From Black Holes to the Cosmos: Matt Visser's journey through space and time, Trieste, Italy

Cosmological Singularities and Quantum Particles

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Based on

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Introduction

- Appearance of singularities is one of the most important phenomena in General Relativity and its generalizations and modifications.
- The singularities were first discovered in such simple geometries as those of Friedmann and Schwarzschild and later their general character was established (Penrose, Hawking).
- The investigation of the oscillatory approach to the cosmological singularity (Belinsky, Khalatnikov, Lifshitz) known also as Mixmaster universe (Misner) has opened the way to the birth of a new branch of the mathematical physics chaotic cosmology and hyperbolic Kac-Moody algebras (Damour, Henneaux, Nicolai).

Introduction

- What happens when a universe approaching the cosmological singularity?
- One can try to study the opportunity to cross the singularity.
- Very interesting effects arise at the singularities crossing.
- What happens with quantum particles at the singularity crossing?

Singularities

We shall consider a flat Friedmann universe

$$ds^2 = dt^2 - a^2(t)dl^2.$$

Big Bang - Big Crunch

$$a(t) \to 0, \quad t \to t_0,$$
$$\frac{\dot{a}(t)}{a(t)} \to \pm\infty, \quad t \to t_0.$$

Big Rip

$$
\begin{aligned}
&a(t) \to \infty, \quad t \to t_0 < \infty, \\
&\frac{\dot{a}(t)}{a(t)} \to \infty, \quad t \to t_0, \\
&w = \frac{p}{\rho} < -1.
\end{aligned}
$$

Big Brake and other soft singularities

$$a(t) \to a_0 < \infty, \quad t \to t_0 < \infty,$$
$$\dot{a}(t) \to \dot{a}_0 < \infty, \quad t \to t_0 < \infty,$$
$$\ddot{a}(t) \to -\infty, \quad t \to t_0 < \infty.$$

If $\dot{a}_0 = 0$, it is Big Brake.

For example it can be realized for the anti-Chaplygin gas,

$$p = \frac{A}{\rho}, \quad A > 0.$$

What does it mean to describe cosmological singularity crossing?

On choosing some convenient field parametrization one can provide a matching between the characteristics of the universe before and after the singularity crossing.
On choosing appropriate combinations of the field variables we can describe the passage through the Big Bang - Big Crunch singularity, but this does not mean that the presence of such a singularity is not essential. Indeed, extended objects cannot survive this passage.

Scalar particles

What happens with particles (in quantum field theoretical sense) when the universe passes through the cosmological singularity ?
The scalar field in the flat Friedmann universe satisfies the Klein-Gordon equation:

$$\Box\phi + V'(\phi) = 0.$$

One can consider a spatially homogeneous solution of this equation ϕ_0, depending only on time t as a classical background.
A small deviation from this background solution can be represented as a sum of Fourier harmonics satisfying linearized equations

$$\ddot{\phi}(\vec{k}, t) + 3\frac{\dot{a}}{a}\dot{\phi}(\vec{k}, t) + \frac{\vec{k}^2}{a^2}\phi(\vec{k}, (t)) + V''(\phi_0(t))\phi(\vec{k}, (t)) = 0.$$

The corresponding quantized field is

$$\hat{\phi}(\vec{x},t) = \int d^3\vec{k}(\hat{a}(\vec{k})u(k,t)e^{i\vec{k}\cdot\vec{x}} + \hat{a}^+(\vec{k})u^*(k,t)e^{-i\vec{k}\cdot\vec{x}}),$$

where the creation and the annihilation operators satisfy the standard commutation relations:

$$[\hat{a}(\vec{k}),\hat{a}^+(\vec{k}')] = \delta(\vec{k}-\vec{k}').$$

The basis functions should be normalized so that the canonical commutation relations between the field ϕ and its canonically conjugate momentum $\hat{\mathcal{P}}$ were satisfied

$$[\hat{\phi}(\vec{x},t),\hat{\mathcal{P}}(\vec{y},t')] = i\delta(\vec{x}-\vec{y}).$$

$$u(k,t)\dot{u}^*(k,t) - u^*(k,t)\dot{u}(k,t) = \frac{i}{(2\pi)^3 a^3(t)}.$$

The linearized Klein-Gordon equation has two independent solutions.

To define a particle it is necessary to have two independent non-singular solutions.

It is a non-trivial requirement in the situations when a singularity or other kind of irregularity of the spacetime geometry occurs.

It is convenient also to construct explicitly the vacuum state for quantum particles as a Gaussian function of the corresponding variable. Let is introduce an operator

$$\hat{f}(\vec{k},t) = (2\pi)^3(\hat{a}(\vec{k})u(k,t) + \hat{a}^+(-\vec{k})u^*(k,t)).$$

Its canonically conjugate momentum is

$$\hat{p}(\vec{k},t) = a^3(t)(2\pi)^3(\hat{a}(\vec{k})\dot{u}(k,t) + \hat{a}^+(-\vec{k})\dot{u}^*(k,t)).$$

We can express the annihilation operator as

$$\hat{a}(\vec{k}) = i\hat{p}(\vec{k}, t)u^*(k, t) - ia^3(t)\hat{f}(\vec{k}, t)\dot{u}^+(k, t).$$

Representing the operators $\hat{f}$ and $\hat{p}$ as

$$\hat{f} \to f, \quad \hat{p} \to -i\frac{d}{df},$$

one can write down the equation for the corresponding vacuum state in the following form:

$$\left(u^*\frac{d}{df} - ia^3\dot{u}^*f\right)\Psi_0(f) = 0.$$

$$\Psi_0(f) = \frac{1}{\sqrt{|u(k, t)|}}\exp\left(\frac{ia^3(t)\dot{u}^*(k, t)f^2}{2u^*(k, t)}\right).$$

In the case of the Big Bang - Big Crunch singularity, one of the basis functions in the vicinity of the singularity becomes singular and it is impossible to construct a Fock space.

In the case of the Big Rip singularity, when in finite interval of time the universe achieves an infinite volume and infinite time derivative of the scale factor, the Fock space can be constructed for a spectator scalar field, but it does not exist for the phantom scalar field driving the expansion.

In the case of the model with Big Rip singularity one can construct the Fock space for scalar particles.

Fermions

The action for the Dirac fermions in a curved spacetime is

$$S_f = \int d^4x \sqrt{-g} \left[\frac{i}{2} \left(\bar{\psi} \gamma^\mu \nabla_\mu \psi - \nabla_\mu \bar{\psi} \gamma^\mu \psi \right) - m \bar{\psi} \psi \right].$$

The Dirac equation on the flat Friedmann background is

$$\left[i\gamma^0 \left(\partial_t + \frac{3}{2} \frac{\dot{a}}{a} \right) + \frac{i}{a} \gamma^i \partial_i - m \right] \psi = 0.$$

The spinor field operator ψ can be represented in the following form:

$$\hat{\psi}(\vec{x},t) = a^{-3/2}(t)\int \frac{d^3k}{(2\pi)^{3/2}}\sum_{s=1}^{2}\Big[\hat{a}_s(\vec{k})u_s(\vec{k},t)e^{i\vec{k}\cdot\vec{x}}$$

$$+\hat{b}_s^\dagger(\vec{k})v_s(\vec{k},t)e^{-i\vec{k}\cdot\vec{x}}\Big].$$

Here $\hat{a}_s(\vec{k}), \hat{a}_s^\dagger(\vec{k})$ and $\hat{b}_s(\vec{k}), \hat{b}_s^\dagger(\vec{k})$ are the operators of annihilation and creation for particles and antiparticles respectfully, while $u_s(\vec{k},t)$ and $v_s(\vec{k},t)$ are the corresponding basis functions.

The weight factor $a^{-3/2}$ is especially chosen in order to write the action in a special parametrization reflecting the conformal properties of the spinor field. It is conformally invariant in the massless case and, therefore, decouples in this parametrization from the gravitational background.

Each four-component Dirac spinor consists of two two-component Weyl spinors:

$$u_s(\vec{k},t) = \begin{pmatrix} u_k(t)\xi_s(\vec{k}) \\ -\tilde{u}_k(t)\xi_s(\vec{k}) \end{pmatrix}, \qquad v_s(\vec{k},t) = \begin{pmatrix} v_k(t)\xi_s(\vec{k}) \\ -\tilde{v}_k(t)\xi_s(\vec{k}) \end{pmatrix},$$

The two-component spinors $\xi(\vec{k})$ satisfy the following equations

$$\vec{\sigma} \cdot \hat{k}\, \xi_s(\vec{k}) = \lambda_s\, \xi_s(\vec{k}), \qquad \lambda_s = \pm 1,$$

and to the normalisation condition

$$\xi_s^\dagger(\vec{k})\xi_{s'}(\vec{k}) = \delta_{ss'}.$$

The basis functions satisfy the following equations:

$$i\partial_t\, u_s(\vec{k}, t) = \left(\frac{\vec{\alpha} \cdot \vec{k}}{a(t)} + \beta m\right) u_s(\vec{k}, t),$$

$$i\partial_t\, v_s(\vec{k}, t) = \left(-\frac{\vec{\alpha} \cdot \vec{k}}{a(t)} + \beta m\right) v_s(\vec{k}, t).$$

In the Weyl representation

$$\beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \qquad \vec{\alpha} = \begin{pmatrix} -\vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix},$$

we have

$$i\dot{u}_k = -\nu_k u_k - m\tilde{u}_k,$$
$$i\dot{\tilde{u}}_k = \nu_k \tilde{u}_k - m u_k,$$

$$i\dot{v}_k = \nu_k v_k - m\tilde{v}_k,$$
$$i\dot{\tilde{v}}_k = -\nu_k \tilde{v}_k - m v_k,$$

where

$$\nu_k(t) = \frac{k}{a(t)}.$$

The corresponding second order equation for the scalar function u_k is

$$\ddot{u}_k + \big(-i\dot{\nu}_k + \nu_k^2 + m^2\big)u_k = 0,$$

and for the scalar function v_k

$$\ddot{v}_k + \big(i\dot{\nu}_k + \nu_k^2 + m^2\big)v_k = 0.$$

Now we are in a position to study the behavior of fermion quantum particles in the vicinity of different kinds of singularities.

$$\ddot{u}_k + \left(i\frac{k\dot{a}}{a^2} + \frac{k^2}{a^2} + m^2\right) u_k = 0,$$
$$\ddot{v}_k + \left(-i\frac{k\dot{a}}{a^2} + \frac{k^2}{a^2} + m^2\right) v_k = 0.$$

It is enough to consider one of these two equations, because the asymptotic behavior of their solutions in the vicinity of singularity is the same.

Results

For all three types of the cosmological singularities (Big-Bang–Big Crunch, Big Rip, Big Brake and other soft singularities) the second-order equations for the fermion basis functions have two independent non-singular solutions and one can construct the Fock space for the quantum particles in the vicinity (or at the crossing) of these singularities.

Conclusions

- Fermions are “resistant” at the passing through all the types of the singularities in contrast to the scalar particles.
- This result was not expected.
- It would be interesting to understand if there are some deep reasons behind of it.