Gravitational-wave memory effects from binary-black-hole mergers

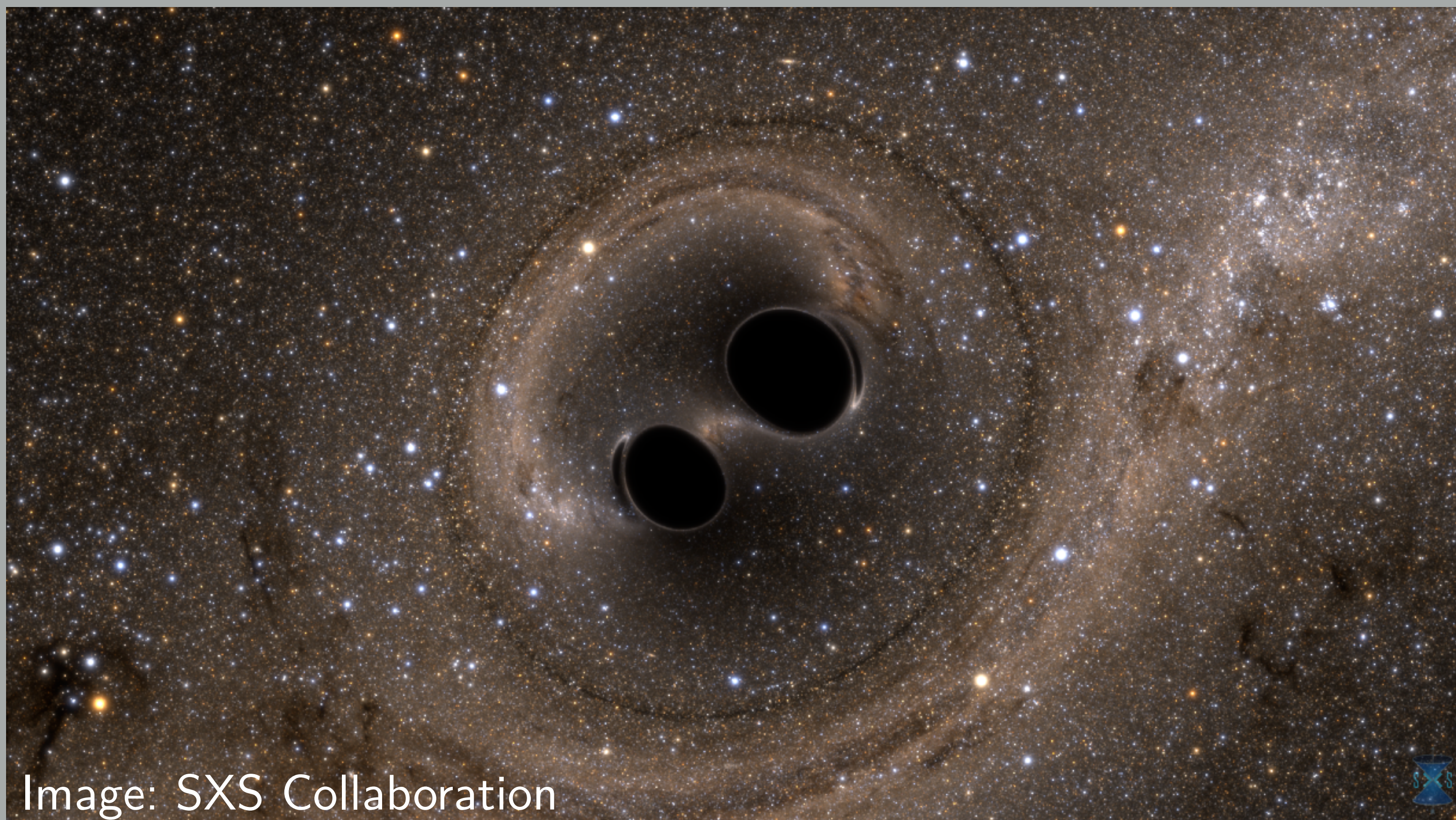


Image: SXS Collaboration

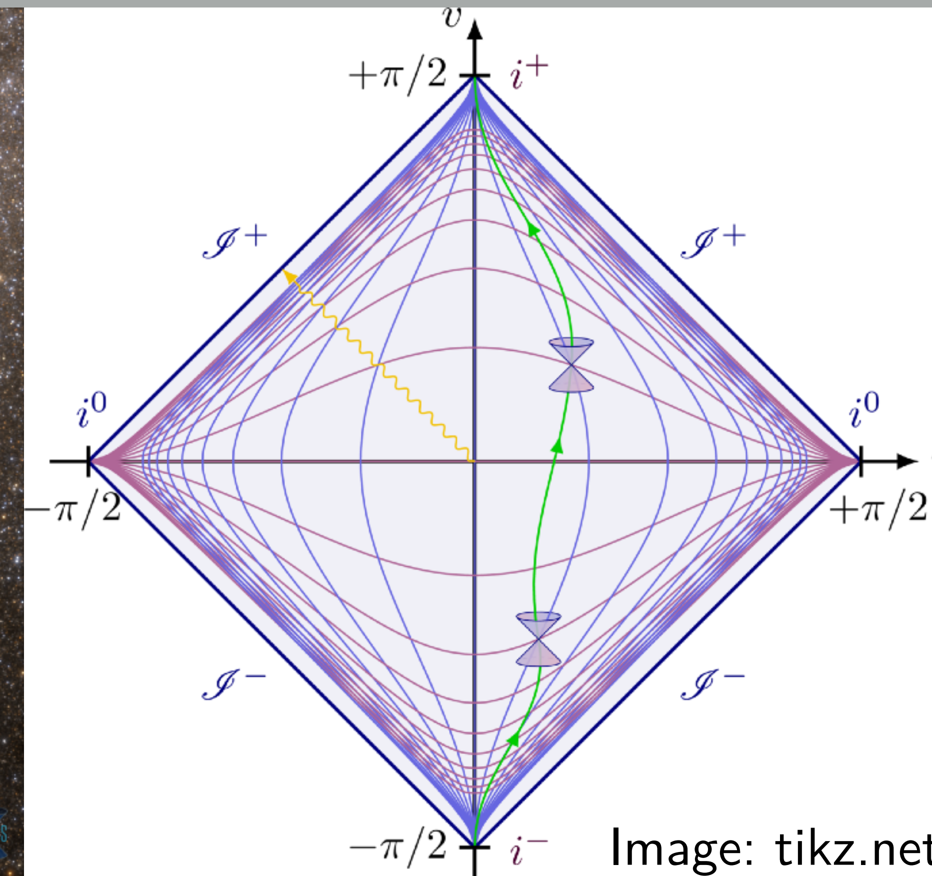
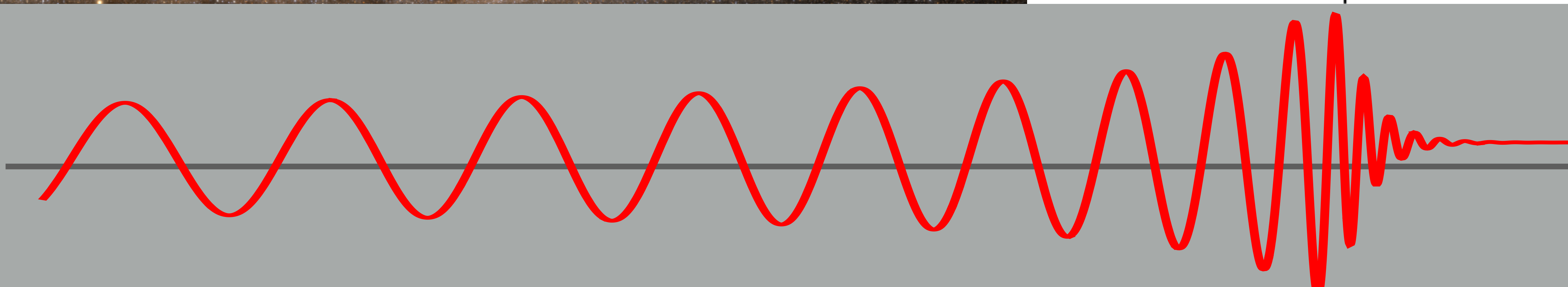


Image: tikz.net



David A. Nichols, University of Virginia, Dept. of Physics

IFPU Workshop: Perspectives on Gravity

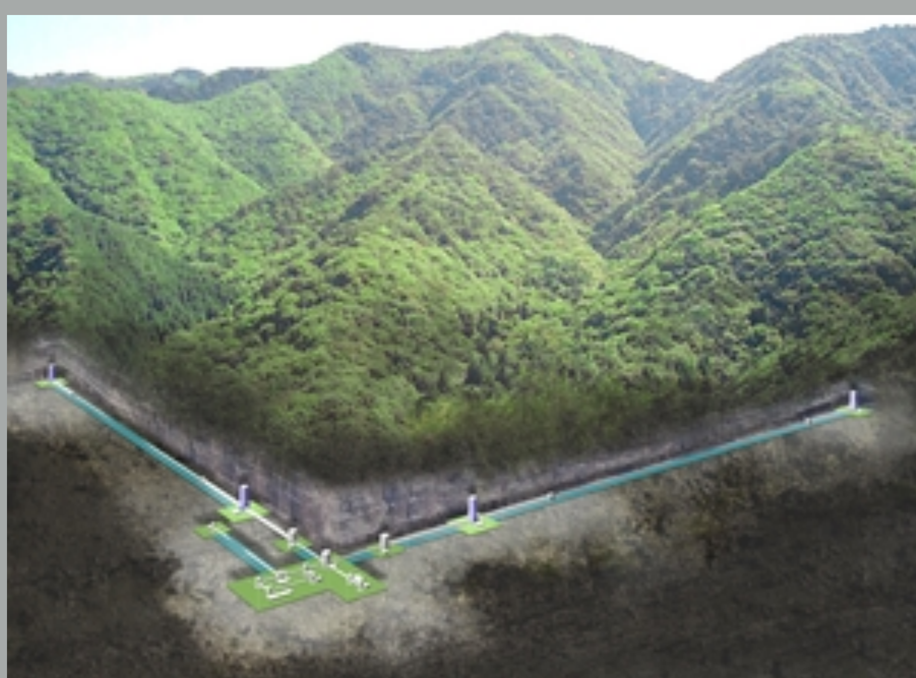
11 June 2026

LIGO, Virgo, and KAGRA (LVK) detectors

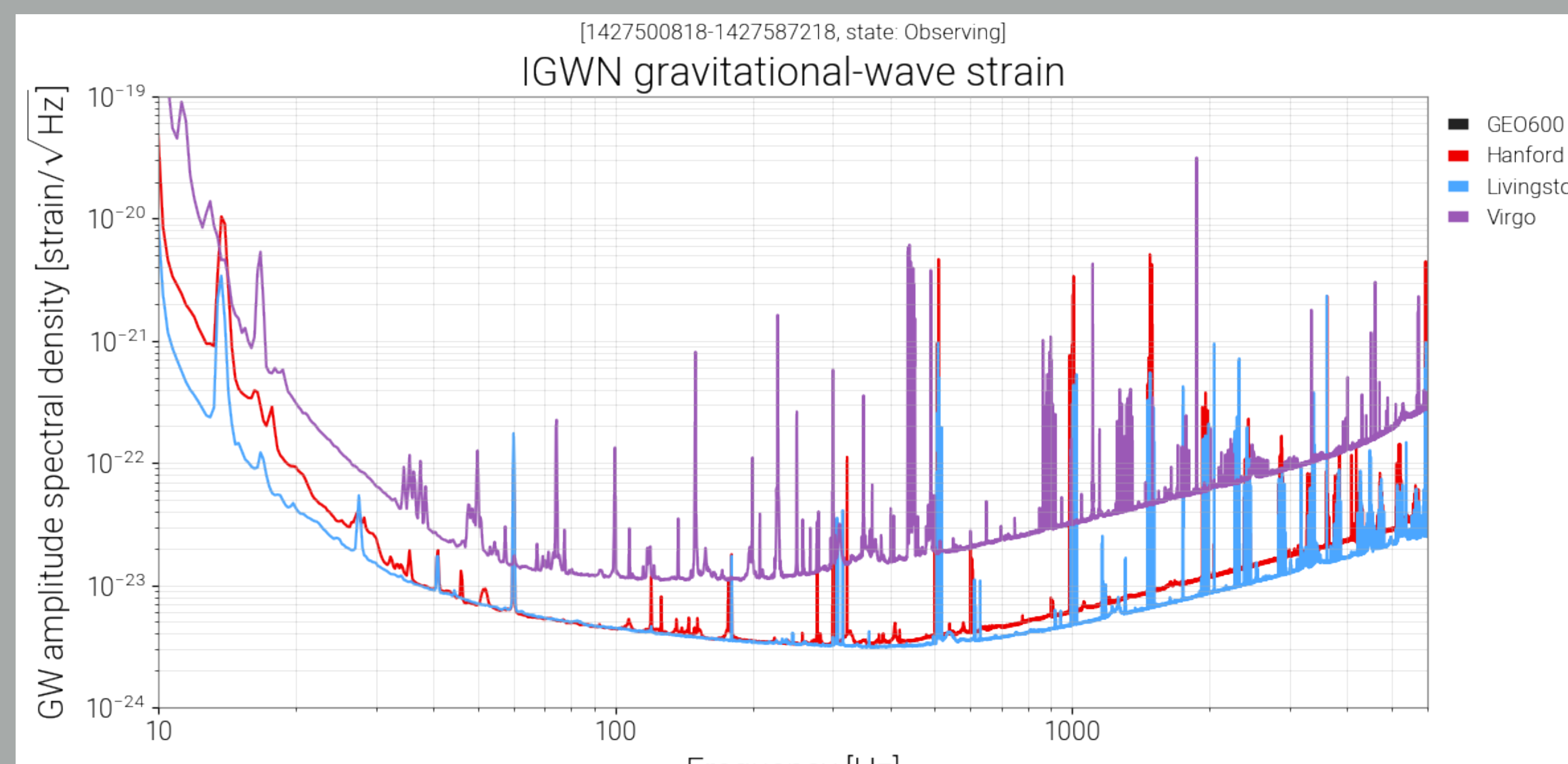
LIGO Hanford

LIGO Livingston

Virgo



KAGRA

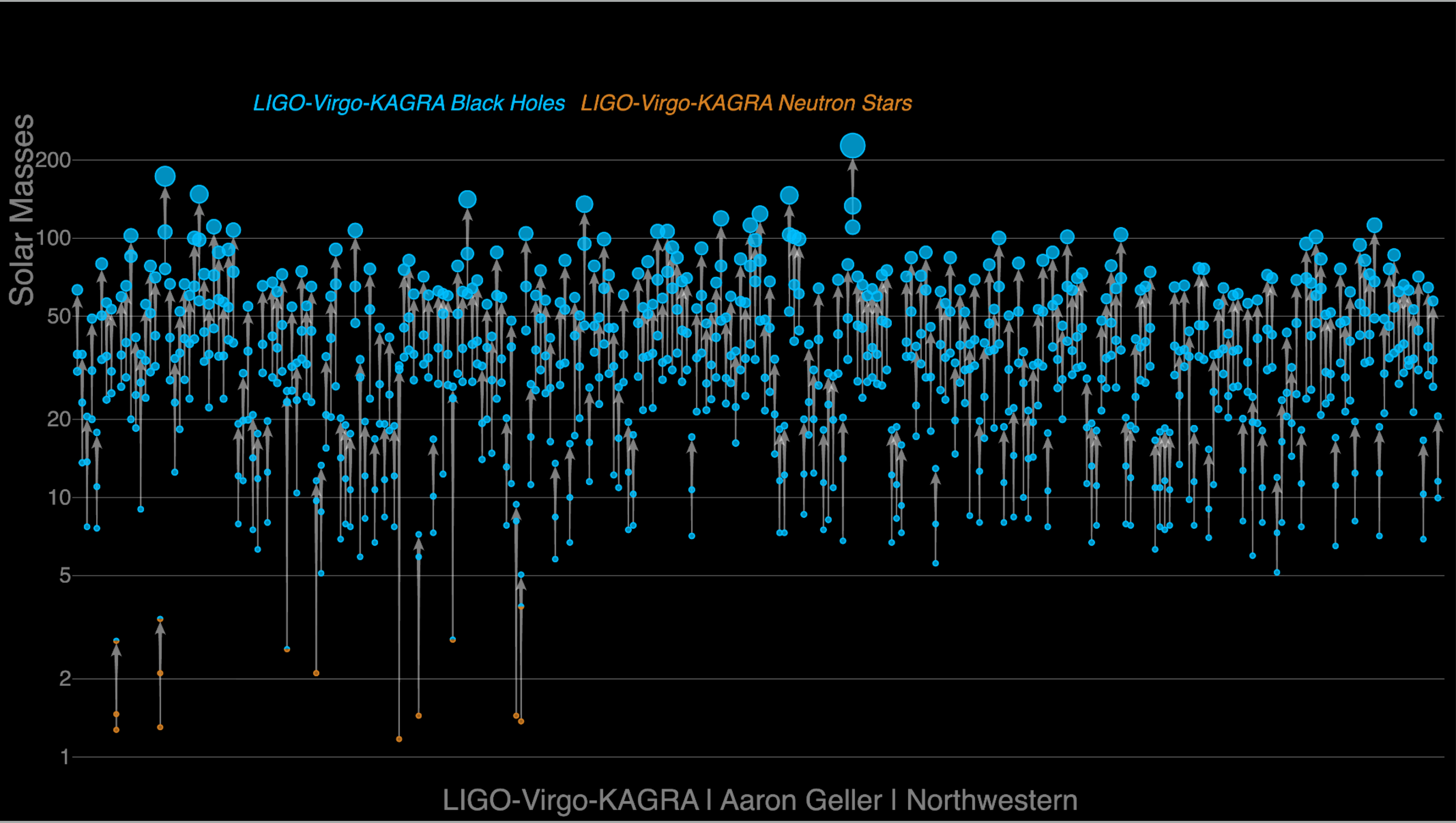


GWOSC (2025)

Noise curves circa 2025

Images: LIGO, Virgo, KAGRA

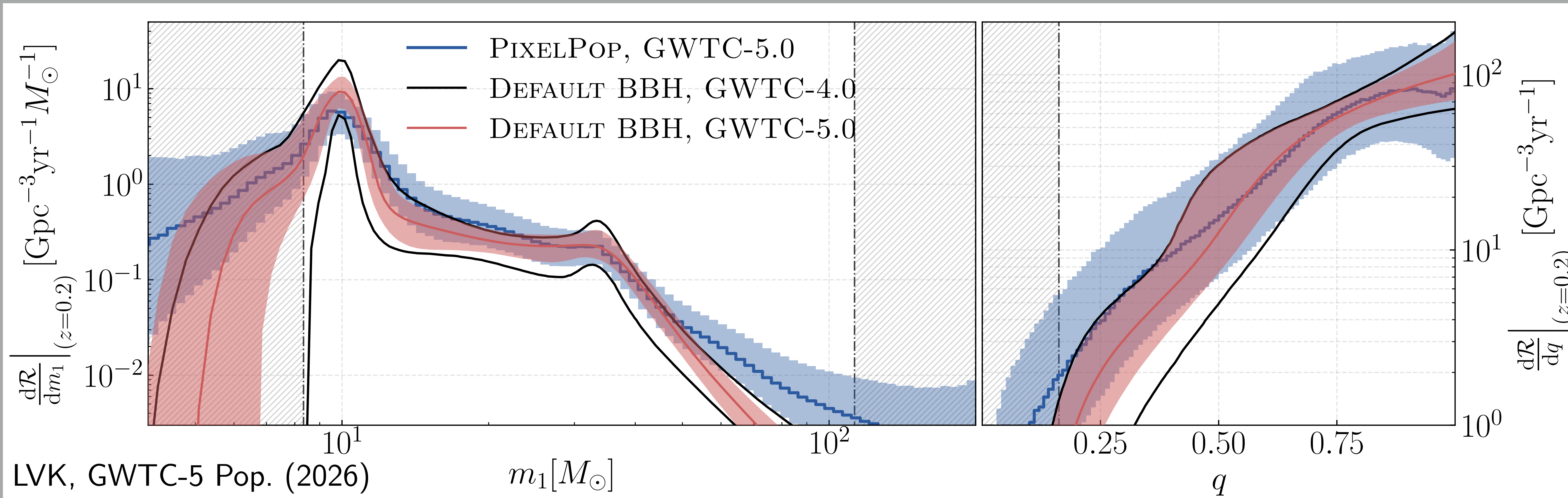
LVK detections of binary black holes (BBHs)



O1+O2:	O3a:	O3b:	O4a:	O4b:
10 events	45 events	35 events	128 events	161 events

LVK BBH population properties

- Prior to LVK detections, BH merger rate and mass distribution unknown; now pinned down to a factor of ~2



LVK, GWTC-5 Pop. (2026)

- Now much more confident about what will be measured and what we can learn about GR with BBH mergers from individual events and populations!

GW sources and the "Planck luminosity"

Image Credit: LIGO

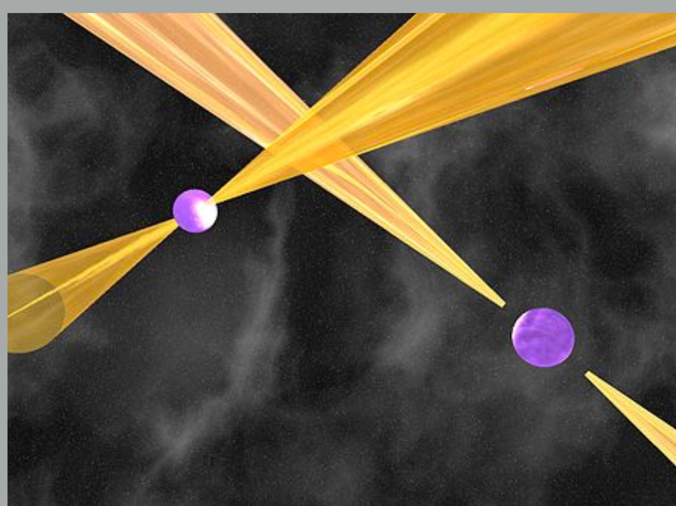


Image Credit: M. Kramer

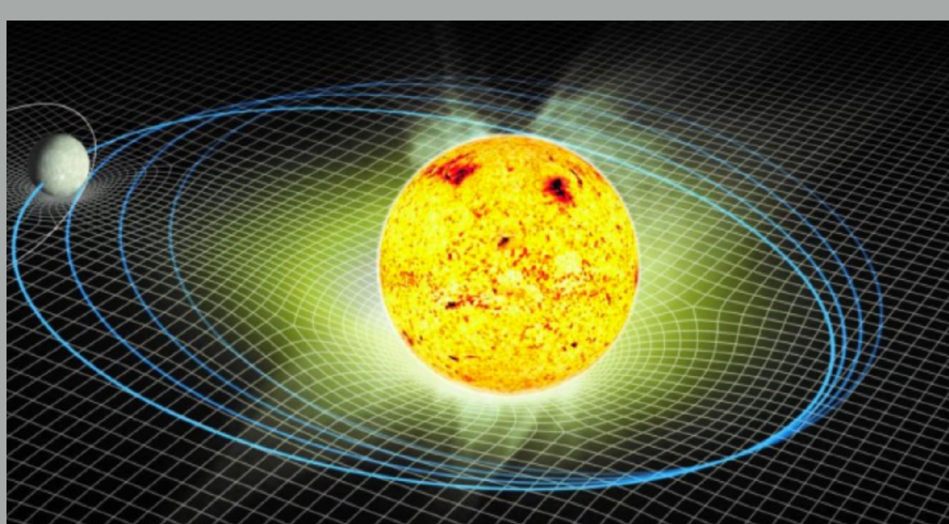


Image Credit: NASA

Luminosity (Planck units): $\frac{c^5}{G} \approx 3.6 \times 10^{52}$W

10^{-3} *Extragalactic*

(binary black holes)

Radiate $\sim 0.01Mc^2$ in $\sim 10GM/c^3$

10^{-27} *Galactic*

(binary pulsars)

10^{-52} *Solar system*

(Mercury & Sun)

Nonlinear dynamics important even in the wave zone for BBHs!

Outline

1. Computing memory effects in the Bondi-Sachs framework
2. Measuring memory with GW detectors
3. Detection prospects of GW memory effects
4. Generalizations of GW memory: “Higher” memory effects
5. Detection prospects of higher memory effects

1. Bondi-Sachs framework

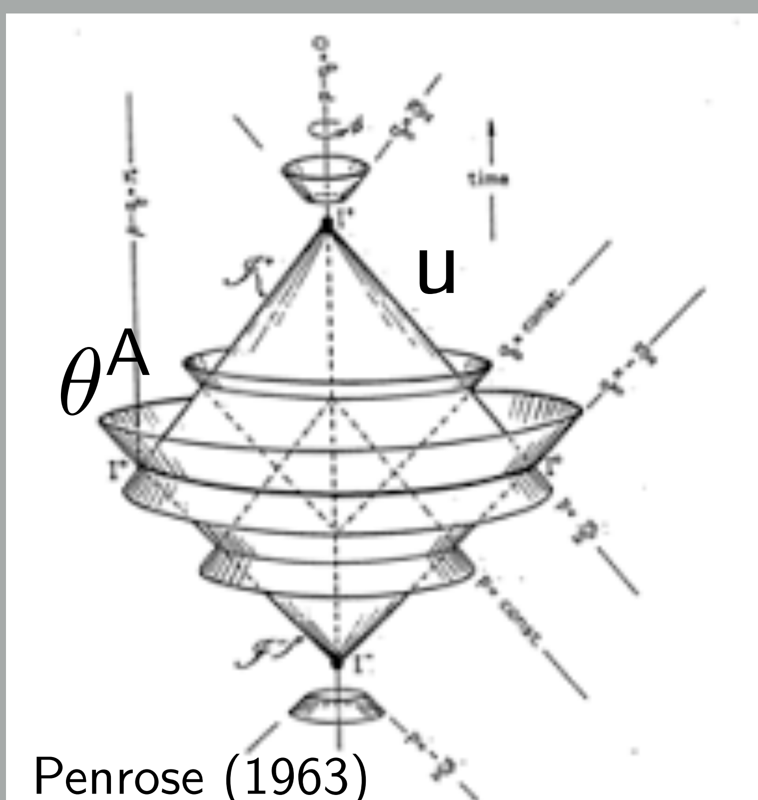
Bondi-Sachs coordinates

Bondi+ (1962), Sachs (1962)

- Bondi coordinates (u, r, θ^A)
- u: retarded time
- r: areal radius

θ^A : coordinates on 2-sphere

h_{AB}: metric on S^2



Penrose (1963)

Bondi gauge conditions

$$g_{rr} = g_{rA} = 0 \qquad \det(h_{AB}) = q(\theta^C)$$

- ∂_r is a null vector field and angular coords held fixed along integral curves
- 2-metric determinant r & u independent

Bondi-Sachs metric

$$ds^2 = -Ve^{2\beta}du^2 - 2e^{2\beta}dudr + r^2h_{AB}(d\theta^A - U^Adu)(d\theta^B - U^Bdu)$$

Asymptotically flat spacetimes

Metric expanded in 1/r, "on shell"

$$h_{AB} = [1 + O(r^{-2})]q_{AB} + \frac{1}{r}C_{AB} + \frac{1}{r^3}\mathcal{E}_{AB} + O(r^{-4})$$

q_{AB} : Round 2-sphere metric $\qquad$ D_A : covariant derivative of q_{AB}

$$V = 1 + \frac{2m}{r} + O(r^{-2}) \qquad \beta = O(r^{-2})$$

$$U^A = -\frac{1}{2r^2}D_B C^{AB} - \frac{2}{3r^3}N^A + \frac{1}{16r^3}D^A C^2 + \frac{1}{2r^3}C^{AB}D^C C_{BC} + O(r^{-4})$$

Bondi metric functions: radiative data & "charges"

$C_{AB}(u, \theta^C)$: shear tensor $\qquad$ $m(u, \theta^A)$: Bondi mass aspect

$N_{AB} = \partial_u C_{AB}$: Bondi news $\quad$ $N_A(u, \theta^B)$: angular momentum aspect

$\mathcal{E}_{AB}(u, \theta^C)$: first "higher" aspect (quadrupole)

Dynamical Einstein equations in vacuum

"Conservation-law"-type Einstein equations (w/ G & c)

$$\dot{m}c^2 = \frac{c^5}{G}\left(-\frac{1}{8c^2}N_{AB}N^{AB} + \frac{1}{4c}D_A D_B N^{AB}\right)$$

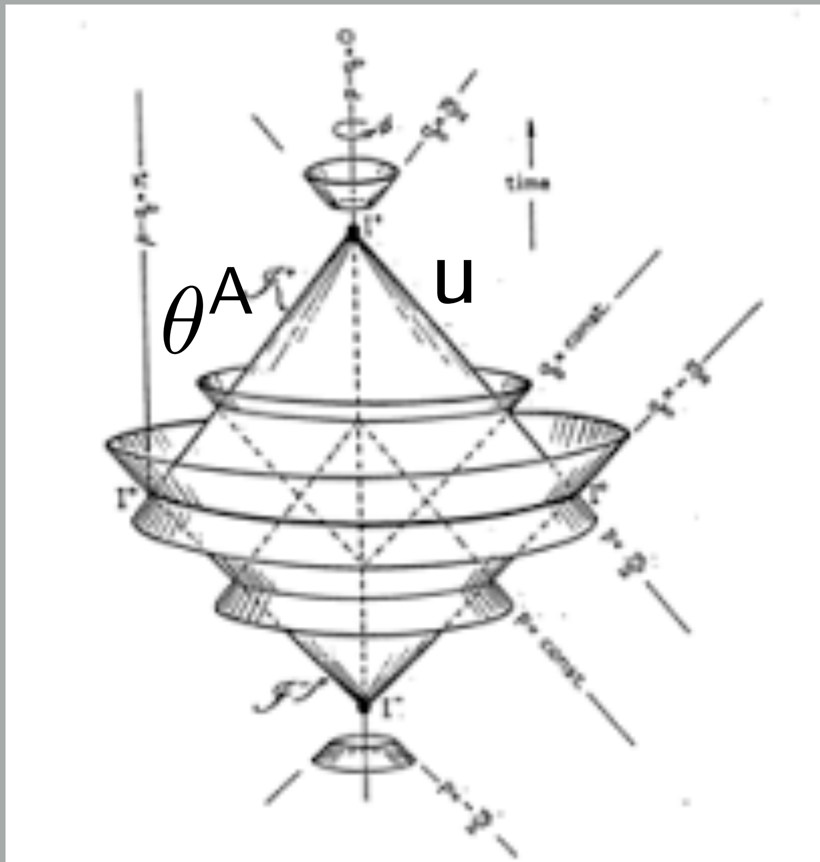
$$\dot{N}_A = D_A mc^2 + \frac{c^4}{4G}(D_B D_A D_C C^{BC} - D^2 D^B C_{AB}) + \frac{c^4}{4G}\left(\frac{3}{c}D_B N^{BC} C_{CA} + \frac{1}{c}N^{BC} D_B C_{CA}\right)$$

- N_{AB} can be of order $10^{-1.5}$ c for BBH mergers
- Y_{lm} moments of conservation laws related to Bondi-Metzner-Sachs (BMS) charges

"Higher" evolution-type Einstein equations (quadrupole)

$$\dot{\mathcal{E}}_{AB} = \frac{c}{2}mC_{AB} + \frac{1}{3}D_{\langle A}N_{B\rangle} + \frac{c^3}{4G}\left(\frac{1}{c}N_{CD}C^{CD}C_{AB} + C_A{}^C D_{[B}D^D C_{C]D}\right)$$

Asymptotic (BMS) symmetries



Penrose (1963)

Bondi-Metzner-Sachs (BMS) algebra

Bondi, van der Burg, Metzner (1962), Sachs (1962)

Symmetry (u=0): $\vec{\zeta} = \alpha(\theta^A)\vec{\partial}_u + Y^A\vec{\partial}_A$

$\alpha(\theta^A)$, supertranslation; Y^A, Lorentz

$$u' = u + \alpha(\theta^A), \quad \alpha(\theta^A) = \sum_{\ell m} \alpha_{\ell m} Y_{\ell m}(\theta^A)$$

Supertranslation Charges

Lorentz $\longrightarrow$ Relativistic angular momentum

Supertranslations $\longrightarrow$ Supermomentum

- GWs change charges, but change in charges equals their flux

Memory effect from charge & flux in vacuum

"Electric & magnetic parity" decomposition of C_{AB}

$$C_{AB} = D_{\langle A}D_{B\rangle}\Phi + \epsilon_{C\langle B}D_{A\rangle}D^{C}\Psi \qquad D^{A}D^{B}C_{AB} = \frac{1}{2}\mathcal{D}\Phi$$

- $\mathcal{D} = D^2(D^2+2)$ removes the $\ell \leq 1$ spherical harmonics (Bondi 4-momentum loss)
- For $\ell > 1$ get constraints on the change in the shear ΔC_{AB}

$$\mathcal{D}\Delta\Phi = 8\Delta m + \int_{u_0}^{u} N_{AB}N^{AB}du'$$

GW memory

Supermomentum "charge" term

GW "flux" term

- C_{AB} related to the strain measured by LIGO via

$$r(h_+ - ih_\times) = C_{AB}(e_+^{AB} - ie_\times^{AB})$$

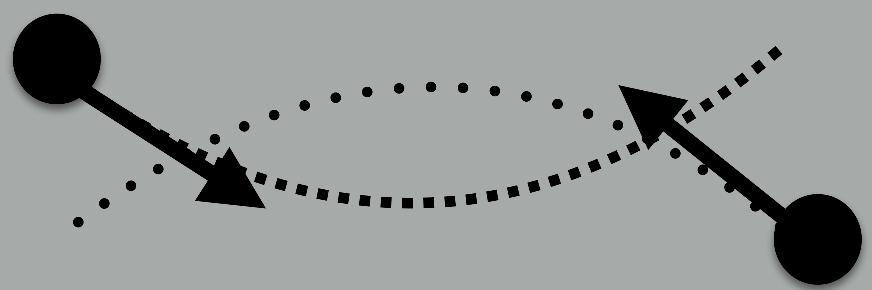
- GR predicts a lasting strain after a burst of waves!

$$r\Delta h_+ = e_+^{AB}D_AD_B\Delta\Phi$$

Sources of GW memory in vacuum

$$\mathcal{D}\Delta\Phi = 8\Delta m + \int_{u_0}^{u} N_{AB}N^{AB}du'$$

"Change in charge"



Ex. NSs/BHs
scattering or BBH Kicks

Zeldovich & Polnarev (1974)

"Hard" flux
of GWs

Image Credit: SXS

Ex: BBH mergers

Christodoulou (1992)

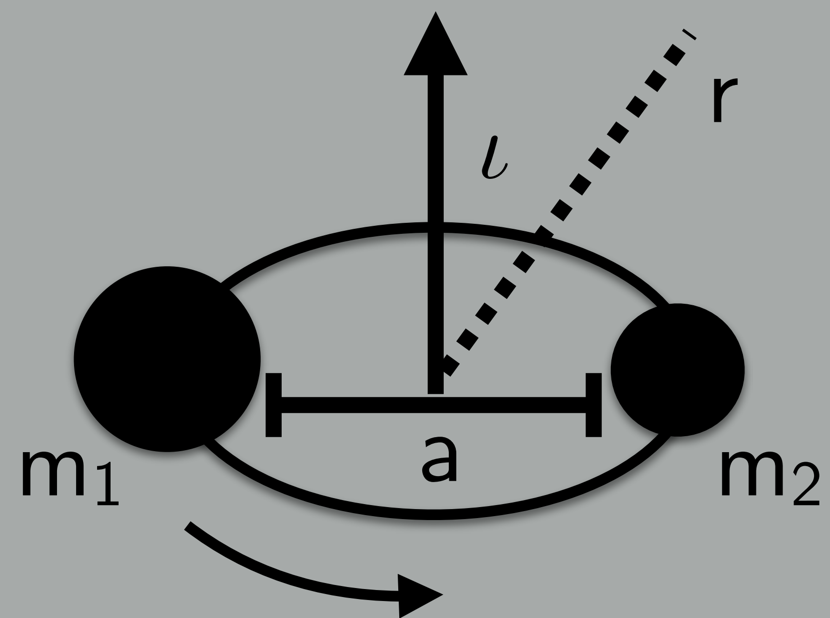
Blanchet & Damour (1992)

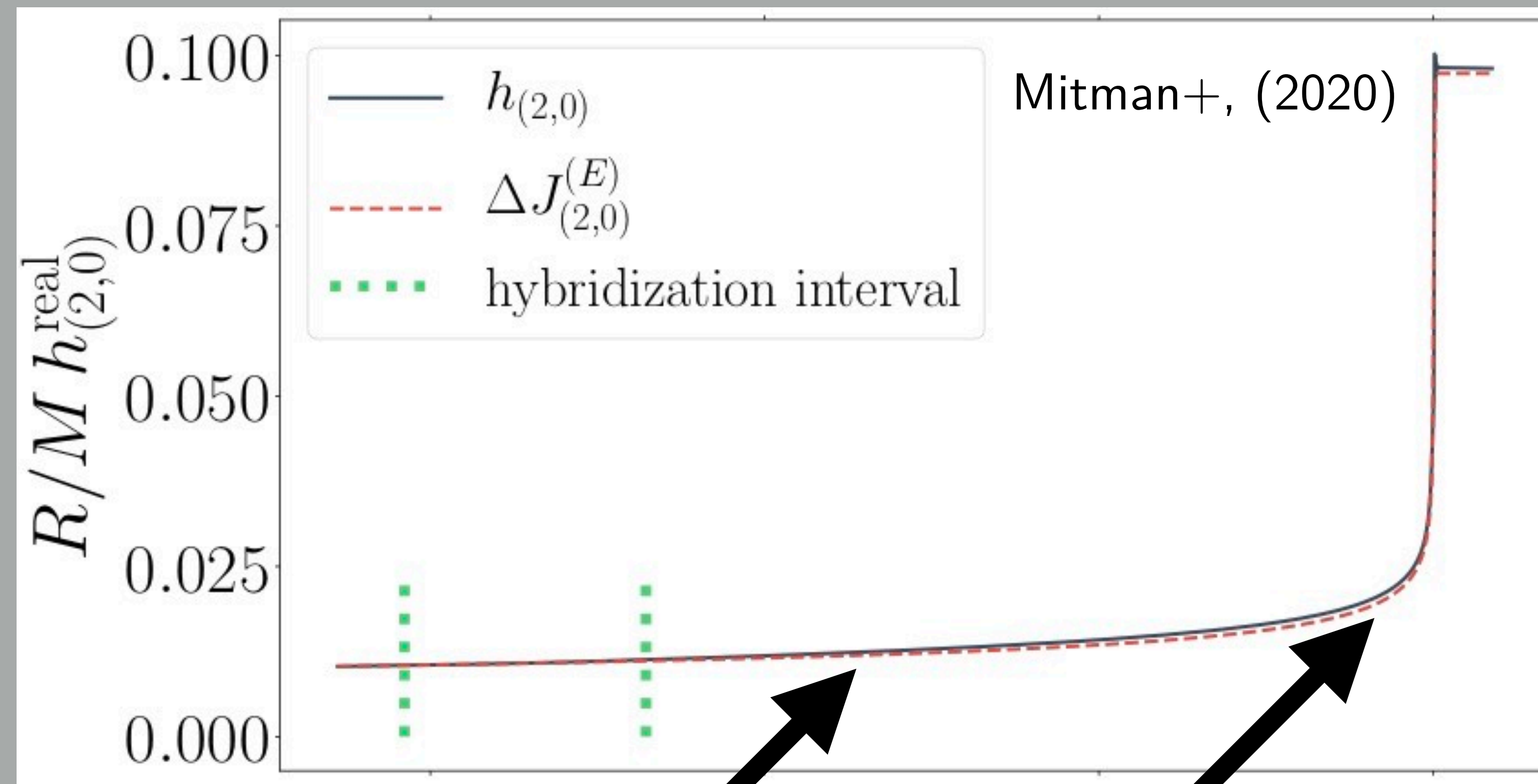
2. Caveat on detecting GW memory

$$\mathcal{D}\Delta\Phi(u) = 8\Delta m(u) + \int_{u_0}^{u} N_{AB}N^{AB}du'$$

- Detectors like LIGO/Virgo measure a finite frequency range; the zero-frequency ΔC_{AB} is not accessible to LIGO.
- When $\Delta C_{AB} \neq 0$, there is a GW memory *signal*, $h_{mem}(u)$, computed from the conservation law that gives rise to ΔC_{AB}, which has frequencies LIGO can measure.
- For binary black holes, the nonlinear GW flux term is the dominant contribution to ΔC_{AB} and it has a distinct frequency, time, polarization and sky pattern from the "main" signal.
- The GW memory *signal* (the nonoscillatory, nonlinear term) will be used as the proxy for detection of memory with LIGO/Virgo in the detection forecasts.

Memory effect from BBH mergers





Inspiral

Merger/
Ringdown

Time

"Newtonian" expectations

$$\Delta h_+ = \frac{G}{c^4}\frac{1}{48r}\Delta\left(\frac{G m_1 m_2}{a}\right)$$
$$\times \sin^2\iota(17+\cos^2\iota)$$

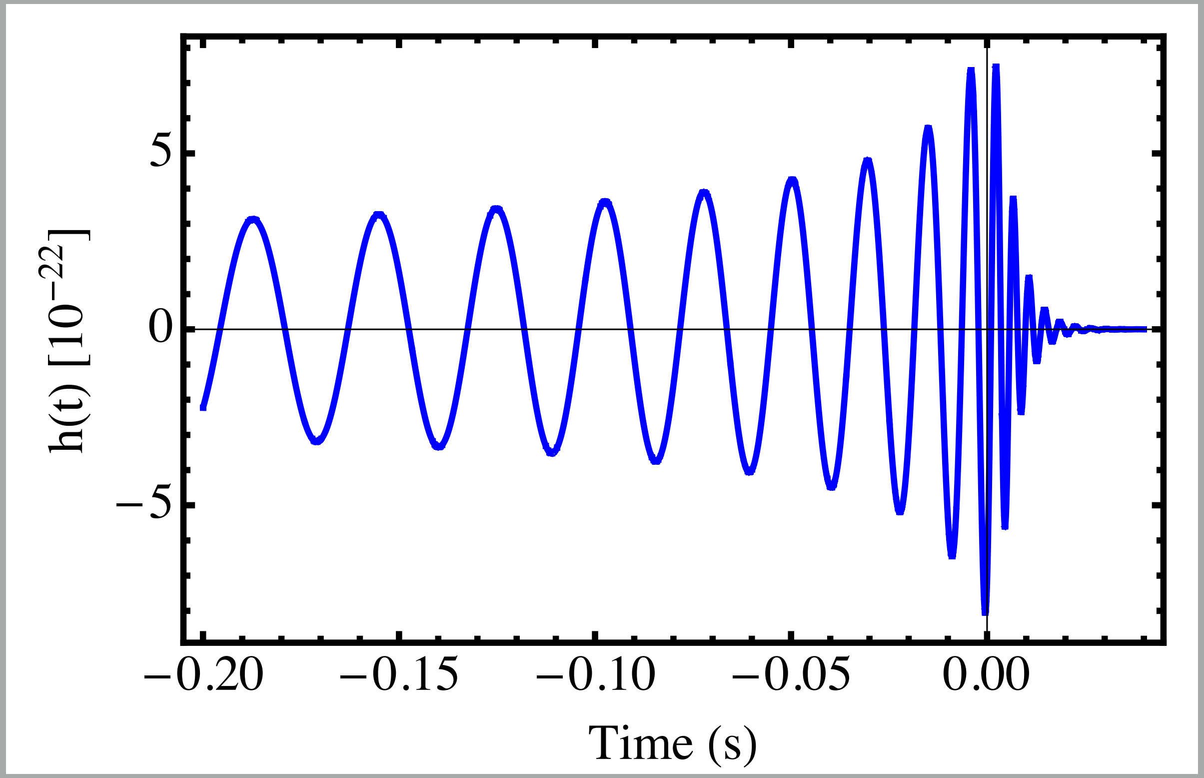
$+$ post-Newtonian corrections

Wiseman & Will (1991), Favata (2009)

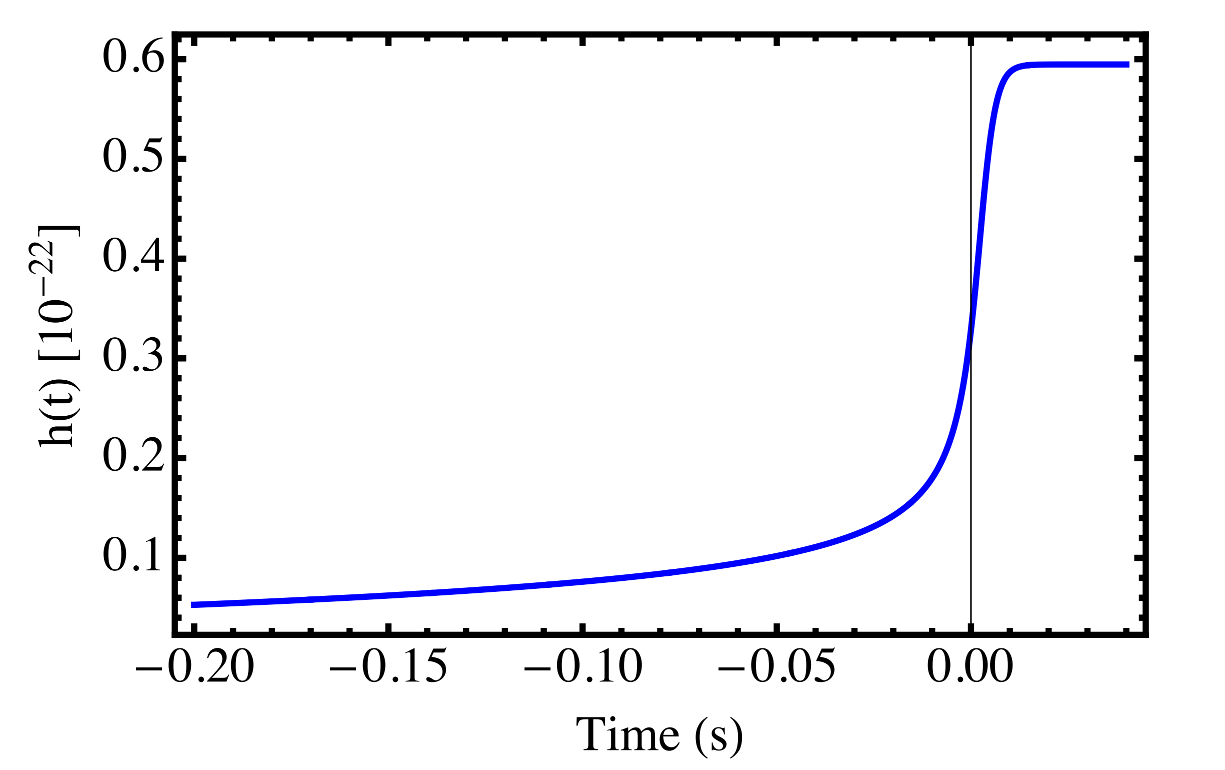
Numerical Relativity (NR)

Memory computed via NR and via flux term in supermomentum conservation agree well

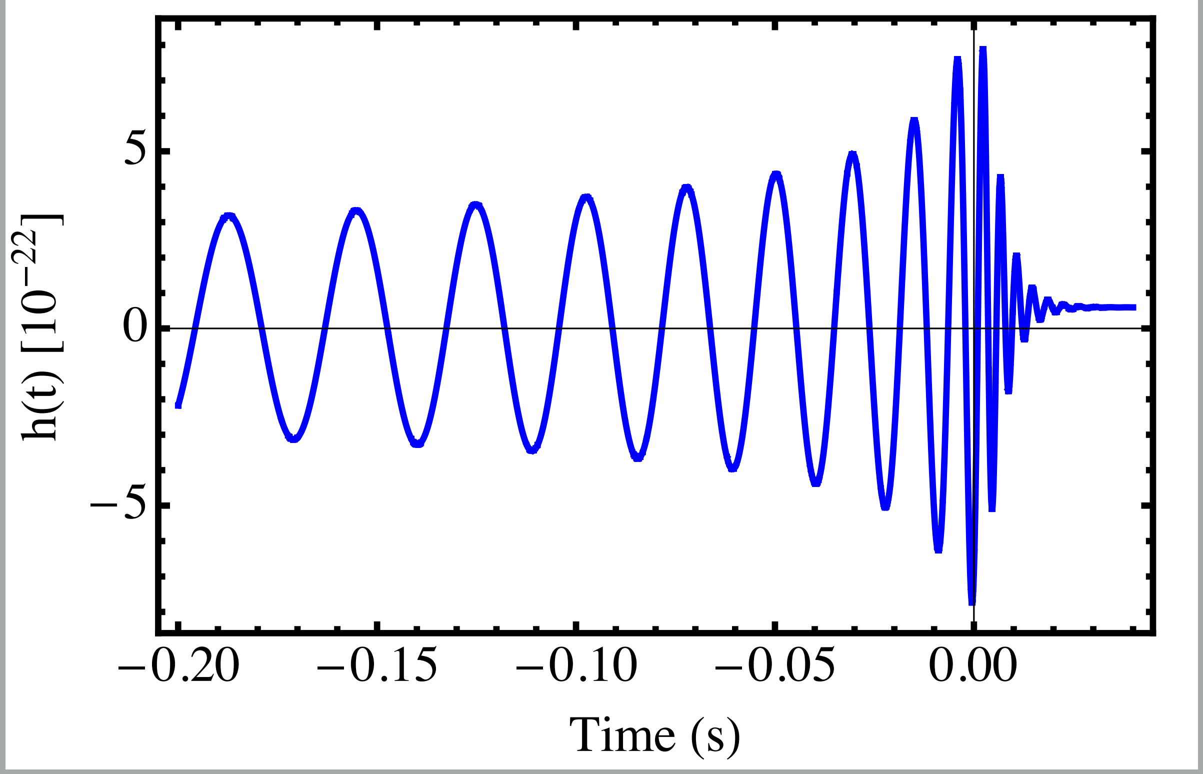
GW memory for GW150914



Waveform, no memory



GW memory waveform



Complete waveform

- *Test*: Determine if waveform with memory is consistent with the GW data
- *Forecast*: Use additional signal-to-noise (S/N) in memory waveform as a proxy

3. Prospects for measuring GW memory

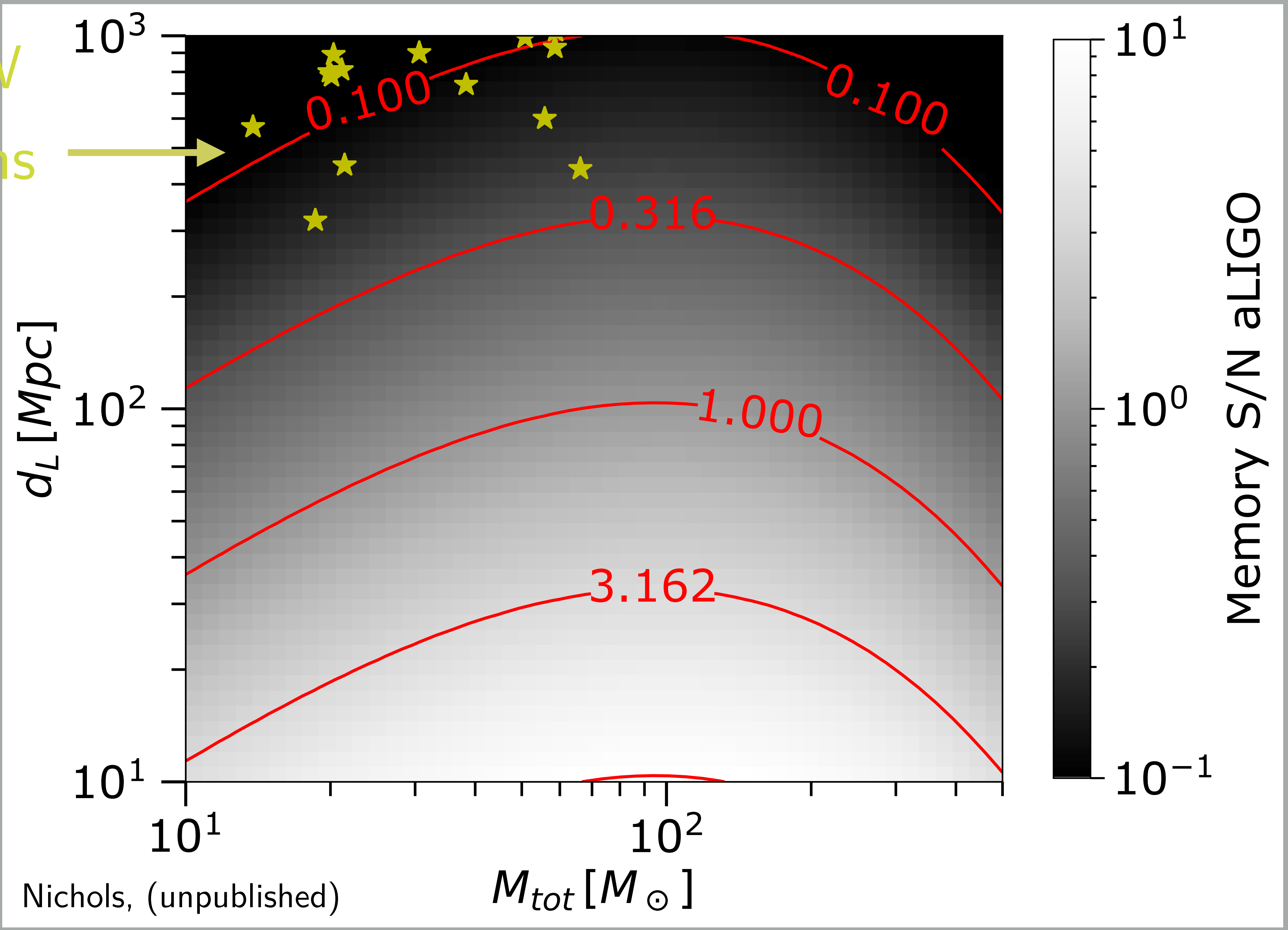
- Signal to noise (ratio) (S/N or SNR); σ of a detection

$$\mathrm{SNR}^2 = 4\int_{f_{\mathrm{low}}}^{f_{\mathrm{high}}} \frac{|h(f)|^2}{S_n(f)}\,df$$

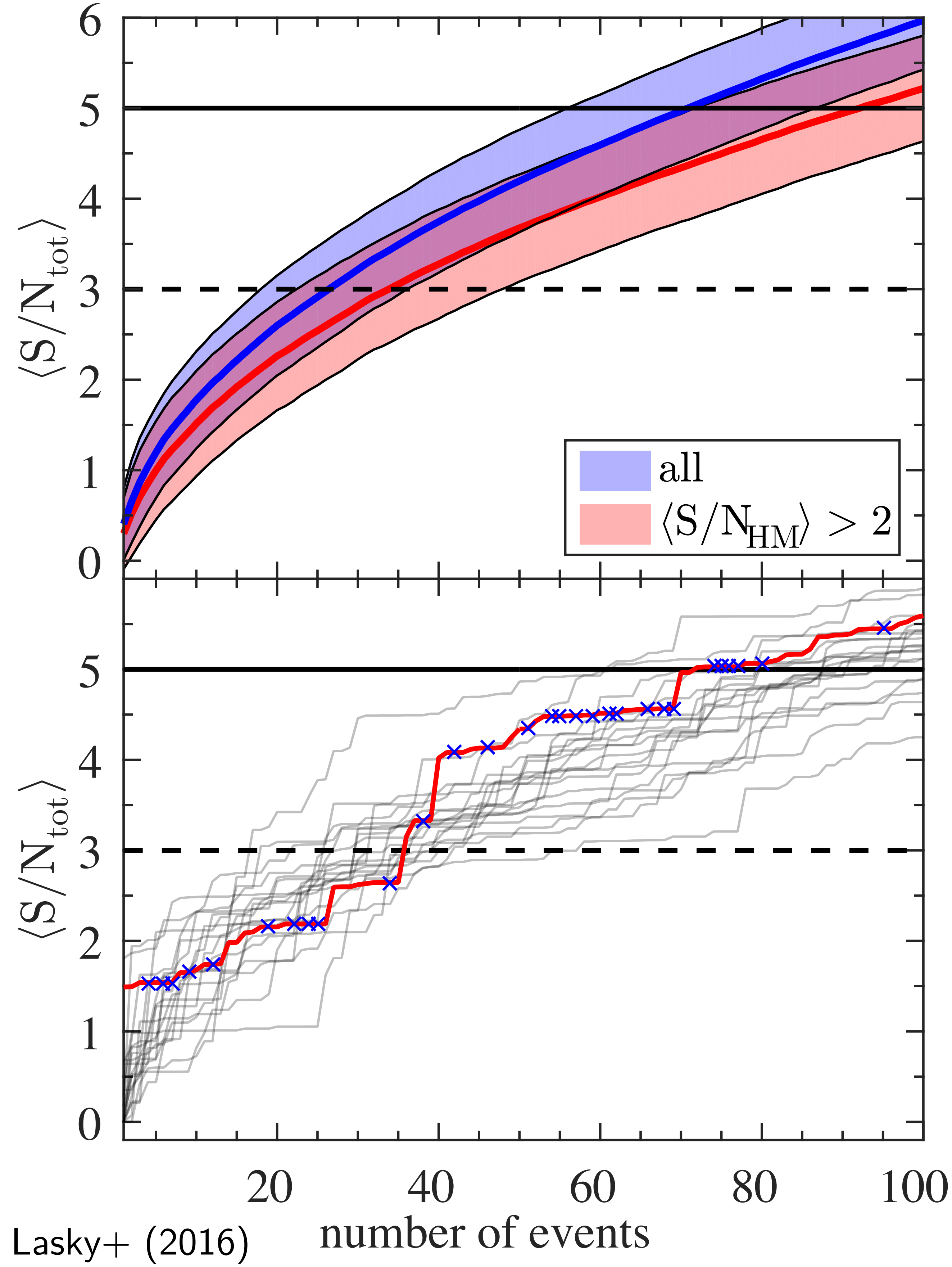
$h(f)$: Fourier transform of $h(t)$

$S_n(f)$: LIGO noise spectrum

BBH GW detections O3a



Detecting memory in populations with LIGO



Lasky+ (2016)

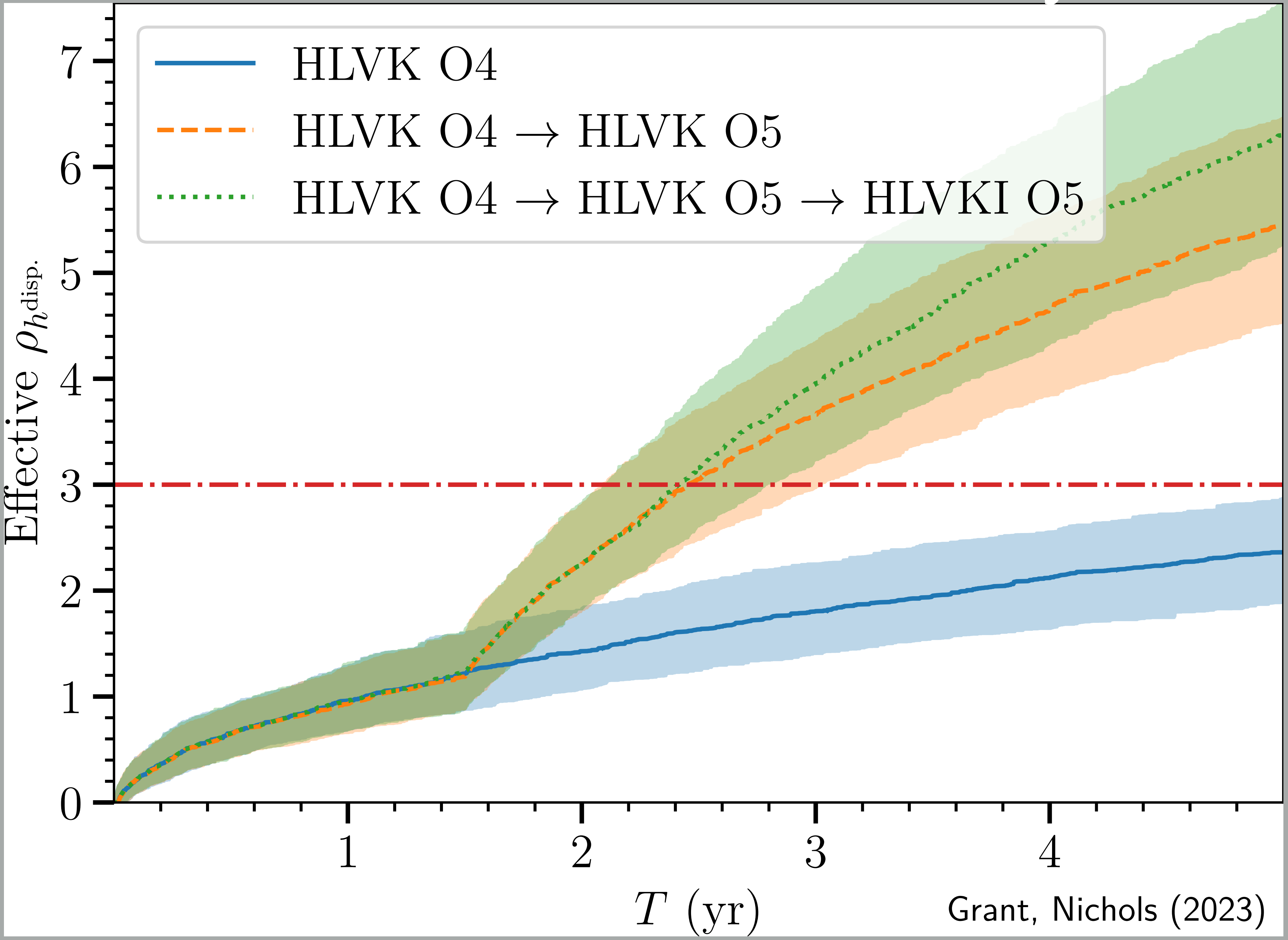
S/N of memory in population:

$$\langle S/N_{tot} \rangle = \left(\sum_{i=1}^{n} \langle S/N_{\Delta h_i} \rangle^2 \right)^{1/2}$$

$$\langle S/N_{tot} \rangle \sim \sqrt{n}$$

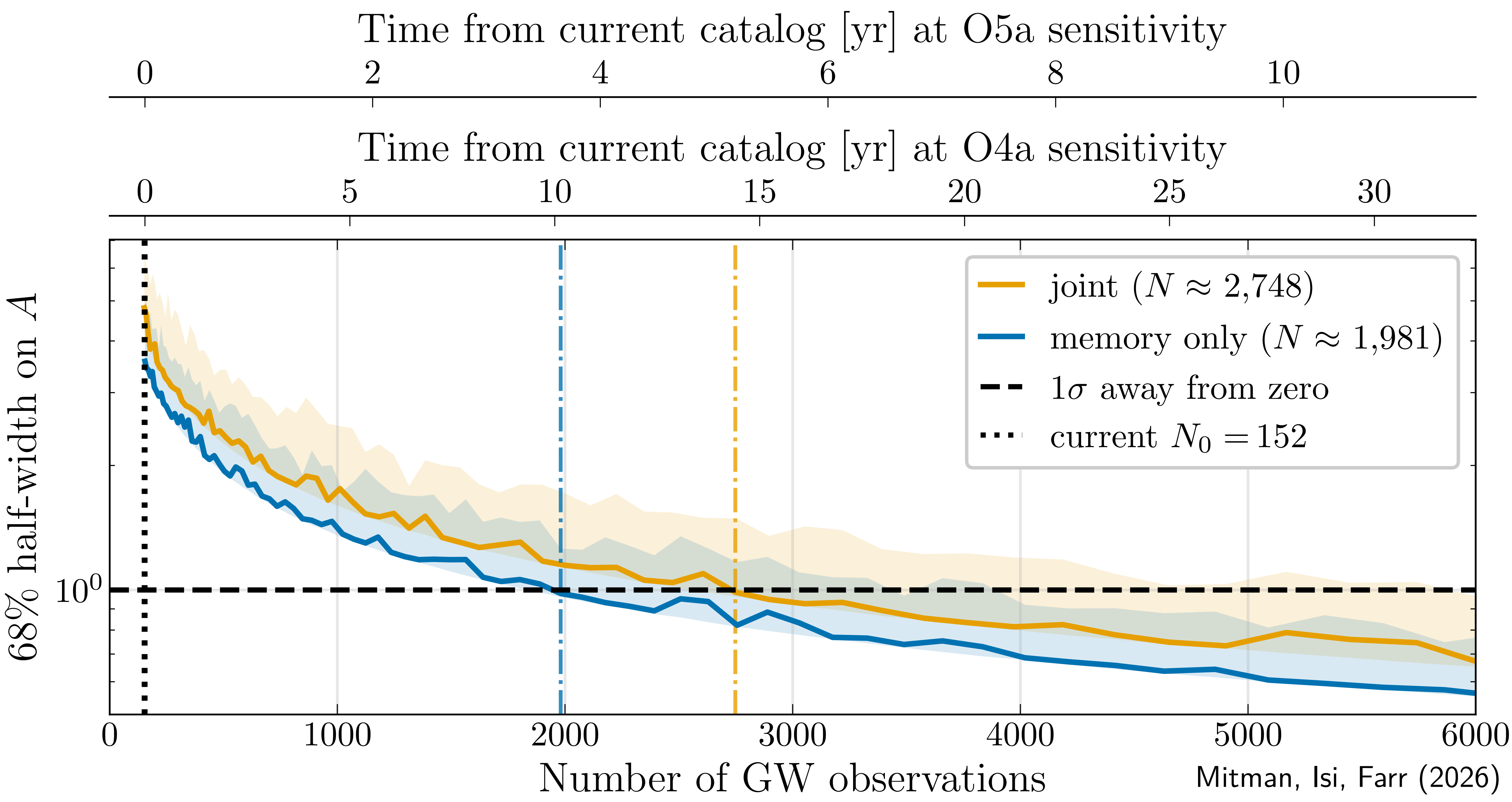
- Population consists of GW150914-like events
- Make multiple realizations of this population
- Only count events with measurable higher harmonics of the GWs

Populations and time to memory detection



- Simulate multiple realizations of a BBH population consistent with O1-O3a LIGO BBHs; take median and 1-sigma error
- Number of BBH detections required is a few times 10^3!

Memory detection and hierarchical inference



- Suppose $h_{\text{tot}} = h_{\text{osc}} + A\,h_{\text{mem}}$ with A normally distributed
- Constrain mean & std. dev. of A to be 1 & 0, respectively

4. Higher GW Memory Effects

"Electric/Magnetic" decomposition of C_{AB}

$$C_{AB} = D_{\langle A}D_{B\rangle}\Phi + \epsilon_{C\langle B}D_{A\rangle}D^{C}\Psi$$

- Mass aspect evolution predicts offset in electric part of C_{AB}, but offset in magnetic shear not predicted for BBHs.
- Do the evolution of higher Bondi aspects constrain C_{AB}?

Angular momentum aspect and time integrals of C_{AB}

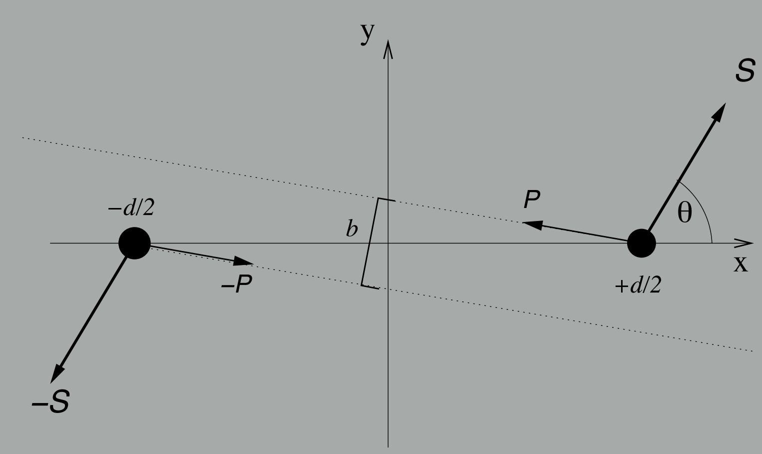
- The angular momentum aspect evolution constrains both electric and magnetic parts of the time integral of C_{AB}
- Magnetic part ("2D curl") is "spin memory" Pasterski+ (2015)
- Electric part ("2D divergence") is "center of mass (CM) memory"; more subtle because C_{AB} has an offset Nichols (2018)

Spin memory sources in vacuum

$$D^2\mathcal{D}\Psi = 2\epsilon^{AE}D_E\left(4\dot{N}_A + 3D_B N^{BC}C_{CA} + N^{BC}D_B C_{CA}\right)$$

Charge term

Flux term for GWs



Ex. Unbound spinning objects

Pasterski+ (2015)

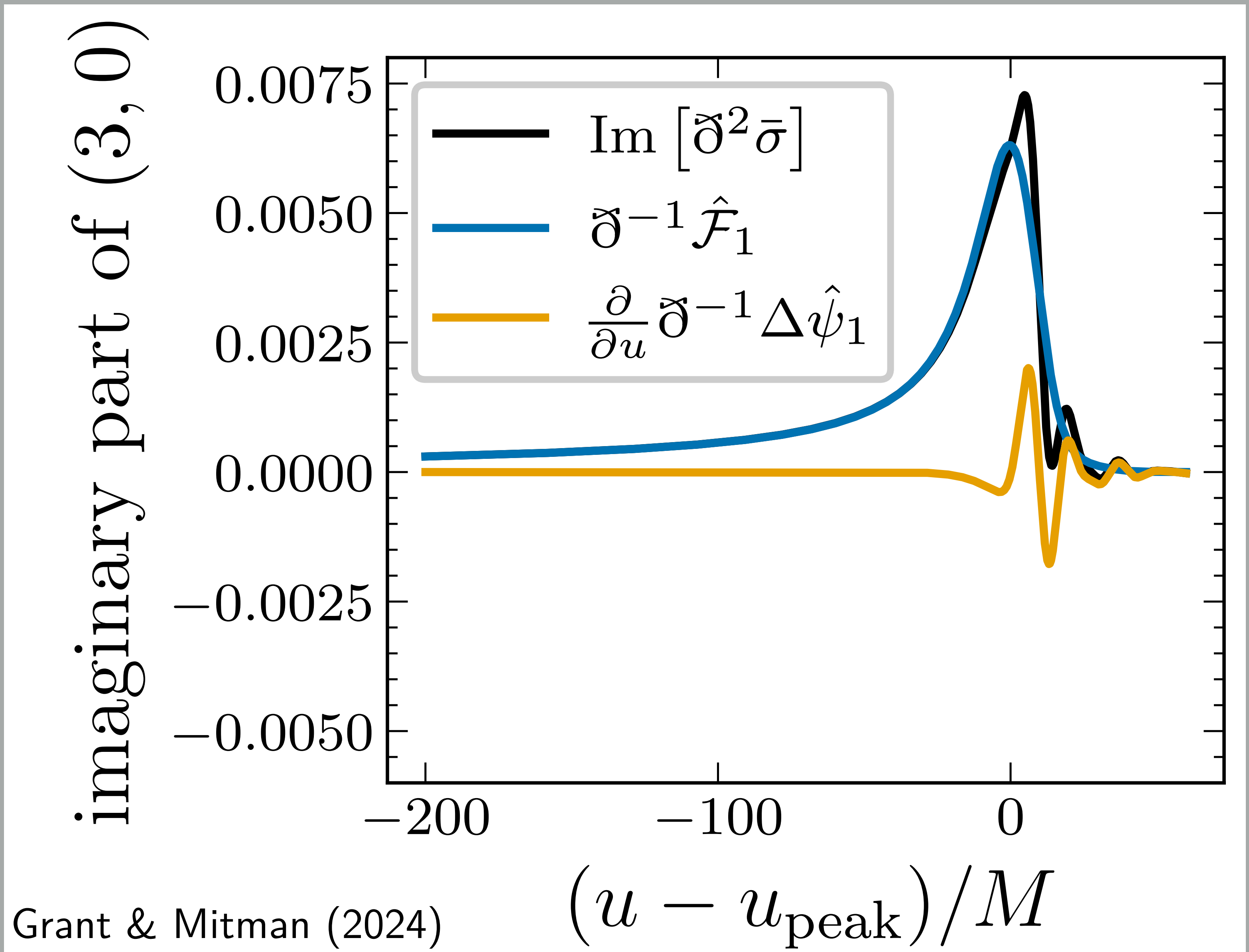
Ex: BBH mergers

Nichols (2017)

- Integrating will lead to a change in the time integral of shear

Spin memory effect: Non-spinning binaries

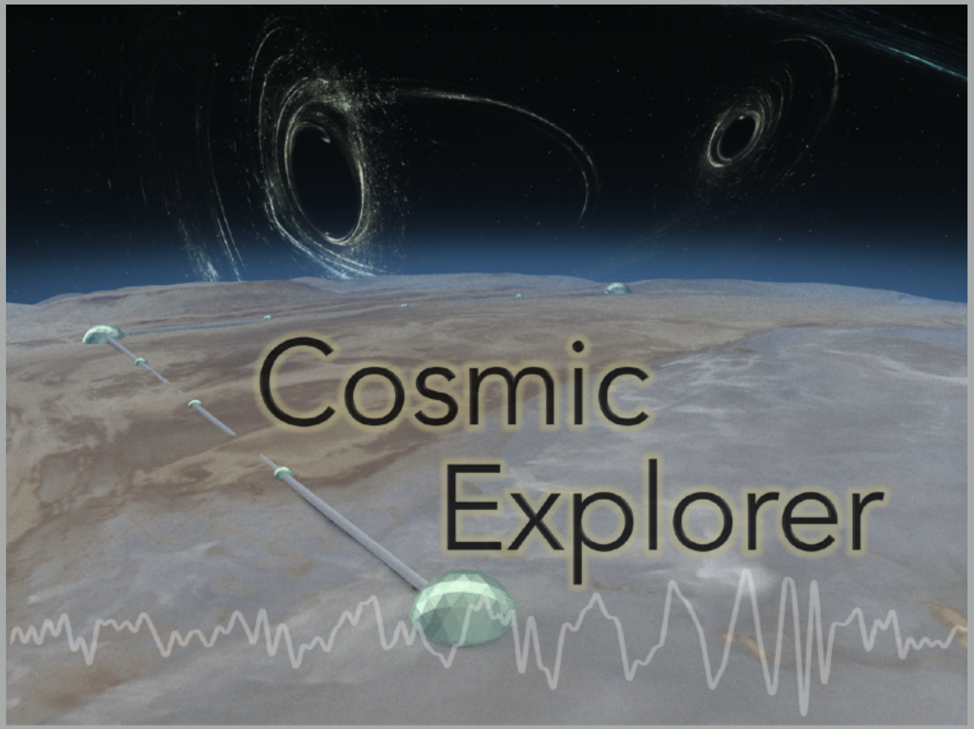
- Flux term is the leading PN contribution, which is proportional to the flux of orbital (not spin!) angular momentum Nichols (2017)
- Appears in shear at 2.5PN order (previously computed) Arun+ (2005)

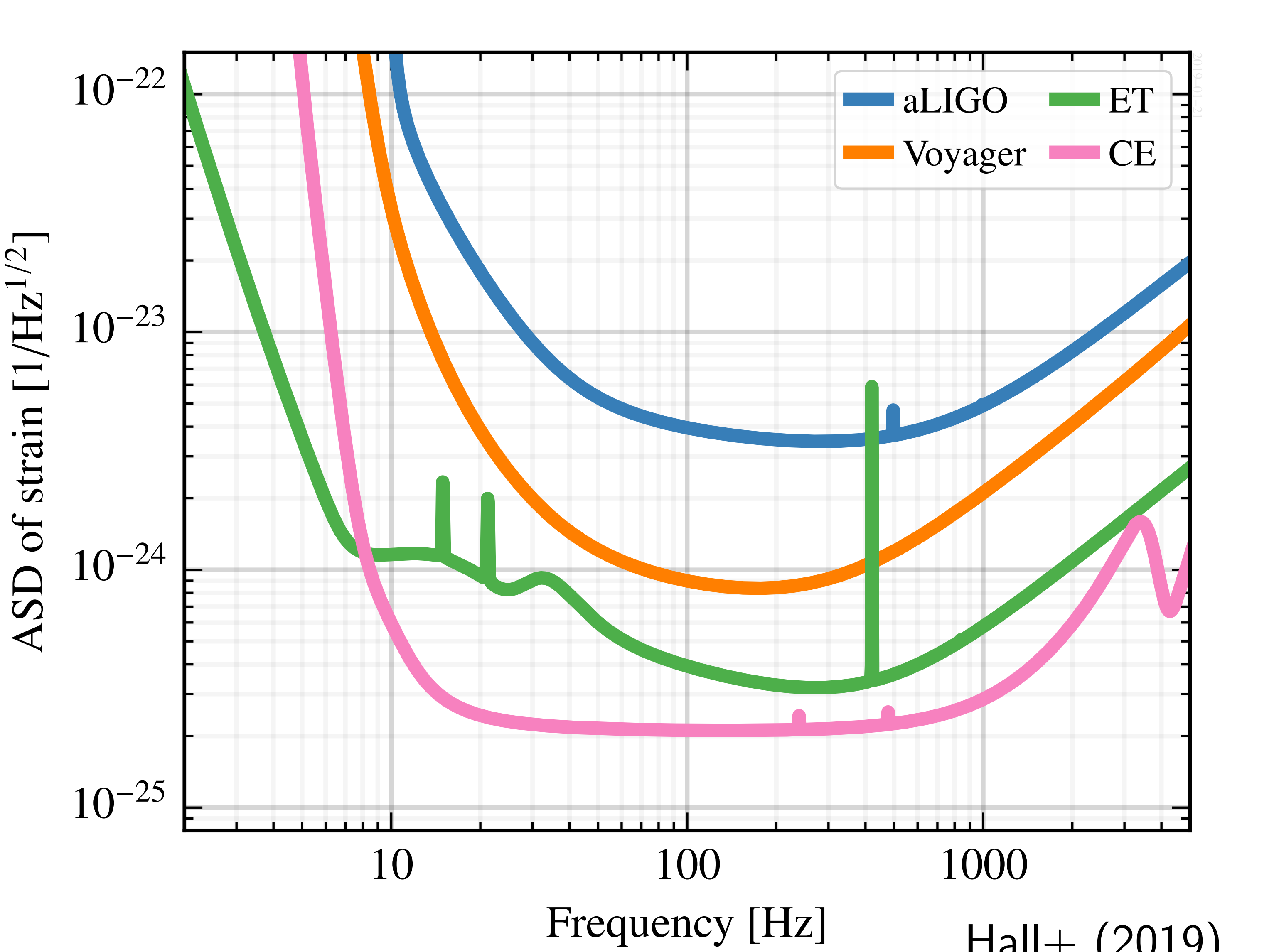


Grant & Mitman (2024)

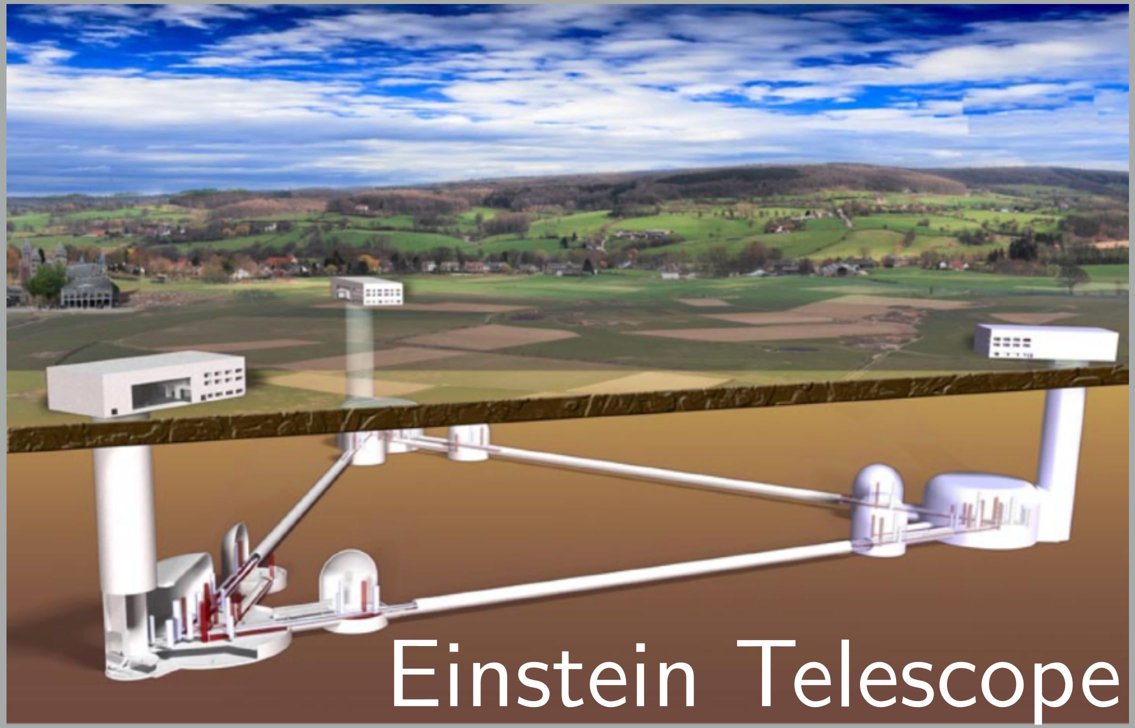
- Flux part of the spin memory is larger than the “charge”, which also won’t produce an offset in integrated shear
- Shear has distinctive, functional dependence in time, freq., polarization & sky pattern

Planned GW detectors on Earth (3G)



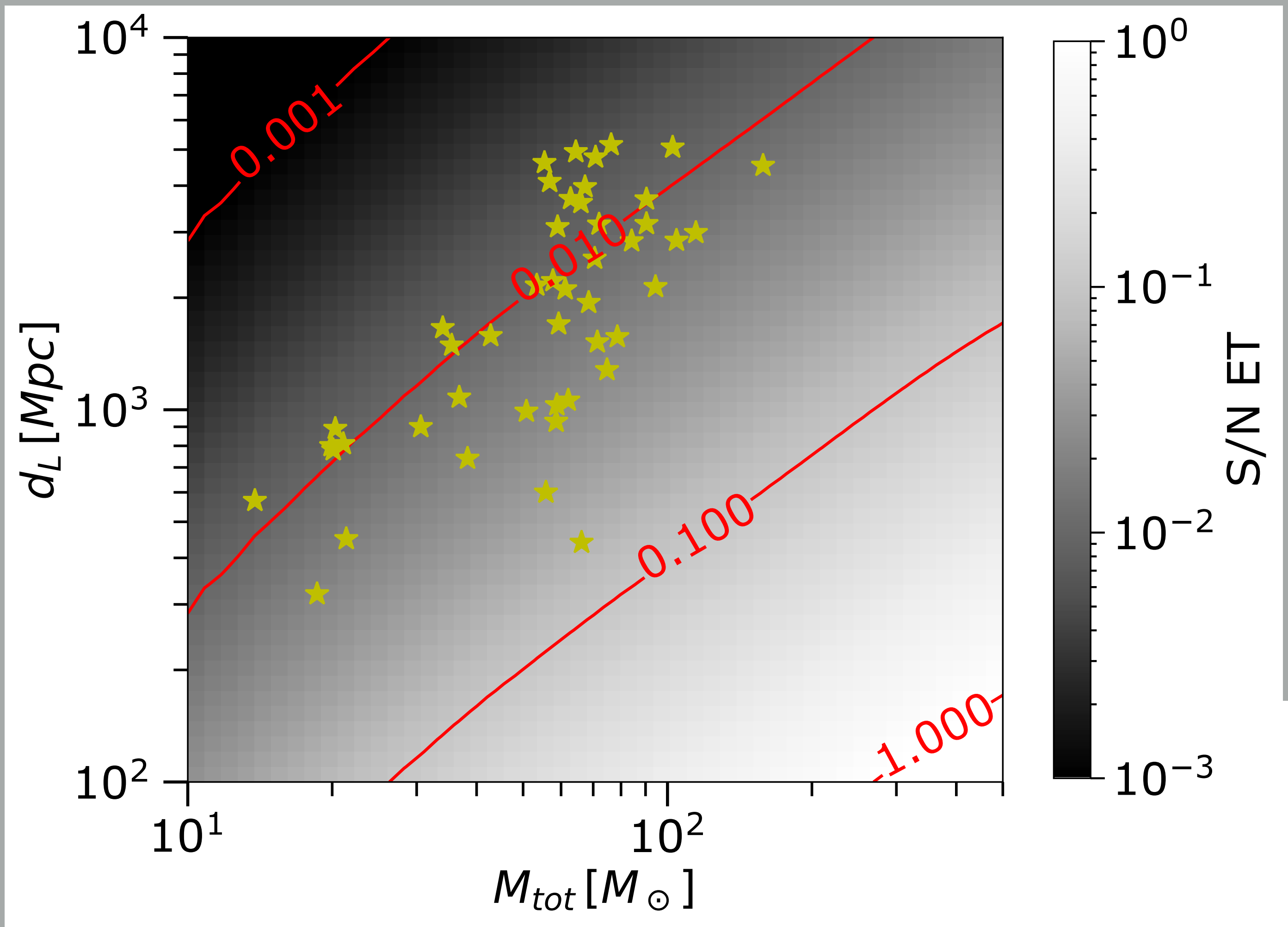


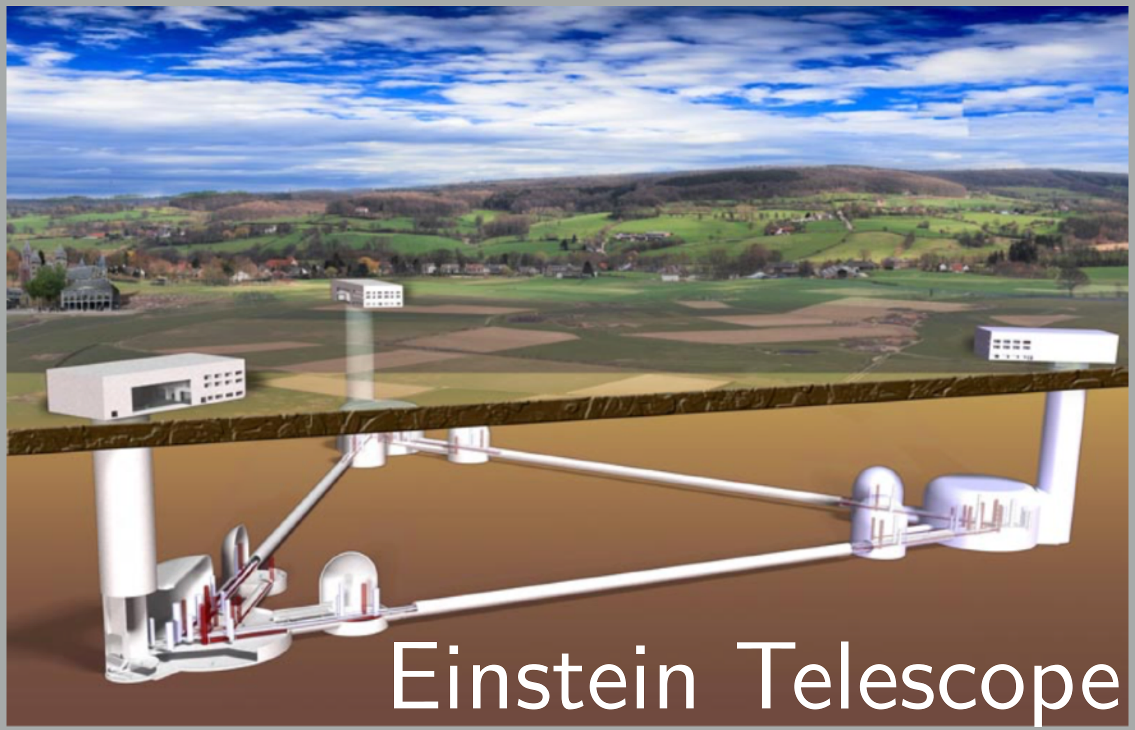
Hall+ (2019)

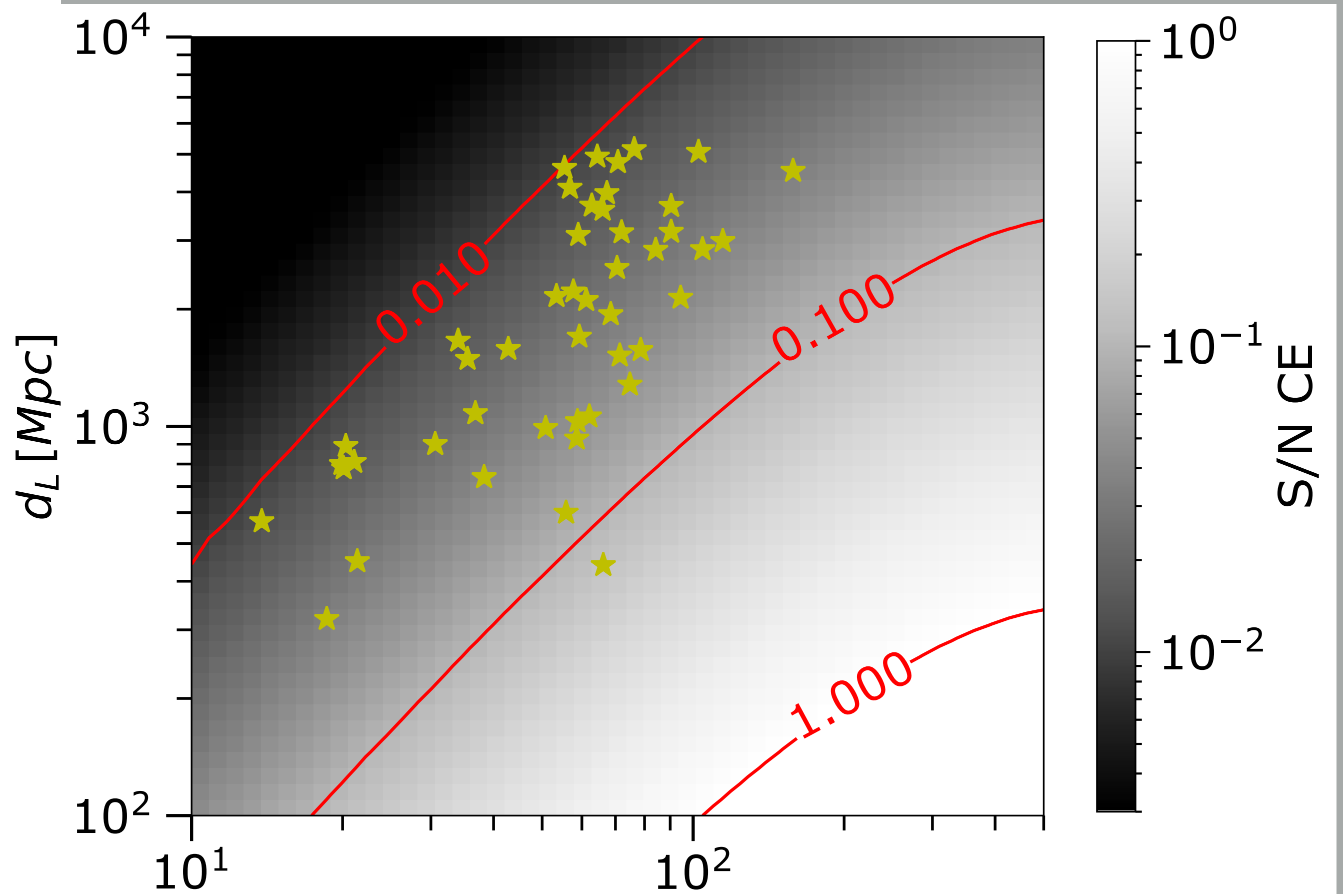


- CE: 40km, above ground, US, stage 1 mid 2030s, stage 2 2040s
- ET: 10km, underground, EU, operational in 2030s

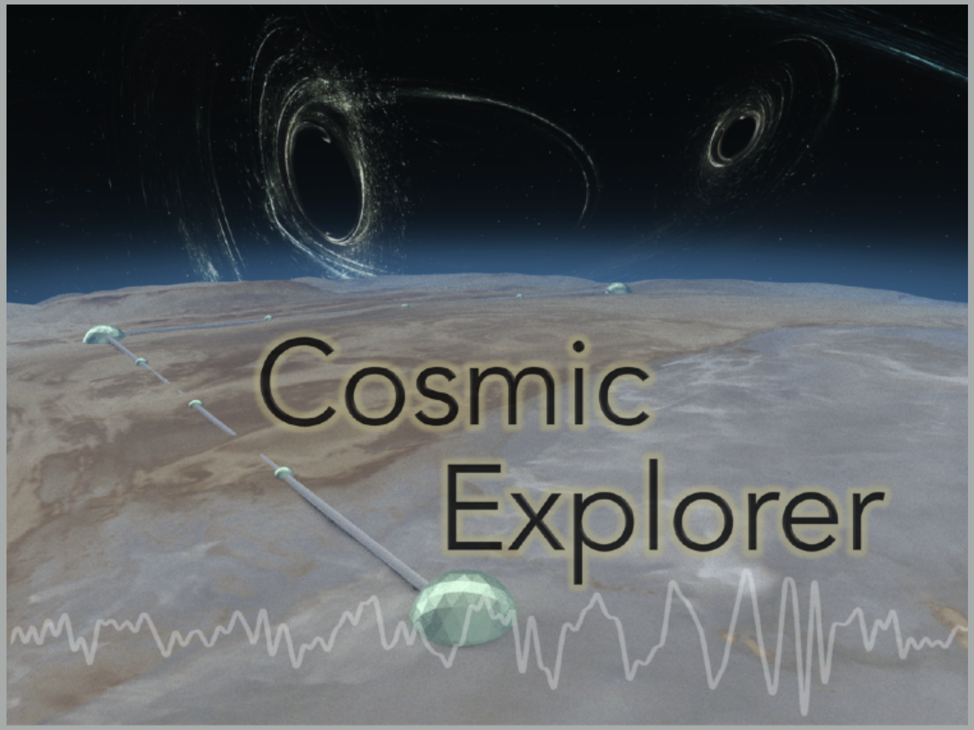
5. Detecting spin memory with ET and CE



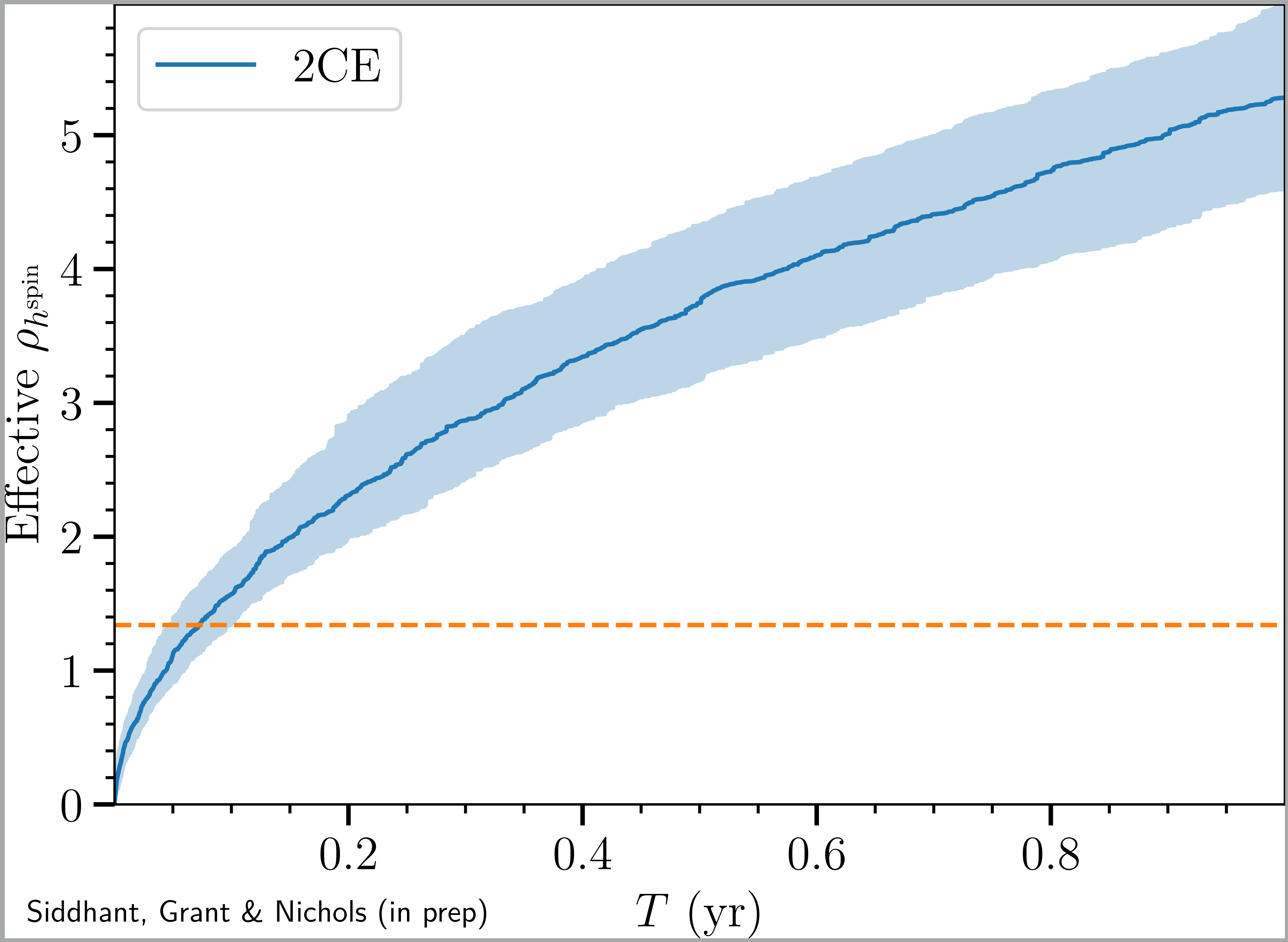




Nichols, (unpublished)



Time to spin memory detection



- Detection possible in O(1 year) with O(10^4) events!

Computing CM Memory

- Compute $D^A N_A$ and use equation for the mass aspect:

$$D^2\mathcal{D}\Delta\Phi = D^2\int N_{AB}N^{AB}du \quad \text{(Displacement memory from before)}$$

$$-2D^A\left(4\dot{N}_A + 3D_B N^{BC}C_{CA} + N^{BC}D_B C_{CA}\right)$$

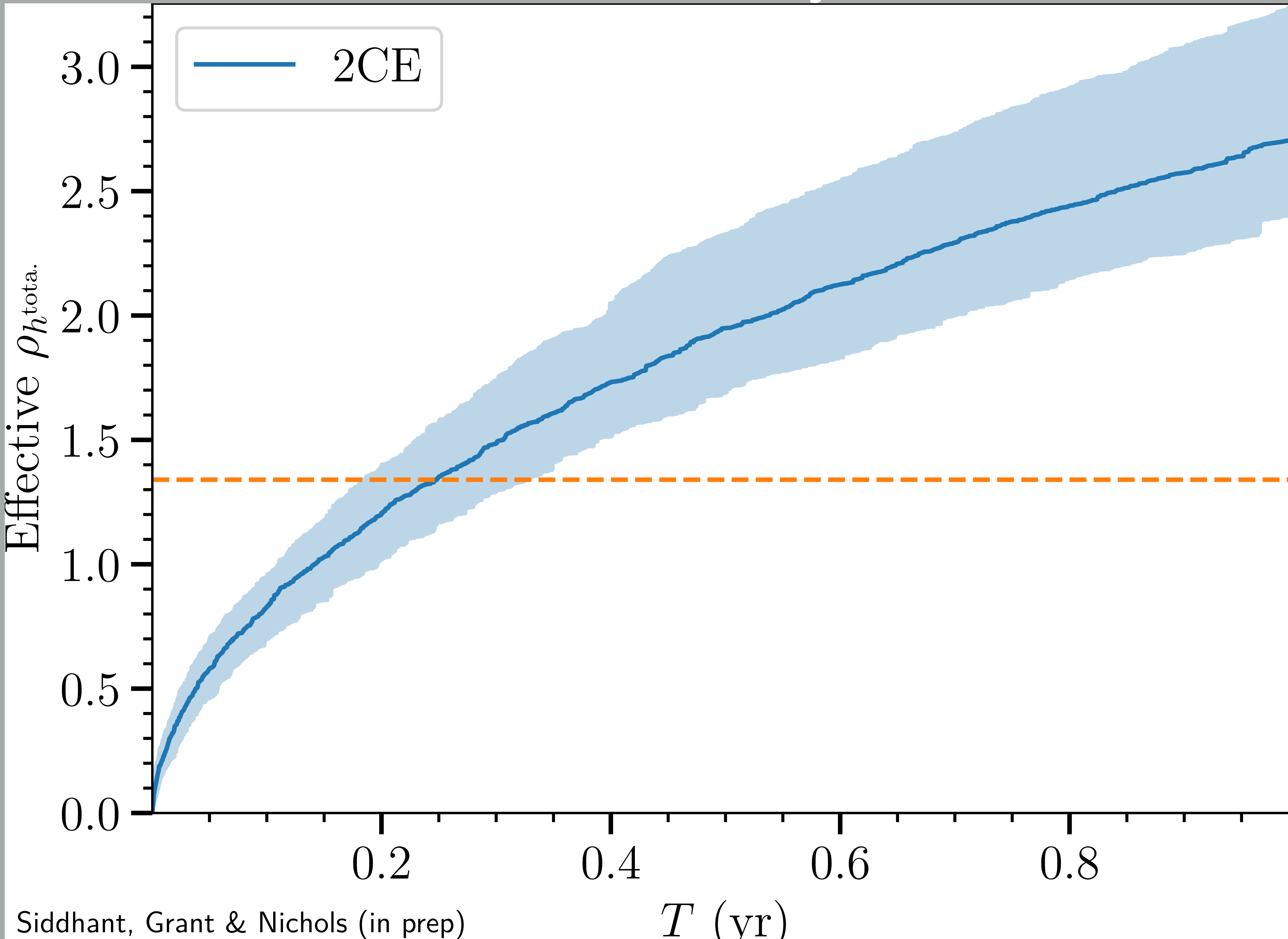
(New flux and charge terms, which when integrated produce an offset in the time integral of shear)

- Dominant non-oscillatory contribution comes from the charge term ("stationary-to-stationary" approximation):

$$D^A\dot{N}_A = -\frac{3}{2}\partial_u\left(m_{(\ell=0)}D^2\Phi_{\mathrm{mem}}\right)$$

- GW memory signal from CM memory looks qualitatively the same as the spin memory, but is ~1/2 the magnitude.

Time to CM memory detection



Siddhant, Grant & Nichols (in prep)

- Detection also possible in O(1 year) with O(10^4) events!

Yet higher memory effects

- Next higher memory comes from the "quadrupole" aspect Grant and Nichols (2022); also called "ballistic memory" Grant (2024)
- PN calculations: displacement, spin, CM and ballistic flux terms all contribute to the nonlinear radiative multipoles in the known 3.5PN waveform (also charge contributions) Siddhant, Grant, Nichols (2024)
- Electric-parity part from

$$\mathcal{D}^2\Delta\Phi = \mathcal{D}\int N_{AB}N^{AB}du - 2(D^2+2)D^A\left(3D_BN^{BC}C_{CA} + N^{BC}D_BC_{CA}\right)$$

(Displacement and CM memory signals from before)

$$+24D_AD_B\ddot{\mathcal{E}}^{AB} + \frac{3}{2}D_AD_B\mathcal{D}\partial_u[\Psi({}^*C)^{AB} - \Phi C^{AB}]$$

(New charge and "pseudo-flux" terms for ballistic memory)

$$-D_AD_B\partial_u[(N_{CD}C^{CD})C^{AB}]$$

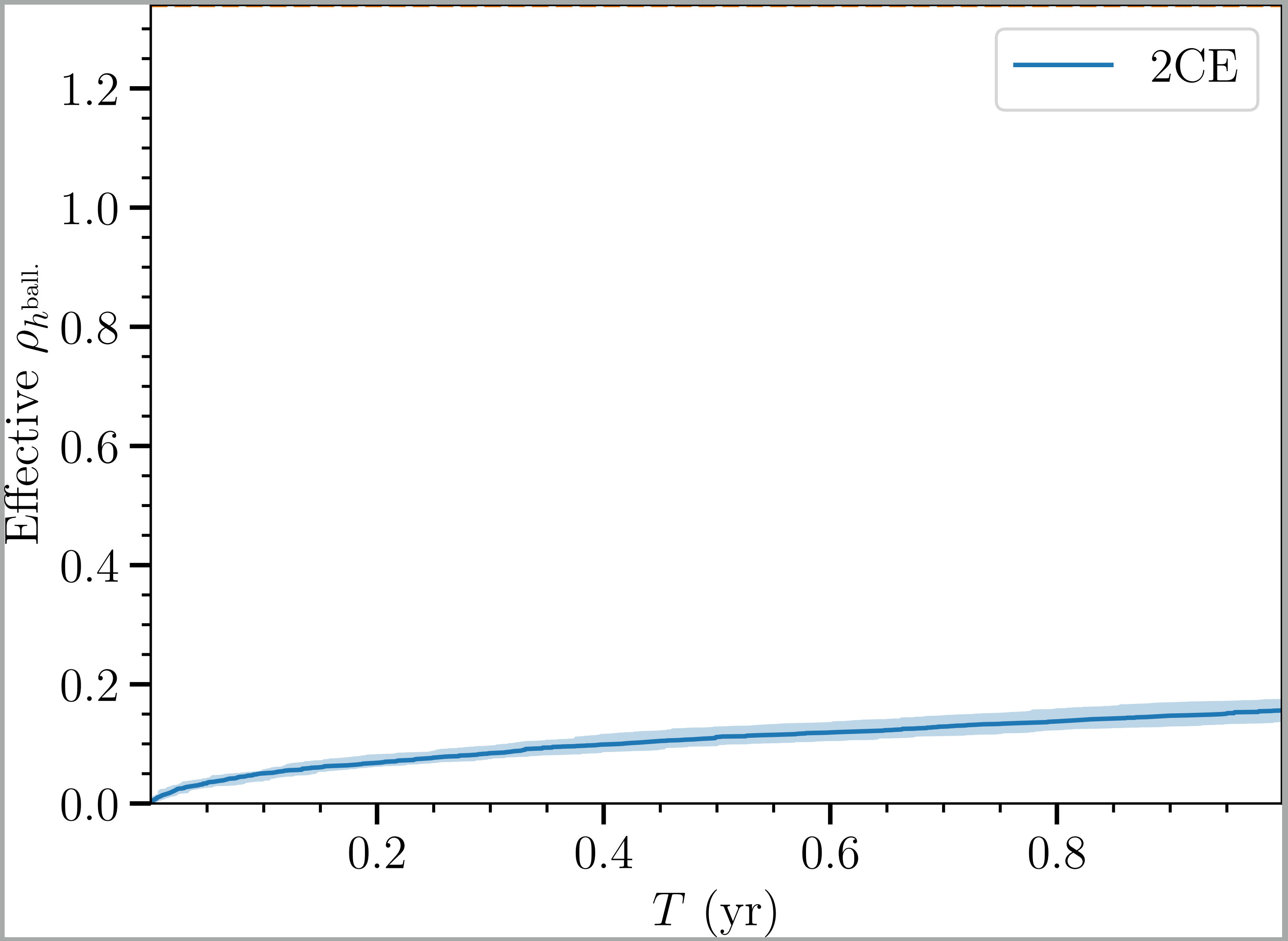
(New flux term for ballistic memory)

Ballistic Memory

- Charge contribution obtained via (for $\ell \geq 3$, stat.-to-stat.)

$$(\mathsf{D}^2+6)\mathsf{D}_\mathsf{A}\mathsf{D}_\mathsf{B}\mathcal{E}^{\mathsf{AB}} = -\frac{2}{3}[2\mathsf{D}^\mathsf{A}\mathsf{D}^\mathsf{B}\mathsf{D}^\mathsf{C}(\mathsf{N}_\mathsf{A}\mathsf{C}_{\mathsf{BC}}) - \mathsf{D}^2\mathsf{D}^\mathsf{B}(\mathsf{N}^\mathsf{C}\mathsf{C}_{\mathsf{BC}})]$$

- Forecasts of ballistic memory in progress
- Have computed charge, but *not* flux or pseudo-flux terms
- Charge part is small:
- Will need to go beyond current approximation to compute the flux terms



Siddhant, Grant & Nichols (in prep)

6. Conclusions

- GW memory from BBH mergers is an observational signature of conservation laws in asymptotic flatness
- LIGO (A+) can detect it in a population of mergers

- Two generalizations of memory effects related to angular momentum evolution: spin and CM memory
- Spin memory and CM are measurable by CE in populations

- A new hierarchy of potentially measurable memory effects related to higher Bondi evolution equations.
- Initial calculations suggest beyond CE, but more work needed