Dynamical perspective on black hole ringdown

Marina De Amicis

PERIMETER INSTITUTE

IFPU Workshop:

Perspectives on gravity: from theory to observations

Framework

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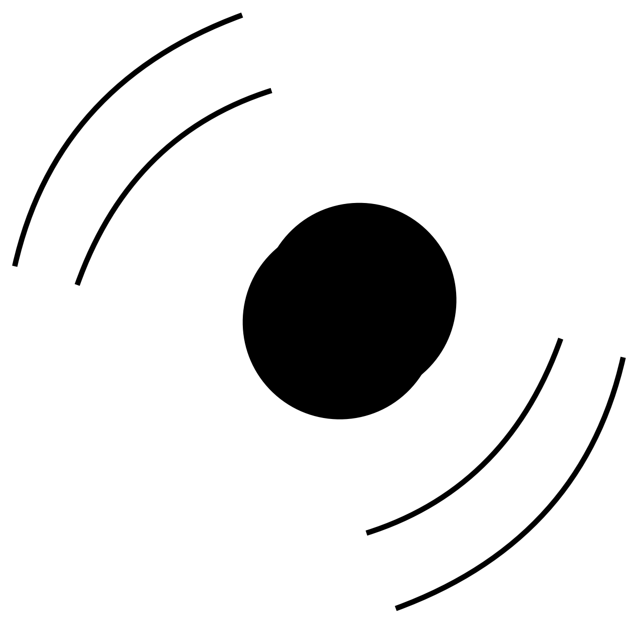
After $\tau_{\text{start}} - \tau_{\text{peak}} \sim 10 - 20M$ → **stationary ringdown:**

~ **One-body perturbation theory valid: BH + small, Gaussian and narrow initial data**



$$h_{\ell m} \sim \theta(\tau - \tau_{\text{start}}) \sum_{n,\pm} A_{\ell m n \pm} e^{i\omega_{\ell m n \pm}(\tau - \tau_{\text{start}})}\ , \quad \underline{\omega_{\ell m n \pm} = \omega^{\text{Re}}_{\ell m n \pm} - i\,|\,\omega^{\text{Im}}_{\ell m n \pm}|}$$

Quasinormal Modes

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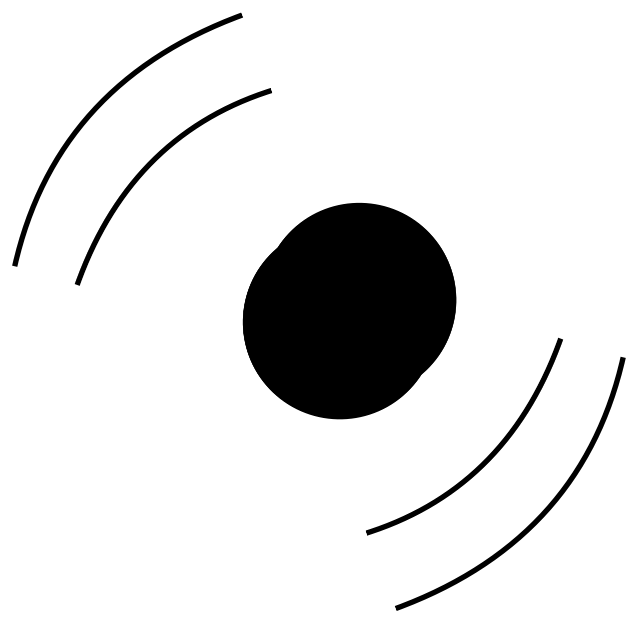

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Stationary ringdown → tests of General Relativity

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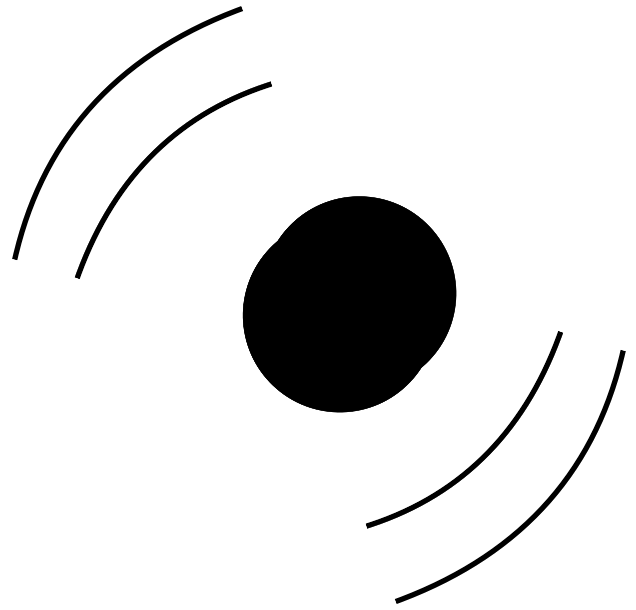

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Quasinormal Modes

Stationary ringdown → tests of General Relativity

- **No-hair conjecture: (astrophysical) BH described by mass and spin**
- **Detection of 2 QNM: test presence of additional parameters/GR deviation**

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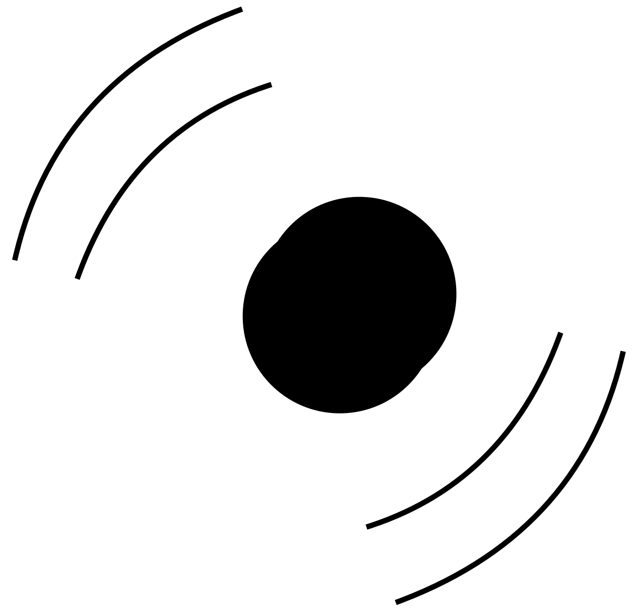

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Stationary ringdown → tests of General Relativity

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Is this model satisfactory?

Ringdown models shortcomings

Stationary ringdown template:

$$h_{\ell m}(\tau) = \theta(\tau - \tau_{\text{start}}) \sum_{n,\pm} A_{\ell mn\pm} e^{-i\omega_{\ell mn\pm}(\tau - \tau_{\text{start}})}$$

Free, unknown ad-hoc parameter τ_{start}

- $\tau_{\text{start}} - \tau_{\text{peak}} \sim 10 - 20M$ $\longrightarrow$ model do not hold where signal is loudest
- Pushing τ_{start} earlier? $\longrightarrow$ could lead to overfitting [Baibhav + (2023)] [Mitman + (2025)][Berti+ (2025)]
  not valid: non-linearities at early times...

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Time dependence for generic ID

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…**fails** already at **linear** level

[Chavda+(2024)]

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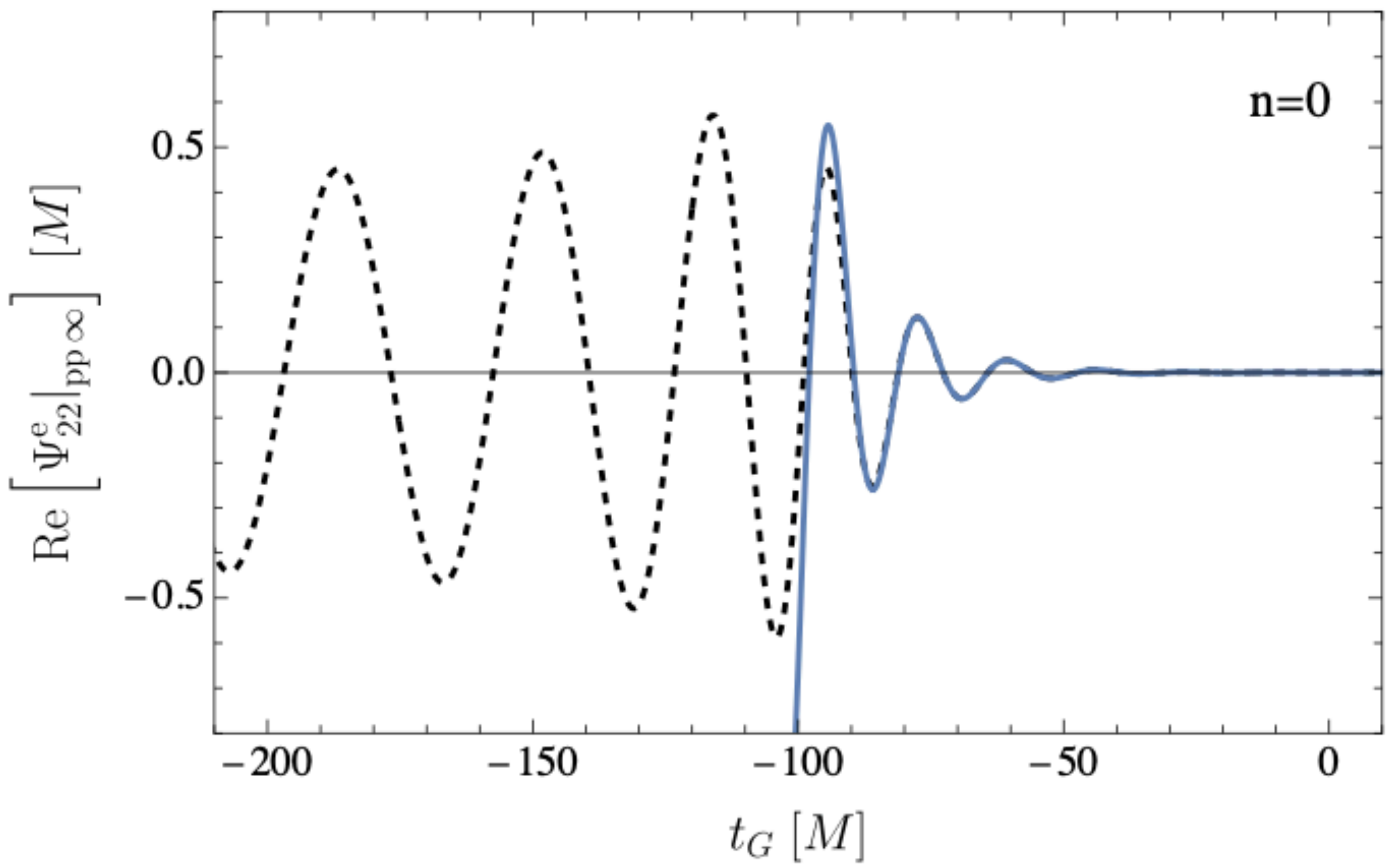

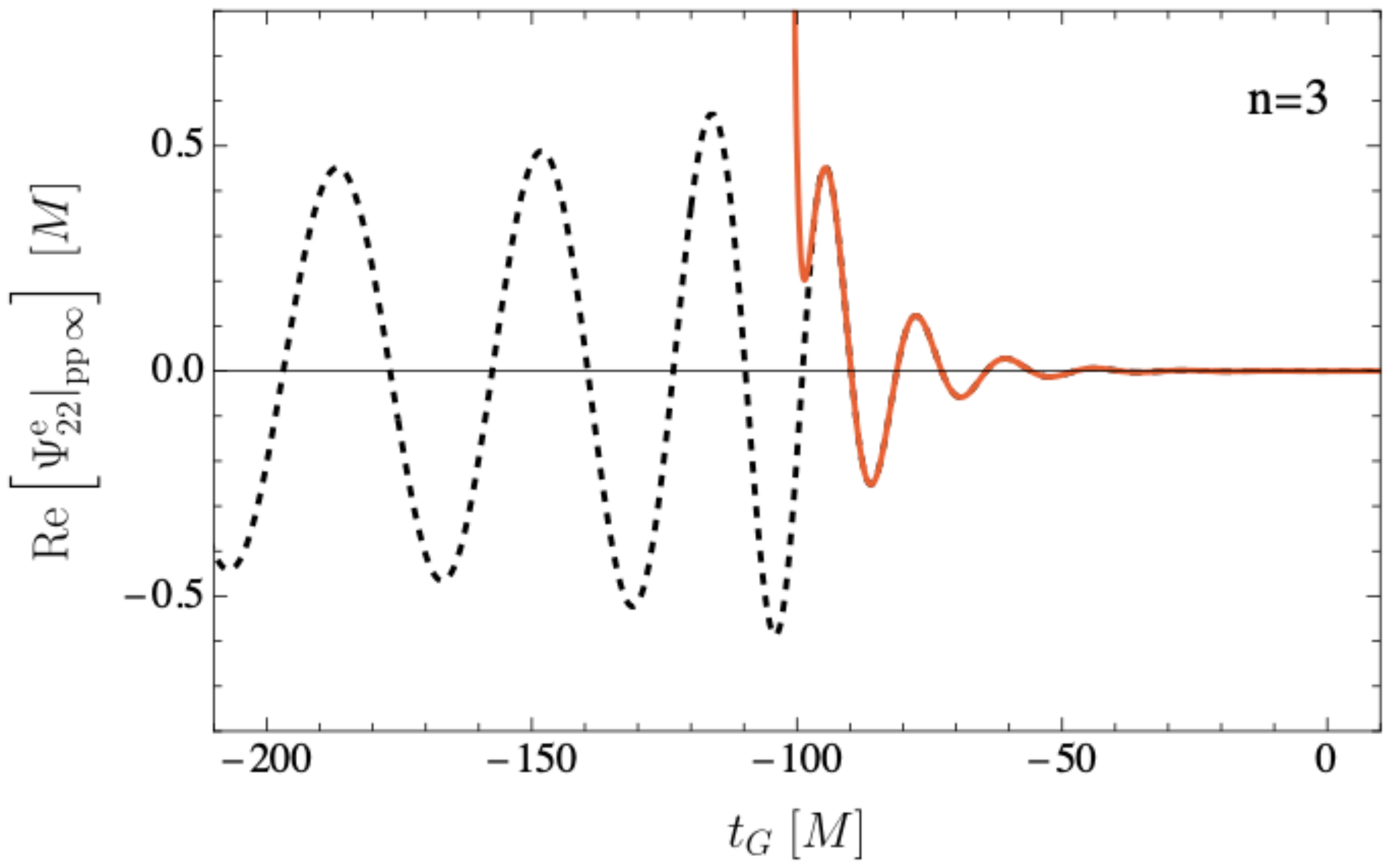

[Kuchler+(2025)]

Ringdown models: refined version

What we have:

$$h_{\ell m}(\tau) = \theta(\tau - \tau_{\text{start}}) \sum_{n,\pm} A_{\ell mn\pm} e^{-i\omega_{\ell mn\pm}(\tau-\tau_{\text{start}})}$$

One-body problem, perturbative:
BH+small, narrow, Gaussian initial data

Extrapolate to $\longrightarrow$

Two-body problem, non-linear:
comparable masses

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$\boxed{\textbf{How can we improve it?}}$

Two-body problem, perturbative:
extreme mass-ratio limit

Extrapolate to →

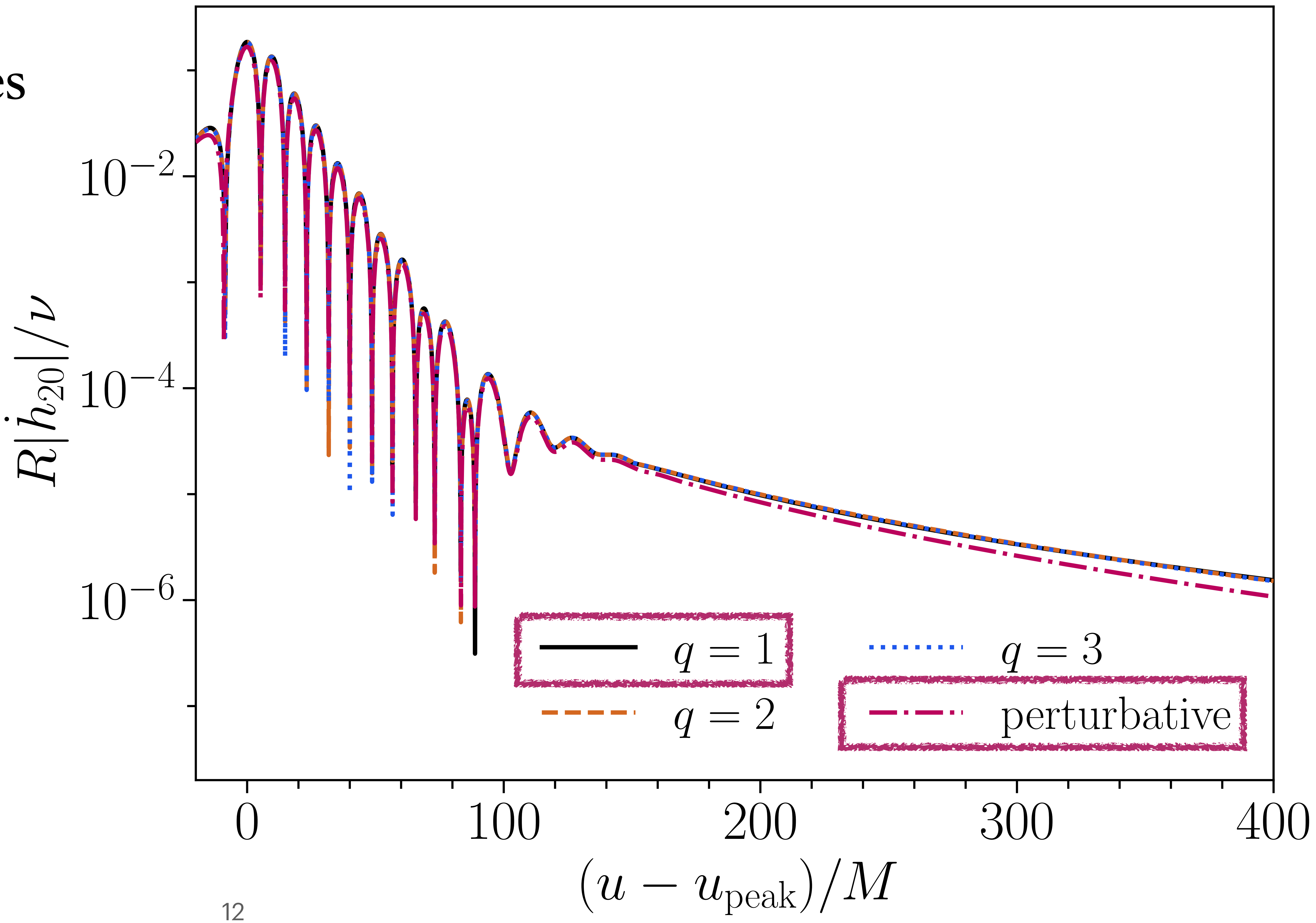
Two-body problem, non-linear:
comparable masses

Why extreme mass-ratio limit?

Similar transient post-merger behaviour between

- Numerical relativity, comparable masses
- Perturbative, extreme mass ratio

Radial infall vs head on for $r_0 = 100M$ [De Amicis+(2024)]



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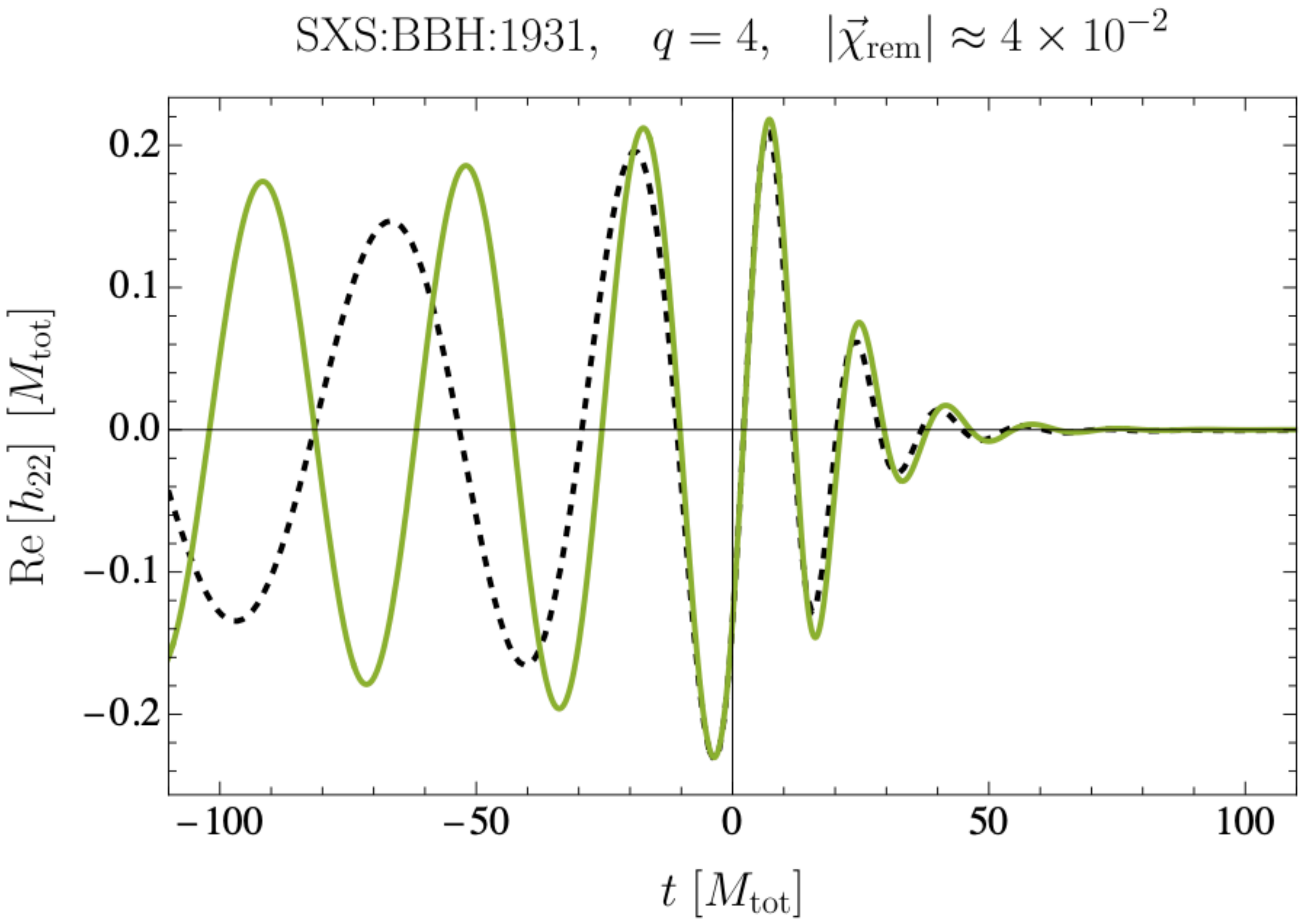
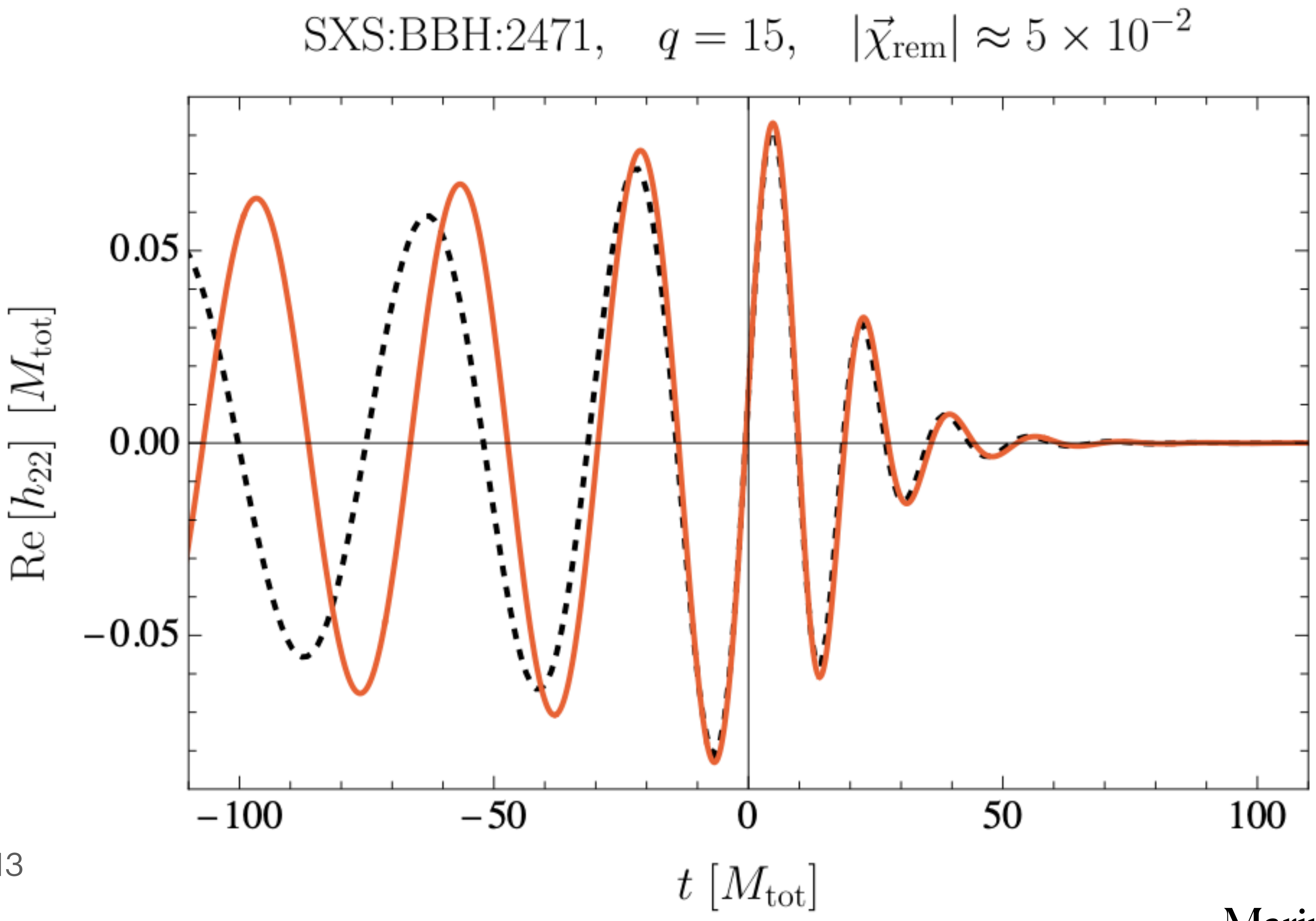




Merger-ringdown phenomenological models

Available merger "early times" templates:

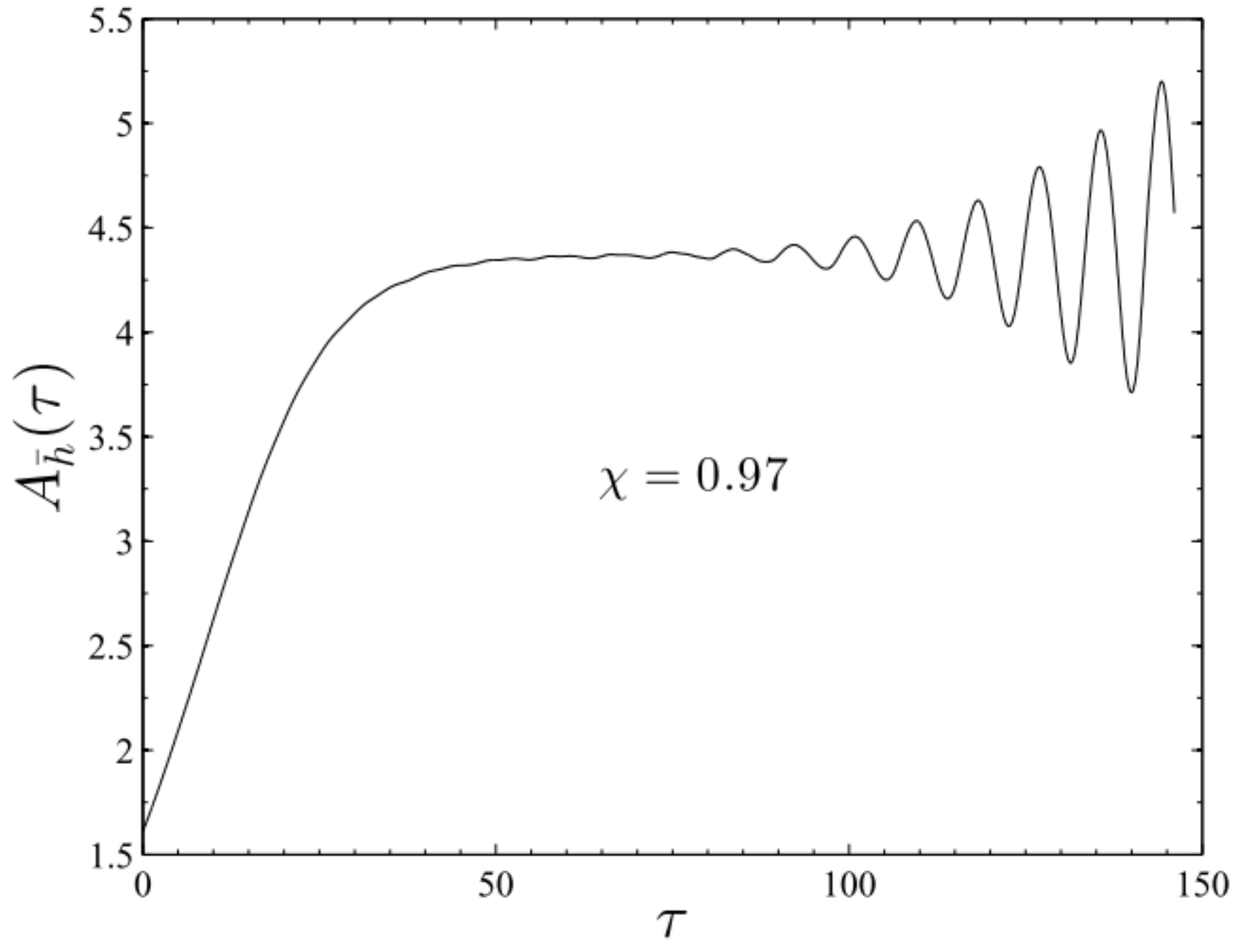
[Damour and Nagar (2015)]

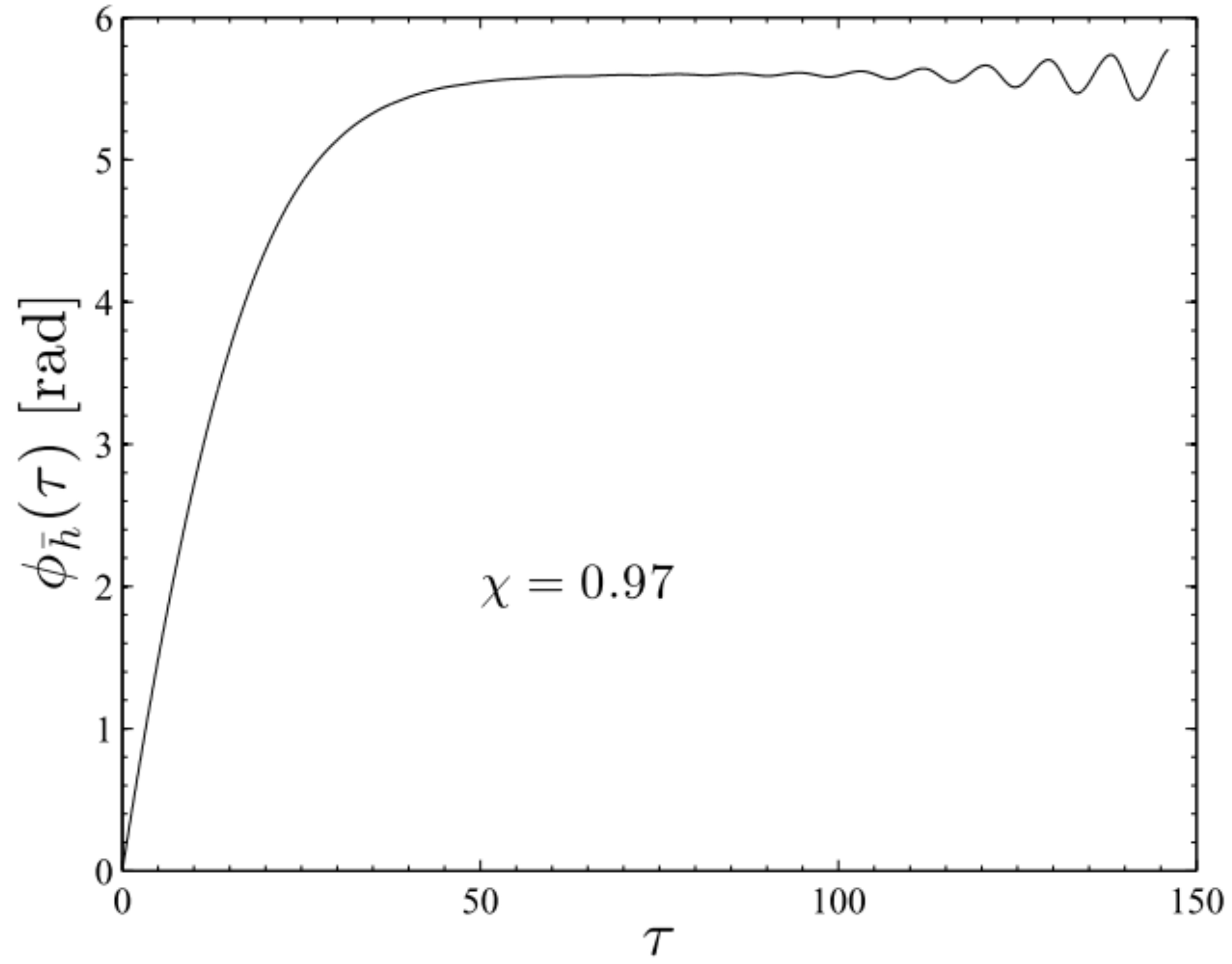
- $h_{\ell m}(t) = \boxed{A_{\ell m}(t)}\ e^{i \boxed{\phi_{\ell m}(t)}} \cdot e^{i\omega_{\ell m 0} t}$

Activation functions:

$$A_{\ell m}(t) = c_1^A \ \tanh\left(c_2^A t + c_3^A\right) + c_4^A \,,$$

$$\phi_{\ell m}(t) = -\, c_1^\phi \ \ln\left(\frac{1 + c_3^\phi e^{-c_2^\phi t} + c_4^\phi\, e^{-2c_2^\phi t}}{1 + c_3^\phi + c_4^\phi}\right) \,,$$





- **Same functional form** both for **non-linear comparable masses** and **linear test-particle cases**.
- **Non-linearities** encapsulated in free **coefficients** of the fit

Ringdown models: refined version

Stationary ringdown template:

$$h_{\ell m}(\tau) = \theta(\tau - \tau_{\rm start}) \sum_{n,\pm} A_{\ell m n \pm} e^{-i\omega_{\ell m n \pm}(\tau - \tau_{\rm start})}$$

How can we improve it?

Work from **perturbative**, **extreme mass-ratio** limit

i) Model the **ringdown** as **dynamically excited** by the **two-body** problem

[De Amicis, Cannizzaro, Carullo, Sberna, 2506.21668]
[De Amicis, Cannizzaro, Carullo, Kuntz, Sberna, 2605.16492]

ii) **Model other linear components** of the signal at early times

[De Amicis, Cannizzaro, 2601.11706]

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$$h_{\ell m}(\tau) = \theta(\tau - \tau_{\rm start}) \sum_{n,\pm} A_{\ell mn\pm} e^{-i\omega_{\ell mn\pm}(\tau - \tau_{\rm start})}$$

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Test-particle on a Schwarzschild background

- Regge-Wheeler/Zerilli equation:

$$\left[\partial_t^2 - \partial_{r_*}^2 + V_{\ell m}(r_*)\right] \Psi_{\ell m}(t, r_*) = S_{\ell m}(t, r_*)$$

$$S_{\ell m}(t, r_*) = e^{-im\varphi(t)} \left[f_{\ell m}(t, r_*)\delta(r_* - r_*(t)) + g_{\ell m}(t, r_*)\partial_{r_*}\delta(r_* - r_*(t))\right]$$

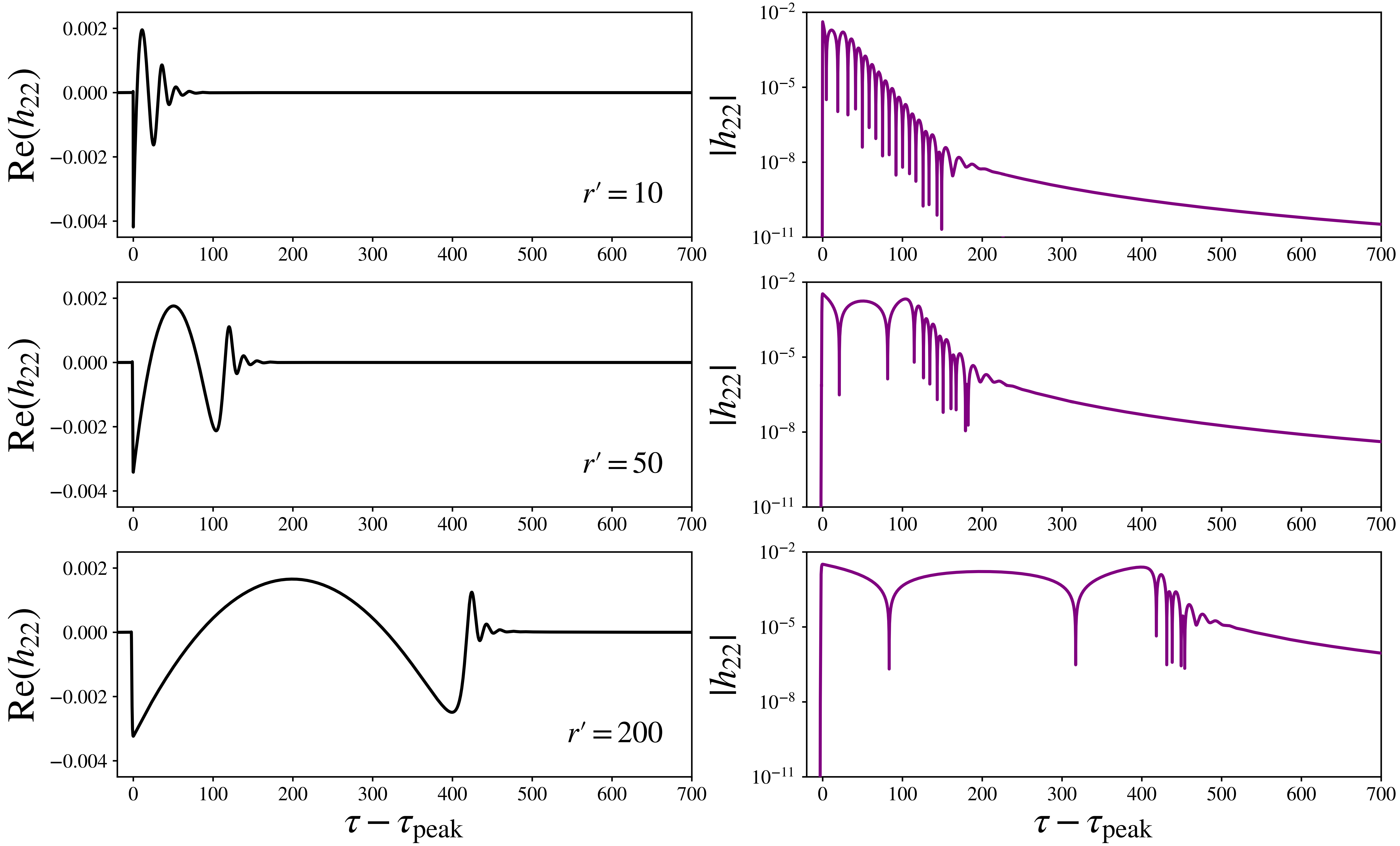
- General solution:

$$\Psi_{\ell m}(t - r_*) = \int_{-\infty}^{\infty} dt' \int_{-\infty}^{\infty} dr'_* \, S_{\ell m}(t', r'_*) \, \underbrace{G_{\ell m}(t, t'; r_*, r'_*)}_{\text{Green's function}}$$

Basic tool: Green's function

Green's function: $\left[\partial_t^2 - \partial_{r_*}^2 + V_{\ell m}(r_*)\right] G_{\ell m}(t,t';r_*,{r_*}') = \delta(t-t')\delta(r_* - {r_*}')$

Dictates how perturbations propagate on the curved background



Early times → prompt response:

- longer the further ${r_*}'$

Late times → tail:

- power-law decay [Price, (1972)]
- larger amplitude the further ${r_*}'$

Intermediate times → QNMs:

- exponentially damped vibrations

[Leaver, (1985)]

Dynamical quasinormal mode excitation

- Regge-Wheeler/Zerilli equation:

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- QNMs-propagated solution:

Retarded Green's function → **Physical signals travel on/inside light-cone**

$$\Psi_{\ell m}^{\mathrm{QNMs}}(t - r_*) = \int_{-\infty}^{\infty} dt' \int_{-\infty}^{\infty} dr'_*\, S_{\ell m}(t', r'_*)\, G_{\ell m}^{\mathrm{QNMs}}(t, t'; r_*, r'_*)$$

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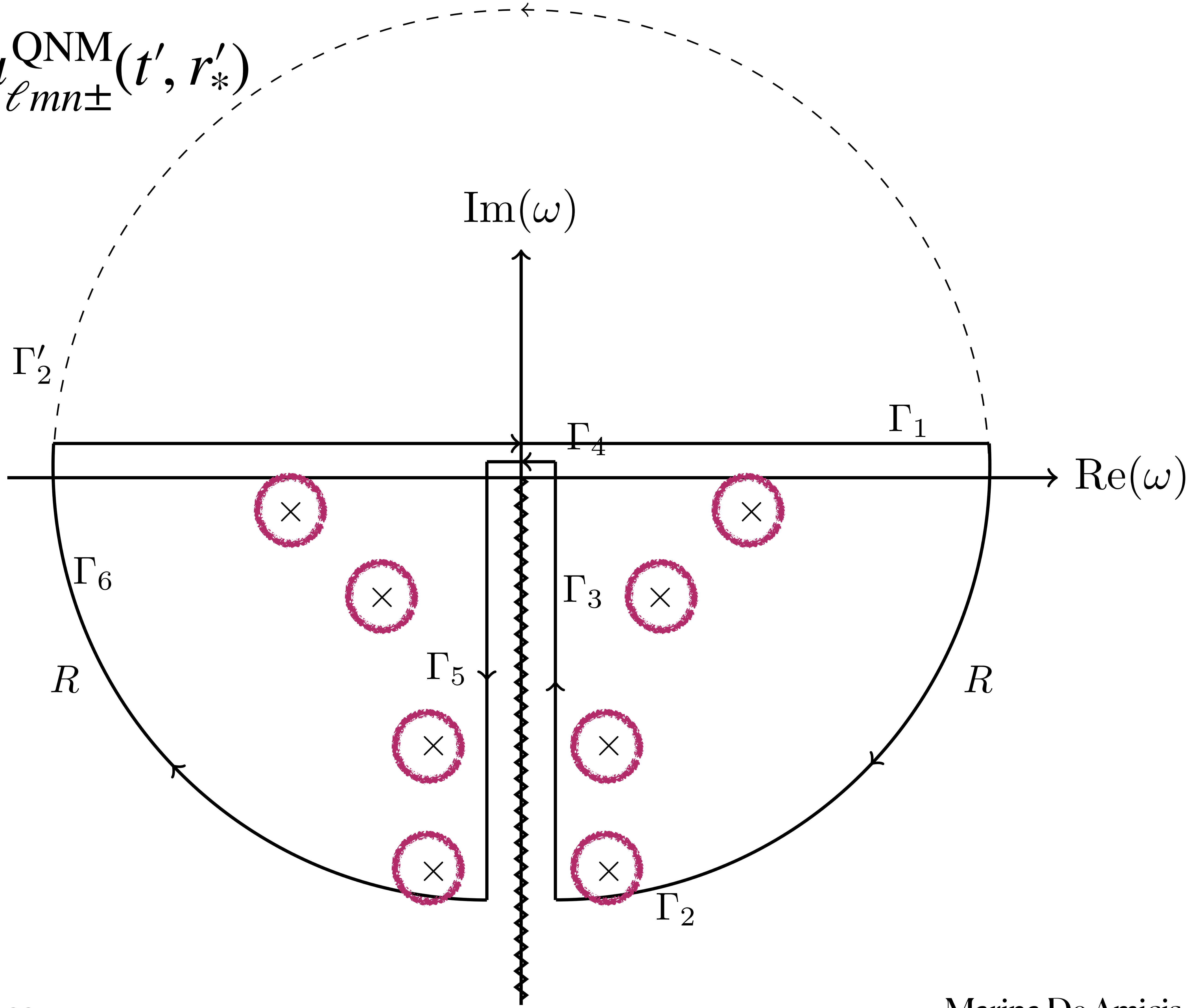
Retarded QNMs Green's function

$$G^{\text{QNMs}}_{\ell m}(t-t'; r_*, r'_*) = \sum \text{Res}\,.\left\{e^{-i\omega(t-r')}\tilde{G}_{\ell m}(\omega; r_*, r'_*)\right\} =$$

$$= \sum_{n,\pm} \; e^{-i\omega_{\ell m n\pm}(t-r_*)}\; B_{\ell m n\pm}\; u^{\text{QNM}}_{\ell m n\pm}(t', r'_*)$$

[Leaver, Phys. Rev. D 34, 384]

- Poles only present in lower half plane
- Time-domain QNMs only present when the contour is closed in the lower half plane



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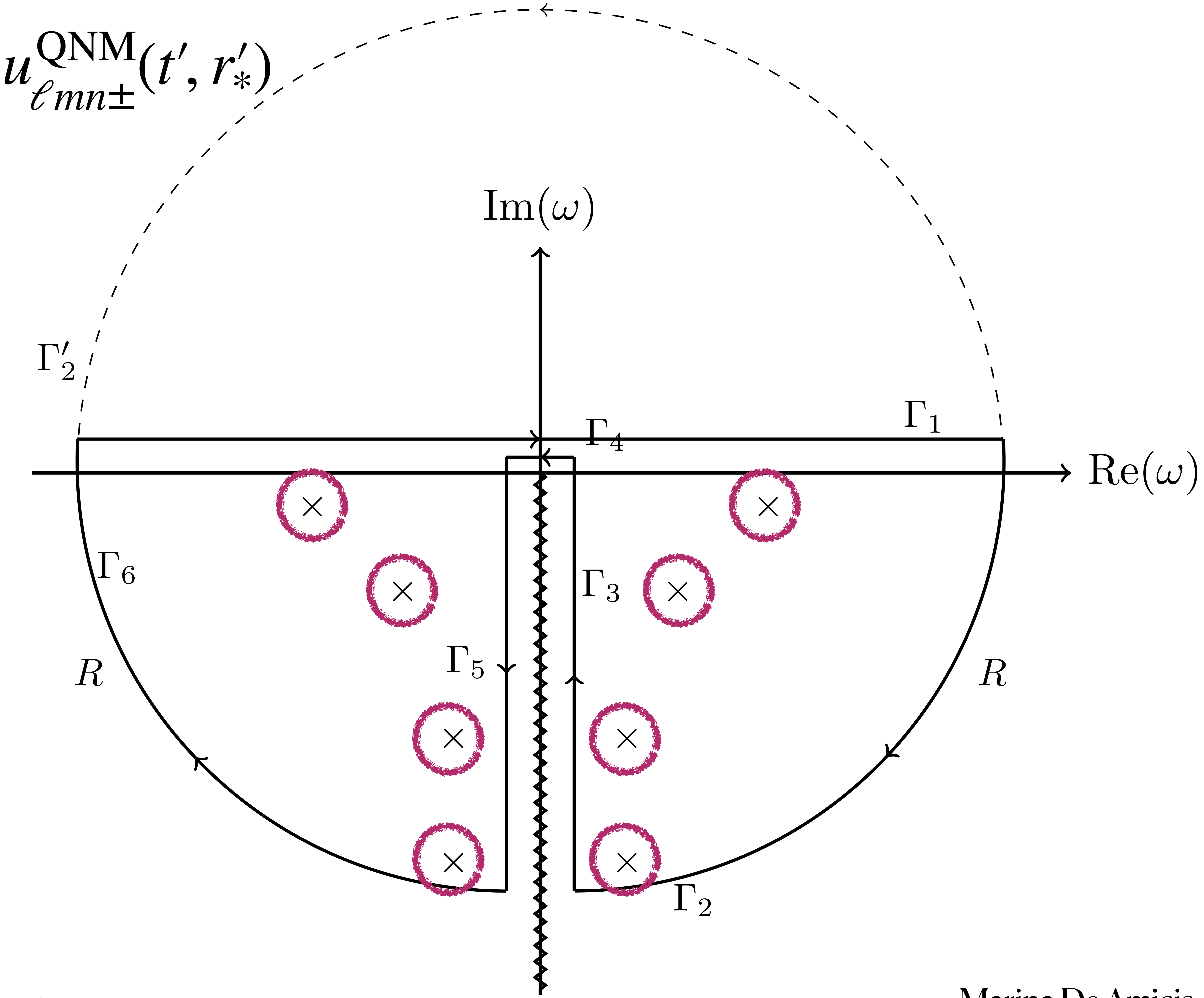

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Retarded QNMs Green's function

[De Amicis+, 2506.21668]

Condition to close in lower-half plane

$$G^{\mathrm{QNM}}_{\ell m}(t-t'; r_*, r'_*) = \underline{\theta[t - r_* - C(t', r'_*)]} \times \underbrace{\sum_{n,\pm} e^{-i\omega_{\ell mn\pm}(t-r_*)} \, B_{\ell mn\pm} \, u^{\mathrm{QNM}}_{\ell mn\pm}(t', r'_*)}_{\text{Leaver's GF}}$$

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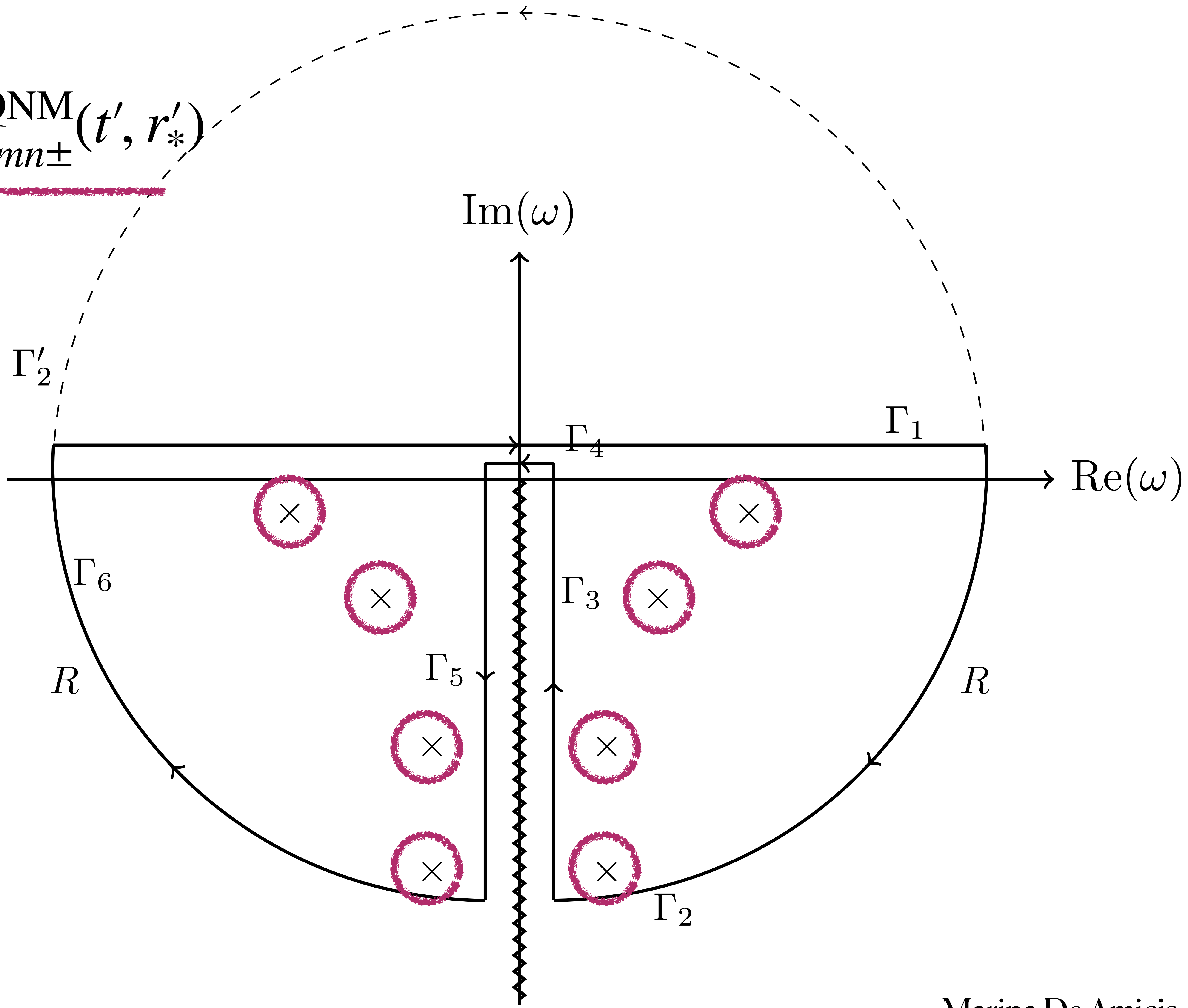

QNMs-propagated waveform

[De Amicis+, 2506.21668]

$$G^{\mathrm{QNM}}_{\ell m}(t-t';r_*,r'_*) = \theta[t-r_*-C(t',r'_*)]\sum_{n,\pm} e^{-i\omega_{\ell mn\pm}(t-r_*)}\, B_{\ell mn\pm}\, u^{\mathrm{QNM}}_{\ell mn\pm}(t',r'_*)$$

$$S_{\ell m}(t,r_*) = e^{-im\varphi(t)}\left[f_{\ell m}(t,r_*)\delta(r_*-r_*(t)) + g_{\ell m}(t,r_*)\partial_{r_*}\delta(r_*-r_*(t))\right]$$

Convolution result:

$$\Psi^{\mathrm{QNM}}_{\ell m}(t-r_*) = \sum_{n,\pm} e^{-i\omega_{\ell mn\pm}(t-r_*)}\, B_{\ell mn\pm}\left[c_{\ell mn\pm}(t-r_*) + i_{\ell mn\pm}(t-r_*)\right]$$

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Trajectory $(r_*(t),\varphi(t))$, apparent trajectory $(\bar{t},\bar{r}_*)$ / $t-r_* = C(\bar{t},\bar{r}_*)$

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- **Activation** coefficient: **hereditary**, **accumulates** in time

$$c_{\ell mnp}(t-r_*) = -\int_{\bar{r}_*(t-r_*)}^{\infty} dr'_* \frac{1}{\dot{r}_*(t(r'_*))}\left[u_{\ell mnp}f_{\ell m} - \partial_{r'_*}\left(u_{\ell mnp}g_{\ell m}\right)\right]_{(t(r'_*),r'_*)}$$

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- **Impulsive** coefficient: **local** term

$$i_{\ell mnp}(t-r_*) = \frac{u_n(\bar{t},\bar{r}_*)\, g(\bar{t},\bar{r}_*)\,\mathrm{sign}(\bar{r}_*)}{|\,1+\mathrm{sign}(\bar{r}_*)\,\dot{r}_*(\bar{r})\,|}$$

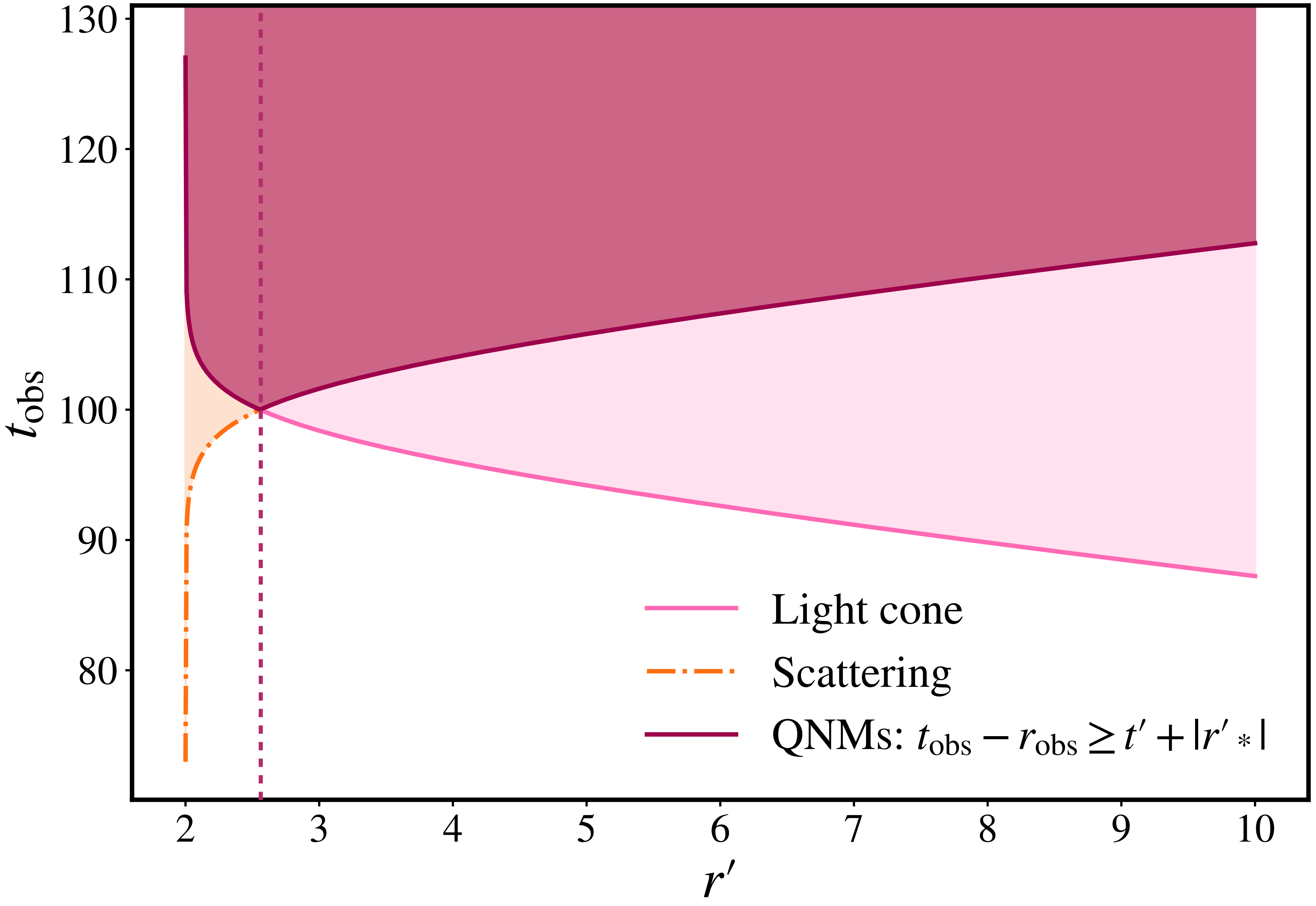
QNMs propagation condition

[Kuntz, 2510.17954]
[Arnaudo+, 2510.18956]
[Arnaudo+,2511.17703]
[De Amicis+, 2605.16492]

- For a source at (t', r'_*)
  QNM signal observed at

$$t_{\rm obs} - r_{*,\rm obs} \geq C(t', r'_*) \equiv t' + |\, r'_* \,|$$

$r_{*,\rm obs} = 100M$



- Define **bounce radius** $r_* = 0$

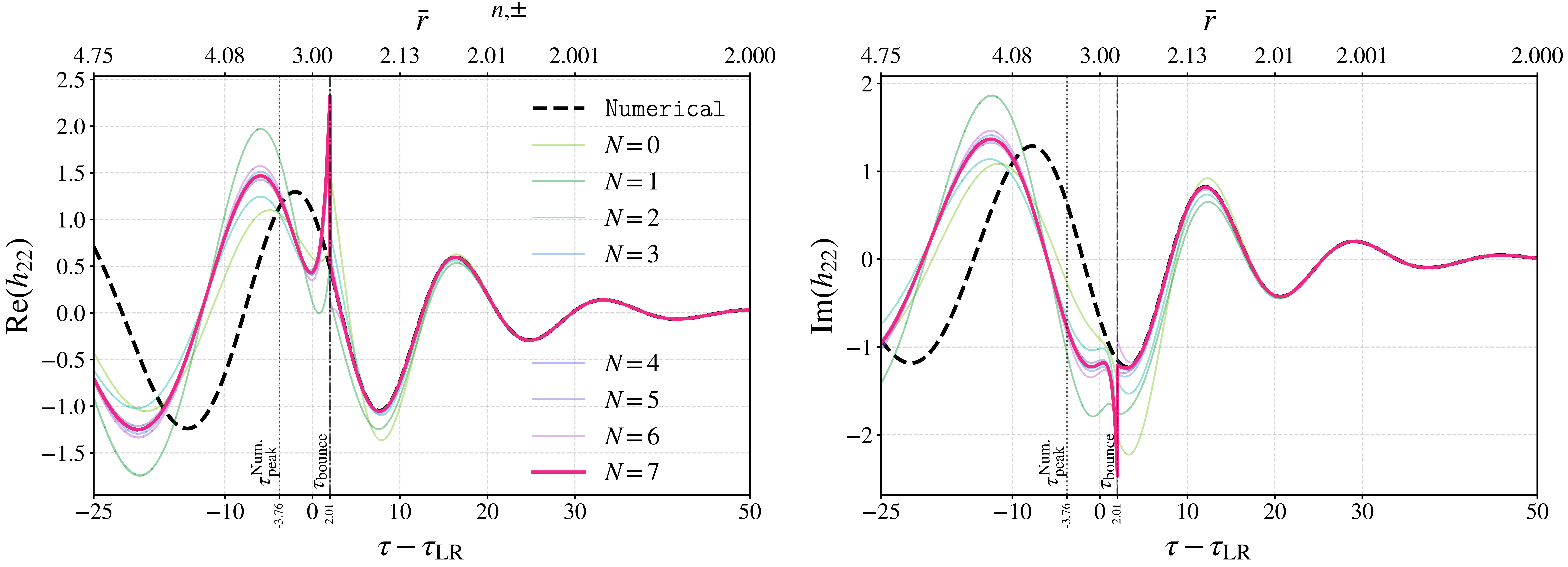
$r'_* < 0 \quad , \quad t - r_* \geq t' - r'_*$ On light-cone propagation

$r'_* > 0 \quad , \quad t - r_* \geq t' + r'_*$ Scattering off **bounce radius** $r_* = 0$, ~ potential peak

QNMs propagated signal: plunge-merger-ringdown

[De Amicis+, 2506.21668]
[De Amicis+, 2605.16492]

$$\Psi^{\rm QNM}_{\ell m} = \sum_{n,\pm}^{N} e^{-i\omega_{\ell mn\pm}(t-r_*)}\, B_{\ell mn\pm}\left[c_{\ell mn\pm}(t-r_*) + i_{\ell mn\pm}(t-r_*)\right]$$



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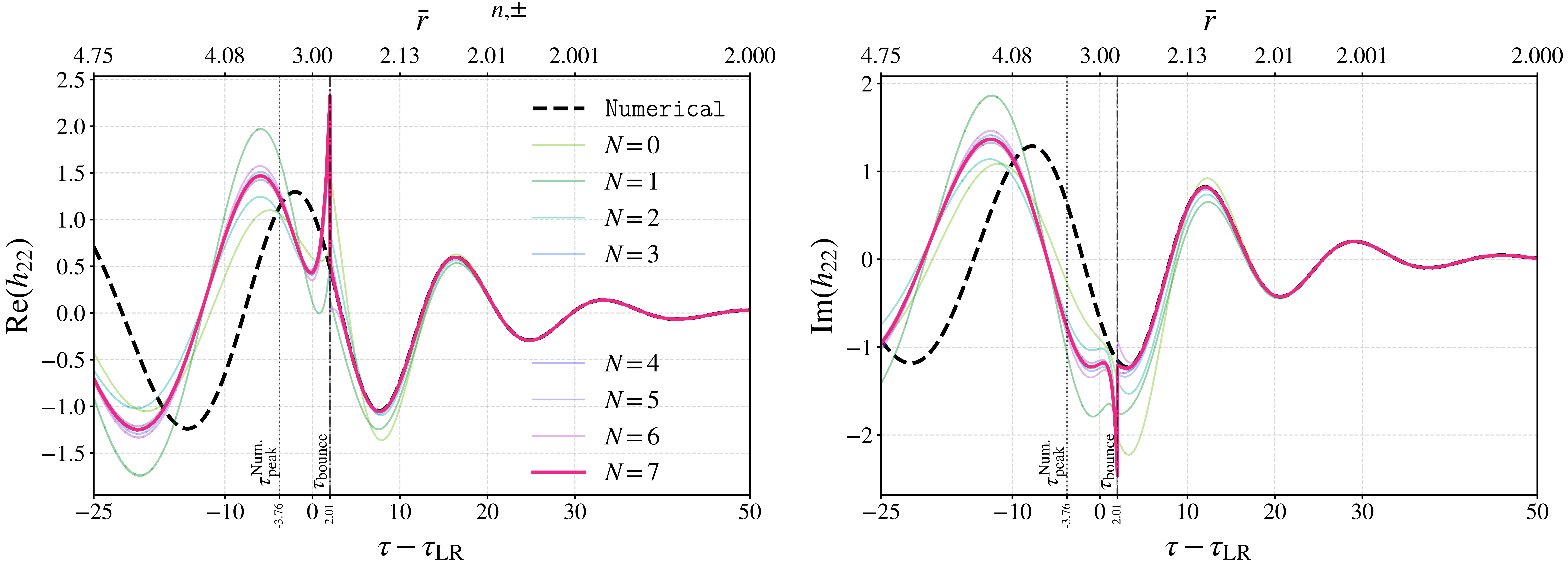

- After bounce $\bar{r}_* = 0$ crossing, signal completely propagated by QNMs, ~ $8M$ after waveform peak

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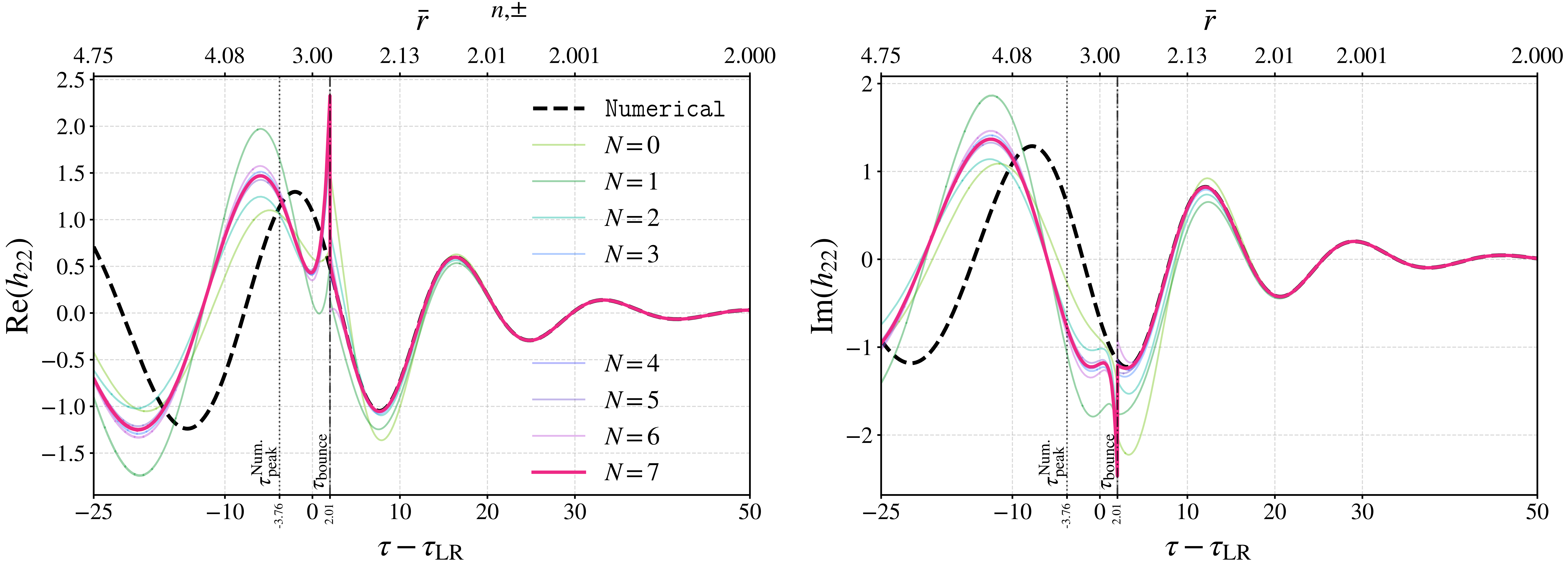

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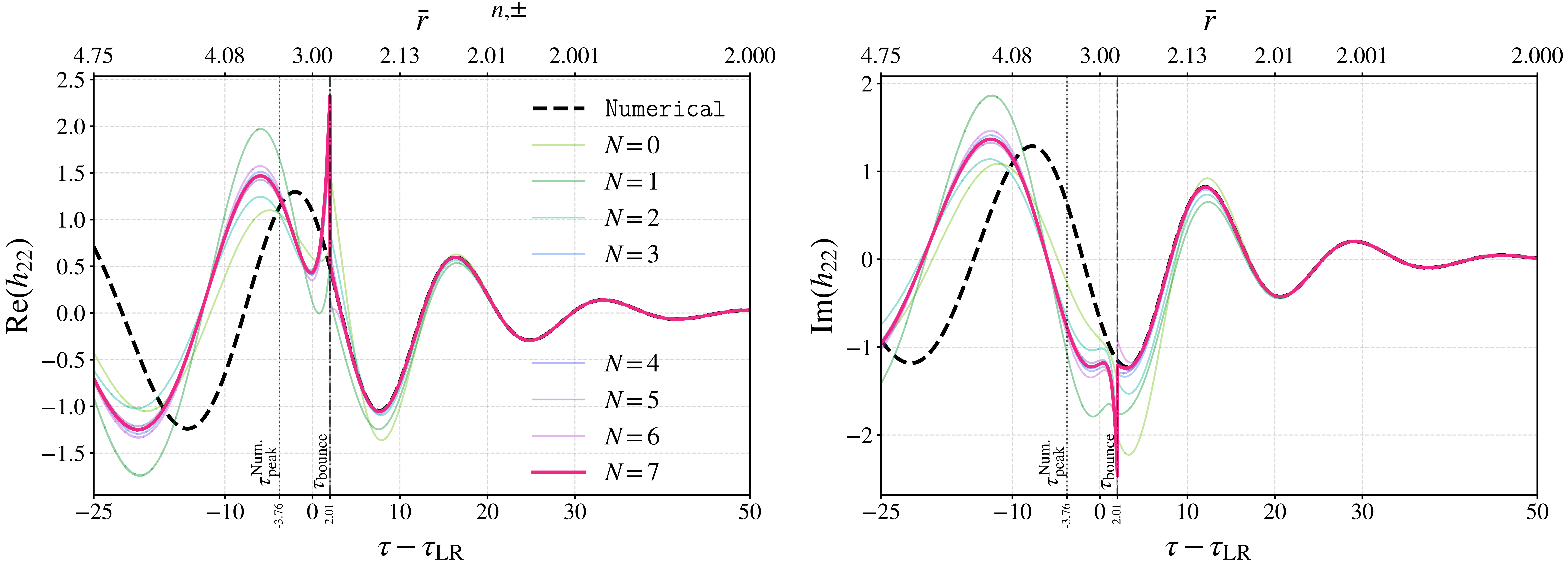

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- Higher overtones are more relevant during plunge
- Prompt needed before bounce $\bar{r}_* = 0$ crossing **[Ma, 2604.08680]**

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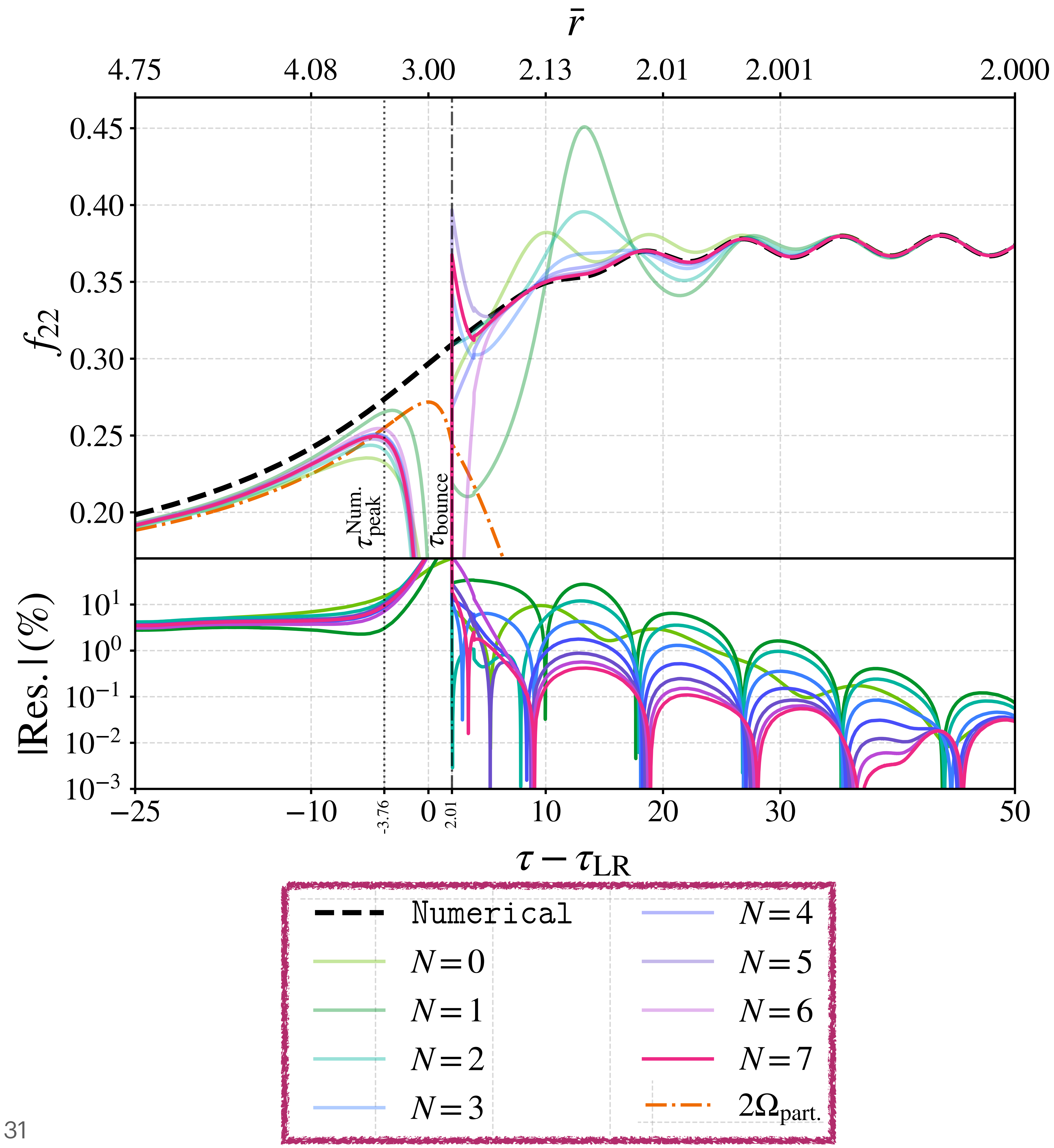
How QNMs are excited

[De Amicis+, 2506.21668]
[De Amicis+, 2605.16492]



$$\left[\partial_t^2 - \partial_{r_*}^2 + V_{\ell m}(r_*)\right]\Psi_{\ell m}(t, r_*) = e^{-im\varphi(t)}F_{\ell m}(t, r_*)$$

- Frequency driven by particle orbital frequency until bounce $\bar{r}_* = 0$ crossing

  $\longrightarrow$ system behaves as driven oscillator

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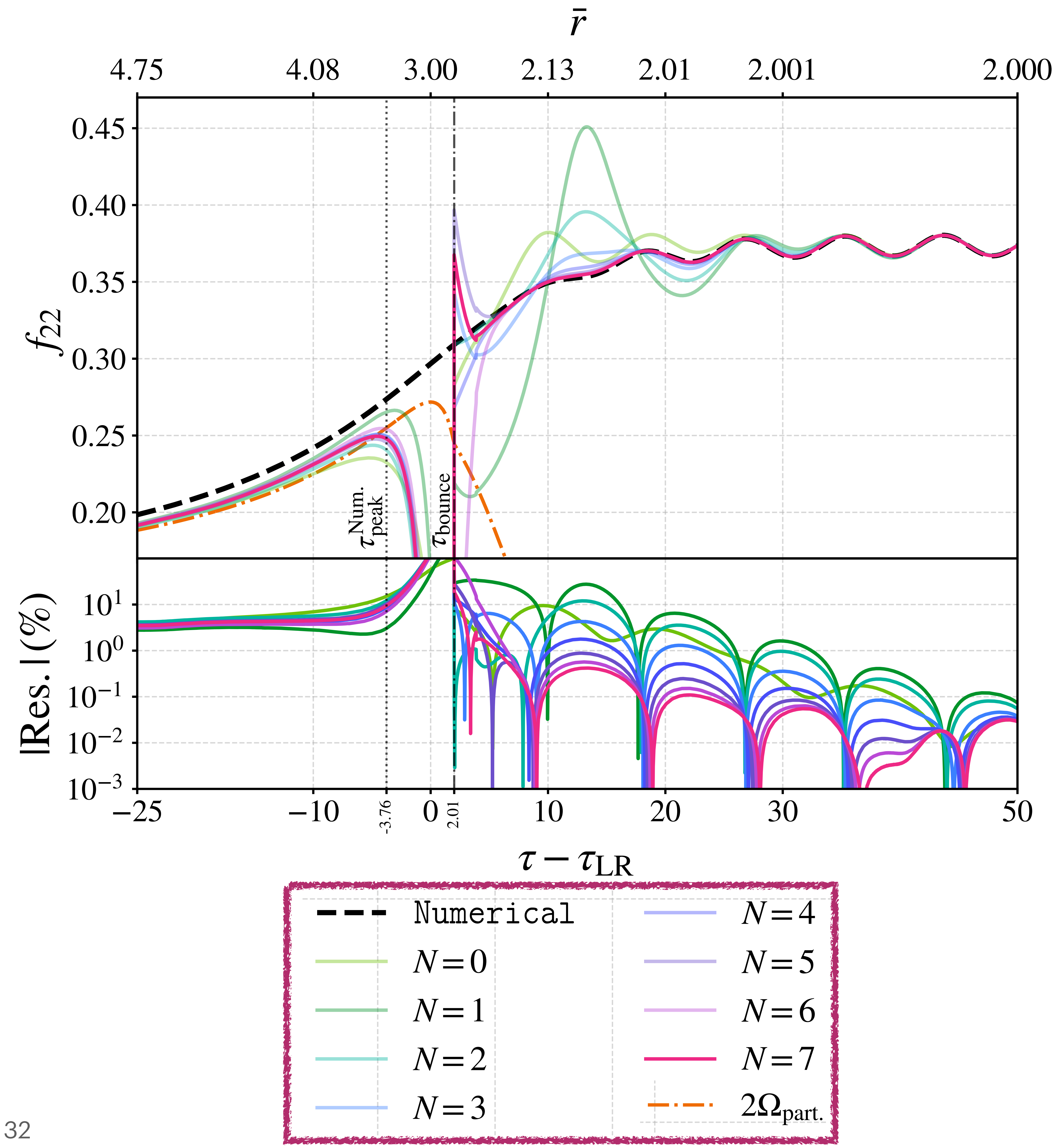
How QNMs are excited

[De Amicis+, 2506.21668]
[De Amicis+, 2605.16492]



$$\left[\partial_t^2 - \partial_{r_*}^2 + V_{\ell m}(r_*)\right]\Psi_{\ell m}(t, r_*) = e^{-im\varphi(t)}F_{\ell m}(t, r_*)$$

- Frequency driven by particle orbital frequency until bounce $\bar{r}_* = 0$ crossing

  $\longrightarrow$ system behaves as driven oscillator

- After QNMs excitation behaves as free oscillator + redshift as particular solution

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Ringdown models: refined version

How we can improve it

Stationary ringdown template:

$$h_{\ell m}(\tau) = \theta(\tau - \tau_{\rm start}) \sum_{n,\pm} \cancel{A_{\ell mn\pm}} e^{-i\omega_{\ell mn\pm}(\tau-\tau_{\rm start})}$$

Work from **perturbative**, **extreme mass-ratio** limit

i) Model the **ringdown** as **dynamically excited** by the **two-body** problem

[De Amicis+, 2506.21668]
[De Amicis+, 2605.16492]

- $$\Psi^{\rm QNM}_{\ell m}(t - r_*) = \sum_{n,\pm} e^{-i\omega_{\ell mn\pm}(t-r_*)} B_{\ell mn\pm} \left[c_{\ell mn\pm}(t - r_*) + i_{\ell mn\pm}(t - r_*)\right]$$

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- $\boxed{\text{What can we say about the stationary regime?}}$

Activation and impulsive coefficients

[De Amicis+, 2506.21668]

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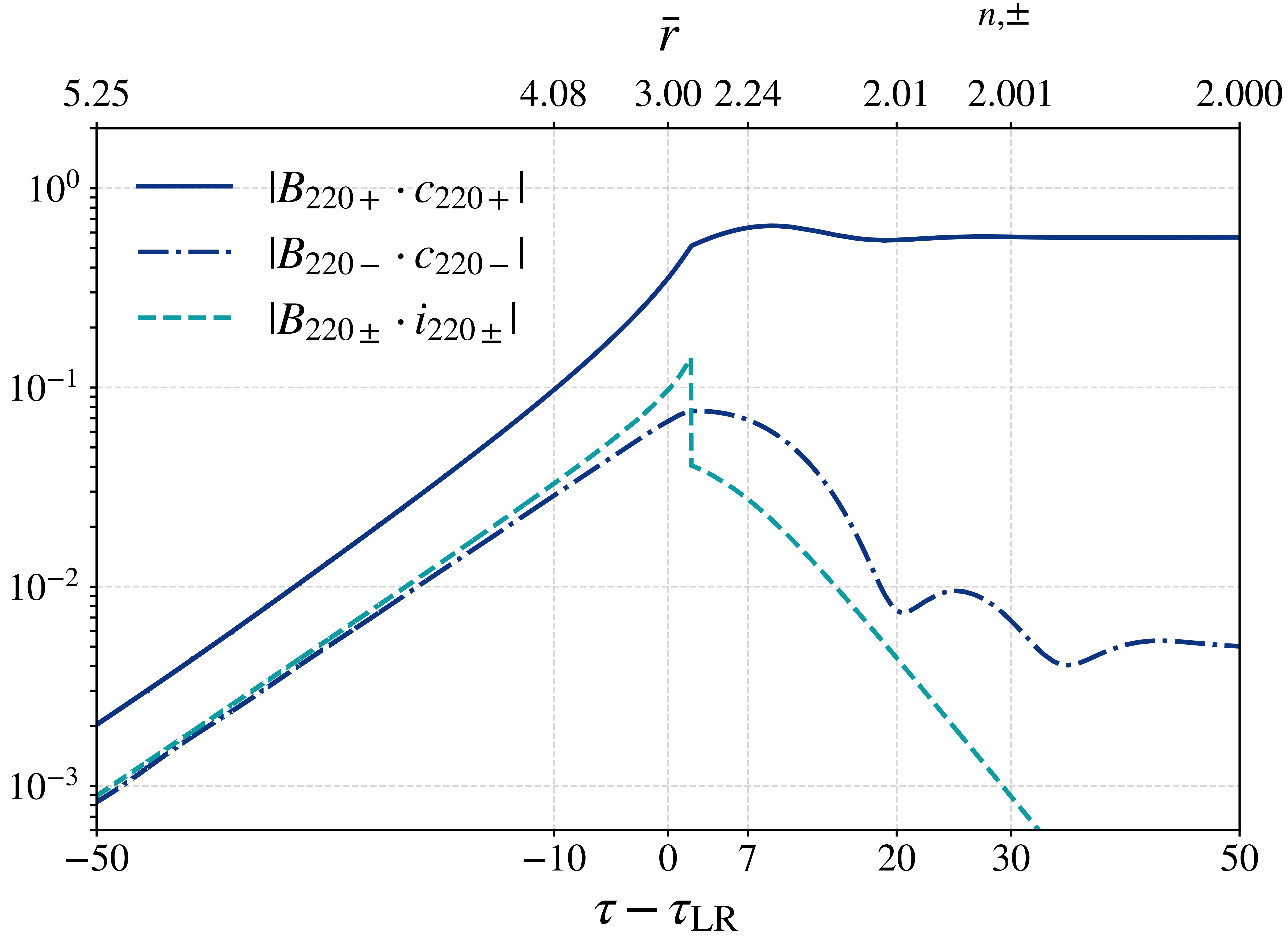

Fundamental mode:

- Activation saturates to a constant at late times
- Impulsive vanishes at late times

Activation and impulsive coefficients

[De Amicis+, 2506.21668]

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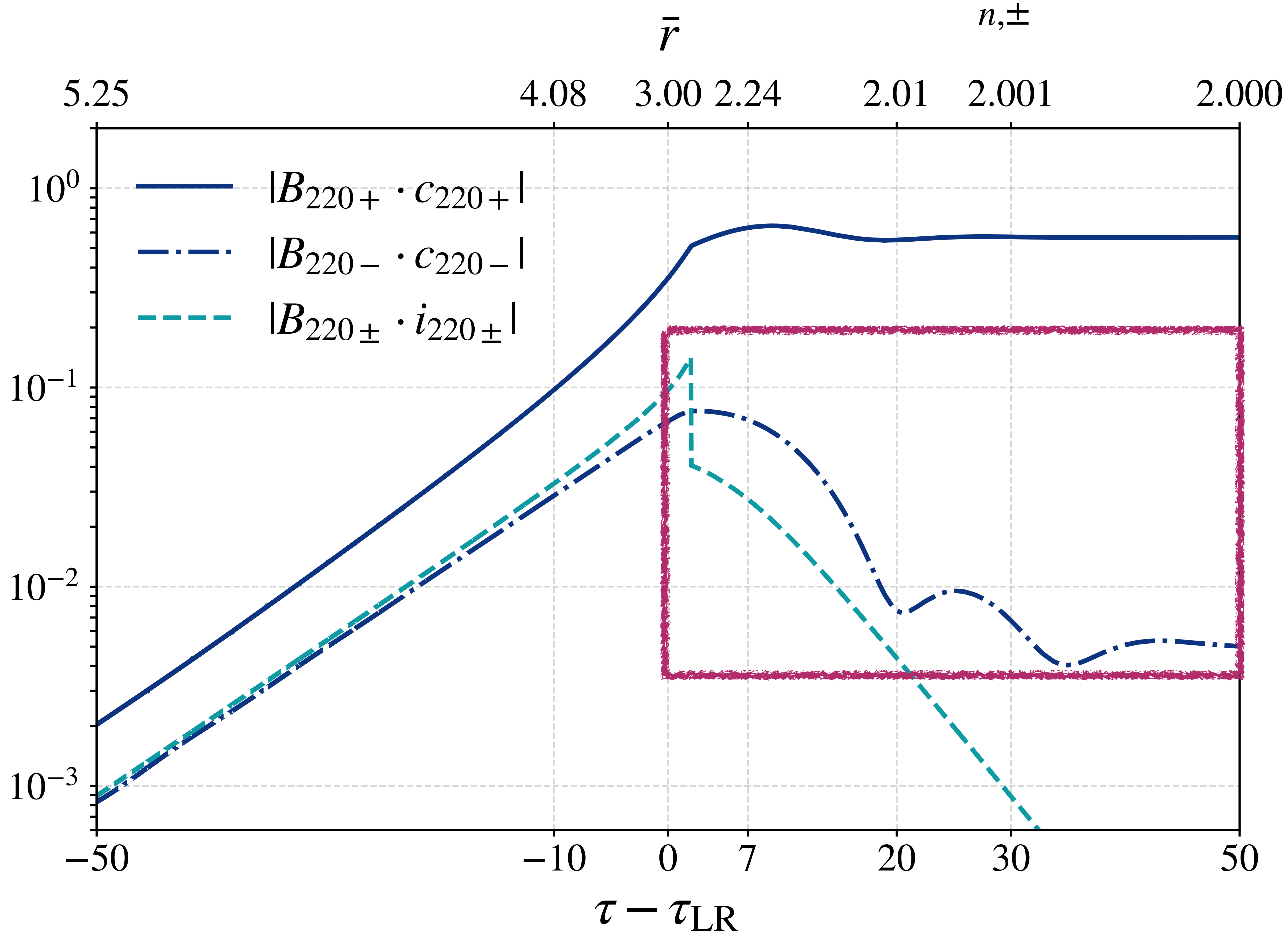

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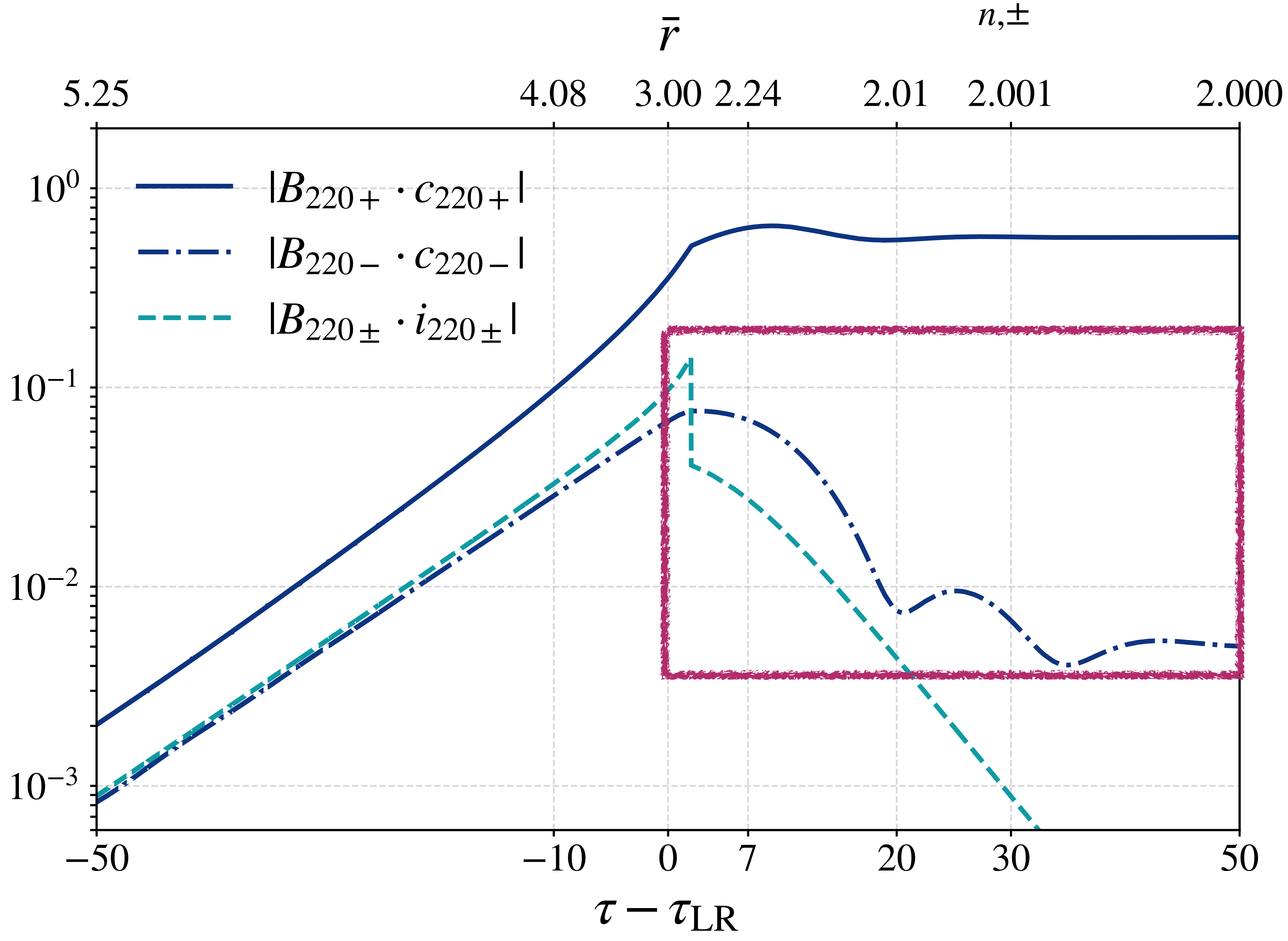

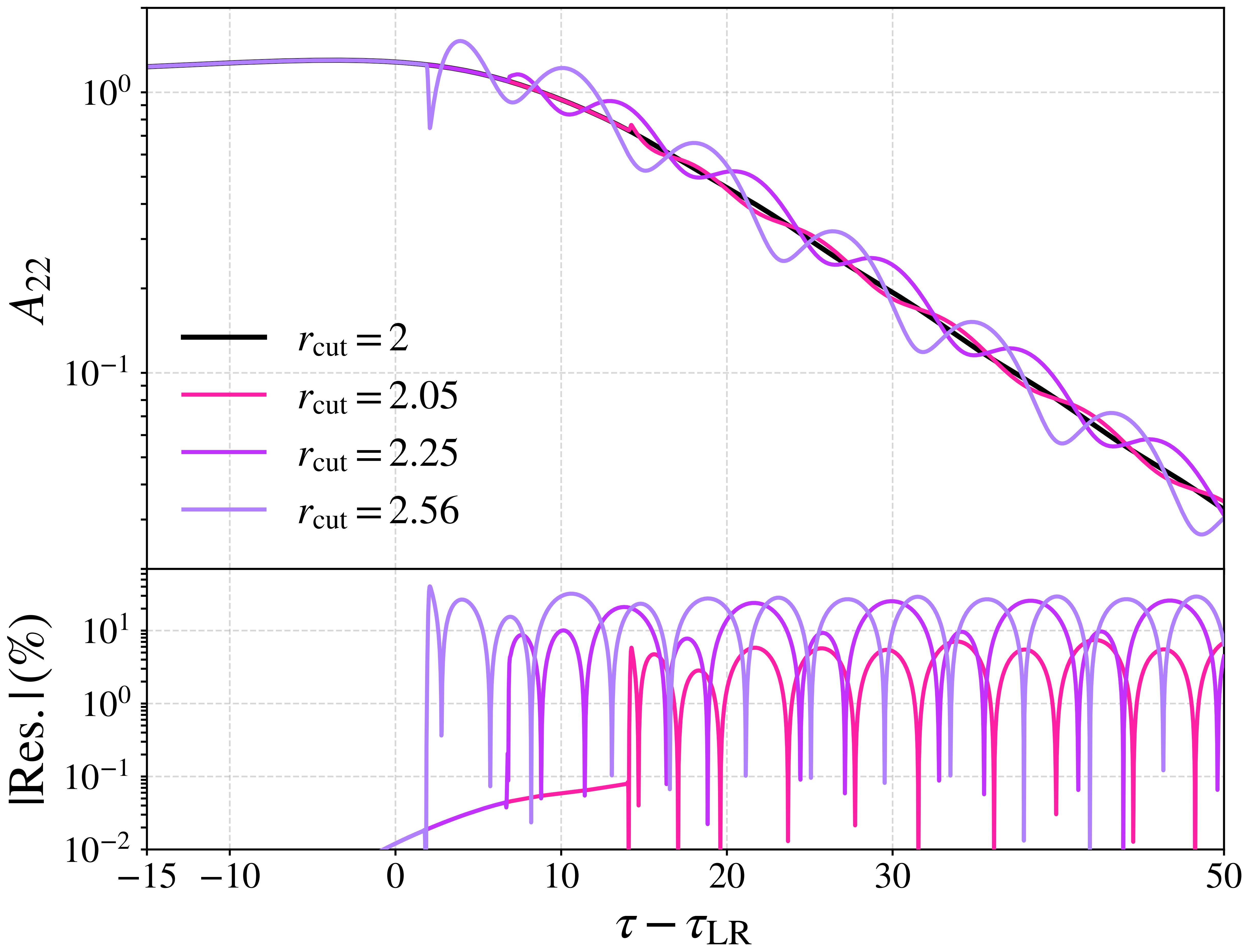

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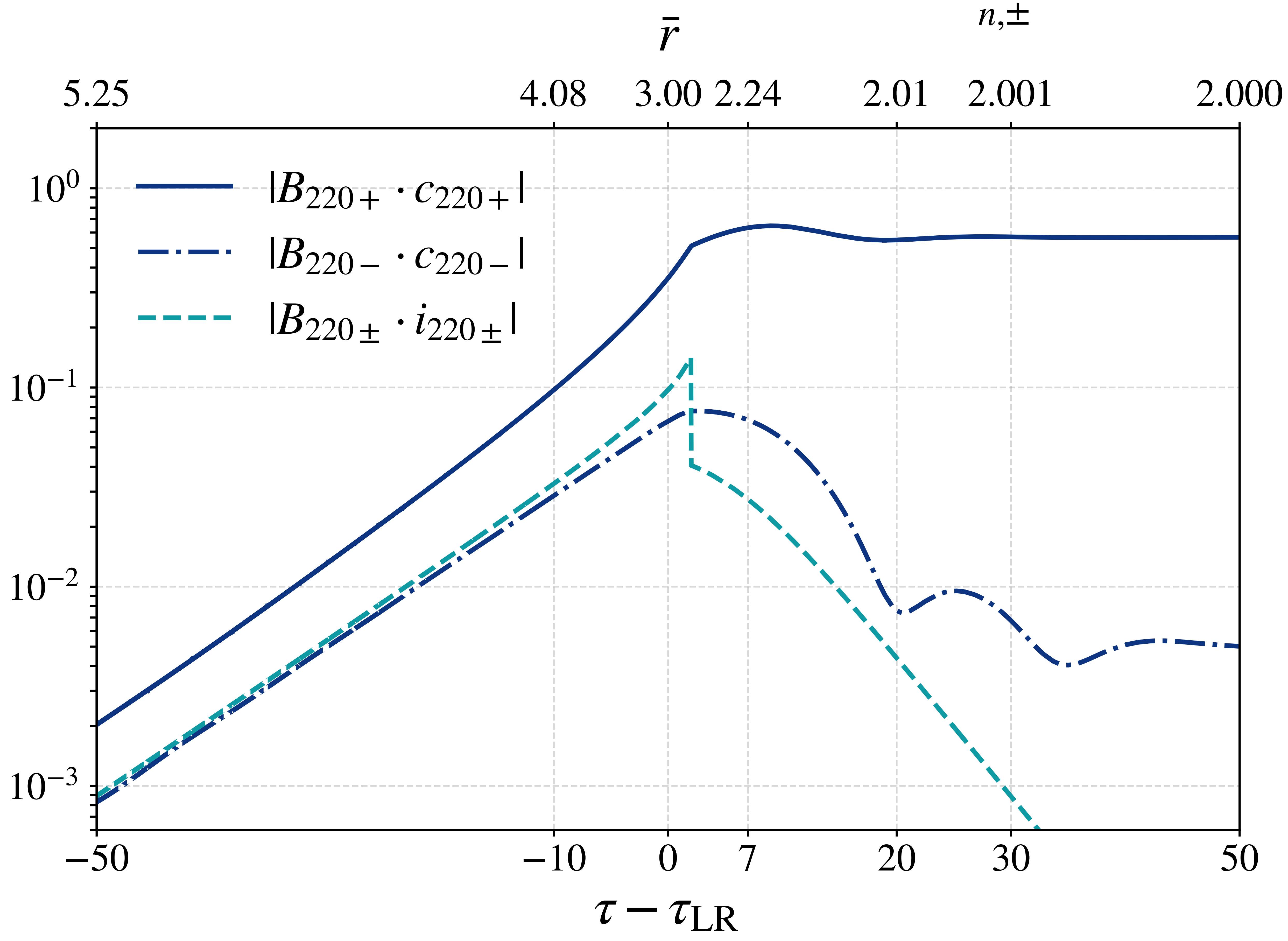

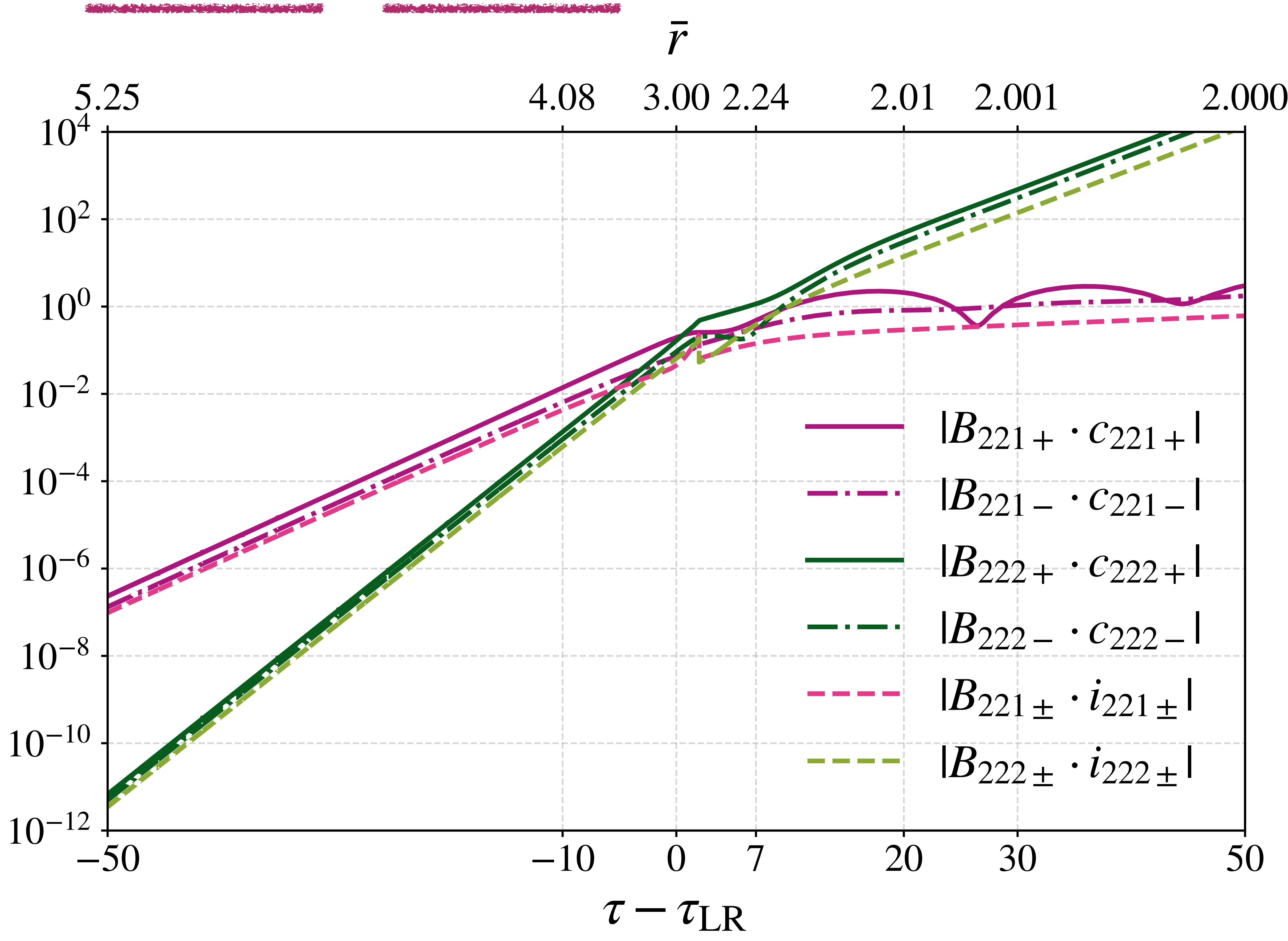

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Overtones:

- Activation and impulsive grow exponentially at late times

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Activation and impulsive coefficients

[De Amicis+, 2506.21668]

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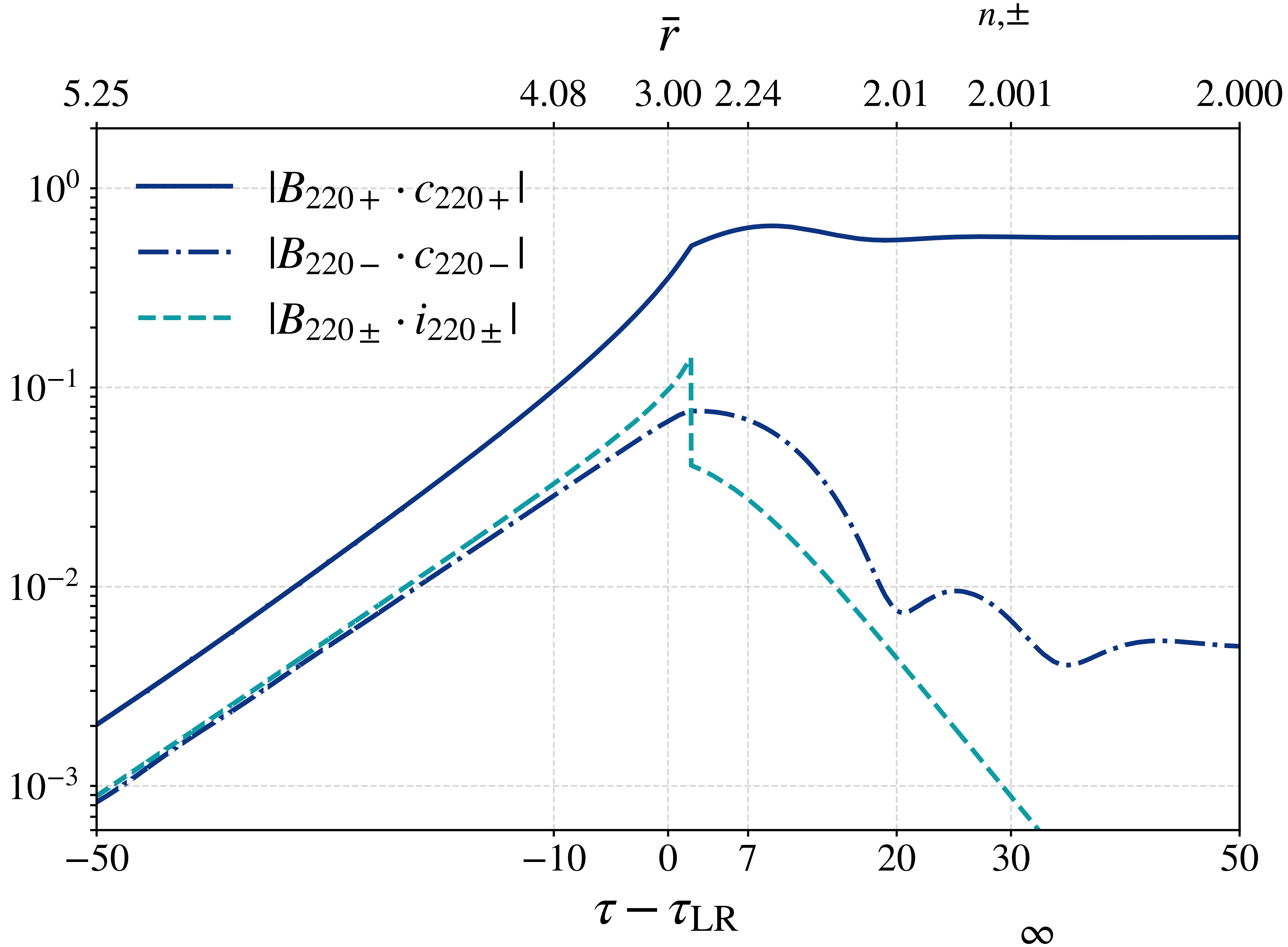

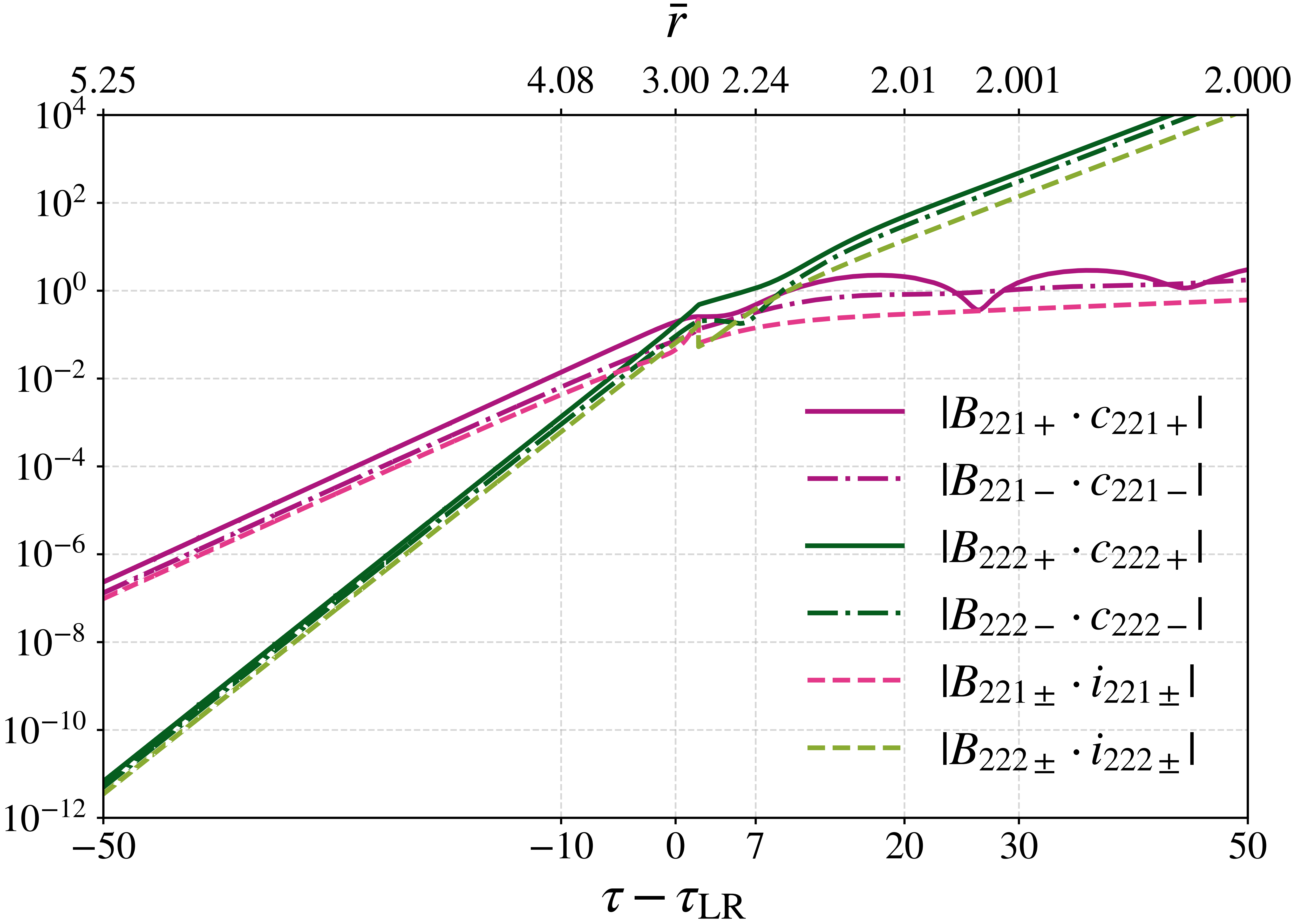

$$c^{\bar{r}<r_{\rm LR}}_{\ell mn\pm}(t-r_*) \simeq A^{\rm hereditary}_{\ell mn\pm} + A^{\rm near-horizon}_{\ell mn\pm} - \sum_{j=0}^{\infty} a_{j,\ell mn\pm} e^{-\frac{j+1}{2r_h}(t-r_*)} e^{i\omega_{\ell mn\pm}(t-r_*)}$$

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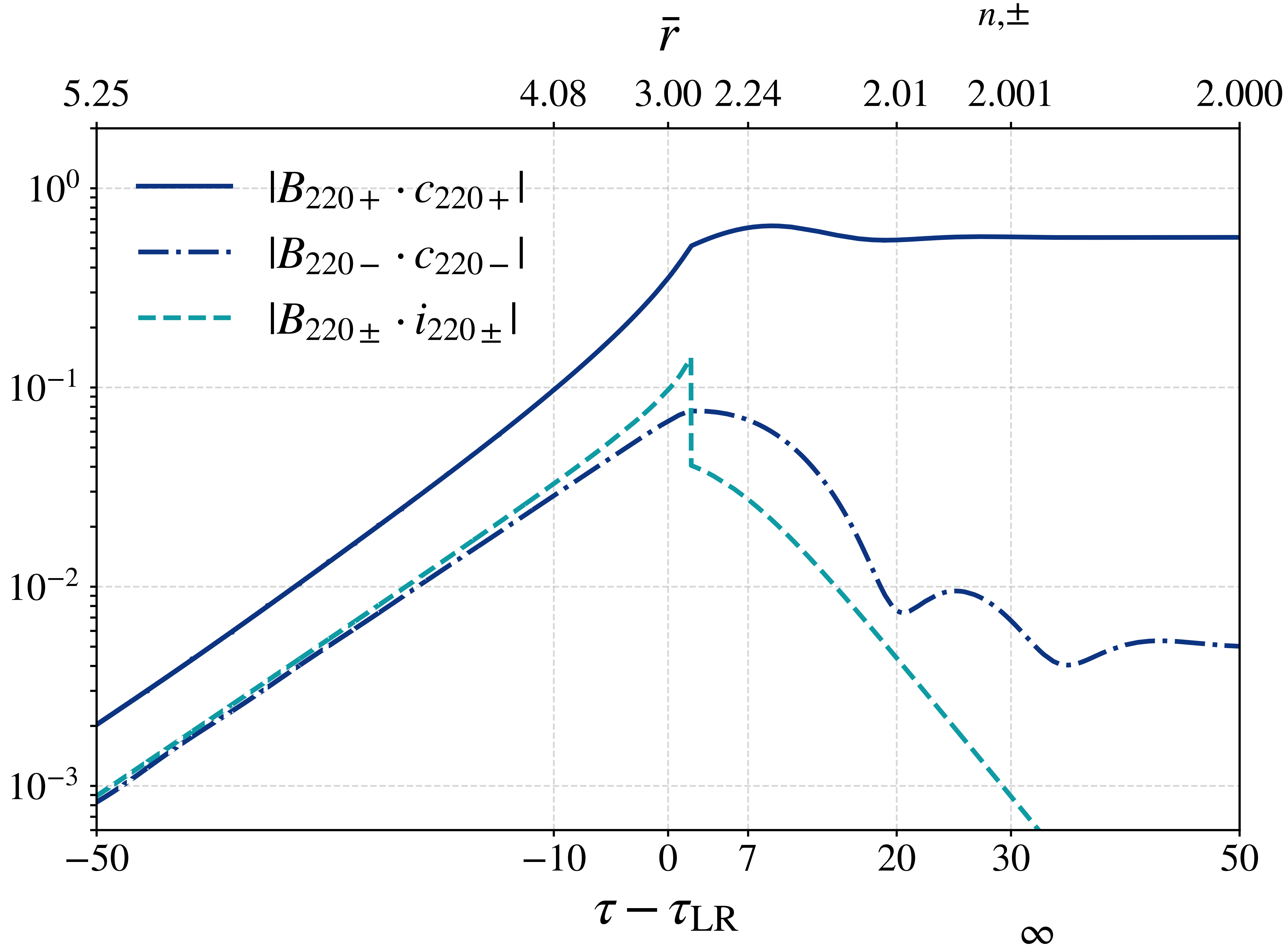

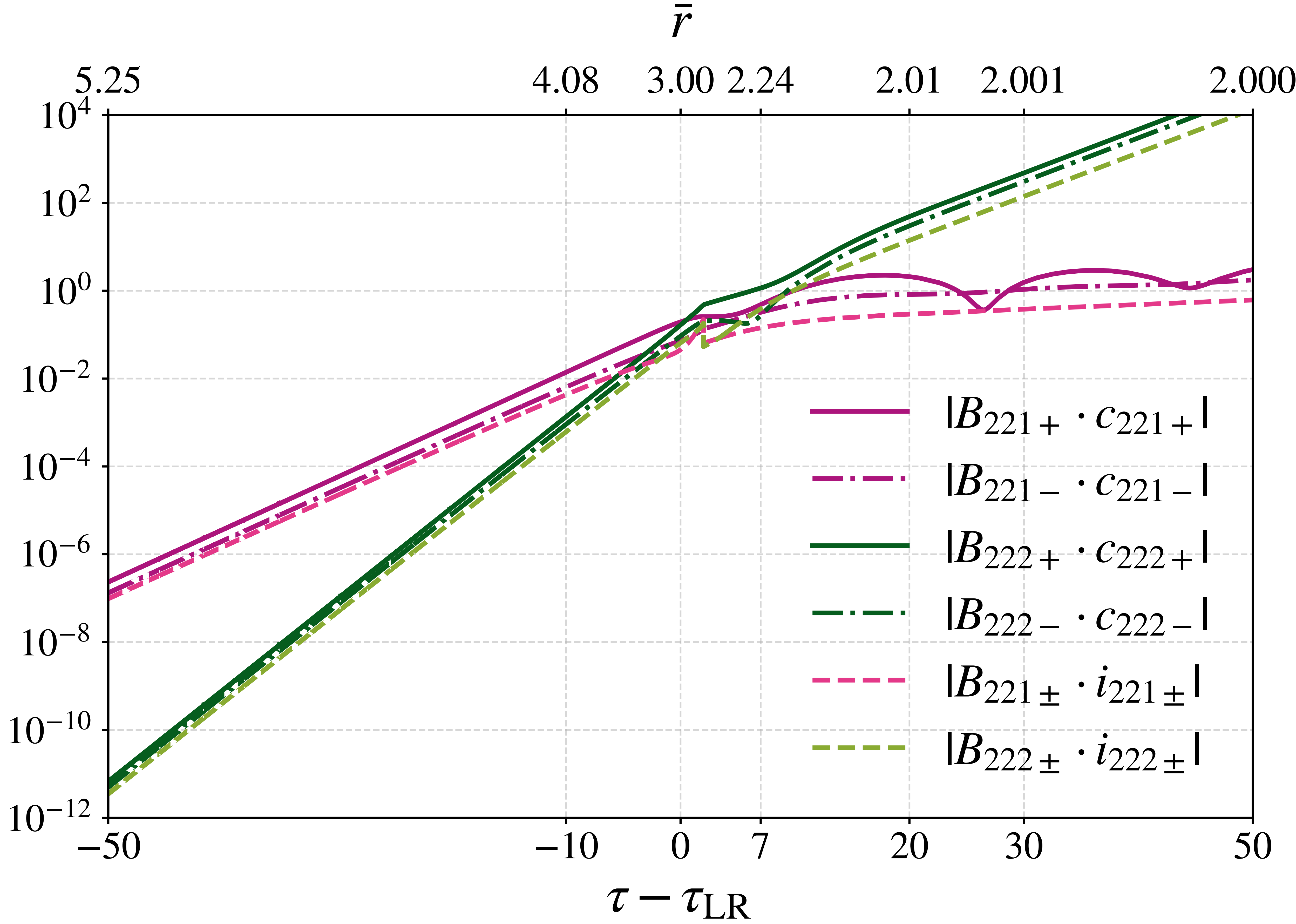

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Activation and impulsive coefficients

[De Amicis+, 2506.21668]

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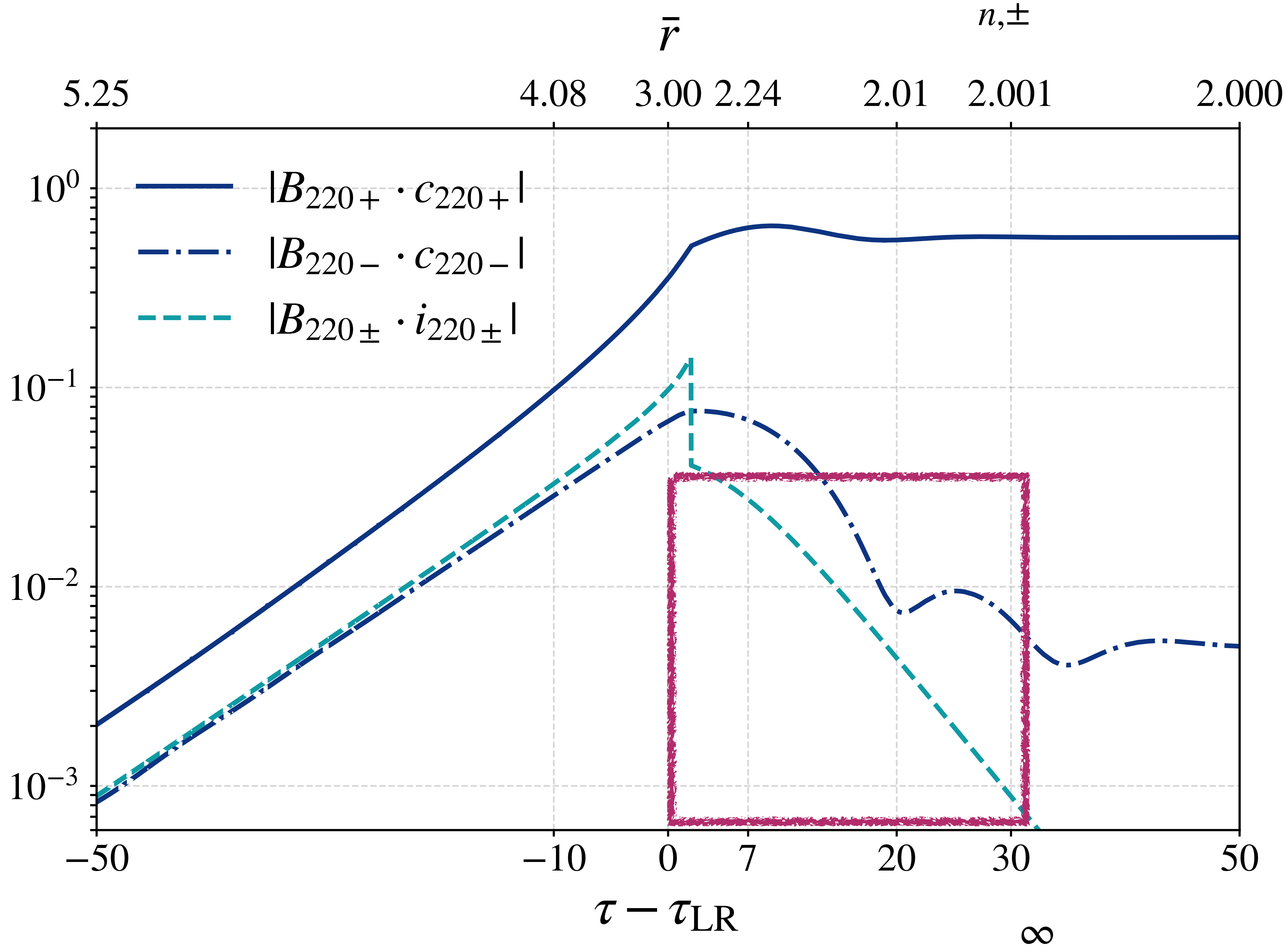

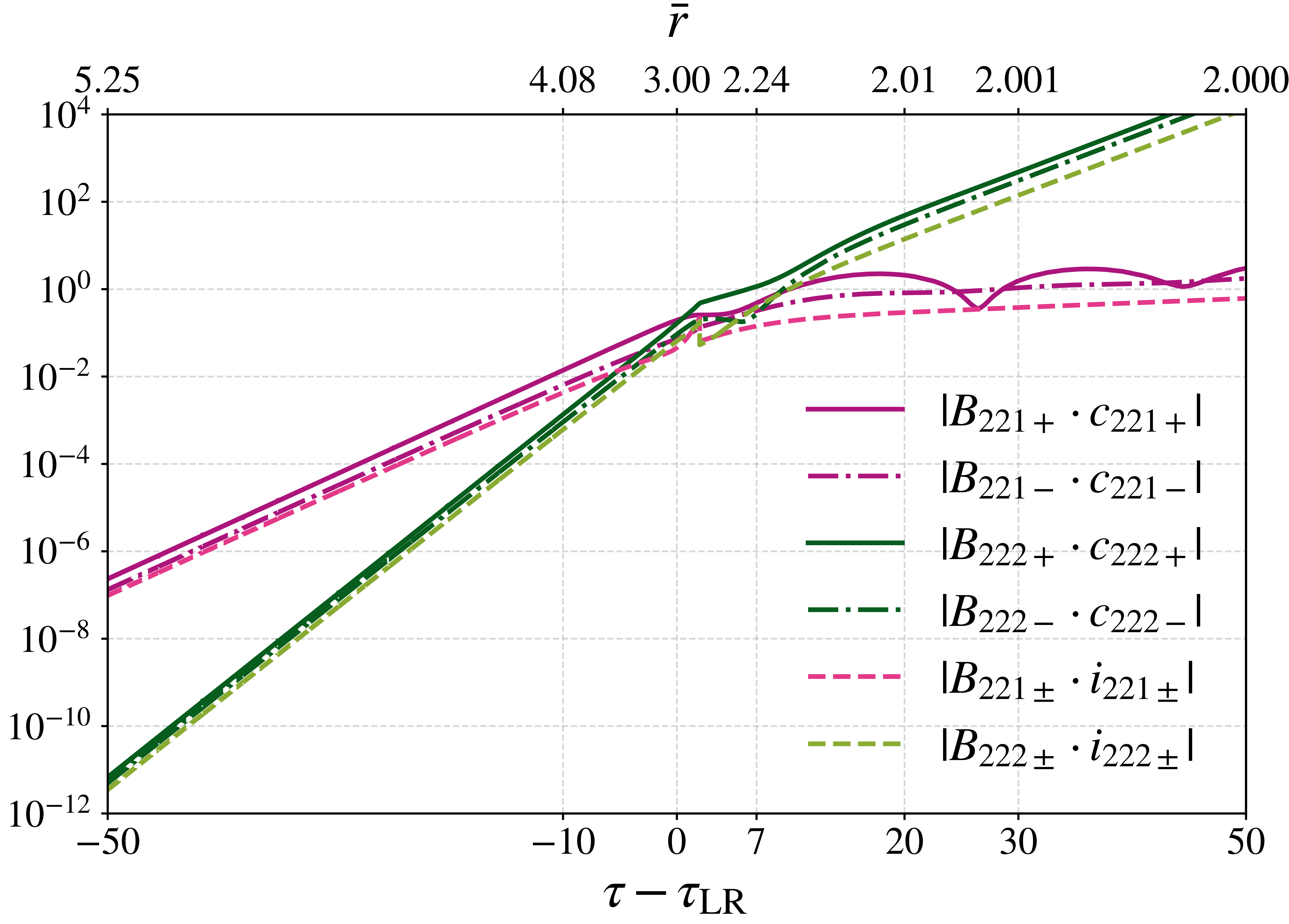

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Activation and impulsive coefficients

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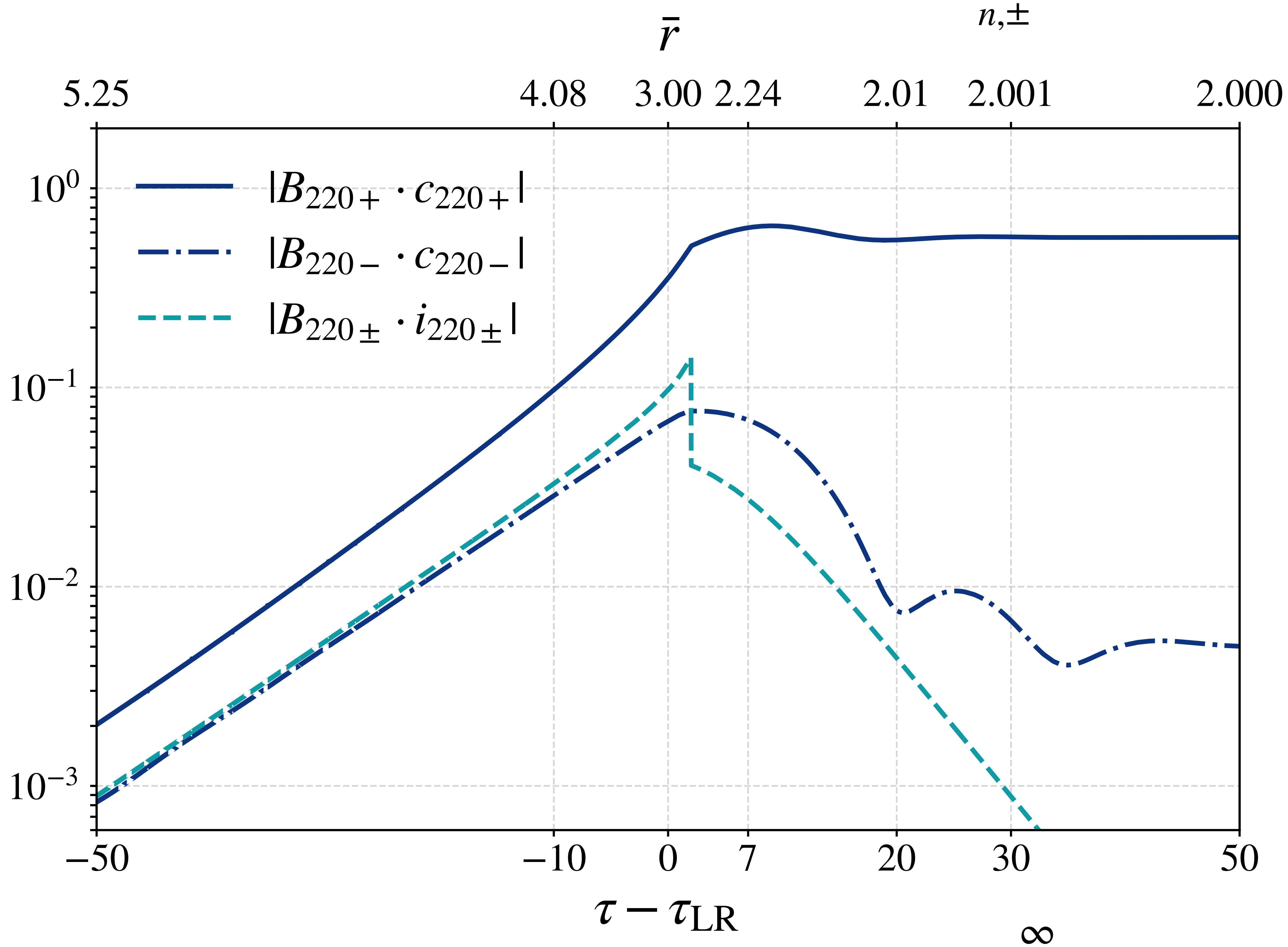

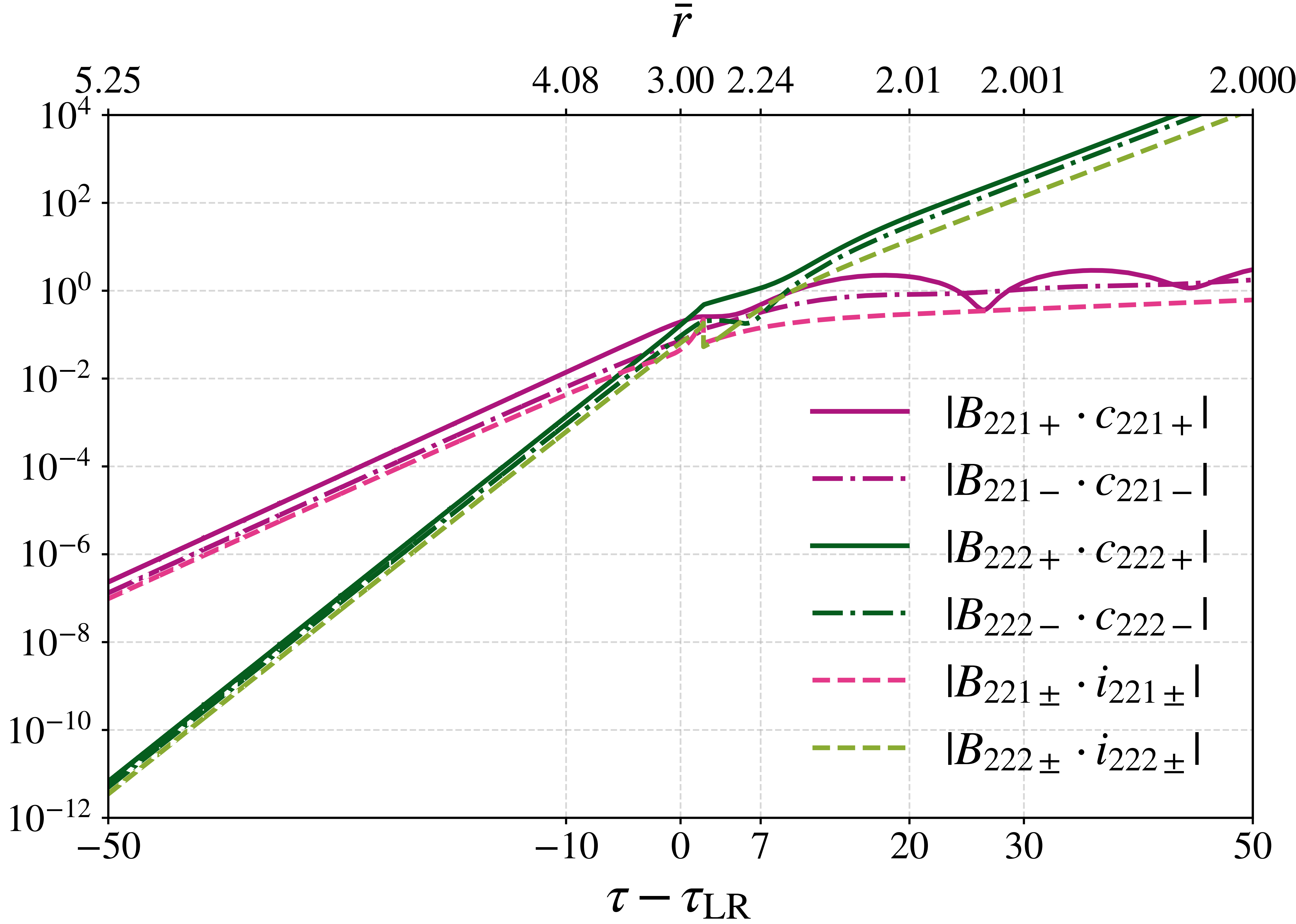

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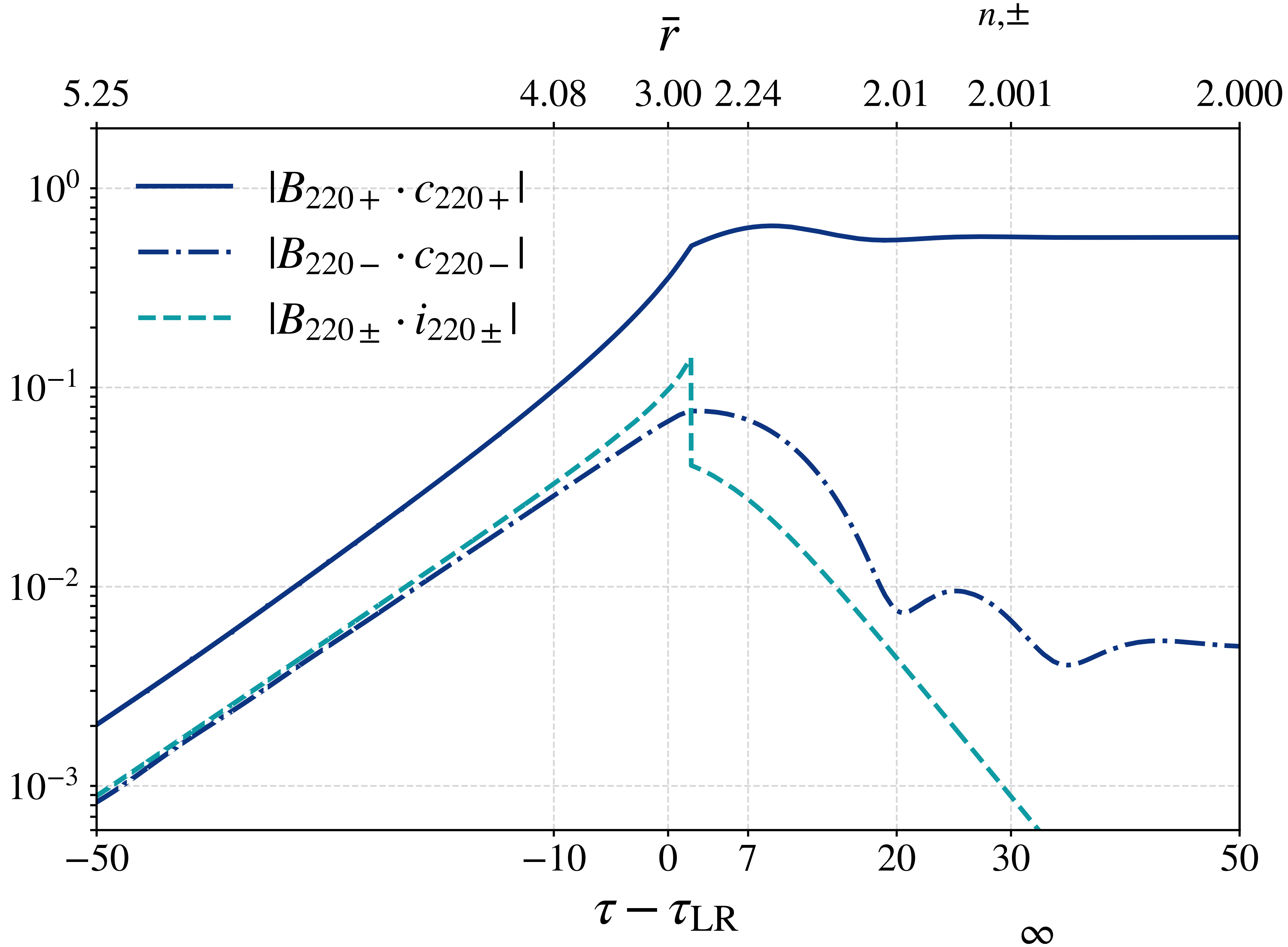

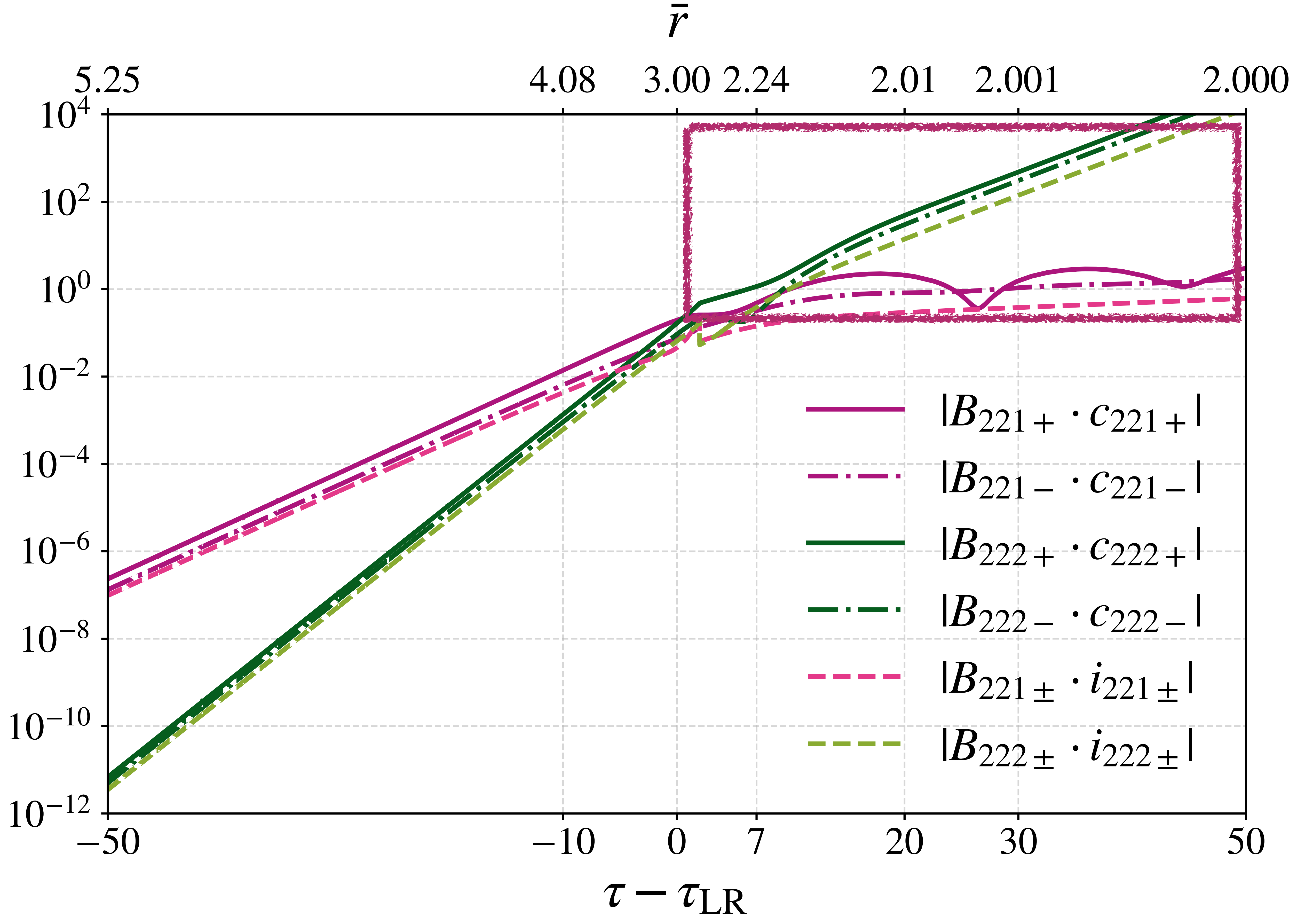

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QNMs propagated signal: late times

[De Amicis+, 2506.21668]

- Late times / near horizon expansion

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- Redshift identically cancelled once summing over all QNM contributions

[Kuntz+,2606.07173]

Ringdown models: refined version

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ii) **Constant-amplitude** model only valid at **late times,** clear dependence on **inspiral 2-body**

[De Amicis+, 2506.21668]

$$\Psi^{\text{QNM, late times}}_{\ell m}(t - r_*) = \sum_{n\pm} \left(A^{\text{hereditary}}_{\ell mn\pm} + A^{\text{near-horizon}}_{\ell mn\pm}\right) e^{-i\omega_{\ell mn\pm}(t-r_*)}$$

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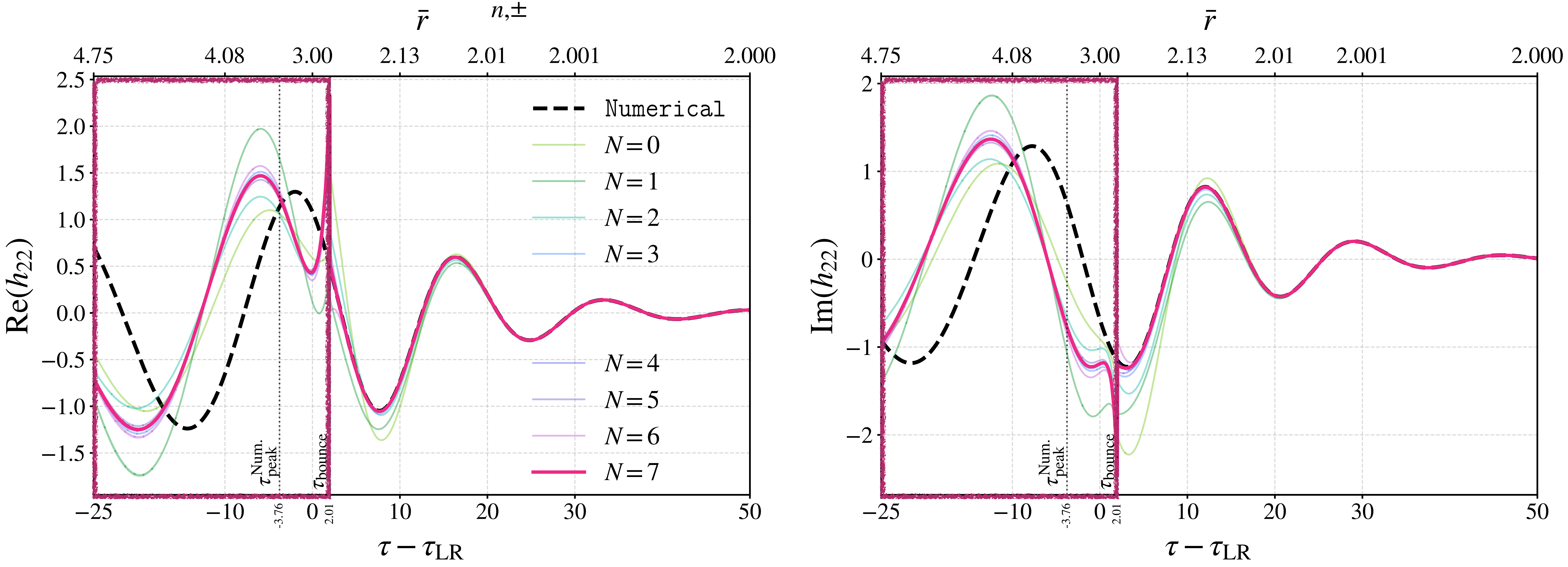
iii) **Other linear contributions** at **early** times

[Kuntz, 2510.17954]
[Arnaudo+, 2510.18956]
[De Amicis+, 2601.11706]
[Su, 2601.22015]
[Ma, 2604.08680]

QNMs propagated signal: plunge-merger-ringdown

$$\Psi^{\rm QNM}_{\ell m} = \sum_{n,\pm}^{N} e^{-i\omega_{\ell mn\pm}(t-r_*)} B_{\ell mn\pm} \left[c_{\ell mn\pm}(t-r_*) + i_{\ell mn\pm}(t-r_*)\right]$$



- Prompt contribution needed before bounce $\bar{r}_* = 0$ crossing

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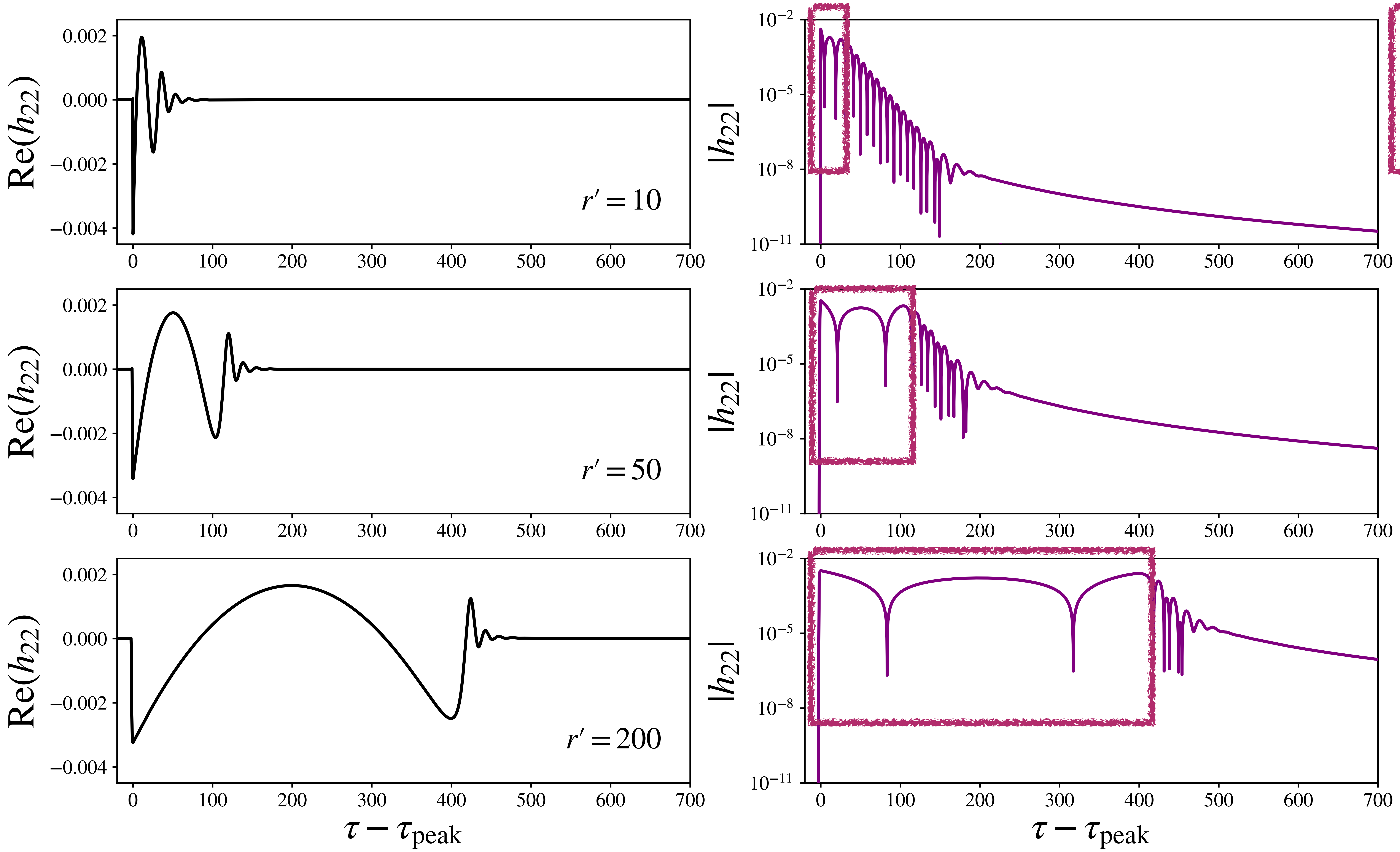
Basic tool: Green's function

Green's function: $\left[\partial_t^2 - \partial_{r_*}^2 + V_{\ell m}(r_*)\right] G_{\ell m}(t, t'; r_*, r_*') = \delta(t - t')\delta(r_* - r_*')$

Dictates how perturbations propagate on the curved background



Early times → prompt response:

- longer the further r_*'

→ Need a closed form model

[Kuntz, 2510.17954]
[Arnaudo+, 2510.18956]
[De Amicis, Cannizzaro, 2601.11706]
[Su+, 2601.22015]
[Ma, 2604.08680]
[Motohashi+, 2605.01964]

Some useful equations

[De Amicis+, 2601.11706]

$$G_{\ell m,\, r_*>r'_*}(t-t'; r_*, r'_*) = \frac{i}{2\pi}\int_{-\infty}^{\infty} d\omega\, e^{-i\omega(t-r_*-t')}\tilde{G}_1(\omega; r_*, r'_*) + \frac{i}{2\pi}\int_{-\infty}^{\infty} d\omega\, e^{-i\omega(t-r_*-t')}\tilde{G}_2(\omega; r_*, r'_*)$$

Some useful equations

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- **Post-Minkowskian** limit $|\omega| \ll 1$

$$\tilde{G}_1(\omega;r_*,r_*') \approx \underline{\sum_{j=1}^{\ell} A_1^{(j)}(\omega;r_*,r_*')\,\omega^{-1-j}} + \underline{\mathcal{O}[\log(i\omega G)]} + \mathcal{O}[(\omega G)^0]$$

Simple poles of order up to $\ell+1$

Branch cut on positive imaginary axis

$$\tilde{G}_2(\omega;r_*,r_*') \approx \underline{\sum_{j=1}^{\ell} A_2^{(j)}(\omega;r_*,r_*')\,\omega^{-1-j}} + \underline{\mathcal{O}[\log(i\omega G)]} + \underline{\mathcal{O}[\log(-i\omega G)]} + \mathcal{O}[(\omega G)^0]$$

Branch cut on negative imaginary axis

Some useful equations

[De Amicis+, 2601.11706]

$$G_{\ell m,\, r_*>r'_*}(t-t';r_*,r'_*) = \frac{i}{2\pi}\int_{-\infty}^{\infty} d\omega\, e^{-i\omega(t-r_*-t')}\tilde{G}_1(\omega;r_*,r'_*) + \frac{i}{2\pi}\int_{-\infty}^{\infty} d\omega\, e^{-i\omega(t-r_*-t')}\tilde{G}_2(\omega;r_*,r'_*)$$

- **Post-Minkowskian** limit $|\omega| \ll 1$

$$\tilde{G}_1(\omega;r_*,r'_*) \approx \boxed{\underline{\sum_{j=1}^{\ell} A_1^{(j)}(\omega;r_*,r'_*)\,\omega^{-1-j}}} + \underline{\mathcal{O}[\log(i\omega G)]} + \mathcal{O}[(\omega G)^0]$$

Simple poles of order up to $\ell+1$

Branch cut on positive imaginary axis

$$\tilde{G}_2(\omega;r_*,r'_*) \approx \boxed{\underline{\sum_{j=1}^{\ell} A_2^{(j)}(\omega;r_*,r'_*)\,\omega^{-1-j}}} + \underline{\mathcal{O}[\log(i\omega G)]} + \underline{\mathcal{O}[\log(-i\omega G)]} + \mathcal{O}[(\omega G)^0]$$

Branch cut on negative imaginary axis

Some useful equations

[De Amicis+, 2601.11706]

$$G_{\ell m,\, r_*>r'_*}(t-t'; r_*, r'_*) = \frac{i}{2\pi}\int_{-\infty}^{\infty} d\omega\, e^{-i\omega(t-r_*-t')}\tilde{G}_1(\omega; r_*, r'_*) + \frac{i}{2\pi}\int_{-\infty}^{\infty} d\omega\, e^{-i\omega(t-r_*-t')}\tilde{G}_2(\omega; r_*, r'_*)$$

- **Post-Minkowskian** limit $|\omega| \ll 1$

$$\tilde{G}_1(\omega; r_*, r'_*) \approx \underline{\sum_{j=1}^{\ell} A_1^{(j)}(\omega; r_*, r'_*)\,\omega^{-1-j}}$$

→ First to be activated $u > u'$

Simple poles of order up to $\ell + 1$

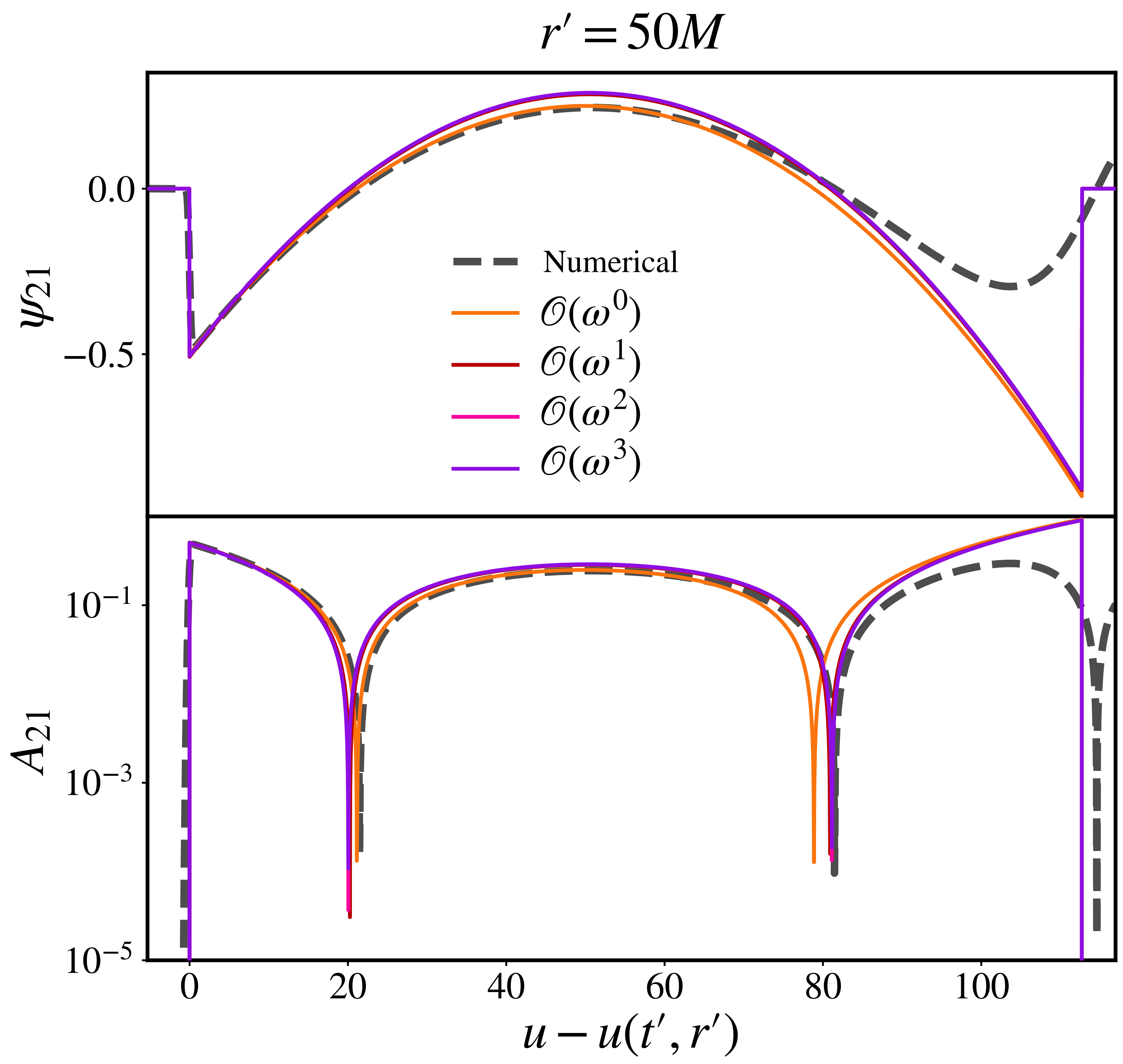
$$\tilde{G}_2(\omega; r_*, r'_*) \approx \underline{\sum_{j=1}^{\ell} A_2^{(j)}(\omega; r_*, r'_*)\,\omega^{-1-j}}$$

→ Activated later $u > v'$, cancels $\tilde{G}_1$ contribution

Corrected post-Minkowskian result

[De Amicis+, 2601.11706]

$$G^{\rm Pole}_{\ell=2\,m}(t-t';r_*,r'_*) = \theta(u-u')\theta(v'-u)\left[p_0(r') + p_1(r')(u-u') + p_2(r')(u-u')^2\right]$$



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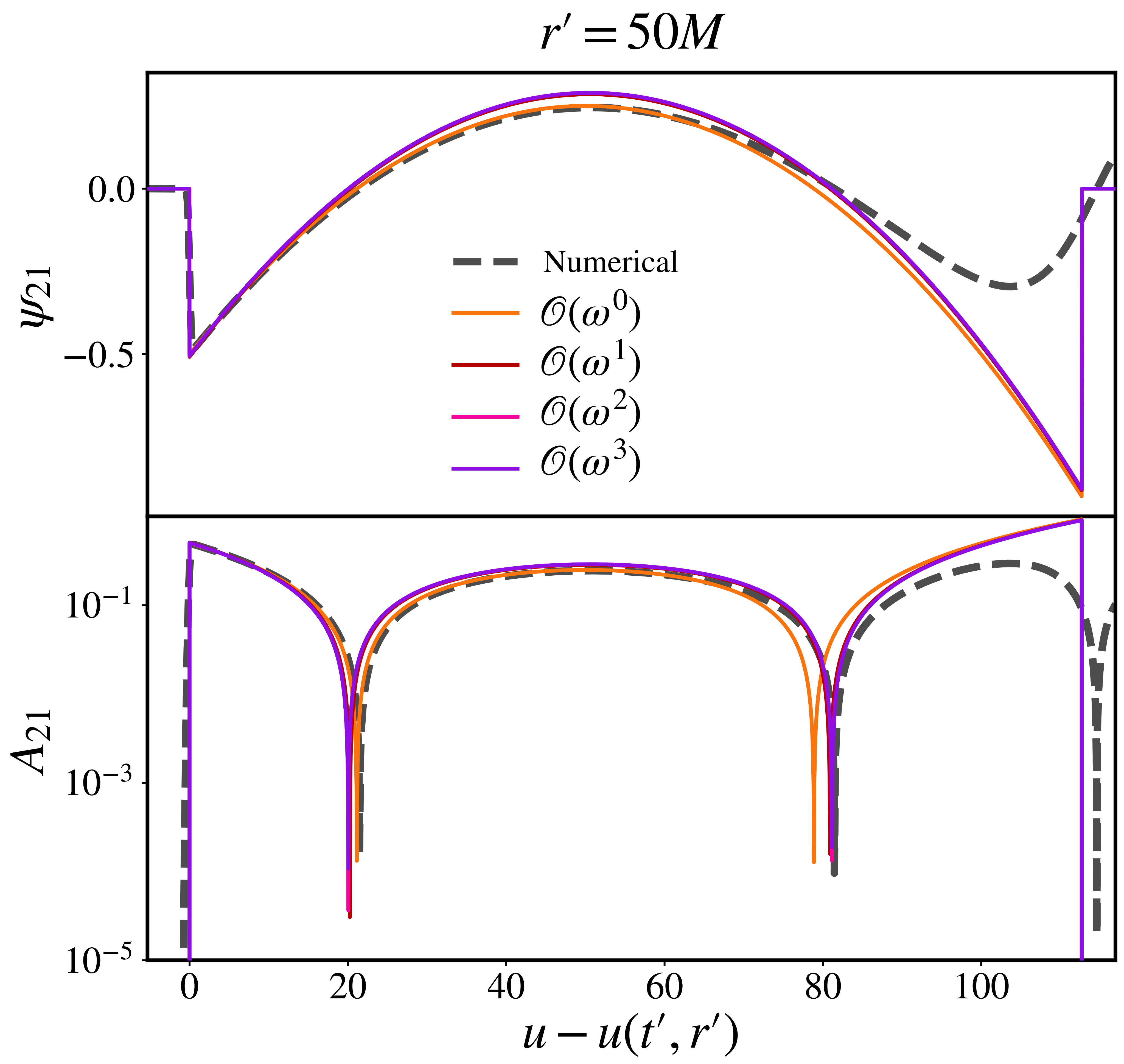
Corrected post-Minkowskian result

[De Amicis+, 2601.11706]

$$G^{\rm Pole}_{\ell=2\,m}(t-t';r_*,r'_*) = \theta(u-u')\theta(v'-u)\left[p_0(r') + p_1(r')(u-u') + p_2(r')(u-u')^2\right]$$



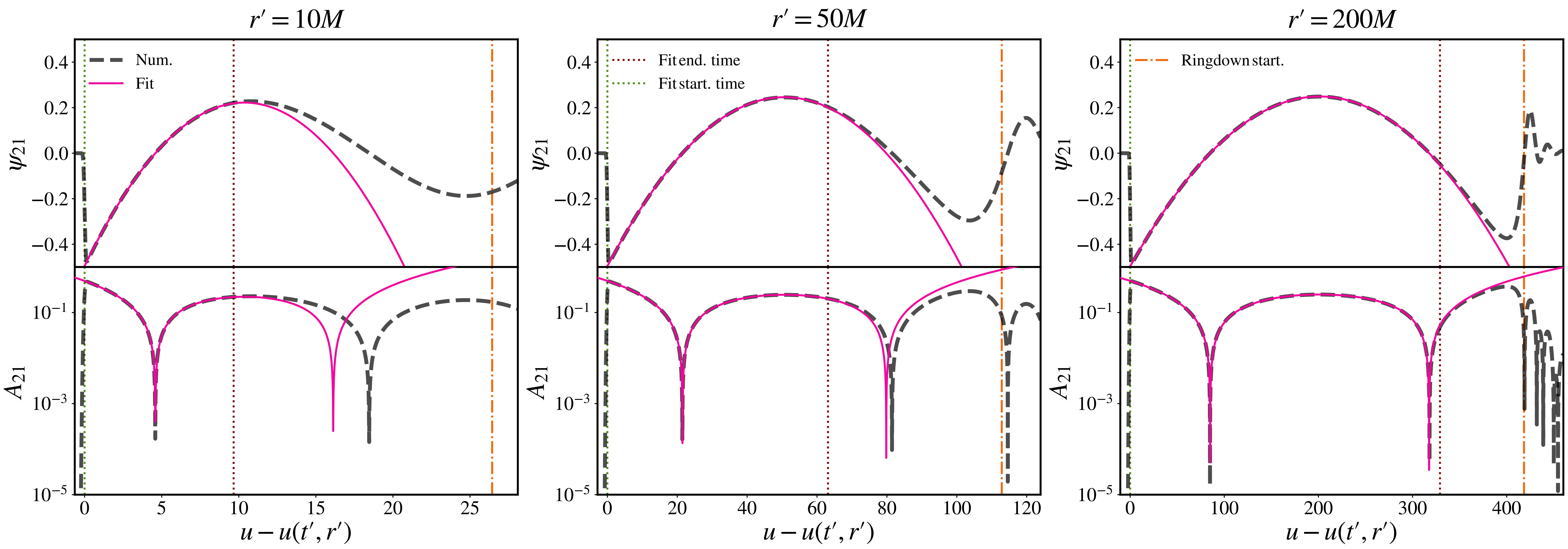
- Expansion of Coulomb wave function solution converges poorly
- Other spectral contribution to the prompt neglected

Small distances

[De Amicis+, 2601.11706]

- Is a polynomial in $u(t, r_*)$ still a good description? → Fit with $p_{0,1,2}(r')$ free parameters

$$G^{\text{Pole}}_{\ell=2\,m}(t-t'; r_*, r'_*) = \theta(u-u')\theta(v'-u)\left[p_0(r') + p_1(r')(u-u') + p_2(r')(u-u')^2\right]$$



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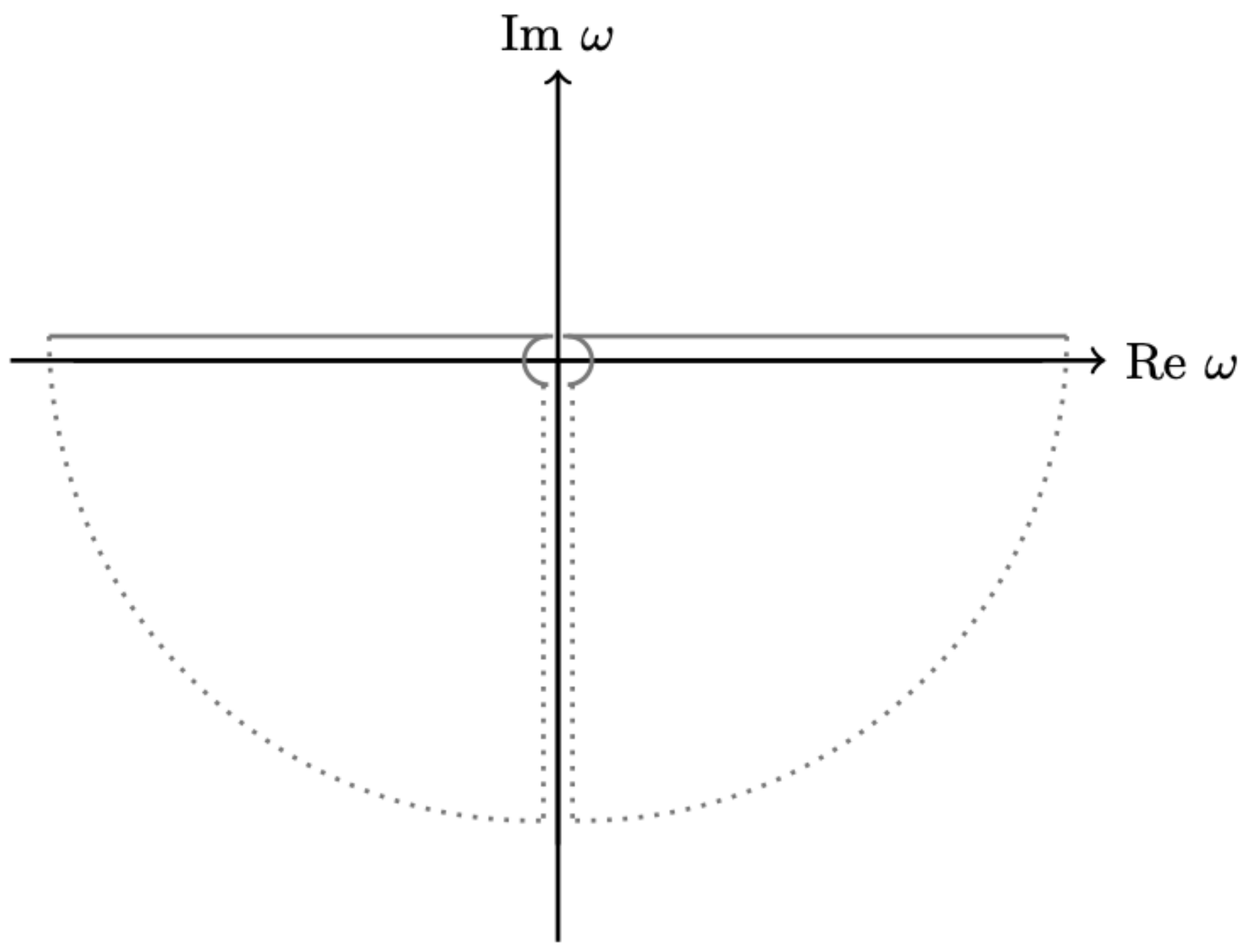
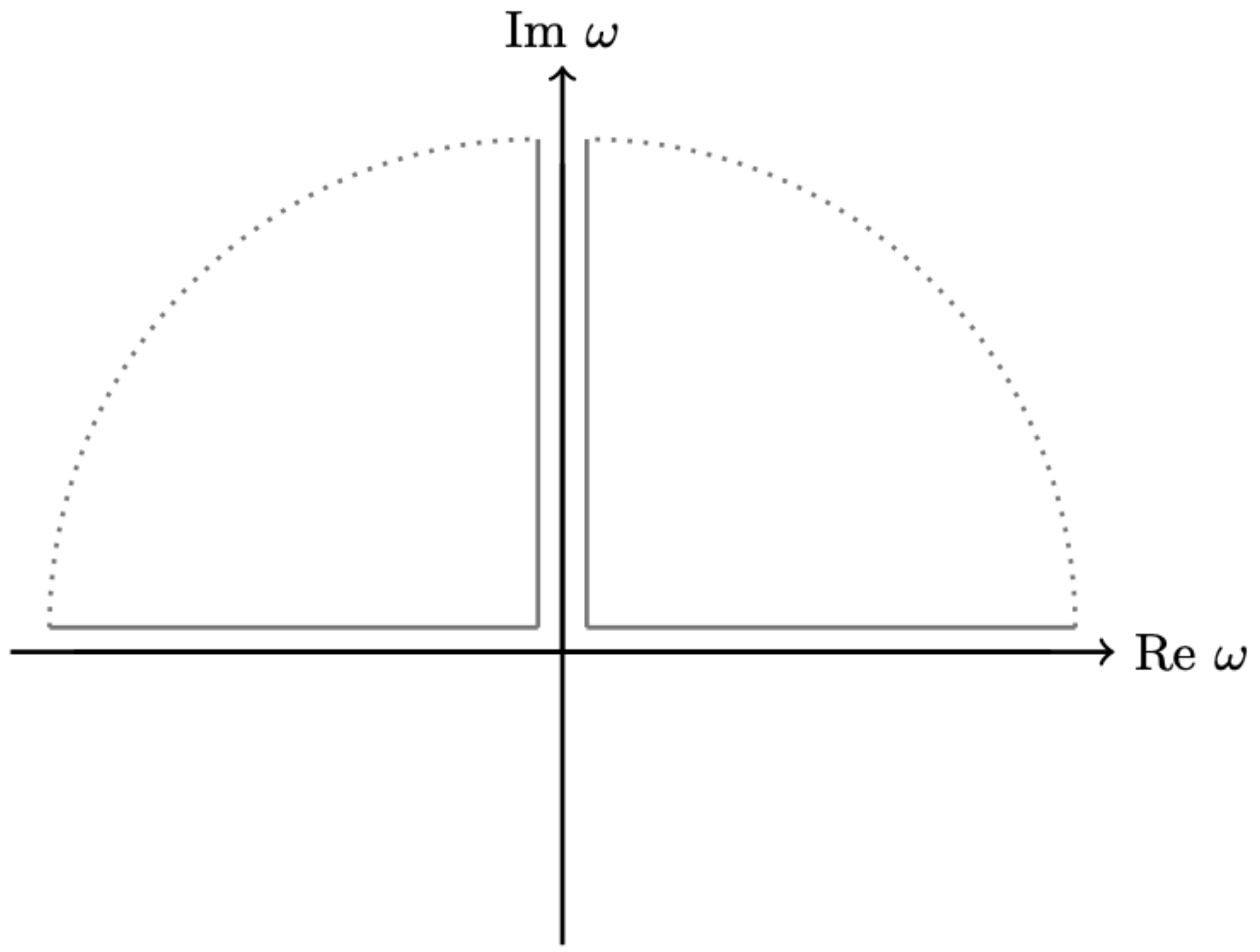
Prompt response: recent progress

[Ma, 2604.08680]



(a) $\tilde{G}_1(\omega; r_*, r'_*)$



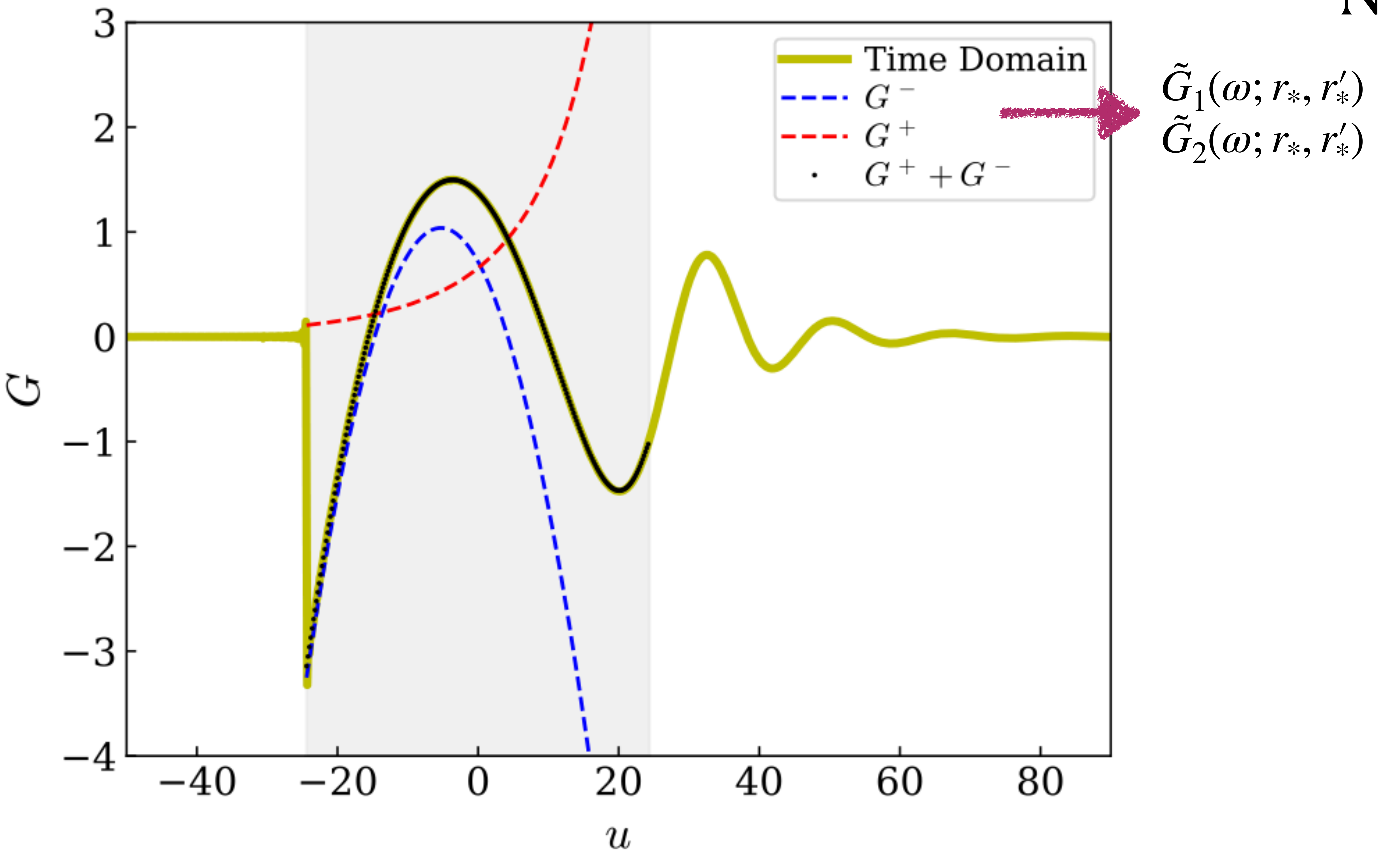
(b) $\tilde{G}_2(\omega; r_*, r'_*)$

Numerical integration in Schwarzschild of:

- Pole of $\tilde{G}_1(\omega; r_*, r'_*)$ in $\omega = 0$
- Branch cut on positive imaginary axis in $\tilde{G}_2(\omega; r_*, r'_*)$

Prompt response: recent progress

[Ma, 2604.08680]



(b) $r' = 20$

Numerical integration in Schwarzschild of:

- Pole of $\tilde{G}_1(\omega; r_*, r'_*)$ in $\omega = 0$
- Branch cut on positive imaginary axis in $\tilde{G}_2(\omega; r_*, r'_*)$

Ringdown models: refined version

Work from **perturbative**, **extreme mass-ratio** limit

i) Model the **ringdown** as **dynamically excited** by the **two-body** problem [De Amicis+, 2506.21668] [De Amicis, 2605.16492]

$$\Psi^{\mathrm{QNM}}_{\ell m}(t - r_*) = \sum_{n,\pm} e^{-i\omega_{\ell mn\pm}(t-r_*)} B_{\ell mn\pm} \left[c_{\ell mn\pm}(t - r_*) + i_{\ell mn\pm}(t - r_*)\right]$$

ii) **Constant-amplitude** model only valid at **late times,** clear dependence on **inspiral 2-body**

[De Amicis+, 2506.21668]

$$\Psi^{\mathrm{QNM,\,late\,times}}_{\ell m}(t - r_*) = \sum_{n\pm} \left(A^{\mathrm{hereditary}}_{\ell mn\pm} + A^{\mathrm{near-horizon}}_{\ell mn\pm}\right) e^{-i\omega_{\ell mn\pm}(t-r_*)}$$

iii) Prompt response relevant at early times,

- a possible contribution: [De Amicis+, 2601.11706]

$$G^{\mathrm{Pole}}_{\ell\, m}(t - t'; r_*, r'_*) = \theta(u - u')\theta(v' - u) \times \sum_{j=0}^{\ell} p_j(r')(u - u')^j$$

- Still missing full closed form

?