Semiclassical effects near the Cauchy horizon of a Reissner-Nordström black hole

From Black Holes to the Cosmos, Trieste

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Joint work with: P. Taylor, M. Casals

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Outline

1 Motivation

2 The renormalization problem

3 Renormalized Vacuum Polarization

4 RSET

5 Conclusions

Table of Contents

1 Motivation

2 The renormalization problem

3 Renormalized Vacuum Polarization

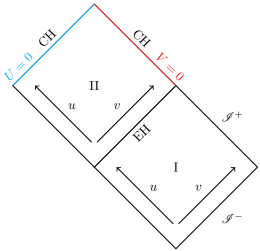
4 RSET

5 Conclusions

Motivation

Motivation
The renormalization problem
Renormalized Vacuum Polarization
RSET
Conclusions

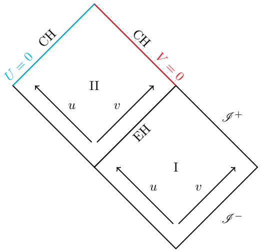
- Quantum fields near the Cauchy horizon of a Reissner-Nordstrom black hole → Cosmic censorship hypothesis



Motivation

Motivation
The renormalization problem
Renormalized Vacuum Polarization
RSET
Conclusions

- Quantum fields near the Cauchy horizon of a Reissner-Nordstrom black hole $\rightarrow$ Cosmic censorship hypothesis



- Classical massless fields$\rightarrow$ weak null curvature singularity

Motivation

Motivation

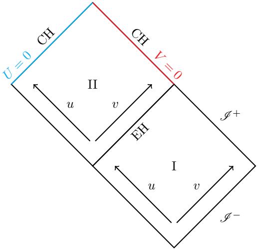
The renormalization problem

Renormalized Vacuum Polarization

RSET

Conclusions

- Quantum fields near the Cauchy horizon of a Reissner-Nordstrom black hole $\rightarrow$ Cosmic censorship hypothesis



- Classical massless fields$\rightarrow$ weak null curvature singularity
- Classical massive fields $\rightarrow$ stronger singularity, but still continuously-extendible beyond the CH

A surprising result

Motivation

The renormalization problem

Renormalized Vacuum Polarization

RSET

Conclusions

PHYSICAL REVIEW D **99**, 061502(R) (2019)

Rapid Communications

Analysis of quantum effects inside spherical charged black holes

Assaf Lanir, Amos Ori, Noa Zilberman, Orr Sela, Ahron Maline, and Adam Levi

Department of Physics, Technion, Haifa 32000, Israel

(Received 8 November 2018; published 19 March 2019; corrected 27 May 2021)

We numerically compute the renormalized expectation value $\langle\hat{\Phi}^2\rangle_{\mathrm{ren}}$ of a minimally coupled massless quantum scalar field in the interior of a four-dimensional Reissner-Nordstrom black hole, in both the Hartle-Hawking and Unruh states. To this end we use a recently developed mode-sum renormalization scheme based on covariant point splitting. In both quantum states, $\langle\hat{\Phi}^2\rangle_{\mathrm{ren}}$ is found to approach a *finite* value at the inner horizon (IH). The final approach to the IH asymptotic value is marked by an inverse-power tail r_*^{-n}, where r_* is the Regge-Wheeler "tortoise coordinate" and with $n = 2$ for the Hartle-Hawking state and $n = 3$ for the Unruh state. We also report here the results of an analytical computation of these inverse-power tails of $\langle\hat{\Phi}^2\rangle_{\mathrm{ren}}$ near the IH. Our numerical results show very good agreement with this analytical derivation (for both the power index and the tail amplitude), in both quantum states. Finally, from this asymptotic behavior of $\langle\hat{\Phi}^2\rangle_{\mathrm{ren}}$ we analytically compute the leading-order asymptotic behavior of the trace $\langle\hat{T}^{\mu}_{\mu}\rangle_{\mathrm{ren}}$ of the renormalized stress-energy tensor at the IH. In both quantum states this quantity is found to diverge like $b(r-r_-)^{-1}r_*^{-n-2}$ (with n specified above and with a known parameter b). To the best of our knowledge, this is the first fully quantitative derivation of the asymptotic behavior of these renormalized quantities at the IH of a four-dimensional Reissner-Nordstrom black hole. In particular, this is the first conclusive result showing the divergence of the renormalized stress-energy tensor at the Cauchy horizon.

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A surprising result

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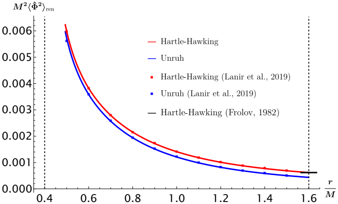

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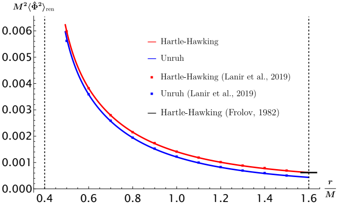

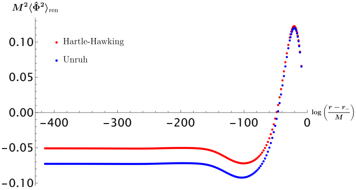

The semiclassical fluxes at the CH

Motivation

The renormalization problem

Renormalized Vacuum Polarization

RSET

Conclusions

PHYSICAL REVIEW LETTERS **124,** 171302 (2020)

Quantum Fluxes at the Inner Horizon of a Spherical Charged Black Hole

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In an ongoing effort to explore quantum effects on the interior geometry of black holes, we explicitly compute the semiclassical flux components $\langle T_{uu}\rangle_{\text{ren}}$ and $\langle T_{vv}\rangle_{\text{ren}}$ (u and v being the standard Eddington coordinates) of the renormalized stress-energy tensor for a minimally coupled massless quantum scalar field, in the vicinity of the inner horizon (IH) of a Reissner-Nordström black hole. These two flux components seem to dominate the effect of backreaction in the IH vicinity, and furthermore, their regularization procedure reveals remarkable simplicity. We consider the Hartle-Hawking and Unruh quantum states, the latter corresponding to an evaporating black hole. In both quantum states, we compute $\langle T_{uu}\rangle_{\text{ren}}$ and $\langle T_{vv}\rangle_{\text{ren}}$ in the IH vicinity for a wide range of Q/M values. We find that both $\langle T_{uu}\rangle_{\text{ren}}$ and $\langle T_{vv}\rangle_{\text{ren}}$ attain finite asymptotic values at the IH. Depending on Q/M, these asymptotic values are found to be either positive or negative (or vanishing in between). Note that having a nonvanishing $\langle T_{vv}\rangle_{\text{ren}}$ at the IH implies the formation of a curvature singularity on its ingoing section, the Cauchy horizon. Motivated by these findings, we also take initial steps in the exploration of the backreaction effect of these semiclassical fluxes on the near-IH geometry.

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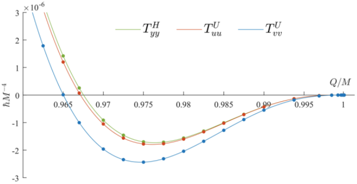

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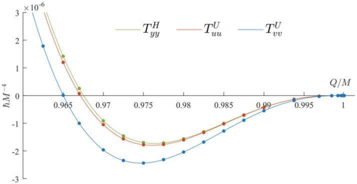

- Do these results hold for different field parameters?

The semiclassical fluxes at the CH

Motivation

The renormalization problem

Renormalized Vacuum Polarization

RSET

Conclusions

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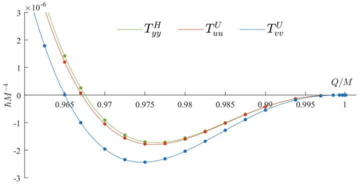

- Do these results hold for different field parameters?
- We consider for the first time quantized massive scalar fields

Table of Contents

1 Motivation

2 The renormalization problem

3 Renormalized Vacuum Polarization

4 RSET

5 Conclusions

The renormalization problem

Motivation
The renormalization problem
Renormalized Vacuum Polarization
RSET
Conclusions

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$$G_{\alpha\beta} + \mathcal{O}(R^2) = 8\pi \left(T^{(\mathrm{cl})}_{\alpha\beta} + \langle \hat{T}_{\alpha\beta} \rangle^{\mathrm{A}}_{\mathrm{ren}} \right).$$

The renormalization problem

Motivation

The renormalization problem

Renormalized Vacuum Polarization

RSET

Conclusions

The semiclassical Einstein equations

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- The renormalized stress-energy tensor

$$\langle \hat{T}_{\alpha\beta} \rangle^{\mathrm{A}}_{\mathrm{ren}} = \lim_{x' \to x} \hat{\tau}_{\alpha\beta} \left[G_{\mathrm{A}}(x, x') - K(x, x') \right].$$

Motivation
The renormalization problem
Renormalized Vacuum Polarization
RSET
Conclusions

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$$\langle \hat{T}_{\alpha\beta} \rangle^{\mathrm{A}}_{\mathrm{ren}} = \lim_{x' \to x} \hat{\tau}_{\alpha\beta} \Big[\underbrace{G_{\mathrm{A}}(x, x')}_{\text{Green's function}} - \underbrace{K(x, x')}_{\text{Hadamard parametrix}} \Big].$$

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Motivation

The renormalization problem

Renormalized Vacuum Polarization

RSET

Conclusions

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$$\langle \hat{T}_{\alpha\beta} \rangle^{\mathrm{A}}_{\mathrm{ren}} = \lim_{x' \to x} \hat{\tau}_{\alpha\beta} \left[G_{\mathrm{A}}(x, x') - K(x, x') \right].$$

$$\boxed{G_{\mathrm{A}}(x, x') = \sum_{lm\omega} e^{-\mathrm{i}\omega\Delta t}\, Y_{lm}(\theta, \phi)\, Y^{*}_{lm}(\theta', \phi') g^{\mathrm{A}}_{\omega l}(r, r')}$$

The renormalization problem

Motivation

The renormalization problem

Renormalized Vacuum Polarization

RSET

Conclusions

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$$\boxed{K(x, x') \sim \frac{U(x, x')}{\sigma_\epsilon(x, x')} + V(x, x') \log \left(\frac{2\sigma_\epsilon(x, x')}{\ell^2} \right)}$$

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Motivation

The renormalization problem

Renormalized Vacuum Polarization

RSET

Conclusions

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- **Computational Problem:** can we express the $K(x, x')$ as a mode-sum in the same form as $G_{\mathrm{A}}(x, x')$ so that we subtract mode-by-mode?

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Motivation
The renormalization problem
Renormalized Vacuum Polarization
RSET
Conclusions

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- **Computational Problem:** can we express the $K(x,x')$ as a mode-sum in the same form as $G_{\mathrm{A}}(x,x')$ so that we subtract mode-by-mode?
- **Recent solution:** Extended coordinate method (ECM)
  Taylor, Breen, 2016, 2017;
  Taylor, Breen, Ottewill, 2022;
  Arrechea, Breen, Ottewill, LP, Taylor, 2025.

Extended Coordinate Method

Motivation

The renormalization problem

Renormalized Vacuum Polarization

RSET

Conclusions

- The ECM allows to express the Hadamard parametrix

$$K(x,x') \sim \frac{U(x,x')}{\sigma_\epsilon(x,x')} + V(x,x') \log\left(\frac{2\sigma_\epsilon(x,x')}{\ell^2}\right)$$

as a mode-sum in the same form as $G_{\mathrm{A}}(x,x')$

$$K(x,x') = \sum_{l=0}^{\infty}(2l+1)P_l(\cos\gamma)\int_0^\infty d\omega\, e^{-\mathrm{i}\omega\Delta t} k_{\omega l}^{(m)}(r,r').$$

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Motivation

The renormalization problem

Renormalized Vacuum Polarization

RSET

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- Mode-by-mode subtraction
- Renormalized vacuum polarization (VP)

$$\begin{aligned}\langle\hat{\Phi}^2\rangle^{\mathrm{A}}_{\mathrm{ren}} &:= \lim_{x'\to x}\left(G_{\mathrm{A}}(x,x') - K(x,x')\right)\\ &= \sum_{l=0}^{\infty}(2l+1)\int_0^\infty d\omega\left(g^{A}_{\omega l}(r) - k^{(m)}_{\omega l}(r)\right).\end{aligned}$$

Extended Coordinate Method

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- Mode-by-mode subtraction
- Renormalized vacuum polarization (VP)

$$\begin{aligned}\langle\hat{\Phi}^2\rangle^{\mathrm{A}}_{\mathrm{ren}} &:= \lim_{x'\to x}\left(G_{\mathrm{A}}(x,x') - K(x,x')\right)\\ &= \sum_{l=0}^{\infty}(2l+1)\int_0^{\infty} d\omega \left(g_{\omega l}^{A}(r) - k_{\omega l}^{(m)}(r)\right).\end{aligned}$$

- The order m of the expansion defines the rate of convergence

Table of Contents

1 Motivation

2 The renormalization problem

3 Renormalized Vacuum Polarization

4 RSET

5 Conclusions

Renormalized Vacuum Polarization

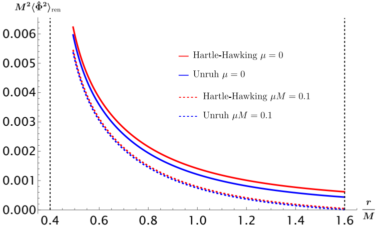
Motivation

The renormalization problem

Renormalized Vacuum Polarization

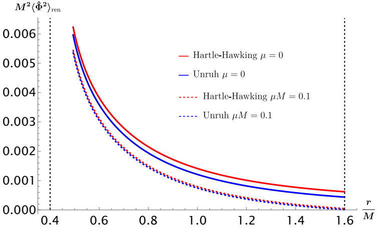
RSET

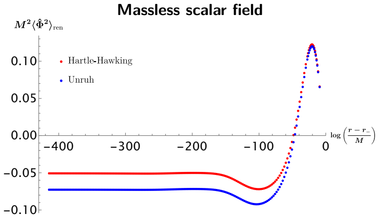
Conclusions



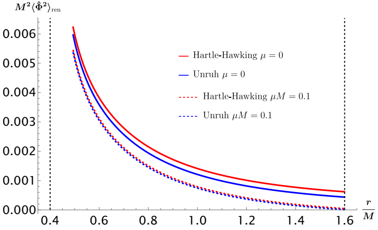
Renormalized Vacuum Polarization

Motivation
The renormalization problem
Renormalized Vacuum Polarization
RSET
Conclusions

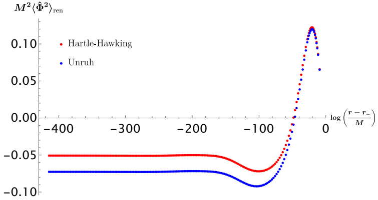




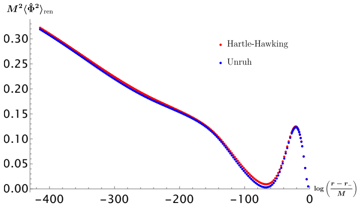
Renormalized Vacuum Polarization



Massless scalar field



Massive scalar field, $\mu M = 0.1$



VP near the Cauchy horizon - numerical results

Motivation

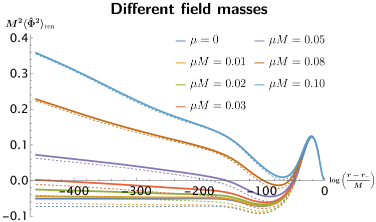
The renormalization problem

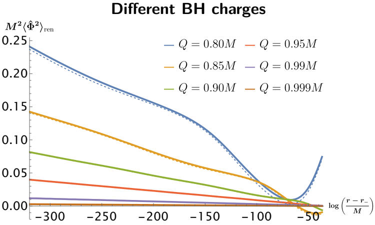
Renormalized Vacuum Polarization

RSET

Conclusions

- Results for the Hartle-Hawking (solid lines) and Unruh (dashed lines) states.





- Numerical evidence that the VP diverges logarithmically at the CH for massive fields

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VP near the Cauchy horizon - semi-analytical analysis

Motivation

The renormalization problem

Renormalized Vacuum Polarization

RSET

Conclusions

- It can be shown that

$$\langle\hat{\Phi}^2\rangle^{\mathrm{A}}_{\mathrm{ren}} \sim \mathrm{w}_< \quad \mathrm{as} \quad r \to r_-,$$

where

$$\mathrm{w}_< = \frac{1}{4\pi^2}\sum_{l=0}^{\infty}(2l+1)\int_0^{\mu} d\omega \left\{ g^<_{\omega l} - k^{(m)}_{\omega l} \right\},$$

VP near the Cauchy horizon - semi-analytical analysis

Motivation
The renormalization problem
Renormalized Vacuum Polarization
RSET
Conclusions

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$$g^<_{\omega l}(r) = \frac{1}{2\,r_-^2\,\omega}\Big[\Big(|A_{\omega l}|^2 + |B_{\omega l}|^2 + A_{\omega l}\,B^*_{\omega l}e^{2i\omega\, r_*} + A^*_{\omega l}B_{\omega l}e^{-2i\omega\, r_*}\Big)\coth(\tfrac{\pi\,\omega}{\kappa_+})$$
$$+\,\Re\Big\{A^{\mathrm{up}}_{\omega l}\Big(A^2_{\omega l}e^{2i\omega r_*} + B^2_{\omega l}e^{-2i\omega r_*} + 2A_{\omega l}B_{\omega l}\Big)\Big\}\mathrm{csch}(\tfrac{\pi\omega}{\kappa_+})\Big] \quad \text{as} \quad r \to r_-\,.$$

VP near the Cauchy horizon - semi-analytical analysis

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- Small-ω expansions (McNamara 1978; Kehle, Shlapentokh-Rothman 2019)

$$A_{\omega l} = \frac{i\,q_l}{\omega} + a^{(0)}_l + \mathcal{O}(\omega), \qquad B_{\omega l} = -\frac{i\,q_l}{\omega} + b^{(0)}_l + \mathcal{O}(\omega), \quad \text{with} \quad |B_{\omega l}|^2 - |A_{\omega l}|^2 = 1.$$

VP near the Cauchy horizon - semi-analytical analysis

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- We find

$$\langle\hat{\Phi}^2\rangle^{\mathrm{A}}_{\mathrm{ren}} \sim k(\mu, Q)\log|f|, \quad r \to r_-. \qquad \boxed{\text{State independent result}}$$

Table of Contents

1 Motivation
2 The renormalization problem
3 Renormalized Vacuum Polarization
4 RSET
5 Conclusions

RSET near the Cauchy horizon

Motivation

The renormalization problem

Renormalized Vacuum Polarization

RSET

Conclusions

- RSET components

$$\langle \hat{T}^{t}{}_{t} \rangle^{\mathrm{A}}_{\mathrm{ren}} = -\langle \hat{T}^{r}{}_{r} \rangle^{\mathrm{A}}_{\mathrm{ren}} \sim \frac{\mathcal{K}^{\mathrm{A}}}{|f|}, \qquad r \to r_{-},$$

$$\langle \hat{T}^{\theta}{}_{\theta} \rangle^{\mathrm{A}}_{\mathrm{ren}} = \langle \hat{T}^{\phi}{}_{\phi} \rangle^{\mathrm{A}}_{\mathrm{ren}} \sim \mathcal{L} \log |f|, \qquad r \to r_{-}.$$

RSET near the Cauchy horizon

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- $\mathcal{L}$ vanishes for $\mu = 0$.

RSET near the Cauchy horizon

Motivation
The renormalization problem
Renormalized Vacuum Polarization
RSET
Conclusions

- RSET components

$$\langle \hat{T}^{t}{}_{t}\rangle^{\mathrm{A}}_{\mathrm{ren}} = -\langle \hat{T}^{r}{}_{r}\rangle^{\mathrm{A}}_{\mathrm{ren}} \sim \frac{\mathcal{K}^{\mathrm{A}}}{|f|}, \qquad r \to r_-,$$
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- $\mathcal{L}$ vanishes for $\mu = 0$.
- The semiclassical fluxes are finite on the Cauchy horizon

$$\langle \hat{T}^{-}_{uu}\rangle^{\mathrm{H}}_{\mathrm{ren}} = \langle \hat{T}^{-}_{vv}\rangle^{\mathrm{H}}_{\mathrm{ren}} = \frac{\mathcal{K}^{\mathrm{H}}}{2}, \quad \langle \hat{T}^{-}_{yy}\rangle^{\mathrm{U}}_{\mathrm{ren}} = \frac{\mathcal{K}^{\mathrm{H}}}{2} + \mathcal{J}_y,$$

RSET near the Cauchy horizon

Motivation
The renormalization problem
Renormalized Vacuum Polarization
RSET
Conclusions

- RSET components

$$\langle \hat{T}^{t}{}_{t} \rangle^{\rm A}_{\rm ren} = -\langle \hat{T}^{r}{}_{r} \rangle^{\rm A}_{\rm ren} \sim \frac{\mathcal{K}^{\rm A}}{|f|}, \qquad r \to r_-,$$

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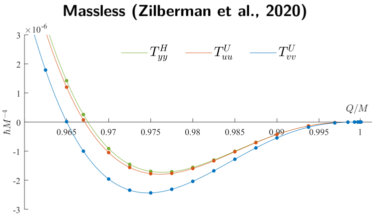
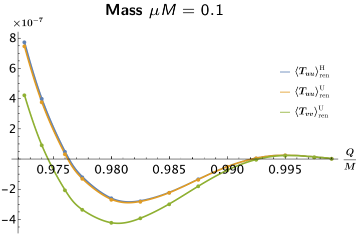
$\rightarrow$ In agreement with previous massless results (Zilberman et al., 2020).

The RSET near the Cauchy horizon

- Numerical computation of the semiclassical fluxes:

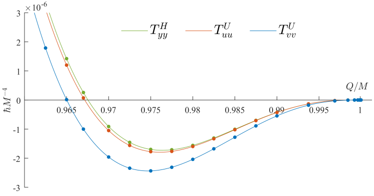




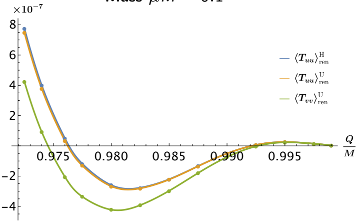
The RSET near the Cauchy horizon

- Numerical computation of the semiclassical fluxes:

Massless (Zilberman et al., 2020)



Mass $\mu M = 0.1$



- Ingoing flux at the CH (in agreement with massless results (Zilberman et al., 2020))

$$\langle \hat{T}^{-}_{VV} \rangle^{\mathrm{A}}_{\mathrm{ren}} = \frac{\langle \hat{T}^{-}_{vv} \rangle^{\mathrm{A}}_{\mathrm{ren}}}{\kappa_-^2 V^2}, \qquad V \to 0^+.$$

Result independent of state and field parameters

→ Classical perturbations: $T^{-}_{VV} \sim \frac{C}{V^2 (\log(V))^p}$, $p > 0$.

→ May convert the CH into a strong singularity (Hollands et al., 2020).

Table of Contents

1 Motivation

2 The renormalization problem

3 Renormalized Vacuum Polarization

4 RSET

5 Conclusions

Conclusions

Motivation

The renormalization problem

Renormalized Vacuum Polarization

RSET

Conclusions

Components as $r \to r_-$	Massless	Massive
$\langle \hat{\Phi}^2 \rangle^{\mathrm{A}}_{\mathrm{ren}}$	Finite	$k(\mu, Q) \log \lvert f \rvert$
$\langle \hat{T}^t{}_t \rangle^{\mathrm{A}}_{\mathrm{ren}} = -\langle \hat{T}^r{}_r \rangle^{\mathrm{A}}_{\mathrm{ren}}$	$\propto \lvert f \rvert^{-1}$	$\propto \lvert f \rvert^{-1}$
$\langle \hat{T}^\theta{}_\theta \rangle^{\mathrm{A}}_{\mathrm{ren}} = \langle \hat{T}^\phi{}_\phi \rangle^{\mathrm{A}}_{\mathrm{ren}}$	Finite	$\mathcal{L} \log \lvert f \rvert$
$\langle \hat{T}^-_{yy} \rangle^{\mathrm{A}}_{\mathrm{ren}}$	Finite	Finite
Near extremality	$\langle \hat{T}^-_{yy} \rangle^{\mathrm{A}}_{\mathrm{ren}} < 0$	$\langle \hat{T}^-_{yy} \rangle^{\mathrm{A}}_{\mathrm{ren}} > 0$
$\langle \hat{T}^-_{VV} \rangle^{\mathrm{A}}_{\mathrm{ren}}$	$\propto V^{-2}$	$\propto V^{-2}$

Conclusions

Motivation
The renormalization problem
Renormalized Vacuum Polarization
RSET
Conclusions

Components as $r \to r_-$	Massless	Massive
$\langle \hat{\Phi}^2 \rangle^{\mathrm{A}}_{\mathrm{ren}}$	Finite	$k(\mu, Q) \log \lvert f \rvert$
$\langle \hat{T}^t{}_t \rangle^{\mathrm{A}}_{\mathrm{ren}} = -\langle \hat{T}^r{}_r \rangle^{\mathrm{A}}_{\mathrm{ren}}$	$\propto \lvert f \rvert^{-1}$	$\propto \lvert f \rvert^{-1}$
$\langle \hat{T}^\theta{}_\theta \rangle^{\mathrm{A}}_{\mathrm{ren}} = \langle \hat{T}^\phi{}_\phi \rangle^{\mathrm{A}}_{\mathrm{ren}}$	Finite	$\mathcal{L} \log \lvert f \rvert$
$\langle \hat{T}^-_{yy} \rangle^{\mathrm{A}}_{\mathrm{ren}}$	Finite	Finite
Near extremality	$\langle \hat{T}^-_{yy} \rangle^{\mathrm{A}}_{\mathrm{ren}} < 0$	$\langle \hat{T}^-_{yy} \rangle^{\mathrm{A}}_{\mathrm{ren}} > 0$
$\langle \hat{T}^-_{VV} \rangle^{\mathrm{A}}_{\mathrm{ren}}$	$\propto V^{-2}$	$\propto V^{-2}$

Thank you, any questions?