingoing flux survives once the horizons are gone — the exterior settles back to flat space.

Backup: ray tracing in the interior

$$\langle T_{u_2 u_2} \rangle_{\text{phys}} = \frac{1}{f^2(r)} \int_{x_1}^{r(u_2,v)} f(r')\, f^{(3)}(r')\, dr'$$



Backup: analytic limits (BH)

Early times ($\Delta v \to 0$)

$$\langle T_{u_2 u_2} \rangle_{\mathrm{phys}} \approx \frac{f^{(3)}(x_1)}{192\pi} (v - \tilde{v}_1)$$

- Sign set by $f^{(3)}(r)$: model-dependent.
- Energy = integral of the curvature variation along the ray.

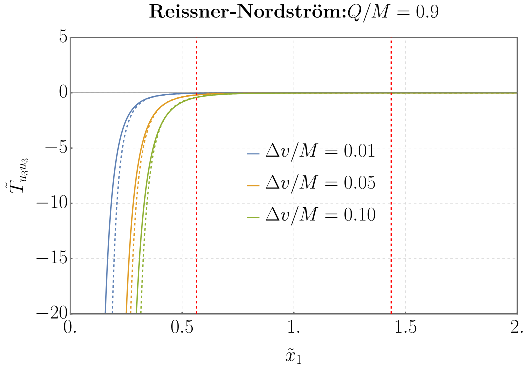
Late times ($\Delta v \to \infty$), interior

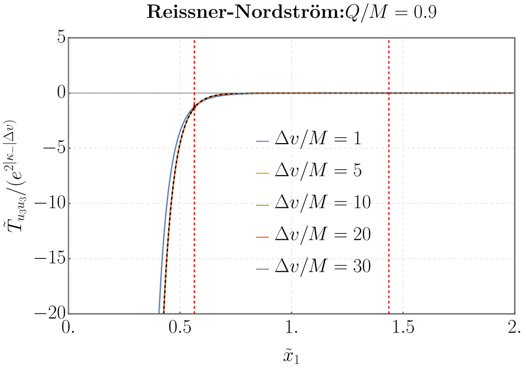
$$\langle T_{u_2 u_2} \rangle \approx \frac{f^{(3)}(r_-)}{384\pi\kappa_-} + C(x_1)\, e^{2|\kappa_-|(v-\tilde{v}_1)}$$

$$C(x_1) = \frac{1}{192\pi r_-^2} \left(\frac{B(x_1)}{4\kappa_-^2} - 1 \right) e^{4|\kappa_-| R_{\mathrm{BH}}(x_1)}$$

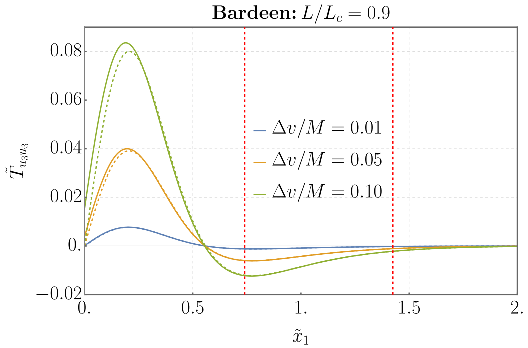
- Controlled by the *inner* horizon κ_-.

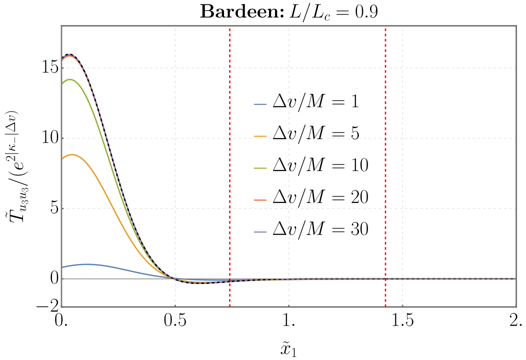
Backup: flux profiles, RN (short and long lifetimes)



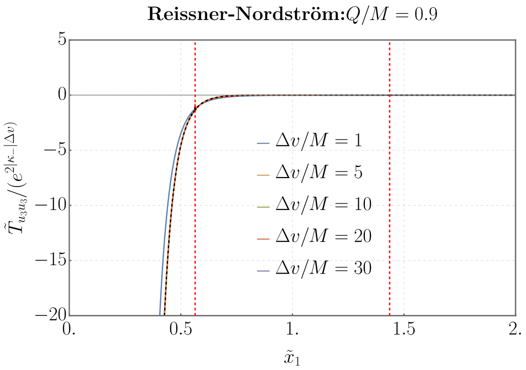


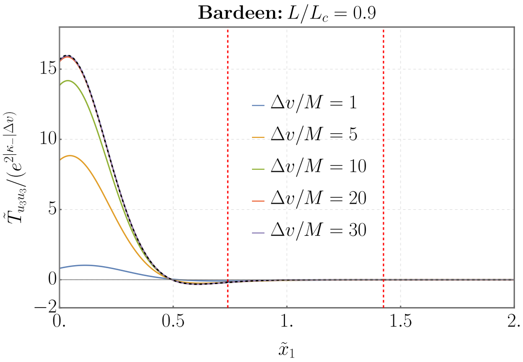
Backup: flux profiles, Bardeen (short and long lifetimes)





Backup: asymptotic coefficient, RN vs Bardeen

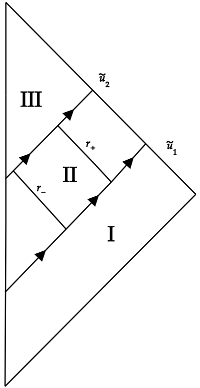




Backup: spacetime model, evanescent white hole

Outgoing shells at $\tilde{u}_1, \tilde{u}_2$; region II is anti-trapped:

Region	Metric	Content
I $(u < \tilde{u}_1)$	$-du\,dv_1$	flat, $\lvert in \rangle$ vacuum
II $(\tilde{u}_1 < u < \tilde{u}_2)$	$-f(r)\,du\,dv_2$	anti-trapped region
III $(u > \tilde{u}_2)$	$-du\,dv_3$	post-evaporation



Israel matching:

$$\left.\frac{dv_2}{dv_1}\right|_{\tilde{u}_1} = \frac{1}{f(y_1)}, \qquad \left.\frac{dv_2}{dv_3}\right|_{\tilde{u}_2} = \frac{1}{f(y_2)}$$

Backup: RSET of the white hole

Region II (inside the WH):

$$\langle T_{uu}\rangle^{\mathrm{WH}}_{\mathrm{phys}} = -\frac{B(r)}{192\pi}$$

$$\langle T_{v_2v_2}\rangle^{\mathrm{WH}}_{\mathrm{phys}} = \frac{B(y_1)-B(r)}{192\pi\, f(r)^2}$$

Region III (post-evaporation):

$$\langle T_{v_3v_3}\rangle^{\mathrm{WH}}_{\mathrm{phys}} = \frac{B(y_1)-B(y_2)}{192\pi\, f(y_2)^2}$$

$$\langle T_{uu}\rangle^{\mathrm{WH}}_{\mathrm{phys}} = 0$$

Time-reversal

$$u \leftrightarrow -v, \quad x_i \leftrightarrow y_i, \quad T_{uu} \leftrightarrow T_{vv}$$

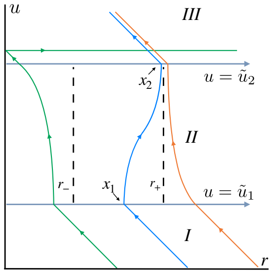
On horizons:

$$\langle T_{uu}\rangle\Big|_{r_h} = -\frac{\kappa_h^2}{48\pi}, \qquad \langle T_{v_3v_3}\rangle\Big|_{r_h} \propto \frac{f^{(3)}(r_h)}{\kappa_h}\Big[1 - e^{2\kappa_h}$$

POTH ($\kappa_h > 0$): $\langle T_{v_3v_3}\rangle \sim e^{2\kappa_+\Delta u} \to \infty.$

Backup: ray tracing and analytic limits (WH)

$$\langle T_{v_2 v_2}\rangle = -\frac{1}{f^2(r)}\int_{y_1}^{r(u,v_2)} f(r')\, f^{(3)}(r')\, dr'$$



Early times ($\Delta u \to 0$)

$$\langle T_{v_3 v_3}\rangle \approx -\frac{f^{(3)}(y_1)}{192\pi}\,\Delta u$$

The minus sign reflects the time-reversed kinematics.

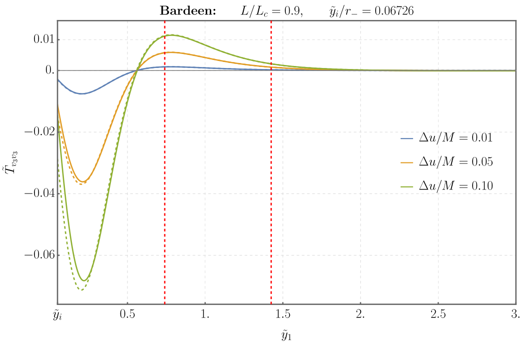
Late times ($\Delta u \to \infty$)

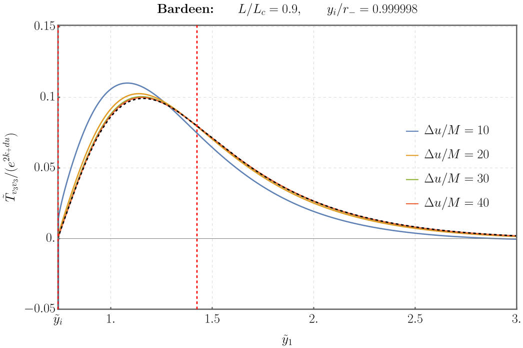
$$\langle T_{v_3 v_3}\rangle \approx \frac{f^{(3)}(r_+)}{384\pi\kappa_+} + C(y_1)\, e^{2\kappa_+\Delta u}$$

$$C(y_1) = \frac{1}{192\pi r_+^2}\left(\frac{B(y_1)}{4\kappa_+^2} - 1\right) e^{-4\kappa_+ R_{\mathrm{WH}}(y_1)}$$

Controlled by the *outer* horizon.

Backup: ingoing fluxes of the Bardeen white hole





Backup: null expansions I – evaporation

The outgoing expansion θ_+ is sourced by the *ingoing* flux T_{vv}.

Region II ($\lambda_+ = v$, non-affine)

$$\frac{d\theta_{+,2}}{dv} = -\frac{1}{2}\theta_{+,2}^2 + \frac{f'(r)}{2}\,\theta_{+,2} - \frac{8\pi}{r^2}\langle T_{vv}^{(2D)}\rangle$$

Near a horizon $\theta_{+,2} \sim 0$:

$$\left.\frac{d\theta_{+,2}}{dv}\right|_{r_h} \simeq \frac{\kappa_h^2}{6r_h^2} > 0$$

$\Rightarrow$ FOTH inward, FITH outward: the trapped region shrinks.

Region III (affine $\lambda_+ = u_3$)

$$\frac{d\theta_{+,3}}{dv} = -\frac{\theta_{+,3}^2}{2} - \frac{8\pi}{r^2}\langle T_{vv}^{(2D)}\rangle$$

Since $\langle T_{vv}\rangle^{\mathrm{III}} = 0$: $\theta_{+,3} = 1/r$ (Minkowski).

BH $\leftrightarrow$ WH duality

The same equation governs θ_- in the WH case ($-u \leftrightarrow v$, $T_{vv} \leftrightarrow T_{uu}$).

Backup: null expansions II – formation of a new region

The ingoing expansion θ_- is sourced by the *outgoing* flux T_{uu}.

Region II (in terms of x_1)

$$\frac{d\theta_{-,2}}{dx_1} = \frac{f(r)}{f(x_1)}\left[\theta_{-,2}^2 + \frac{16\pi}{r^2}\frac{\langle T^{(2D)}_{u_2u_2}\rangle}{f(r)^2}\right]$$

Initially $\theta_{-,2} = -1/r$. Negative T_{uu} near r_- drives $d\theta_{-,2}/dx_1 > 0$:

1. $\theta_- < 0$: fully trapped;
2. $\theta_- \to 0$: anti-trapping horizon (PITH + POTH);
3. $\theta_- > 0$: **anti-trapped region**.

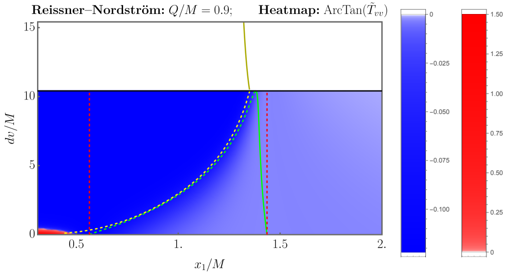
Region III

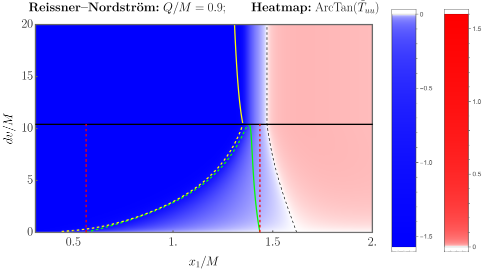
$$\frac{d\theta_{-,3}}{dx_1} = \frac{f(x_2)}{f(x_1)}\left[\theta_{-,3}^2 + \frac{16\pi}{r^2}\langle T^{(2D)}_{u_3u_3}\rangle\right]$$

The 4D factor $1/r^2$ progressively damps the frozen flux $\Rightarrow$ finite anti-trapped region.

WH case (by duality): same equation for θ_+ sourced by T_{vv}; positive T_{vv} near r_+ drives $\theta_+ \to 0$ $\Rightarrow$ **new trapped region**.

Backup: RSET heatmaps – Reissner–Nordström



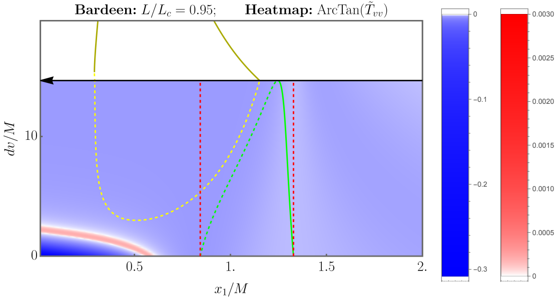


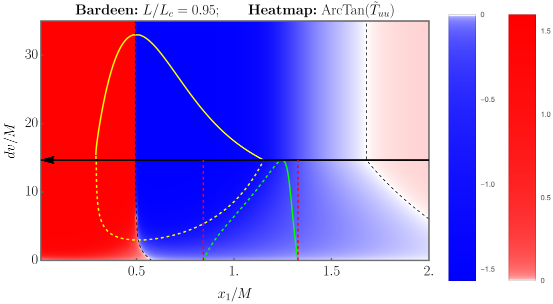
Ingoing (left) and outgoing (right) RSET in the (x_1, dv) plane, overlaid with the marginal surfaces (trapped in green, anti-trapped in yellow; solid = outer, dashed = inner).

What to read off

The ingoing flux is negative throughout $\Rightarrow$ the trapped region only shrinks. The outgoing flux is negative and diverges at $x_1 \to 0 \Rightarrow$ the anti-trapped region expands without bound.

Backup: RSET heatmaps – Bardeen



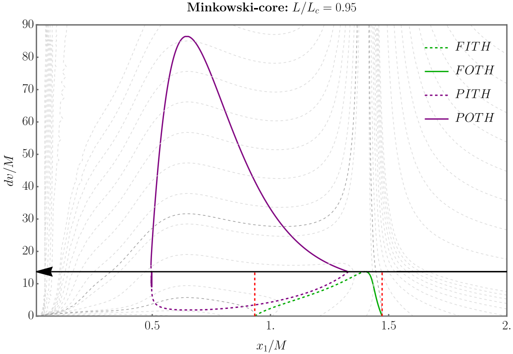


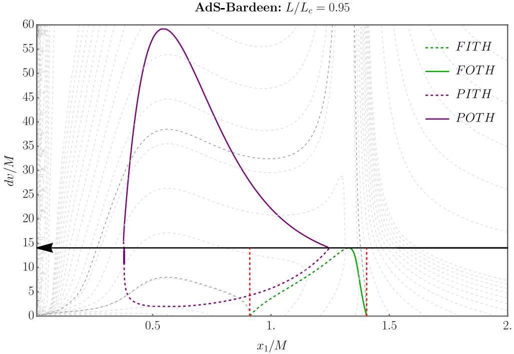
Same quantities as in the RN case, for the Bardeen regular black hole.

What to read off

The outgoing flux (right) changes sign at small x_1: the anti-trapped region expands where it is negative and *contracts* where it is positive. This is what gives the WH region a finite lifetime.

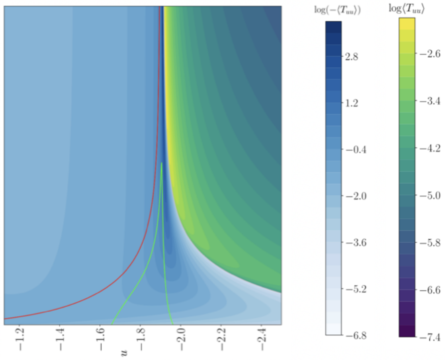
Backup: marginal surfaces – Minkowski and AdS cores

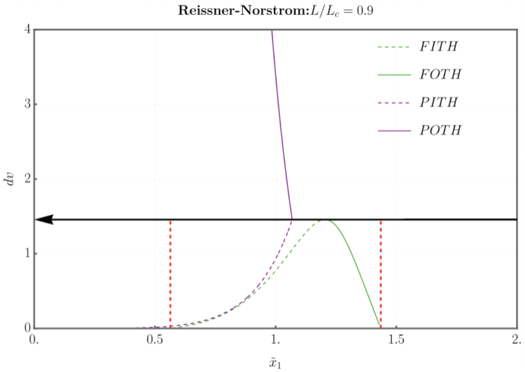




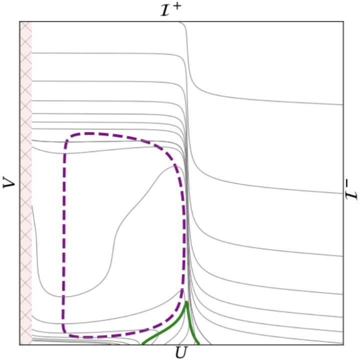
Minkowski core (left) and AdS-core Bardeen (right): different flux profiles, same qualitative fate for the anti-trapped region.

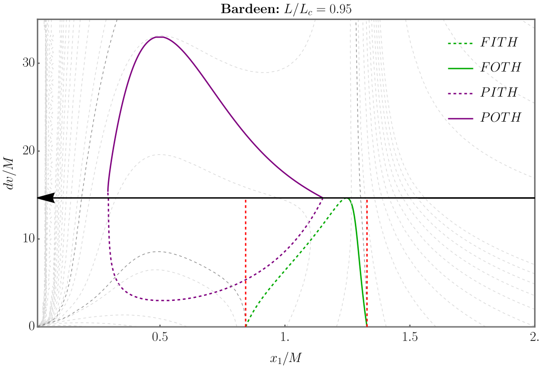
Backup: comparison with the RN numerical evolution





Backup: our diagnostic vs the 2D simulations





Backup: Schwarzschild does not transition

For $f = 1 - 2M/r$ one has $f^{(3)}(r) = 12M/r^4 > 0$ everywhere, so

$$\langle T_{u_2 u_2} \rangle^{\mathrm{BH}}_{\mathrm{phys}} = \frac{1}{192\pi} \int_{\tilde{v}_1}^{v} f^2 f^{(3)}\, dv' \;\geq\; 0 \qquad \text{everywhere in II and III.}$$

Consequence

θ_- can never cross zero: **no anti-trapped region forms**. The single-horizon case does not transition — and, on the FOTH, the bracket $[1 - e^{-2\kappa_h(v-\tilde{v}_1)}]$ saturates: **no exponential amplification**.

This also reproduces Hiscock's classic results [5] as a particular case of our general formulae.