Gravitational collapse and integrable singularities

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From Black Holes to the Cosmos

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Theory and Phenomenology of Fundamental Interactions
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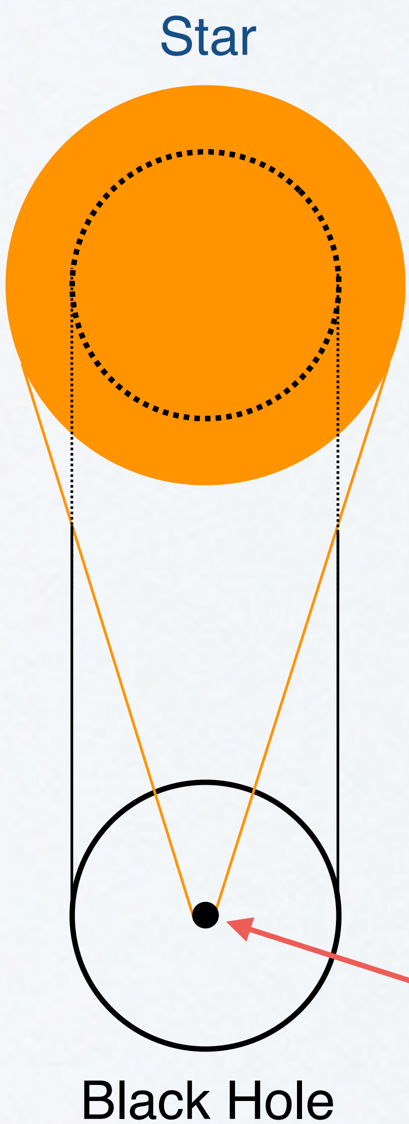
Summary

- Preamble: gravitational collapse
- Regular and integrable densities
- Toy kinematic model: collapsing Schwarzschild mimickers
- Quantum Gravity: gravitational collare and dust core

Works in collaboration with: A. Giusti, A. Y. Kamenshchik, J. Ovalle, …

Preamble - QG and gravitational collapse

BH form from collapse - the classical view:



- $|\,\mathrm{matter}\rangle$ ~ very large number of SM particles ($M_\odot \sim 10^{57}$ neutrons)
- $|\,\mathrm{gravity}\rangle$ ~ very large number of "gravitons" [1] ($N_{\rm G} \sim M_\odot^2/m_{\rm p}^2 \sim 10^{76}$)

$\downarrow$

Reduce and (not over-)simplify

(how…?)

Singularity ~ CED $\longrightarrow$ QED … but … GR ~ QCD

CED $\rightarrow$ "Atomic collapse"

QED $\rightarrow$ Atoms

[1] J.D. Bekenstein, PRD 7 (1973) 2333

Regular and Integrable densities

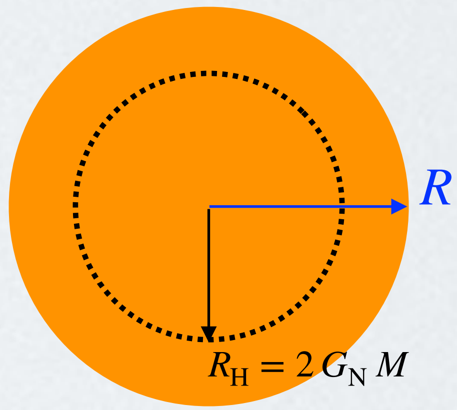
MisnerSharpHernandez mass: $ds^2 = g_{tt}\,dt^2 + g_{rr}\,dr^2 + r^2\,d\Omega^2$

Einstein equations

$$g^{rr} = 1 - \frac{2\,G_N\,m(r)}{r}$$

$$m(r) = 4\,\pi \int_0^r \rho(x)\,x^2\,dx \qquad 0 \le r \le R$$

Star



Regular (central) density: $\rho(r \sim 0) \sim \rho_0 \implies m(r \sim 0) \sim r^3$

Finite geometric invariants ~ Finite tidal forces

Classical configuration

Regular and Integrable densities

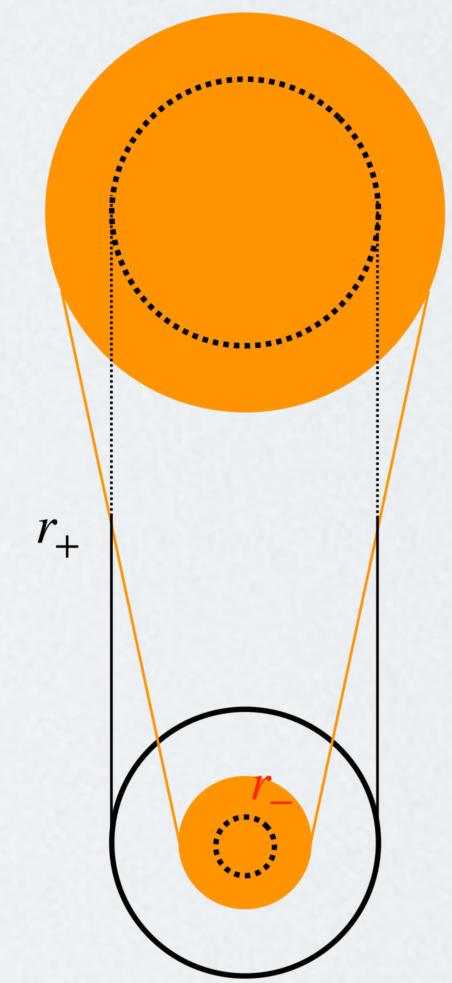
MSH mass function [1]:

$$ds^2 = g_{tt}\,dt^2 + g_{rr}\,dr^2 + r^2\,d\Omega^2$$

Einstein equations

$$g^{rr} = 1 - \frac{2\,G_N\,m(r)}{r}$$

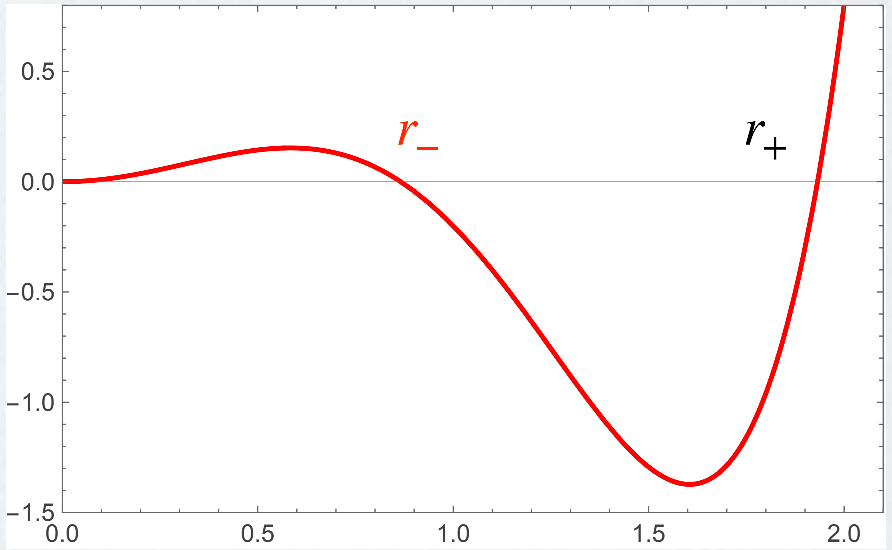
$$m(r) = 4\,\pi \int_0^r \rho(x)\,x^2\,dx \qquad 0 \le r \le R < r_+ = R_H$$



Regular (central) density: $\rho(r \sim 0) \sim \rho_0 \implies m(r \sim 0) \sim r^3$

Horizon(s): $g^{rr} = 0 \implies \Delta = r_\pm^2 - 2\,r_\pm\,m(r_\pm) = 0$

Inner Cauchy horizon



[1] R.C., A. Giusti, J. Ovalle, JHEP 05 (2023) 118 [arXiv:2303.02713]

Regular and Integrable densities

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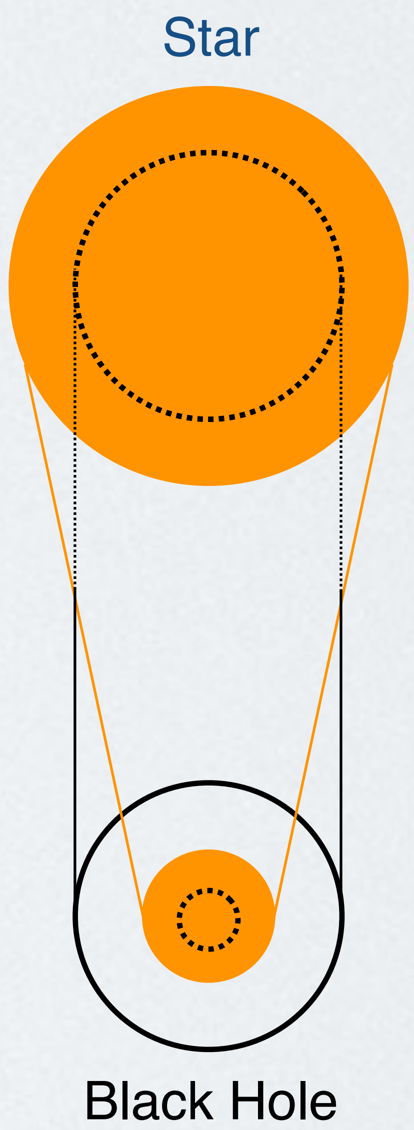
$\Big\downarrow$ Einstein equations

$$g^{rr} = 1 - \frac{2\, G_{\mathrm{N}}\, m(r)}{r}$$

$$m(r) = 4\,\pi \int_0^r \rho(x)\, x^2\, dx \simeq 4\,\pi \int_0^r \mu\, |\psi(x)|^2\, x^2\, dx < \infty$$



Regular (central) density: $\rho(r) \sim \rho_0 \implies m(r) \sim r^3$

Integrable density: $m(r) \sim r^{1+n} < \infty \implies \rho(r) \sim r^{n-2} \qquad (-1 < n < 2)$

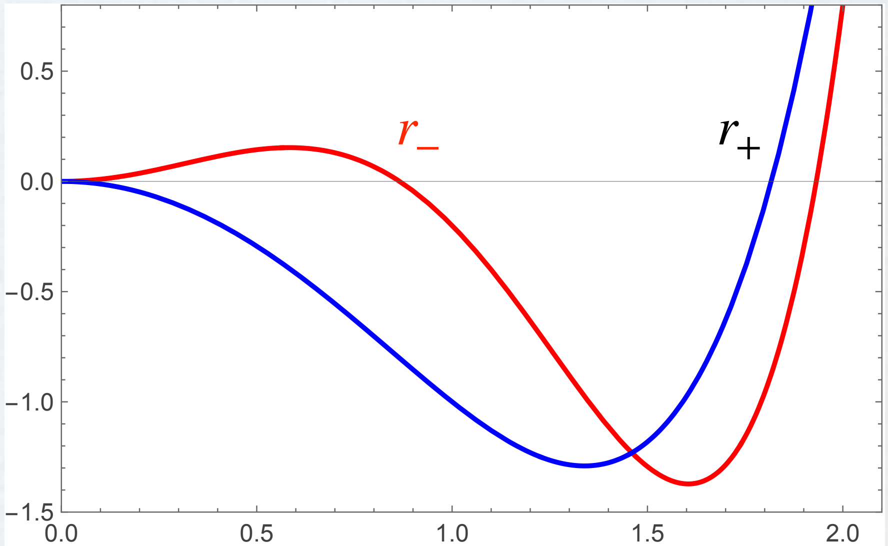
Horizon(s): $g^{rr} = 0 \implies \Delta = r_\pm^2 - 2\, r_\pm\, m(r_\pm) = 0$

$$r = r_\pm \qquad (0 < n < 2)$$

$$r = r_+ \qquad (-1 < n < 0)$$

$\Big\downarrow$

“Quantum” configuration ~ no Cauchy horizon



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Regular and Integrable densities

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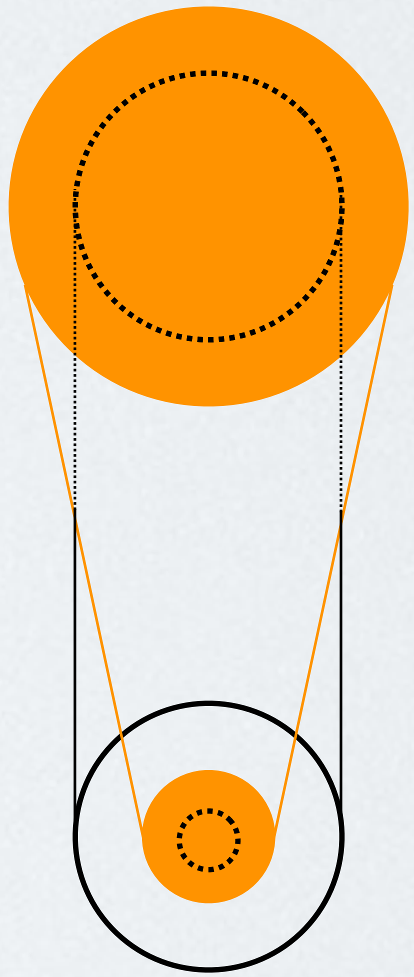

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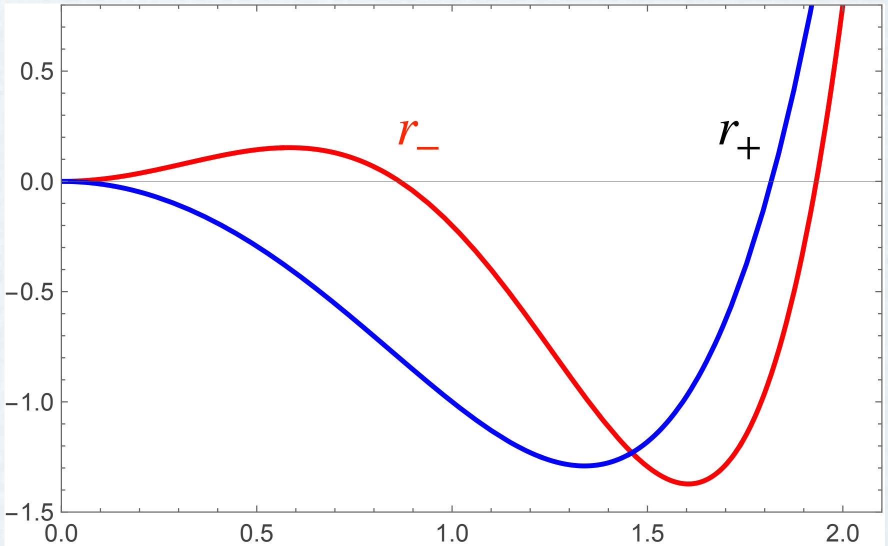


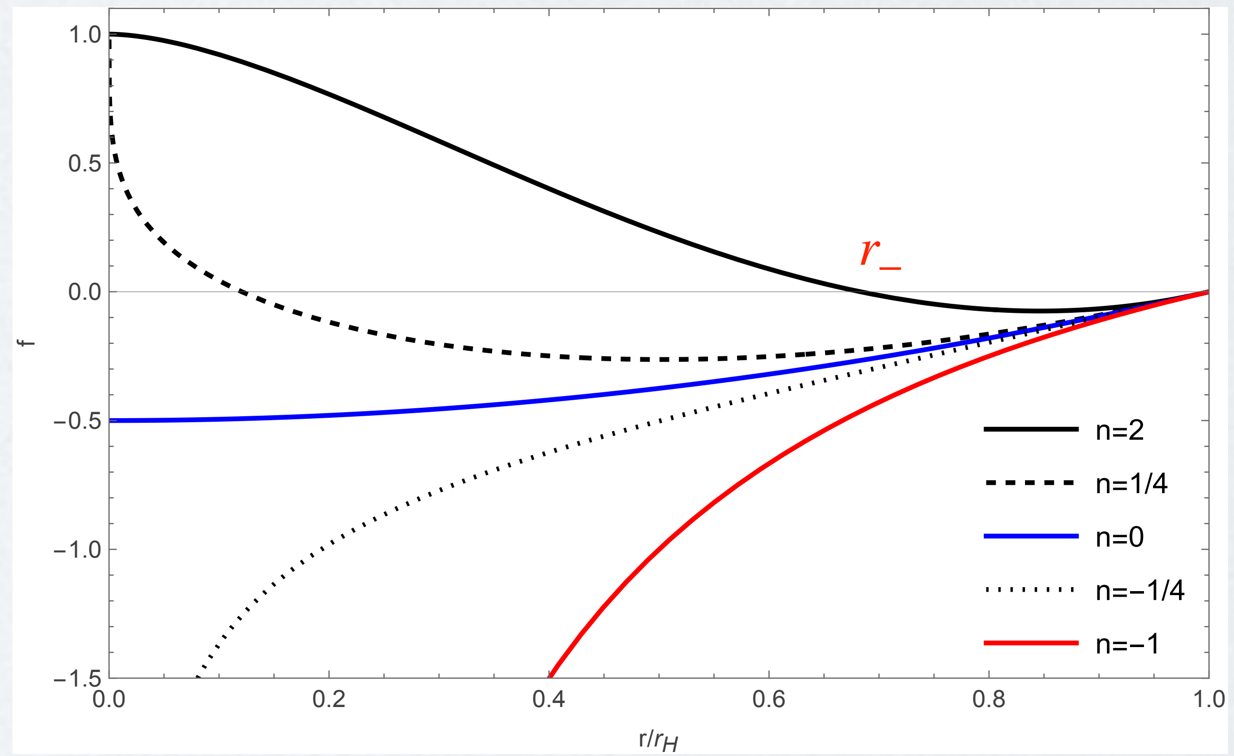
Schwarzschild: $m(r) = M \implies \Delta = r_H^2 - 2\,r_H\,G_N\,M \quad (n \to -1)$

[1] R.C., A. Giusti, J. Ovalle, JHEP 05 (2023) 118 [arXiv:2303.02713]
[2] R.C., A. Giusti, A. Kamenshchik, J. Ovalle, to appear in EPJC [arXiv:2605.01808]

Collapsing Schwarzschild mimickers

Mimicker geometry [1,2,3]: $$ds^2 = -f(r)\,dv^2 + 2\,dv\,dr + r^2\,d\Omega^2$$ (ingoing light rays: v = constant)

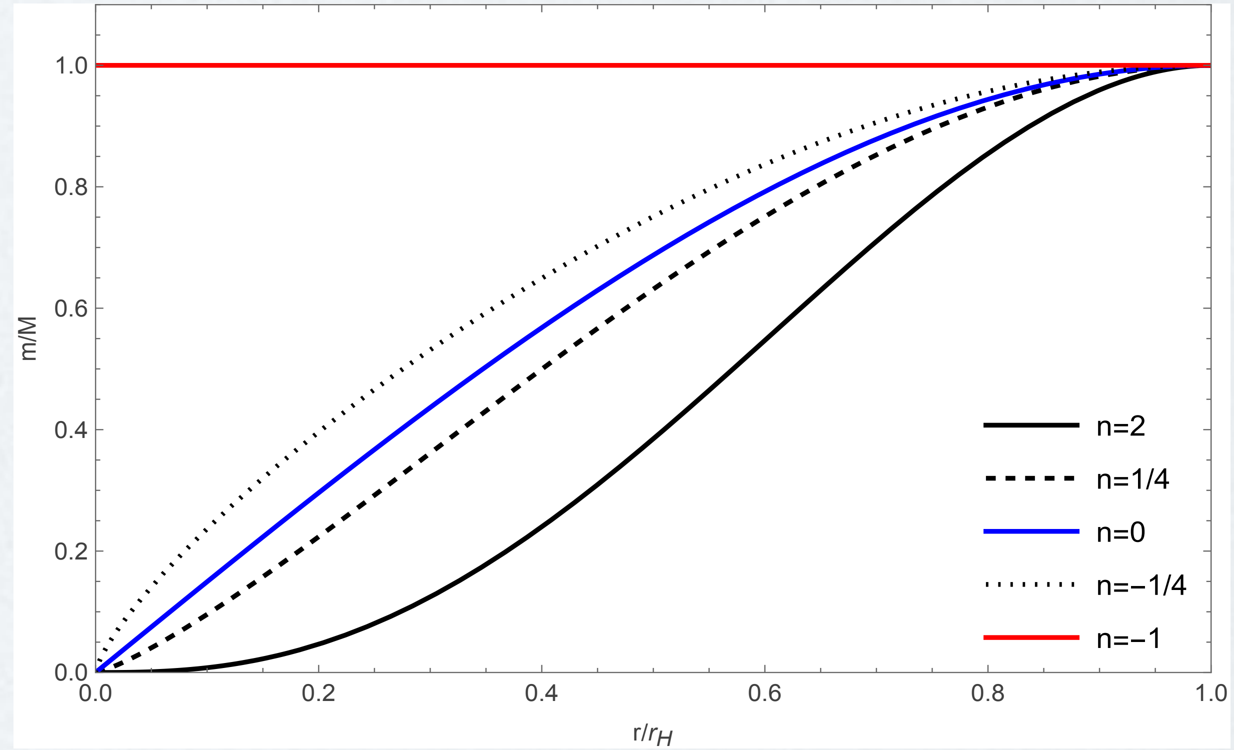
$$f = \begin{cases} 1 - \dfrac{2\,G_{\rm N}\,m(r)}{r} & 0 \le r \le r_{\rm H} = 2\,G_{\rm N}\,M \\ 1 - \dfrac{2\,G_{\rm N}\,M}{r} & r \ge r_{\rm H} \end{cases}$$



Minkowski breaking [2,3]

MSH mass function [1,2,3]: $f(r), m(r) \in C^1(\mathbb{R}^+)$

$$m = \frac{r}{2\,G_{\rm N}\,(n-2)}\left[(n+1)\,\frac{r^2}{r_{\rm H}^2} - 3\left(\frac{r}{r_{\rm H}}\right)^n\right]$$



[1] J. Ovalle, PRD 109 (2024) 104032 [arXiv:2405.06731]; J. Ovalle, arXiv:2509.00816
[2] J. Ovalle, R.C. A. Kamenshchik, PRD113 (2026) 064042 [arXiv:2603.06451]
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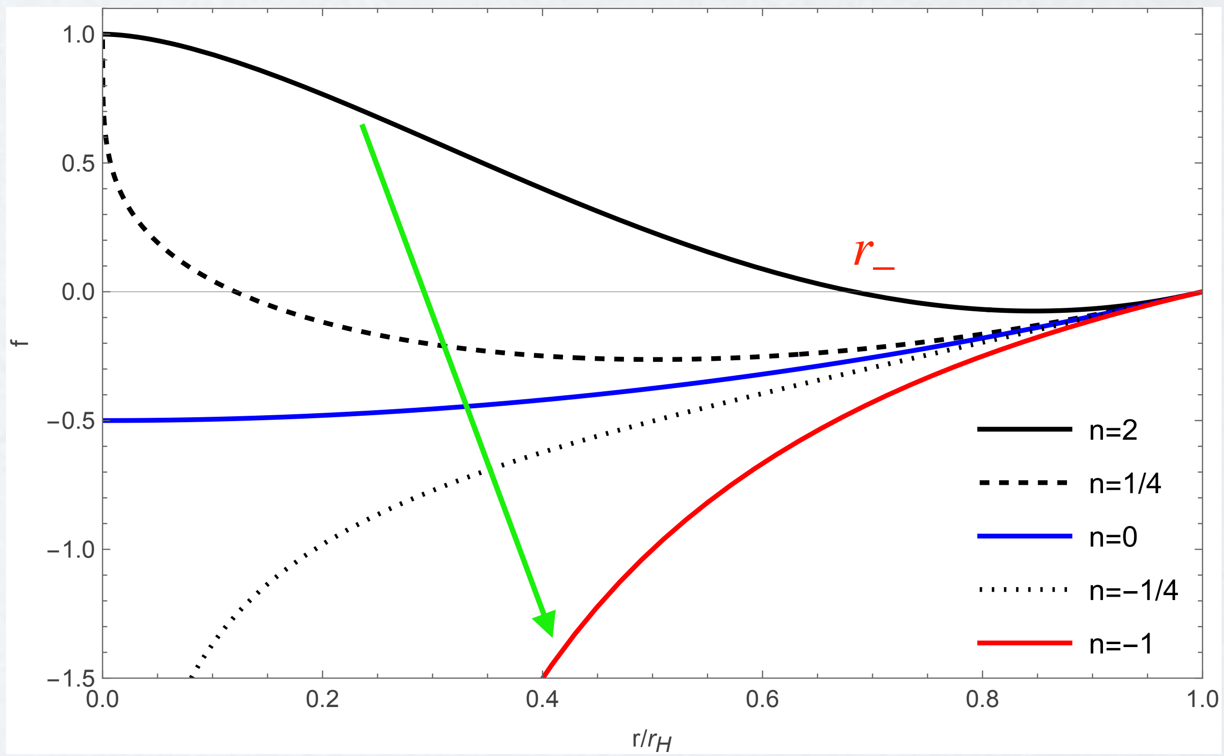

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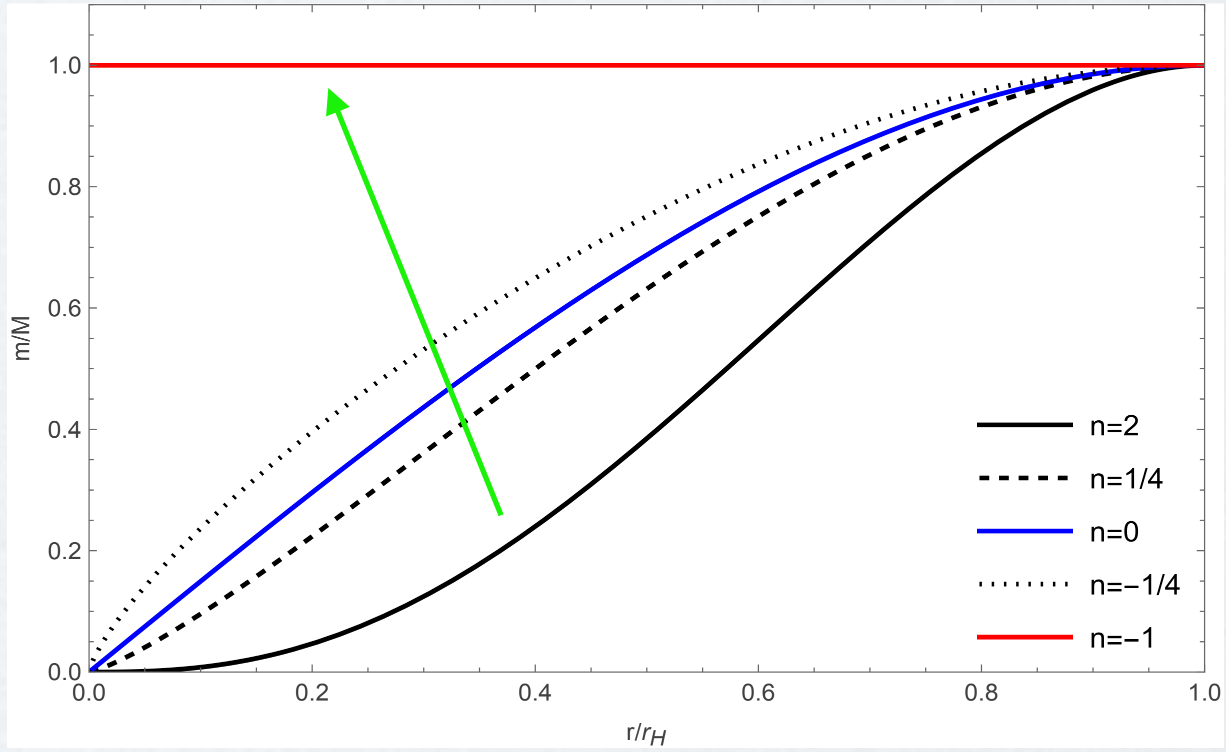
MSH mass function [1,2,3]:

$$m = \frac{r}{2\,G_N\,[n(v)-2]}\left\{[n(v)+1]\,\frac{r^2}{r_H^2} - 3\left(\frac{r}{r_H}\right)^{n(v)}\right\}$$



Time evolution $\sim \dot{n}(v) < 0$: $\quad n > 2 \to -1$

[1] J. Ovalle, PRD 109 (2024) 104032 [arXiv:2405.06731]; J. Ovalle, arXiv:2509.00816
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Collapsing Schwarzschild mimickers

EM tensor [1,2]:

$$ds^2 = -f(v,r)\,dv^2 + 2\,dv\,dr + r^2\,d\Omega^2$$

$$f = 1 - \frac{2\,G_{\rm N}\,m(v,r)}{r} \quad \Longrightarrow \quad G_{ab} = 8\,\pi\,G_{\rm N}\,T_{ab}$$

Tetrad components:

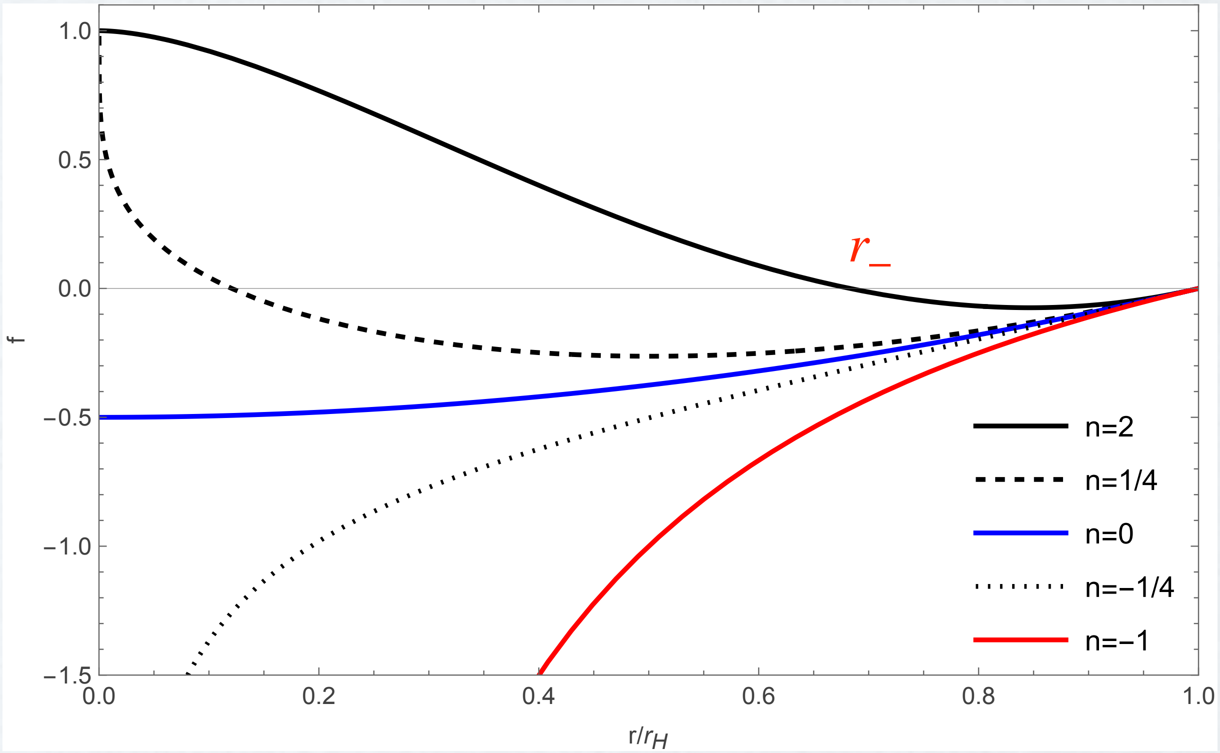
$g_{ab} = \eta_{ab}$

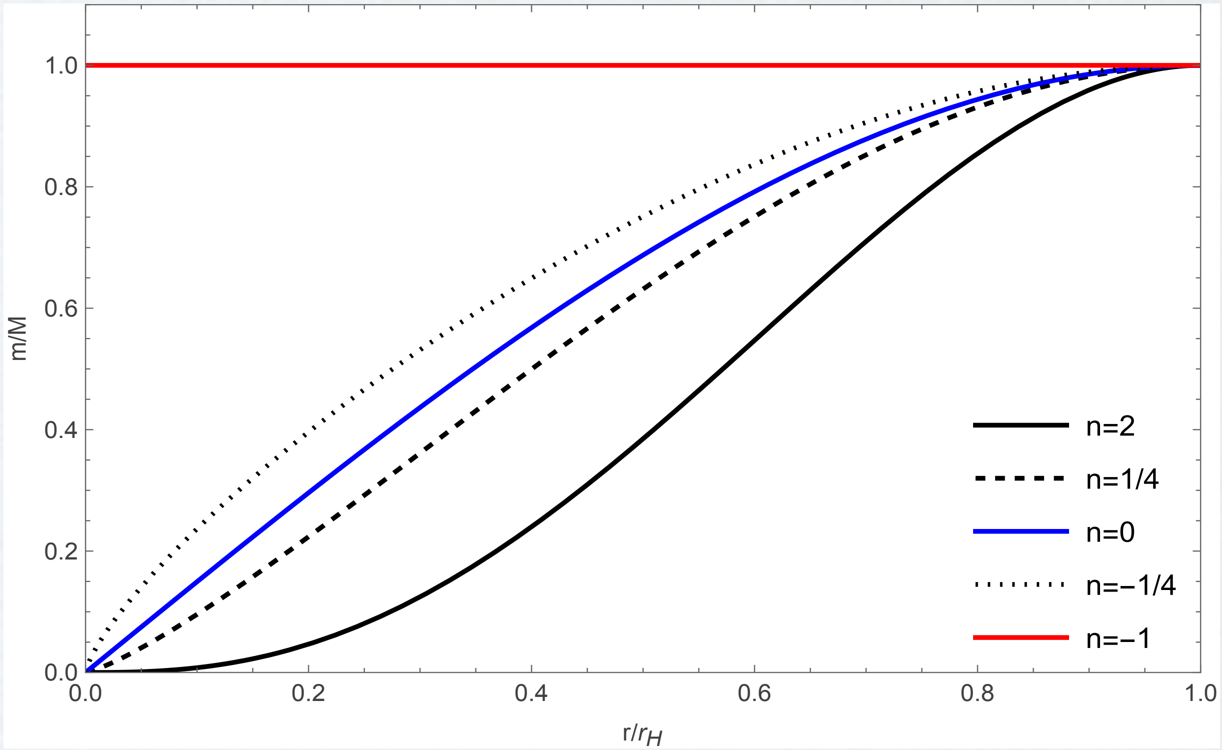
$$T_{ab} = \begin{bmatrix} \rho + \Phi/2 & \Phi/2 & 0 & 0 \\ \Phi/2 & p_{\rm r} + \Phi/2 & 0 & 0 \\ 0 & 0 & p_{\rm t} & 0 \\ 0 & 0 & 0 & p_{\rm t} \end{bmatrix}$$

$$\rho = -p_{\rm r} = \frac{3}{8\,\pi\,G_{\rm N}\,r_{\rm H}^2}\left(\frac{n+1}{n-2}\right)\left[1 - \left(\frac{r}{r_{\rm H}}\right)^{n-2}\right]$$

$$p_{\rm t} = -\frac{3}{8\,\pi\,G_{\rm N}\,r_{\rm H}^2}\left(\frac{n+1}{n-2}\right)\left[1 - \frac{n}{2}\left(\frac{r}{r_{\rm H}}\right)^{n-2}\right]$$

$$\Phi = -\frac{3\,r\,\dot{n}}{8\,\pi\,G_{\rm N}\,r_{\rm H}^2\,(n-2)^2}\left\{1 - \left(\frac{r}{r_{\rm H}}\right)^{n-2}\left[1 + (n-2)\,\ln\left(\frac{r_{\rm H}}{r}\right)\right]\right\}$$





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$$\rho = -p_r = \frac{3}{8\,\pi\,G_N\,r_H^2}\left(\frac{n+1}{n-2}\right)\left[1-\left(\frac{r}{r_H}\right)^{n-2}\right]$$

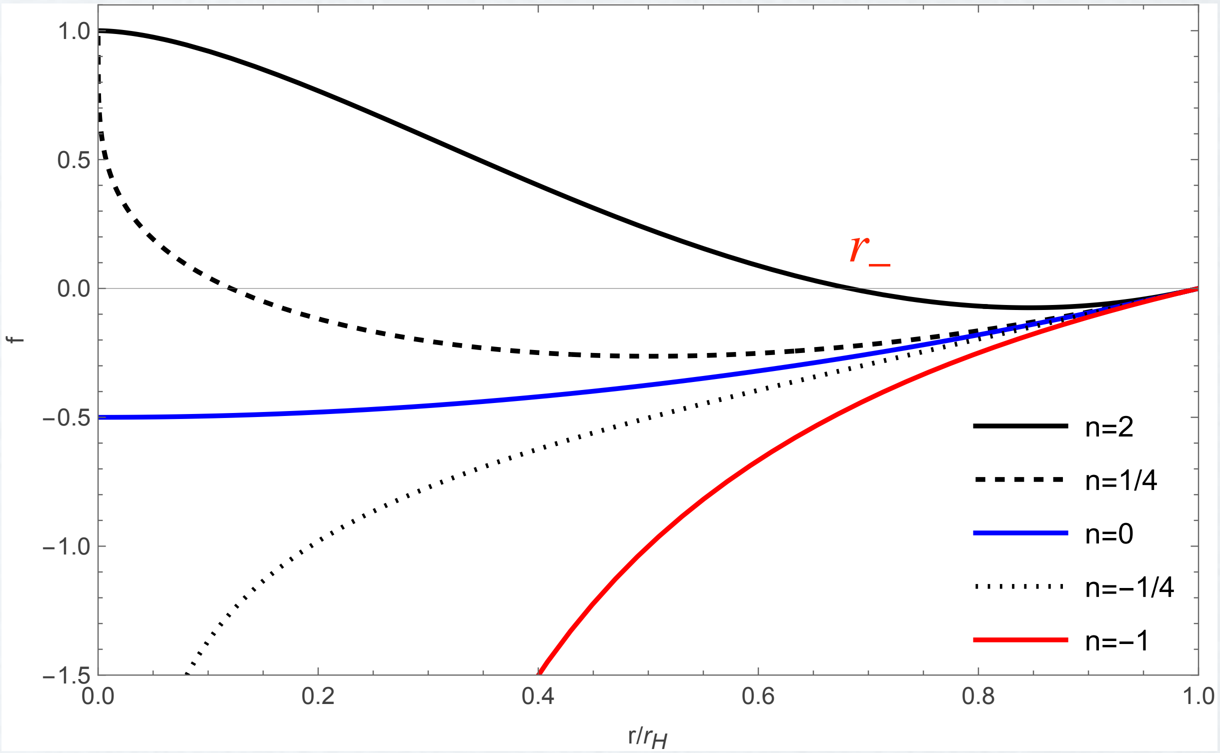
$$m(r) \sim r^3 \qquad (n > 2)$$

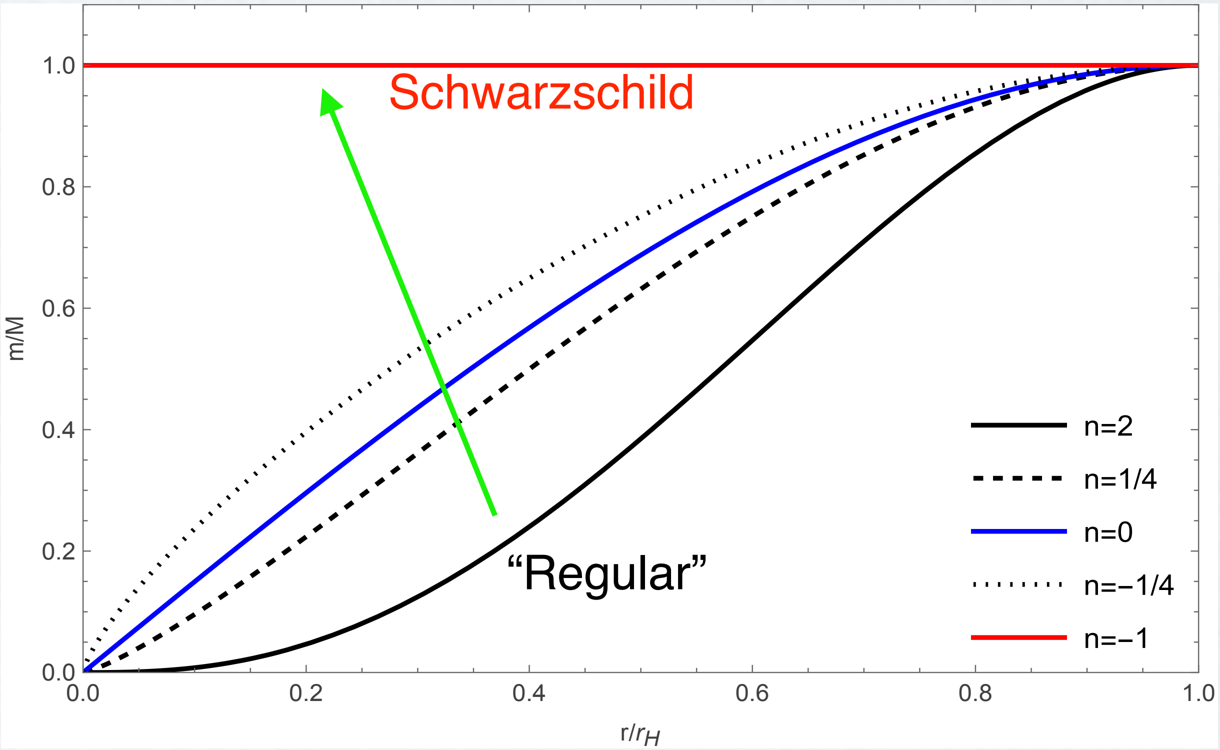
$$m(r) \sim r^3\,\ln(r) \qquad (n = 2)$$

$$m(r) \sim r^{1+n} \qquad (-1 < n < 2)$$

$$m(r) = M \qquad (n \to -1^+)$$

$$4\,\pi\,r^2\,\Phi \sim \dot{n}\,r^{1+n}\,\ln(r) \quad \Longrightarrow \quad \dot{n} \to 0 \quad (n \to -1^+)$$





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Collapsing Schwarzschild mimickers

Classical static end-state (before Schwarzschild singularity) [1]?

Geodesic 4-velocity: $u^\mu \nabla_\mu u^\nu = 0$ $\quad u^\mu u_\mu = -1$

$$u^\mu = \left(-\frac{1}{1+\sqrt{1-f}}, \sqrt{1-f}, 0, 0\right)$$

$$f = f(r)$$

$$h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$$

$$B_{\mu\nu} = \nabla_\mu u_\nu \qquad \theta = B^\mu{}_\mu = 0 \qquad \text{(expansion)}$$

$$\sigma_{\mu\nu} = \frac{1}{2}\left(B_{\mu\nu} + B_{\nu\mu}\right) - \frac{1}{3}\theta h_{\mu\nu} = 0 \qquad \text{(shear)}$$

$$\omega_{\mu\nu} = \frac{1}{2}\left(B_{\mu\nu} - B_{\nu\mu}\right) = 0 \qquad \text{(rotation)}$$

Raychaudhuri equation: $$\frac{d\theta}{d\tau} = -\frac{\theta^2}{3} - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}u^\mu u^\nu = -R_{\mu\nu}u^\mu u^\nu = 0$$

“Regular”

$R_{\mu\nu} = 0$ $\quad$ Schwarzschild vacuum!

[1] R.C., A. Giusti, A. Kamenshchik, J. Ovalle, to appear in EPJC [arXiv:2605.01808]

Collapsing Schwarzschild mimickers

Quantum (Madelung) approximation [1]:

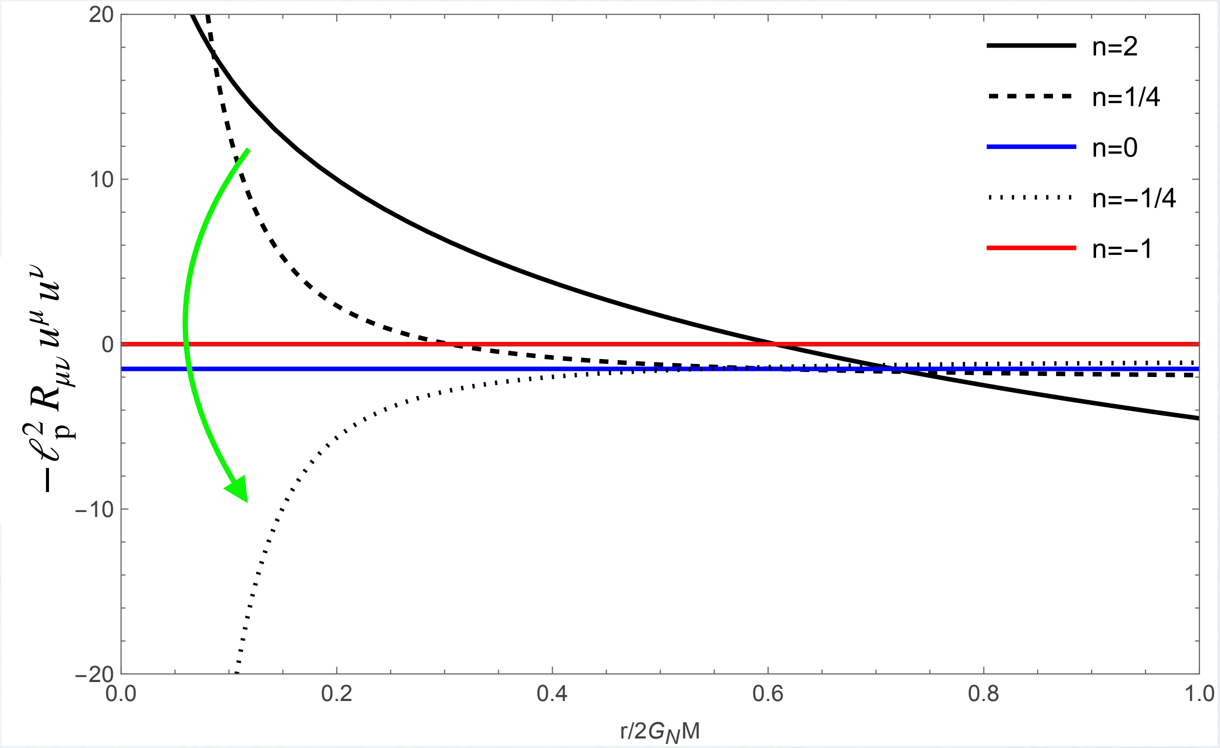
$$\mu\,\left|\psi\right|^2 \simeq \rho = \frac{3}{8\,\pi\,G_{\rm N}\,r_{\rm H}^2}\left(\frac{n+1}{n-2}\right)\left[1-\left(\frac{r}{r_{\rm H}}\right)^{n-2}\right]$$

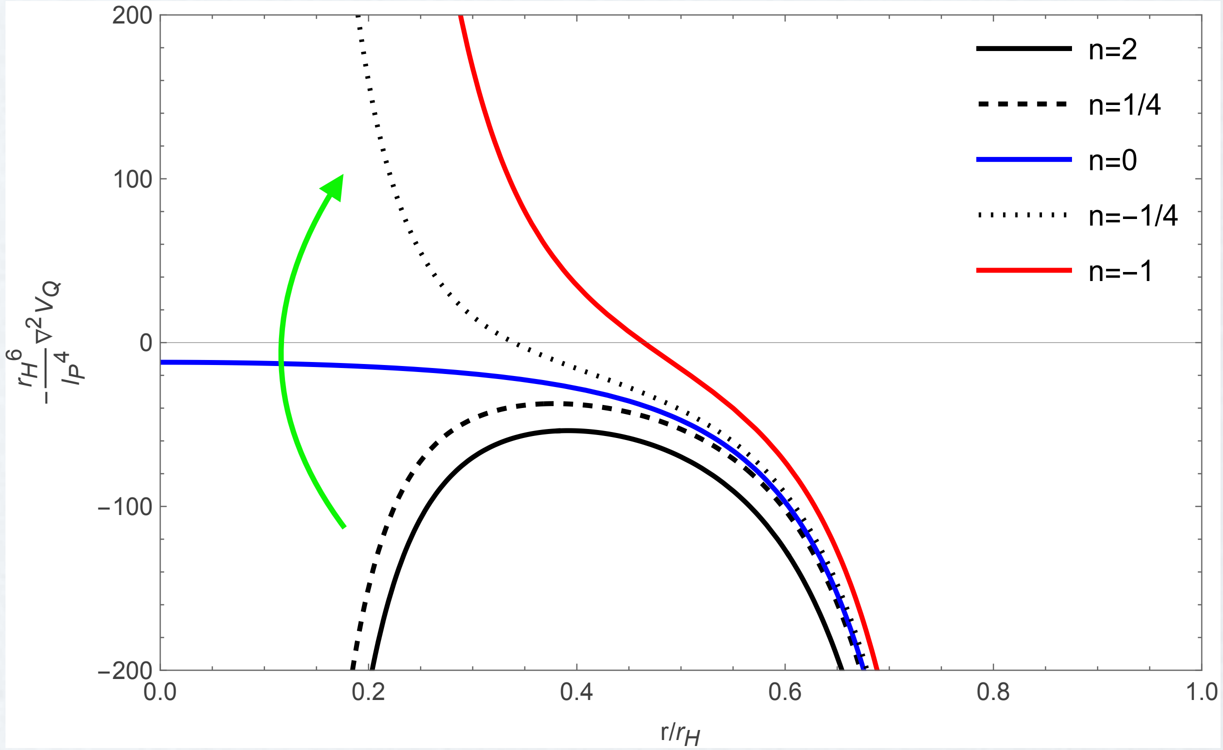
Quantum (Madelung) potential [2,3]:

$$\frac{d\theta}{d\tau} = -\,R_{\mu\nu}\,u^\mu\,u^\nu - \nabla^2 V_{\rm Q}$$

$$V_{\rm Q} \simeq -\,2\,\frac{\ell_{\rm p}^2}{r_{\rm H}^2}\,\frac{\nabla^2|\psi|}{|\psi|} \simeq -\,\frac{\ell_{\rm p}^2}{r_{\rm H}^2}\,\frac{\nabla^2\rho}{\rho}$$

Minkowski breaking [3]





$$-\,\nabla^2 V_{\rm Q} \simeq -\,\frac{n\,\ell_{\rm p}^4}{r_{\rm H}^2\,r^4}$$

$$-\,\nabla^2 V_{\rm Q} \simeq -\,\frac{\ell_{\rm p}^4/r_{\rm H}^2}{(r-r_{\rm H})^4}$$

[1] E. Madelung, Z. Phys. 40 (1927) 322
[2] S. Das, PRD 89 (2014) 084068 [arXiv:1311.6539]
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Unstable horizon
(Outer Boulware vacuum
Hawking radiation?)

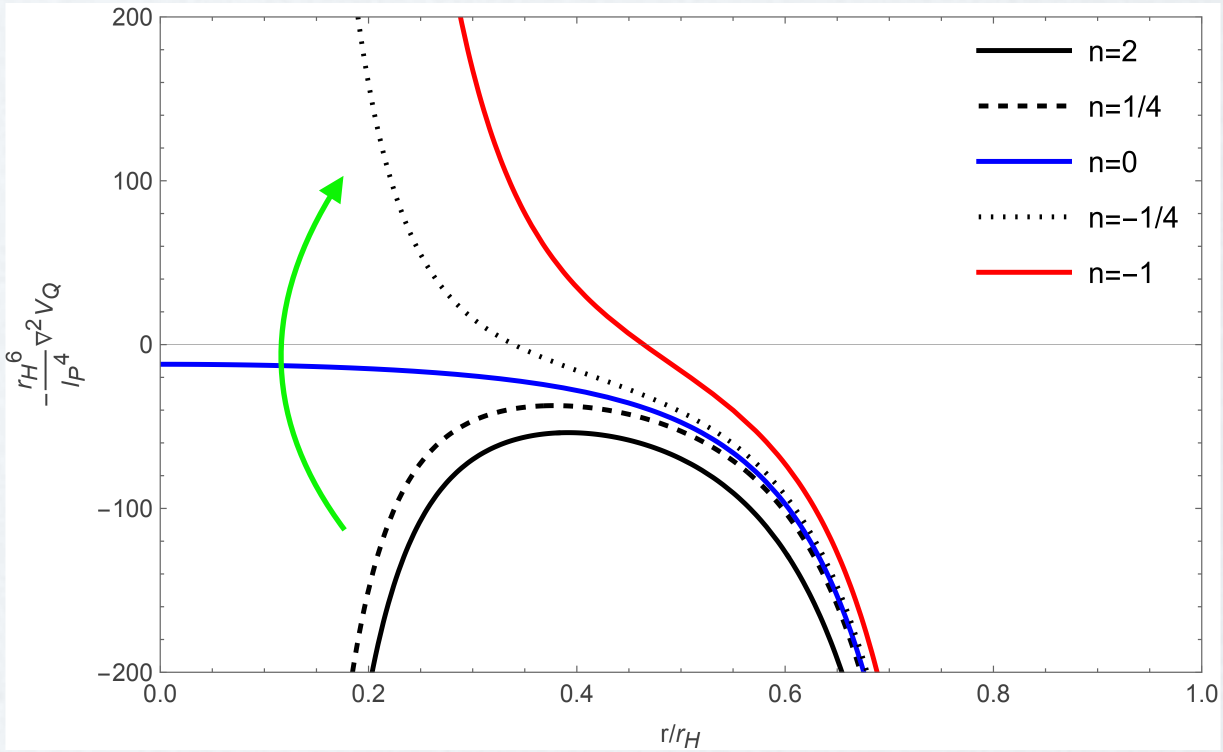
Outward quantum pressure
(after Minkowski breaking)

Quantum (Madelung) potential [2,3]:

$$\frac{d\theta}{d\tau} = -\,R_{\mu\nu}\,u^\mu\,u^\nu - \nabla^2 V_Q$$

$$V_Q \simeq -\,2\,\frac{\ell_p^2}{r_H^2}\,\frac{\nabla^2|\psi|}{|\psi|} \simeq -\,\frac{\ell_p^2}{r_H^2}\,\frac{\nabla^2\rho}{\rho}$$

Minkowski breaking [3]



$$-\nabla^2 V_Q \simeq -\,\frac{n\,\ell_p^4}{r_H^2\,r^4} \quad (n \neq 0)$$

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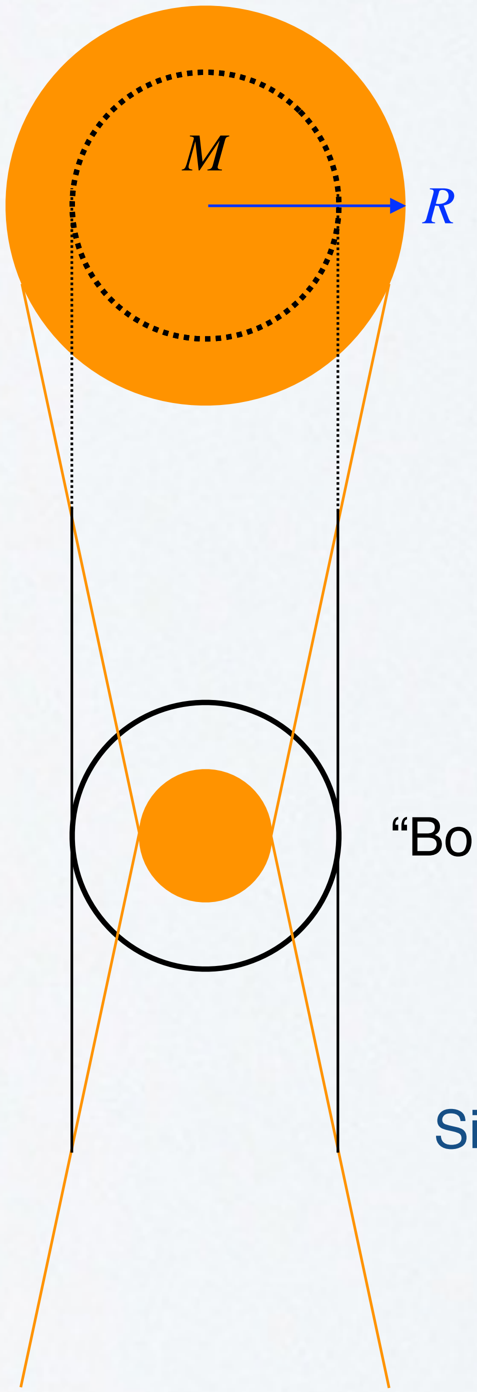
QG and gravitational collapse

BH form from collapse - the semiclassical view:

Star



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- $|\mathrm{gravity}\rangle$ ~ very large number of gravitons ($N_G \sim M_\odot^2/m_p^2 \sim 10^{76}$)

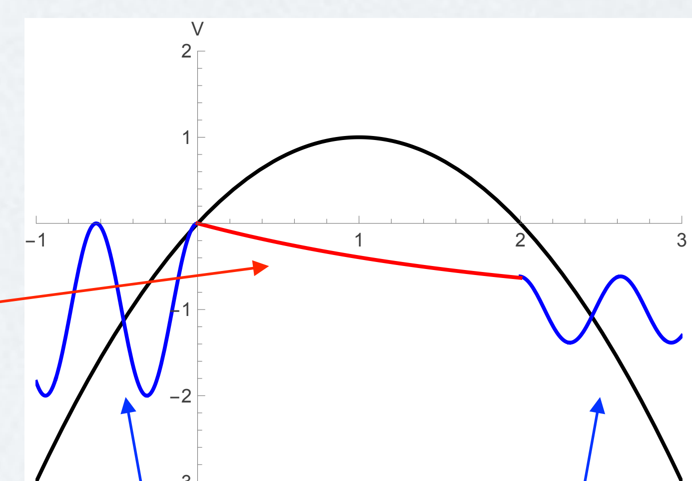
Lemaître-Tolman-Bondi-Oppenheimer-Snyder model

Canonical quantisation of R (and M) [1]

"Bounce" ~ BH to WH tunneling ← Semiclassical approximation [1,2]

Size of "bounce" ($R < R_H$?)

"Quantum" geometry?



Classical geometry

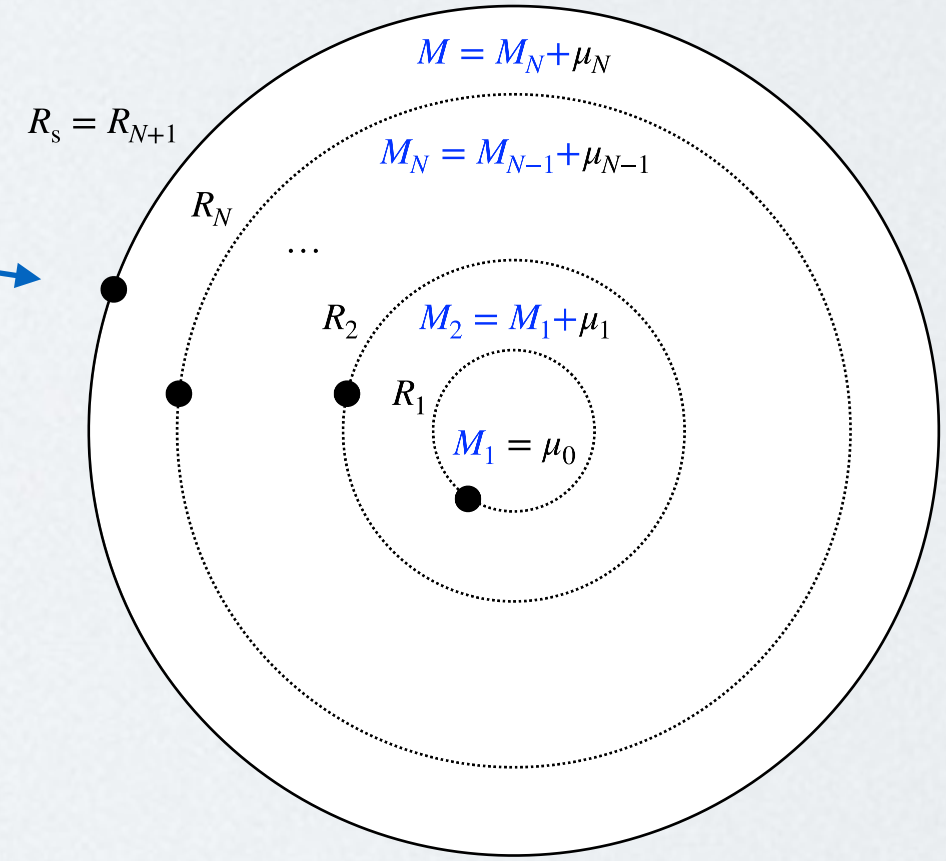
[1] Kuchar, Hajicek, Vaz, Kiefer, Husain, Giesel, Rovelli…
[2] R.C. IJMPD 9 (2000) 511 [arXiv:gr-qc/9810073]

Quantum dust core

Collapsing ball of dust [2]

$$ds^2 = -\left(1 - \frac{2\,G_\mathrm{N}\,M_i}{r}\right)dt^2 + \left(1 - \frac{2\,G_\mathrm{N}\,M_i}{r}\right)^{-1} dr^2 + r^2\,d\Omega^2$$

$$\left(\frac{dR_i}{d\tau}\right)^2 + 1 - \frac{2\,G_\mathrm{N}\,M_i}{R_i} \simeq \frac{E_i^2}{\mu^2}$$



Dust particle Hamiltonians:

$$H_i \equiv \frac{P_i^2}{2\,\mu} - \frac{G_\mathrm{N}\,\mu\,M_i}{R_i} = \frac{\mu}{2}\left(\frac{E_i^2}{\mu^2} - 1\right) \equiv \mathscr{E}_i$$

Schrödinger equation:

$$\hat{H}_i\,\Psi_{n_i} = \mathscr{E}_{n_i}\,\Psi_{n_i}$$

Spectrum of bound states ($n_i \geq 1$):

"GR" [2]

$$\frac{\mathscr{E}_{n_i}}{\mu} \simeq -\frac{G_\mathrm{N}^2\,\mu^3\,M_i}{2\,\hbar^2\,n_i^2} = -\frac{1}{2\,n_i^2}\left(\frac{\mu\,M_i}{m_\mathrm{p}^2}\right)^2 = \frac{1}{2}\left(\frac{E_i^2}{\mu^2} - 1\right)$$

$$\bar{R}_{n_i} \equiv \langle \Psi_{n_i} | \hat{R}_i | \Psi_{n_i} \rangle \simeq n_i^2\,\ell_\mathrm{p}\left(\frac{m_\mathrm{p}^3}{\mu^2\,M_i}\right)$$

Newtonian spectrum

[2] R.C., *Quantum dust cores of black holes*, PLB 843 (2023) 138055 [arXiv:2304.06816]

Quantum dust core

Allowed spectrum [1,2]: $0 \leq \frac{E_i^2}{\mu^2} \simeq 1 - \frac{1}{n_i^2}\left(\frac{\mu M_i}{m_p^2}\right)^2 \rightarrow n_i \geq N_i = \frac{\mu M_i}{m_p^2} \rightarrow \bar{R}_{N_i} \simeq \frac{3}{2} G_N M_i$

Layer thickness [1]: $\frac{\overline{\Delta R}_{n_i}}{\bar{R}_{n_i}} = \frac{\sqrt{n_i^2+2}}{3\,n_i} \simeq \frac{1}{3} \rightarrow \bar{R}_{N_{i+1}} \simeq \bar{R}_{N_i} + \overline{\Delta R}_{N_i} \simeq \frac{4}{3}\bar{R}_{N_i} \rightarrow M_{i+1} \simeq \frac{4}{3} M_i$

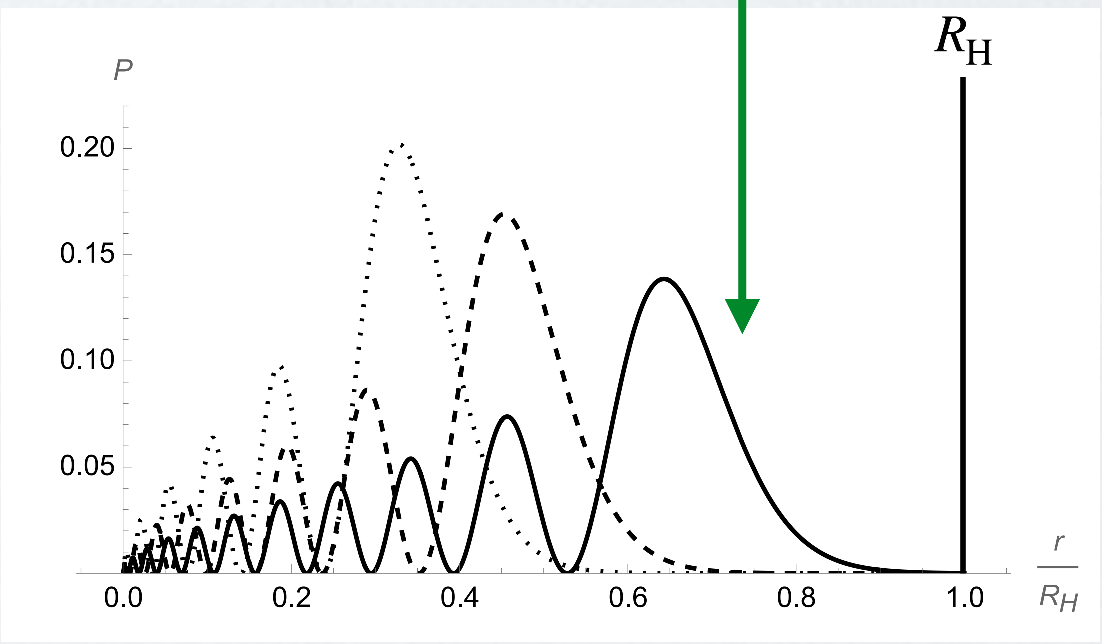
Bekenstein-like area law: $\boxed{N_G = \frac{M}{\mu} N_N \simeq \frac{M^2}{m_p^2}}$

$\boxed{R_s \simeq \frac{3}{2} G_N M = \frac{3}{4} R_H}$

Effective mass and energy density [1]*: $\boxed{m(r) \sim r}$

$\downarrow$

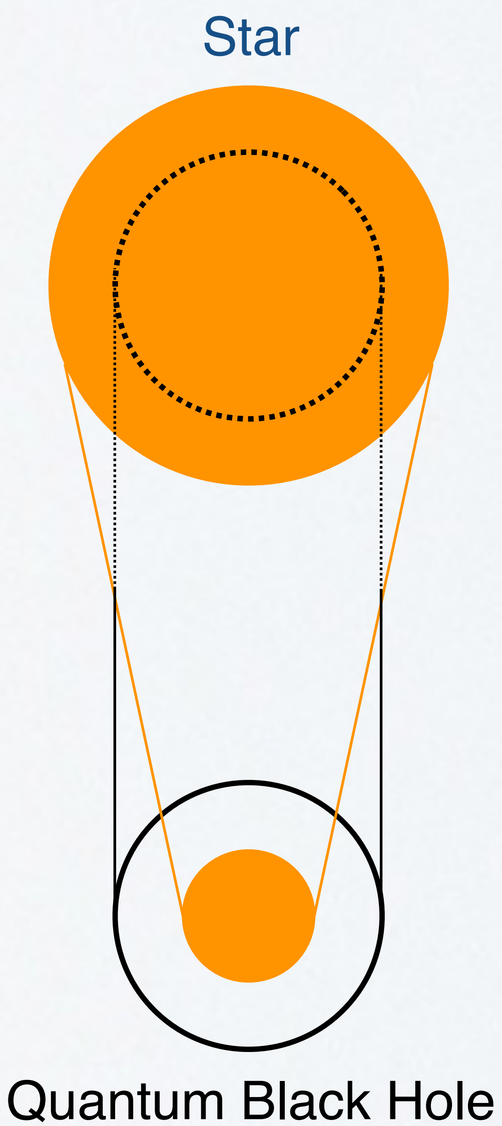
Integrable singularity $(n = 0)$: $\boxed{\rho(r) \sim r^{-2}}$



[1] R.C., *Quantum dust cores of black holes*, PLB 843 (2023) 138055 [arXiv:2304.06816]
[2] R.C., *A quantum bound for the compactness*, EPJC 82 (2022) 1 [arXiv:2103.14582]

Conclusion - QG and gravitational collapse

BH form from collapse - the quantum view:



- $|\mathrm{matter}\rangle$ ~ very large number of SM particles ($M_\odot \sim 10^{57}$ neutrons)
- $|\mathrm{gravity}\rangle$ ~ very large number of gravitons ($N_\mathrm{G} \sim M_\odot^2 \sim 10^{76}$)

Thank you!