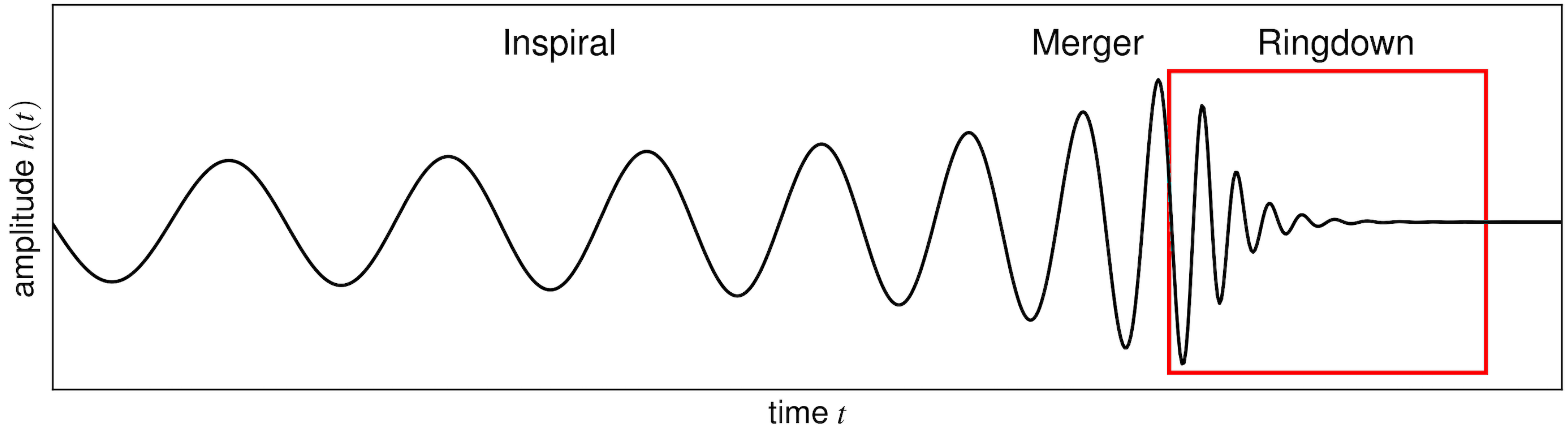
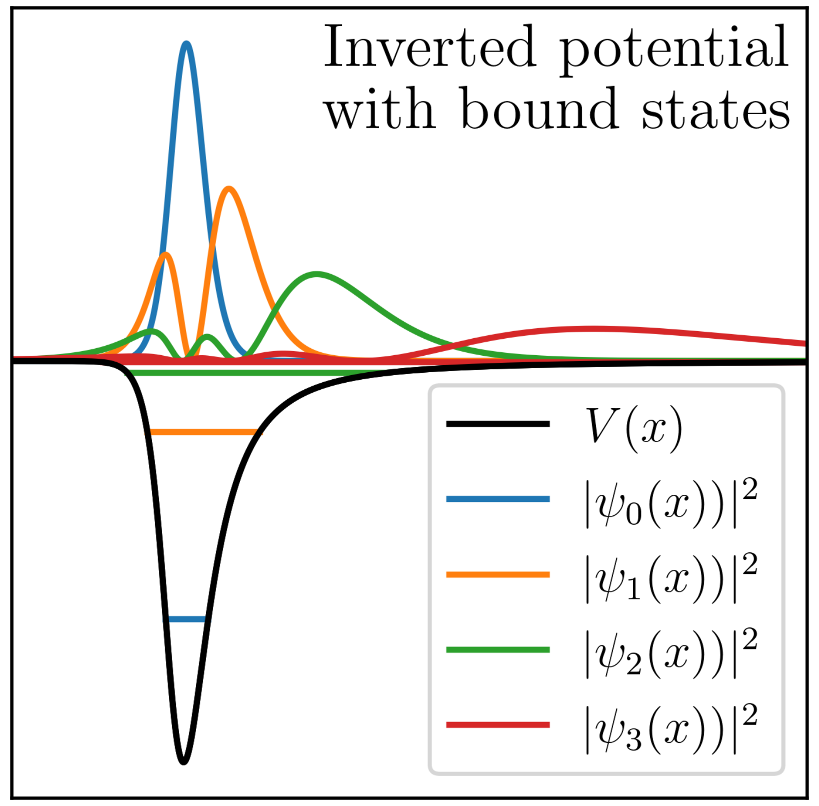
Perspectives on black hole ringdown





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IFPU Focus Week: Perspectives on Gravity: From Theory to Observation

Perspectives on black hole ringdown

Motivation

- Black holes are some of the "most extreme objects" in physics!
  What happens really close to a singularity? Are black holes as GR predicts them?

- Merging black holes are the gravitational physics equivalent to particle scattering
  Probes interactions and dynamics of black holes on different energy and mass scales...

- Probing the relaxation phase of the merger (ringdown) is basically spectroscopy
  Excited black holes emit quasi-normal modes (QNMs) (and more) predicted from GW perturbation theory.

- Astrophysical remnant black hole should be described by mass M and spin a
  Due to no-hair theorems and perturbation-theory, QNMs of final black hole only depend on M and a.

Perspectives on black hole ringdown

Any deviation would question "fundamental assumptions"

- Is there "substantial" matter in the black hole vicinity (standard, dark, exotic,...)?
  Environmental effects can spoil "clean" ringdowns, *Barausse: 1404.7149*, ...

- Is the final object not a black hole?
  Pick your favorite exotic compact object...

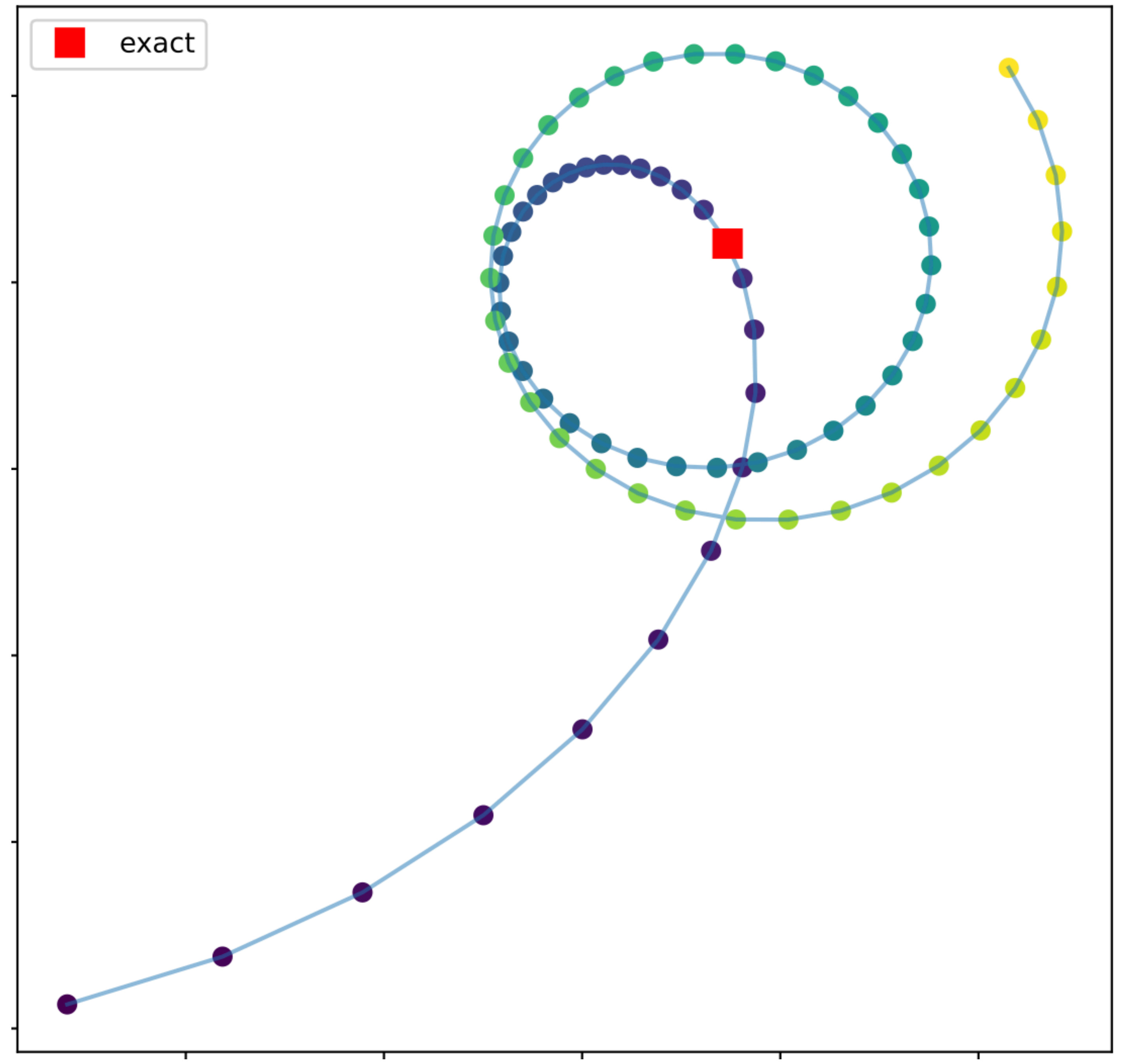
- Is GR not correct/accurate enough in strong-gravity systems?
  Pick your favorite theory... and work it out!

- All of the above are exciting, but probably we just ran into systematics?
  Extracting QNMs is difficult, GW detector noise non-Gaussian, unmodelled GR effects, ...

Overview

Some (biased) selection of topics

- Mapping between bound states and QNMs
- Progress in parametrized QNM frameworks
- Systematic effects in black hole spectroscopy
- *If time is left:*
  - *Spectroscopy of analogue black holes using simulation-based inference*
    Solidoro, SV, Weinfurtner 2604.12800
  - *Black hole spectroscopy of collapsing and merging neutron stars*
    Steppohn, SV, Dietrich 2508.15534



Mapping between bound states and QNMs

Effective perturbation potentials

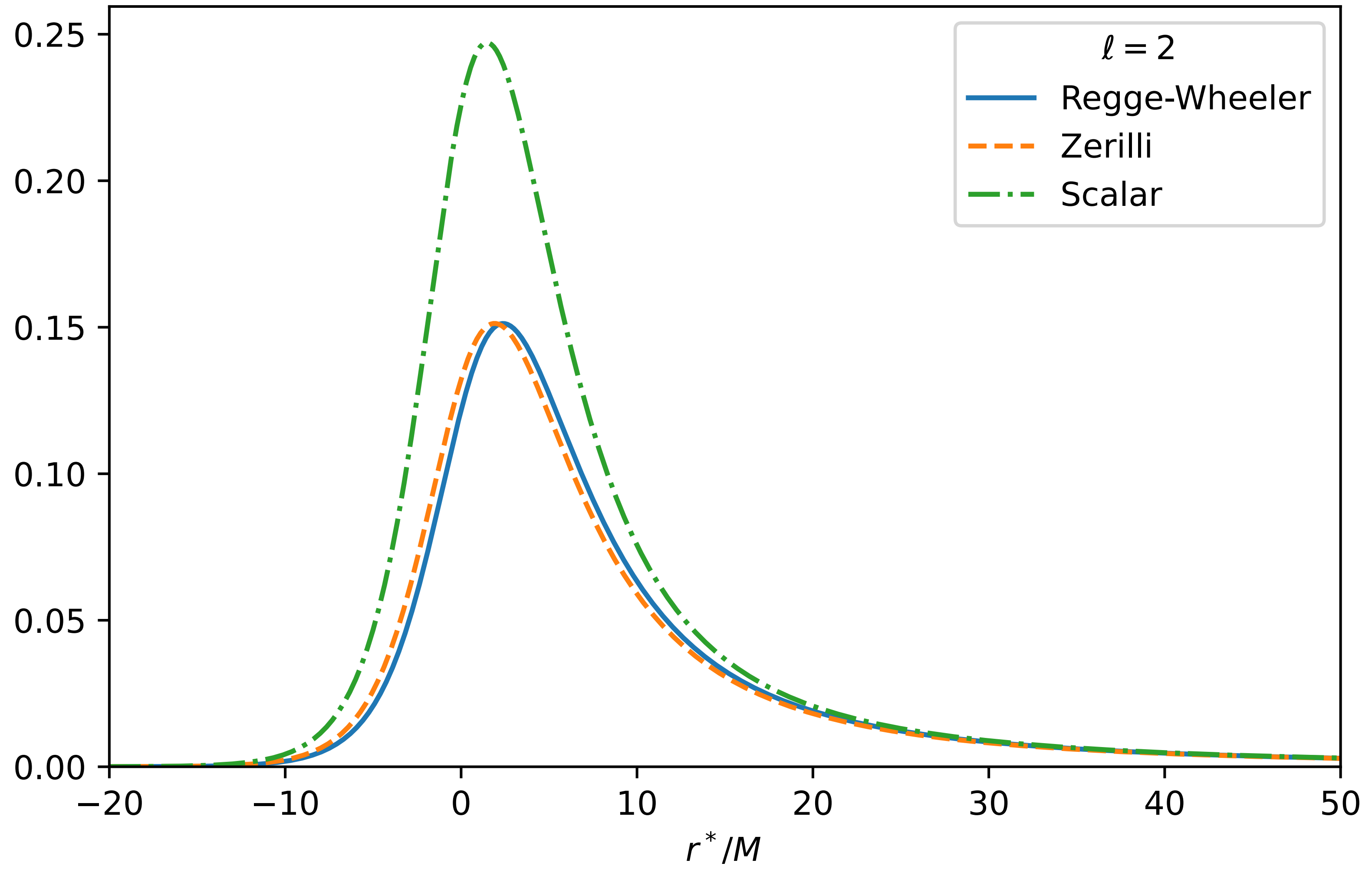
Example for Schwarzschild black holes

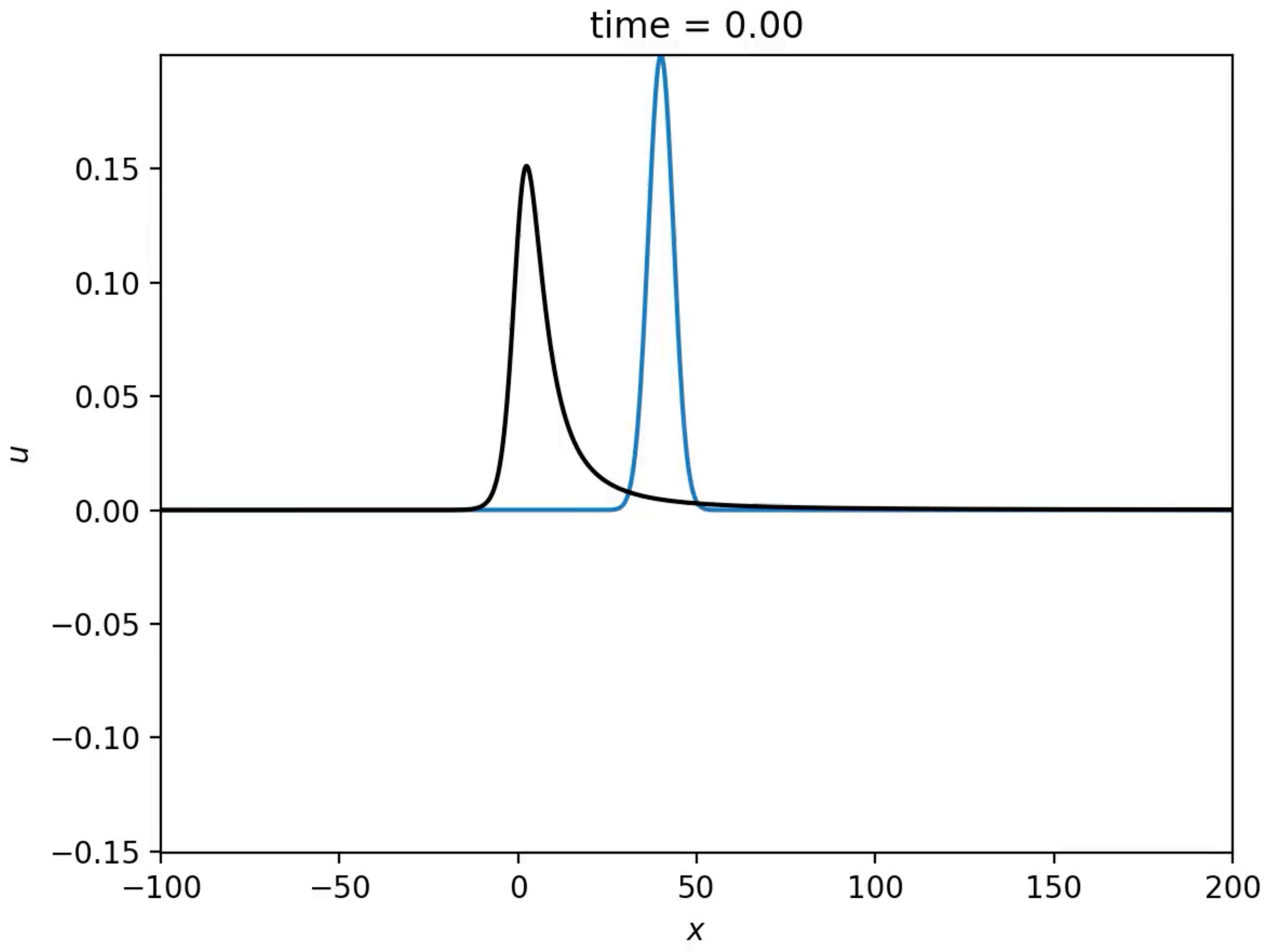
- Perturbations around Schwarzschild described by wave equation:
$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\Psi + \left[\omega_n^2 - V_s(x,P)\right]\Psi(x) = 0$$

- Effective potentials for scalar, axial (Regge-Wheeler 1957), and polar perturbations (Zerilli 1970) of the Schwarzschild black hole:

$$V_{scalar}(r) = \left(1 - \frac{2M}{r}\right)\left(\frac{\ell(\ell+1)}{r^2} + \frac{2M}{r^3}\right)$$
$$V_{RW}(r) = \left(1 - \frac{2M}{r}\right)\left(\frac{\ell(\ell+1)}{r^2} - \frac{6M}{r^3}\right)$$
$$V_Z(r) = \left(1 - \frac{2M}{r}\right)\left(\frac{\ell(\ell+1)}{r^2} - \frac{6M}{r^3}\frac{r^2\lambda(\lambda+4) + 12Mr - 12M^2}{(6M + r\lambda)^2}\right)$$

with $\lambda = \ell(\ell+1) - 2$ and tortoise coordinate $r^*(r) = r + 2M\log\left(\frac{r}{2M} - 1\right)$





QNM spectrum of Schwarzschild

Can you “understand” its structure on complex plain?

- QNM boundary conditions determine eigenvalue spectrum
- Axial and polar QNM spectrum are identical: isospectral!
- Fundamental mode $n = 0$ longest damping time, but overtones $n > 0$ first decrease in frequency, then parallel to the imaginary axis.
- Asymptotic relation $n \to \infty$ (independent of ℓ) given by

$$M\omega_n \approx \frac{\log(3)}{8\pi} - \frac{i}{4}\left(n + \frac{1}{2}\right)$$

Nollert 93 , Hod 98

- Physical interpretation by Maggiore 2007

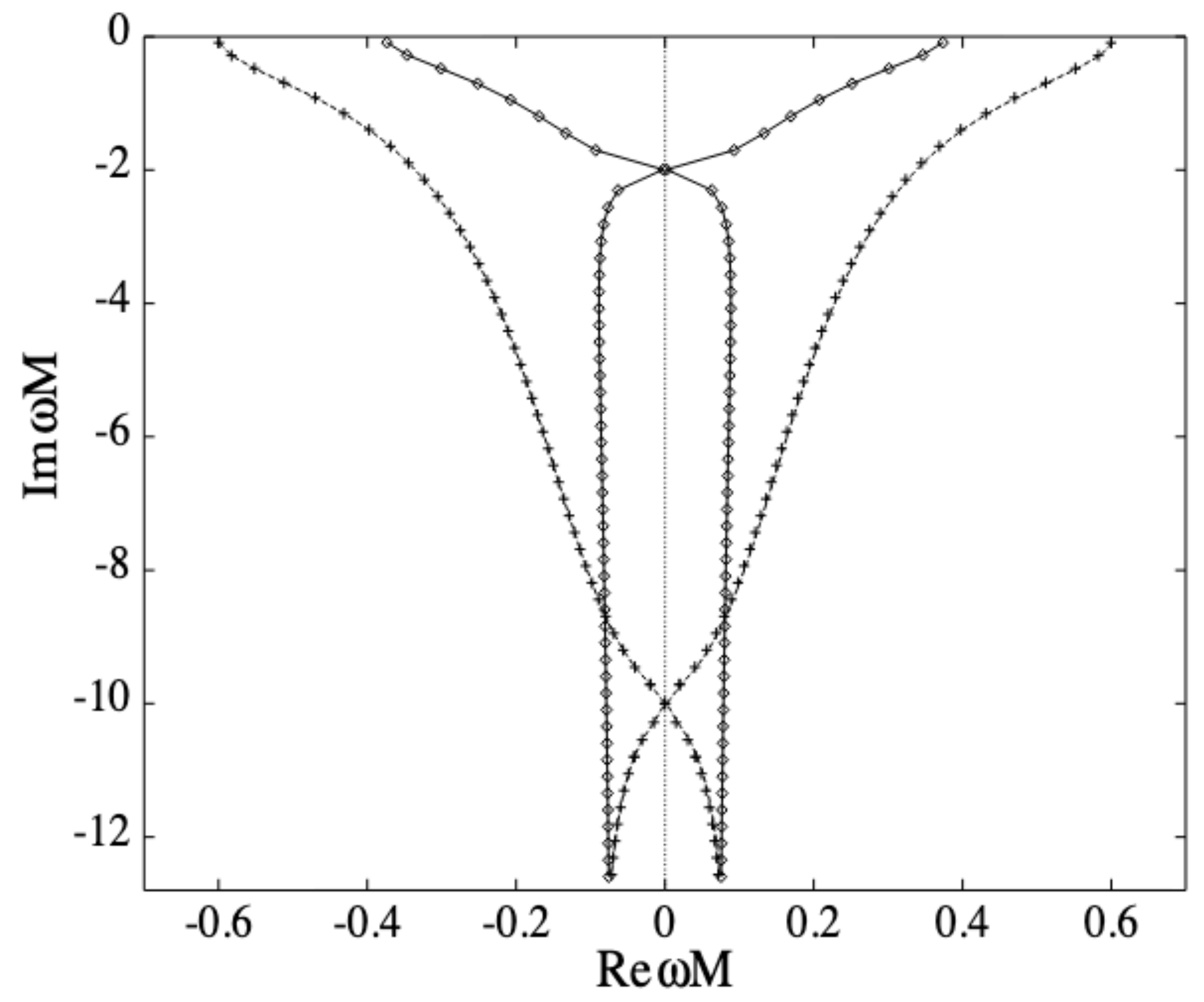


Figure 2: *The spectrum of QNM for a Schwarzschild black-hole, for $\ell = 2$ (diamonds) and $\ell = 3$ (crosses) [25]. The 9th mode for $\ell = 2$ and the 41st for $\ell = 3$ are “special”, i.e. the real part of the frequency is zero ($s = i\omega$).*

Taken from Kokkotas and Schmidt, LivingRev.Rel.2:2,1999

Inverted potential method

Connecting QNMs with bound states

- Proposed by *Mashhoon* (1982), extended (*Blome & Mashhon* 84, Ferrari & *Mashoon* 84).

- Key idea: QNMs from a potential barrier can be mapped to bound states in a potential well.

- Consider the wave equation

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\Psi + \left[\omega_n^2 - V(x,P)\right]\Psi(x) = 0$$ with a potential barrier $V(x,P) > 0$, P are parameters.

- Note for black holes: RW/Z potential is complicated, often explicit calculations use approximate potentials, e.g. Pöschl-Teller.

Inverted potential method

Outline of the recipe

- Apply coordinate transformation $x \to -ix$, and find $P \to \pi(P)$ such that

  $V(-ix, \pi(P)) = V(x, P)$,

- then

  $\frac{\mathrm{d}^2}{\mathrm{d}x^2}\Phi + \left[E_n - \bar{V}(x, P)\right]\Phi(x) = 0$, with $\bar{V}(x, P) = -V(x, P) < 0$,

- and

  $\omega_n^2(P) = -E_n(\pi^{-1}(P))$.

Inverted potential method

Important remarks

- QNMs of original potential related to bound states of inverted potential

  $\omega_n^2(P) = -E_n(\pi^{-1}(P))$, computed at $\pi^{-1}(P)$, **not** $\omega_n^2(P) = -E_n(P)$!

- One might only compute QNMs with associated bound states ($\omega_n \leftrightarrow E_n$), there can be potentials with a finite number of bound states but infinitely many QNMs (e.g. Pöschl-Teller).

- Analytic continuation may not hold in general, do not blindly use it for any type of problem.

- Mashhoon's method widely used for different potentials (hundreds of citations, applications).

- Main bottleneck: explicit analytic knowledge of E_n for inverse mapping is needed.

Inverted potential method

Recent improvements

- Two approaches recently suggested to extend the method to non-analytic E_n:
  - a) semi-analytic: apply transformation in higher-order WKB expansion for bound states.

    -Works for exact Schwarzschild case to high accuracy for first few n.

    -Limited to validity of WKB expansion, less valid for larger n.
    Hatsuda, Phys. Rev. D 101, 024008 (2020)

  - b) numerical: compute the Taylor expansion of the bound states as a function of the parameters P numerically (bound states are much easer to compute numerically).
    -Demonstrated for simple potentials with small number of parameters.
    SV, Phys. Rev. D 106 (2022) 12, 124009

Non-trivial aspects of QNMs

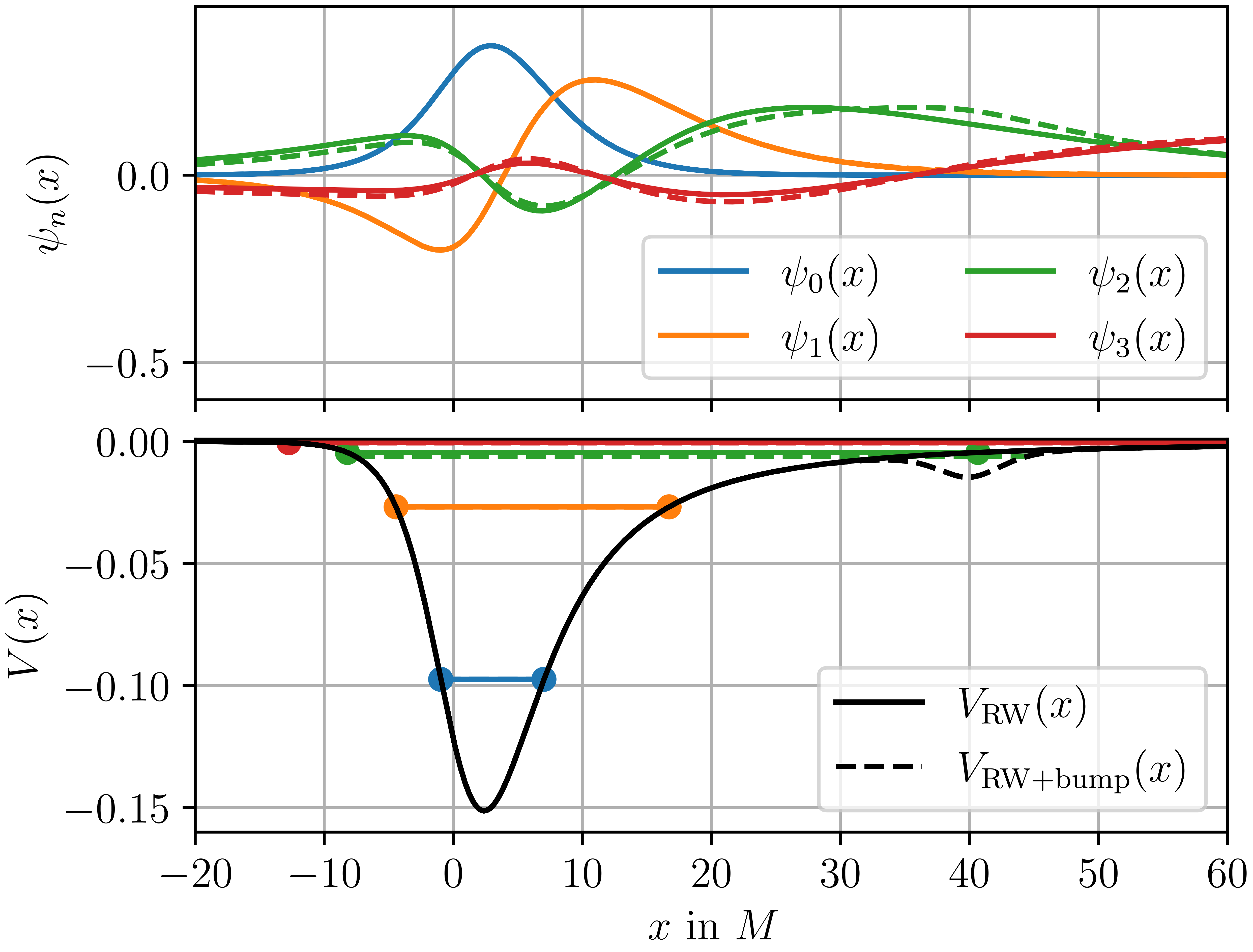
Motivation for revisiting bound-state mapping

- No fundamental technical problem, but can one get more "physical insights" about QNMs?
  - WKB analysis shows that small n QNMs come from local properties of potential barrier.
  - The inverted potential method maps low n QNMs to bound states of a potential well.
- *Can we learn something interesting from the bound states themselves?*
- Widely unexplored question, focus has always been on QNMs.
- Many open QNM questions to "rethink" in the bound state picture.

Bound states of the inverted potential

SV Phys. Rev. Lett. 134 (2025) 24, 24

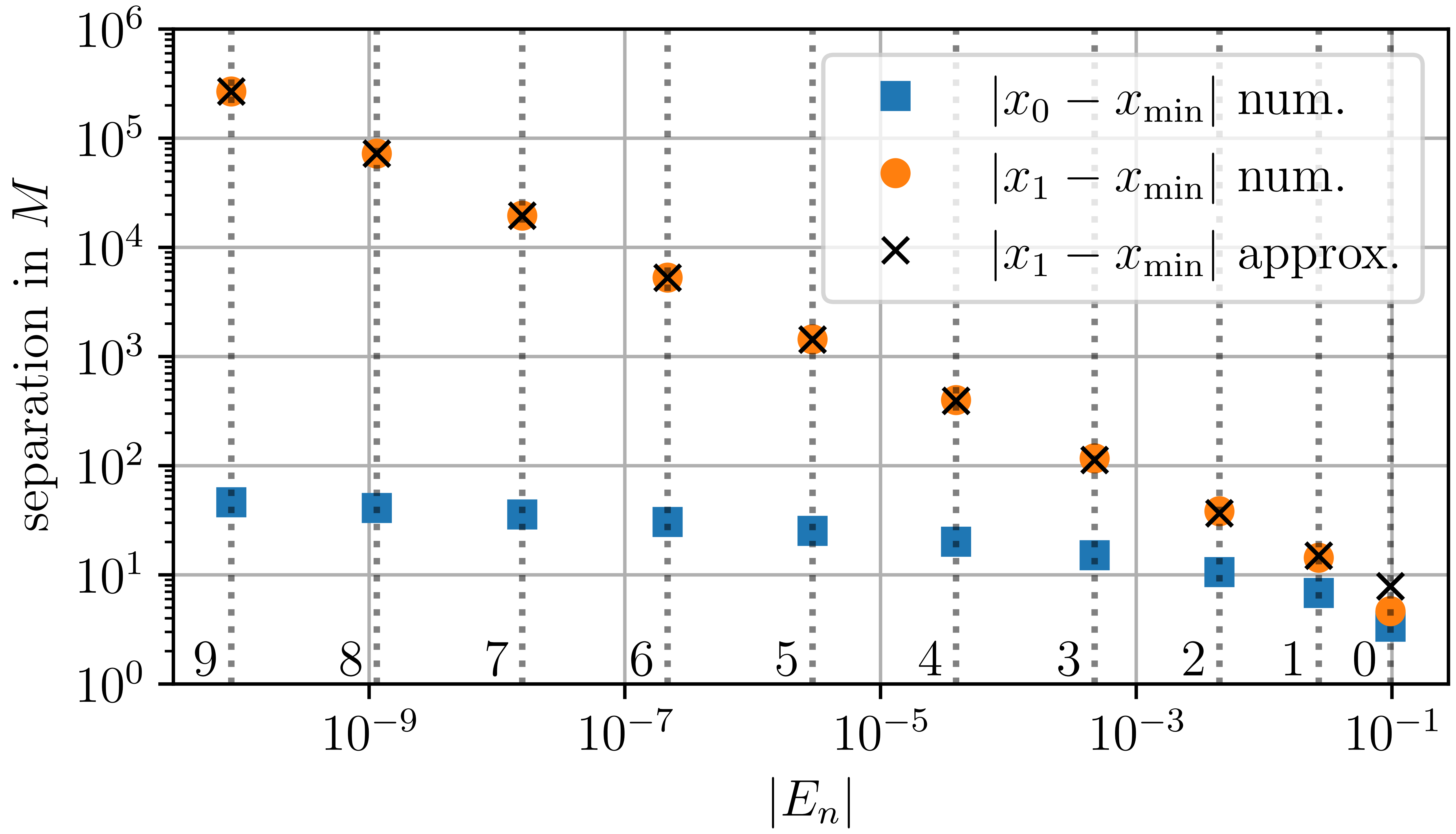
- First numerical bound-state calculation for Schwarzschild BH.
- Direct shooting method stable for bound states (unstable for QNMs!).
- Here for $\ell = 2$ for the inverted Regge-Wheeler potential (solid) and one with a small bump (dashed).
- Bound state eigenfunction with $n \gtrsim 1$ quickly delocalize!



Turning points

Delocalization of the classical turning points

- Classical turning points mark transition from oscillatory to exponentially decaying regions $E_n = V(x_{n,i})$.
- Outer turning point $(x_1 > x_{\min})$ grows exponentially!
- Indicates that eigenfunctions for large n are "marginally bound".
- Implications for QNM overtones?



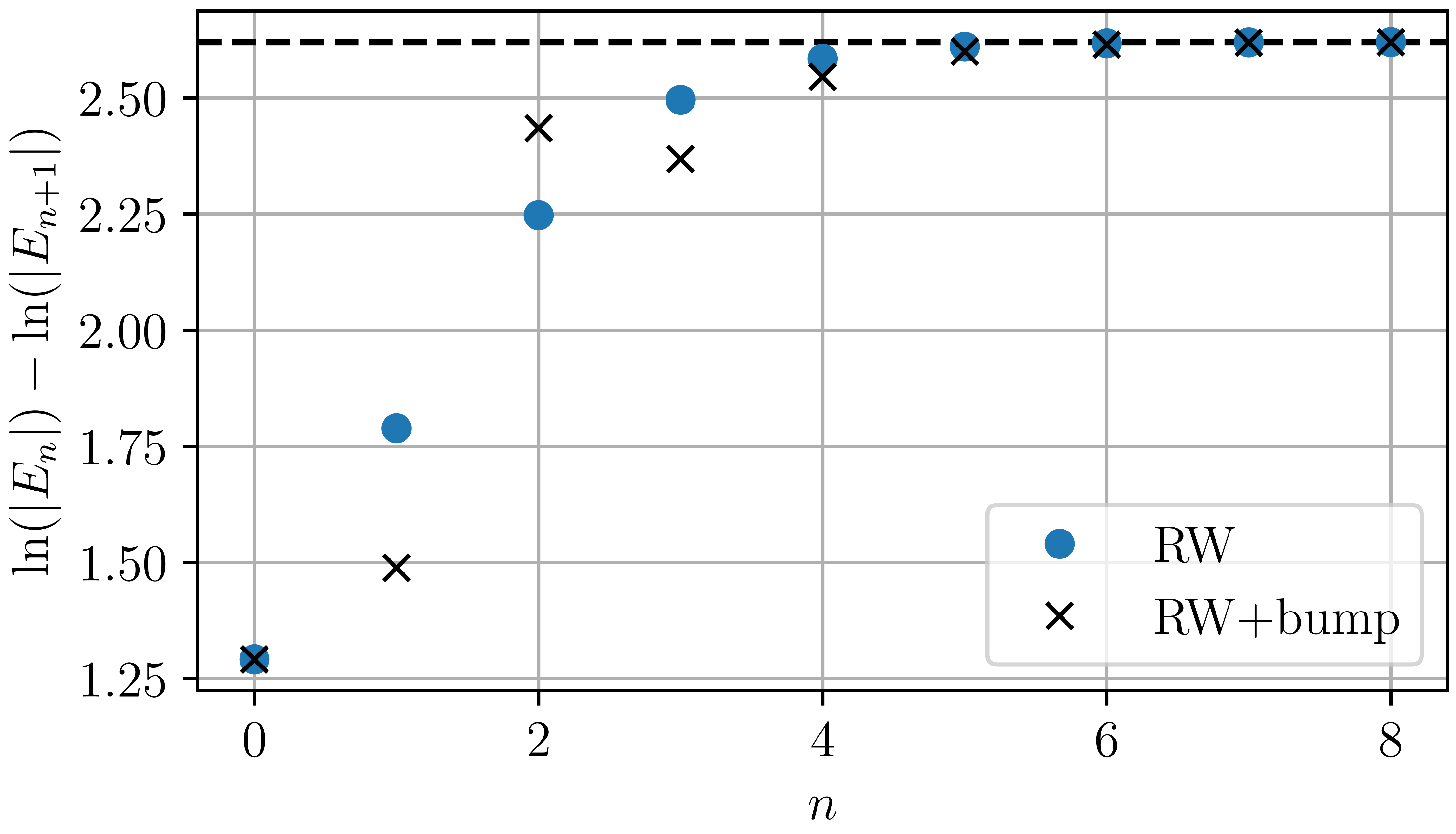
Spectrum of bound states

Asymptotic behavior for large n

- For large n, bound states E_n scale as

$$E_{n+1} \approx E_n e^{-K}, \quad K = \frac{2\pi}{\sqrt{\ell(\ell+1) - 1/4}}$$

- E_n approaches 0 exponentially fast!
- Explains rapid delocalization of eigenfunctions.
- K is also related to properties of the late-time tail!
- Full asymptotic solution obtained afterwards
  Hod, Phys. Rev. D 112 (2025) 6, 064040.
- There are infinitely many E_n.

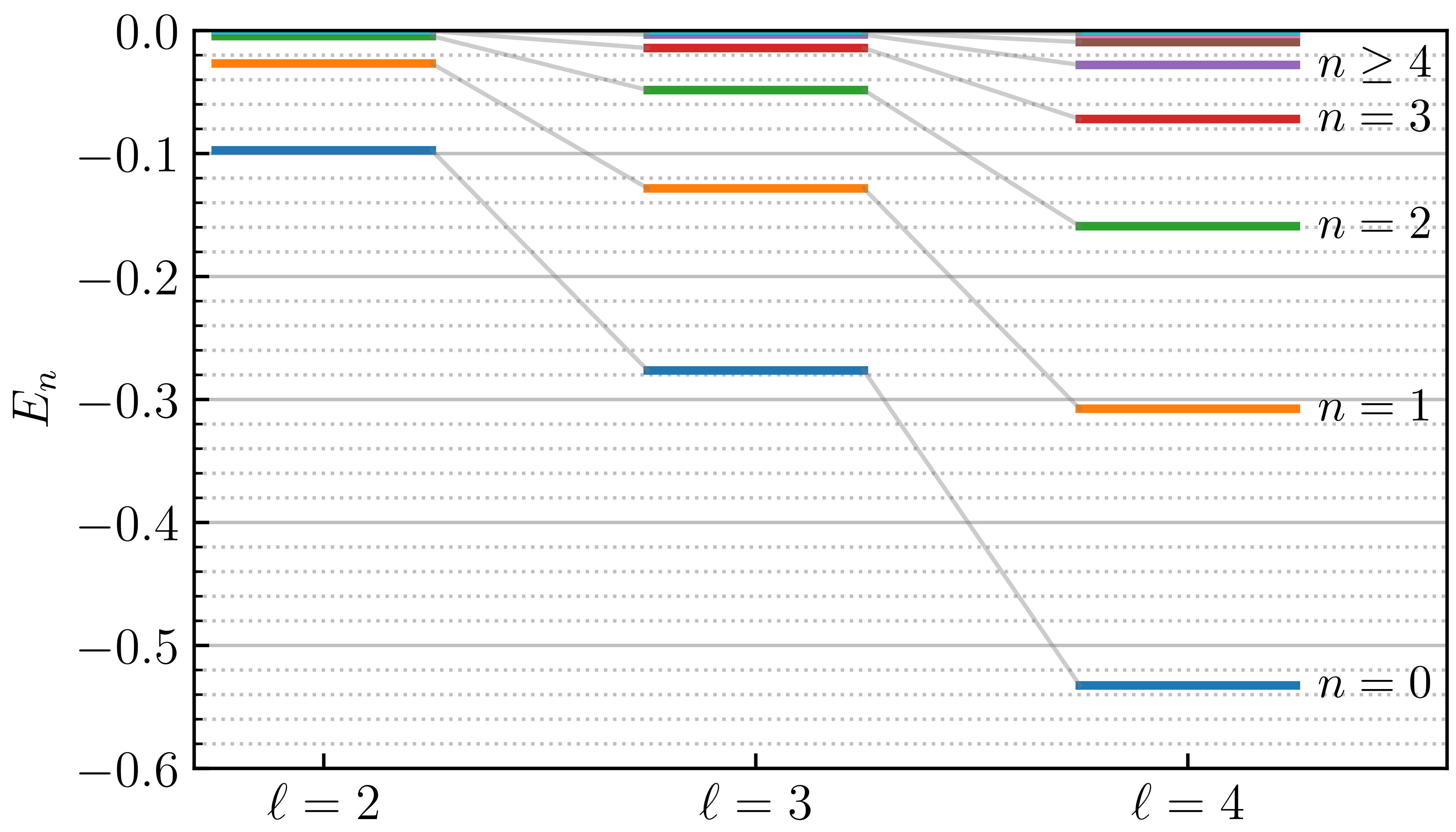


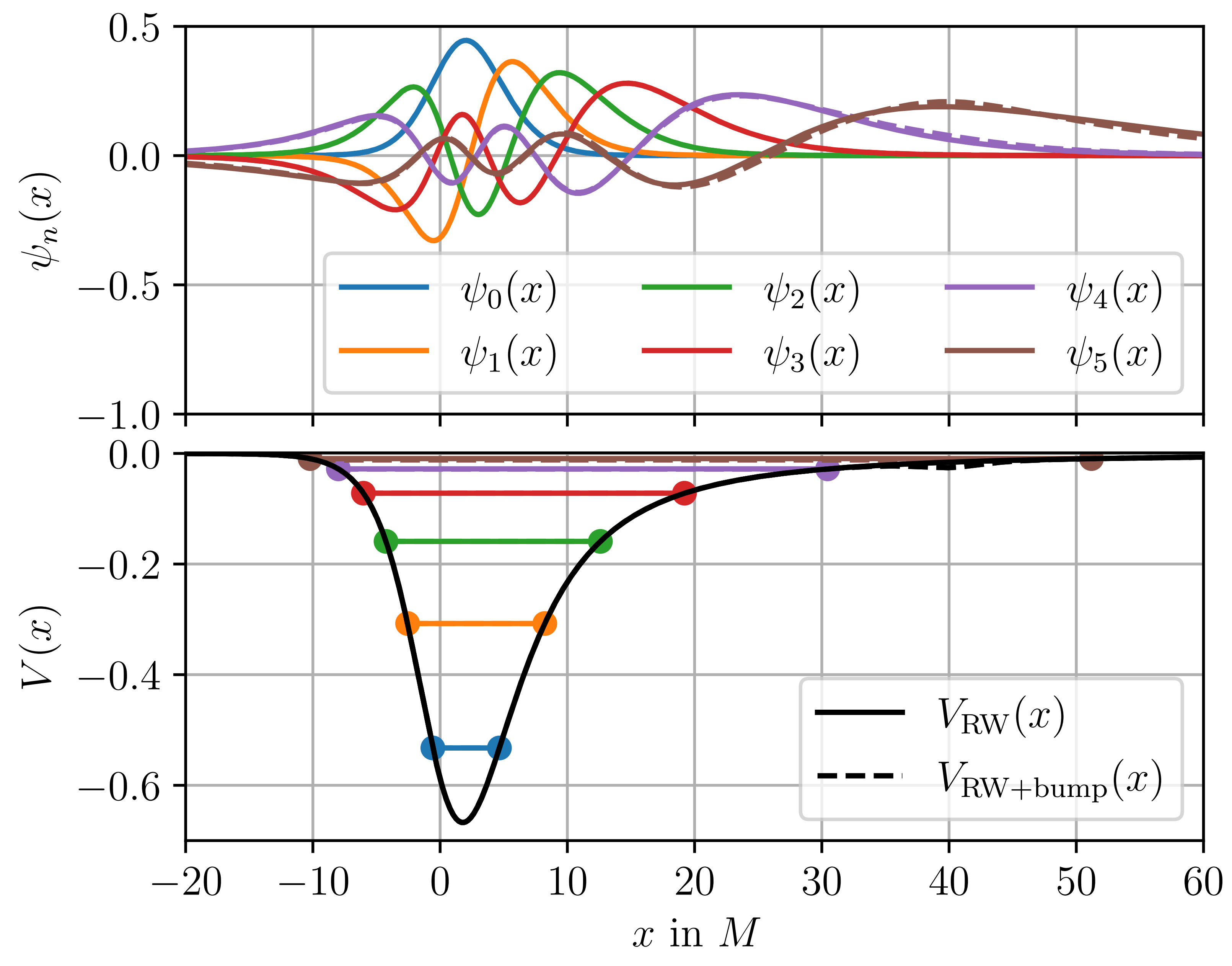
Here for $\ell = 2$, dashed line is K.

Spectrum of bound states

Scaling for $\ell \geq 2$

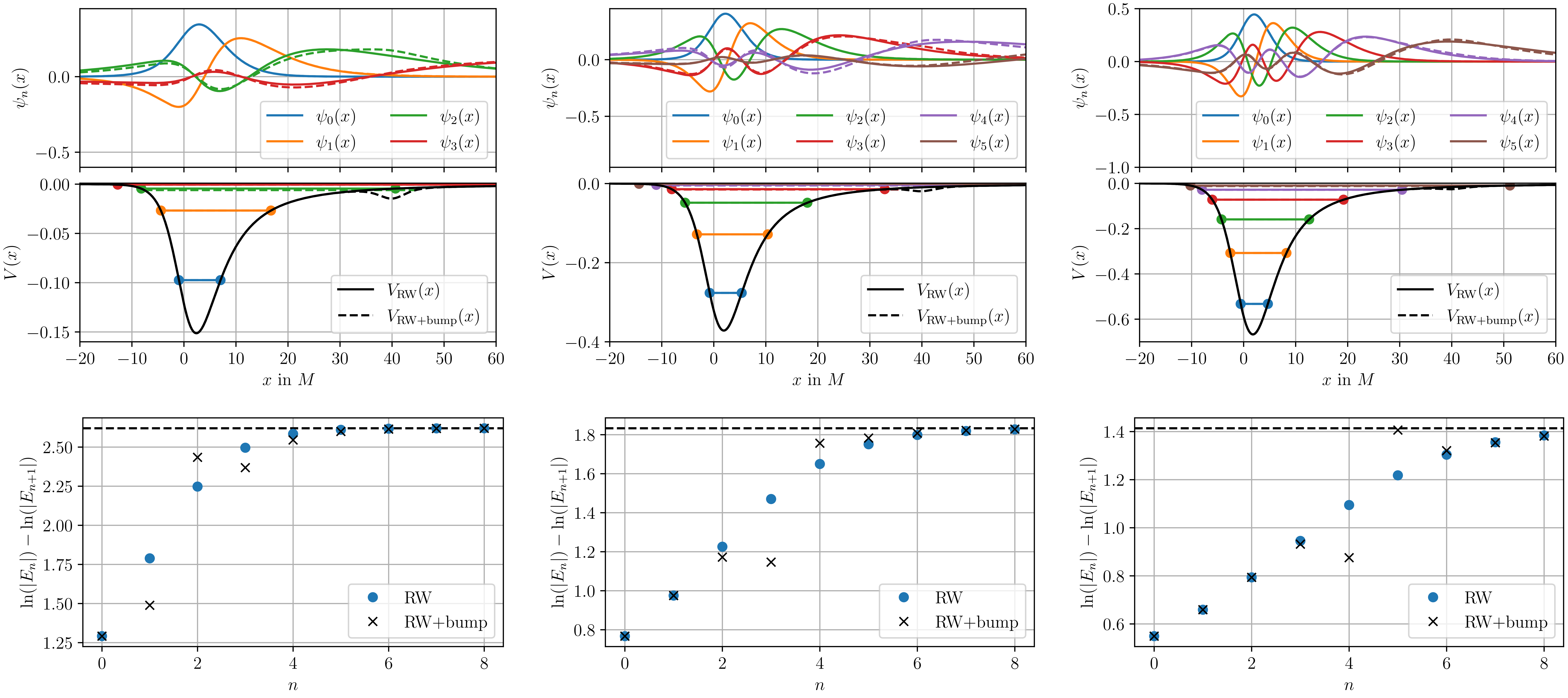
- Potential well gets deeper with increasing ℓ, more negative E_n possible.
- More localized eigenfunctions, transition to delocalized ones for larger n.





$\ell = 4$

Comparison for different ℓ



$\ell = 2$ $\ell = 3$ $\ell = 4$

Conclusions: bound states

- Mashhoon connected QNMs with bound states of the inverted potential in the 1980's.
- Here, first quantitative computation of bound states of the inverted Regge-Wheeler potential:
  - Bound states E_n approach 0 exponentially fast for $n \to \infty$.
  - "Loosely bound" eigenfunctions, outer turning point grows exponentially.
  - E_n stable to small perturbations (in contrast to QNM spectral instability) [not discussed here]
  - Asymptotic level spacing seems related to late-time power law tail.
- Intriguing/provocative/naive question:
  Can overtones be "physically" excited in local time-domain simulations, if associated bound states are sensitive to much larger region of the potential than the one probed in the time domain?

Parametrized QNM framework

Parametrized QNM framework

- For non-rotating black holes, typical structure is

$$f\frac{d}{dr}\left(f\frac{d\mathbf{\Phi}}{dr}\right) + \left[\omega^2 - f\mathbf{V}\right]\mathbf{\Phi} = 0$$

- with $f(r) = 1 - r_0/r$, and r_0 being the location of the event horizon and

$$\mathbf{\Phi} = \left[\Phi^{\text{scalar}}, \Phi^{\text{polar}}, \Phi^{\text{axial}}, \ldots\right]$$

- in GR, V is diagonal with Regge-Wheer/Zerilli/scalar potential
- parametrized QNM framework modifies potentials (Cardoso+ 1901.01265, McManus+ 1906.05155)

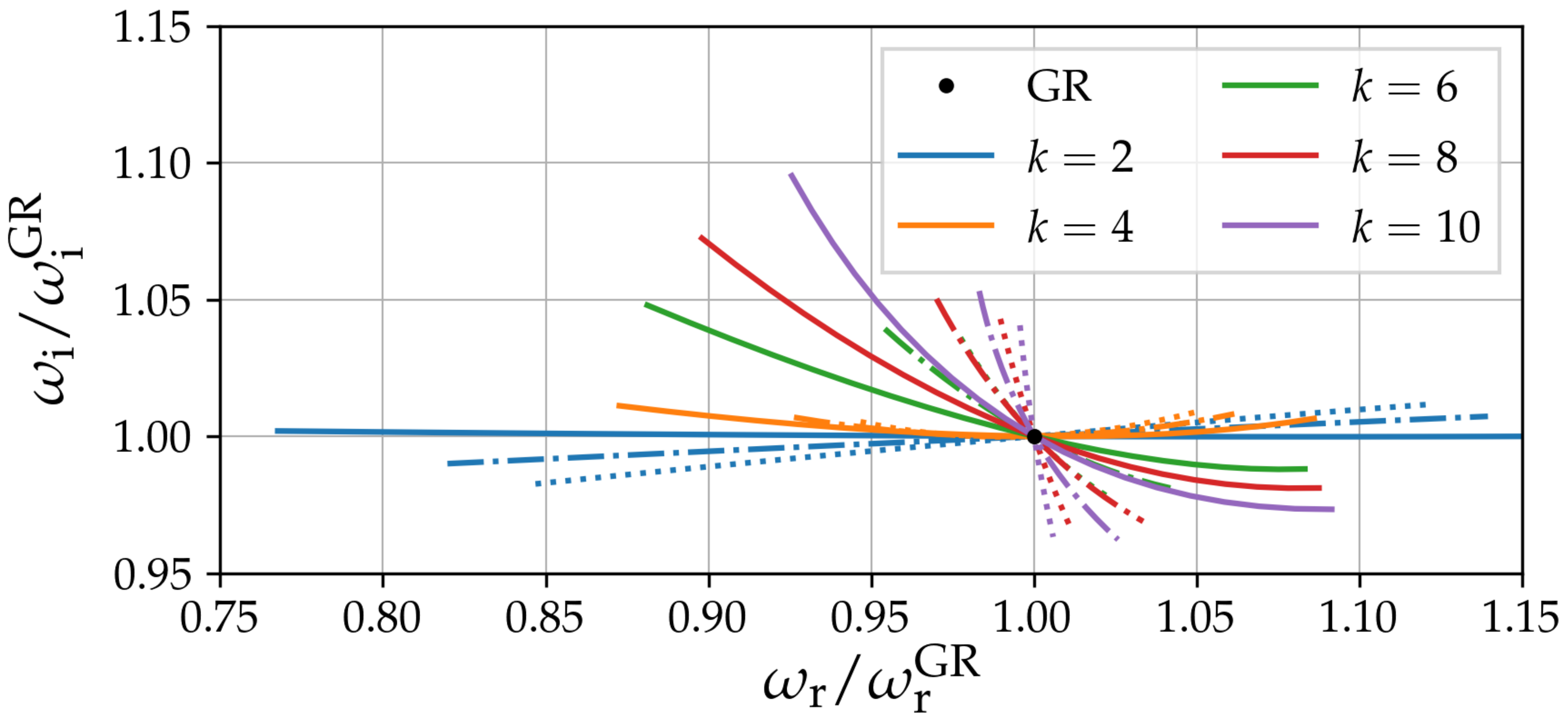
$$V_{ij} = V_{ij}^{\text{GR}} + \delta V_{ij}, \qquad \delta V_{ij} = \frac{1}{r_H^2}\sum_{k=0}^{\infty}\alpha_{ij}^{(k)}\left(\frac{r_H}{r}\right)^k$$

Parametrized QNM framework

- non-zero off-diagonal δV_{ij} couple (new) fields
- for small deviations, approximate up to quadratic order in $\alpha^{(k)}$

$$\omega = \omega^0 + \alpha_{ij}^{(k)} d_{(k)}^{ij} + \alpha_{ij}^{(k)} \delta_\omega \alpha_{pq}^{(s)} d_{(k)}^{ij} d_{(s)}^{pq} + \frac{1}{2} \alpha_{ij}^{(k)} \alpha_{pq}^{(s)} e_{(ks)}^{ijpq}$$

- coefficients $d_{(k)}$, $e_{(ks)}$ universal (compute once)
- $\alpha^{(k)}$ theory dependent or free parameters
- once coefficients are known, QNM prediction is trivial



Relative change of the QNM spectrum along small $\alpha^{(k)}$ range.
Völkel, Franchini and Barausse, PRD 105 084046 (2022)

Parametrized QNM framework

Literature overview (not complete)

	QNM-shift order	involved fields	time evolution
Schwarzschild	lin. (a) & quadr. (b)	single (a,b) & coupled (b)	single axial (h)
non-Schwarzschild*	lin. & quadr. (e)	single (e)	-
Kerr (Teukolsky)	lin. (g)	single (g)	*(i)

- (a) Cardoso+ 1901.01265
  (b) McManus+ 1906.05155
  (c) Kimura+ 2001.09613
  (d) Völkel+ 2202.08655
  (e) Franchini+ 2210.14020
  (f) Hirano+ 2404.09672
  (g) Cano+ 2407.15947
  (h) Thomopoulos+ 2504.17848

Related works:

- important observation for field redefinitions (c)
- computations of overtones in (d,f,g)
- WKB analysis beyond-Teukolsky Tang+ 2512.17786
- Greybody factors (non-rotating, single field) Antoniou 2604.05084
- work in progress (i) De Simone+
- public code: https://github.com/sebastianvoelkel/parametrized_qnm_framework

Parametrized QNM framework

re-sampling QNM “measurements”

- Agnostic GW ringdown tests provide us with constraints on QNMs (e.g., measurement uncertainties on complex frequencies from damped sinusoids).
- Full Bayesian analysis is time-consuming, including beyond-GR models (specific or agnostic).
- But, at high SNR, one can “resample”, i.e., make QNMs the “observable” with uncertainties from posterior of damped sinusoid analysis (e.g., approximated with multivariate Gaussian).
- Allows to constrain theories/models on QNM level (no explicit amplitudes, phases).
- If deviations from GR are small, include IM(R) information on total mass and spin as priors.
  see also Dey+ 2605.18595 for GR case

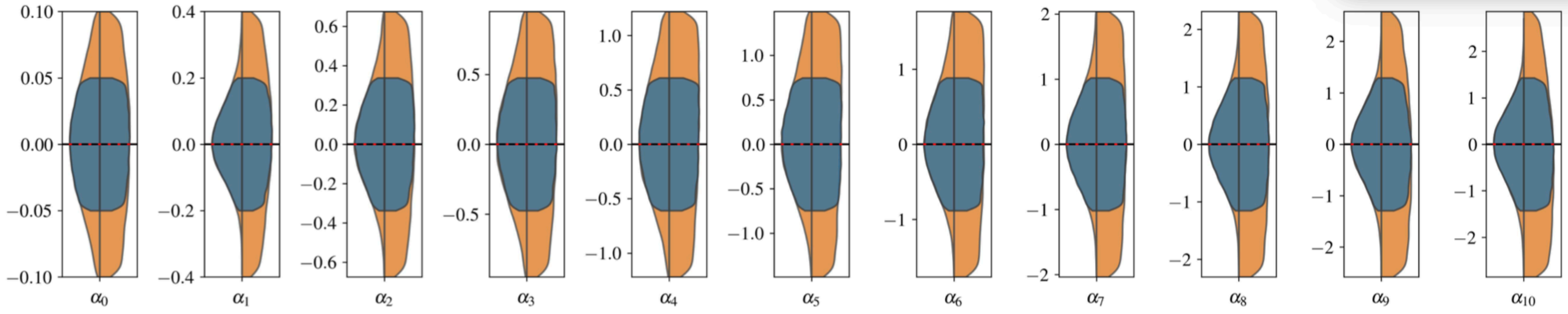


FIG. 2. MCMC results for simulated observations of the Schwarzschild $n = 0, 1$ QNMs with relative errors described in the main text, and assuming a Gaussian prior for $r_{\rm H}$ corresponding to 5 % at 1σ confidence. In each panel, on the left side we plot the posteriors in the optimistic case (only one $\alpha^{(k)}$ is varied at a time), and on the right side we plot the posteriors in the pessimistic case (all $\alpha^{(k)}$'s are varied simultaneously). The blue (orange) color corresponds to a prior range of $\alpha^{(k)}_{\rm P20}$ ($\alpha^{(k)}_{\rm P10}$).

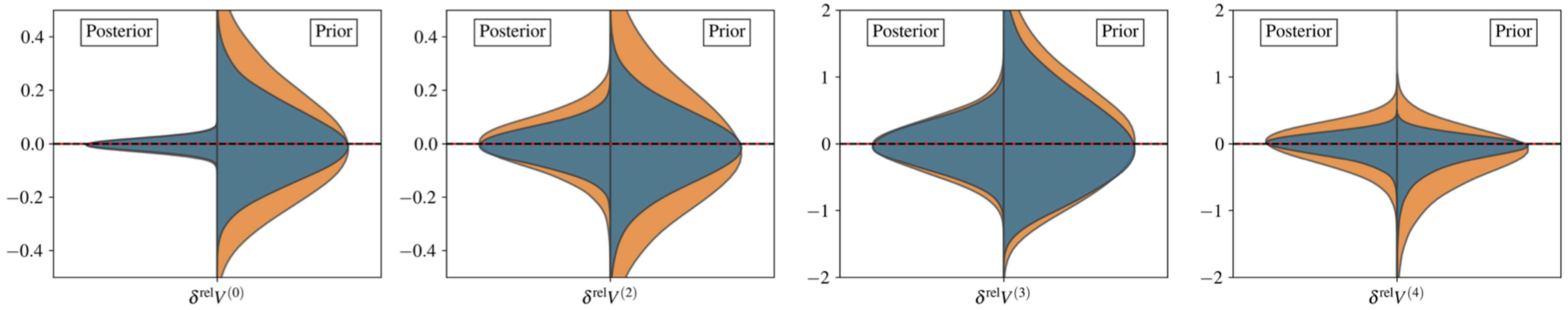


FIG. 3. Same MCMC results as in Fig. 2, but here we sample the relative errors of various derivatives of the effective potential $\delta^{\rm rel}V^{(n)}$ with respect to the tortoise coordinate evaluated at the maximum. The right side of each violin plot shows the sampling from the prior distribution of all $\alpha^{(k)}$ simultaneously, with colors label the different $\alpha^{(k)}$ prior ranges: $\alpha^{(k)}_{\rm P20}$ (blue) and $\alpha^{(k)}_{\rm P10}$ (orange). The left side of each violin plot shows the sampling from the posterior distributions, with colors labeling the prior assumptions. We mark the injections (here corresponding to a Schwarzschild black hole) by red horizontal lines.

SV, Franchini, Barausse, Berti 2209.10564

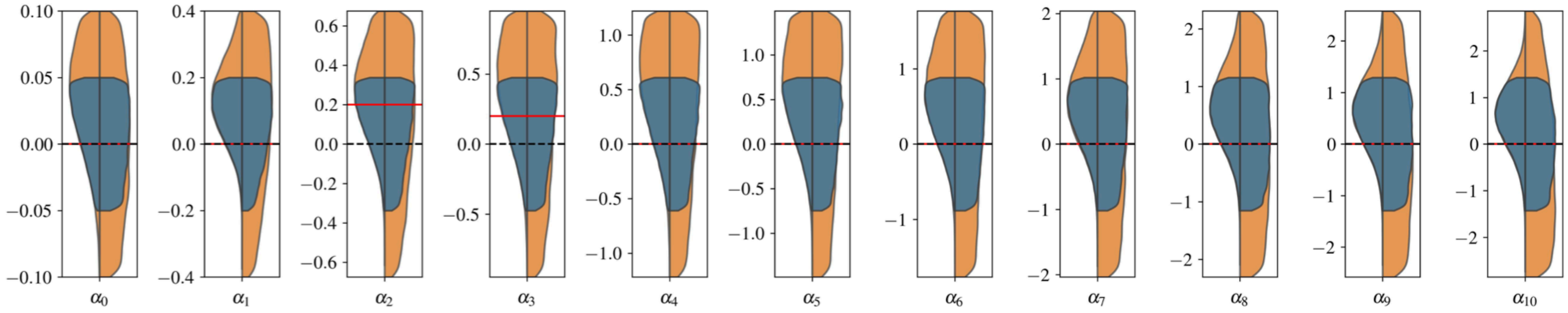


FIG. 4. MCMC results for simulated observations of a modified Schwarzschild $n = 0, 1$ QNMs with relative errors described in the main text and assuming a prior for $r_{\rm H}$ corresponding to $5\,\%$ at 1σ confidence. The injected values (shown as horizontal red lines) are $\alpha^{(2)} = \alpha^{(3)} = 0.2$, with all other values of $\alpha^{(k)}$ set equal to zero. From left to right we show the potential coefficients $\alpha^{(k)}$. In each panel, on the left side we plot the posteriors in the optimistic case (only one $\alpha^{(k)}$ is varied at a time), and on the right side we plot the posteriors in the pessimistic case (all $\alpha^{(k)}$'s are varied simultaneously). The blue (orange) color corresponds to a prior range of $\alpha^{(k)}_{\rm P20}$ ($\alpha^{(k)}_{\rm P10}$).

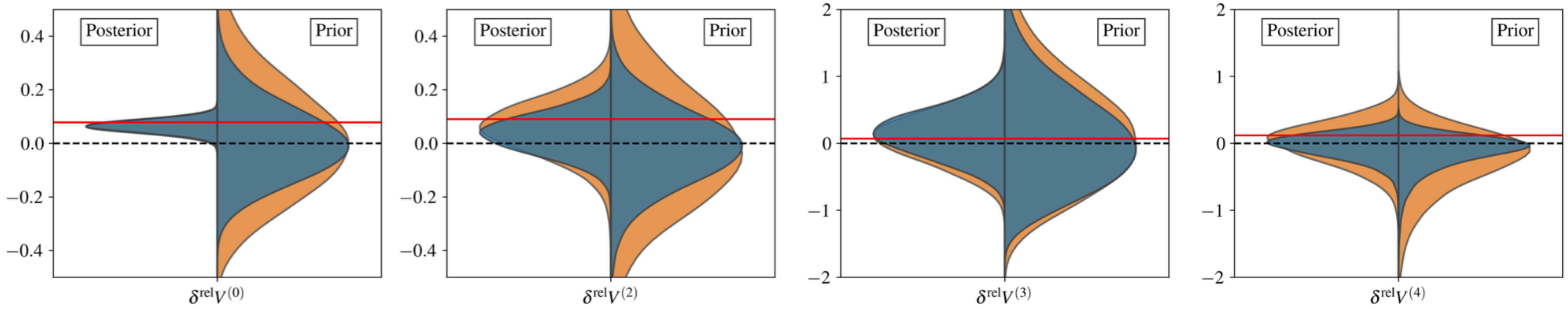


FIG. 5. Same MCMC results as in Fig. 4, but here we sample the relative errors of various derivatives of the effective potential $\delta^{\rm rel}V^{(n)}$ with respect to the tortoise coordinate evaluated at the maximum. The right side of each violin plot shows the sampling from the prior distribution of all $\alpha^{(k)}$ simultaneously. The colors label the different $\alpha^{(k)}$ prior ranges, $\alpha^{(k)}_{\rm P20}$ (blue) and $\alpha^{(k)}_{\rm P10}$ (orange). The left side of each violin plot shows the sampling from the posterior distributions, with colors labeling the prior assumptions. We mark the injections by horizontal red lines.

Parametrized QNM framework in time-domain

check eigenvalue predictions, spectral stability, WKB etc.

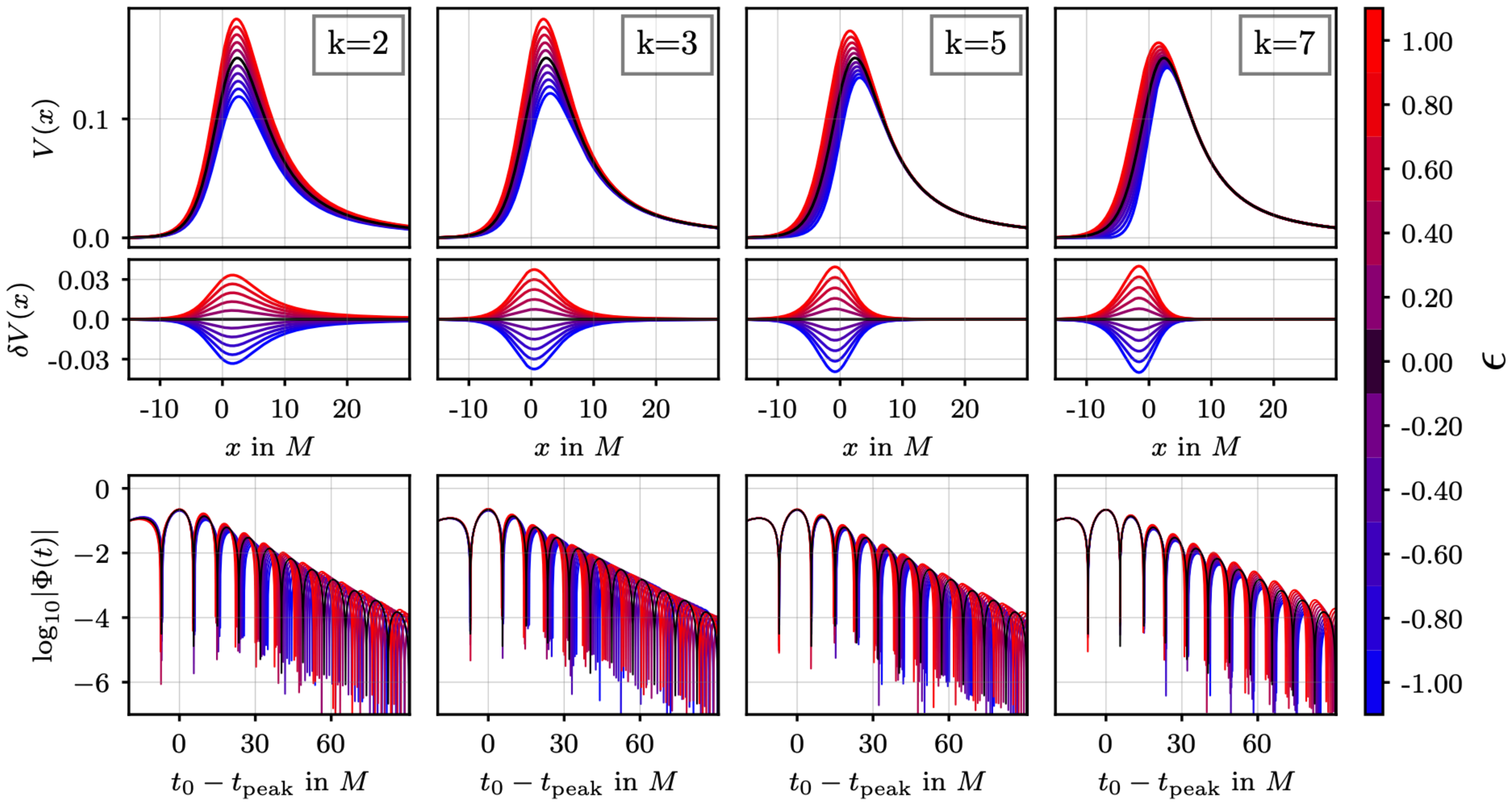


FIG. 2. Modifications to GR potential and respective waveforms. Top row: we show the effective potentials when varying a single $\alpha^{(k)} = \epsilon\alpha^{(k)}_{\rm max}$ modification at a time. Middle row: we plot the corresponding deviations from the original GR potential. Bottom row: we provide the corresponding waveforms. In each panel, we fix the order k and show a family of curves corresponding to increasing the strength of the modification (different colors). The GR case is shown for comparison (black line).

Thomopoulos, SV, Pfeiffer 2504.17848

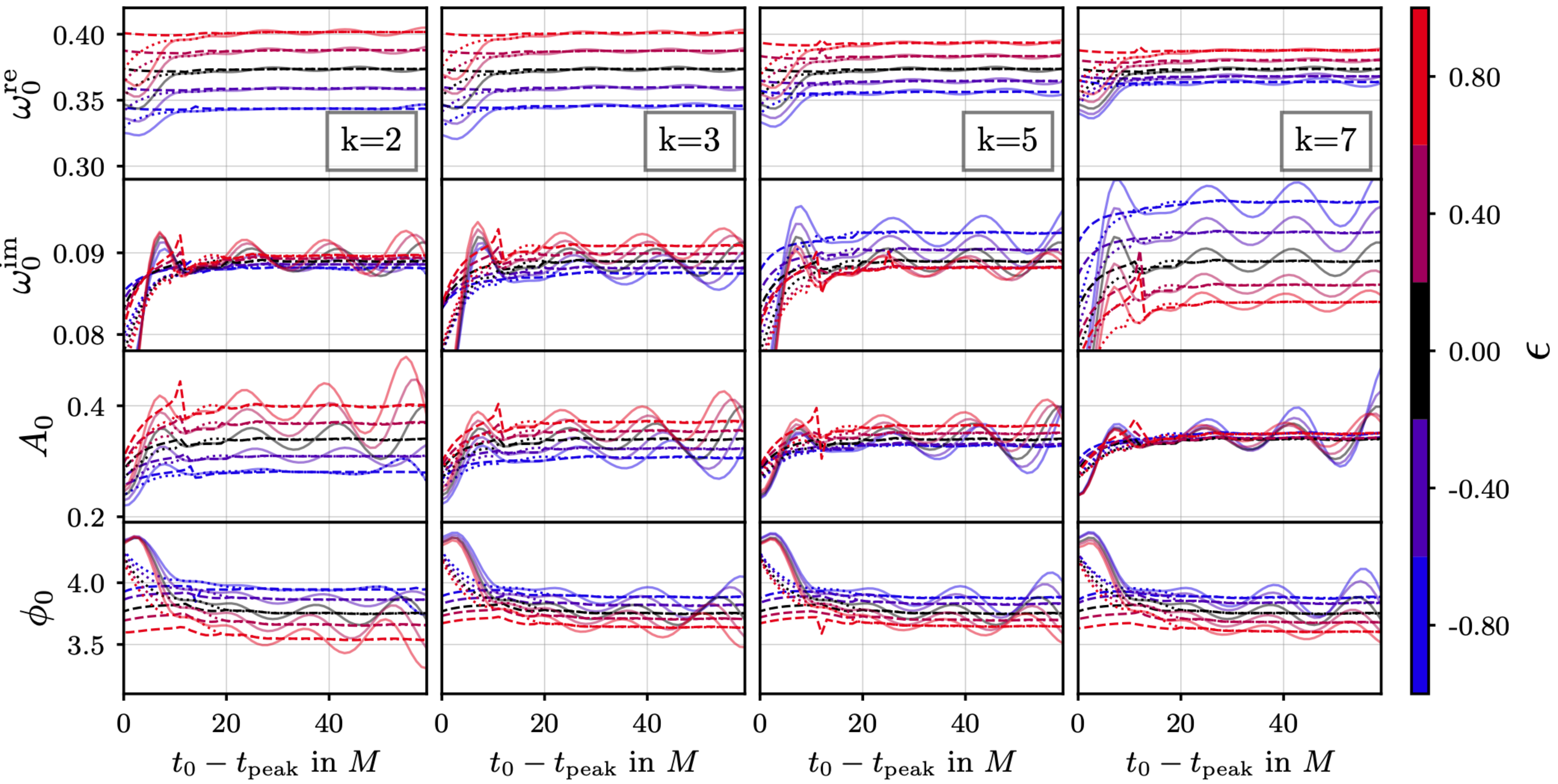


FIG. 4. Extracted parameters from gravitational waveforms. We show from top to bottom the extracted real and imaginary parts of the fundamental QNM, its amplitude A_0 and phase ϕ_0, as function of the starting time $t_0 - t_{\mathrm{peak}}$ when using different fitting models, $n = 0$ (solid lines), $n = 0, 1$ (dashed lines) and $n = 0$, tail (dotted lines). From left to right, we consider different powers $k \in [2, 3, 5, 7]$ of the modifications $\alpha^{(k)}$.

Thomopoulos, SV, Pfeiffer 2504.17848

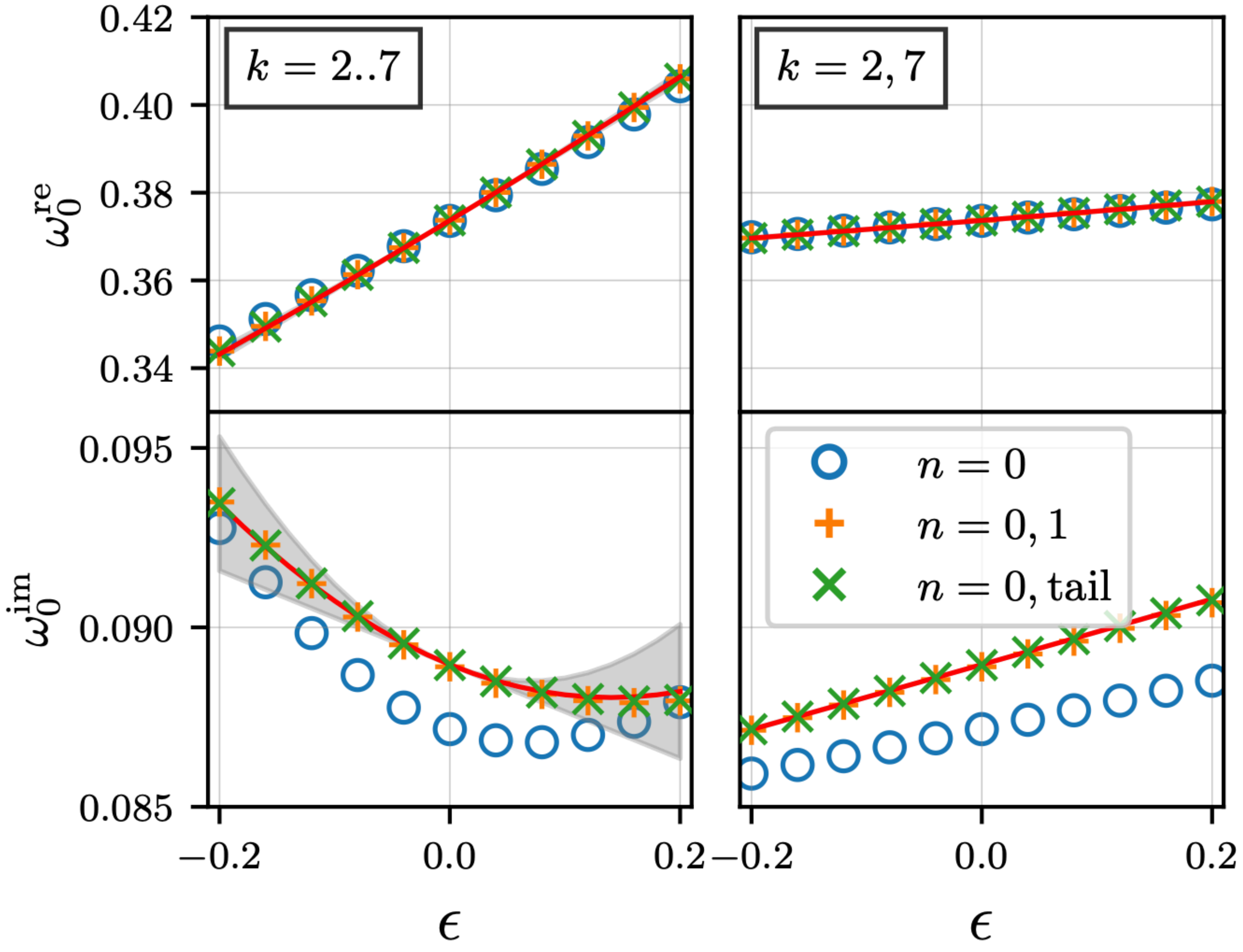
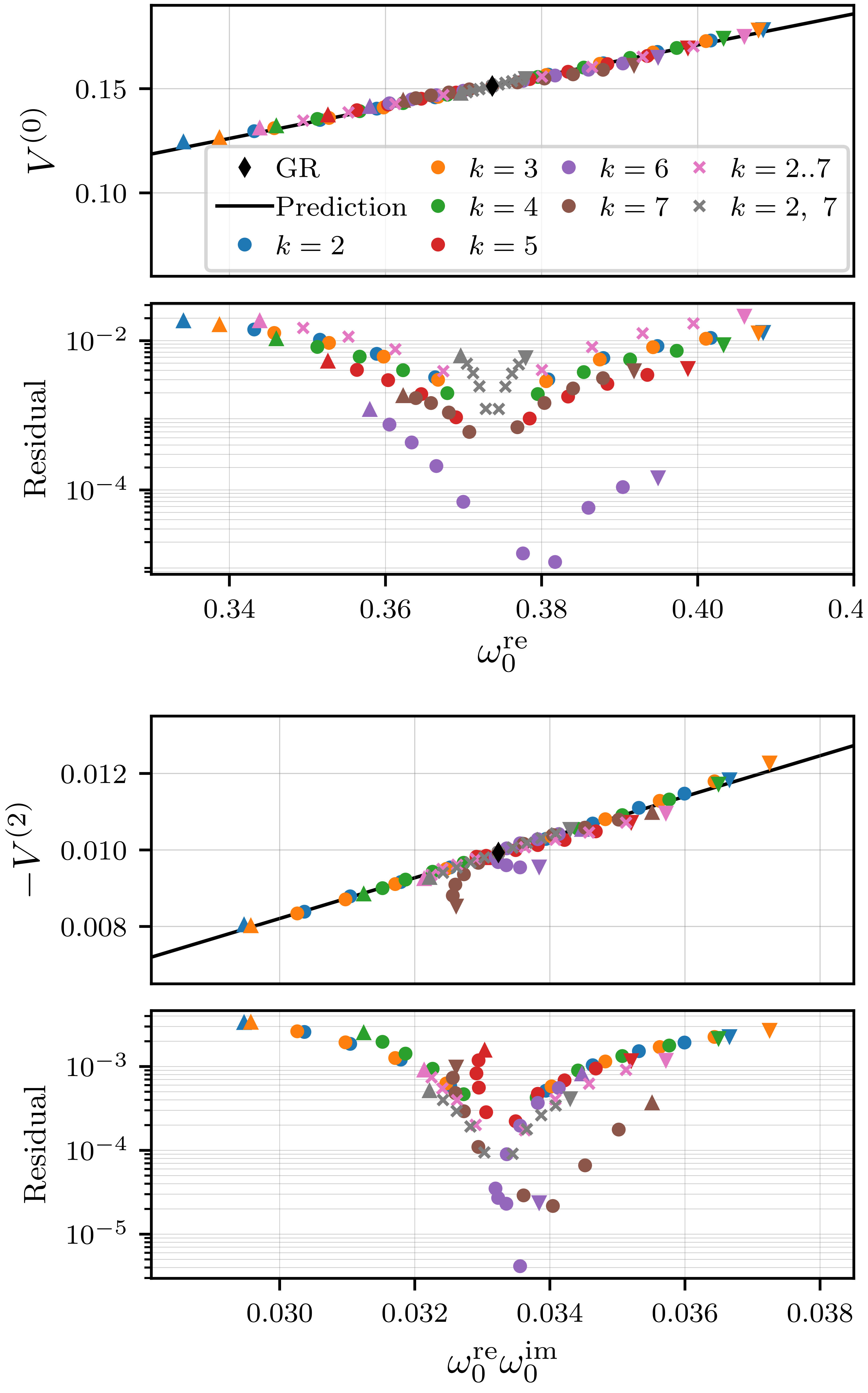


FIG. 9. Extraction of the $n = 0$ QNM for $t_0 - t_{\rm peak} = 50M$ and comparison with the perturbative prediction. Here we show similar results as in Fig. 5, but when varying multiple $\alpha^{(k)}$ including $k = 2...7$ (left) and $k = 2, 7$ (right) at the same time.

Thomopoulos, SV, Pfeiffer 2504.17848



Thomopoulos, SV, Pfeiffer 2504.17848

Systematic effects in black hole spectroscopy

Black hole spectroscopy

Important limitations of the simple picture

- Even within the validity of linear perturbation theory:
  - QNMs do not form a complete basis!
  - Other effects like the "prompt response" and late-time tails are missing.
  - Transition from non-linear merger to linear ringdown is a blurred line
- But, when is linear theory valid for analyzing binary merger signals:
  - How important are non-linear effects?
  - When can we even talk about a single black hole?
- All of these problems call for systematic effects in black hole spectroscopy!

Perturbations of the Schwarzschild black hole

The standard problem

- Perturbation theory gives wave equation

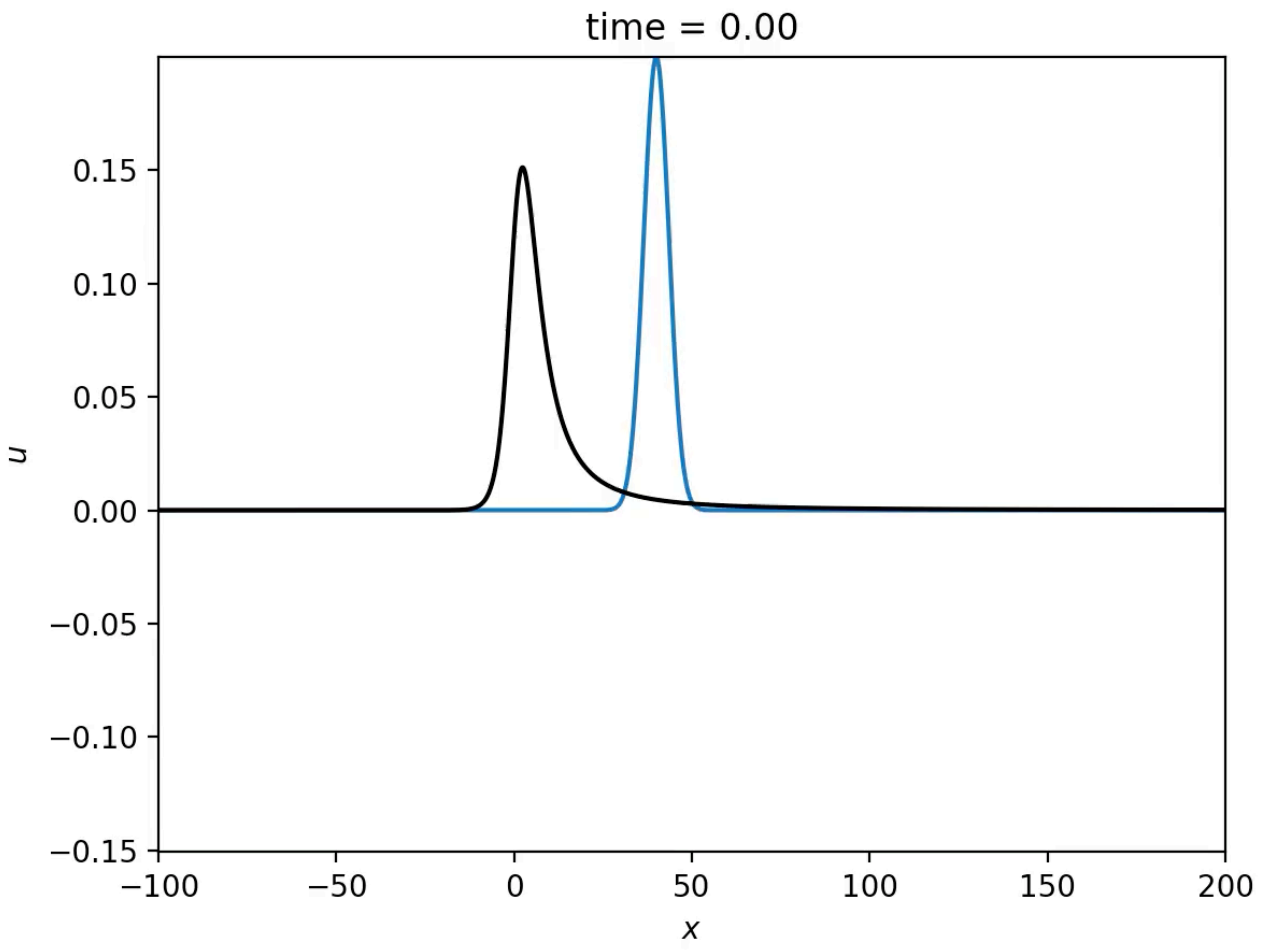
$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}u(t,x) - \frac{\mathrm{d}^2}{\mathrm{d}t^2}u(t,x) + V(x)u(x,t) = 0$$

  in a re-scaled radial coordinate.

- Can be studied as an initial value problem.

- Alternative computation as a boundary value problem (defines QNM spectrum).

- Many QNM problems are already present here!

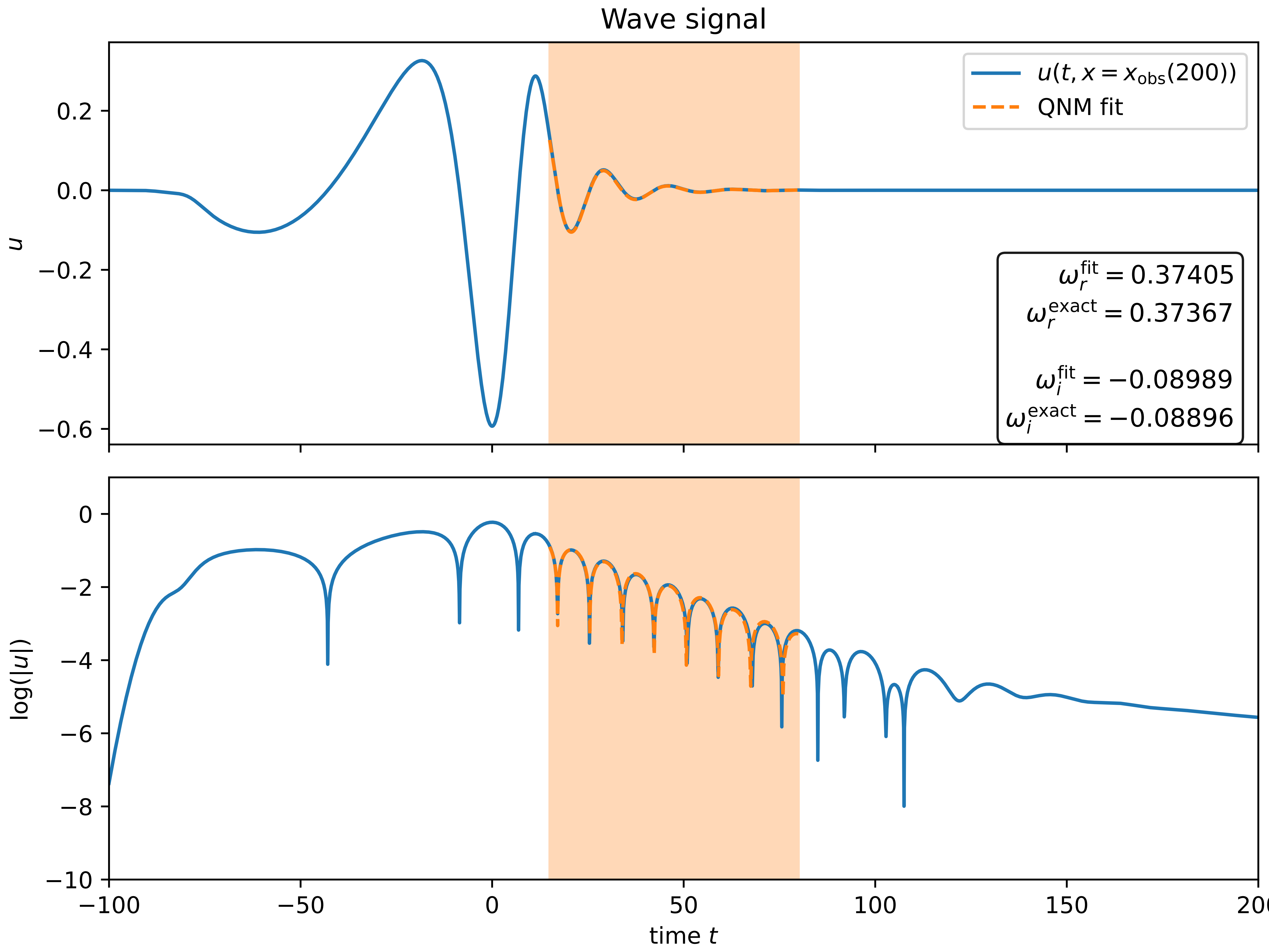


Analysis of initial data

What does an observer far away see?

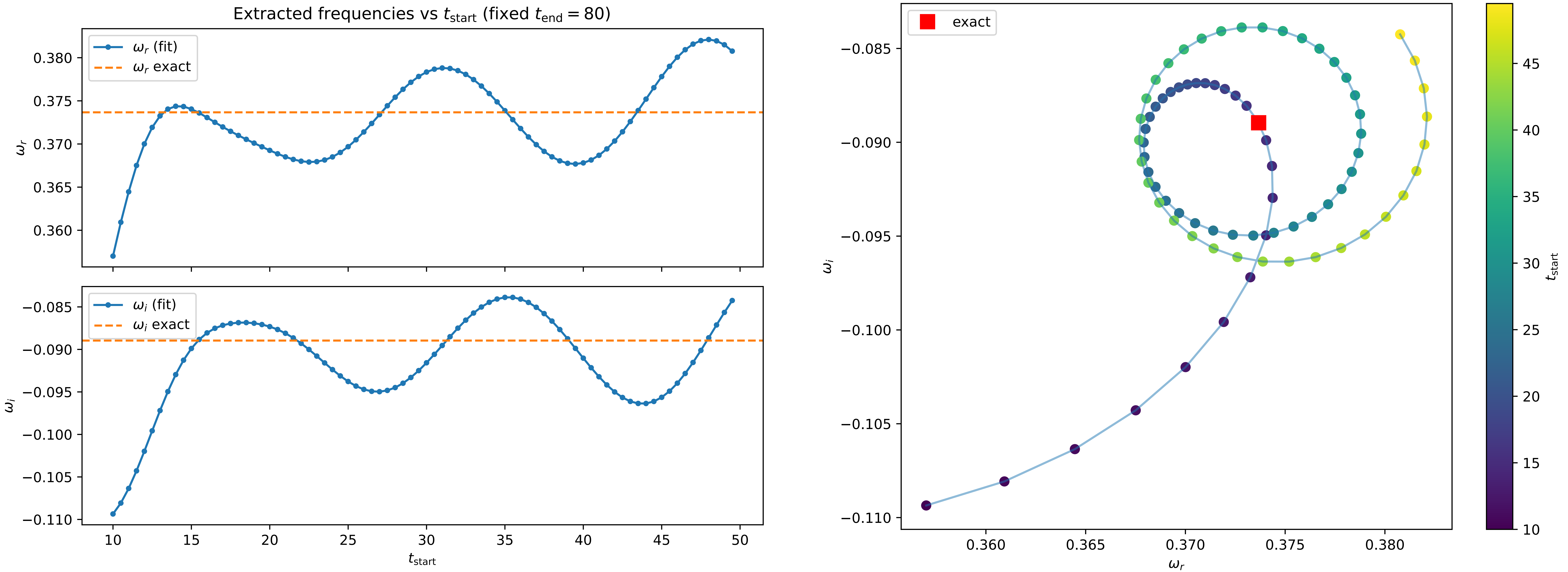
- Analy… $= x_{obs}$
- Three…
  - “pr… that de…
  - “QN… clean da…
  - “ta… below a po… ce tai…

Really that simple?!



Analysis of initial data

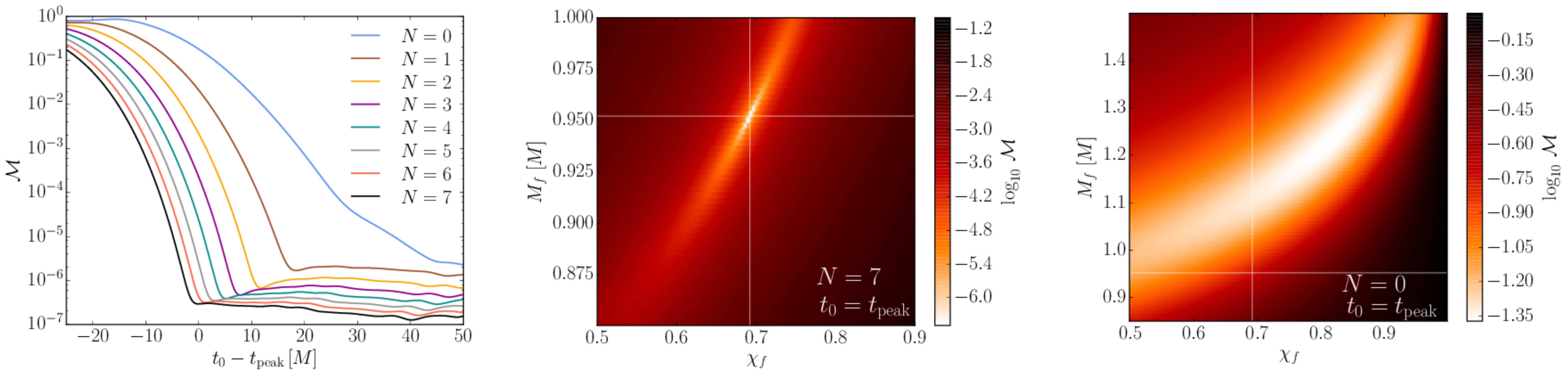
Dependency on when we start the fit



Do we just need to add more QNMs?

There are infinitely many QNMs in the spectrum

- Including many overtones sufficient to model ringdown (Giesler+ Phys.Rev.X 9 (2019) 4, 041060):

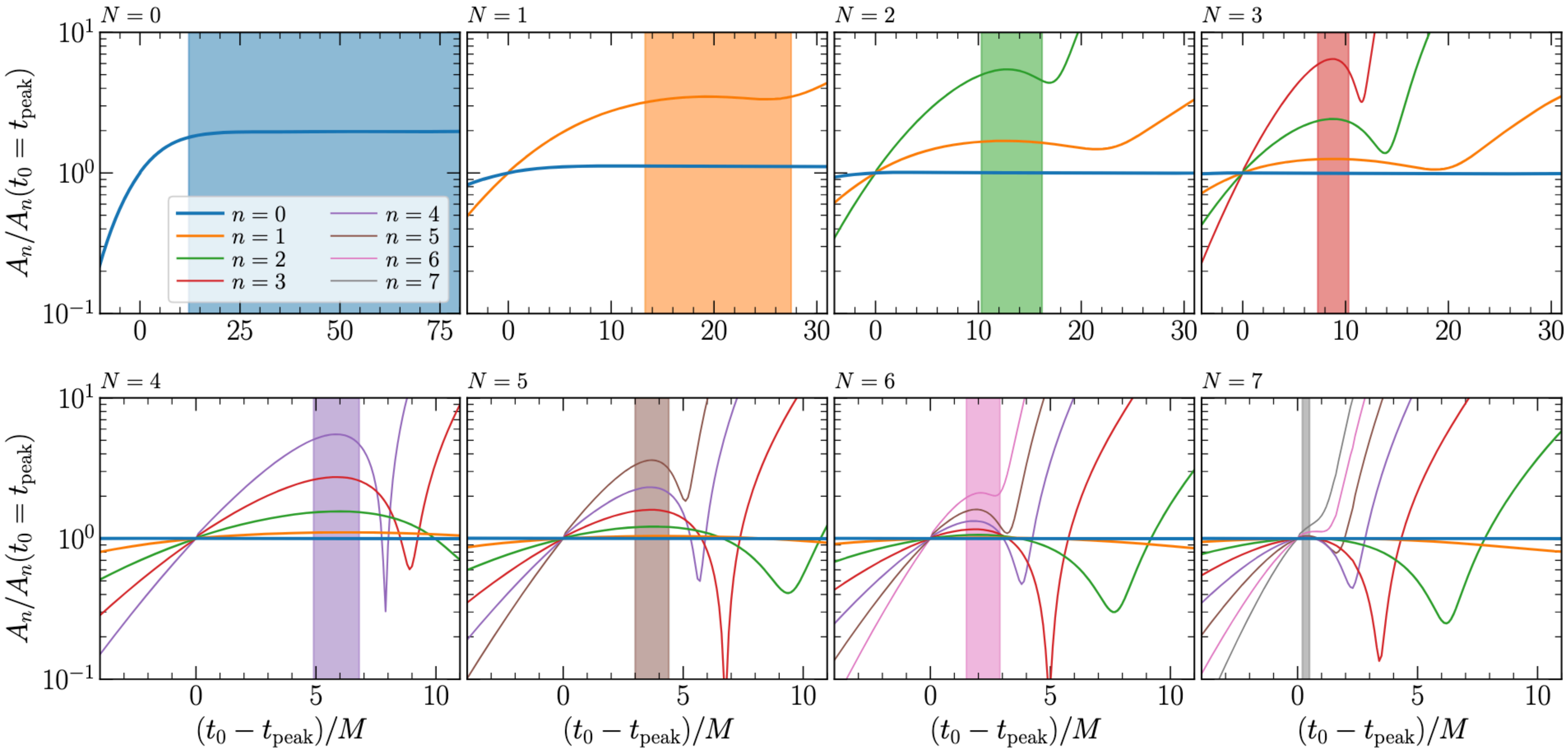


- Controversial claim, many works challenge (at least partially) this claim!

- Here one "assumes the theory" $\omega_{\ell mn} = \omega_{\ell mn}(M, a)$

QNM fitting problem is unstable!

The more QNMs are added, the more unreliable become their fits

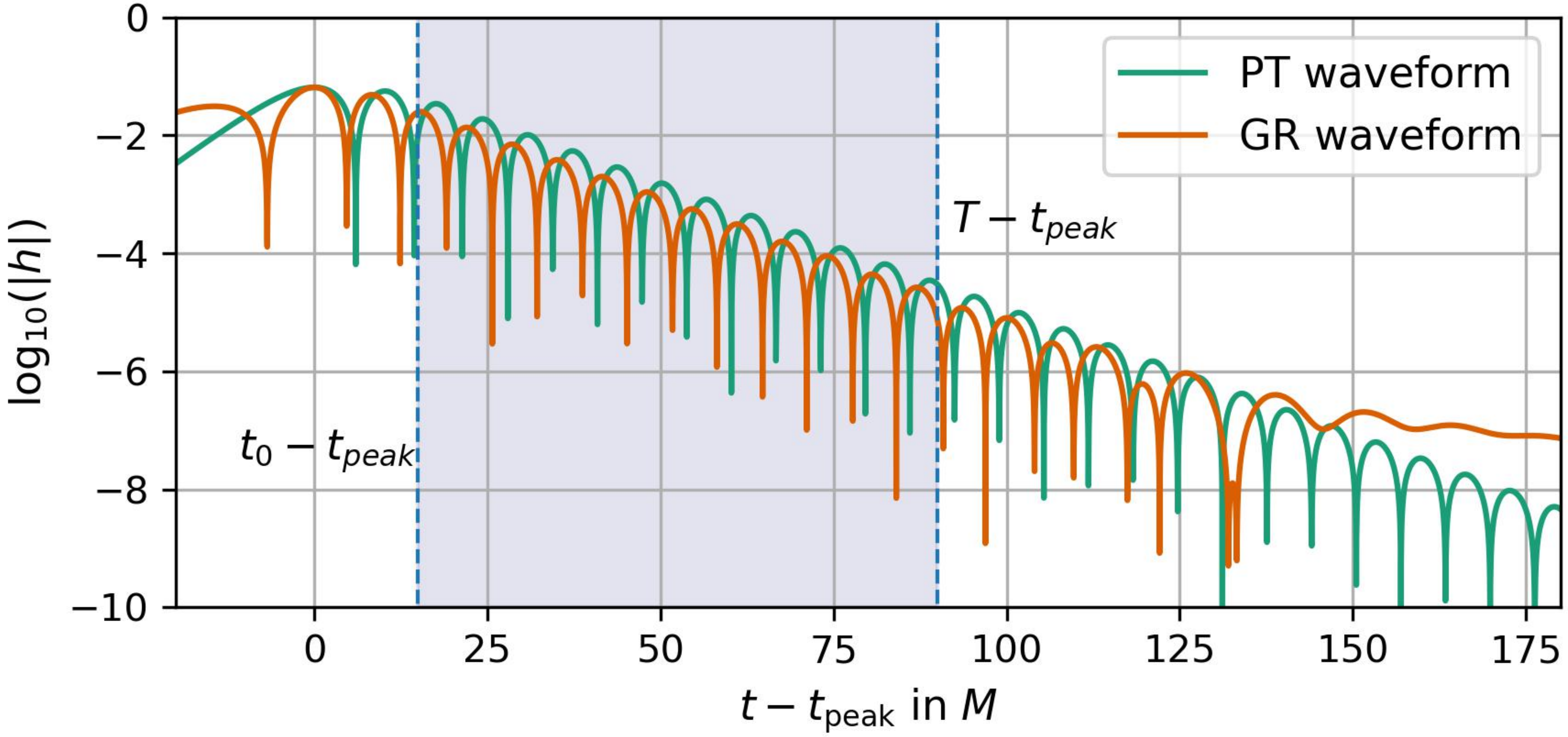


Extracted QNM amplitudes as a function of starting time of the fit when analyzing a numerical relativity simulation
(Figure 6 in Baibhav+, *Phys.Rev.D* 108 (2023) 10, 104020)

Is including more overtones "physical"?

- Check by comparing two simple and clean theoretical setups with different QNMs in the time domain Nee, SV, Pfeiffer, Phys.Rev.D 108 (2023) 4, 044032.

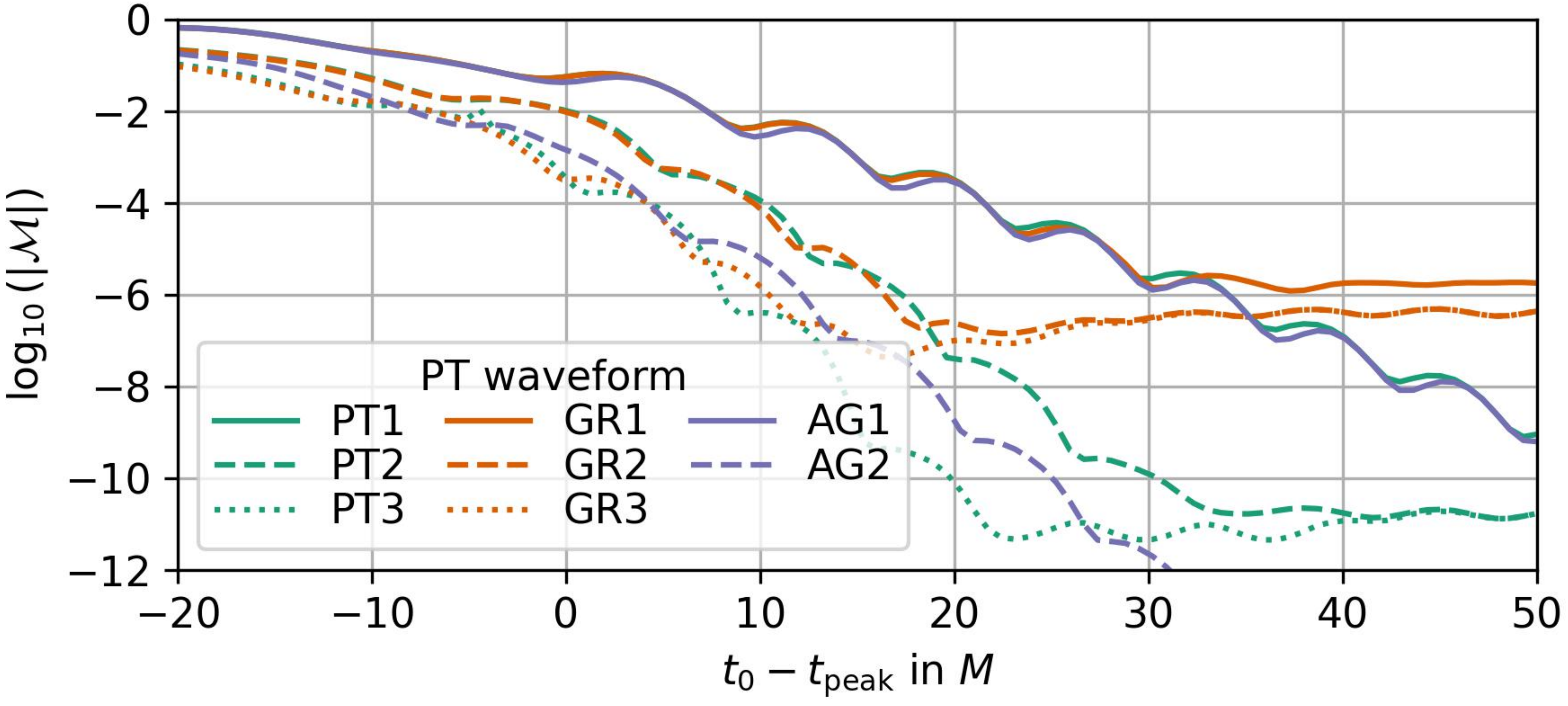
- Can one get "physical" results by including more QNMs in the "wrong" model?



Nee, SV, Pfeiffer, Phys.Rev.D 108 (2023) 4, 044032

Is including more overtones "physical"?

- First, vary the start of the analysis from the peak of the signal $t_0 - t_{\text{peak}}$.
- Compute the "mismatch" for different theory-models (PT or GR) or agnostic QNMs.
- Also vary the number of included QNMs.

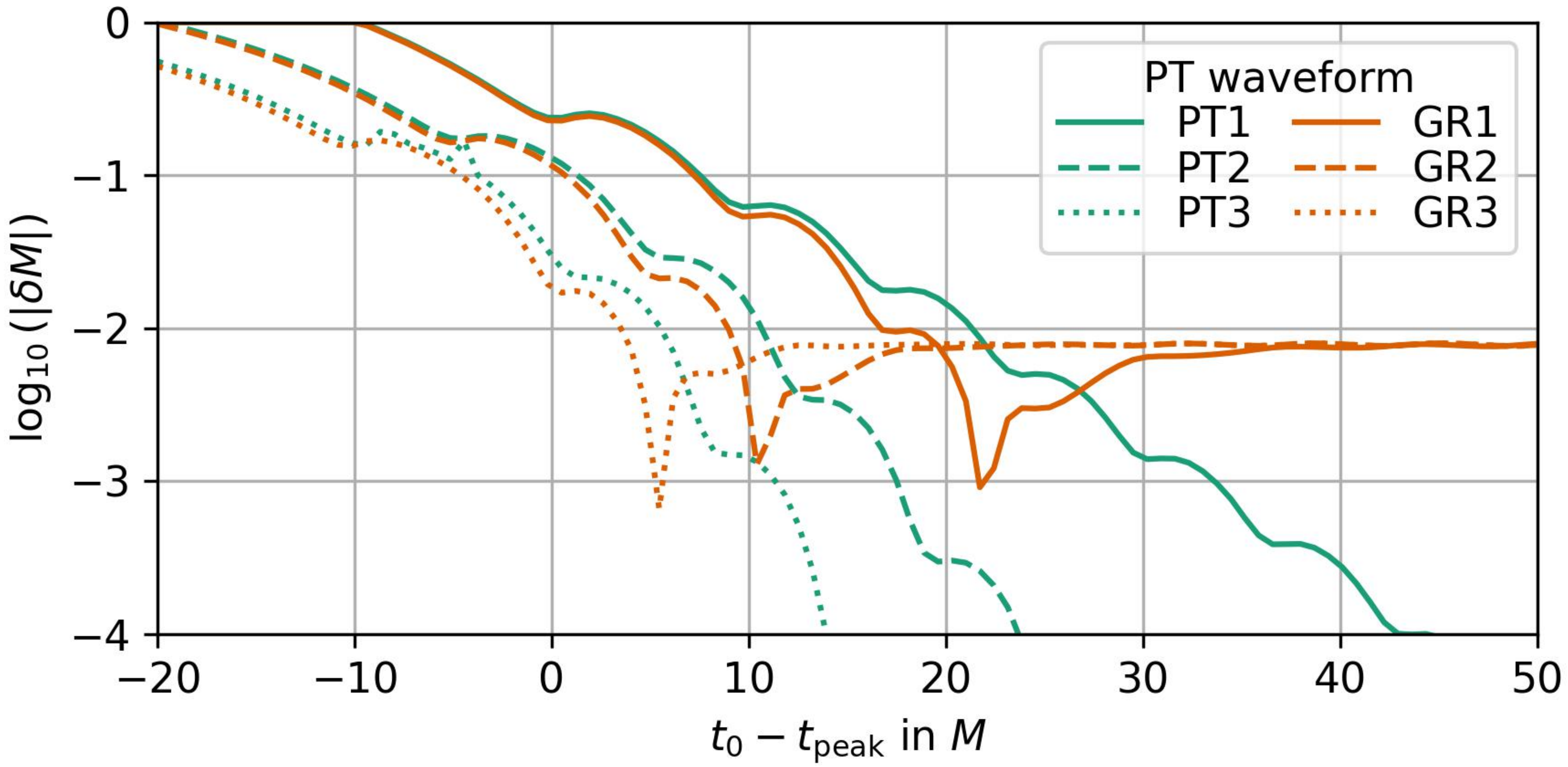


Nee, SV, Pfeiffer, Phys.Rev.D 108 (2023) 4, 044032

Is including more overtones "physical"?

Extracting the black hole mass

- For both models, including overtones reduces error on final mass at early times.
- Small bias in the mass at late times using the "wrong" model.



Nee, SV, Pfeiffer, Phys.Rev.D 108 (2023) 4, 044032

Quantifying modeling inaccuracies

Systematic and efficient way to predict biases

- Assume that a model h_m used to analyze a signal h_s is not accurate.
- Model and signal both depend on parameters θ.
- Analyzing $h_s(\theta_{tr})$ with h_m can cause systematic errors $\delta\theta = \theta_{tr} - \theta_{bs}$.
- Naturally arise when "best-fit" $h_m(\theta_{bf})$ does not occur at true parameter θ_{tr}.
- For large signal-to-noise ratio (SNR), this can be studied with linear-signal analysis (LSA).
- Can we predict the systematic errors from unmodelled effects?

Basics of GW data analysis

Overview of key equations

- Measured data d contains noise and the actual GWs.
- Log-likelihood $\log(p) \propto \langle y | y \rangle$ with $y = h_m(\theta) - d$, where $m(\theta)$ is the GW model.
- Inner product defined as $\langle a, b \rangle = 4\mathrm{Re} \int \frac{ab^*}{S(f)} \mathrm{d}f$, detector described by $S(f)$.
- Linear-signal analysis (LSA) (assuming high SNR, expansion of log-likelihood at injected parameter) yields:
  - Inverse of Fisher matrix $\Gamma_{ij} = \left\langle \partial_i h \,|\, \partial_j h \right\rangle$ can be used to approximate statistical errors of parameters.
  - bias formula $\delta\theta_i = \left(\Gamma_{ij}\right)^{-1} \left\langle \partial_j h \,|\, h_{\mathrm{signal}} - h \right\rangle$ for estimating systematic errors of parameters

    Flanagan, Hughes PRD 57 (1998), Cutler, Vallisneri PRD 76 (2007), ...

Quantifying systematic biases in black hole spectroscopy

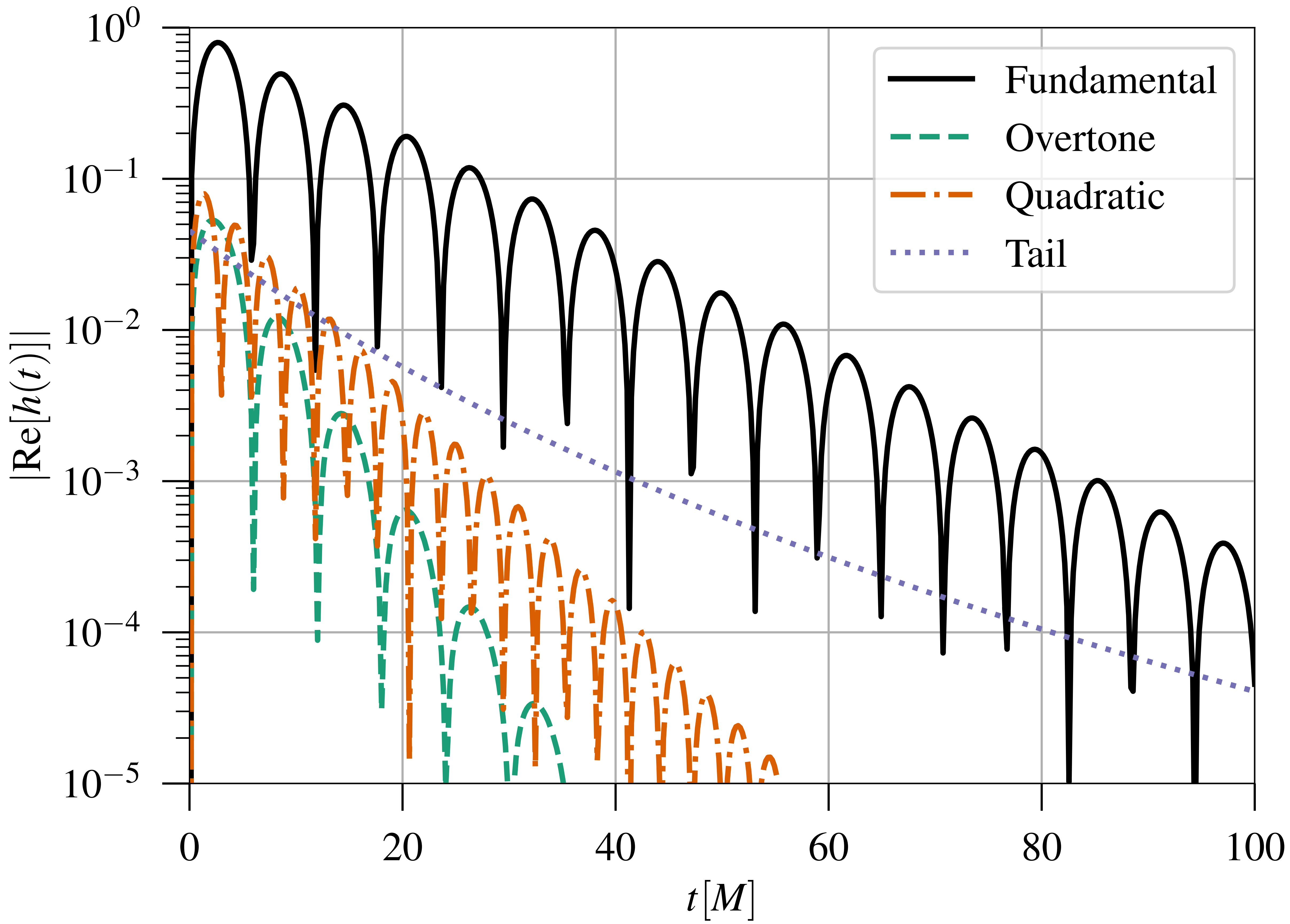
SV & Dhani, *Phys.Rev.D* 112 (2025) 8, 084076

- Application of linear-signal analysis for bias prediction from unmodelled effects.

- Clean injection/recovery setup with
  -LSA (Fisher matrix and bias formula)
  -Bayesian MCMC for validation

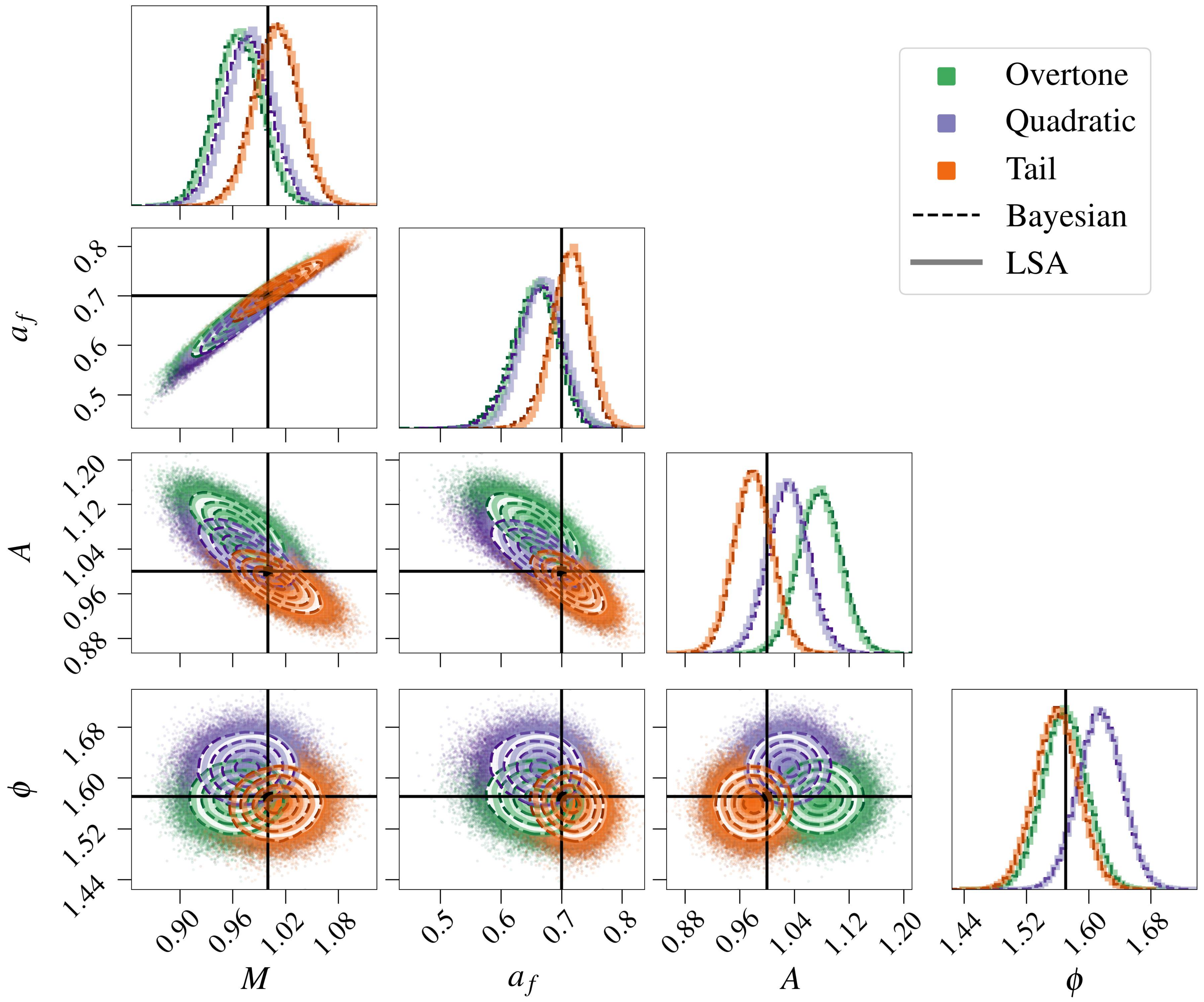
- Assume model is only the fundamental mode, how much does an unmodelled:
  -overtone,
  -quadratic mode,
  -or tail
  bias the fundamental mode's parameters?

- Complementary for LISA: Capuano+ 2506.21181

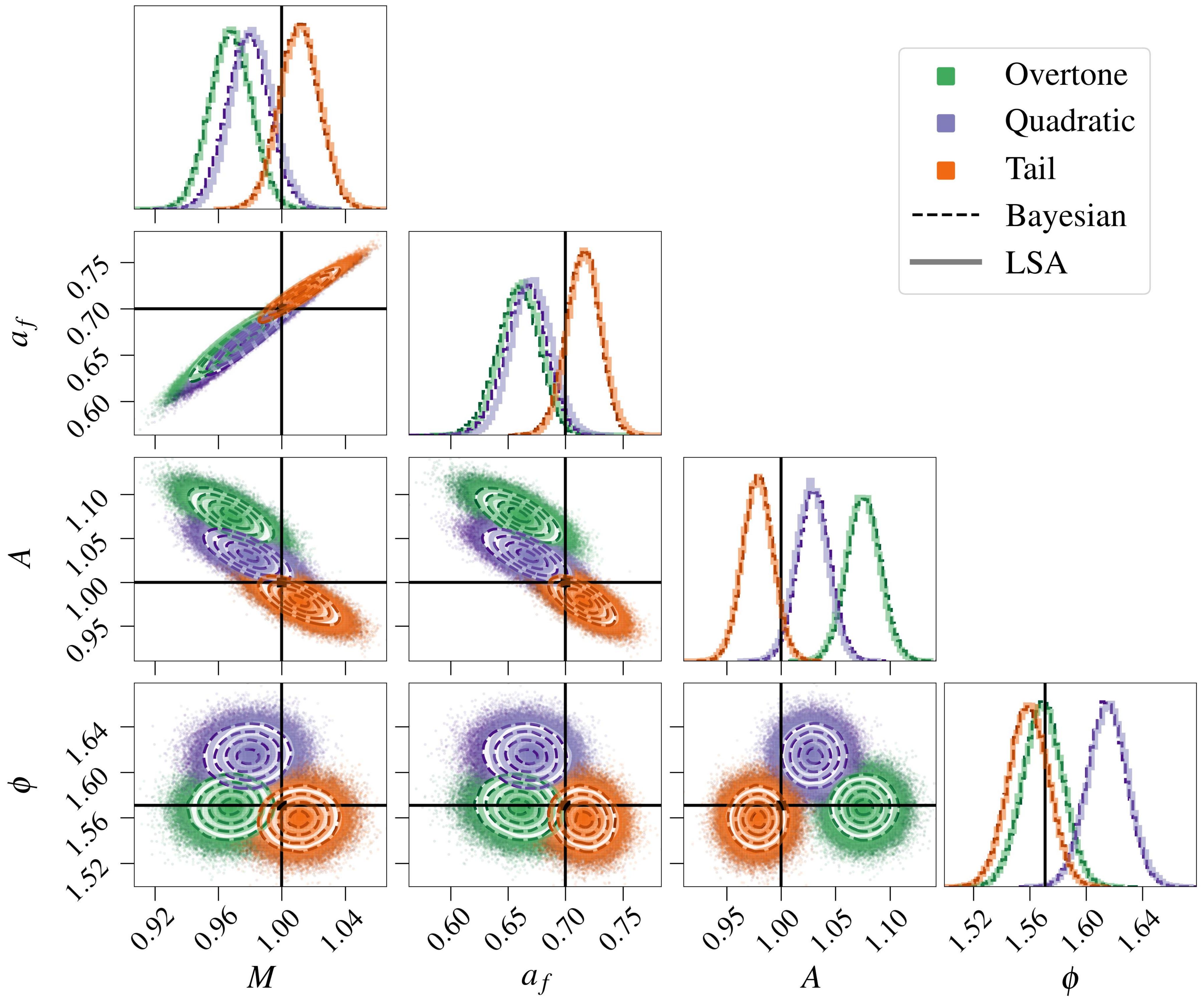


Theory-specific model

fundamental mode modeled as: $\omega_{220}(M, a_f)$



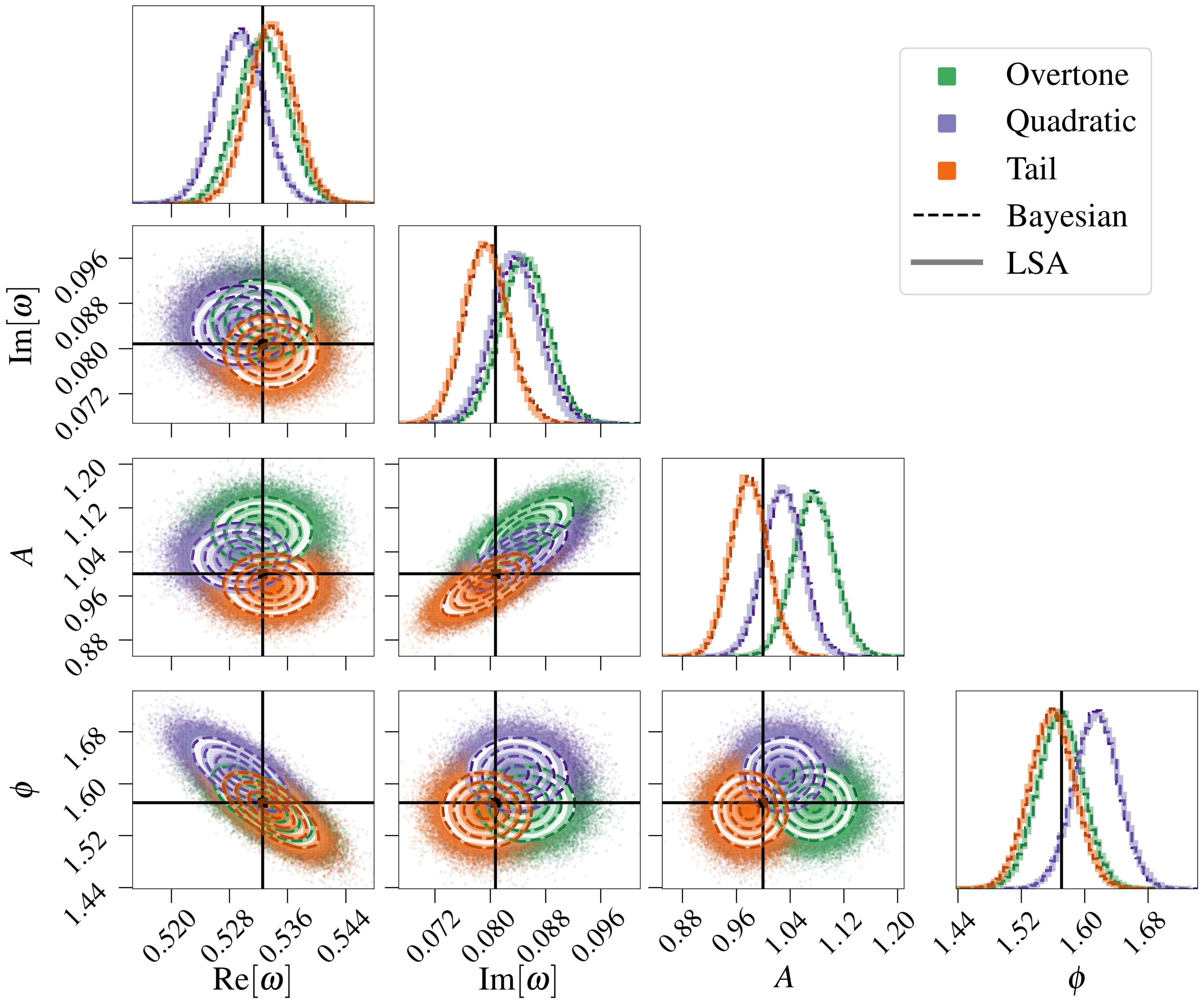
SNR=50



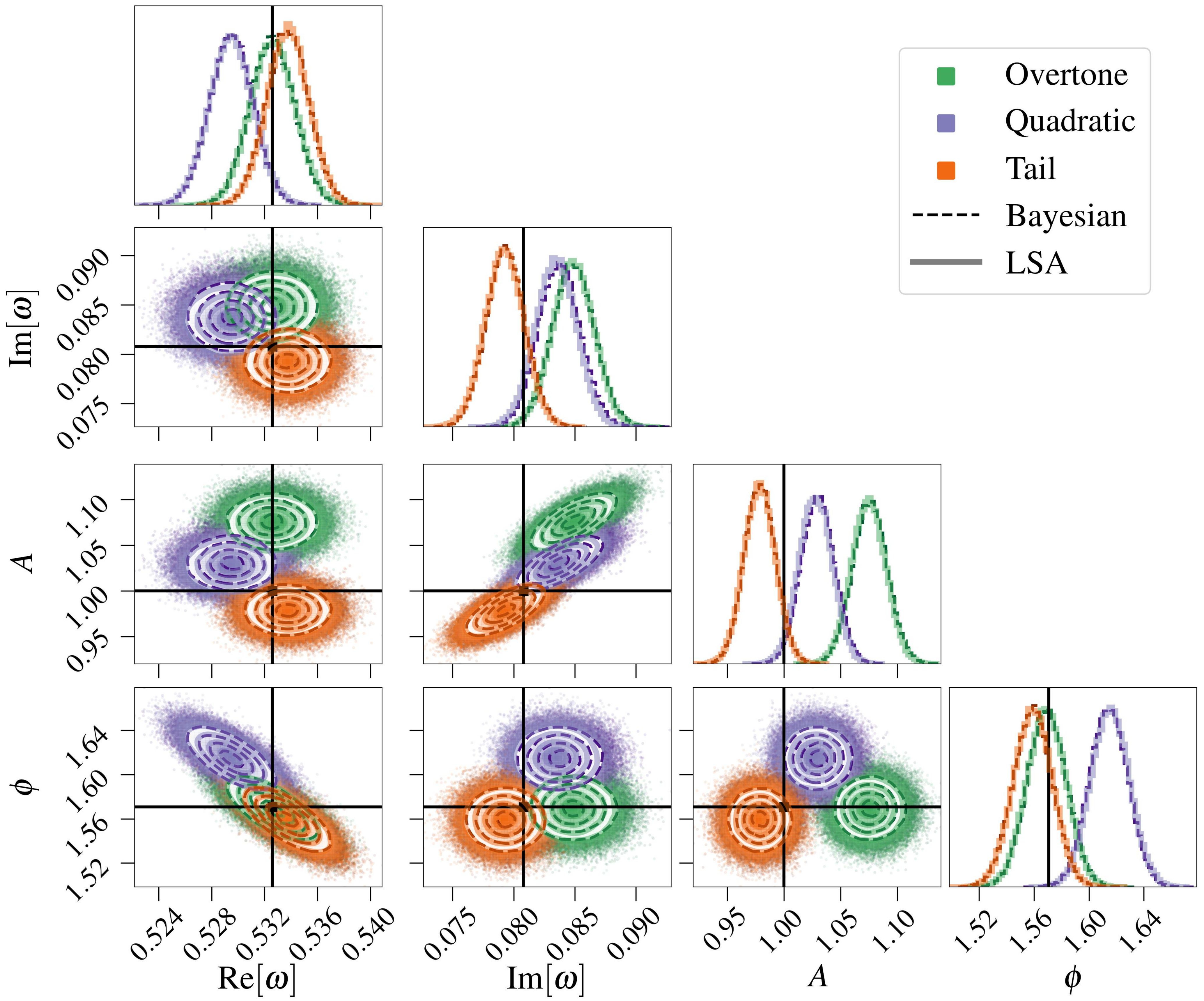
SNR=100

Theory-agnostic model

fundamental mode modeled as: $\omega_{220} = \omega_r + i\omega_i$



SNR=50

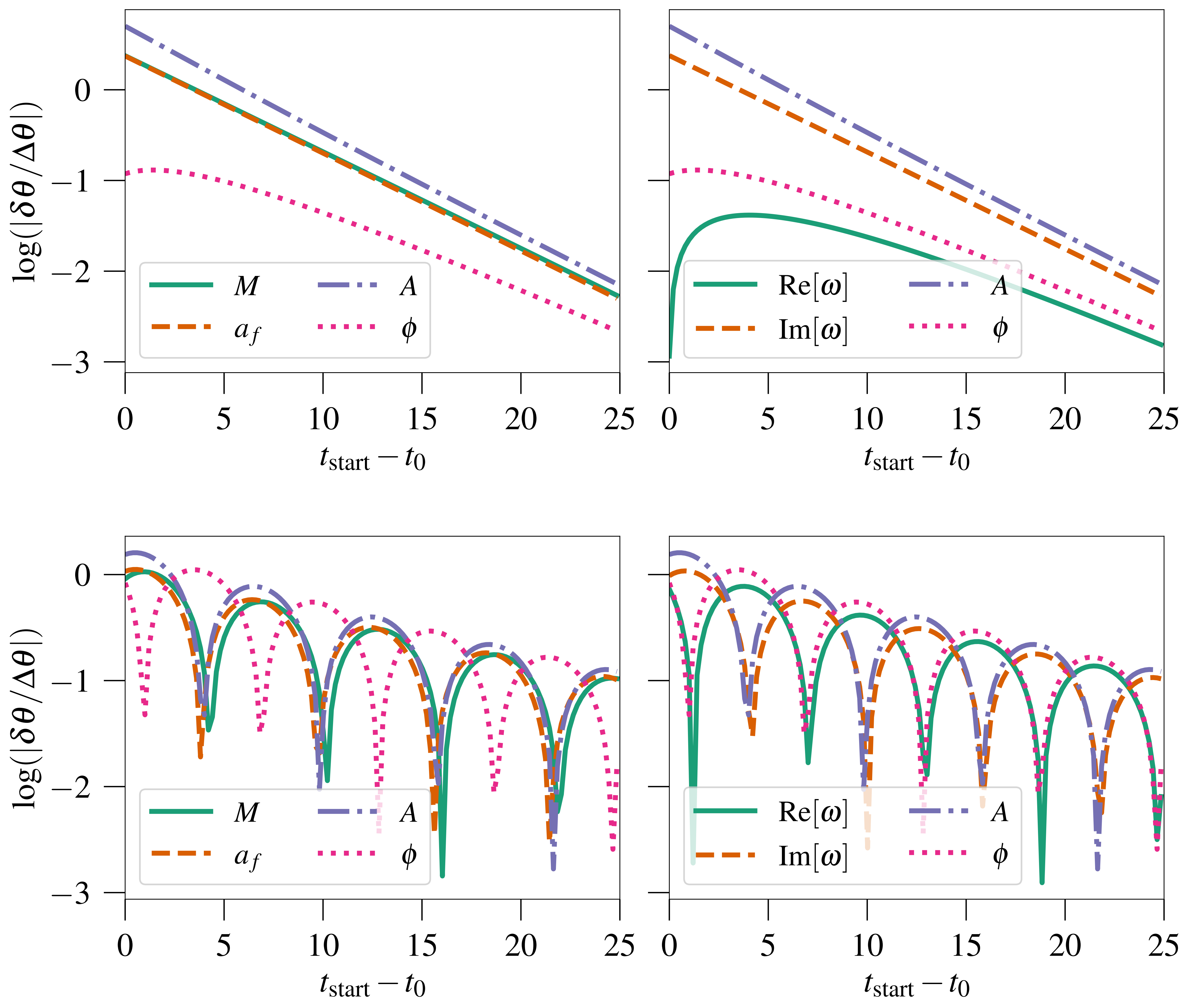


SNR=100

Impact of starting time

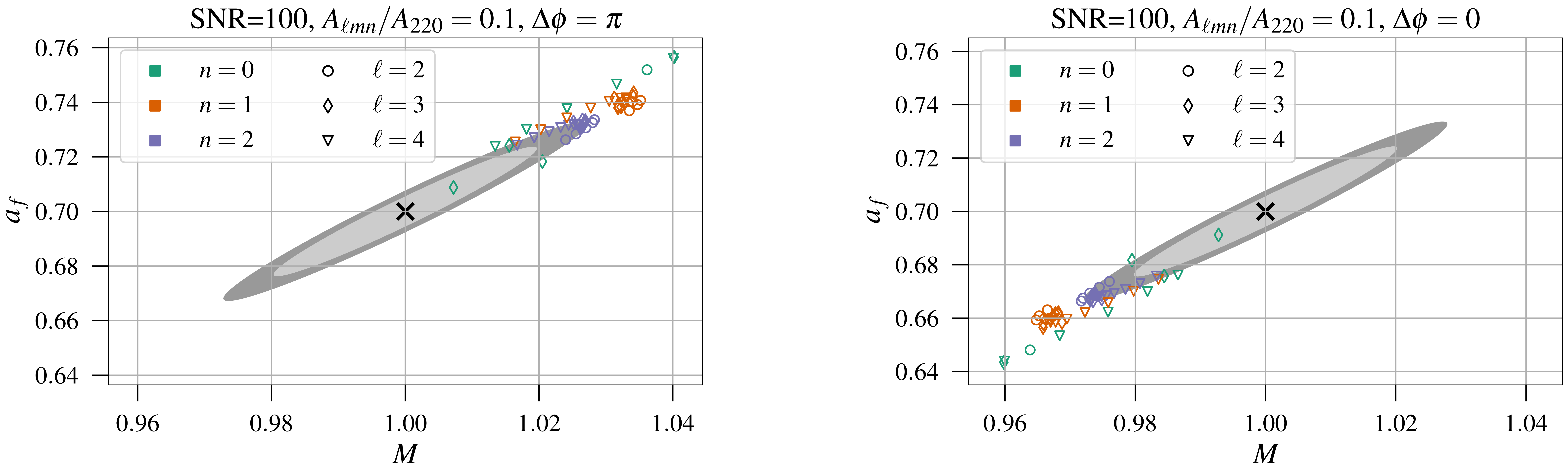
Expensive for Bayesian analysis, cheap for LSA

- Introduce the bias ratio $|\Delta\theta/\delta\theta|$ (systematic over statistical error)
- How does it depend on the start of the analysis?
- Decreasing statistical error and increasing/decreasing systematic error (competition)
- Top: overtone. Bottom: tail. Different behavior.
- Increasing SNR would just lift up lines.



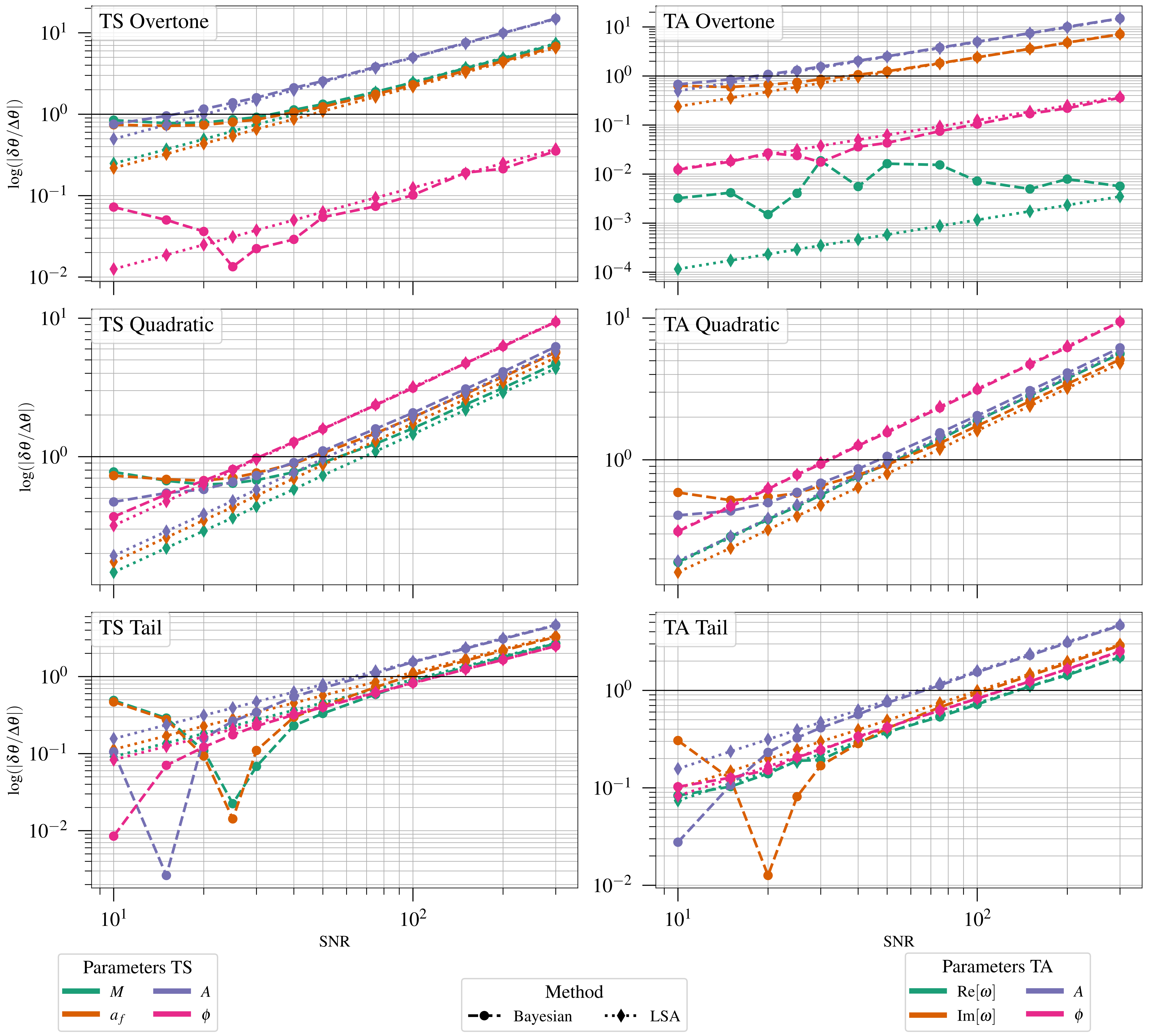
Other possibly unmodelled QNMs

Model is only fundamental mode



Biases at high-SNR

- Basic QNM ringdown tests are in trouble!
- Biases already at tens to hundreds of SNR.
- Overall, bayesian and LSA agree very well.
- Unreliable for very small biases (not physically interesting).
- LSA method can be used for future studies!
- Note: alignment procedure needed when comparing IMR waveform models/families
  Dhani, SV, Buonanno, Estelles, Gair, Pfeiffer, Pompili, Toubiana Phys.Rev.X 15 (2025) 3, 3



Relation to approximate QNM methods

Eikonal approximation, post-Kerr

- Approximate QNM methods useful for "theory-agnostic" tests of BH metrics, e.g., Dey+2212.10725 (often only for QNM shifts).
- However, difficult to assess validity beyond null-tests without knowing true gravitational QNMs, especially for rotating BHs.
- Recently obtained 2+1 scalar-field evolution in modified-Kerr background to extract fundamental scalar QNMs (Prony).
- As a proxy, compared pure Eikonal and post-Kerr results with "exact" time-domain QNMs (caveat scalar vs gravitational)
- Quantified statistical errors with systematic ones!
- What SNR, what ℓ, what method, what spin a, what deviation, ...

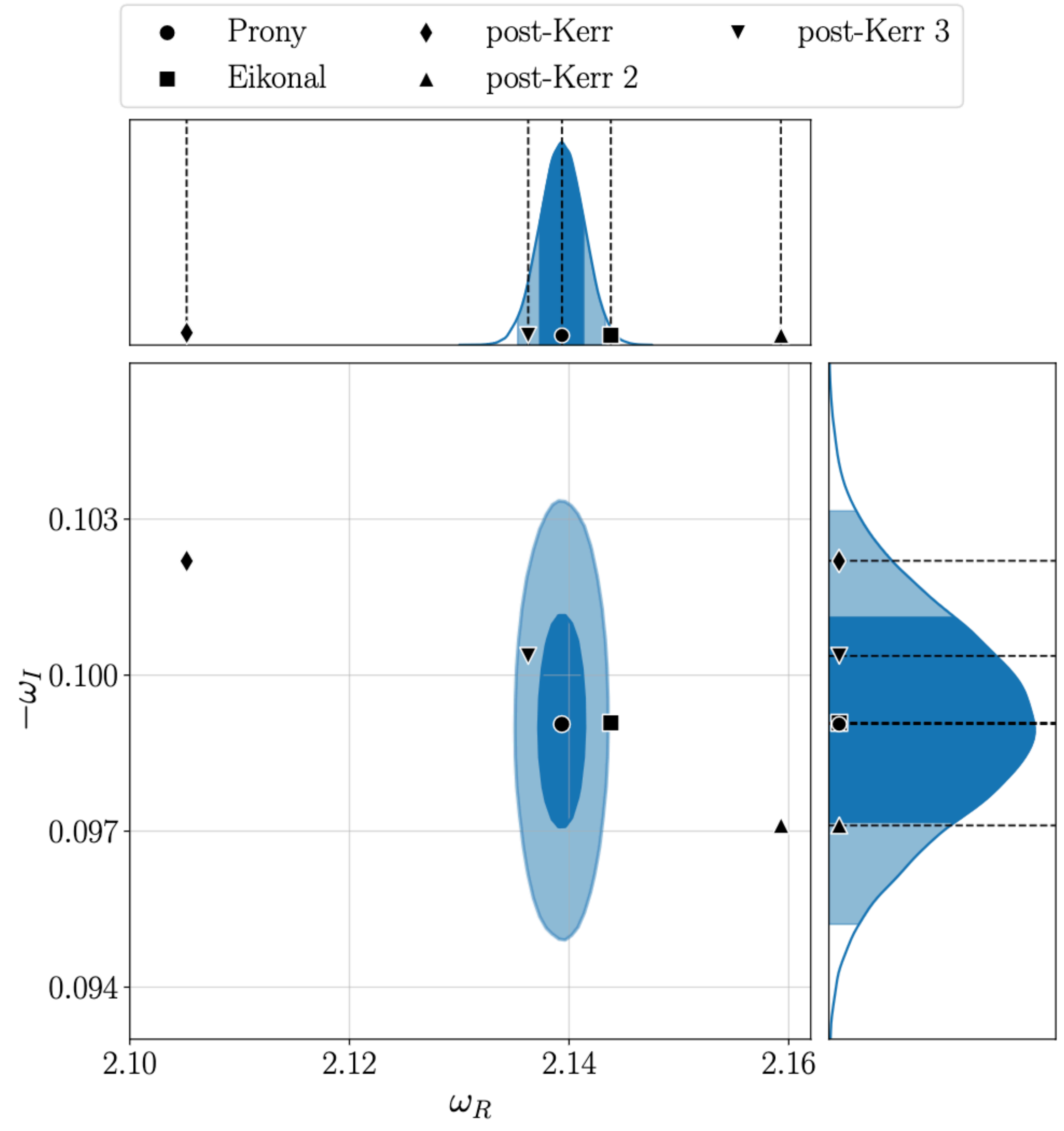


FIG. 6. Corner plot for ω_I vs ω_R, obtained for the mode ($\ell = 10$, $a = 0.3$, $\epsilon = 1$) at SNR $\rho = 100$, with the 1σ and 2σ confidence intervals. The Kerr ($\epsilon = 0$) QNM $\omega^{\rm Kerr} = 2.28 - 0.095\,i$ is well outside the confidence interval.

De Simone, SV, Kokkotas, De Falco, Capozziello
2601.09607 (PRD)

Conclusions systematics

Statistical errors may not be the limiting factor for fundamental physics with QNMs.

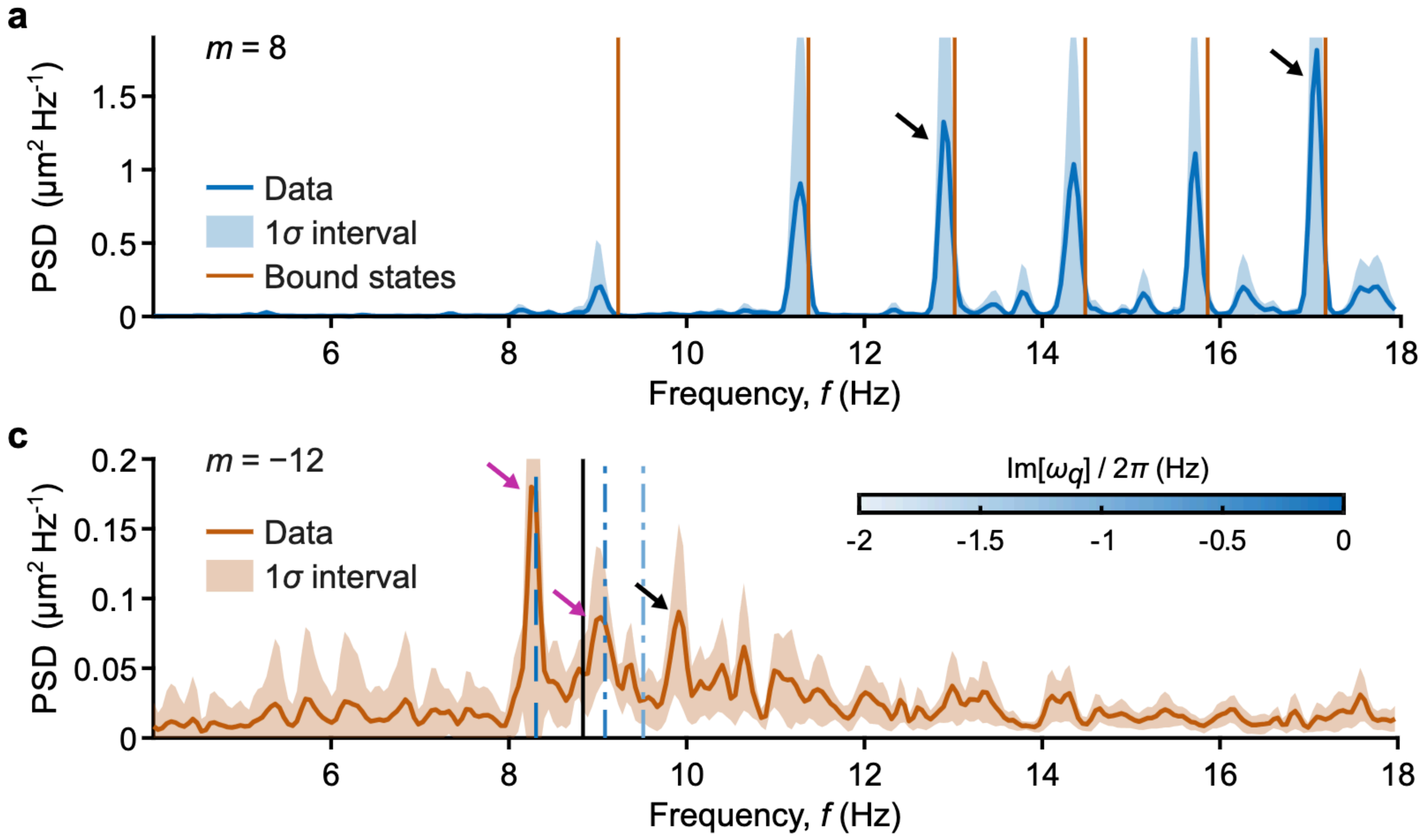
- Direct extraction of QNMs is more difficult than anticipated for a long time.
- Studies of recent years demonstrate a significant presence of systematic errors.
- Biases will overtake at sufficienlty high SNR, crucial for future detectors.
- Our work shows that small unmodelled effects are important for QNM extractions.
- BH Spectroscopy is rich: overtones, quadratic QMMs, mode-mixing, prompt-response, precession, tails, memory, …but it needs more robust methods for extraction!

Spectroscopy of analogue black holes using simulation-based inference

Spectroscopy of analogue black holes

Complementary physics with new challenges

- Analogue black holes allow controlled laboratory studies of QNMs, boundary effects, etc.
- Realistic experiments often driven by noise, not by controlled evolution of "initial data"
  Švančara+ Nature 628 (2024)
- Recently studied in theoretical framework with noise-driven, finite-size Pöschl-Teller potential.
  Solidoro+ Phys. Rev. Lett. 135 (2025) 5, 051401
- Very different from standard astrophysical black hole ringdown (QNM spectrum) and GW detectors
- Open questions about "horizon" properties and more...



Ringdown spectroscopy of a confined analogue black hole
Smaniotto+ 2502.11209

Spectroscopy of analogue black holes

Avoid the likelihood problem

- We propose simulation-based inference (SBI) as alternative [specifically neural posterior estimation (NPE)]
  Solidoro, SV, Weinfurtner 2604.12800
- SBI learns the posterior directly from simulations, avoiding the need to write down an explicit likelihood
- Only requires to "simulate" data, it does not explicitly require to distinguish between signal and noise!
- We train NPE directly on simulated noisy spectra generated from stochastic wave equation.
- We study two models with partially reflective boundaries:
  - Pöschl-Teller potential barrier (simplest system)
  - Shallow-water waves on a vortex flow (closer to hydrodynamical black hole models)

Spectroscopy of analogue black holes

Selected results

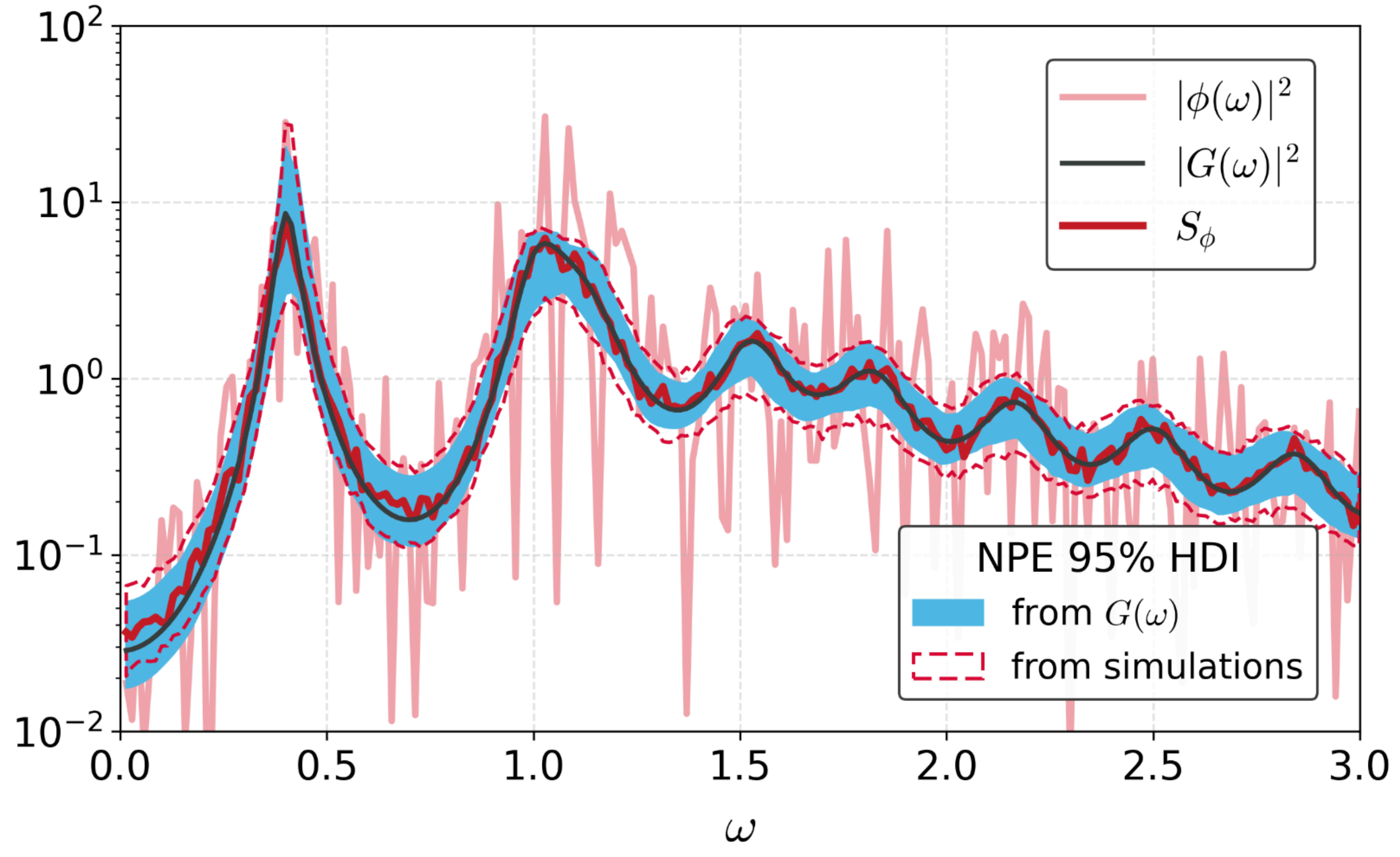
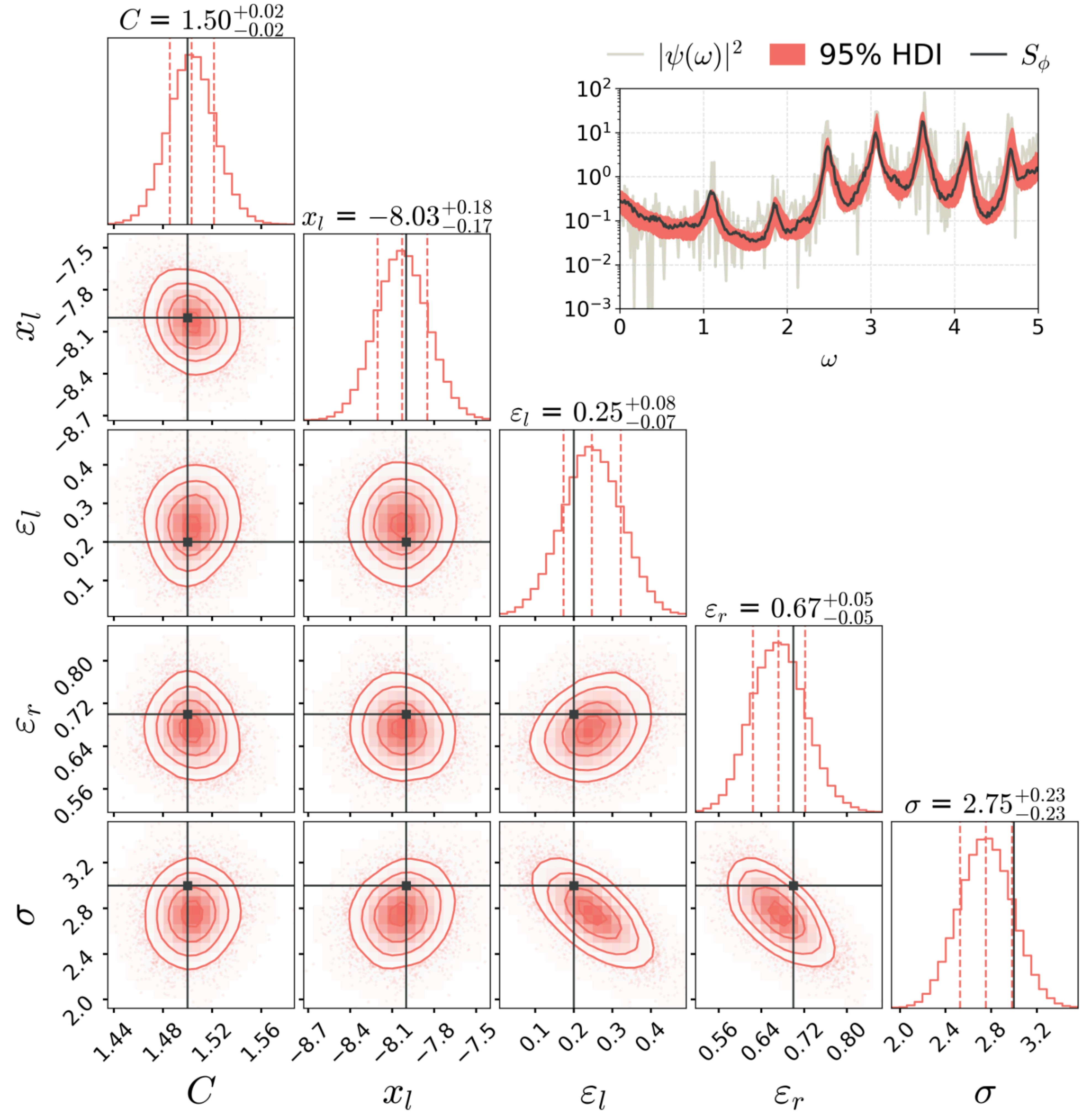


FIG. 1. Application of NPE to the spectrum of Pöschl-Teller noise with $\theta_{\mathrm{PT}} = [1.2, 0.9, -0.5, -5, 0.8, 0.2, 3]$ and $x_r = 4$. The normalized power spectrum obtained from a single noisy wave realisation (light red curve) is shown in log scale and compared with the noise-averaged power spectral density S_ϕ (red curve) and the squared modulus of the analytic Green's function $|G(\omega)|^2$ (dark curve). The regions within the blue band (dashed lines) indicate the 95% credible interval built from the Green's functions (the numerical PSD) with parameters drawn from the posterior distribution estimated by the NPE.



Spectroscopy of analogue black holes

More on shallow-water system

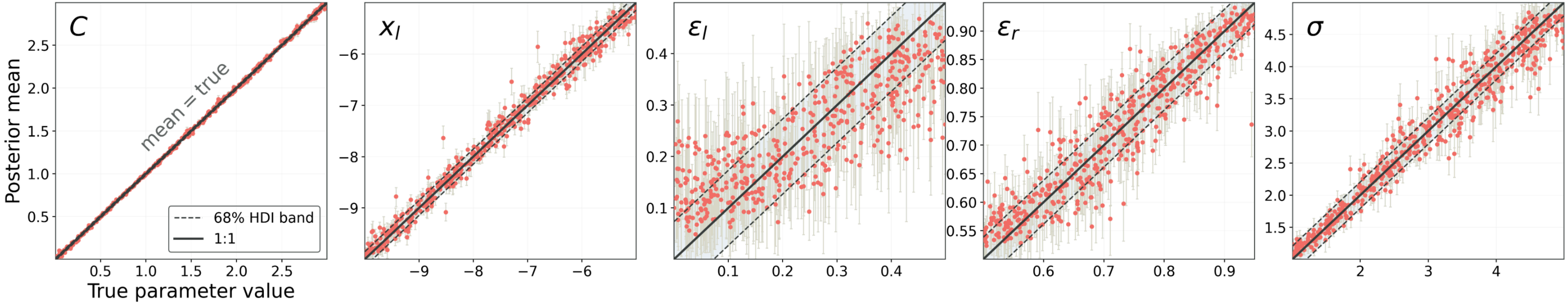


FIG. 3. Validation of the NPE for shallow-water perturbations with $m = 4$ using 500 samples from the test set. The mean of the posterior is plotted against its corresponding true value (orange markers), with 68% highest-density interval (HDI) error bars shown for every sample. The solid diagonal indicates perfect recovery, while the dashed lines denote the average 68% HDI width around the identity.

Spectroscopy of analogue black holes

Key take aways/open problems

- SBI can be used to infer parameters from noisy spectrum without access to likelihood!
- Proof of principle because data was “synthetic”, future extensions should include real data.
- Relation to GW science cases:
  - Current ground-based detectors can be “calibrated” without signals
  - SBI widely used (e.g. DINGO code), but mainly for speed, likelihood known (see importance sampling)
  - LISA will have many overlapping signals (global fit problem) and cannot easily be calibrated
  - ET/CE might be easier to calibrate, but could have similar problems?

Black hole spectroscopy of collapsing and merging neutron stars

Matter effects in the ringdown

Does matter matter?

- Simulated many collapsing neutron stars and several binary neutron star mergers (BAM code)
  Steppohn, SV, Dietrich, Phys. Rev. D 113 (2026)

- Analyzed ringdown carefully, similar to overtone analyses for black holes

- Categorized ringdowns into 3 classes: “clean”, “modified” and “distorted”

- *Key takeaways:*

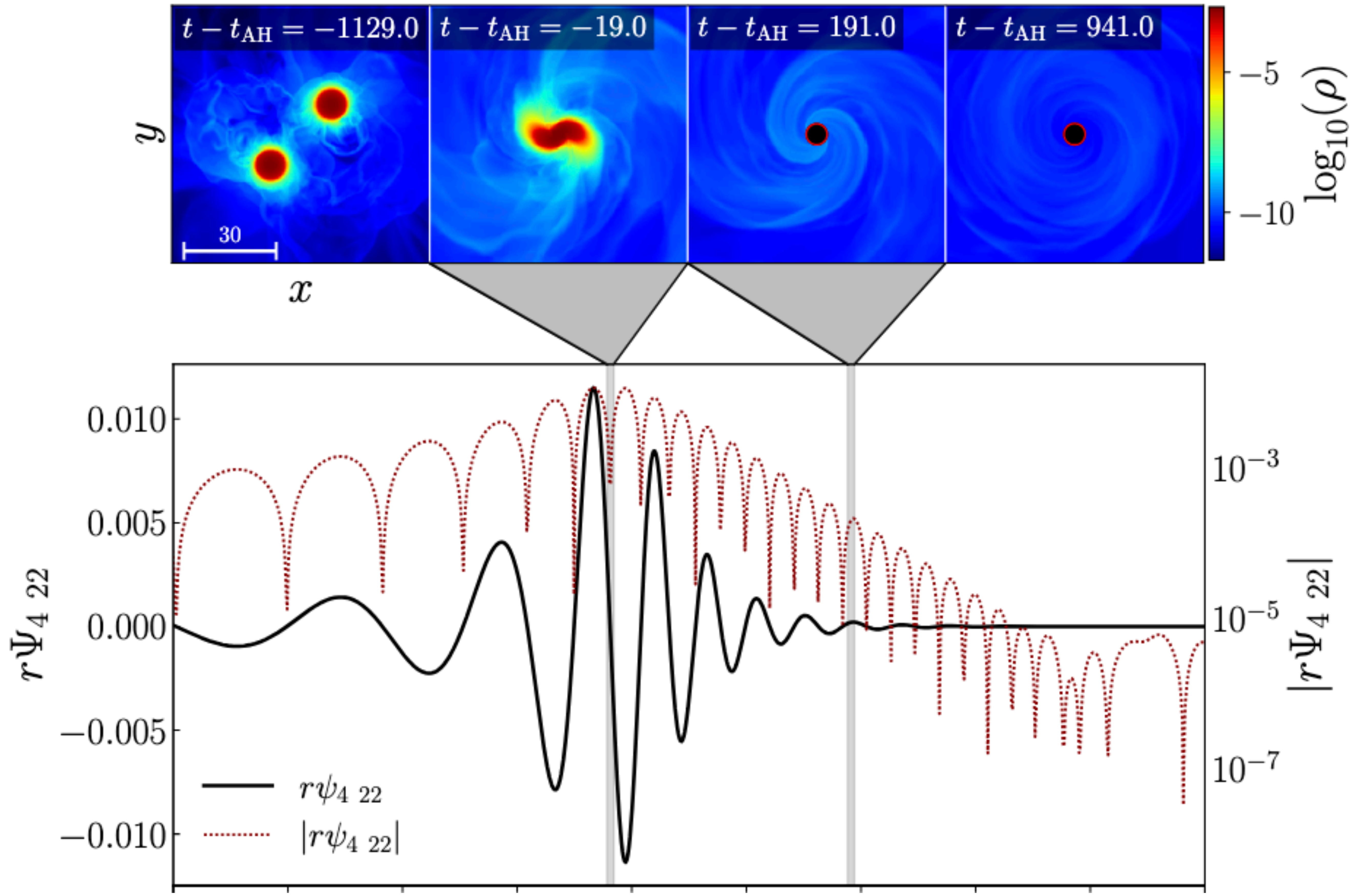
  - Matter effects are generally subdominant to ringdown-modeling systematics.

  - Including multiple QNMs reduces mismatches and improves final-mass estimates at early times.

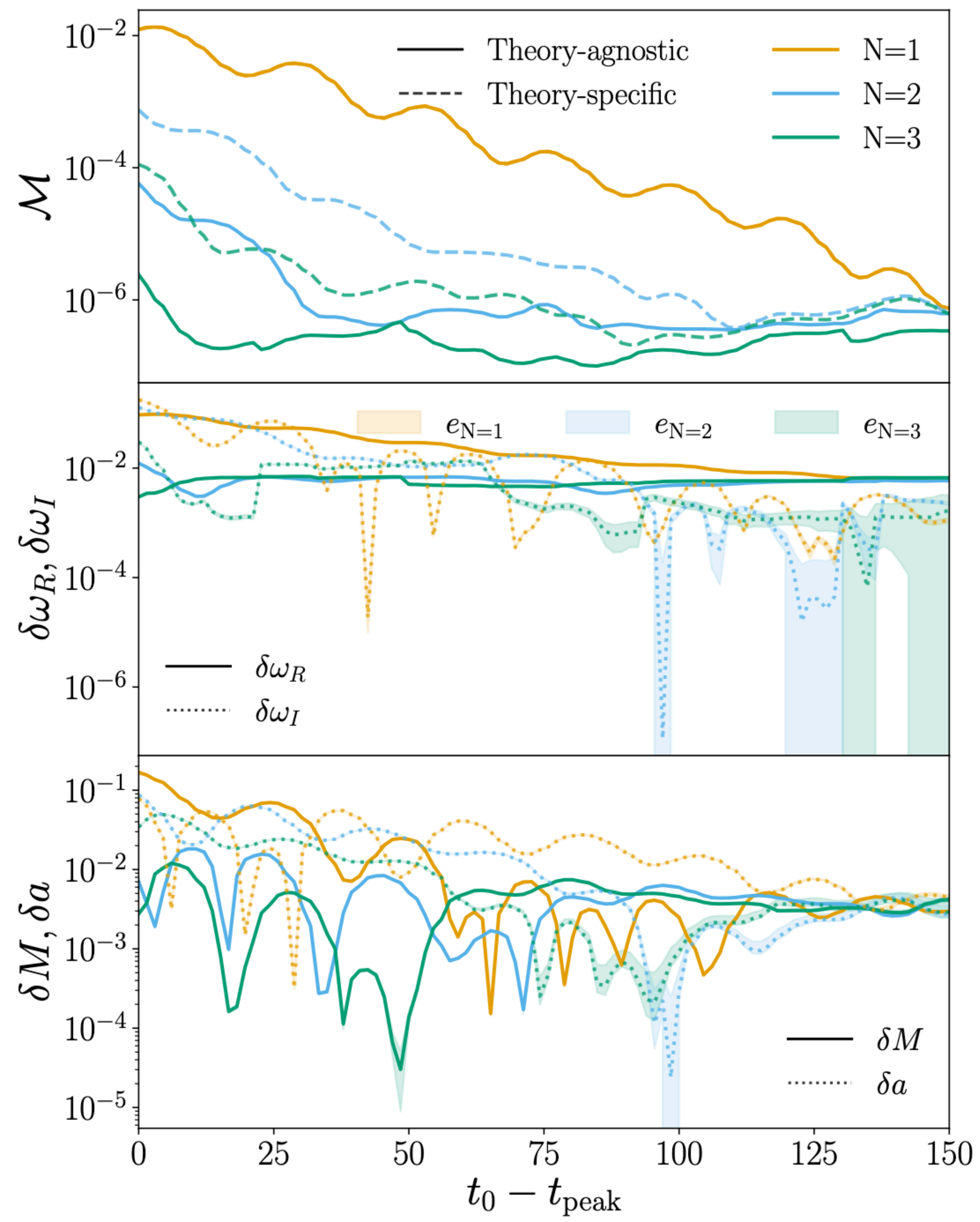
  - Single-mode fits can overestimate deviations and falsely suggest matter effects or GR violations.

Matter effects in the ringdown

Does matter matter?



Steppohn, Völkel, Dietrich, Phys.Rev.D 113 (2026) 2, 024011



Steppohn, Völkel, Dietrich, Phys.Rev.D 113 (2026) 2, 024011

Final Conclusions

Final Conclusions

Black hole spectroscopy is great!

- It provides such an exciting landscape for theoretical and observational perspectives:
  - Bound-state to QNM mapping should be explored in more detail
    Might reveal interesting insights: other objects (ECOs, wormholes,...), spectral stability, greybody-QNM correspondence,...
  - Parametrized QNM framework provides a baseline for exploring new phenomena "agnostically"
    More work in time-domain, exploration of bound states, excitation coefficients, ...
  - Systematic effects (simulations/data) challenge the simple idea of spectroscopy
    Extracting damped oscillations from high-SNR signals is elegant, but due to the nature of black holes, fragile.
  - Analog gravity experiments come with new perspectives and challenges
    Noise-driven dynamics, model and signal conspire!
  - More nonlinear simulations for matter environments and ringdown are needed
    Matter effects in linear-theory alone might miss relevant time-scales and non-stationary features