Gravity from a modified Bekenstein entropy law: cosmological consequences

Marco de Cesare

Scuola Superiore Meridionale (SSM), Napoli, Italy

INFN, Sezione di Napoli, Italy

m.decesare@na.infn.it

talk based on:

MdC, G. Gubitosi (UNINA),V. Kushwaha (LMU Munich), arXiv:2608.23680

MdC, G. Gubitosi, JCAP 01, 046 (2024)

From Black Holes to the Cosmos: Matt Visser's journey through space and time

SISSA (Miramare Campus), Trieste, 27 August 2026

Scuola Superiore Meridionale

Corrections to the Bekenstein entropy

$$S = k_{\mathrm{B}} \frac{A}{4\ell_{\mathrm{Pl}}^2}$$

logarithmic corrections have been obtained in
a number of different approaches to quantum gravity
(string theory, AdS/CFT, loop quantum gravity, GUP, …)

$$S = k_{\mathrm{B}} \frac{A}{4\ell_{\mathrm{Pl}}^2} + k_{\mathrm{B}} C \log\left(\frac{A}{A_0}\right) + \mathcal{O}\left(\frac{A_0}{A}\right)$$

theory-specific
constant C

$A_0 \sim \ell_{\mathrm{Pl}}^2$

Corrections to the Bekenstein entropy

$$S = k_{\mathrm{B}} \frac{A}{4\ell_{\mathrm{Pl}}^2}$$

logarithmic corrections have been obtained in
a number of different approaches to quantum gravity
(string theory, AdS/CFT, loop quantum gravity, GUP, …)

The case of loop quantum gravity

In LQG one finds a modified entropy with log corrections

[Kaul Majumdar; Meissner; Engle, Noui, Perez, Pranzetti; …]

$$S = k_{\mathrm{B}} \frac{A}{4\ell_{\mathrm{Pl}}^2} - \frac{3}{2} k_{\mathrm{B}} \log\left(\frac{A}{A_0}\right) + \mathcal{O}\left(\frac{A_0}{A}\right)$$

Modified gravity from modified entropy

Gravitational dynamics is reconstructed from local equilibrium conditions, generalising Jacobson's spacetime thermodynamics approach

[Alonso-Serrano, Liška, JHEP 12, 196 (2020)]

The idea: modify spacetime thermodynamics to include leading order QG effects, in order to derive an *emergent gravitational dynamics* that is able to capture general features of low-energy QG.

$$\boxed{S_{\mu\nu} - \alpha\,\kappa\, S_{\mu\lambda}S^{\lambda}_{\ \nu} + \frac{\alpha\,\kappa}{4}\left(R_{\kappa\lambda}R^{\kappa\lambda} - \frac{1}{4}R^2\right) g_{\mu\nu} = \kappa\left(T_{\mu\nu} - \frac{1}{4}T g_{\mu\nu}\right)}$$

$$S_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{4}R\, g_{\mu\nu} \qquad \kappa \equiv 8\pi G$$

$\alpha \sim C$ in the entropy formula **treated phenomenologically**

For $\alpha \to 0$ one gets the trace-free Einstein equations (unimodular gravity)

As in unimodular gravity, energy-momentum conservation $\nabla^{\mu} T_{\mu\nu} = 0$ *does not* follow from the field equations. It is a *separate assumption*.

Also, as in unimodular gravity: the cosmological constant Λ is just an *integration constant*

Modified gravity from modified entropy

Gravitational dynamics is reconstructed from local equilibrium conditions, generalising Jacobson's spacetime thermodynamics approach

[Alonso-Serrano, Liška, JHEP 12, 196 (2020)]

The idea: modify spacetime thermodynamics to include leading order QG effects, in order to derive an *emergent gravitational dynamics* that is able to capture general features of low-energy QG.

Some further developments:

- spacetime thermodynamics and foundations: Alonso-Serrano, Liška, Universe 8 (2022) 50; IJGMMP 19 (2022) 2230002
- LQC-inspired cosmology: Alonso-Serrano, Liška, Vicente-Becerril, Phys. Lett. B 839 (2023) 137827; Alonso-Serrano, Mena Marugán, Vicente-Becerril, IJMPD 34 (2025) 2450062
- cosmological background dynamics and perturbations: MdC, Gubitosi, JCAP 01 (2024) 046
- black-hole solutions: Alonso-Serrano, MdC, Del Piano, Phys. Rev. D 112 (2025) 084031

Cosmological Background:

dynamical system analysis

Cosmological background

Spatially flat FLRW geometry

$$ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j$$

Assume a perfect fluid as matter

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p\, g_{\mu\nu}$$

constant equation of state: $w = p/\rho$

NEC: $w + 1 > 0$

From the field equations we obtain just one independent equation

$$\boxed{\dot{H}(1 - \alpha\kappa\dot{H}) = -\frac{\kappa}{2}(\rho + p)}$$

If we also assume energy-momentum conservation, then we have

$$\boxed{\dot{\rho} + 3H(\rho + p) = 0}$$

Cosmological dynamics can be studied as a 2D **dynamical system.**

We need to distinguish between two cases: $\alpha > 0$ and $\alpha < 0$.

[MdC, G. Gubitosi, JCAP 01, 046 (2024)]

Dynamical system analysis: case $\alpha > 0$

Dynamical system techniques represent a powerful tool in cosmology, with several applications to general relativity and modified gravity

[Coley; Wainwright, Ellis; …]

The starting point is to regard $\dot{H}(1 - \alpha\,\kappa\,\dot{H}) = -\frac{\kappa}{2}(\rho + p)$ as a *constraint* on $\dot{H},\ \rho$.

This way, we can better deal with multiple solution branches.

This motivates introducing a new variable ψ such that

$$\dot{H} = \frac{1}{2\alpha\kappa}\left(1 \pm \cosh\psi\right)\ ,\quad \rho = \frac{1}{2\alpha\kappa^2(1+w)}(\sinh\psi)^2$$

$$\dot{\psi} = -\frac{3}{2}(1+w)H\tanh\psi$$

All we have to do now is study the dynamical system for H and ψ

Dynamical system analysis: case $\alpha > 0$

$$\dot{H} = \frac{1}{2\alpha\kappa}\left(1 \pm \cosh\psi\right) \qquad \dot{\psi} = -\frac{3}{2}(1+w)H\tanh\psi$$

First consequences

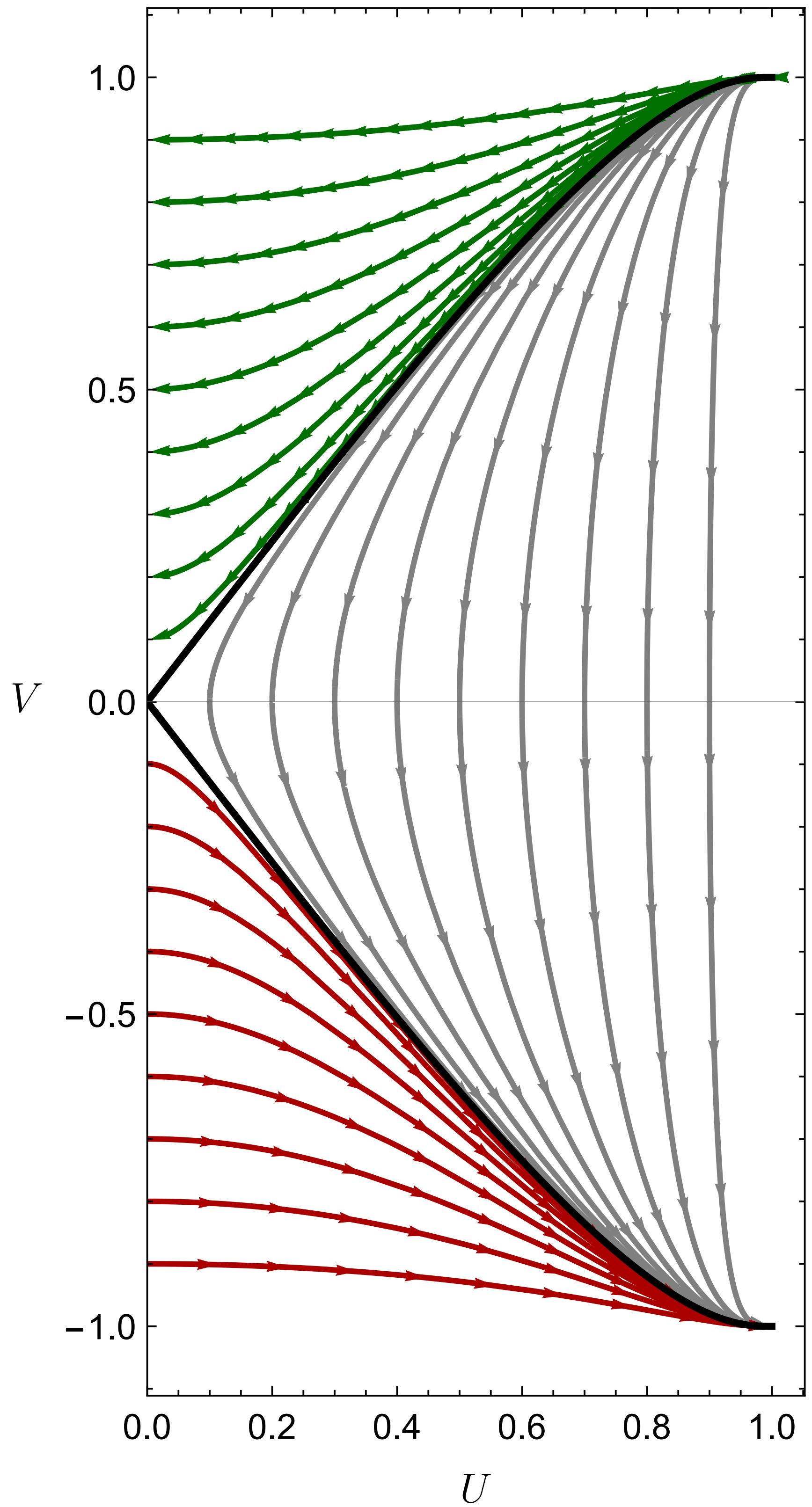
There are two branches:

- one with $\dot{H} > 0$ at all times $\Longrightarrow$ not viable
- one with $\dot{H} \leq 0$ (as in standard cosmology), which we focus on.

Note: in the latter branch there is **no bounce**,
since this would require that H has a zero, around which $\dot{H} > 0$

Are there viable cosmological solutions in this model?

Phase portrait ($\alpha > 0$)



$$\boxed{\dot{H} = \frac{1}{2\alpha\kappa}\left(1 \pm \cosh\psi\right) \quad , \quad \dot{\psi} = -\frac{3}{2}(1+w)H\tanh\psi}$$

Compactify the "phase space":

$U = \tanh\psi$,

$V = \tanh(\sqrt{\kappa}H)$

Orbits fall in three classes:

- ever-collapsing
- re-collapsing
- ever-expanding, with a late-time de Sitter era

At early times, potentially viable orbits approach a past attractor corresponding to **power-law inflation** $a(t) \sim t^q$, with $q = 4/(3(1+w))$

However, the slow-roll condition $\epsilon \ll 1$ and $N_{\text{e-folds}} \approx 60$ can only be satisfied with fine-tuning:

$w + 1 \ll 1$ and $\alpha/(1+w) \approx \frac{3}{32\pi}$

(def. $\epsilon \equiv -\dot{H}/H^2 = 1/q$)

Dynamical system analysis: case $\alpha < 0$

In this case we use a different parametrisation

$$\dot{H} = \frac{1}{2|\alpha|\kappa}\left(\cos\psi - 1\right) \;\; , \quad \rho = \frac{1}{2|\alpha|\kappa^2(1+w)}(\sin\psi)^2$$

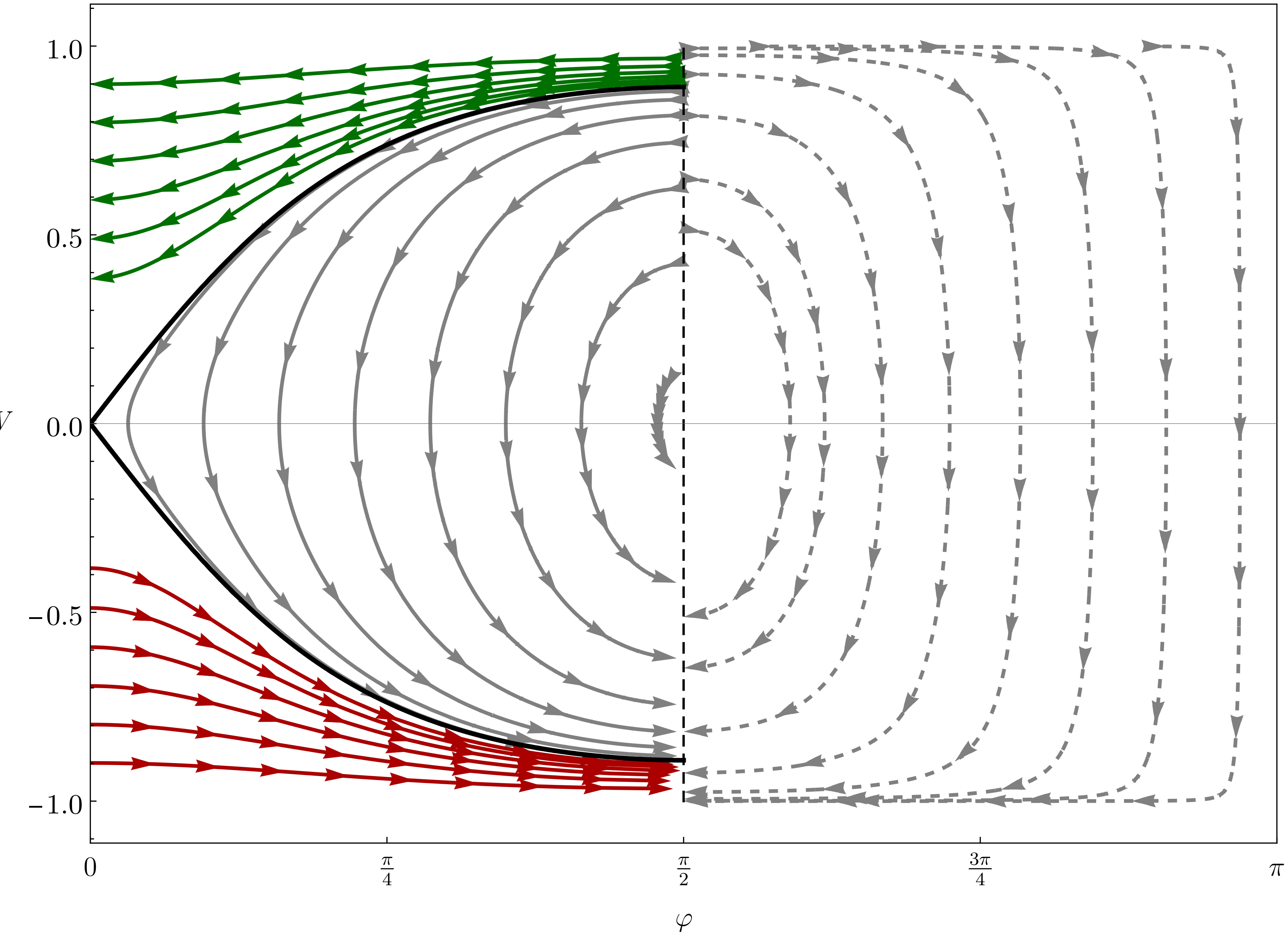
The continuity equation then gives

$$\dot{\psi} = -\frac{3}{2}(1+w)H\tan\psi$$

In this case there is only one branch, where $\dot{H} \leq 0$ identically

Again, $\dot{H} \leq 0$ implies that there is **no bounce**.

Phase portrait ($\alpha < 0$)



$$\dot{H} = \frac{1}{2|\alpha|\kappa}\left(\cos\psi - 1\right) \ ,$$
$$\rho = \frac{1}{2|\alpha|\kappa^2(1+w)}(\sin\psi)^2$$

Compactify the "phase space":
$$W = \tanh\left(\sqrt{\kappa}H\right)$$

Again, orbits fall in three classes:

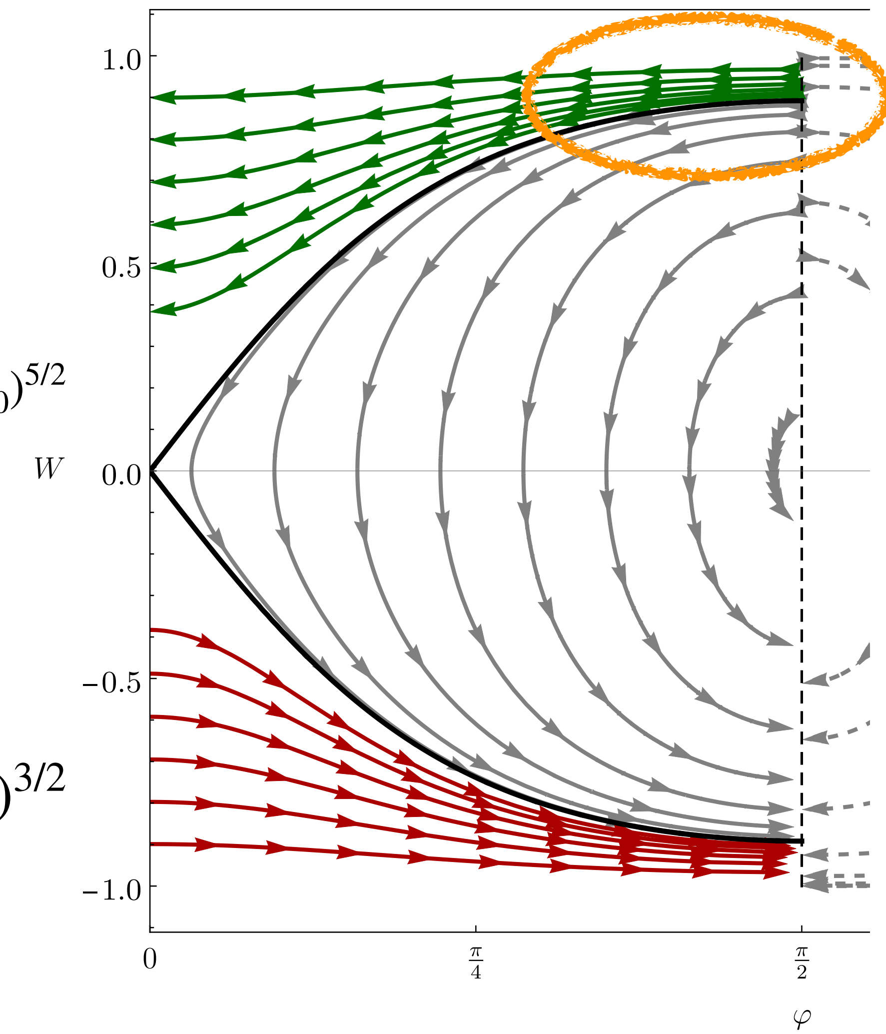
- ever-collapsing
- re-collapsing
- ever-expanding, with a late-time de Sitter era

Also in this case, potentially viable solutions approach an inflationary attractor in the past

Past attractor ($\alpha < 0$)

In the far past, the Hubble rate has the following asymptotics
(the singularity is at $t = t_0$):

$$H \approx H_{\rm i} - \frac{t-t_0}{2|\alpha|\kappa} + \sqrt{\frac{(1+w)H_{\rm i}}{3}}\frac{(t-t_0)^{3/2}}{|\alpha|\kappa} - \frac{1}{20}\left[\frac{1}{2\alpha^2\kappa^2}\sqrt{\frac{3(w+1)}{H_{\rm i}}} + \frac{1}{|\alpha|\kappa}\left(3(w+1)H_{\rm i}\right)^{3/2}\right](t-t_0)^{5/2}$$



The slow-roll parameters read:

$$\epsilon = -\dot{H}/H^2 = \frac{1}{2|\alpha|\kappa H_{\rm i}^2}\left(1 - \sqrt{3(1+w)H_{\rm i}}(t-t_0)^{1/2} + \frac{(t-t_0)}{|\alpha|\kappa H_{\rm i}}\right) + \mathcal{O}(t-t_0)^{3/2}$$

$$\eta = \frac{\dot{\epsilon}}{H\epsilon} = -\frac{1}{2}\sqrt{\frac{3(1+w)}{H_i}}(t-t_0)^{-1/2} + \frac{1}{|\alpha|\kappa H_{\rm i}^2} - \frac{3}{2}(1+w) + \mathcal{O}(t-t_0)^{1/2}\ .$$

Therefore, the slow-roll conditions $\epsilon, |\eta| \ll 1$ cannot be satisfied simultaneously
(unless we fine-tune $w + 1 \ll 1$, which however amounts to assuming inflation)

Theoretical bound on α from the background dynamics

So far, we have identified potentially viable background solutions that:

- are dominated by a cosmological constant at late times
- approach an inflationary attractor at early times. However, they cannot satisfy slow-roll conditions or predict the correct number of e-folds—unless we assume matter with $w \approx -1$

Therefore, the model is incomplete in the early universe:
it does not replace inflation

A conservative bound on α is obtained by requiring that departures from general relativity be strongly suppressed after reheating.

$$\dot{H}(1-\alpha\,\kappa\,\dot{H}) = -\frac{\kappa}{2}(\bar{\rho}+\bar{p})$$

$$|\alpha\,\kappa\,\dot{H}| \ll 1 \qquad \Longrightarrow \qquad \boxed{|\alpha| \ll \frac{1}{2\kappa H_{\rm reh}^2} = \frac{M_{\rm Pl}^2}{2H_{\rm reh}^2}}$$

Dynamics of Cosmological Perturbations

Tensor perturbations

$$ds^2 = a(\eta)^2\Big(-d\eta^2 + (\delta_{ij} + h_{ij}(\eta, x))dx^i dx^j\Big)$$

h_{ij} transverse and traceless: $h^i{}_i = 0$, $\partial^i h_{ij} = 0$

The linearised field equations for tensor perturbations read:

$$h''_{ij} + 2\mathscr{H} h'_{ij} - \Delta h_{ij} = 2\kappa\left(1 + \frac{\alpha\kappa}{a^2}\left(\mathscr{H}' - \mathscr{H}^2\right)\right)^{-1} a^2 \pi_{ij}$$

In the tensor sector, α-dependent corrections only affect the coupling to anisotropic stress (as well as the background quantities a, $\mathscr{H}$)

Scalar perturbations

$$ds^2 = a(\eta)^2\Big(-(1+2\Phi(\eta,x))d\eta^2 + (1-2\Psi(\eta,x))\delta_{ij}dx^i dx^j\Big)$$

Assume matter with no anisotropic stress for simplicity. As in GR, this *implies* $\Psi = \Phi$

$$\delta T^0{}_0 = -\delta\rho\ , \qquad \delta T^0{}_i = (\bar\rho + \bar p)v_{,i}\ , \qquad \delta T^i{}_j = \delta p\,\delta^i{}_j$$

The perturbed field equations in the scalar sector read:

$$\left[1+\frac{\alpha\kappa}{a^2}\left(\mathscr{H}^2-\mathscr{H}'\right)\right]\left(\Phi'+\mathscr{H}\Phi\right) = -\frac{\kappa}{2}a^2(\bar\rho+\bar p)v\ ,$$

$$\left[1+\frac{2\alpha\kappa}{a^2}\left(\mathscr{H}^2-\mathscr{H}'\right)\right]\left[\Phi''+2\left(\mathscr{H}'-\mathscr{H}^2\right)\Phi+\Delta\Phi\right] = \frac{\kappa}{2}a^2(\delta\rho+\delta p)\ .$$

In the following, we will specialise to single-field inflation

Inflaton perturbations

Assume an inflaton field, homogeneous at the background level, with inhomogeneous perturbations on top:

$$\varphi(\eta, \mathbf{x}) = \bar{\varphi}(\eta) + \delta\varphi(\eta, \mathbf{x})$$

$$\bar{\rho} = \frac{\bar{\varphi}'^2}{2a^2} + V(\bar{\varphi}) \ , \qquad \bar{p} = \frac{\bar{\varphi}'^2}{2a^2} - V(\bar{\varphi}) \ , \qquad \bar{\rho} + \bar{p} = \frac{\bar{\varphi}'^2}{a^2}$$

Background dynamics:

$$\left(\mathscr{H}^2 - \mathscr{H}'\right)\left[1 + \frac{\alpha\kappa}{a^2}\left(\mathscr{H}^2 - \mathscr{H}'\right)\right] = \frac{\kappa}{2}\bar{\varphi}'^2 \qquad \bar{\varphi}'' + 2\mathscr{H}\bar{\varphi}' + a^2 V_{,\varphi} = 0 \ .$$

Perturbed Klein-Gordon equation:

$$\Box\,\varphi - V_{,\varphi} = 0 \implies \delta\varphi'' + 2\mathscr{H}\,\delta\varphi' - \nabla^2\delta\varphi - 4\Phi'\bar{\varphi}' + a^2 V_{,\varphi\varphi}\delta\varphi + 2a^2 V_{,\varphi}\Phi = 0$$

Modified Mukhanov-Sasaki equation

Comoving curvature perturbation (standard definition):

$$\mathscr{R} \equiv \Phi + \frac{\mathscr{H}}{\bar{\varphi}'}\delta\varphi$$

Using the perturbed field equations and the above definition, we obtain a dynamical equation for the comoving curvature perturbation (in Fourier space):

$$\mathscr{R}_k'' + \left[2\left(\mathscr{H} - \frac{\mathscr{H}'}{\mathscr{H}} + \frac{\bar{\varphi}''}{\bar{\varphi}'}\right) - \Gamma_{\rm ent}(\eta)\right]\mathscr{R}_k' + k^2\mathscr{R}_k = 0$$

$$\Gamma_{\rm ent}(\eta) \equiv \frac{\left(\mathscr{H}^2 - \mathscr{H}'\right)\left(\sqrt{a^2 + 2\alpha\kappa^2\bar{\varphi}'^2} - a\right)}{\mathscr{H}\sqrt{a^2 + 2\alpha\kappa^2\bar{\varphi}'^2}}$$

With the following definitions:

$$v_k \equiv z_{\rm eff}\mathscr{R}_k$$

$$\frac{z_{\rm eff}'}{z_{\rm eff}} \equiv \frac{z_{\rm GR}'}{z_{\rm GR}} - \frac{1}{2}\Gamma_{\rm ent}(\eta) = \mathscr{H} - \frac{\mathscr{H}'}{\mathscr{H}} + \frac{\bar{\varphi}''}{\bar{\varphi}'} - \frac{1}{2}\Gamma_{\rm ent}(\eta) \implies z_{\rm eff}(\eta) = z_{\rm GR}(\eta)\exp\left[-\frac{1}{2}\int_{\eta_\circ}^{\eta}\Gamma_{\rm ent}(\tilde{\eta})\,{\rm d}\tilde{\eta}\right]$$

$$\boxed{v_k'' + \left(k^2 - \frac{z_{\rm eff}''}{z_{\rm eff}}\right)v_k = 0}$$

Primordial power spectra at $\mathbf{N^3LO}$

(next-to-next-to-next-to-leading order)

Green's function method at $\mathrm{N^3LO}$

method developed in: [Auclair, Ringeval (2022)]

applications to effective theories of inflation: [Bianchi, Gamonal (2024)]

Change of variables: $x \equiv -k\eta > 0\,,\qquad y(x) \equiv \sqrt{2k}\, v_k(\eta)$

The modified Mukhanov-Sasaki equation reads: $$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left[1 - \frac{1}{k^2}\frac{z''_{\mathrm{eff}}}{z_{\mathrm{eff}}}\right] y = 0$$

Deviations from de Sitter encoded in the source term $g(\ln x)$: $$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left(1 - \frac{2}{x^2}\right) y = \frac{g(\ln x)}{x^2} y \qquad g(\ln x) \equiv \eta^2 \frac{z''_{\mathrm{eff}}}{z_{\mathrm{eff}}} - 2$$

General solution in integral form: $$y(x) = y_0(x) + \int_x^\infty \frac{\mathrm{d}u}{u^2} G(x,u)\, g(\ln u)\, y(u)$$

Solution in dS with Bunch-Davies initial conditions: $$y_0(x) = \left(1 + \frac{i}{x}\right) e^{ix}$$

Green's function: $$G(x,u) = \frac{i}{2}\left[y_0^*(u) y_0(x) - y_0(u) y_0^*(x)\right] \Theta(u-x)\ .$$

Expand the solution and the source around $\ln x_\flat = 0$:

$$g(\ln x) = g_{1\flat} + g_{2\flat} \ln x + g_{3\flat} \ln^2 x + \ldots$$

$$y(x) = y_0(x) + y_1(x) + y_2(x) + y_3(x) + \ldots$$

Source expansion coefficients

$$g_{1\flat} = \delta_\flat \left[-6\epsilon_{1\flat}^3 - 6\epsilon_{1\flat}^2 - \frac{9}{2}\epsilon_{2\flat}\epsilon_{1\flat}^2 + \epsilon_{1\flat}\epsilon_{2\flat}^2 + \frac{3}{2}\epsilon_{1\flat}\epsilon_{2\flat} + \frac{1}{2}\epsilon_{1\flat}\epsilon_{2\flat}\epsilon_{3\flat} \right] + 5\epsilon_{1\flat}^3 + \frac{35}{2}\epsilon_{2\flat}\epsilon_{1\flat}^2 + 4\epsilon_{1\flat}^2 + \frac{15}{2}\epsilon_{2\flat}^2\epsilon_{1\flat} + \frac{13}{2}\epsilon_{2\flat}\epsilon_{1\flat} + 5\epsilon_{2\flat}\epsilon_{3\flat}\epsilon_{1\flat} + 3\epsilon_{1\flat} + \frac{1}{4}\epsilon_{2\flat}^2 + \frac{3}{2}\epsilon_{2\flat} + \frac{1}{2}\epsilon_{2\flat}\epsilon_{3\flat} ,$$

$$g_{2\flat} = \delta_\flat \left[-12\epsilon_{1\flat}^3 + 15\epsilon_{2\flat}\epsilon_{1\flat}^2 - \frac{3}{2}\epsilon_{1\flat}\epsilon_{2\flat}^2 - \frac{3}{2}\epsilon_{1\flat}\epsilon_{2\flat}\epsilon_{3\flat} \right] - 11\epsilon_{2\flat}\epsilon_{1\flat}^2 - \frac{13}{2}\epsilon_{2\flat}^2\epsilon_{1\flat} - 3\epsilon_{2\flat}\epsilon_{1\flat} - 8\epsilon_{2\flat}\epsilon_{3\flat}\epsilon_{1\flat} - \frac{1}{2}\epsilon_{2\flat}\epsilon_{3\flat}^2 - \frac{1}{2}\epsilon_{2\flat}^2\epsilon_{3\flat} - \frac{3}{2}\epsilon_{2\flat}\epsilon_{3\flat} - \frac{1}{2}\epsilon_{2\flat}\epsilon_{3\flat}\epsilon_{4\flat} ,$$

$$g_{3\flat} = \frac{3}{2}\epsilon_{1\flat}\epsilon_{2\flat}^2 + \frac{3}{4}\epsilon_{3\flat}^2\epsilon_{2\flat} + \frac{3}{2}\epsilon_{1\flat}\epsilon_{3\flat}\epsilon_{2\flat} + \frac{3}{4}\epsilon_{3\flat}\epsilon_{4\flat}\epsilon_{2\flat} .$$

Scalar power spectrum

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2} \left| \frac{v_k}{z_{\text{eff}}} \right|^2_{x\to 0} = \frac{1}{4\pi^2} \lim_{x\to 0^+} \left| x\, y(x) \right|^2 \left| \frac{k}{x z_{\text{eff}}} \right|^2 .$$

The time corresponding to $x_\flat = 1$ is k-dependent, so we must perform a further logarithmic expansion around a pivot scale $k_\diamond$:

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{\kappa H_\diamond^2}{8\pi^2 \epsilon_{1\diamond}} \left[P_0 + P_1 \ln\left(\frac{k}{k_\diamond}\right) + P_2 \ln^2\left(\frac{k}{k_\diamond}\right) + P_3 \ln^3\left(\frac{k}{k_\diamond}\right) + \dots \right]$$

Power-spectrum expansion coefficients

$$P_0 = 1 - 2\left(\mathscr{C} + 1 + \frac{\delta_\diamond}{2}\right)\epsilon_{1\diamond} - \mathscr{C}\epsilon_{2\diamond} + \left[\frac{\pi^2}{2} + 2\mathscr{C}^2 + 2\mathscr{C} - 3 + \delta_\diamond(6\mathscr{C}+2)\right]\epsilon_{1\diamond}^2 + \left(\frac{7\pi^2}{12} + \mathscr{C}^2 - \mathscr{C} - 6\right)\epsilon_{1\diamond}\epsilon_{2\diamond} + \frac{1}{8}\left(\pi^2 + 4\mathscr{C}^2 - 8\right)\epsilon_{2\diamond}^2 + \frac{1}{24}\left(\pi^2 - 12\mathscr{C}^2\right)\epsilon_{2\diamond}\epsilon_{3\diamond}$$
$$-\frac{1}{24}\left[4\mathscr{C}^3 + 3(\pi^2-8)\mathscr{C} + 14\zeta(3) - 16\right]\left(8\epsilon_{1\diamond}^3 + \epsilon_{2\diamond}^3\right) + \frac{1}{12}\left[13\pi^2 - 8(\pi^2-9)\mathscr{C} + 36\mathscr{C}^2 - 84\zeta(3) - 12\delta_\diamond\left(\frac{3}{2}\pi^2 - 2\mathscr{C}^2 - 2\mathscr{C} - 10\right)\right]\epsilon_{1\diamond}^2\epsilon_{2\diamond}$$
$$-\frac{1}{24}\left[8\mathscr{C}^3 - 15\pi^2 + 6(\pi^2-4)\mathscr{C} - 12\mathscr{C}^2 + 100\zeta(3) + 16 - 4\delta_\diamond(\pi^2-6)\right]\epsilon_{1\diamond}\epsilon_{2\diamond}^2 + \frac{1}{24}\left(\pi^2\mathscr{C} - 4\mathscr{C}^3 - 8\zeta(3) + 16\right)\left(\epsilon_{2\diamond}\epsilon_{3\diamond}^2 + \epsilon_{2\diamond}\epsilon_{3\diamond}\epsilon_{4\diamond}\right)$$
$$+\frac{1}{24}\left[12\mathscr{C}^3 + (5\pi^2-48)\mathscr{C}\right]\epsilon_{2\diamond}^2\epsilon_{3\diamond} - \delta_\diamond\epsilon_{1\diamond}^3\left(\frac{13}{6}\pi^2 + 14\mathscr{C}^2 + 6\mathscr{C} - 19\right) + \frac{1}{12}\left[8\mathscr{C}^3 + \pi^2 + 6(\pi^2-12)\mathscr{C} - 12\mathscr{C}^2 - 8\zeta(3) - 8\right]\epsilon_{1\diamond}\epsilon_{2\diamond}\epsilon_{3\diamond}\ ,$$

$$P_1 = -2\epsilon_{1\diamond} - \epsilon_{2\diamond} + 2(2\mathscr{C} + 1 + 2\delta_\diamond)\epsilon_{1\diamond}^2 + (2\mathscr{C}-1)\epsilon_{1\diamond}\epsilon_{2\diamond} + \mathscr{C}\epsilon_{2\diamond}^2 - \mathscr{C}\epsilon_{2\diamond}\epsilon_{3\diamond} - \frac{1}{8}\left(\pi^2 + 4\mathscr{C}^2 - 8\right)\left(8\epsilon_{1\diamond}^3 + \epsilon_{2\diamond}^3\right) - 4\delta_\diamond(6\mathscr{C}+1)\epsilon_{1\diamond}^3$$
$$-\frac{2}{3}\left[\pi^2 - 9\mathscr{C} - 9 - 3\delta_\diamond(3\mathscr{C}+1)\right]\epsilon_{1\diamond}^2\epsilon_{2\diamond} - \frac{1}{4}\left(\pi^2 + 4\mathscr{C}^2 - 4\mathscr{C} - 4\right)\epsilon_{1\diamond}\epsilon_{2\diamond}^2 + \frac{1}{2}\left(\pi^2 + 4\mathscr{C}^2 - 4\mathscr{C} - 12\right)\epsilon_{1\diamond}\epsilon_{2\diamond}\epsilon_{3\diamond} + \frac{1}{24}\left(\pi^2 - 12\mathscr{C}^2\right)\left(\epsilon_{2\diamond}\epsilon_{3\diamond}^2 + \epsilon_{2\diamond}\epsilon_{3\diamond}\epsilon_{4\diamond}\right)$$
$$+\frac{1}{24}\left(5\pi^2 + 36\mathscr{C}^2 - 48\right)\epsilon_{2\diamond}^2\epsilon_{3\diamond}\ ,$$

$$P_2 = 2\epsilon_{1\diamond}^2 + \epsilon_{1\diamond}\epsilon_{2\diamond} + \frac{1}{2}\epsilon_{2\diamond}^2 - \frac{1}{2}\epsilon_{2\diamond}\epsilon_{3\diamond} + \left(3 + 2\delta_\diamond\right)\epsilon_{1\diamond}^2\epsilon_{2\diamond} - \left(\mathscr{C} - \frac{1}{2}\right)\left(\epsilon_{1\diamond}\epsilon_{2\diamond}^2 - 2\epsilon_{1\diamond}\epsilon_{2\diamond}\epsilon_{3\diamond}\right) - \left(4\mathscr{C} + 8\delta_\diamond\right)\epsilon_{1\diamond}^3 - \frac{\mathscr{C}}{2}\left(\epsilon_{2\diamond}^3 - 3\epsilon_{2\diamond}^2\epsilon_{3\diamond} + \epsilon_{2\diamond}\epsilon_{3\diamond}^2 + \epsilon_{2\diamond}\epsilon_{3\diamond}\epsilon_{4\diamond}\right)\ ,$$

$$P_3 = \frac{1}{6}\left(-8\epsilon_{1\diamond}^3 - 2\epsilon_{1\diamond}\epsilon_{2\diamond}^2 + 4\epsilon_{1\diamond}\epsilon_{2\diamond}\epsilon_{3\diamond} - \epsilon_{2\diamond}^3 + 3\epsilon_{2\diamond}^2\epsilon_{3\diamond} - \epsilon_{2\diamond}\epsilon_{3\diamond}^2 - \epsilon_{2\diamond}\epsilon_{3\diamond}\epsilon_{4\diamond}\right)$$

$\mathscr{C} \equiv \gamma_E + \ln 2 - 2\ ,$ $\qquad \delta_\diamond \equiv \alpha\kappa H_\diamond^2$

explicit α-dependent corrections appear through this combination

Scalar Power Spectrum

Next, we map the results obtained to the standard phenomenological parametrisation:

$$\ln \mathscr{P}_{\mathscr{R}}(k) = \ln A_s + (n_s - 1)\ln\left(\frac{k}{k_\diamond}\right) + \frac{1}{2}\alpha_s \ln^2\left(\frac{k}{k_\diamond}\right) + \frac{1}{6}\beta_s \ln^3\left(\frac{k}{k_\diamond}\right) + \dots$$

which gives:

$$A_s = \frac{\kappa H_\diamond^2}{8\pi^2 \epsilon_{1\diamond}} P_0 \,, \qquad n_s - 1 = \frac{P_1}{P_0} \,,$$

$$\alpha_s = 2\left(\frac{P_2}{P_0} - \frac{1}{2}\frac{P_1^2}{P_0^2}\right) \,, \qquad \beta_s = 6\left(\frac{P_3}{P_0} - \frac{P_1 P_2}{P_0^2} + \frac{1}{3}\frac{P_1^3}{P_0^3}\right)$$

We expand each of these quantities to N3LO and use the results for the expansion coefficients of P_n

$$n_s = 1 + n_s^{(1)} + n_s^{(2)} + n_s^{(3)} + \dots \,, \qquad \alpha_s = \alpha_s^{(2)} + \alpha_s^{(3)} + \dots \,, \qquad \beta_s = \beta_s^{(3)} + \dots$$

$$n_s^{(1)} = -2\epsilon_{1\diamond} - \epsilon_{2\diamond} \,,$$

$$n_s^{(2)} = (-2 + 4\delta_\diamond)\epsilon_{1\diamond}^2 - (2\mathscr{C} + 3 + \delta_\diamond)\epsilon_{1\diamond}\epsilon_{2\diamond} - \mathscr{C}\epsilon_{2\diamond}\epsilon_{3\diamond} \,,$$

$$\begin{aligned} n_s^{(3)} = {} & \left[-2 - 4\delta_\diamond(2\mathscr{C} - 1)\right]\epsilon_{1\diamond}^3 + \left[\pi^2 - 6\mathscr{C} - 15 + \delta_\diamond(10\mathscr{C} - 1)\right]\epsilon_{1\diamond}^2\epsilon_{2\diamond} \\ & + \frac{1}{12}\left[7\pi^2 - 12\mathscr{C}^2 - 36\mathscr{C} - 84 - 12\mathscr{C}\delta_\diamond\right]\epsilon_{1\diamond}\epsilon_{2\diamond}^2 + \left[\frac{7\pi^2}{12} - \mathscr{C}^2 - 4\mathscr{C} - 6 - \mathscr{C}\delta_\diamond\right]\epsilon_{1\diamond}\epsilon_{2\diamond}\epsilon_{3\diamond} \\ & + \frac{1}{24}\left(\pi^2 - 12\mathscr{C}^2\right)\left(\epsilon_{2\diamond}\epsilon_{3\diamond}^2 + \epsilon_{2\diamond}\epsilon_{3\diamond}\epsilon_{4\diamond}\right) + \left(\frac{\pi^2}{4} - 2\right)\epsilon_{2\diamond}^2\epsilon_{3\diamond} \,. \end{aligned}$$

$$\alpha_s^{(2)} = -2\epsilon_{1\diamond}\epsilon_{2\diamond} - \epsilon_{2\diamond}\epsilon_{3\diamond} \,,$$

$$\begin{aligned} \alpha_s^{(3)} = {} & -8\delta_\diamond\epsilon_{1\diamond}^3 + (-6 + 10\delta_\diamond)\epsilon_{1\diamond}^2\epsilon_{2\diamond} - (2\mathscr{C} + 3 + \delta_\diamond)\epsilon_{1\diamond}\epsilon_{2\diamond}^2 - (2\mathscr{C} + 4 + \delta_\diamond)\epsilon_{1\diamond}\epsilon_{2\diamond}\epsilon_{3\diamond} \\ & - \mathscr{C}\left(\epsilon_{2\diamond}\epsilon_{3\diamond}^2 + \epsilon_{2\diamond}\epsilon_{3\diamond}\epsilon_{4\diamond}\right) \,, \end{aligned}$$

$$\beta_s^{(3)} = -2\epsilon_{1\diamond}\epsilon_{2\diamond}^2 - 2\epsilon_{1\diamond}\epsilon_{2\diamond}\epsilon_{3\diamond} - \epsilon_{2\diamond}\epsilon_{3\diamond}^2 - \epsilon_{2\diamond}\epsilon_{3\diamond}\epsilon_{4\diamond}$$

Takeaway

$$A_s\text{: } \mathcal{O}(\delta_\diamond \epsilon) \,,$$

$$n_s - 1\text{: } \mathcal{O}(\delta_\diamond \epsilon^2) \,,$$

$$\alpha_s\text{: } \mathcal{O}(\delta_\diamond \epsilon^3) \,,$$

β_s: no $\delta_\diamond$ corrections at N3LO

Tensor-to-scalar ratio

We perform analogous expansions for the tensor power spectrum (details in the paper), and then compute the tensor-to-scalar ratio, as well as departures from the leading-order consistency relation $r = -8n_T$

$$r = 16\epsilon_{1\diamond}\left(1 + r^{(1)} + r^{(2)} + r^{(3)} + \dots\right)$$

$$r^{(1)} = \mathscr{C}\epsilon_{2\diamond} + \delta_\diamond \epsilon_{1\diamond}\,,$$

$$r^{(2)} = -\frac{1}{8}\left(\pi^2 - 4\mathscr{C}^2 - 8\right)\epsilon_{2\diamond}^2 - 4\mathscr{C}\delta_\diamond\epsilon_{1\diamond}^2 - \left(\frac{\pi^2}{2} - \mathscr{C} - 4 - 2\mathscr{C}\delta_\diamond\right)\epsilon_{1\diamond}\epsilon_{2\diamond} - \frac{1}{24}\left(\pi^2 - 12\mathscr{C}^2\right)\epsilon_{2\diamond}\epsilon_{3\diamond}\,,$$

$$\begin{aligned} r^{(3)} = {} & \frac{1}{24}\left[4\mathscr{C}^3 - 3(\pi^2-8)\mathscr{C} + 14\zeta(3) - 16\right]\epsilon_{2\diamond}^3 + \frac{1}{3}\delta_\diamond\epsilon_{1\diamond}^3\left(5\pi^2 + 12\mathscr{C}^2 - 12\mathscr{C} - 48\right) - \frac{1}{24}\left(\pi^2\mathscr{C} - 4\mathscr{C}^3 - 8\zeta(3) + 16\right)\left(\epsilon_{2\diamond}\epsilon_{3\diamond}^2 + \epsilon_{2\diamond}\epsilon_{3\diamond}\epsilon_{4\diamond}\right) \\ & + \left[\frac{\mathscr{C}^3}{2} - \frac{1}{24}(7\pi^2 - 48)\mathscr{C}\right]\epsilon_{2\diamond}^2\epsilon_{3\diamond} - \left[\pi^2 - \mathscr{C} - 7\zeta(3) - \delta_\diamond\left(\frac{5\pi^2}{12} - 9\mathscr{C}^2 + 2\mathscr{C}\right)\right]\epsilon_{1\diamond}^2\epsilon_{2\diamond} - \frac{1}{12}\left[\pi^2 - 12\mathscr{C}^2 + 6\mathscr{C}(\pi^2 - 8) + \delta_\diamond\left(\pi^2 - 12\mathscr{C}^2\right)\right]\epsilon_{1\diamond}\epsilon_{2\diamond}\epsilon_{3\diamond} \\ & - \frac{1}{24}\left[19\pi^2 - 36\mathscr{C}^2 + 24(\pi^2-9)\mathscr{C} - 84\zeta(3) - 48 + 2\delta_\diamond\left(5\pi^2 - 24\mathscr{C}^2 - 36\right)\right]\epsilon_{1\diamond}\epsilon_{2\diamond}^2 \end{aligned}$$

$$\Delta_c \equiv r + 8n_T \qquad\qquad \Delta_c = 16\epsilon_{1\diamond}\left(\Delta_c^{(1)} + \Delta_c^{(2)} + \Delta_c^{(3)} + \dots\right)$$

$$\Delta_c^{(1)} = (-1 + \delta_\diamond)\epsilon_{1\diamond} - \epsilon_{2\diamond}\,,$$

$$\Delta_c^{(2)} = (-1 - 4\mathscr{C}\delta_\diamond)\epsilon_{1\diamond}^2 - (2\mathscr{C} + 3 - 2\mathscr{C}\delta_\diamond)\epsilon_{1\diamond}\epsilon_{2\diamond} - \left(\frac{\pi^2}{12} + \mathscr{C}\right)\epsilon_{2\diamond}^2 - (\mathscr{C}+1)\epsilon_{2\diamond}\epsilon_{3\diamond}\,,$$

$$\begin{aligned} \Delta_c^{(3)} = {} & \left[-1 + \delta_\diamond\left(\frac{5\pi^2}{3} + 4\mathscr{C}^2 - 4\mathscr{C} - 16\right)\right]\epsilon_{1\diamond}^3 + \left[\pi^2 - 5\mathscr{C} - 14 + \delta_\diamond\left(\frac{5\pi^2}{12} - 9\mathscr{C}^2 + 2\mathscr{C}\right)\right]\epsilon_{1\diamond}^2\epsilon_{2\diamond} + \left[\frac{7\pi^2}{12} - \mathscr{C}^2 - 4\mathscr{C} - 8 - \delta_\diamond\left(\frac{\pi^2}{12} - \mathscr{C}^2\right)\right]\epsilon_{1\diamond}\epsilon_{2\diamond}\epsilon_{3\diamond} + \left(\frac{\pi^2}{24} - \frac{\mathscr{C}^2}{2} - \mathscr{C} - 1\right)\left(\epsilon_{2\diamond}\epsilon_{3\diamond}^2 + \epsilon_{2\diamond}\epsilon_{3\diamond}\epsilon_{4\diamond}\right) \\ & + \left[\frac{\pi^2}{8} - \frac{3\mathscr{C}^2}{2} - \frac{\mathscr{C}}{6}(\pi^2 + 6) - \zeta(3) - 1\right]\epsilon_{2\diamond}^2\epsilon_{3\diamond} + \left[\frac{\pi^2}{24} - \frac{\mathscr{C}^2}{2} - \frac{\mathscr{C}\pi^2}{12} + \frac{\zeta(3)}{4} - 1\right]\epsilon_{2\diamond}^3 + \left[\frac{\pi^2}{2} - 2\mathscr{C}^2 - 7\mathscr{C} + \frac{7\zeta(3)}{2} - 13 - \delta_\diamond\left(\frac{5\pi^2}{12} - 2\mathscr{C}^2 - 3\right)\right]\epsilon_{1\diamond}\epsilon_{2\diamond}^2 \end{aligned}$$

Conclusions

- Scalar perturbations retain the Mukhanov–Sasaki form, with $z_{\rm GR}$ replaced by $z_{\rm eff}$, and $c_s = 1$
- Free tensor modes keep the standard propagation equation and are modified only through the background evolution
- Explicit entropy corrections are progressively slow-roll suppressed: $A_s : \mathcal{O}(\delta_\diamond \epsilon)$; $n_s - 1 : \mathcal{O}(\delta_\diamond \epsilon^2)$; $\alpha_s : \mathcal{O}(\delta_\diamond \epsilon^3)$. N3LO is therefore essential: lower orders do not capture the first explicit correction to the running α_s
- Scalar and tensor sectors are affected differently, leading to corrections to r and to the single-field consistency relation

Conclusions

- Scalar perturbations retain the Mukhanov–Sasaki form, with z_{GR} replaced by z_{eff} , and $c_s = 1$
- Free tensor modes keep the standard propagation equation and are modified only through the background evolution
- Explicit entropy corrections are progressively slow-roll suppressed: $A_s : \mathcal{O}(\delta_\diamond \epsilon)$; $n_s - 1 : \mathcal{O}(\delta_\diamond \epsilon^2)$; $\alpha_s : \mathcal{O}(\delta_\diamond \epsilon^3)$. N3LO is therefore essential: lower orders do not capture the first explicit correction to the running α_s
- Scalar and tensor sectors are affected differently, leading to corrections to r and to the single-field consistency relation

Future directions

- Study concrete inflationary potentials and quantify how the modified background changes inflationary predictions
- Evolve perturbations through reheating and later cosmological epochs, and implement the model in a Boltzmann code
- Constrain the model using a joint analysis of n_s, α_s, r and, when available in the future, n_T

Conclusions

- Scalar perturbations retain the Mukhanov–Sasaki form, with $z_{\rm GR}$ replaced by $z_{\rm eff}$, and $c_s = 1$
- Free tensor modes keep the standard propagation equation and are modified only through the background evolution
- Explicit entropy corrections are progressively slow-roll suppressed: $A_s : \mathcal{O}(\delta_\diamond \epsilon)$; $n_s - 1 : \mathcal{O}(\delta_\diamond \epsilon^2)$; $\alpha_s : \mathcal{O}(\delta_\diamond \epsilon^3)$. N3LO is therefore essential: lower orders do not capture the first explicit correction to the running α_s
- Scalar and tensor sectors are affected differently, leading to corrections to r and to the single-field consistency relation

Conclusions

- Scalar perturbations retain the Mukhanov–Sasaki form, with $z_{\rm GR}$ replaced by $z_{\rm eff}$, and $c_s = 1$
- Free tensor modes keep the standard propagation equation and are modified only through the background evolution
- Explicit entropy corrections are progressively slow-roll suppressed: $A_s : \mathcal{O}(\delta_\diamond \epsilon)$; $n_s - 1 : \mathcal{O}(\delta_\diamond \epsilon^2)$; $\alpha_s : \mathcal{O}(\delta_\diamond \epsilon^3)$. N3LO is therefore essential: lower orders do not capture the first explicit correction to the running α_s
- Scalar and tensor sectors are affected differently, leading to corrections to r and to the single-field consistency relation

Thank you for your attention!