Noa Zilberman
Princeton University
Princeton gravity Initiative

The semiclassical energy outflux emerging from a collapsing shell

based on joint work with Amos Ori

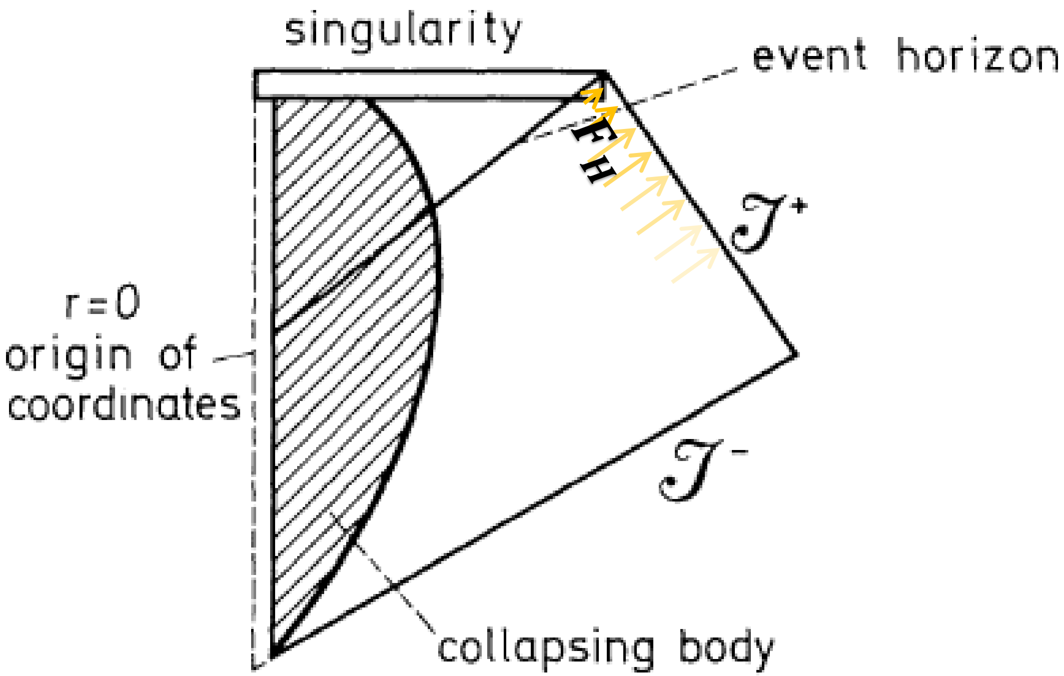
PRL 135, 021401 (2025) | arXiv:2503.00622

From Black Holes to the Cosmos: Matt Visser's journey through space and time

SISSA, August 2026

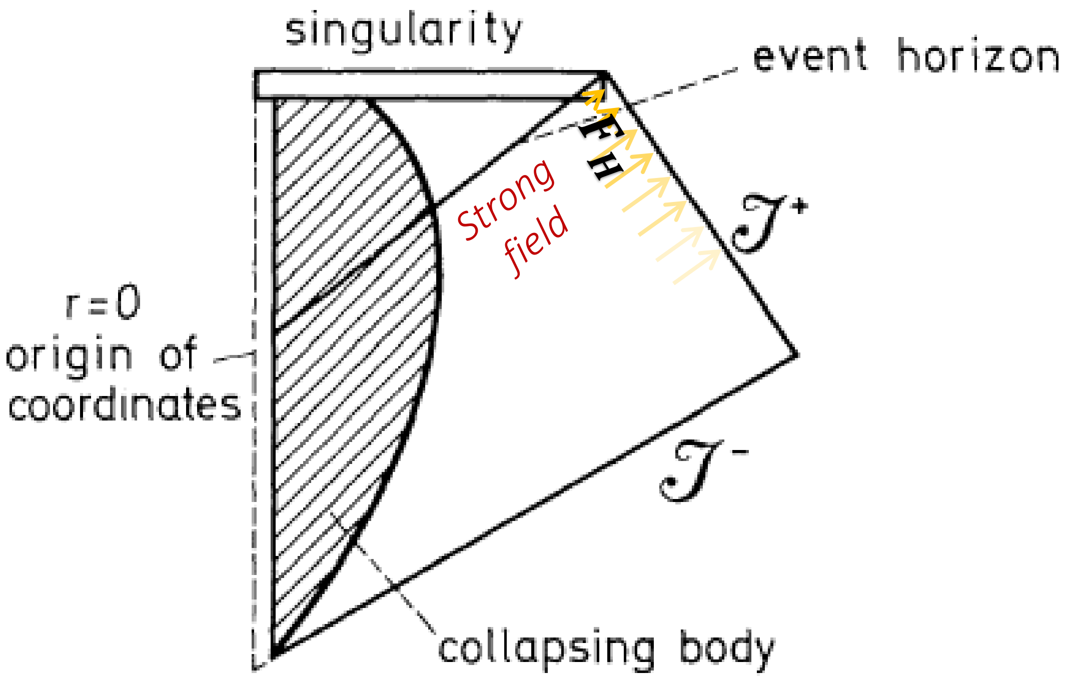
When a compact object collapses to form a BH, QFT predicts the emission of massless particles which carry energy to $\mathfrak{I}^+$.

The emission process relaxes to a steady state known as *Hawking radiation*.



Where (in spacetime) does this Hawking radiation energy come from?

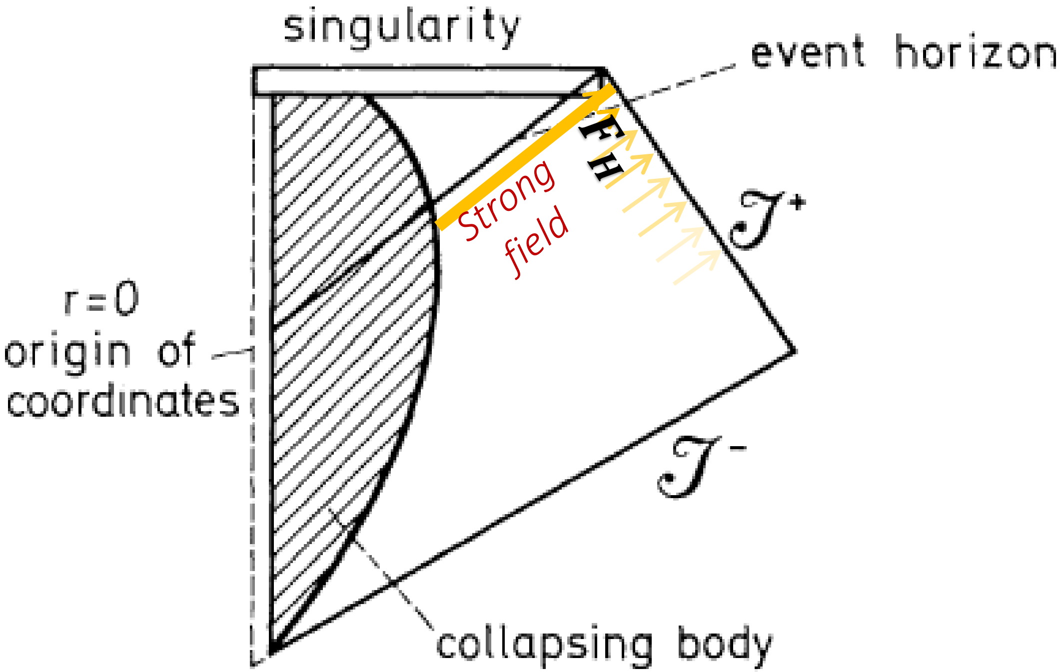
(Hawking, 1974)



"The origin of the energy in black hole evaporation has been a longstanding issue, which is still not entirely settled."

- Chen, Unruh and collaborators (2017)

Clearly, the Hawking-radiation energy comes from the strong field region. But exactly from what part?



Option $\boldsymbol{i}$: the outgoing radiation starts from zero at (extremely close to) the event horizon, and gradually develops over a fairly broad strong-field region

Option $\boldsymbol{ii}$: the energy emerges from the collapsing body itself and propagates in a conserved manner through the surrounding strong-field region

Proc. R. Soc. Lond. A. **351**, 129–139 (1976)
Printed in Great Britain

On the origin of black hole evaporation radiation

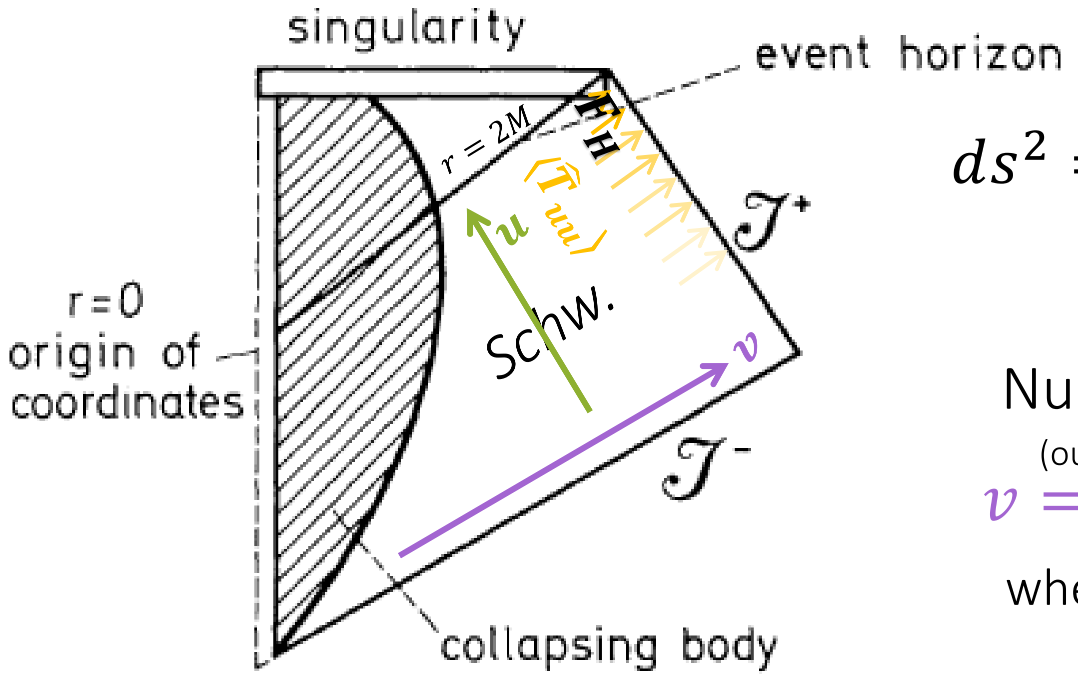
BY P. C. W. DAVIES
Department of Mathematics, King's College, London

(*Communicated by R. Penrose, F.R.S. – Received* 26 *February* 1976)

The physical basis underlying the black hole evaporation process is clarified by a calculation of the expectation value of the energy-momentum tensor for a massless scalar field in a completely general two dimensional collapse scenario. It is found that radiation is produced inside the collapsing matter which propagates both inwards and outwards. The ingoing component eventually emerges from the star after travelling through the centre. The outgoing energy flux appears at infinity as the evaporation radiation discovered by Hawking. At late times, outside the star, the former component fades out exponentially, and the latter component approaches a value which is independent of the details of the collapse process. In the special case of a collapsing hollow, thin shell of matter, all the radiation is produced at the shell. These results are independent of regularization ambiguities, which enter only the static vacuum polarization terms in the energy-momentum tensor.

The significance of an earlier remark about black hole explosions is discussed in the light of these results.

Spherically symmetric collapsing object, surrounded by a vacuum region – Schwarzschild geometry (Birkhoff)



$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 d\Omega^2$$
$$f = 1 - 2M/r$$

Null Eddington coordinates

(outside the star and outside the horizon)

$v = t + r_*$, $u = t - r_*$

where $r_* = r + 2M \ln\left(\frac{r}{2M} - 1\right)$

(in Minkowski $r_* = r$)

A key component of the stress-energy tensor:

$\langle \hat{T}_{uu} \rangle_{\text{ren}}(u, v)$ – outward energy flux density

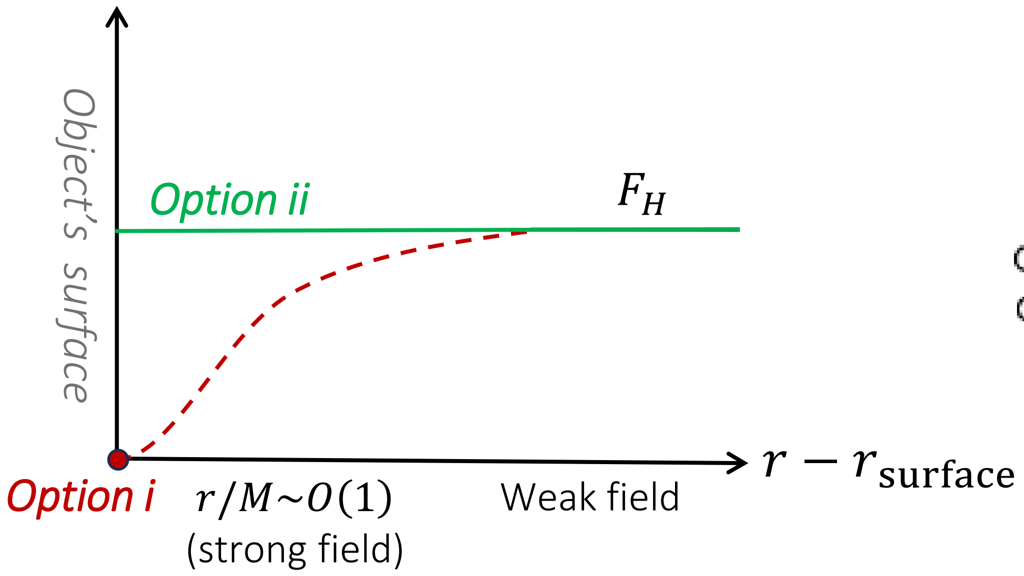
$F \equiv 4\pi r^2 \langle \hat{T}_{uu} \rangle_{\text{ren}}$ - the quantum energy outflux

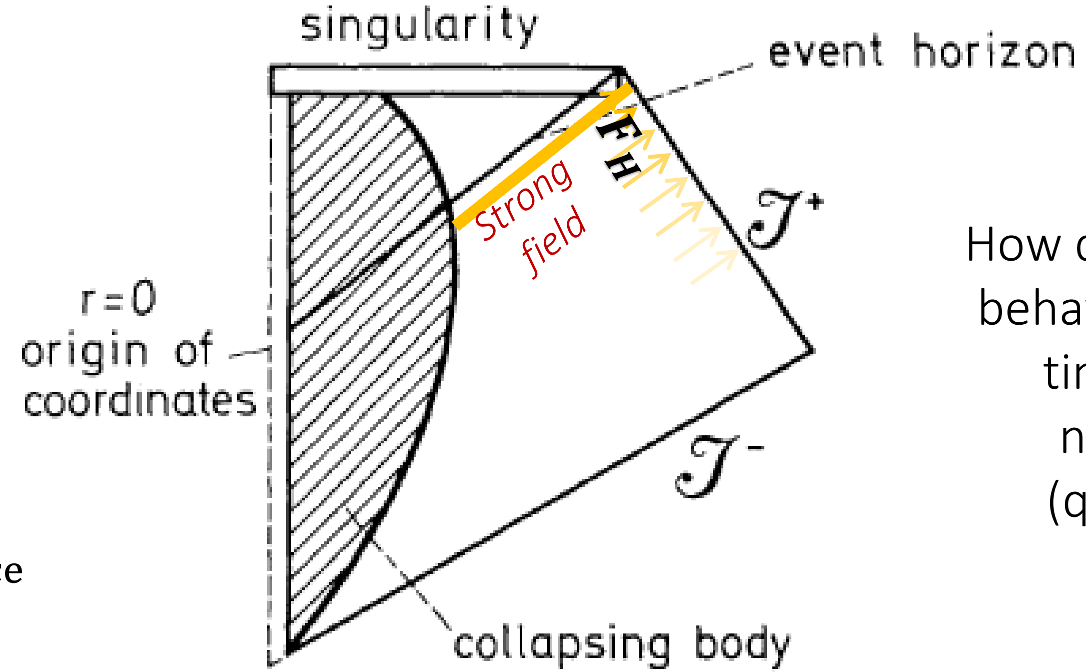
Late time domain $u \gg M$: outflux at $\mathfrak{I}^+$ has already settled down to its constant value

Hawking radiation (F at $\mathfrak{I}^+$): $\lim_{r\to\infty} F = F_H \nearrow \hbar P/M^2$

The surface of the collapsing object has reached its near-horizon r value $r \approx 2M$

$F \equiv 4\pi r^2 \langle \hat{T}_{uu} \rangle$ along $u = u_0 \gg M$



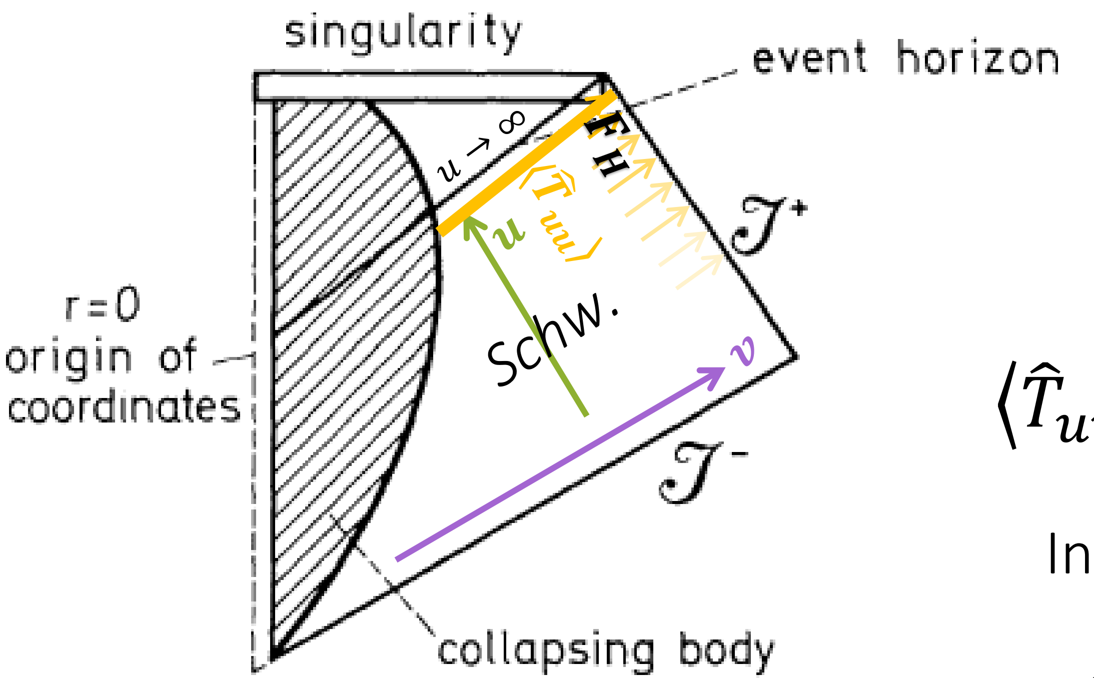


How does the **outflux** behave along a late-time outgoing null geodesic (qualitatively)

Option $\boldsymbol{i}$: the outgoing radiation starts from zero at (extremely close to) the event horizon, and gradually develops over a fairly broad strong-field region

Option $\boldsymbol{ii}$: the energy emerges from the collapsing body itself and propagates in a conserved manner through the surrounding strong-field region

Option *ii*: the energy emerges from the collapsing body itself and propagates in a conserved manner through the surrounding strong-field region



$F \equiv 4\pi r^2 \langle \hat{T}_{uu} \rangle_{\text{ren}}$ - the energy outflux is conserved along a late-time null geodesic

$$\langle \hat{T}_{uu} \rangle_{\text{ren}}^{(ii)} = F_H / 4\pi r^2 \quad \text{at} \quad u \gg M$$

$$\langle \hat{T}_{uu} \rangle_{\text{ren}}^{(ii,\text{surface})} = F_H / 4\pi (2M)^2 \quad \text{at the surface}$$

In a regular (Kruskal) coordinate $U \propto -e^{-u/4M}$:

$$\langle \hat{T}_{UU} \rangle_{\text{ren}}^{(ii,\text{surface})} \propto \langle \hat{T}_{uu} \rangle_{\text{ren}}^{(ii,\text{surface})} / U^2 \quad \to \infty \text{ at the EH}$$

at $U \approx 0$ $(u \gg M)$

Significant backreaction effects

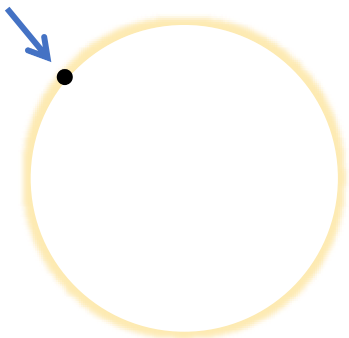
Halting the collapse process? ($dM/du = -4\pi r^2 T_{uu}$ and $u \to \infty$)

Preventing horizon formation (horizon avoidance)? 😱

In the absence of concrete calculations (in **4d** gravitational collapse), it is hard to prove which of the two options is the correct one.

Our main goal is to present such a concrete analysis.

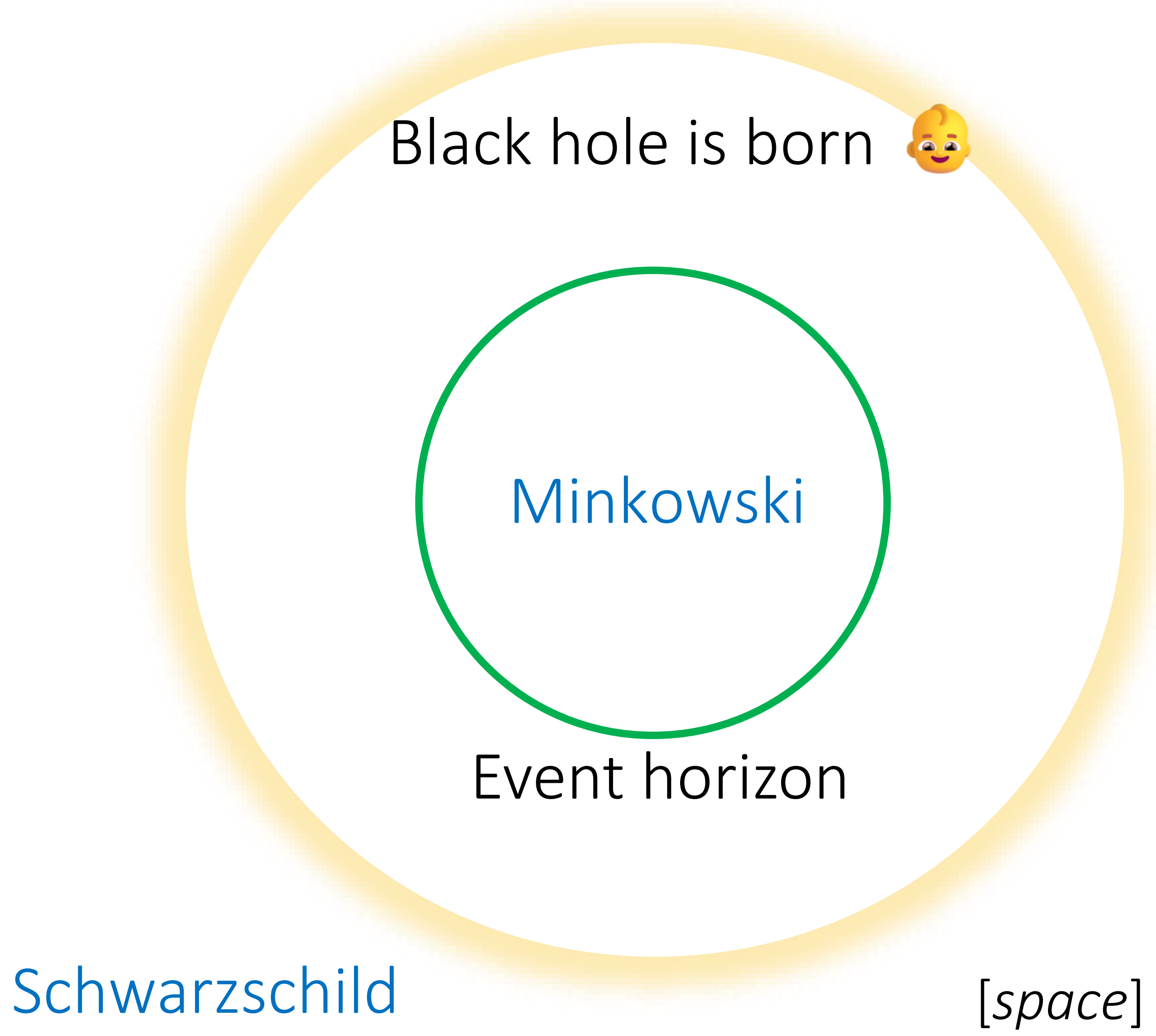
(we will derive a simple, explicit, analytical expression for $\langle \hat{T}_{uu} \rangle_{\mathrm{ren}}$ in the Schwarzschild region just outside a collapsing object.)

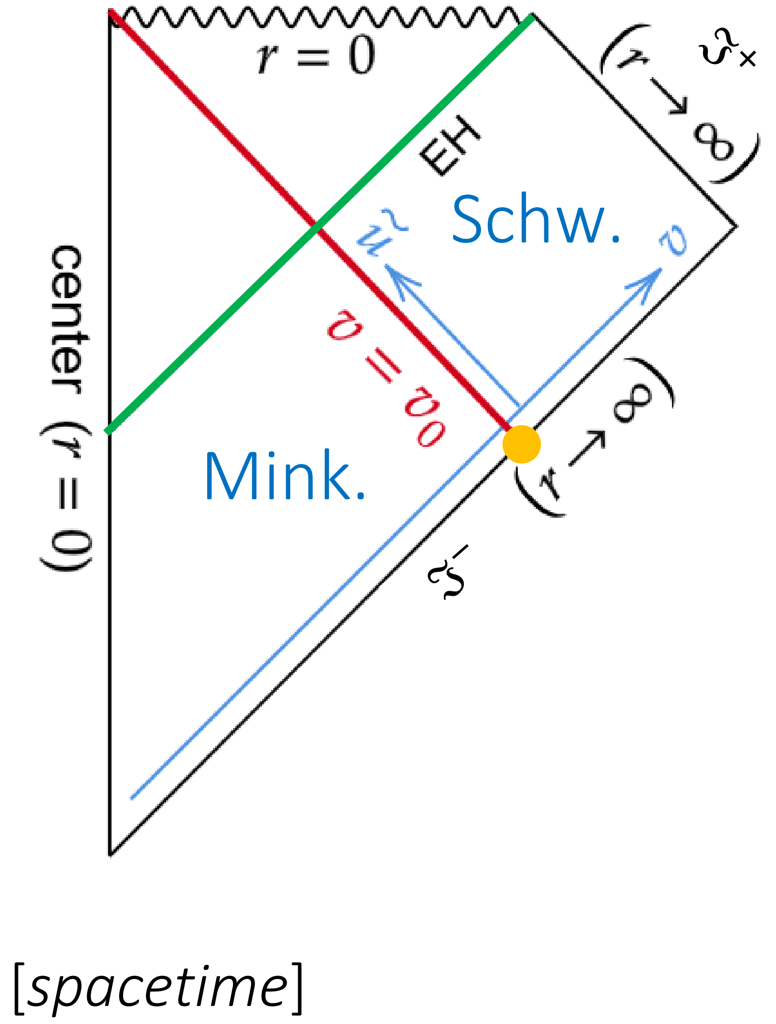


$$\langle \hat{T}_{uu} \rangle_{\mathrm{ren}}^{(\mathrm{surface})} \underset{\text{at late times}}{\neq} \langle \hat{T}_{uu} \rangle_{\mathrm{ren}}^{(ii,\mathrm{surface})} = F_H / 4\pi (2M)^2$$

$\rightarrow 0$ in accordance with option i

*The spherically-symmetric collapsing **thin null** shell model*

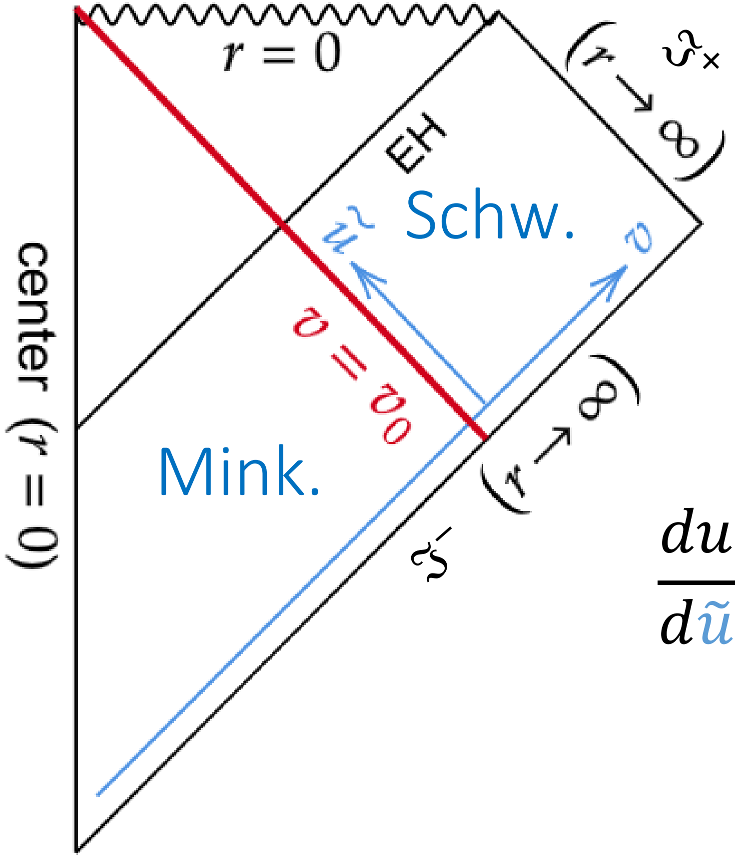




The spherically-symmetric collapsing thin null shell model (4d)

[Andersson, Siahmazgi, Clark, Fabbri and others have considered this model]

Shell propagating inward along an incoming null ray $v = v_0$.



$$M = M_0 \Theta(v - v_0)$$

Mass of the shell

The two different geometries are continuously matched across the shell.

Metric in (u, v, θ, φ):

$$ds^2 = -f du dv + r^2 d\Omega^2$$

$$f = 1 - (2M_0/r)\Theta(v - v_0) \quad \text{discontinuity}$$

<u>Metric continuity across the shell:</u>

$$\frac{du}{d\tilde{u}} = \begin{cases} 1 & , \quad v > v_0 \ \text{(Schwarzschild)} \\ f_0 \equiv 1 - \dfrac{2M_0}{r(u, v_0)} & , \quad v < v_0 \ \text{(Minkowski)} \end{cases}$$

where $r(u, v_0) = \frac{1}{2}(v_0 - u)$

$$g_{\tilde{u}v} = -\frac{1}{2}\left(1 - \frac{2M_0}{r}\right)$$ at the shell (approaching from both sides)

The quantum field and vacuum state

Minimally-coupled massless scalar field $\hat{\Phi}$, Klein-Gordon: $\square\,\hat{\Phi} = 0$

Decompose into **normalized mode solutions**, defined by **initial data**:

$$\Phi_{\omega\ell m}(t,r,\theta,\varphi) = \frac{1}{r\sqrt{4\pi\omega}}\, Y_{\ell m}(\theta,\varphi)\, e^{-i\omega v} \quad \text{at } \mathfrak{I}^- \quad + \text{ regularity condition at } r = 0.$$

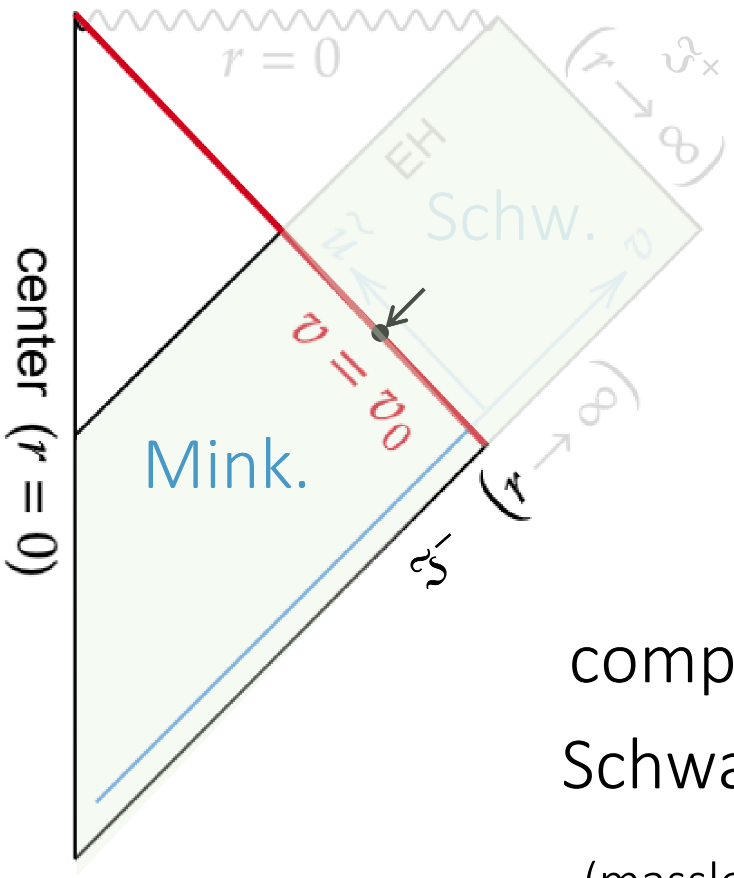
$$\hat{\Phi}(x) = \int_0^\infty d\omega \sum_{\ell m} \left[\Phi_{\omega\ell m}(x)\hat{a}_{\omega\ell m} + \Phi^*_{\omega\ell m}(x)\hat{a}^\dagger_{\omega\ell m}\right]$$

Vacuum state taken with respect to the positive-frequency modes $\Phi_{\omega\ell m}$.

The "in" vacuum state $|0\rangle_{\text{in}}$: $\forall \omega,\ell,m$: $\hat{a}_{\omega\ell m}|0\rangle_{\text{in}} = 0$

no incoming excitations from $\mathfrak{I}^-$ (the natural state which Hawking analyzed – coincides with vacuum in Minkowski)

In Minkowski: $\langle \hat{T}_{uu} \rangle_{\text{ren}}$ in the "in" vacuum vanishes everywhere



→ by causality, also in our model at $v < v_0$.

$$\langle \hat{T}_{uu} \rangle_{\text{ren}} = \begin{cases} 0 & v < v_0 \\ ? & v > v_0 \end{cases}$$

Non-zero, due to spacetime curvature

Our goal:

compute $\langle \hat{T}_{uu} \rangle_{\text{ren}}$ at the outer surface of the shell, (on the Schwarzschild side, approaching from the exterior $v \to v_0^+$)

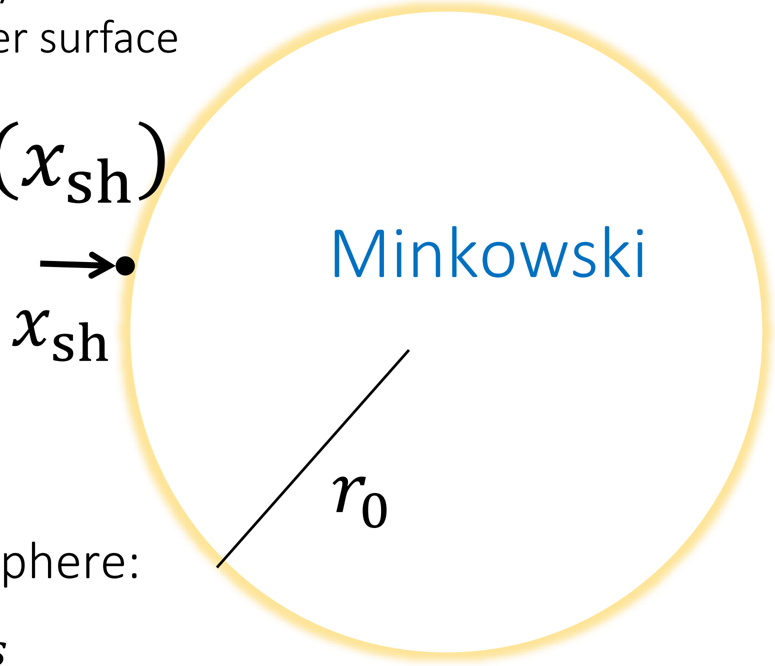
(massless scalar field in vacuum – no ambiguities related to the renormalization.)

(The paper also does $\langle \hat{\Phi}^2 \rangle_{\text{ren}}$, but I'll skip it in this talk)

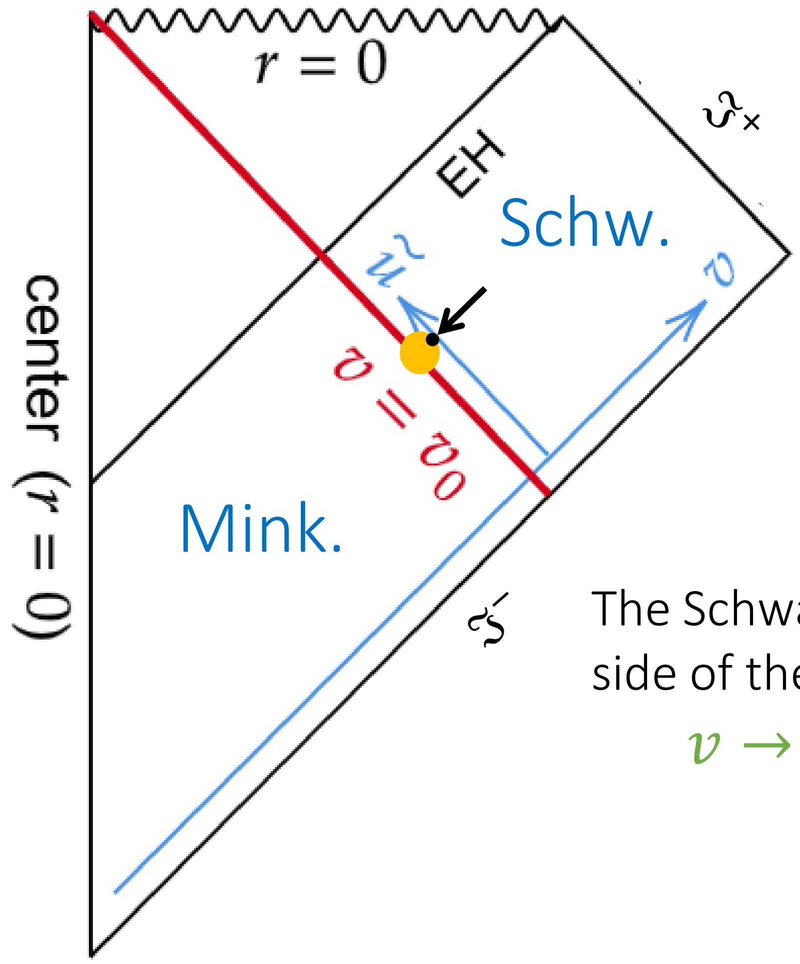
An illustration of our goal

Quantum energy outflux
at the shell's outer surface

$$\langle \hat{T}_{uu} \rangle^{+}_{\text{ren}} (x_{\text{sh}})$$



Radius of the sphere:
$r_0 \geq r_s$



The Schwarzschild
side of the shell
$v \to v_0^+$

Schwarzschild $\quad x = x_{\text{sh}}$ on the shell, with $v = v_0$, $r = r_0$ (and $\tilde{u} = \tilde{u}_0(v_0, r_0)$)

Classically: $T_{uu} = \Phi_{,u}\Phi_{,u}$

Naïve quantum version: $\langle \hat{T}_{uu} \rangle = \langle \hat{\Phi}_{,u}\hat{\Phi}_{,u} \rangle$

Point-splitting regularization – schematic overview

$$\langle \hat{T}_{uu} \rangle_{\mathrm{ren}}(x) = \lim_{x' \to x} \left[\frac{1}{2} \langle \{ \overbrace{\hat{\Phi}_{,u}(x)\hat{\Phi}_{,u}(x')}^{G_{uu}(x,x') \equiv} \} \rangle - \tilde{L}_{uu}(x,x') \right]$$

Symmetrization

$x \leftarrow x'$

Depends on the (non-static) **quantum state**

Constructed from modes $\Phi_{\omega lm}$
Requires numerical computation
(notoriously demanding)

Depends only on the (locally static) **geometry**

Known analytically

$$\langle \hat{T}_{uu} \rangle_{\text{ren}}(x) = \lim_{x' \to x} \left[G_{uu}(x, x') - \tilde{L}_{uu}(x, x') \right]$$

Constructed from modes $\Phi_{\omega l m}$
Requires numerical computation
(notoriously demanding)

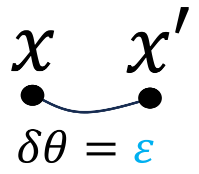
$$G_{uu}(x, x') \equiv \frac{1}{2} \langle \{ \hat{\Phi}_{,u}(x) \hat{\Phi}_{,u}(x') \} \rangle$$

Symmetrization

Known analytically

$\tilde{L}_{uu}(x, x')$ is the *coordinate based counterterm*
obtained from the (covariant) Christensen's counterterm $C_{\mu\nu}(x, x')$
Levi (2017)

θ-splitting: $\quad x = (\tilde{u}, v, \theta = 0, \varphi), \ x' = (\tilde{u}, v, \theta = \varepsilon, \varphi)$
(spherical symmetry)



$$\langle \hat{T}_{uu} \rangle_{\text{ren}}(x) = \lim_{\varepsilon \to 0} \left[G_{uu}(\tilde{u}, v; \varepsilon) - \tilde{L}_{uu}(r; \varepsilon) \right]$$

Depends on the (non-static) **quantum state**

Depends only on the (locally static) **geometry**

$x = x_{\mathrm{sh}}$ on the shell, with $v = v_0$, $r = r_0$ (and $\tilde{u} = \tilde{u}_0(v_0, r_0)$)

x_{sh} is approached from the Schwar. side of the shell $v \to v_0^+$

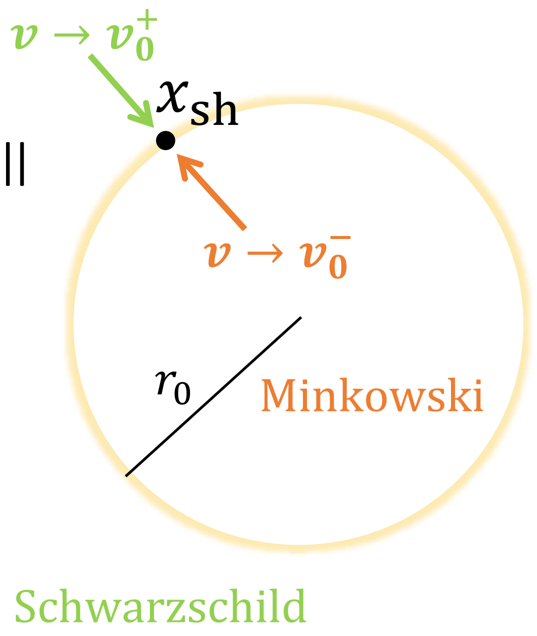
Requires numerics | Known analytically

$$\langle \hat{T}_{uu} \rangle^{+}_{\mathrm{ren}}(x_{\mathrm{sh}}) = \lim_{\varepsilon \to 0} \left[G^{+}_{uu}(\tilde{u}_0, v_0; \varepsilon) - \tilde{L}^{+}_{uu}(r_0; \varepsilon) \right]$$

x_{sh} is approached from the Mink. side of the shell $v \to v_0^-$

$$\langle \hat{T}_{uu} \rangle^{-}_{\mathrm{ren}}(x_{\mathrm{sh}}) = \lim_{\varepsilon \to 0} \left[G^{-}_{uu}(\tilde{u}_0, v_0; \varepsilon) - \tilde{L}^{-}_{uu}(r_0; \varepsilon) \right]$$

Want to use continuity at the shell



$x = x_{\text{sh}}$ on the shell, with $v = v_0, r = r_0$ (and $\tilde{u} = \tilde{u}_0(v_0, r_0)$)

x_{sh} is approached from the Schwar. side of the shell $v \to v_0^+$

Requires numerics — Known analytically

$$\langle \hat{T}_{uu} \rangle_{\text{ren}}^{+}(x_{\text{sh}}) = \lim_{\varepsilon \to 0}\left[G_{uu}^{+}(\tilde{u}_0, v_0; \varepsilon) - \tilde{L}_{uu}^{+}(r_0; \varepsilon)\right]$$

x_{sh} is approached from the Mink. side of the shell $v \to v_0^-$

$$\langle \hat{T}_{uu} \rangle_{\text{ren}}^{-}(x_{\text{sh}}) = \lim_{\varepsilon \to 0}\left[G_{uu}^{-}(\tilde{u}_0, v_0; \varepsilon) - \tilde{L}_{uu}^{-}(r_0; \varepsilon)\right]$$

Problem: G_{uu} is composed of $\hat{\Phi}_{,u}$ and is *not* continuous across the shell.
To proceed, *transform to* $\tilde{u}$ ($\hat{\Phi}_{,\tilde{u}}$ is continuous across the shell):

$$G_{\tilde{u}\tilde{u}}(x, x') \equiv \frac{1}{2}\left\langle\left\{\hat{\Phi}_{,\tilde{u}}(x)\hat{\Phi}_{,\tilde{u}}(x')\right\}\right\rangle$$

$$\tilde{L}_{\tilde{u}\tilde{u}}(r_0; \varepsilon) \equiv \left(\frac{du}{d\tilde{u}}\right)^2 \tilde{L}_{uu}(r_0; \varepsilon)$$

$x = x_{sh}$ on the shell, with $v = v_0, r = r_0$ (and $\tilde{u} = \tilde{u}_0(v_0, r_0)$)

Want:

x_{sh} is approached from the Schwar. side of the shell $v \to v_0^+$

Requires numerics

Known analytically

$$\langle \hat{T}_{\tilde{u}\tilde{u}} \rangle_{\text{ren}}^{+}(x_{sh}) = \lim_{\varepsilon \to 0} \left[G_{\tilde{u}\tilde{u}}^{+}(\tilde{u}_0, v_0; \varepsilon) - \tilde{L}_{\tilde{u}\tilde{u}}^{+}(r_0; \varepsilon) \right]$$

$\|$ Continuity across the shell

x_{sh} is approached from the Mink. side of the shell $v \to v_0^-$

$$\underbrace{\langle \hat{T}_{\tilde{u}\tilde{u}} \rangle_{\text{ren}}^{-}(x_{sh})}_{=0} = \lim_{\varepsilon \to 0} \left[G_{\tilde{u}\tilde{u}}^{-}(\tilde{u}_0, v_0; \varepsilon) - \tilde{L}_{\tilde{u}\tilde{u}}^{-}(r_0; \varepsilon) \right]$$

Subtracting these equations:

$$\langle \hat{T}_{\tilde{u}\tilde{u}} \rangle_{\text{ren}}^{+}(x_{sh}) = -\lim_{\varepsilon \to 0} \left[\tilde{L}_{\tilde{u}\tilde{u}}^{+}(r_0; \varepsilon) - \tilde{L}_{\tilde{u}\tilde{u}}^{-}(r_0; \varepsilon) \right]$$

We got rid of the notoriously demanding numerical part! 💃

$$\langle \hat{T}_{\tilde{u}\tilde{u}} \rangle^{+}_{\mathrm{ren}}(x_{\mathrm{sh}}) = -\lim_{\varepsilon\to 0}\left[\tilde{L}^{+}_{\tilde{u}\tilde{u}}(r_0;\varepsilon) - \tilde{L}^{-}_{\tilde{u}\tilde{u}}(r_0;\varepsilon)\right]$$

Recall:

$$\frac{du}{d\tilde{u}} = \begin{cases} 1 & , \quad v > v_0 \quad (\text{Schw.}) \\ f_0 \equiv 1 - \dfrac{2M_0}{r(u,v_0)} & , \quad v < v_0 \quad (\text{Mink.}) \end{cases} \qquad \left[r(u,v_0) = \frac{1}{2}(v_0 - u) \right]$$

Back to u:

$$\langle \hat{T}_{uu} \rangle^{+}_{\mathrm{ren}}(x_{\mathrm{sh}}) = -\lim_{\varepsilon\to 0}\Big[\overbrace{\tilde{L}^{+}_{uu}(r_0;\varepsilon)}^{\text{Schw.}} - \overbrace{f_0^2\,\tilde{L}^{-}_{uu}(r_0;\varepsilon)}^{\text{Mink.}}\Big]$$

Reminder:

$$f = 1 - \frac{2M}{r}$$

$$f_0 = 1 - \frac{2M_0}{r_0}$$

$$\langle \hat{T}_{uu} \rangle^{+}_{\mathrm{ren}}(x_{\mathrm{sh}}) = -\lim_{\varepsilon \to 0} \left[\tilde{L}^{+}_{uu}(r_0; \varepsilon) - f_0^2 \tilde{L}^{-}_{uu}(r_0; \varepsilon) \right]$$

In Schwarzschild

(counterterm expanded in the point-separation ε)

$$\tilde{L}_{uu}(r; \varepsilon) = \hbar \left[\frac{f^2}{64\pi^2 r^4} \sin^{-2}\left(\frac{\varepsilon}{2}\right) - \frac{3Mf^2}{160\pi^2 r^5} \right]$$

NZ, A. Levi, A. Ori (2020)

Schwarzschild side:
Plug in $M = M_0$

$$\tilde{L}^{+}_{uu}(r_0; \varepsilon) = \hbar \left[\cancel{\frac{f_0^2}{64\pi^2 r_0^4} \sin^{-2}\left(\frac{\varepsilon}{2}\right)} - \boxed{\frac{3M_0 f_0^2}{160\pi^2 r_0^5}} \right]$$

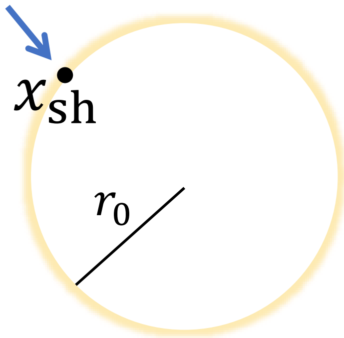
The finite counterterm

Minkowski side:
Plug in $M = 0$

$$\tilde{L}^{-}_{uu}(r_0; \varepsilon) = \hbar \left[\cancel{\frac{1}{64\pi^2 r_0^4} \sin^{-2}\left(\frac{\varepsilon}{2}\right)} \right]$$

The ε dependence drops off completely! 💃🏾

The outflux density emerging from the shell (at a point where $r = r_0$)



Schwarzschild

$$\boxed{\langle \hat{T}_{uu} \rangle^{+}_{\rm ren}(r_0) = \hbar \frac{3M_0}{160\pi^2 r_0^7}(r_0 - 2M_0)^2}$$

The component most directly related to the question of EH regularity

As the shell approaches the **horizon**, the outflux density vanishes *quadratically* with the remaining distance $r_0 - 2M_0$.

Regularity of the Kruskal-based $\langle \hat{T}_{UU} \rangle^{+}_{\rm ren}$:

$U \propto -e^{-\frac{u}{4M}}$

Near EH $\propto r - 2M$

(with $v = const$):

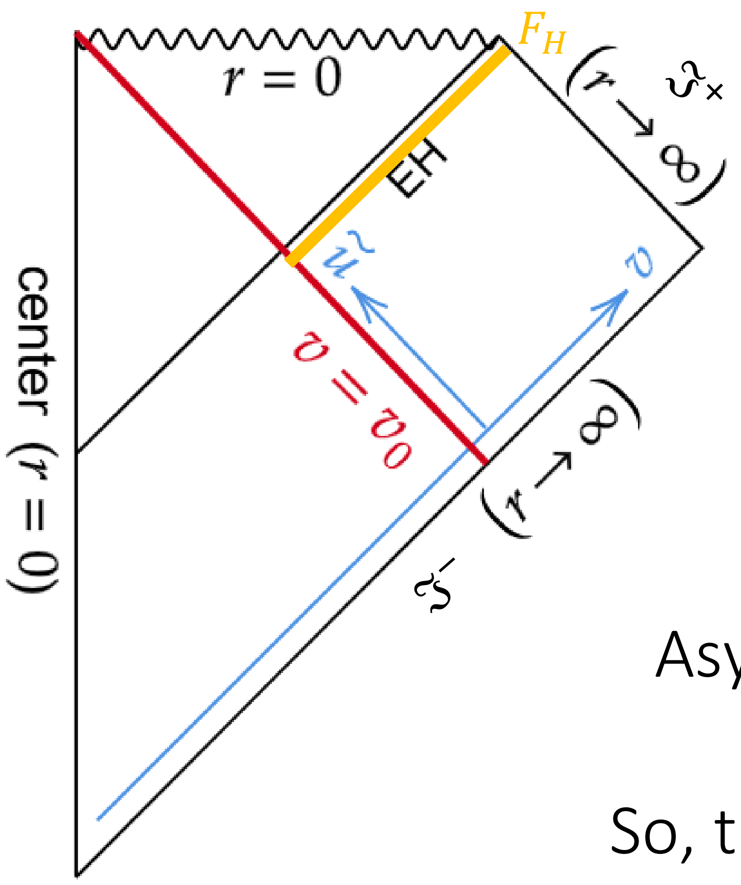
$$\langle \hat{T}_{UU} \rangle \propto \langle \hat{T}_{uu} \rangle / U^2 \propto \langle \hat{T}_{uu} \rangle / (r_0 - 2M_0)^2 \quad \text{(finite)}$$

Option *ii* $\left\{ \begin{array}{l} \text{Significant backreaction effects at the horizon} \\ \text{Halting the collapse process?} \\ \text{Preventing horizon formation (horizon avoidance)?} \end{array} \right.$

Nope.

The evolution of the outflux along a late-time outgoing null ray

$F = 4\pi r^2 \langle \hat{T}_{uu} \rangle_{\mathrm{ren}}$ $\qquad$ $u = u_0 \gg M_0$

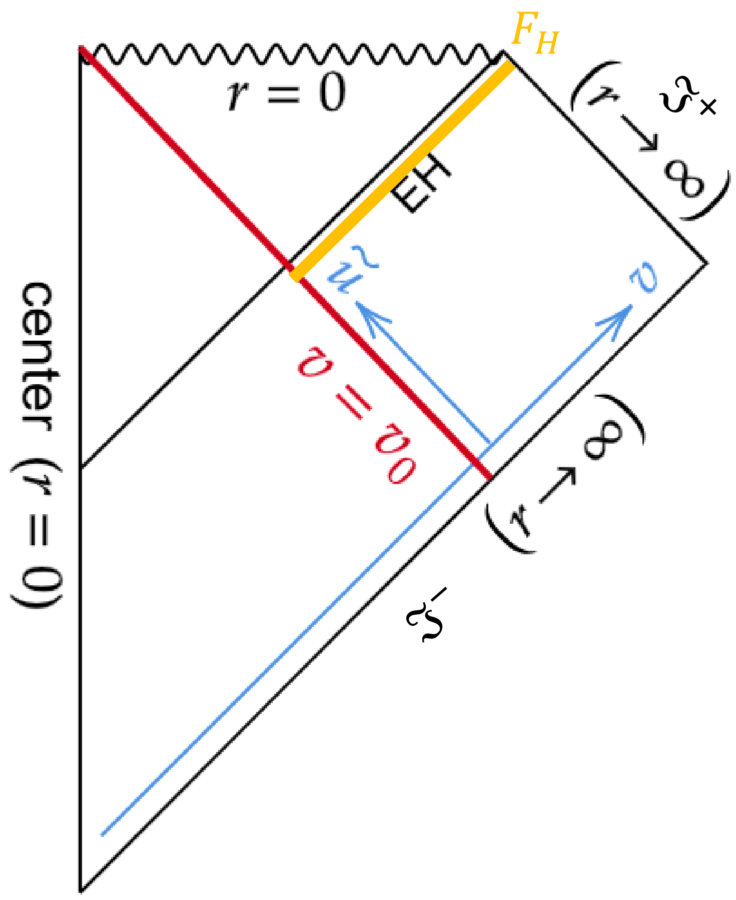


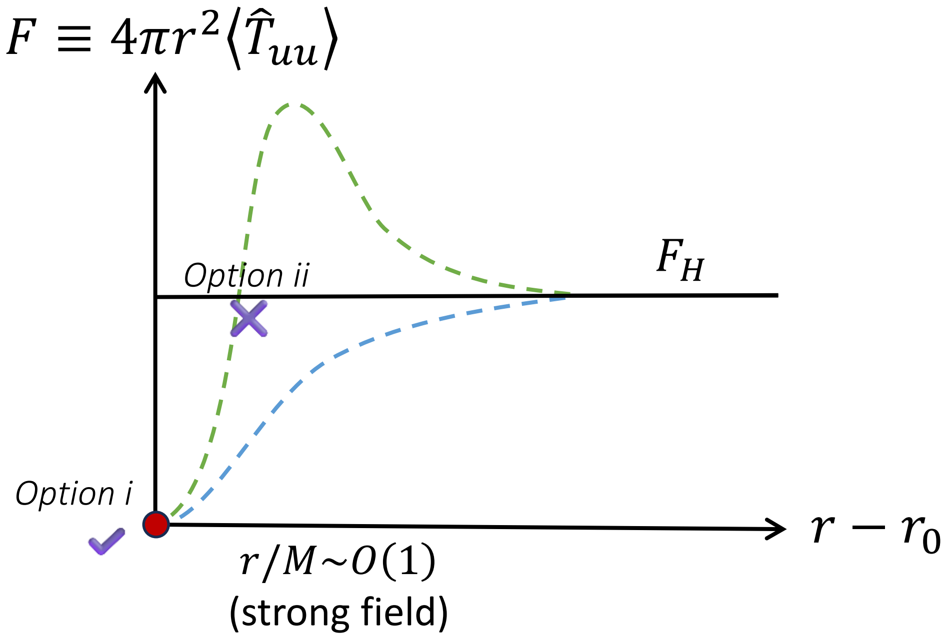
The geodesic leaves the shell at $r = r_0 \approx 2M_0$ and propagates towards $\mathfrak{I}^+$.

The outflux emerging from the shell is *negligibly small* for such a geodesic ($\propto \delta r^2$).

Asymptotic value in the weak field region $r \gg M_0$, $F \to F_H$.

So, the evolution of the outflux from 0 at the shell to F_H at $\mathfrak{I}^+$ *occurs gradually as the ray traverses the strong-field region*.

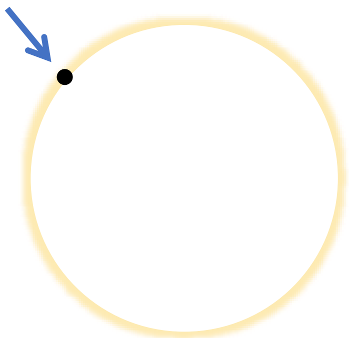




The outfluxes along a late-time outgoing null geodesic $(u = u_0 \gg M_0)$ leaving the shell at $r = r_0 \approx 2M_0$

In the absence of concrete calculations (in **4d** gravitational collapse), it is hard to prove which of the two options is the correct one.

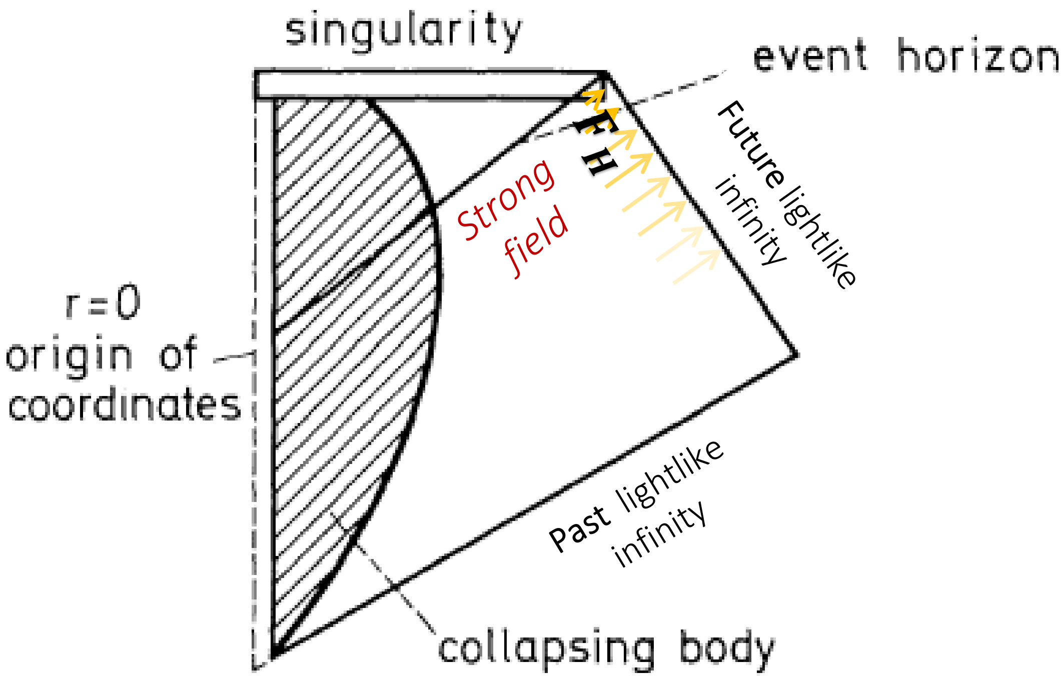
Our main goal is to present such a concrete analysis.



Schwarzschild

$$\langle \hat{T}_{uu} \rangle_{\mathrm{ren}}^{(\mathrm{surface})} \underset{\text{at late times}}{\neq} \langle \hat{T}_{uu} \rangle_{\mathrm{ren}}^{(ii,\mathrm{surface})} = F_H/4\pi(2M)^2$$

$\to 0$ in accordance with option i



Option $\boldsymbol{i}$: the outgoing radiation starts from zero at (extremely close to) the event horizon, and gradually develops over a fairly broad strong-field region

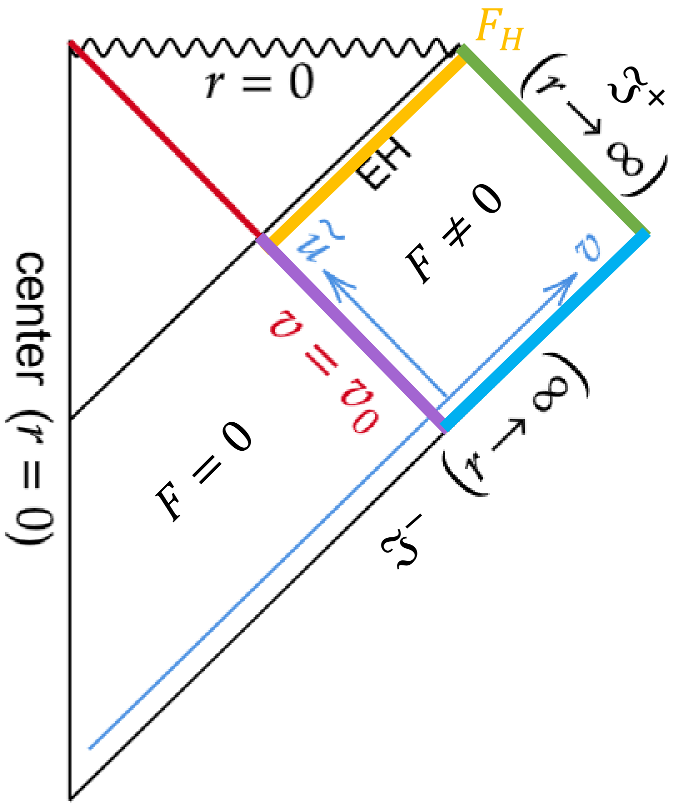
Option $\boldsymbol{ii}$: the energy emerges from the collapsing body itself and propagates in a conserved manner through the surrounding strong-field region

Additional insights

(time permitting)

Bonus topic I: what is now known

for the outflux in the collapsing shell spacetime



The energy outflux

$$F = 4\pi r^2 \langle \hat{T}_{uu} \rangle_{\text{ren}}$$

Along a late-time outgoing null geodesic $u = u_0 \gg M_0$ the outflux $F^{\text{late}}(r)$ is a function of r, and

$$\lim_{r \to 2M} F^{\text{late}}(r) = 0\,, \qquad \lim_{r \to \infty} F^{\text{late}}(r) = F_H$$

Along $\mathfrak{I}^-$ the outflux vanishes (the "in" vacuum state)

Along $\mathfrak{I}^+$ the outflux $F^{\mathfrak{I}^+}(u)$ is a (monotonic?) function of u, and

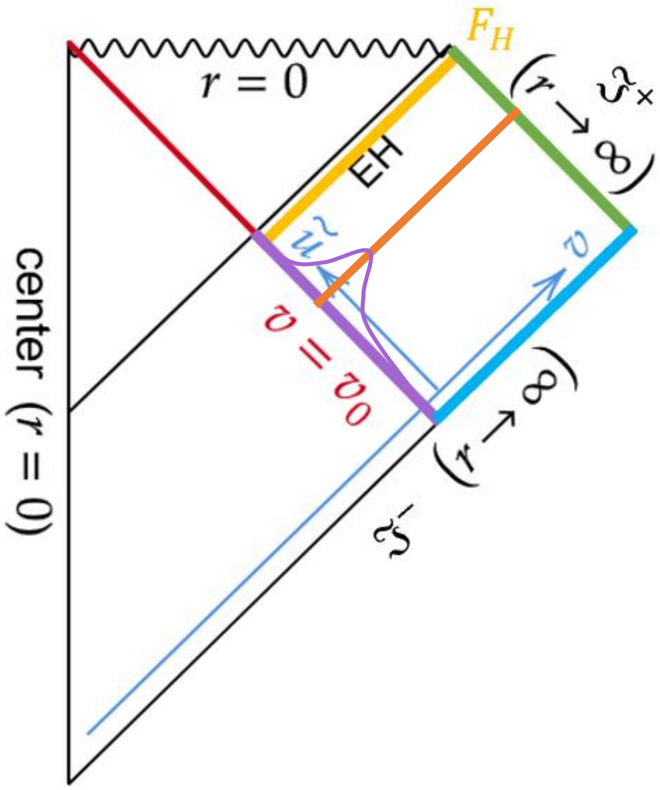
$$\lim_{u \to -\infty} F^{\mathfrak{I}^+}(u) = 0, \qquad \lim_{u \to \infty} F^{\mathfrak{I}^+}(u) = F_H$$

Along the shell $v = v_0$ the outflux $F^+(r_0)$ is now known, and

$$\lim_{r_0 \to \infty} F^+(r_0) = 0\,, \qquad \lim_{r_0 \to 2M} F^+(r_0) = 0$$

Bonus topic I: what is now known

for the outflux in the collapsing shell spacetime



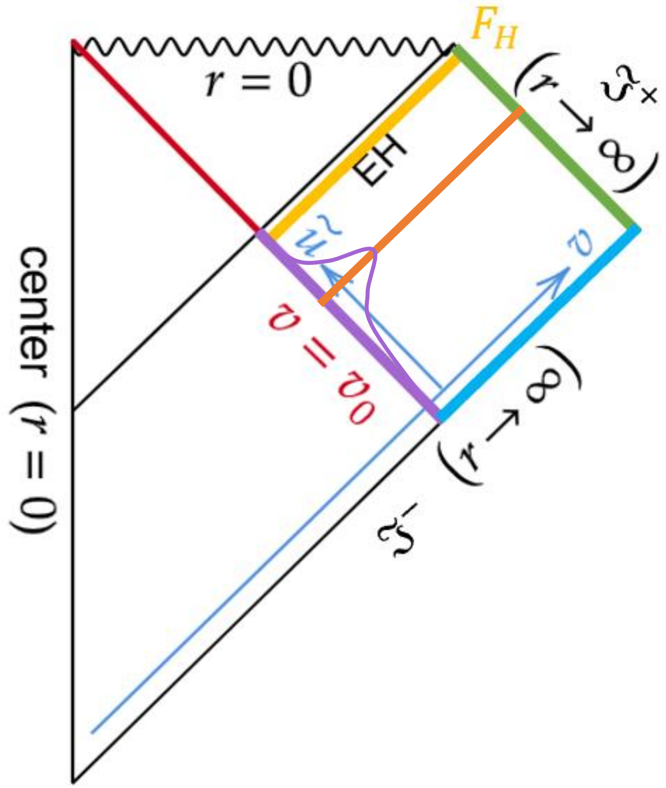
The outflux at the shell $F^+(r_0) = 4\pi r_0^2 \langle \hat{T}_{uu}\rangle^+_{\mathrm{ren}} = \hbar \dfrac{3M_0}{40\pi r_0^5}\left(1-\dfrac{2M_0}{r_0}\right)^2$

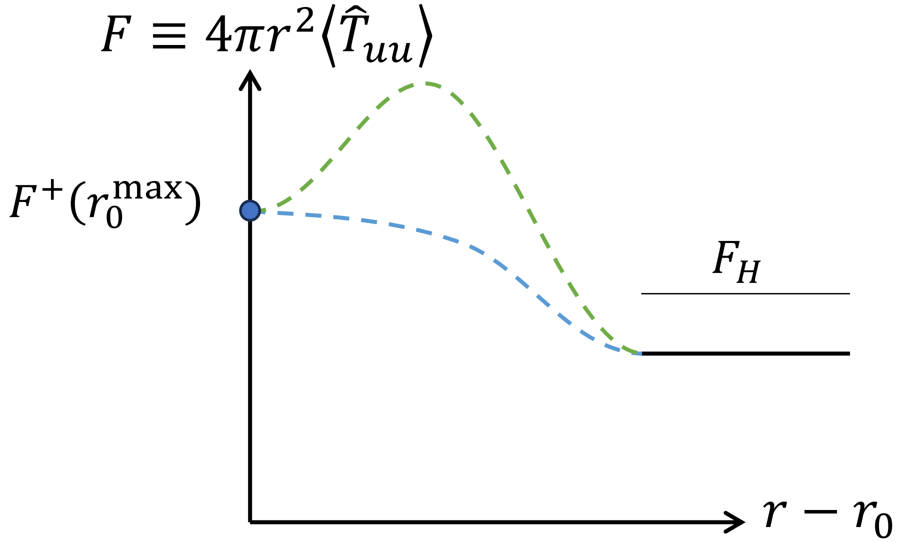
has a maximal value at $r_0^{\mathrm{max}} = \dfrac{10}{3}M$

Maximum outflux at the shell $F^+(r_0^{\mathrm{max}}) = \dfrac{81}{250000\pi}\hbar M^{-2} \approx 1.031\times 10^{-4}\hbar M^{-2}$

The **Hawking** outflux $F^H \approx 7.44\times 10^{-5}\hbar M^{-2}$ Elster (1983)

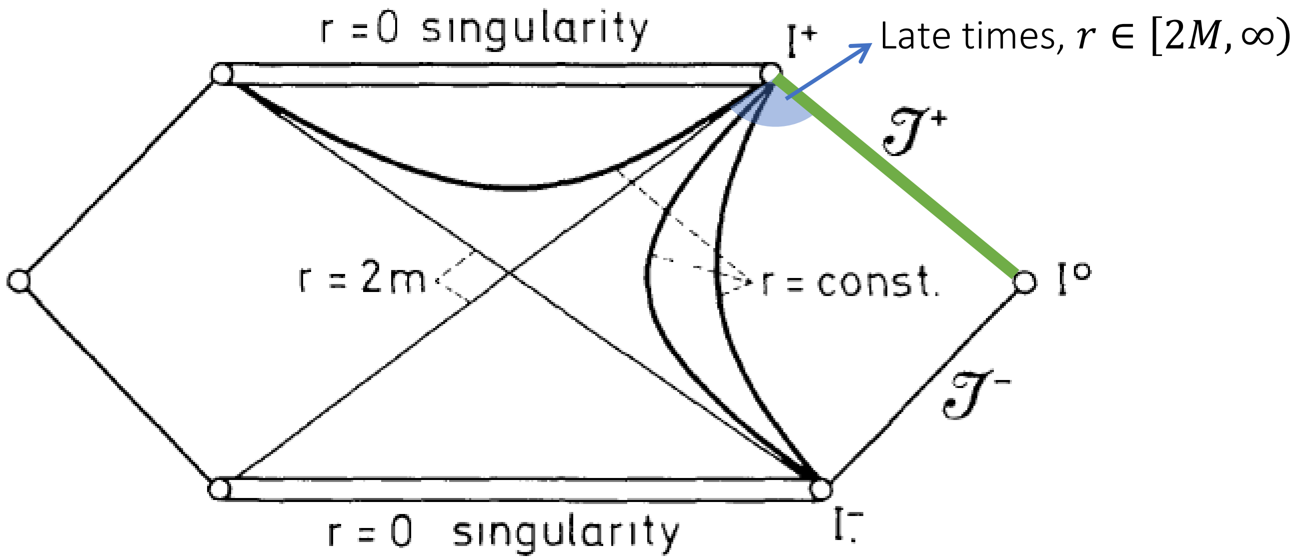
The outflux F may evolve ***non-monotonically*** (with r) along the outgoing null geodesic emanating from the shell at r_0^{max}





The outfluxes along the outgoing null geodesic leaving the shell at $r = r_0^{\max}$

The situation in the Unruh state in Schwarzschild



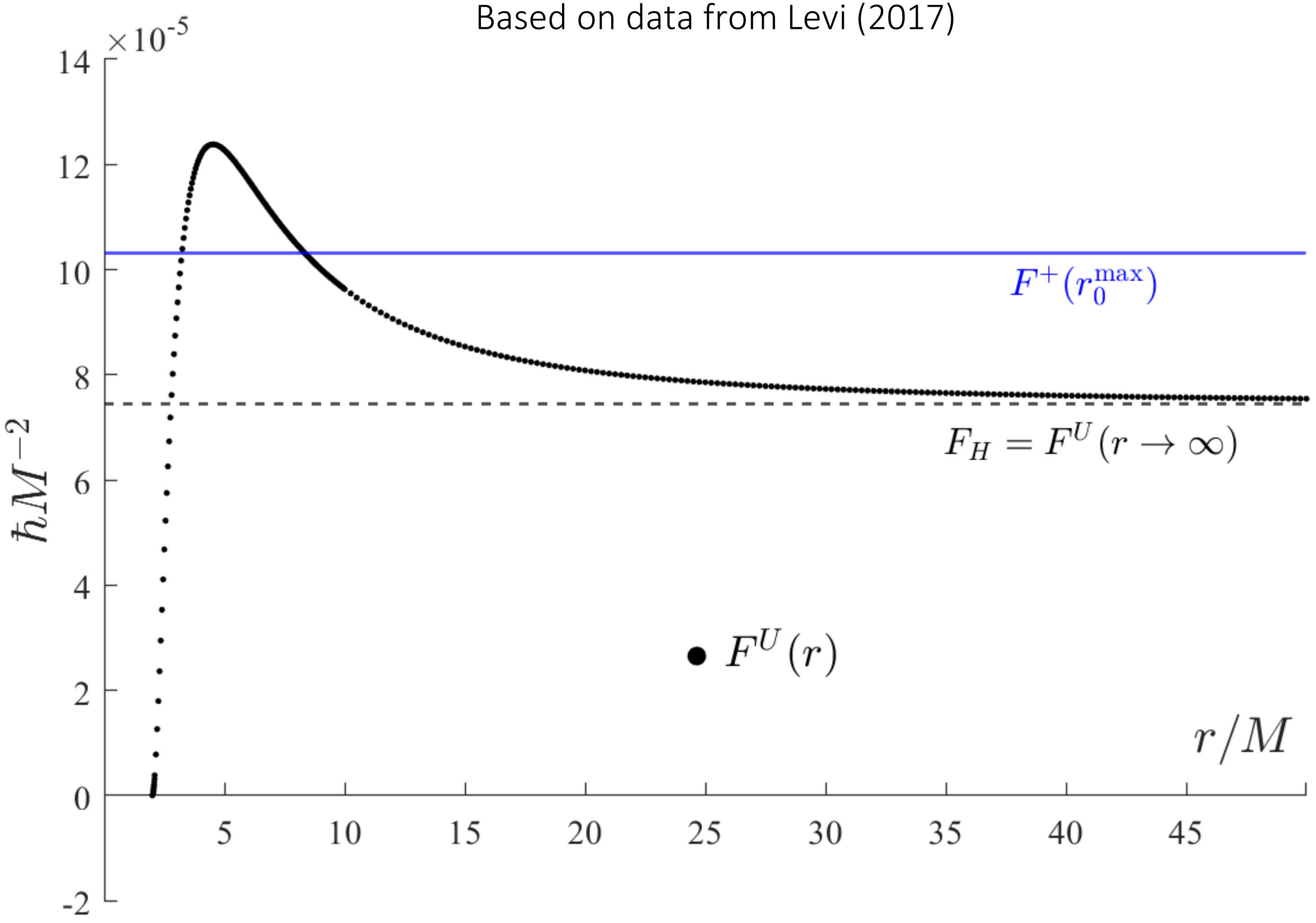
The Unruh state (some facts):

- Is defined on an eternal stationary background (e.g. Schwarzschild)
- Is constructed to mimic gravitational collapse at $u \to \infty, v \to \infty$
- Is stationary (unlike the state in collapse) – everything depends *only* on r
- Involves Hawking radiation at infinity

$$\lim_{r \to \infty} F^U(r) = F^H$$

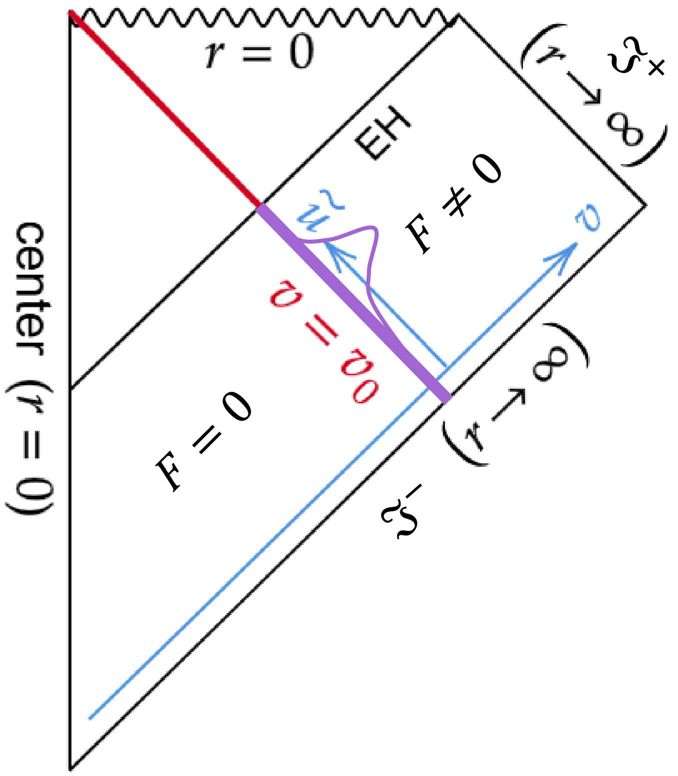
The situation in the Unruh state in Schwarzschild

Based on data from Levi (2017)



Given this behavior in the **Unruh state**, it is not too surprising if the outflux along outgoing null geodesics in the **collapsing-shell** spacetime exhibits a similar non-monotonic trend.

Bonus topic II: Comparison to the situation in 1+1



The energy outflux
$F = 4\pi r^2 \langle \hat{T}_{uu} \rangle_{\text{ren}}$

The outflux at the shell $F^+(r_0) = 4\pi r_0^2 \lim_{v \to v_0^+} \langle \hat{T}_{uu} \rangle_{\text{ren}}$:

<u>3+1:</u>

$$F^+(r_0) = \hbar \frac{3M_0}{40\pi r_0^5}\left(1 - \frac{2M_0}{r_0}\right)^2$$

A step-function discontinuity at the shell!

<u>1+1:</u> (Hiscock, 1981)

PHYSICAL REVIEW D VOLUME 23, NUMBER 12 15 JUNE 1981

Models of evaporating black holes. I

William A. Hiscock

$$ds^2 = -\frac{\bar{u}(v_0 - U - 4m)}{(\bar{u}+4m)(v_0 - U)}\, d\bar{u}\, d\bar{v} \quad (v > v_0). \tag{B10}$$

The stress-energy tensor of the quantized field may now be calculated by the usual procedure. The result in region I is the obvious one,

$$T_{\mu\nu} = 0 \quad (v < 0). \tag{B11}$$

In region II ($v_0 > v > 0$) the stress-energy is

$$T_{u*u*} = (24\pi)^{-1}\left(-\frac{m}{r^3} + \frac{3}{2}\frac{m^2}{r^4} - \frac{8m}{\bar{u}^3} - \frac{24m^2}{\bar{u}^4}\right), \tag{B12}$$

$$T_{vv} = (24\pi)^{-1}\left(-\frac{m}{r^3} + \frac{3}{2}\frac{m^2}{r^4}\right), \tag{B13}$$

$$T_{u*v} = (24\pi)^{-1}\left(-\frac{m}{r^3}\right)\left(1 - \frac{2m}{r}\right). \tag{B14}$$

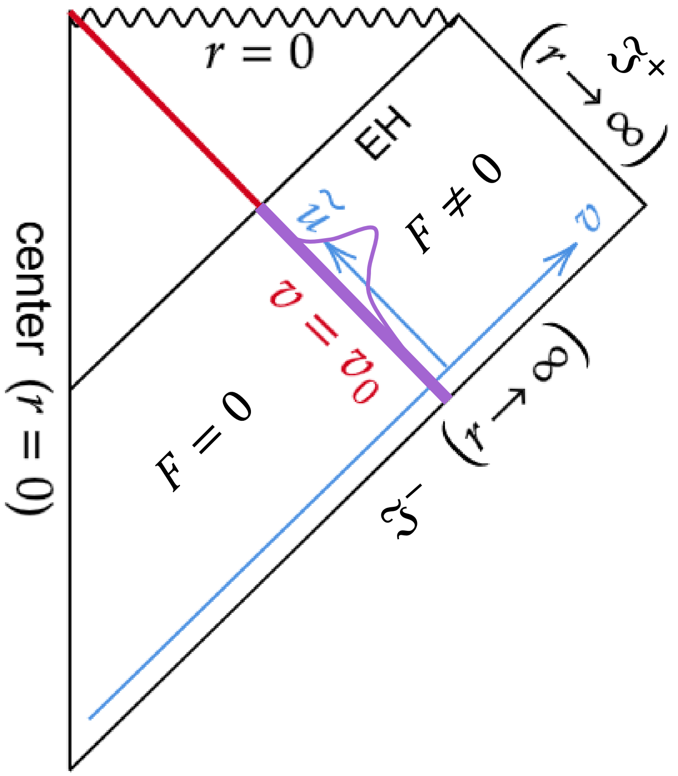
```
(*Result in FabbriNavarro-Salas book Eq. (5.104),
 taking r0=(v0-uin)/2;
 also Hiscock1981 Eq. (B12) where \overline
   {u} is our \tilde{u} with v0=
  0 (where the shell appears here) and thus
   equals 2r0 at the shell,
 and u*=
  v-2r* is the usual Eddington coordinate in
    the Schwarzschild region)
```

$$\text{Tuu} = \frac{1}{24\pi}\left(\frac{-M}{r\hat{\ }3} + \frac{3}{2}\frac{M^2}{r^4} - 8\frac{M}{(-2\,r0)^3} - 24\frac{M^2}{(-2\,r0)^4}\right);$$

$$\boxed{\begin{array}{l}\text{Tuu /. r} \to \text{r0}\\ 0\end{array}}$$

Vanishes on both sides
of the collapsing shell

Bonus topic II: Comparison to the situation in 1+1



The energy outflux
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3+1:

$$F^+(r_0) = \hbar \frac{3M_0}{40\pi r_0^5}\left(1 - \frac{2M_0}{r_0}\right)^2$$

A step-function discontinuity at the shell!

1+1: (Hiscock, 1981)

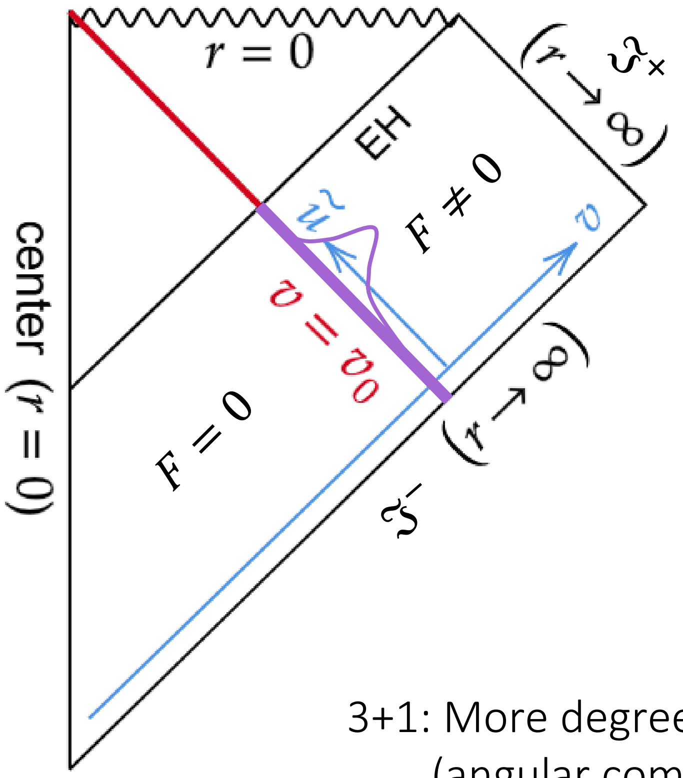
$$F^+(r_0) = 0$$

Continuity at the shell is enforced in the **1+1** case:

$$\langle \hat{T}_{uv} \rangle_{\text{ren}} \propto \langle \hat{T}^{\mu}_{\mu} \rangle_{\text{ren}} \propto R \;\; \text{bounded at the shell}$$

Energy-momentum conservation: $\langle \hat{T}_{uu} \rangle_{\text{ren}}$ **must** be continuous at the null shell
$(T^{\beta}{}_{\alpha;\beta} = 0)$

Bonus topic II: Comparison to the situation in 1+1



The energy outflux
$F = 4\pi r^2 \langle \hat{T}_{uu} \rangle_{\text{ren}}$

The outflux at the shell $F^+(r_0) = 4\pi r_0^2 \lim_{v \to v_0^+} \langle \hat{T}_{uu} \rangle_{\text{ren}}$:

3+1:

$$F^+(r_0) = \hbar \frac{3M_0}{40\pi r_0^5}\left(1 - \frac{2M_0}{r_0}\right)^2$$

A step-function discontinuity at the shell!

1+1: (Hiscock, 1981)

$$F^+(r_0) = 0$$

Continuity at the shell is NOT enforced in the ~~1+1~~ 3+1 case:

3+1: More degrees of freedom (angular components)

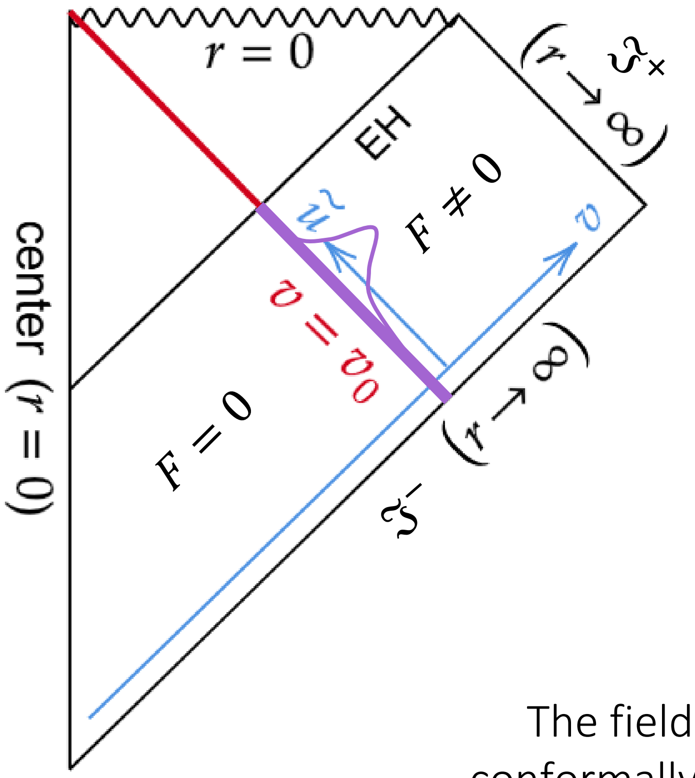
$\langle \hat{T}_{uv} \rangle_{\text{ren}}$ NOT $\propto \langle \hat{T}^\mu_\mu \rangle_{\text{ren}} \propto R$ bounded at the shell

Energy-momentum conservation: $\langle \hat{T}_{uu} \rangle_{\text{ren}}$ **must** be continuous at the null shell

$(T^\beta{}_{\alpha;\beta} = 0)$

Bonus topic II: Comparison to the situation in 1+1

The outflux at the shell $F^+(r_0) = 4\pi r_0^2 \lim_{v\to v_0^+} \langle \hat{T}_{uu} \rangle_{\text{ren}}$:



<u>3+1:</u>

$$F^+(r_0) = \hbar \frac{3M_0}{40\pi r_0^5}\left(1 - \frac{2M_0}{r_0}\right)^2$$

A step-function **discontinuity** at the shell!

<u>1+1:</u> (Hiscock, 1981)

$$F^+(r_0) = 0$$

Continuity at the shell is NOT enforced in the ~~1+1~~ 3+1 case:

The field is not conformally-coupled

$\langle \hat{T}_{uv} \rangle_{\text{ren}}$ NOT $\propto \langle \hat{T}^\mu_\mu \rangle_{\text{ren}}$ NOT $\propto R$ bounded at the shell

local term $-\frac{1}{2}\Box\langle\hat{\Phi}^2\rangle_{\text{ren}}$

Energy-momentum conservation: $\langle \hat{T}_{uu} \rangle_{\text{ren}}$ NOT **must** be continuous at the null shell in 3+1

$(T^\beta{}_{\alpha;\beta} = 0)$

The energy outflux
$F = 4\pi r^2 \langle \hat{T}_{uu} \rangle_{\text{ren}}$

Summary

- We computed the outflux at the outer surface of a collapsing thin null shell:

$$F^+(r_0) = 4\pi r_0^2 \langle \hat{T}_{uu} \rangle^+_{\mathrm{ren}} = \hbar \frac{3M_0}{160\pi^2 r_0^7} (r_0 - 2M_0)^2$$

- Crucially, as the shell approached the horizon, the emitted outflux vanishes quadratically with the remaining distance $r_0 - 2M_0$ (*exponentially* in u).

- This result confirms the Hawking energy outflux does not emerge from the collapsing body, as is sometimes speculated, but develops in a broad strong-field region outside. This corroborates the more conventional picture of Hawking radiation as a gradual, steady process that *cannot* interfere with black hole formation in gravitational collapse.

Take away message:

Semiclassical backreaction does not significantly modify the classical horizon geometry and does not give rise to horizon avoidance.

Thank you for listening!

The paper presented in this talk:
PRL 135, 021401 (2025) | arXiv:2503.00622

Questions?

nz3745@princeton.edu

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The trace of a minimally-coupled massless scalar field in 3+1

$$\langle T^{\alpha}_{\alpha} \rangle_{ren} = T_{\text{anomaly}} - \frac{1}{2} \Box \left\langle \hat{\Phi}^2 \right\rangle_{ren} \qquad \text{(minimally coupled massless field)}\,.$$

$$T_{\text{anomaly}} \equiv \frac{1}{2880\pi^2} \left(R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} - R_{\alpha\beta} R^{\alpha\beta} + \frac{5}{2} R^2 + 6 \Box R \right)$$

Energy-momentum conservation in 3+1

$$0 = T^{\beta}_{\alpha;\beta} = \frac{1}{\sqrt{-g}}\left(\sqrt{-g}T^{\beta}_{\alpha}\right)_{,\beta} - \frac{1}{2}g_{\beta\mu,\alpha}T^{\beta\mu}$$

$$0 = \left(r^2 T_{\tilde{u}v}\right)_{,\tilde{u}} + \left(r^2 T_{\tilde{u}\tilde{u}}\right)_{,v} - r^2 g^{\tilde{u}v} g_{\tilde{u}v,\tilde{u}} T_{\tilde{u}v} - g_{\tilde{u}v} g_{\theta\theta,\tilde{u}} T^{\theta}_{\theta}$$

Plugging the step function behavior of $T_{\tilde{u}\tilde{u}}$ (which is exactly that of T_{uu}) at the shell:

$$r^2 T_{\tilde{u}\tilde{u}} = \alpha\left(\tilde{u}, v\right)\Theta\left(v - v_0\right)$$

$$0 = \left(r^2 T_{\tilde{u}v}\right)_{,\tilde{u}} + \left(\alpha\Theta\left(v - v_0\right)\right)_{,v} - r^2 g^{\tilde{u}v} g_{\tilde{u}v,\tilde{u}} T_{\tilde{u}v} - g_{\tilde{u}v} g_{\theta\theta,\tilde{u}} T^{\theta}_{\theta}$$

$$0 = \left(r^2 T_{\tilde{u}v}\right)_{,\tilde{u}} + \alpha\left(\tilde{u}, v\right)_{,v}\Theta\left(v - v_0\right) + \alpha\delta\left(v - v_0\right) - r^2 g^{\tilde{u}v} g_{\tilde{u}v,\tilde{u}} T_{\tilde{u}v} - g_{\tilde{u}v} g_{\theta\theta,\tilde{u}} T^{\theta}_{\theta}$$

We see the δ-function along $v = v_0$ can come from either $T_{\tilde{u}v}$ (both green expressions, since the derivative in the first is wrt $\tilde{u}$ and not v, can give a $\delta\left(v - v_0\right)$ if $T_{\tilde{u}v} \propto \delta\left(v - v_0\right)$), or from $T^{\theta}_{\theta} = T^{\varphi}_{\varphi}$. So there are two possible sources for the required δ function. This is unlike the 2D case, in which the δ function *has to* come from $T_{\tilde{u}v}$, which is entirely determined by the trace anomaly.

Our result for the vacuum polarization at the shell's exterior surface

$$\langle \hat{\Phi}^2 \rangle^{+}_{\mathrm{ren}}(r_0) = \frac{\hbar}{24\pi^2} \frac{M_0}{r_0^3}$$

Incidentally, when evaluating this result at $r_0 = 2M_0$, one obtains $\hbar/192\pi^2 M_0^2$, which is precisely the value of $\langle \hat{\Phi}^2 \rangle_{\mathrm{ren}}$ at the EH of a Schwarzschild BH in the Hartle-Hawking state :

P. Candelas, Vacuum polarization in Schwarzschild spacetime, Phys. Rev. D **21**, 2185 (1980).

The θ-splitting PMR counter-terms, obtained by processing Christensen's counter-terms and expanding them in the splitting parameter $\epsilon \equiv \delta\theta = \theta' - \theta$, have the general form [8]:

$$\frac{1}{\hbar}\tilde{L}_{\mu\nu}(\epsilon) = a_{\mu\nu}\sin^{-4}(\epsilon/2) + b_{\mu\nu}\sin^{-2}(\epsilon/2) + c_{\mu\nu}\sin^{-1}(\epsilon/2) + d_{\mu\nu}\log(\sin(\epsilon/2)) + e_{\mu\nu}$$

For the two flux components, corresponding to $\mu = \nu = y$, we have:

$(y = u$ or $v)$

$$a_{yy} = 0$$

$$b_{yy} = f^2(r)\frac{1}{64\pi^2 r^4}$$

$$c_{yy} = 0$$

$$\boxed{d_{yy} = f^2(r)\frac{Q^2}{240\pi^2 r^6}}$$

The method presented for computing $\langle T_{uu}\rangle^{+}_{\mathbf{ren}}$ only holds for $Q = 0$.
For $Q \neq 0$, the ε dependence does **not** drop in the difference between the interior and exterior counterterms.

$$e_{yy} = f^2(r)\frac{2Q^2(7+\gamma) + Q^2\log\left(r^2\mu^2\right) - 9Mr}{480\pi^2 r^6}$$

where γ is the Euler constant, μ is an unknown scale parameter related to the ambiguity in the regularization procedure (see more in Sec. 6), and, recall, $f(r) = (r - r_-)(r - r_+)/r^2$. This leaves us with