Probing astrophysical environments of stellar mass black hole binaries through the LISA stochastic signal

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IFPU workshop seminar [2507.00694, 2605.05537]

SISSA

Early inspiral, low frequency regime of stellar mass black hole binaries: window into astrophysical environment

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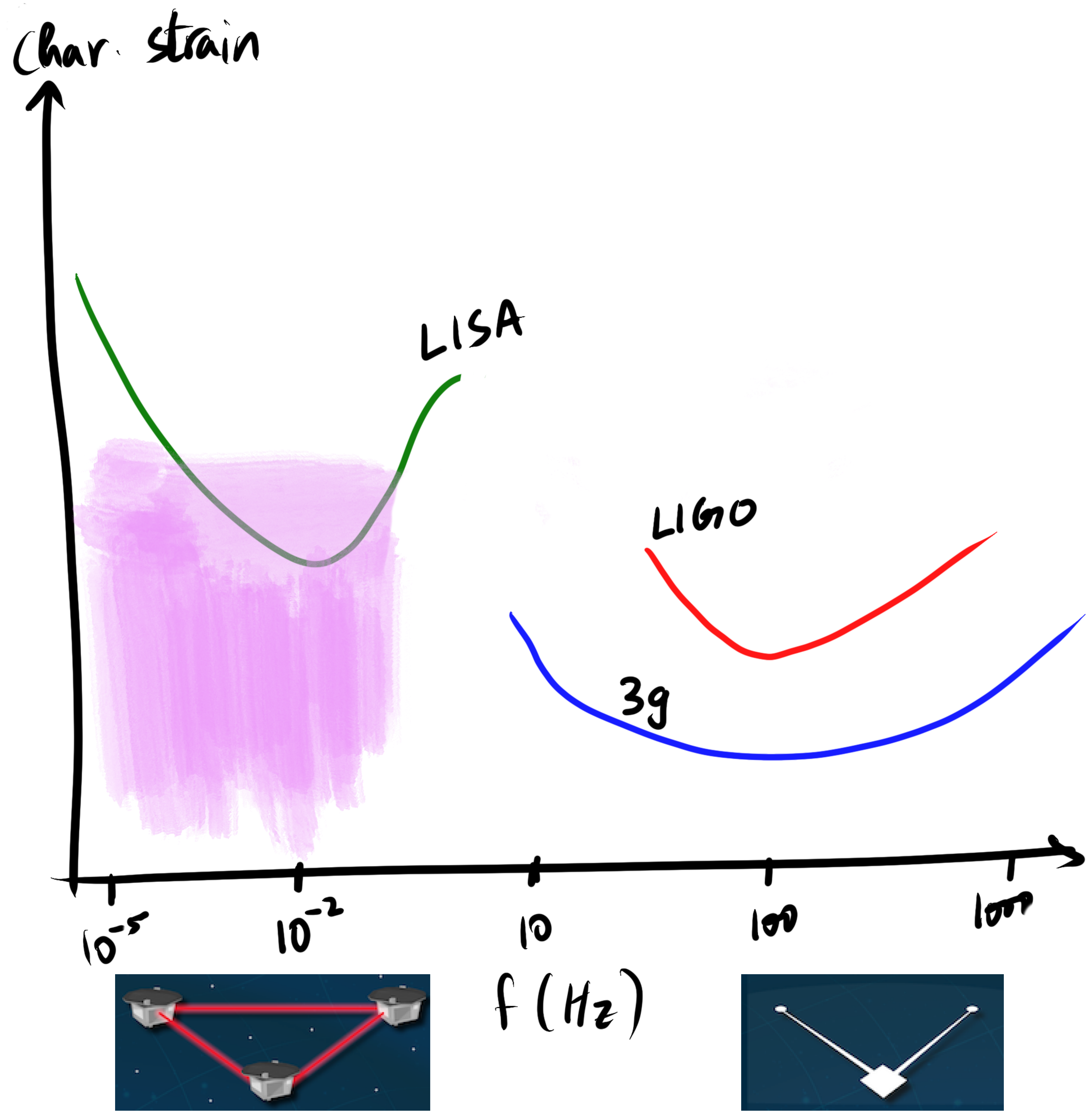

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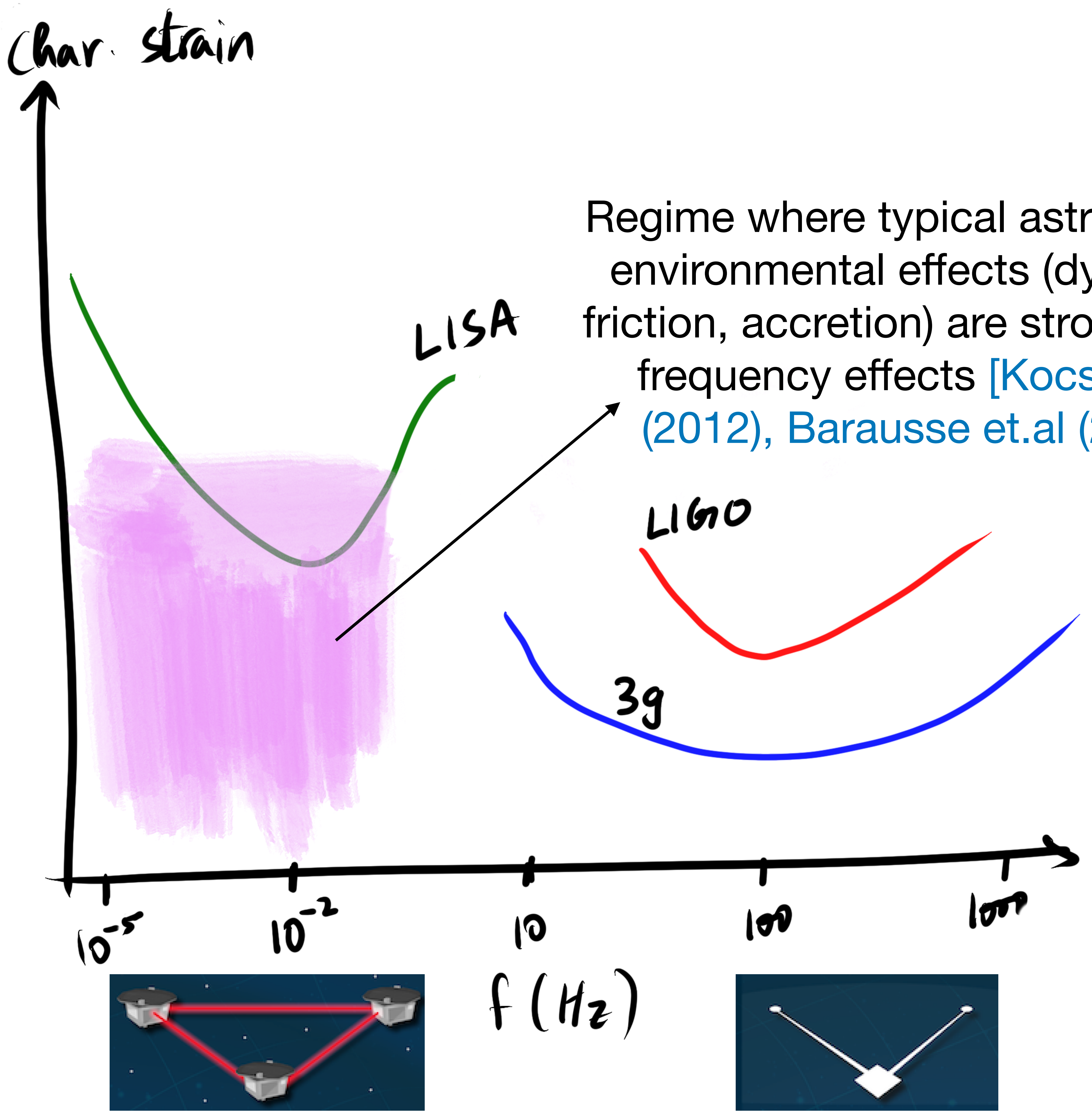

Regime where typical astrophysical environmental effects (dynamical friction, accretion) are strong — low frequency effects [Kocsis et.al (2012), Barausse et.al (2014)].

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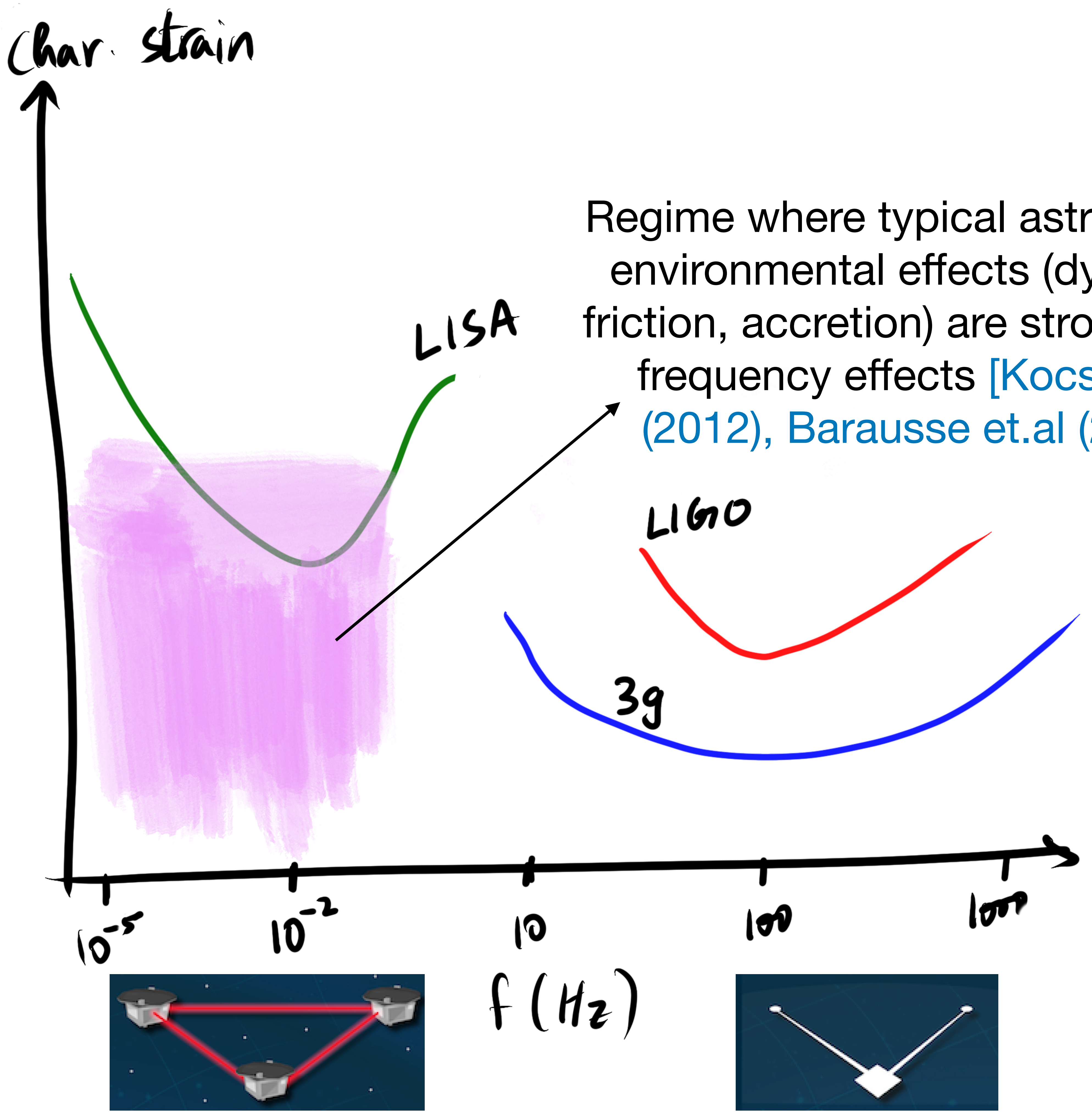

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- Probing environmental effects with resolvable stellar-mass BBH (sBBH) in LISA:
  - Dynamical friction and accretion [Toubiana et.al (2021), Caputo et.al (2020)]
  - Third body effects [Sberna et.al (2022)]

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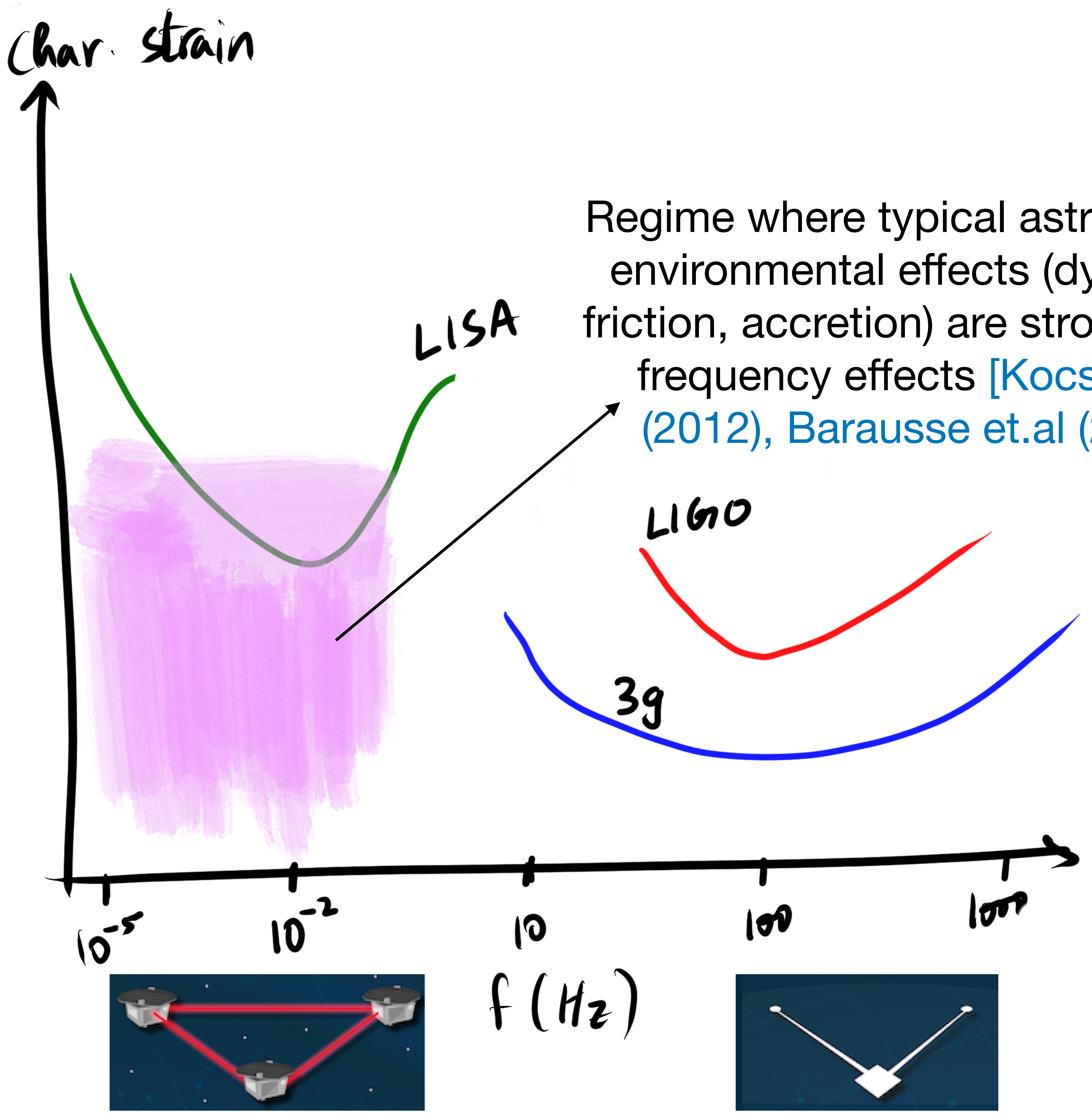

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- **Can we probe environmental effects from unresolved sBBH in LISA through the resulting stochastic GW background (SGWB)?**

Primer on SGWB from vacuum sBBH

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SGWB spectrum is characterized by the dimensionless, normalized GW energy density (per logarithmic frequency) $\Omega_{\rm GW}(f)$

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Comoving merger rate

Source params (masses, spins etc.)

[Phinney (2001), Allen & Romano (2001)]

$$\Omega_{\rm GW}(f) = \frac{f}{\rho_c H_0} \iint dz d\boldsymbol{\phi} \frac{R_{\rm GW}(z)}{(1+z)\mathscr{E}(z)} p(\boldsymbol{\phi}) \frac{dE_{\rm GW}}{df_s}(\boldsymbol{\phi}) \Bigg|_{f_s = f(1+z)}$$

Prob. distribution of source params

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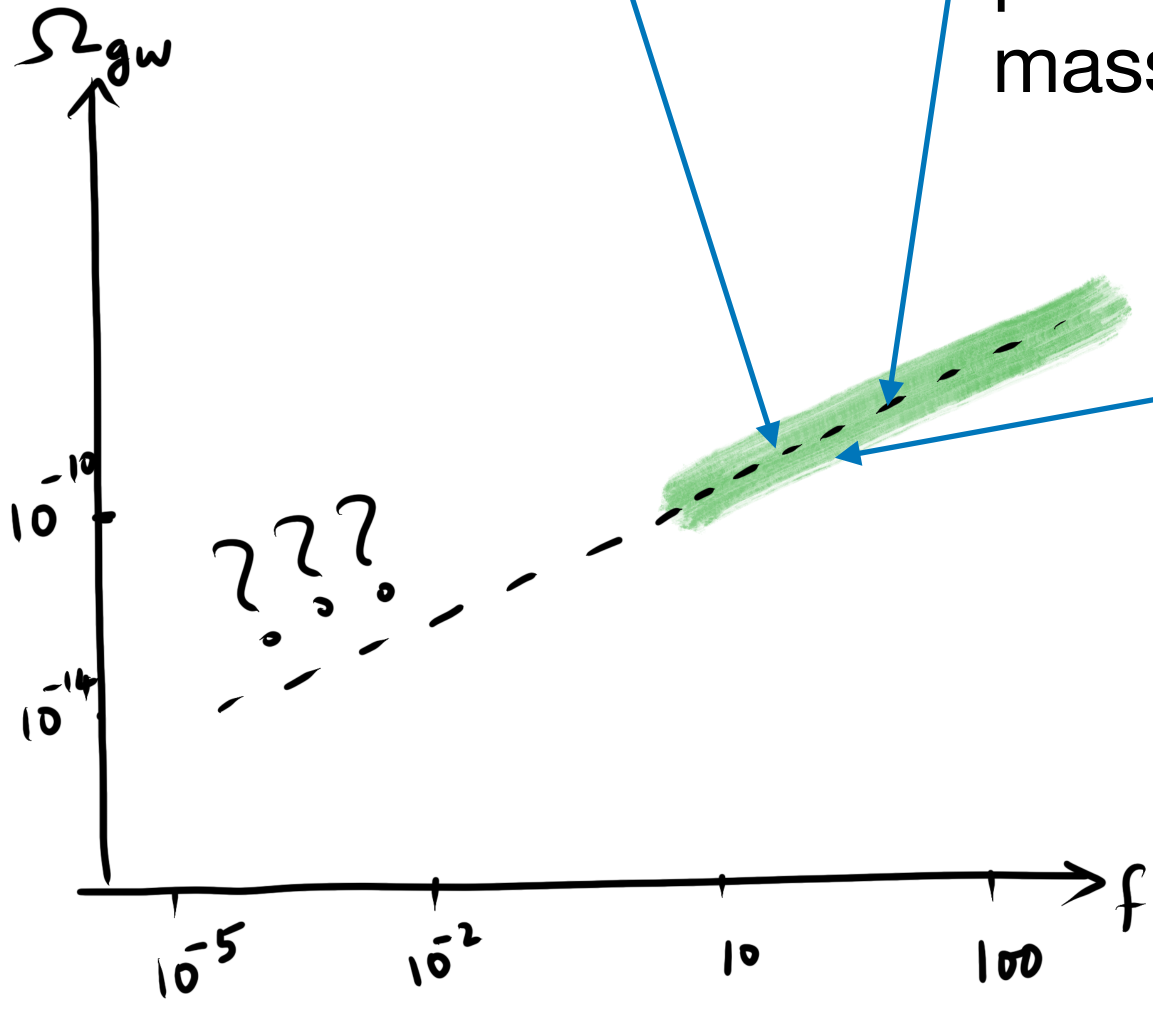
Prob. distribution of source params

If sBBH were in vacuum, $\Omega_{\rm GW}(f) = A_{\rm vac} f^{2/3}$

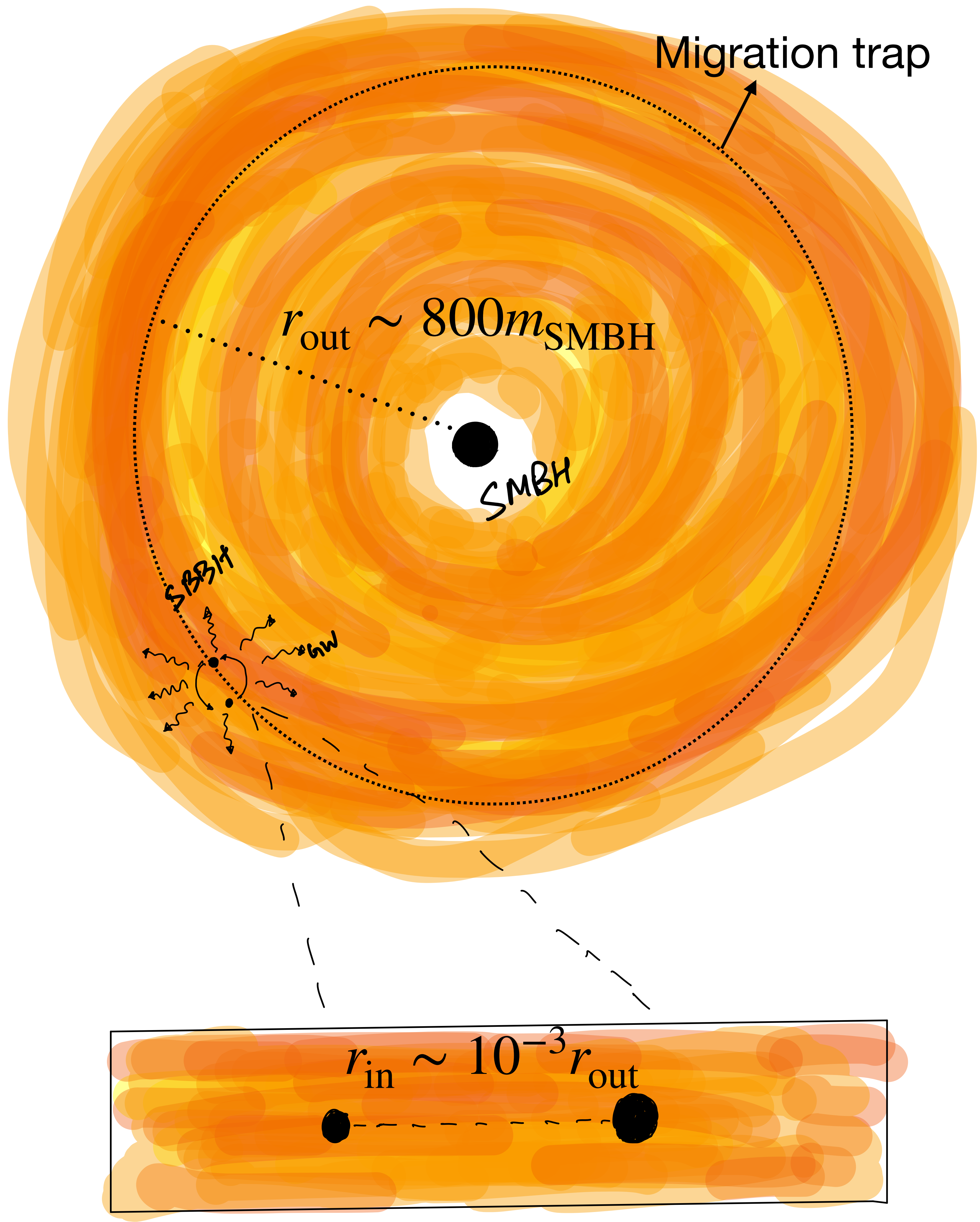


Posterior predictive from LVK posteriors on merger rate and mass distribution

$A_{\rm vac} \approx 5 \times 10^{-11} [Hz^{-2/3}]$ from median of posterior predictive

Primer on environmental effects

Concrete example: sBBH in AGN disk



Primer on environmental effects

Typically dominant effects induced by environment: [Kocsis etal (2012), Barausse etal (2014)]

- **Dynamical friction**: Gravitational drag due to density wakes produced by each BH [-5.5PN order].
- **Accretion**: Adiabatic change in masses of sBBH + hydrodynamic drag due to momentum exchange [-4PN order]
- Line of sight CoM acceleration / Doppler delay effect [-4PN order]

Primer on environmental effects: dynamical friction

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[Kim & Kim (2007)]

$$\overrightarrow{F}_{\mathrm{DF},A} = -\frac{4\pi\rho m_A^2}{v_A^2} I_A \hat{v}_A, \qquad I_A = \ln\left(\frac{100 r_A v_s}{11 v_A r_{\mathrm{min},A}}\right)$$

"Coulomb logarithm" in high Mach number limit

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"Coulomb logarithm" in high Mach number limit

$$\frac{df_s}{dt} = \left(\frac{df_s}{dt}\right)_{\mathrm{vac}} + \frac{12\rho}{m^3\eta^2}\left[m_1^3 \ln\left(\frac{f_1^*}{f_s}\right) + (m_1 \leftrightarrow m_2)\right]$$

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$$\frac{f_{\mathrm{turn}}}{\left(\ln[1\mathrm{Hz}/\mathrm{f}_{\mathrm{turn}}]\right)^{3/11}} \approx (1.69 \times 10^{-3}\mathrm{Hz})\left(\frac{m}{50M_\odot}\right)^{-5/11}\left(\frac{\rho}{10^{-10}\mathrm{g\,cm^{-3}}}\right)^{3/11}$$

Primer on environmental effects: gas accretion

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[Caputo et.al (2020)]

Adiabatic change in masses parametrized by Eddington ratio and Salpeter time ~ $10^7 yr$

$$m_A(t) \approx m_{A,0}\left(1 + \frac{f_{\rm Edd} t}{\tau}\right), \quad \overrightarrow{F}_{{\rm Acc},A} = -\xi \dot{m}_A \vec{v}_A$$

Hydrodynamic drag from accreting matter, parametrized by linear drag coefficient

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$$f_{\rm turn} \approx 10^{-4}{\rm Hz}\left(\frac{f_{\rm edd}}{1}\right)^{3/8}\left(\frac{m}{50\,M_\odot}\right)^{-5/8}\left(\frac{5+3\xi}{8}\right)^{3/8}\left(\frac{\tau}{4.5\times 10^7\,{\rm yr}}\right)^{-3/8}$$

Is your stochastic signal detectable?

[Pozzoli, Gair, Buscicchio, Speri, PRD (2025)]

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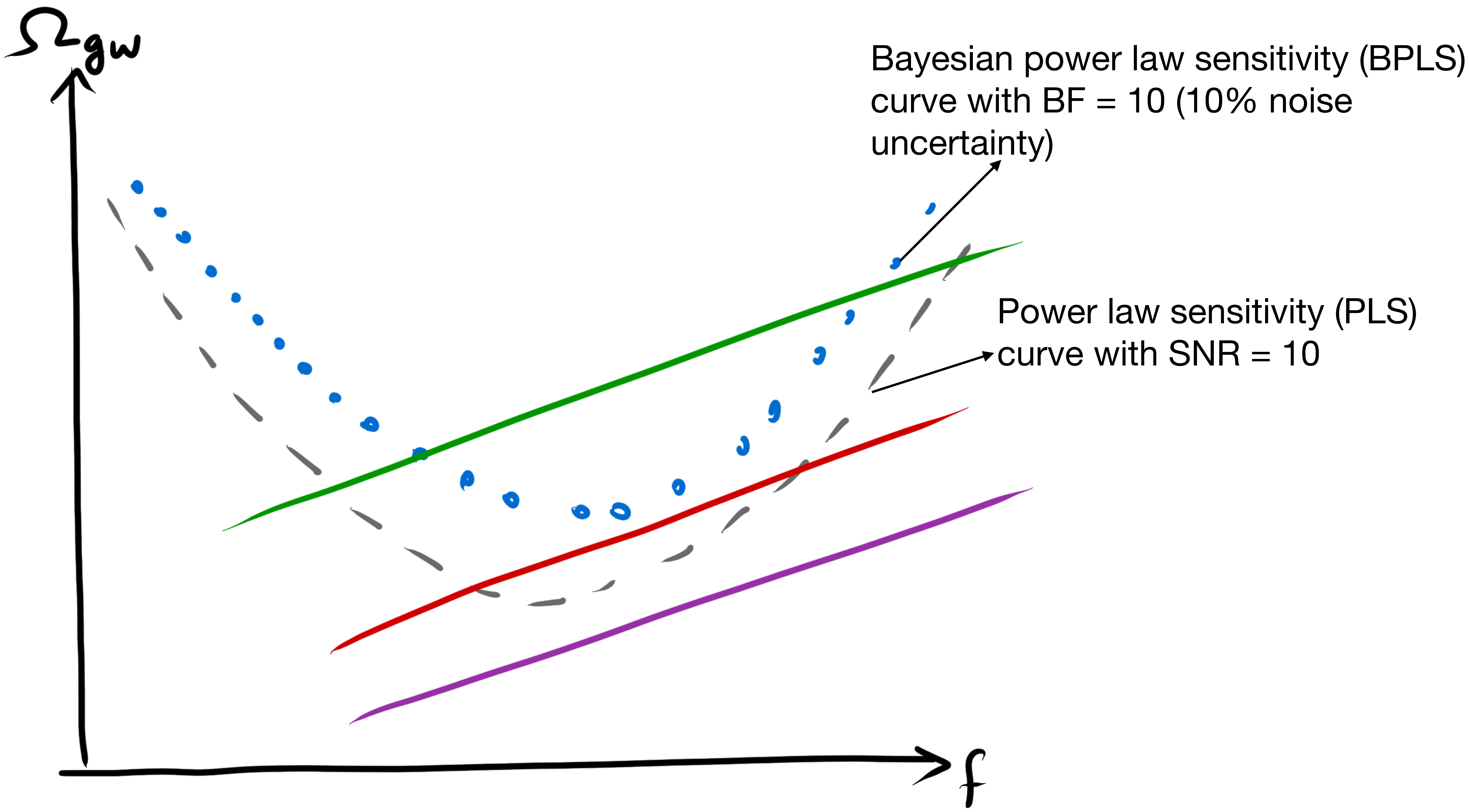
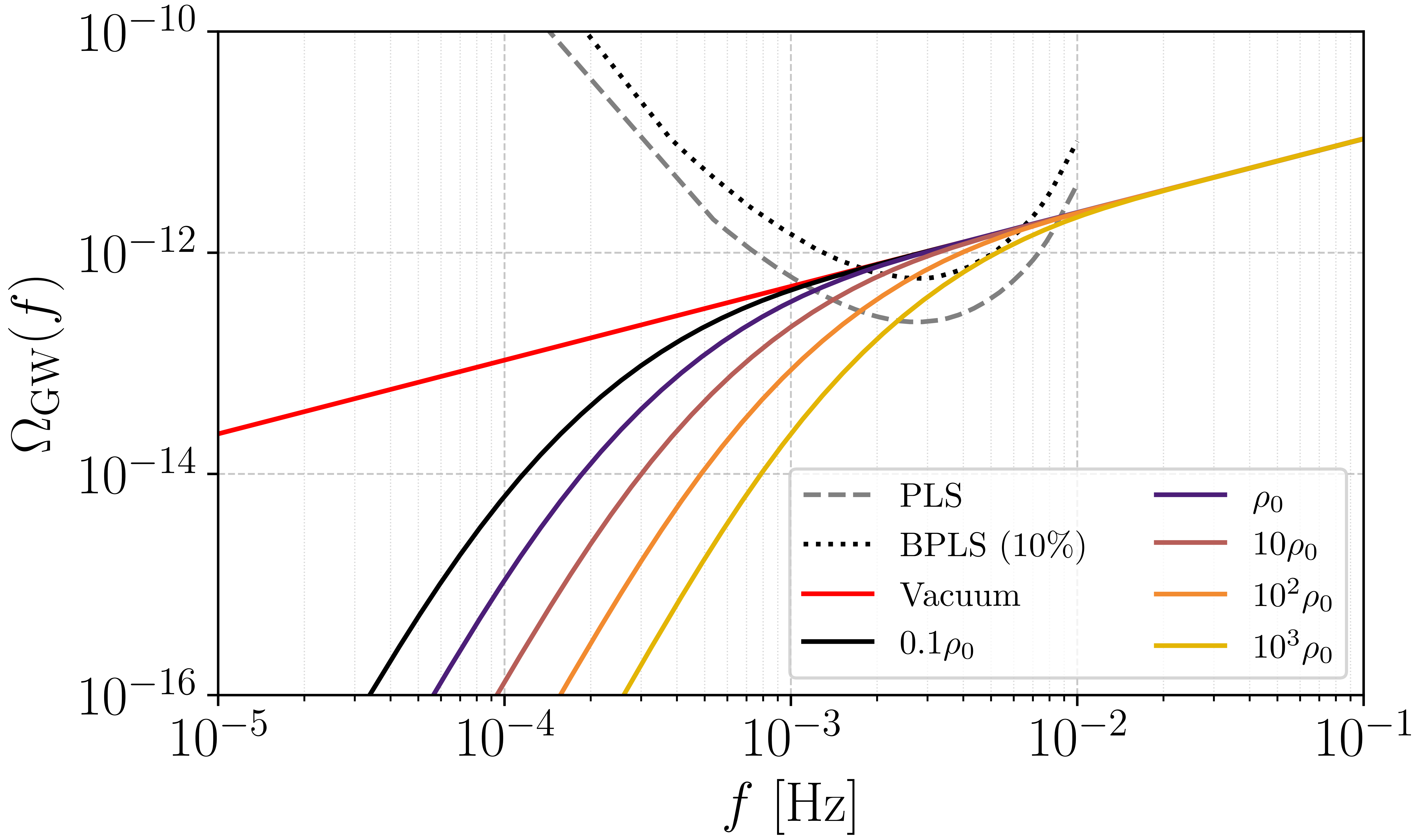


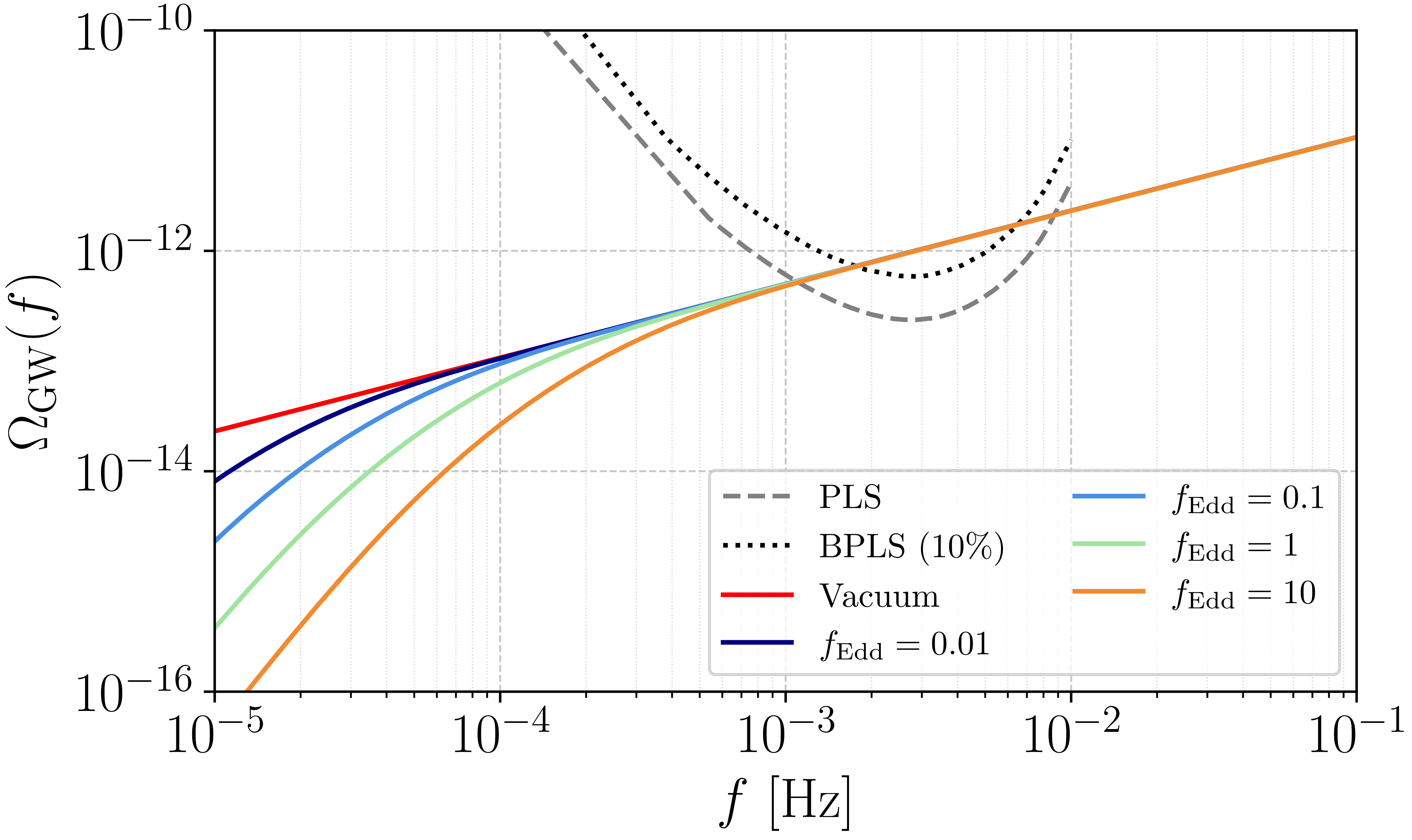
SGWB of sBBH with dynamical friction effects in LISA: qualitative detectability



- Vacuum case is above both PLS and BPLS curves in a significant part of the LISA band —> vacuum signal potentially detectable

- Dynamical friction effects are potentially detectable due to turning point in sensitive part of LISA band

SGWB of sBBH with gas accretion effects in LISA: qualitative detectability



- Vacuum case is above both PLS and BPLS curves in a significant part of the LISA band —> vacuum signal potentially detectable

- Gas accretion effects are potentially not detectable due to turning point outside sensitive part of LISA band

Phenomenological parametric models for environmental effects in SGWB

For “ready-to-use” in parameter estimation, we developed new phenomenological parametric models of the SGWB that can be mapped to different environmental effects

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Rational power law model:

$$\Omega_{\mathrm{RPL}} = \frac{A_{\mathrm{vac}} f^{\gamma}}{1 + A_{\mathrm{m}} \alpha f^{\beta} [\ln(f/1\mathrm{Hz})]^{\kappa}}$$

Captures the SGWB at both low and high frequencies. Coefficients and spectral indices determined from asymptotic matching

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Rational power law + peak model:

$$\Omega_{\rm RPLP} = \frac{\Omega_{\rm RPL}}{1 + \mathscr{G}(f, \alpha)}$$

Gaussian "peak" correction to fit better at intermediate frequencies

Parameter	**Dynamic Friction**	**Accretion**
α	ρ	$(5+3\xi)\, f_{\rm Edd}/\tau$
β	$-11/3$	$-8/3$
κ	1	0
Asymptotic matching parameters		
$A_{\rm vac}$	4.97×10^{-11}	4.97×10^{-11}
A_m	-2.73×10^{-7}	4.35×10^{2}
Gaussian spectral correction parameters		
A_g	$1.02 - 0.0262 \log_{10}(\alpha/\alpha_0)$	0.904
$f_{\rm peak}$	$0.0269\,(\alpha/\alpha_0)^{0.262}$	$9.73\,(\alpha/\alpha_0)^{0.374}$
σ	0.222	0.321

Bayesian parameter estimation of LISA SGWB

Coarse-grained likelihood [Appourchaux (2003)] implemented in BAHAMAS [Pozzoli etal (2025)]:

$$\ln \mathscr{L}(\hat{\boldsymbol{S}} \mid \theta) = \sum_{i=1}^{D} \sum_{k} \left[-\ln \Gamma(N) - N \ln \left(\frac{S_k(f_i;\theta)}{N} \right) + (N-1) \ln \hat{S}_k(f_i) - \frac{N \hat{S}_k(f_i)}{S_k(f_i;\theta)} \right],$$

$$S_k(f;\theta) = S_{\mathrm{GW}}(f;\theta) + S_{k,n}(f;\theta),$$

PSD of channel k

SGWB PSD

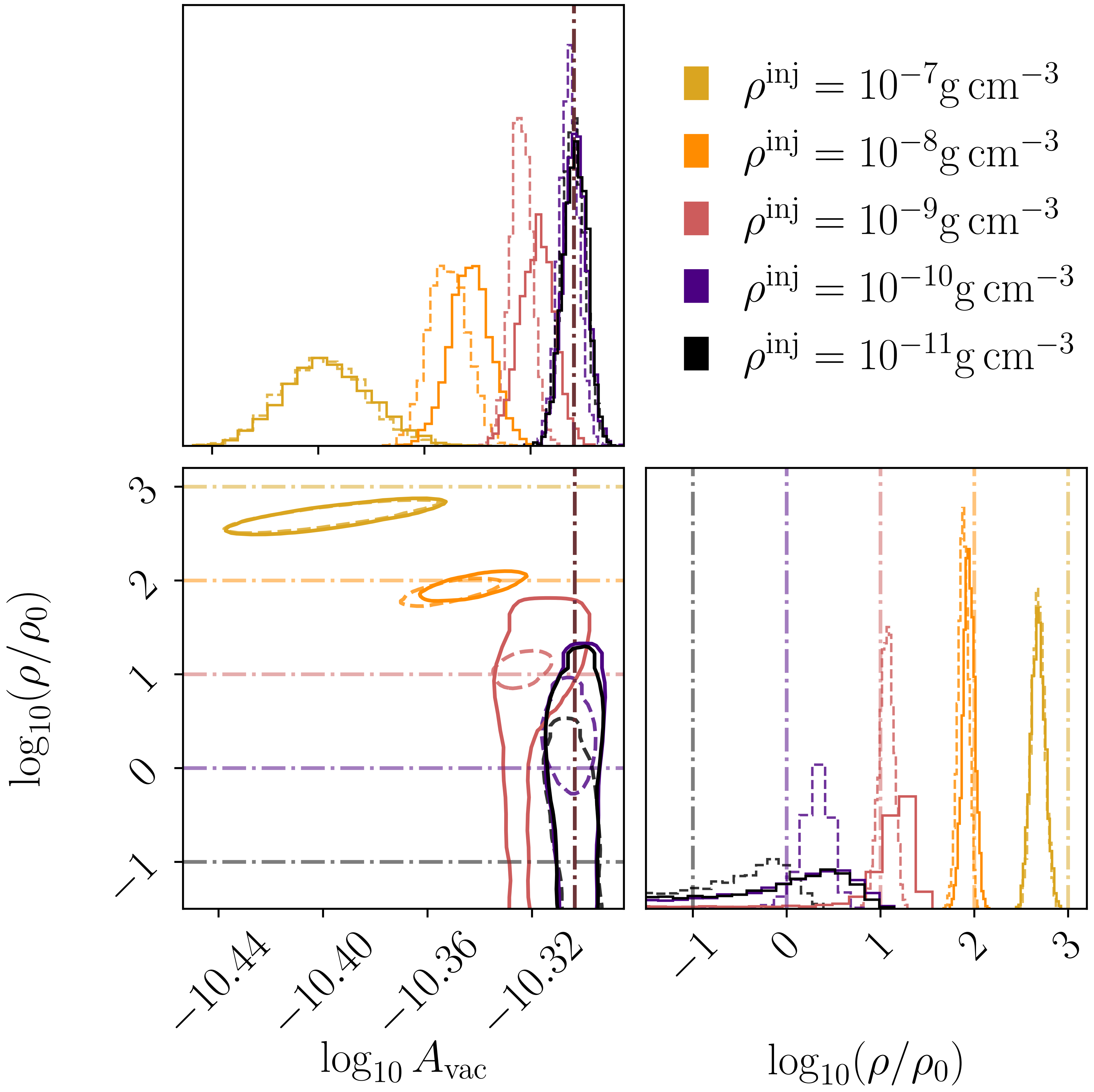
Instrumental PSD

LISA response for channel k

PSD of astrophysical SGWB (both from sBBH and GB)

$$S_{k,\mathrm{GW}}(f;\theta) = R_k(f;\theta)\, S_{\mathrm{GW}}(f;\theta), \qquad S_{\mathrm{GW}}(f;\theta) = \frac{3H_0^2}{4\pi^2 f^3} \Omega_{\mathrm{GW}}(f;\theta)$$

Detection and parameter estimation of dynamical friction from SGWB in LISA



$\rho_0 = 10^{-10}{\rm g\ cm^{-3}}$

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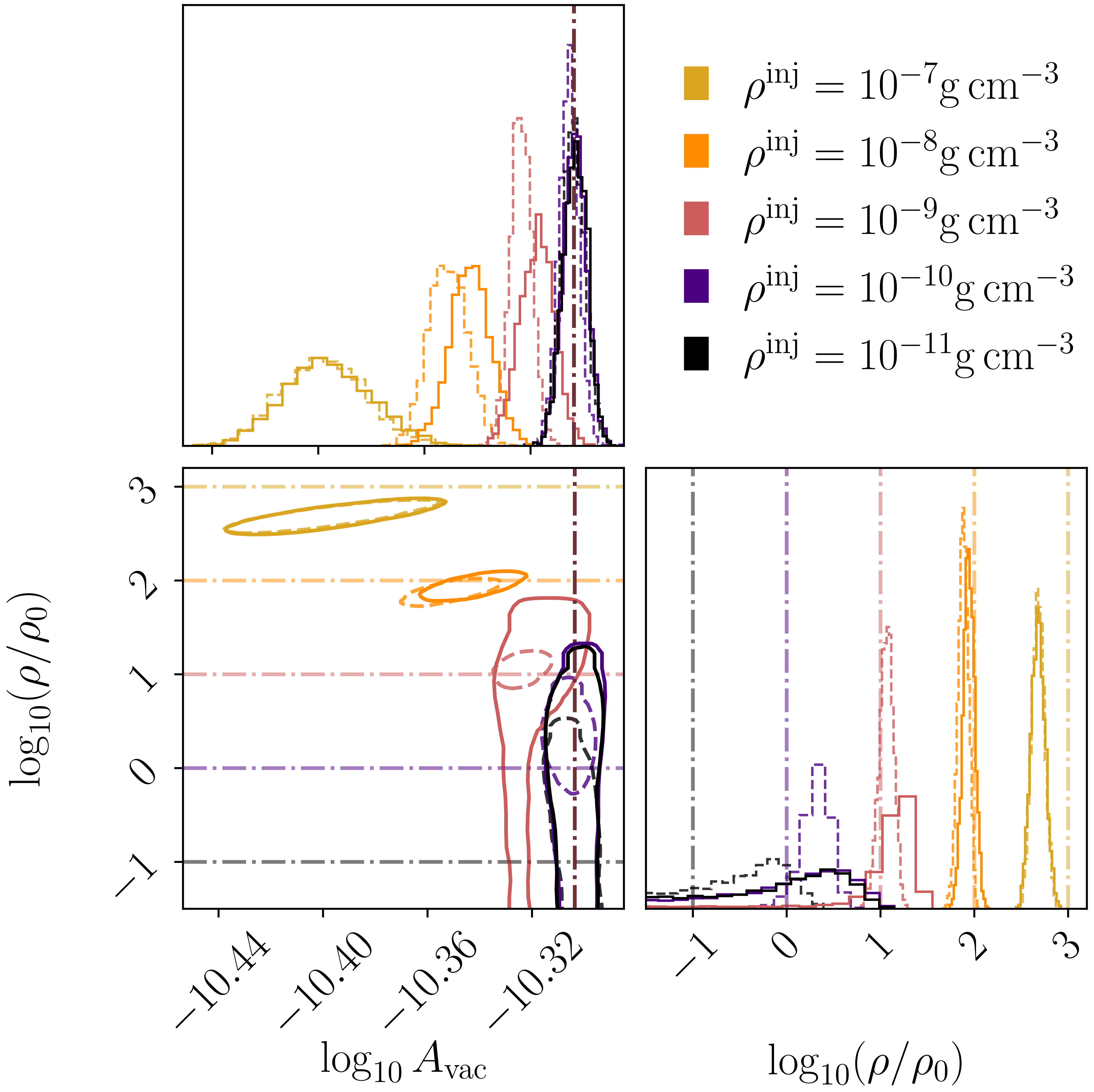
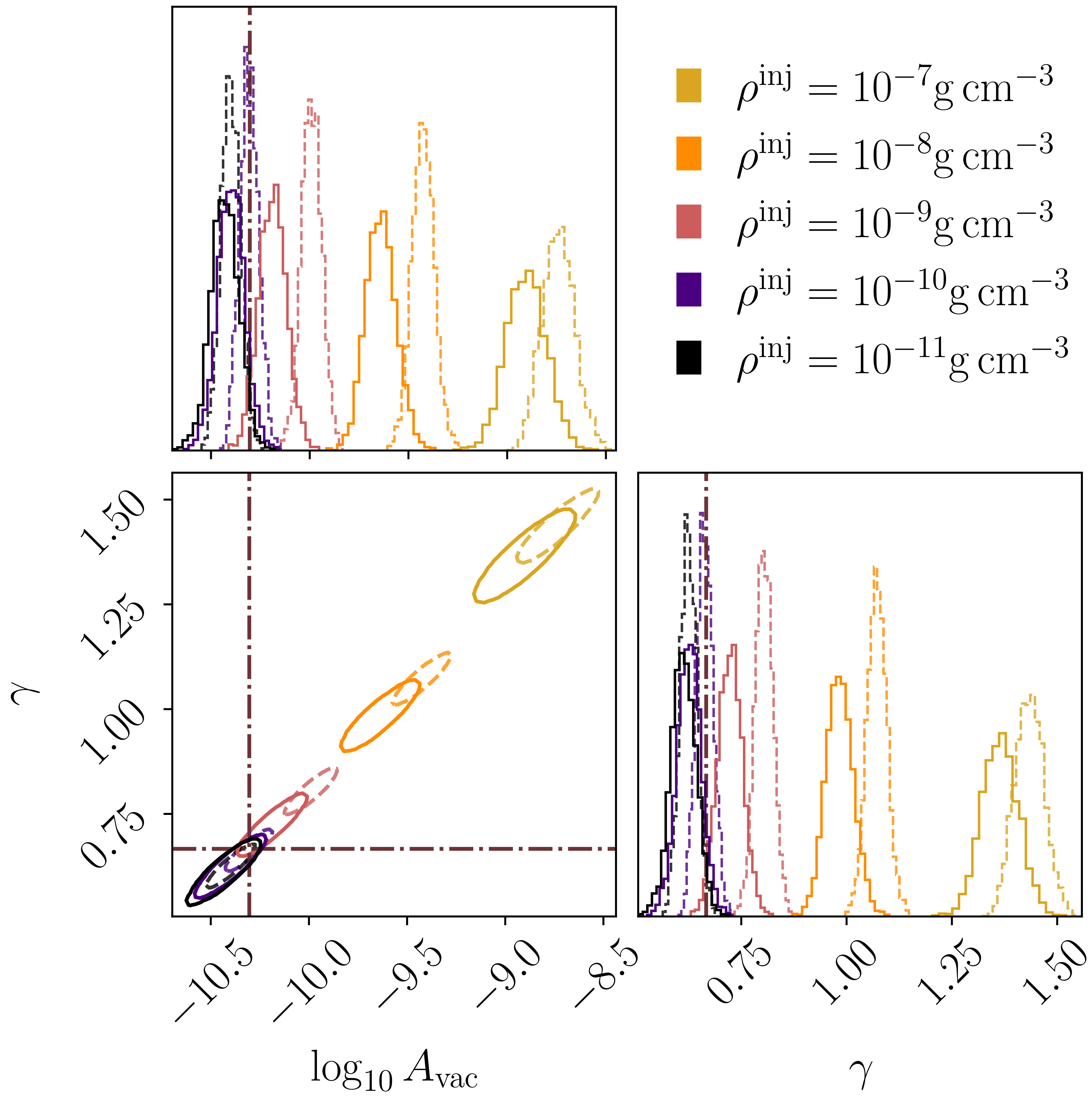


$\rho_0 = 10^{-10}$g cm^{-3}

ρ^{inj}/ρ_0	$\delta\rho/\rho^{inj}$		$\delta A_{vac}/A_{vac}^{inj}$		$\log_{10} \mathcal{B}_{vac}^{non-vac}$	
	With	Without	With	Without	With	Without
10^{-1}	37.85	8.218	0.02188	0.01727	1.424	1.010
1	4.248	1.4670	0.02075	0.01490	1.146	3.965
10	1.118	0.4042	0.02114	0.01430	1.777	8.240
10^2	0.3362	0.2033	0.03404	0.02478	1.281	10.16
10^3	0.1606	0.1340	0.05334	0.0493	1.427	7.570

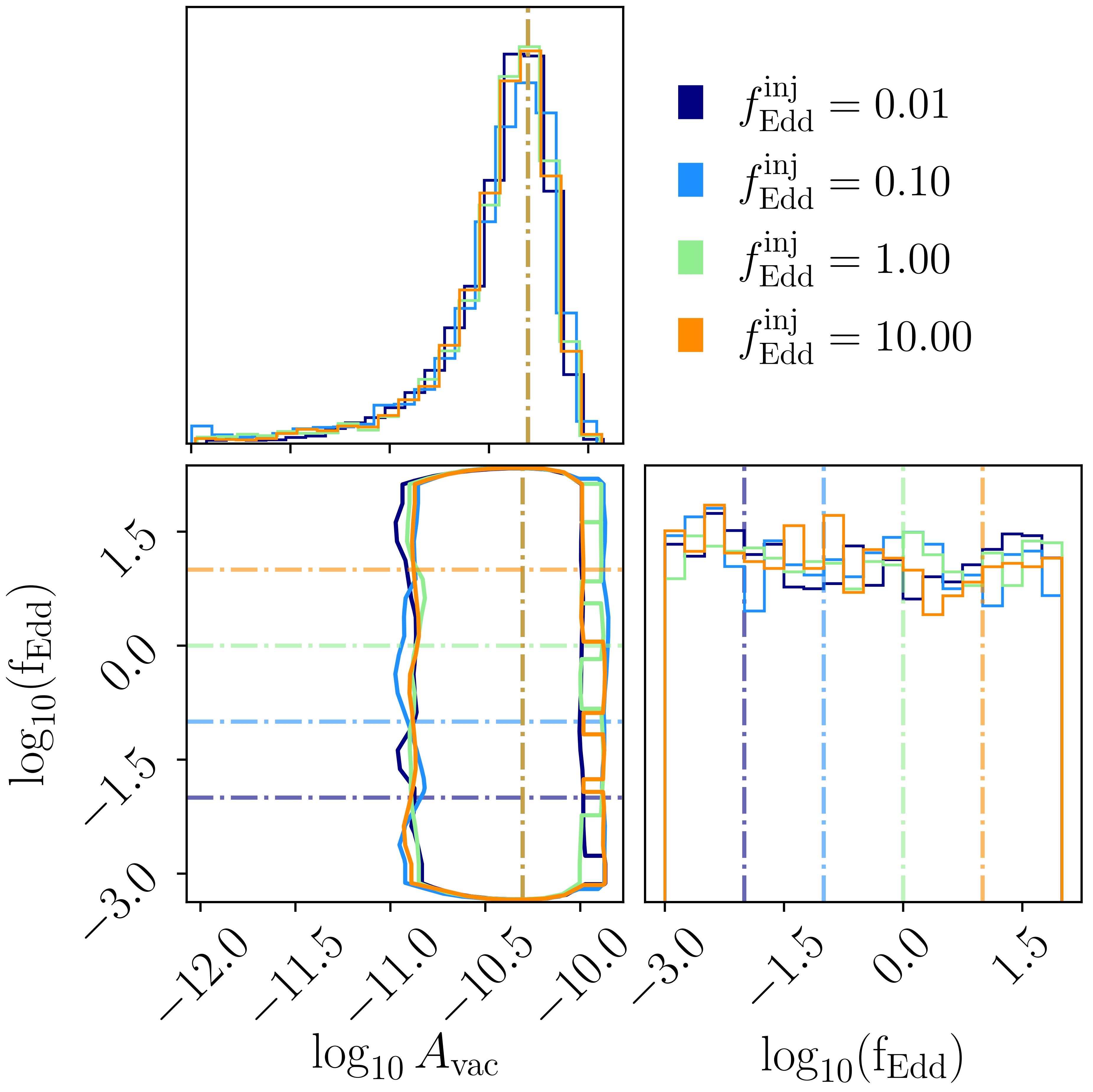
Dynamical friction effects are measurable and can be distinguished from vacuum

Systematic biases due to neglecting dynamical friction



Parameter estimation of accretion effects from SGWB in LISA

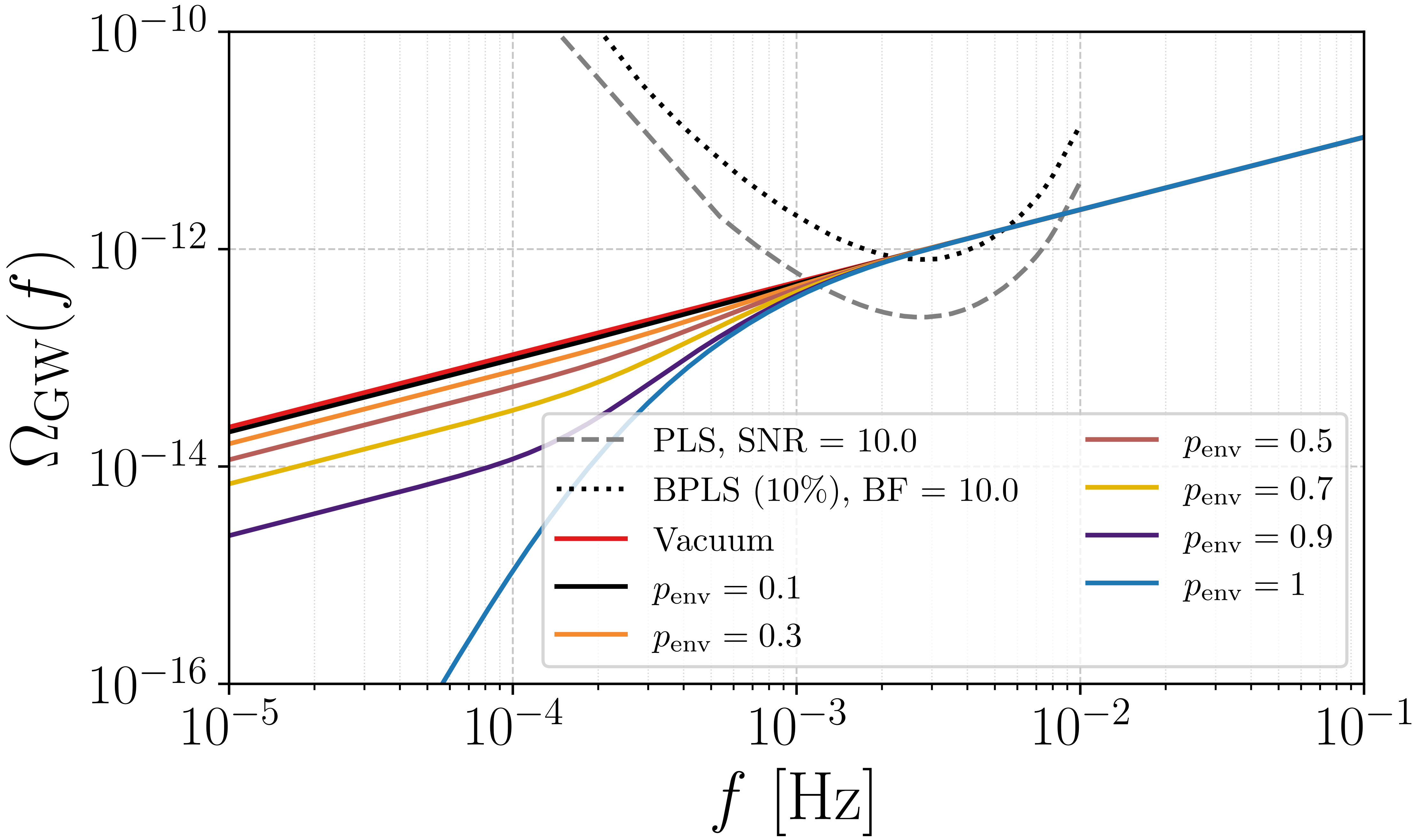
Gas accretion effects cannot be measured even for large Eddington ratio



Detecting dynamical friction from a sub-population of sBBH

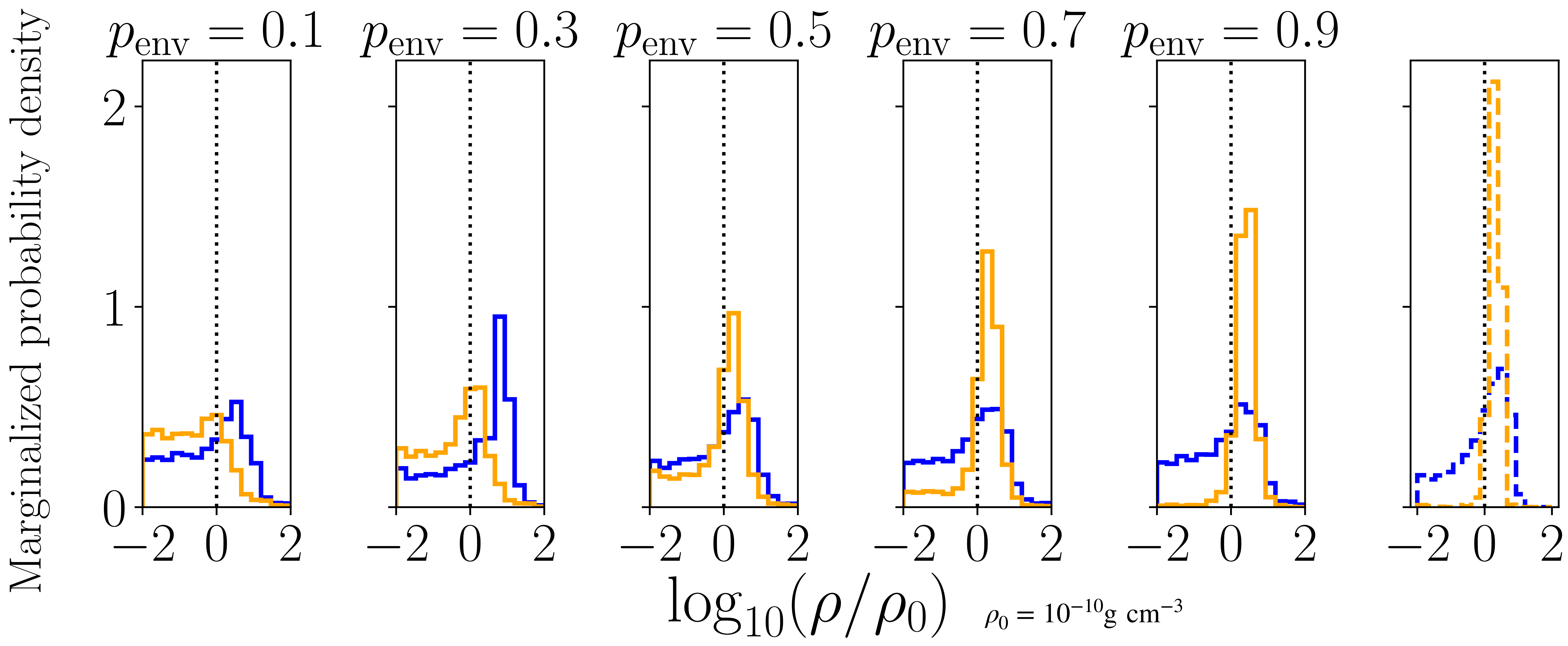
Mixture model to capture sub-population dependence:

$$\Omega_{\rm Frac} = p_{\rm env}\Omega_{\rm env} + (1 - p_{\rm env})\Omega_{\rm vacuum}$$



Detecting dynamical friction from a sub-population of sBBH

With Galactic foreground (blue histograms): $\log_{10} \mathscr{B}^{\mathrm{Frac}}_{\mathrm{vac}} \sim 0.79 - 0.95$

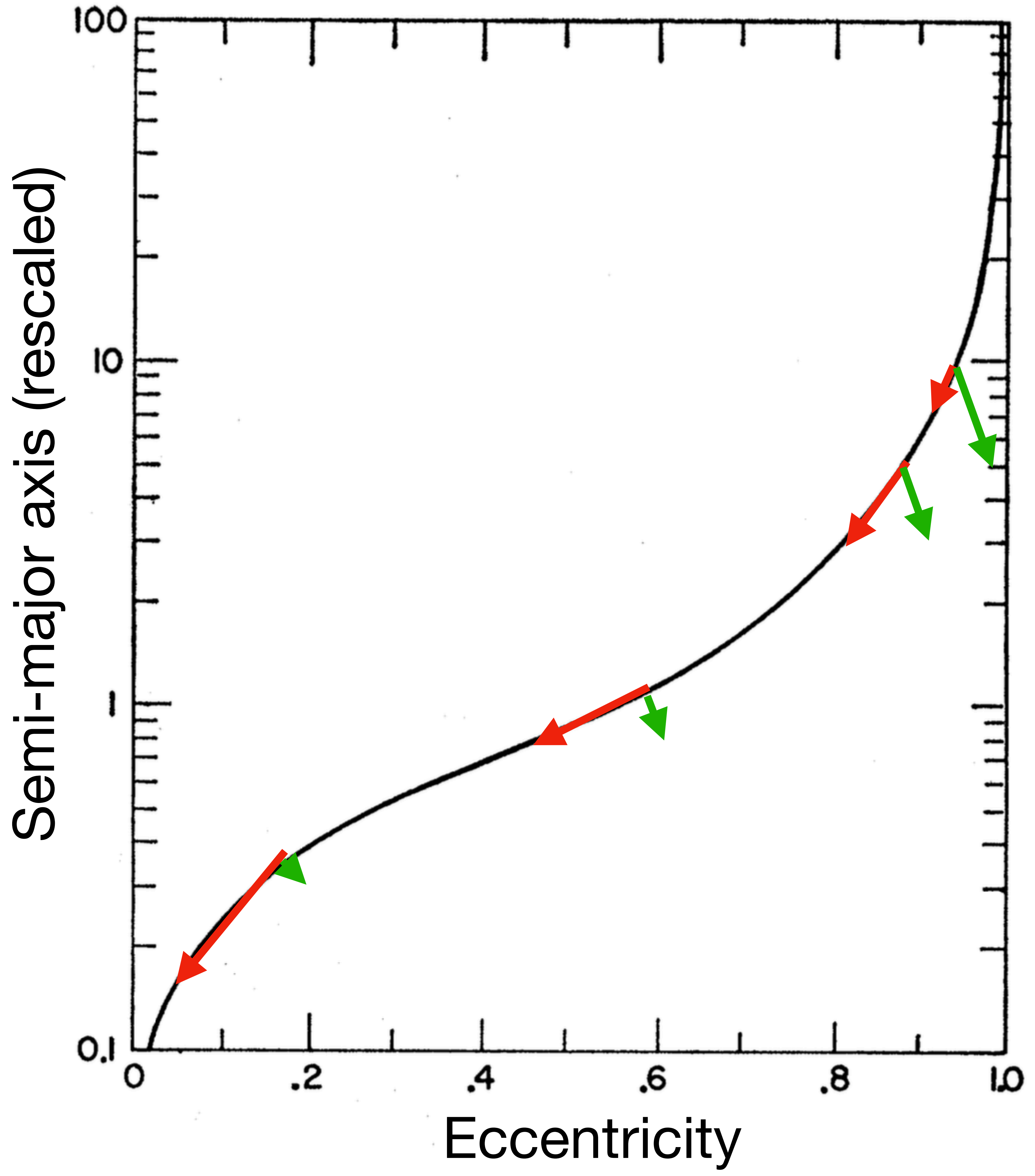


Substantial preference for mixed fraction model over vacuum—
dynamical friction in sub-populations can be measured

Impact of eccentricity in detecting environmental effects and probing dynamical formation

Early inspiral, low frequency regime of stellar mass black hole binaries: window into eccentricity

- In vacuum, GW radiation reaction leads to shrinking and circularizing of orbits [Peters (1964)]
- In an astrophysical environment, eccentricity can be excited due to gas and third body torques: potentially compete with GW radiation reaction [Gair et.al (2011), Rodig et.al (2011), Cardoso et.al (2021), RSC & Yunes (2022), Duque et.al (2024)]
- Thus, early inspiral probes eccentricity potentially caused by astrophysical environment



[adapted from Peters (1964)]

Primer on gravitational waves from eccentric binaries

GW signal is thus a sum of harmonics

$$h_{+,\times}(t) = -\frac{m\eta}{R}\left(2\pi m F\right)^{2/3}\sum_{j=1}^{\infty}\left[C_{+,\times}^{(j)}\cos jl + S_{+,\times}^{(j)}\sin jl\right]$$

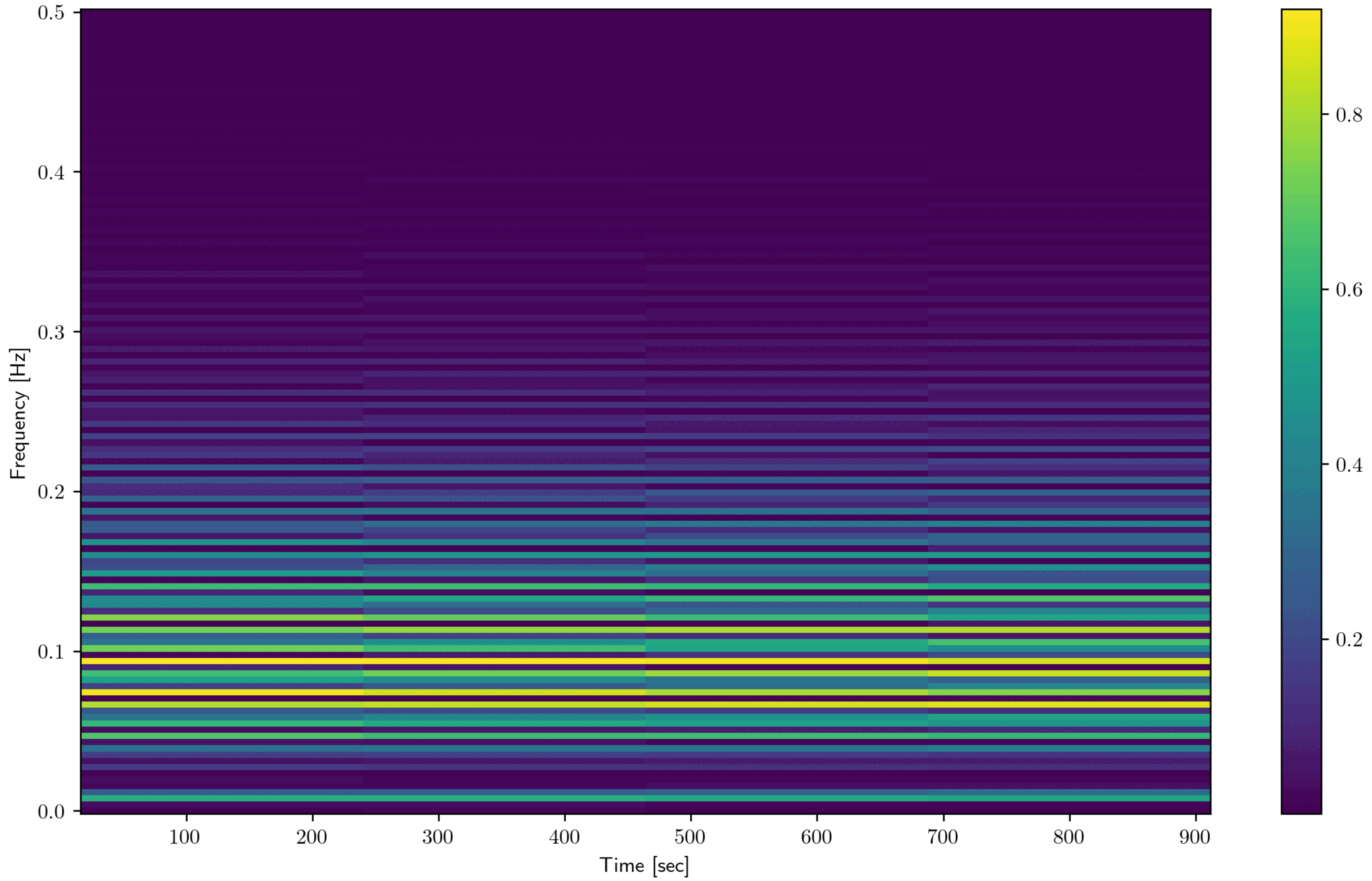
[Moore etal (2018)]

Power emitted into each harmonic:

$$P_{\mathrm{GW},j} \equiv \frac{dE_{\mathrm{GW},j}}{dt} = \frac{32}{5}\left(2\pi f_{\mathrm{orb}}\mathcal{M}\right)^{10/3} g(j,e_j),$$

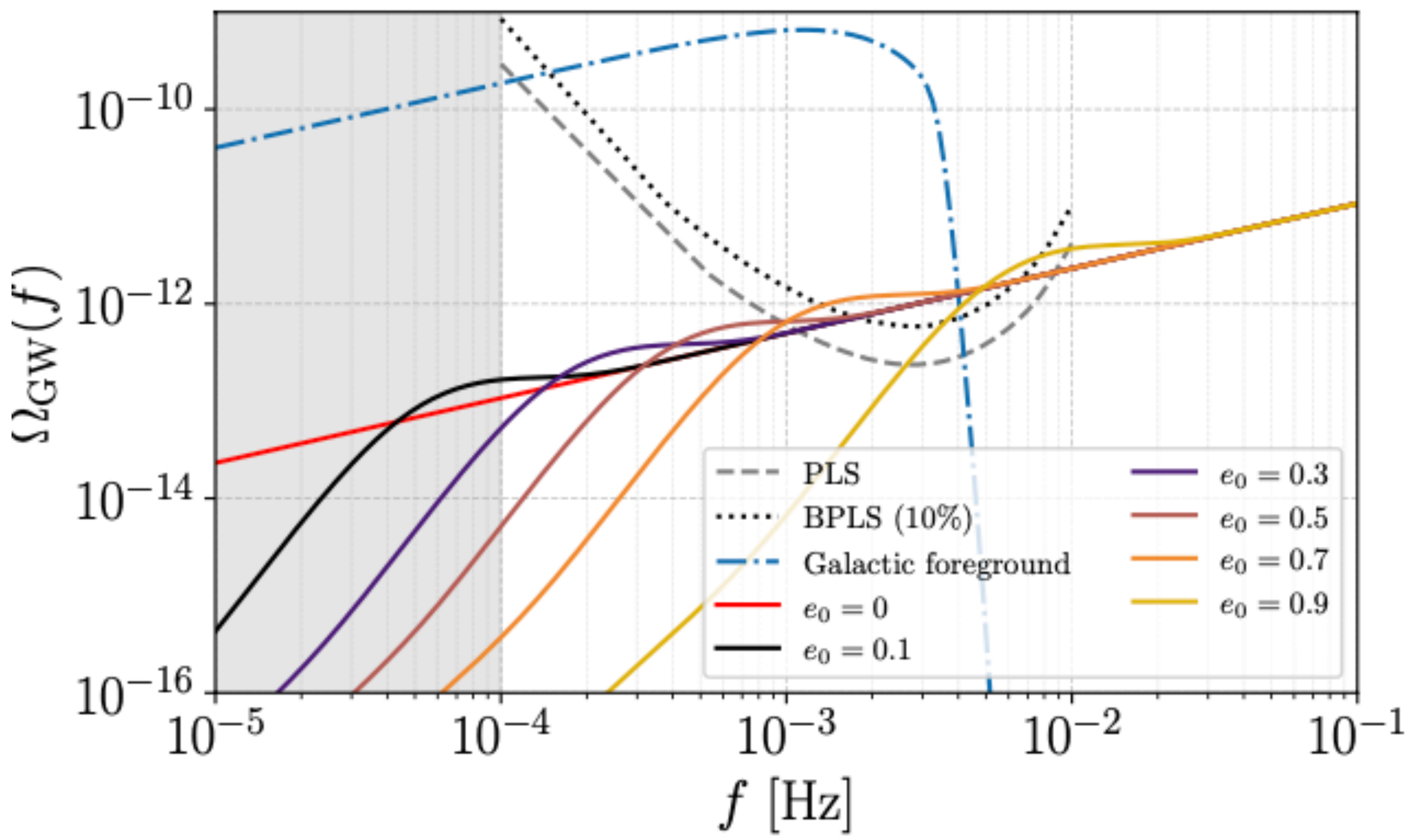
[Peters & Mathews (1963)]

$$\begin{aligned} g(j,e) = \frac{j^4}{32}\Big[&\Big\{J_{j-2}(je) - 2eJ_{j-1}(je) \\ &+\frac{2}{j}J_j(je) + 2eJ_{j+1}(je) - J_{j+2}(je)\Big\}^2 \\ &+\left(1-e^2\right)\left\{J_{j-2}(je) - 2J_j(je) + J_{j+2}(je)\right\}^2 \\ &+\frac{4}{3j^2}J_j^2(je)\Big]. \end{aligned}$$



Signatures of eccentricity in LISA band

Dirac delta distribution



Initial eccentricity $e_0 \equiv e(f_{\rm orb,0} = 10^{-4} Hz)$

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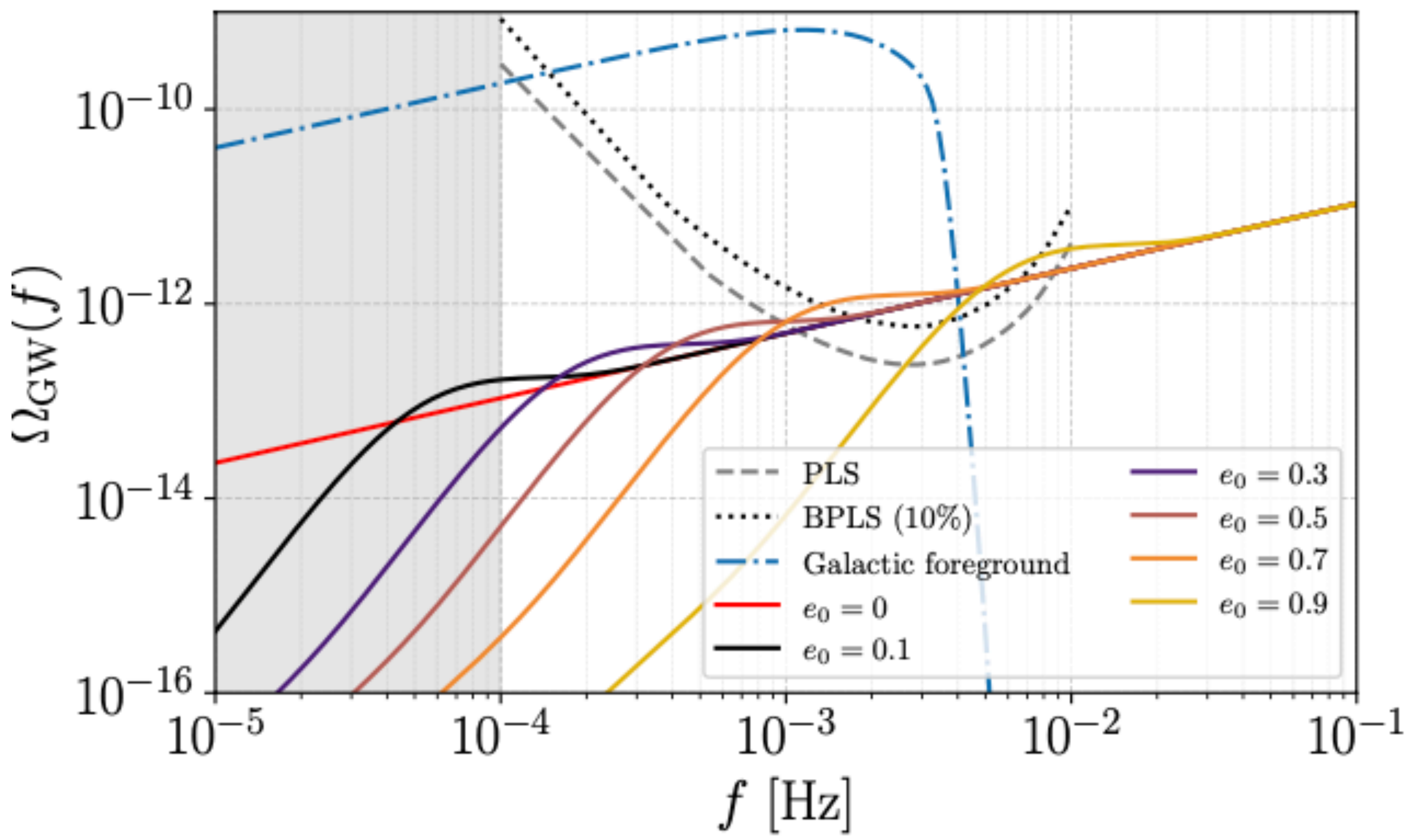

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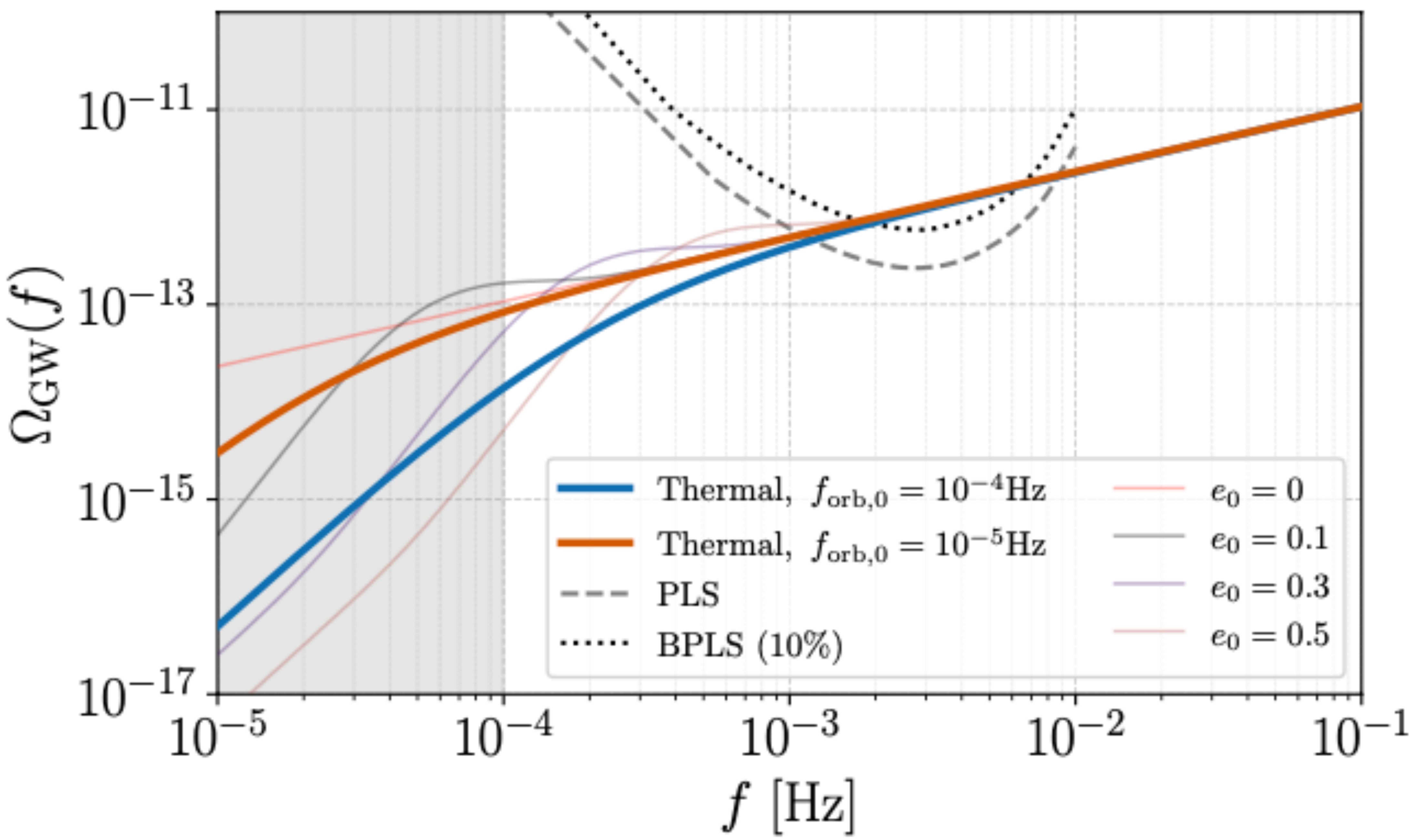
Thermal distribution

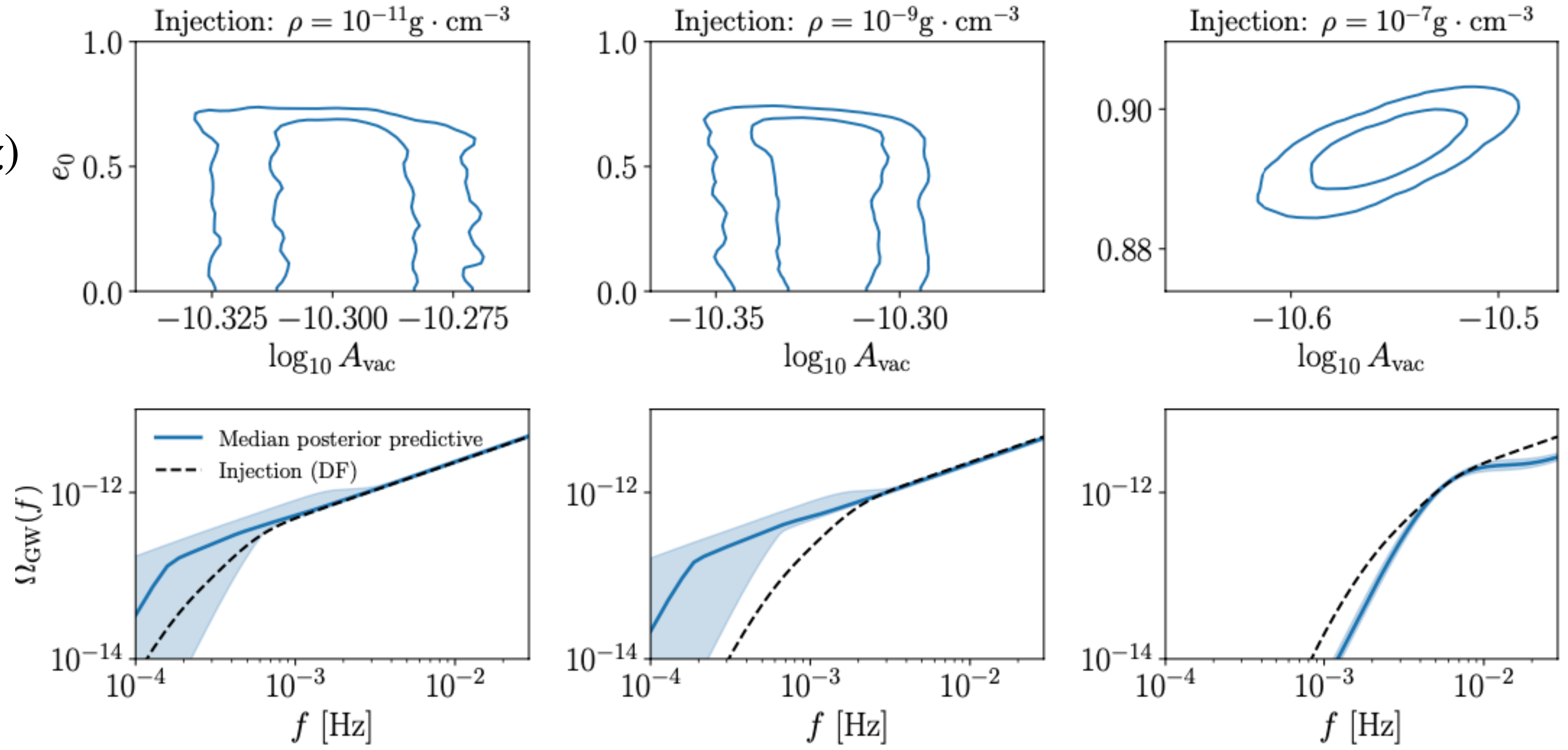


Initial eccentricity $e_0 \equiv e(f_{\rm orb,0})$ follows $p(e_0) \sim 2e_0$

Detecting dynamical friction when allowing for eccentricity in vacuum model

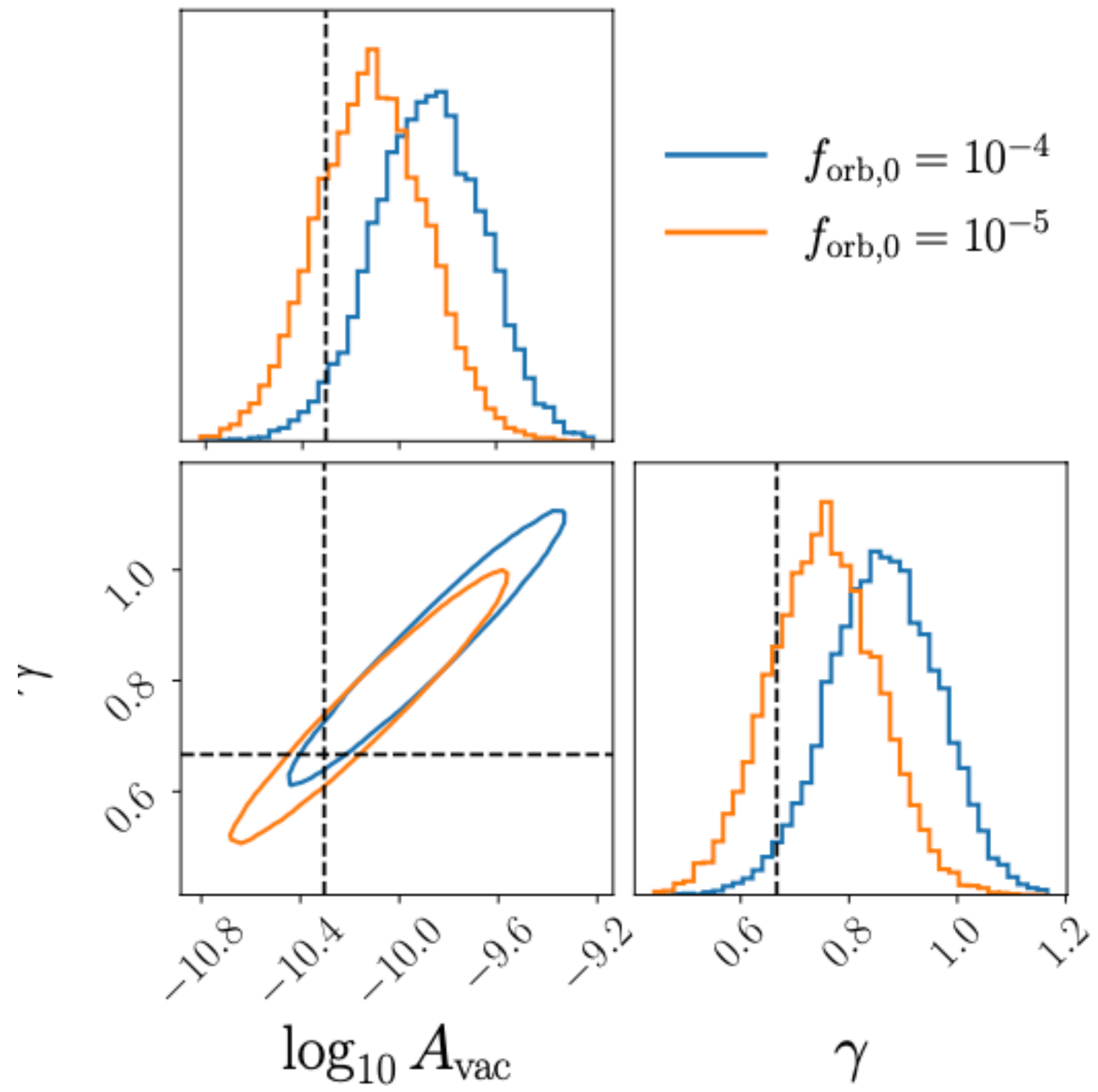
Initial eccentricity $e_0 \equiv e(f_{\mathrm{orb},0} = 10^{-4} Hz)$ follows Dirac-delta distribution

Reconstructed posterior predictive of the spectrum



ρ (g cm^{-3})	10^{-11}	10^{-10}	10^{-9}	10^{-8}	10^{-7}
$\log_{10} \mathcal{B}^{\mathrm{DF}}_{\mathrm{ecc}}$	-0.01275	0.01033	-0.09733	-0.1328	1.337
$\log_{10} \mathcal{B}^{\mathrm{DF}}_{\mathrm{cir}}$	1.397	1.375	1.407	1.012	1.474

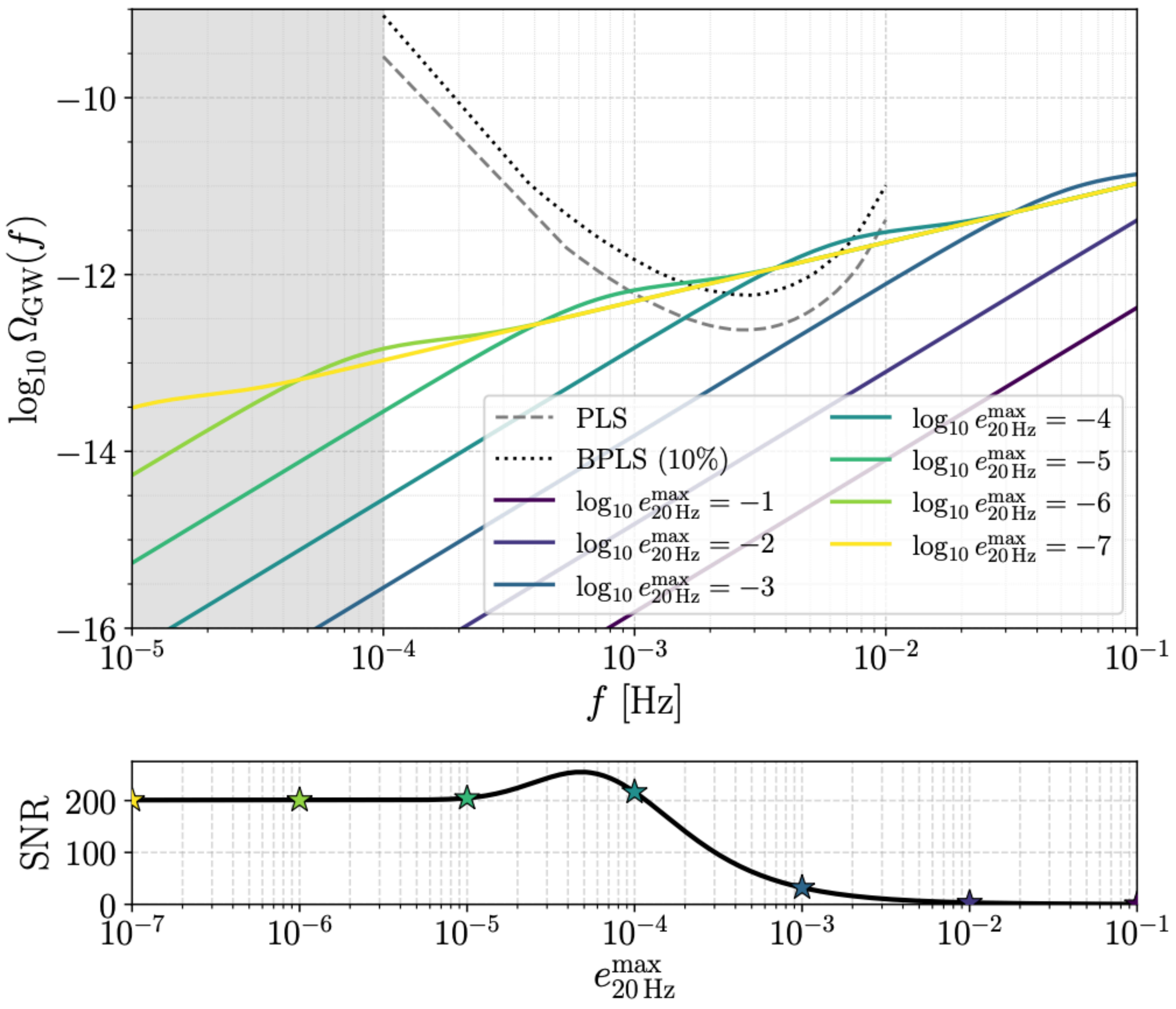
Bias in circular model parameters: thermal distribution injection



Bias in circular model parameters increases with $f_{\rm orb,0}$

Constraining eccentricity in LVK band through non-detection of SGWB in LISA band

Eccentricity $e(f_{(2,2)} = 20Hz) \sim U(0, e^{\max}_{20Hz})$



For $e^{\max}_{20Hz} \gtrsim 10^{-2} \rightarrow \mathrm{SNR} \lesssim 3$ and thus the signal will be difficult to detect

Takeaways

- For typical gas densities dynamical friction effects can be measured in the LISA stochastic signal. Dynamical friction model is preferred only when $\rho \gtrsim 10^{-7}\mathrm{g\ cm}^{-3}$ if including eccentricity in the vacuum model.
- For typical Eddington ratios, accretion effects cannot be measured in LISA stochastic signal.
- Improved modeling: “ready-to-use” parametric models for environmental or eccentricity effects in SGWB

Future work

- Interplay of eccentricity and environmental effects in the orbital evolution and stochastic background
  - Improved modeling of environmental effects such as dynamical friction (mutual wake effect, disk profile etc.)
  - Expected distribution of eccentricities (and semi-major axes) under different astrophysical channels and interactions with environment
- Role of extragalactic foreground on the detection of the SGWB from sBBHs

My interests / background

Eccentric binaries (post-Newtonian, multiple timescales methods)

Waveform modeling and systematics (inspiral-merger-ringdown)

Astrophysical environments and dynamics of binaries

Tidal effects in neutron star binaries

Extra slides

Primer on SGWB from vacuum sBBH: quantitative

[Allen & Romano (2001), Phinney (2001), Abbott etal (2021)]

Comoving merger rate

Source params (masses, spins etc.)

$$\Omega_{\mathrm{GW}}(f) = \frac{f}{\rho_c H_0} \iint dz d\boldsymbol{\phi} \frac{R_{\mathrm{GW}}(z)}{(1+z)\mathscr{E}(z)} p(\boldsymbol{\phi}) \frac{dE_{\mathrm{GW}}}{df_s}(\boldsymbol{\phi}) \Bigg|_{f_s = f(1+z)}$$

Prob. distribution of source params

GW energy flux

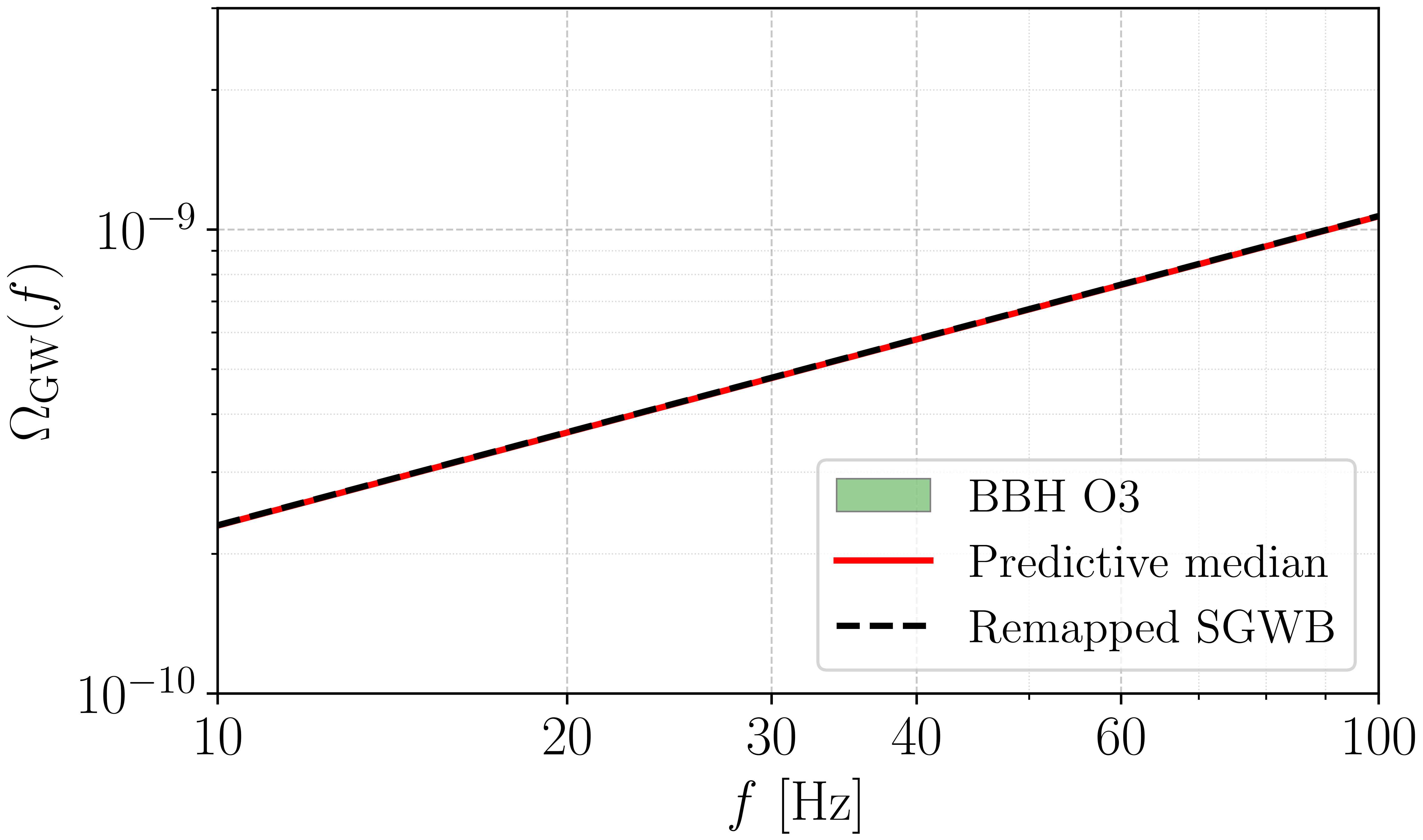
$$\frac{dE_{\mathrm{GW}}}{df_s} \equiv \frac{\dot{E}_{\mathrm{GW}}}{\dot{f}_s} = \frac{\eta m^{5/3} \pi^{2/3}}{3 f_s^{1/3}}$$

$$\Omega_{\mathrm{GW}}(f) = A_{\mathrm{vac}} f^{2/3}$$

Median amplitude informed by LVK constraints on merger rate and mass distribution

$$A_{\mathrm{vac}} = \frac{\pi^{2/3}}{3\rho_c H_0} \iiint dm_1 dq dz \frac{R_{\mathrm{GW}}(z) p(m_1, q)}{(1+z)^{4/3} \mathscr{E}(z)} \eta m^{5/3} \approx 4.97 \times 10^{-11} [Hz^{-2/3}]$$

SGWB from unresolved (vacuum) sBBH in LVK band



Improved model for eccentric SGWB

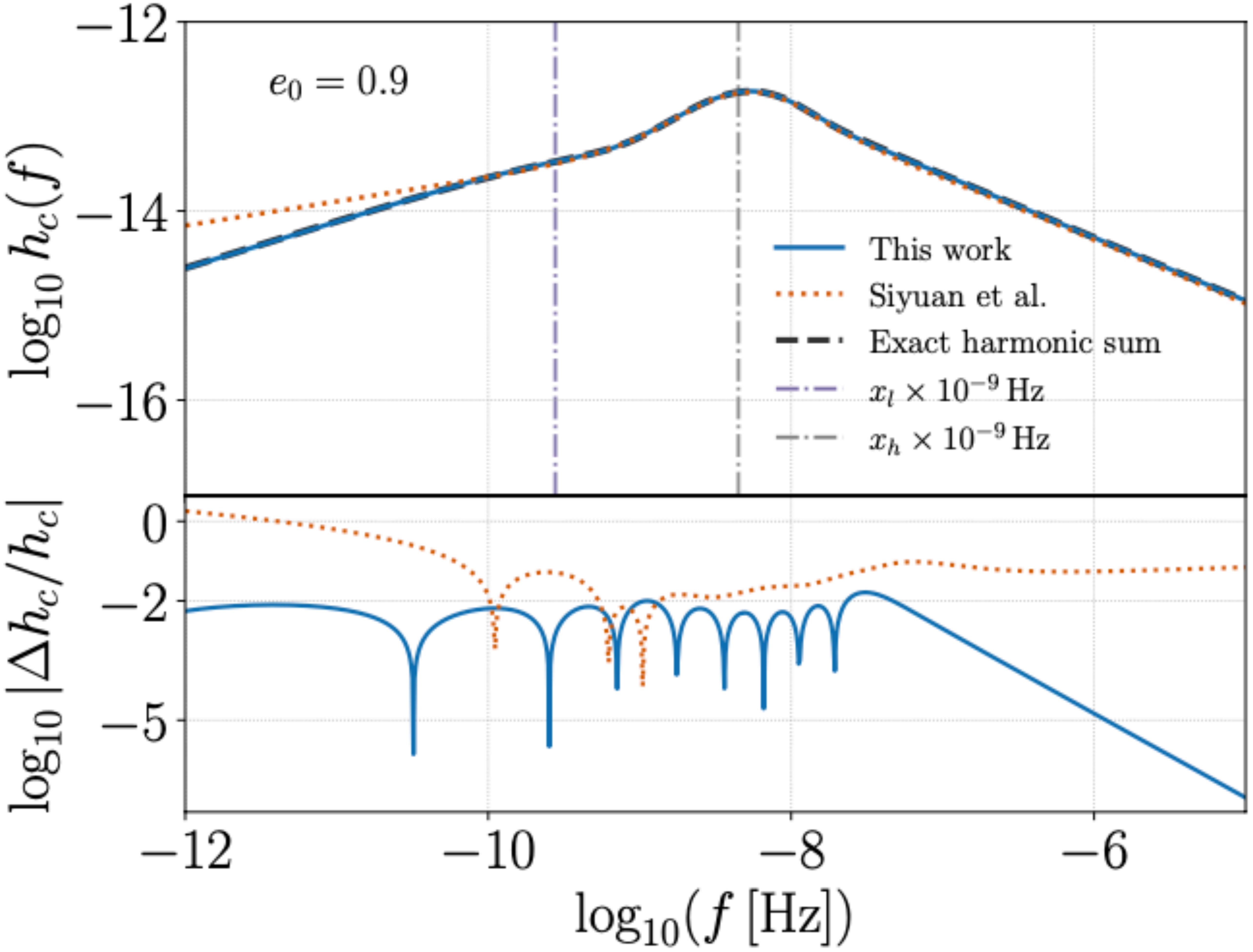
$$h^2_{c,\rm fit}(x) = h^2_{c,\rm circ}(x) S_{\rm fit}(x), \tag{20a}$$

$$S_{\rm fit}(x) = \frac{(x/x_a)^{q_a}}{1+(x/x_a)^{q_a}} \times \left[1 + A_l \left(\frac{x}{x_l}\right)^{p_l} \exp\left(-x/x_l\right)\right] \times \left[1 + A_h \left(\frac{x}{x_h}\right)^{p_h} \exp\left(-x/x_h\right)\right], \tag{20b}$$

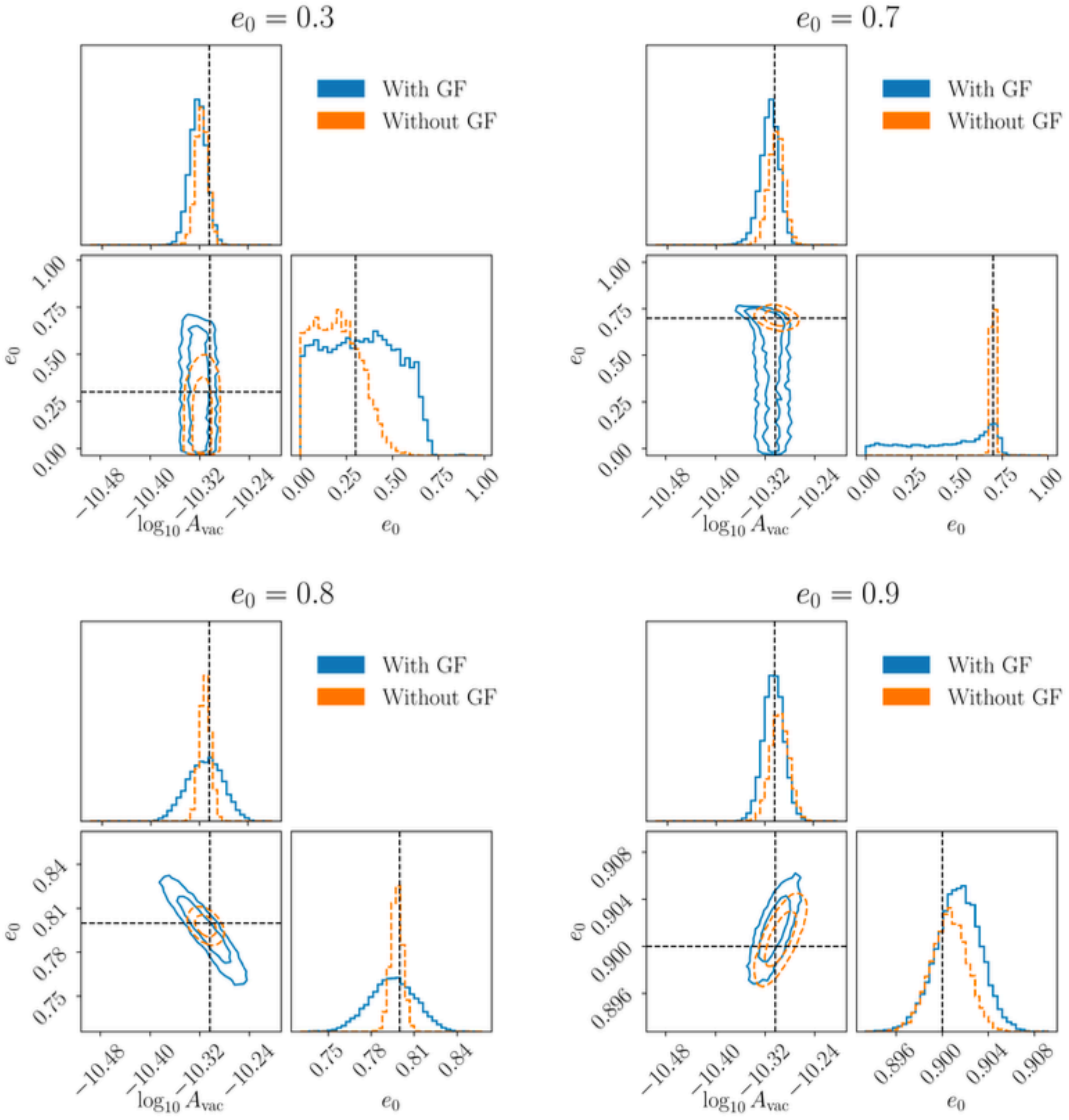
$$h_c^2(f) = A_{\rm pop} \times h^2_{c,\rm fit}(f\frac{f_{p,\rm ref}}{f_p}) \left(\frac{f_p}{f_{p,\rm ref}}\right)^{-4/3}, \tag{24}$$

with the dimensionless amplitude $A_{\rm pop}$ given by

$$A_{\rm pop} = \int dz dm_1 dq \frac{R_{\rm GW}(z) p\left(m_1, q\right)}{H_0 \mathcal{E}(z)(1+z) {\rm Mpc}^{-3}} \times \left(\frac{\mathcal{M}}{\mathcal{M}_0}\right)^{5/3} \left(\frac{1+z}{1+z_0}\right)^{-1/3}. \tag{25}$$



Measuring eccentricity: Dirac delta distribution injection-recovery



$e_0^{\rm inj}$	$e_{f_{\rm orb}=10\,{\rm Hz}}$	SNR	$\delta e_0/e_0^{\rm inj}$	$\delta A_{\rm vac}/A_{\rm vac}^{\rm inj}$	$\log_{10}\mathcal{B}_{\rm cir}^{\rm ecc}$
0.1	5.372×10^{-7}	200	3.069	0.05113	0.7360
0.2	1.136×10^{-6}	200	1.557	0.05049	0.9060
0.3	1.877×10^{-6}	200	0.9955	0.05064	0.6035
0.4	2.886×10^{-6}	201	0.7568	0.05172	0.7175
0.5	4.402×10^{-6}	201	0.6461	0.05049	0.8518
0.6	6.952×10^{-6}	206	0.5232	0.05081	0.9513
0.7	1.193×10^{-5}	228	0.4814	0.05650	0.8157
0.8	2.437×10^{-5}	282	0.03269	0.1073	0.2662
0.9	7.783×10^{-5}	181	0.003499	0.06279	11.12

Energy spectrum of eccentric binaries

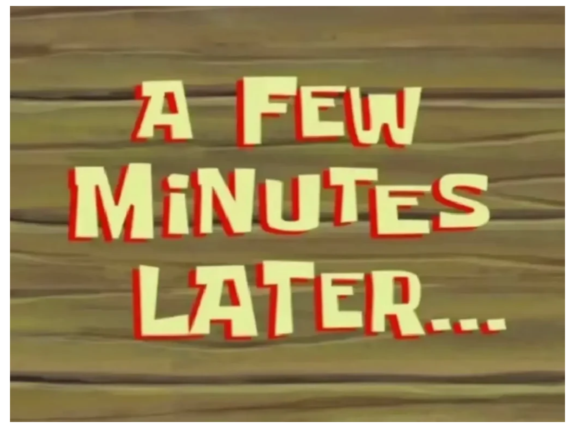
Waveform in frequency domain (stationary phase approximation)

$$\tilde{h}_{+,\times} = -\frac{\eta m}{2D_L}\sum_{j=1}^{\infty}\frac{[2\pi m f_{\rm orb}(t_j^*)]^{2/3}}{\sqrt{j\dot{f}_{\rm orb}(t_j^*)}} \times \left[C_{+,\times}^{(j)}(t_j^*) + iS_{+,\times}^{(j)}(t_j^*)\right] e^{-i\psi_j(t_j^*)},$$

For expressions of mode amplitudes that do not contain typos, refer to [RSC & Yunes (2022)]

Relation between energy spectrum and polarizations [Phinney (2001)]

$$\frac{dE_{\rm GW}}{df_r} = 2\pi^2\frac{D_L^2}{(1+z)^2}f^2 \times \left(\langle|\tilde{h}_+(f_r)|^2\rangle + \langle|\tilde{h}_\times(f_r)|^2\rangle\right)\Big|_{f_r=f(1+z)},$$



$$\boxed{\frac{dE_{\rm GW}}{df_r} = \frac{\mathcal{M}^{5/3}\pi^{2/3}}{3f^{1/3}(1+z)^{1/3}}\sum_{j=1}^{\infty}\frac{g(j,e_j)}{F(e_j)(j/2)^{2/3}}}$$

See [Enoki & Nagashima (2007)] for a different derivation

Primer on eccentric binaries

- Multi-periodic (epicyclic) orbital motion

$$r = a(1 - e\cos u)$$

$$\tan(v/2) = \sqrt{\frac{1+e}{1-e}}\tan(u/2)$$

$$l = n(t - t_{\rm p}) = u - e\sin(u)$$ Kepler equation

[Yunes et.al (2008)]

$$\cos\phi = -e + \frac{2}{e}(1-e^2)\sum_{k=1}^{\infty} J_k(ke)\cos kl, \qquad (2.13)$$

$$\sin\phi = (1-e^2)^{1/2}\sum_{k=1}^{\infty}[J_{k-1}(ke) - J_{k+1}(ke)]\sin kl. \qquad (2.14)$$

- GW Radiation reaction —>loss of energy and angular momentum —> shrinking and circularizing of orbit

$$\left\langle\frac{da}{dt}\right\rangle = -\frac{64}{5}\frac{G^3 m_1 m_2(m_1+m_2)}{c^5 a^3(1-e^2)^{7/2}}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)$$

[Peters (1964)]

$$\left\langle\frac{de}{dt}\right\rangle = -\frac{304}{15}e\frac{G^3 m_1 m_2(m_1+m_2)}{c^5 a^4(1-e^2)^{5/2}}\left(1+\frac{121}{304}e^2\right).$$

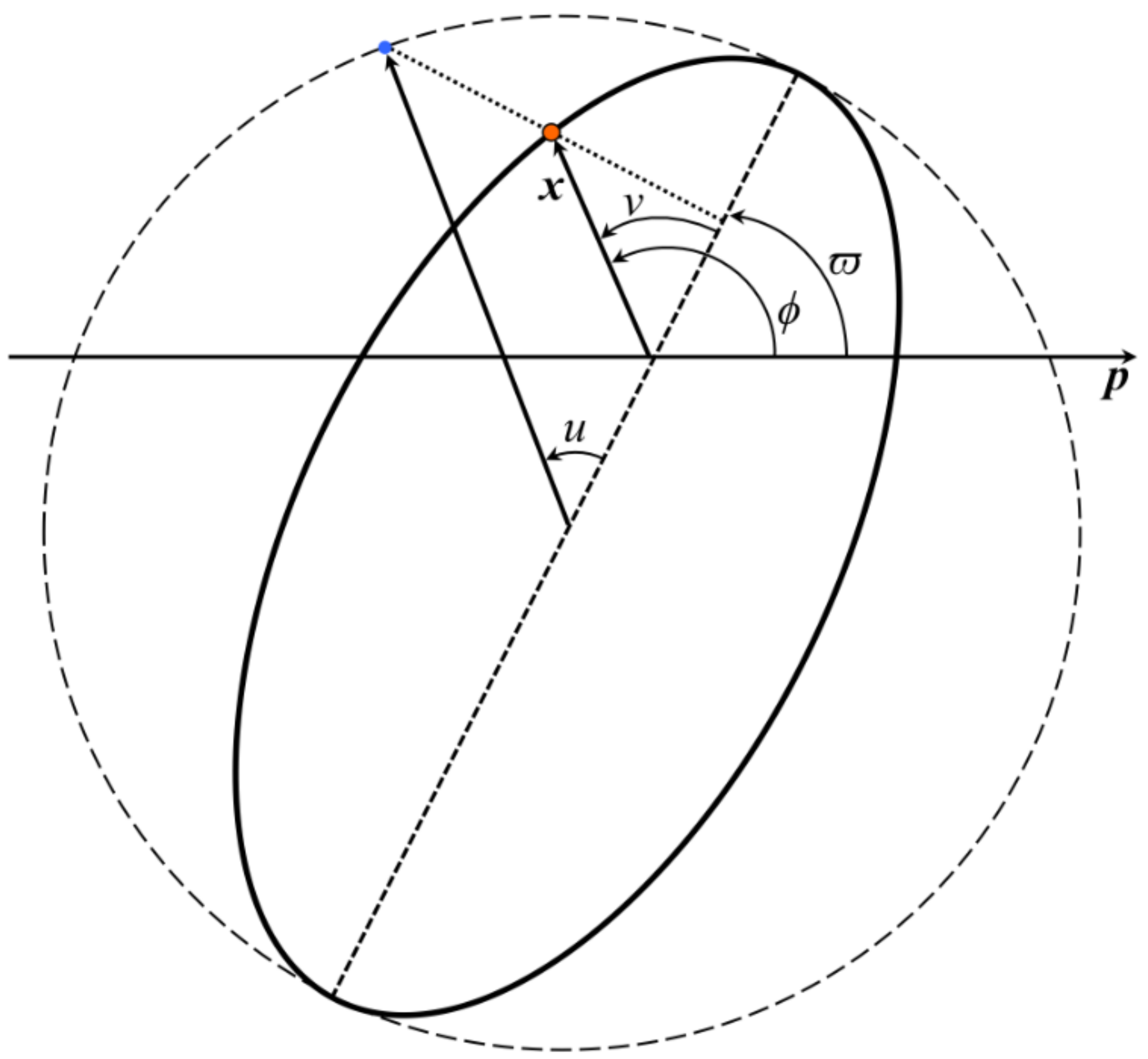


Figure adapted from Moore etal (2016)

Bayesian parameter estimation of SGWB in LISA

$$\ln \mathscr{L}(\hat{\boldsymbol{S}} \mid \theta) = \sum_{i=1}^{D} \sum_{k} \left[-\ln \Gamma(N) - N \ln \left(\frac{S_k(f_i;\theta)}{N} \right) + (N-1) \ln \hat{S}_k(f_i) - \frac{N\, \hat{S}_k(f_i)}{S_k(f_i;\theta)} \right],$$

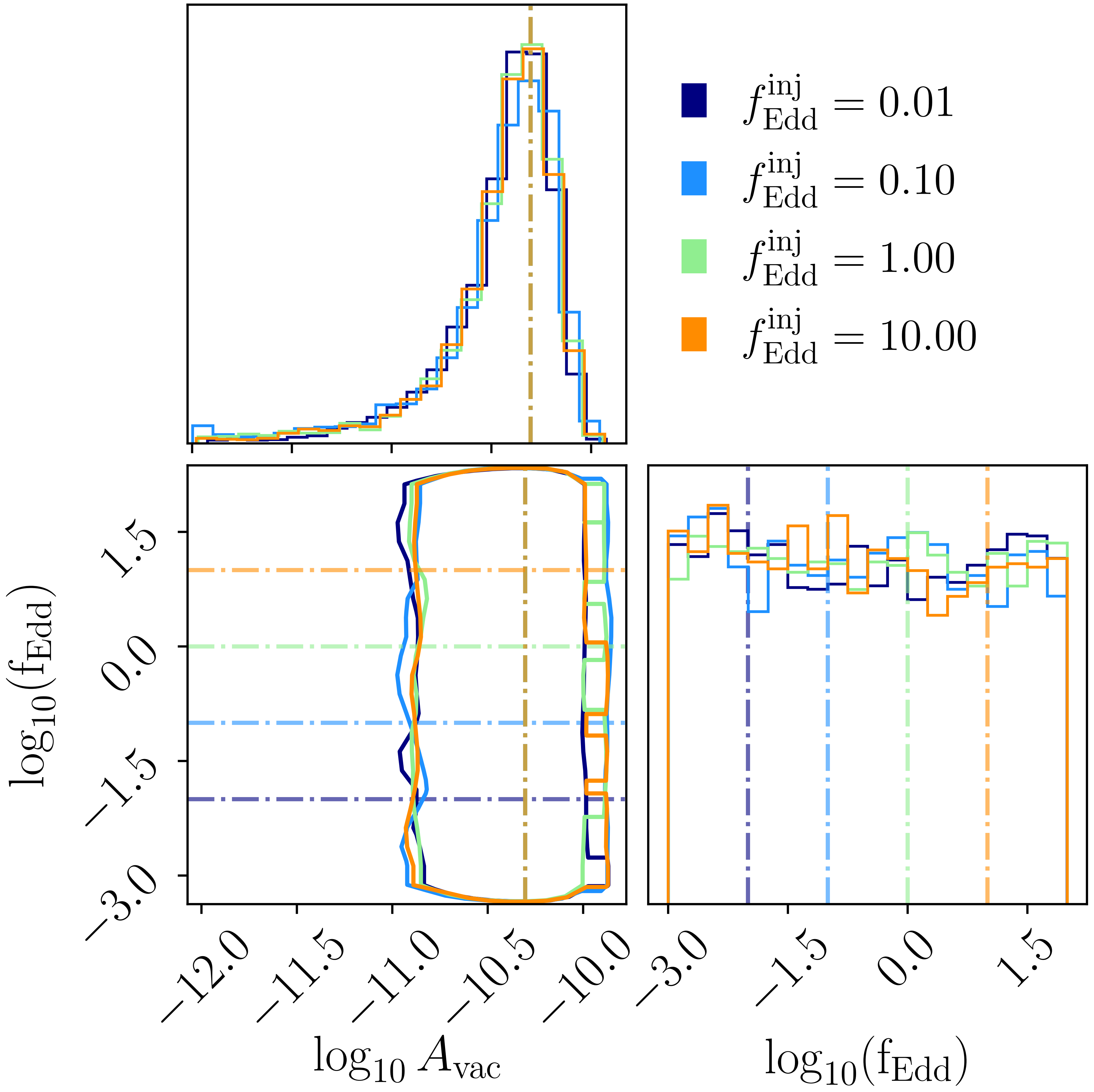
$$S_{k,\mathrm{GW}}(f;\theta) = R_k(f;\theta)\, S_{\mathrm{GW}}(f;\theta) \qquad S_{\mathrm{GW}}(f;\theta) = \frac{3H_0^2}{4\pi^2 f^3} \Omega_{\mathrm{GW}}(f;\theta)$$

$$S_{\mathrm{GW,Gal}}(f;\theta_{\mathrm{Gal}}) = \frac{A_{\mathrm{Gal}}}{2} f^{-7/3} \exp\left[-(f/f_1)^{\alpha_{\mathrm{Gal}}} \right] \left[1 + \tanh((f_{\mathrm{kn}} - f)/f_2) \right]$$

$$\mathrm{SNR} = \sqrt{T \sum_k 4 \int_0^\infty df \left(\frac{S_{\mathrm{k,GW}}(f)}{S_{\mathrm{k,n}}(f)} \right)^2}$$

Parameter estimation of accretion effects from SGWB in LISA

Gas accretion effects cannot be measured even for large Eddington ratio



More than 10% change compared to quasi circular vacuum prediction above $f \sim 10^{-3} Hz$ when:

$$\frac{f_{\rm Edd}}{\tau} \sim \frac{a_{||}}{c} \sim \frac{\dot{G}}{G} \gtrsim 10^{-4}\mathrm{yr}^{-1}$$

Order of magnitude constraints of -4PN effects:

$$f_{\rm Edd} \lesssim 10^3$$

$$a_{||} \lesssim 10^{-3}\mathrm{m\,s}^{-2}$$

$$\dot{G}/G \lesssim 10^{-4}\mathrm{yr}^{-1}$$