Post-Minkowskian, Numerical Relativity and Effective-one-body formalism

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IFPU Workshop - June 2026

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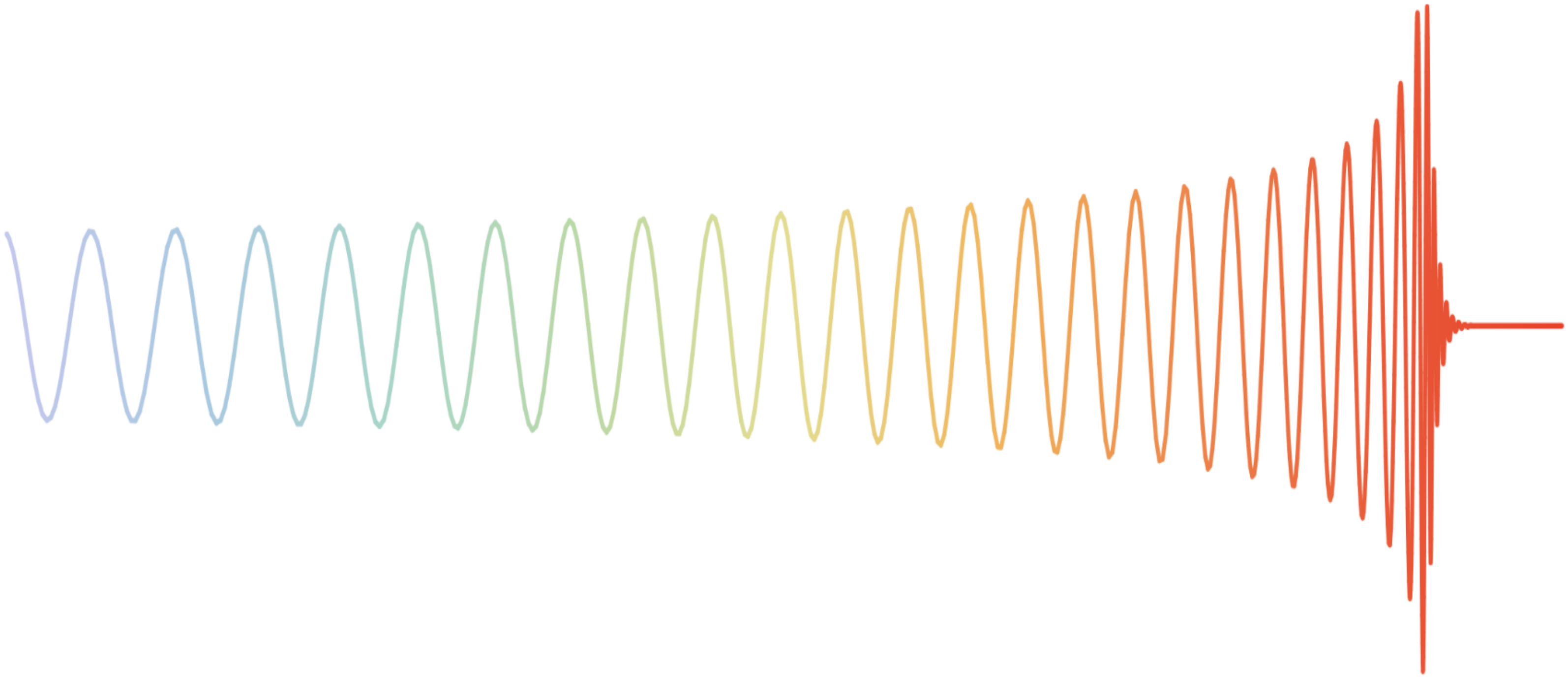
Goals

Gravitational waves (GWs) observations → Strong field tests of gravity

Key ingredient for GW physics → waveform templates (numerical+analytical) and parameter estimation (PE) to infer the source properties and test fundamental fields

Anatomy of an inspiral



TEOBResumS
Effective-one-body model for waveforms from compact binaries

Anatomy of an inspiral

Weak field approximations

post-Newtonian (PN)
Small velocity, weak gravity
(v^2/c^2)

post-Minkowski (PM)
weak gravity (G)

Gravitational self-force (GSF)
(m_2/m_1)

Inspiral

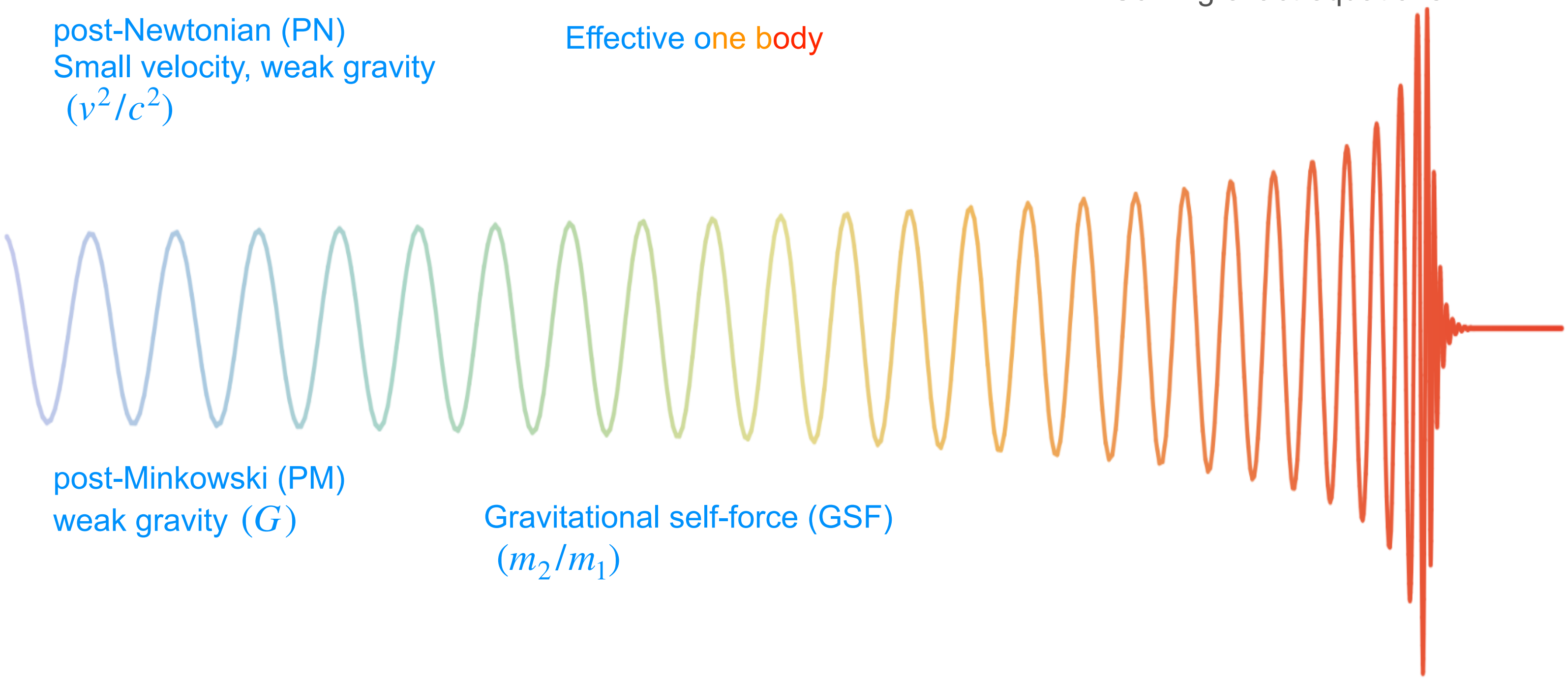
TEOBResumS
Effective-one-body model for waveforms from compact binaries

Anatomy of an inspiral

Numerical Relativity
Solving exact equations

post-Newtonian (PN)
Small velocity, weak gravity
(v^2/c^2)

Effective one body



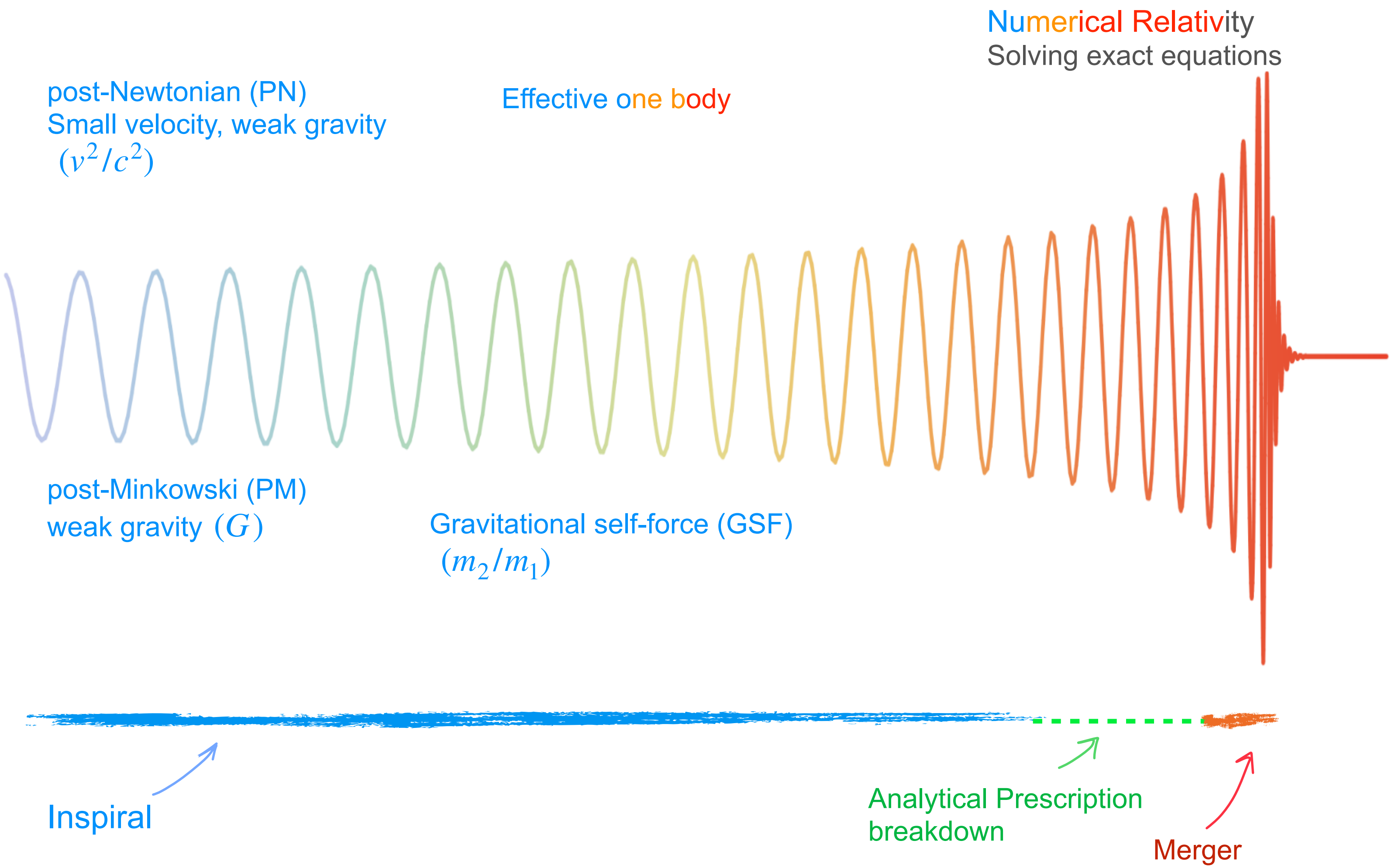
post-Minkowski (PM)
weak gravity (G)

Gravitational self-force (GSF)
(m_2/m_1)

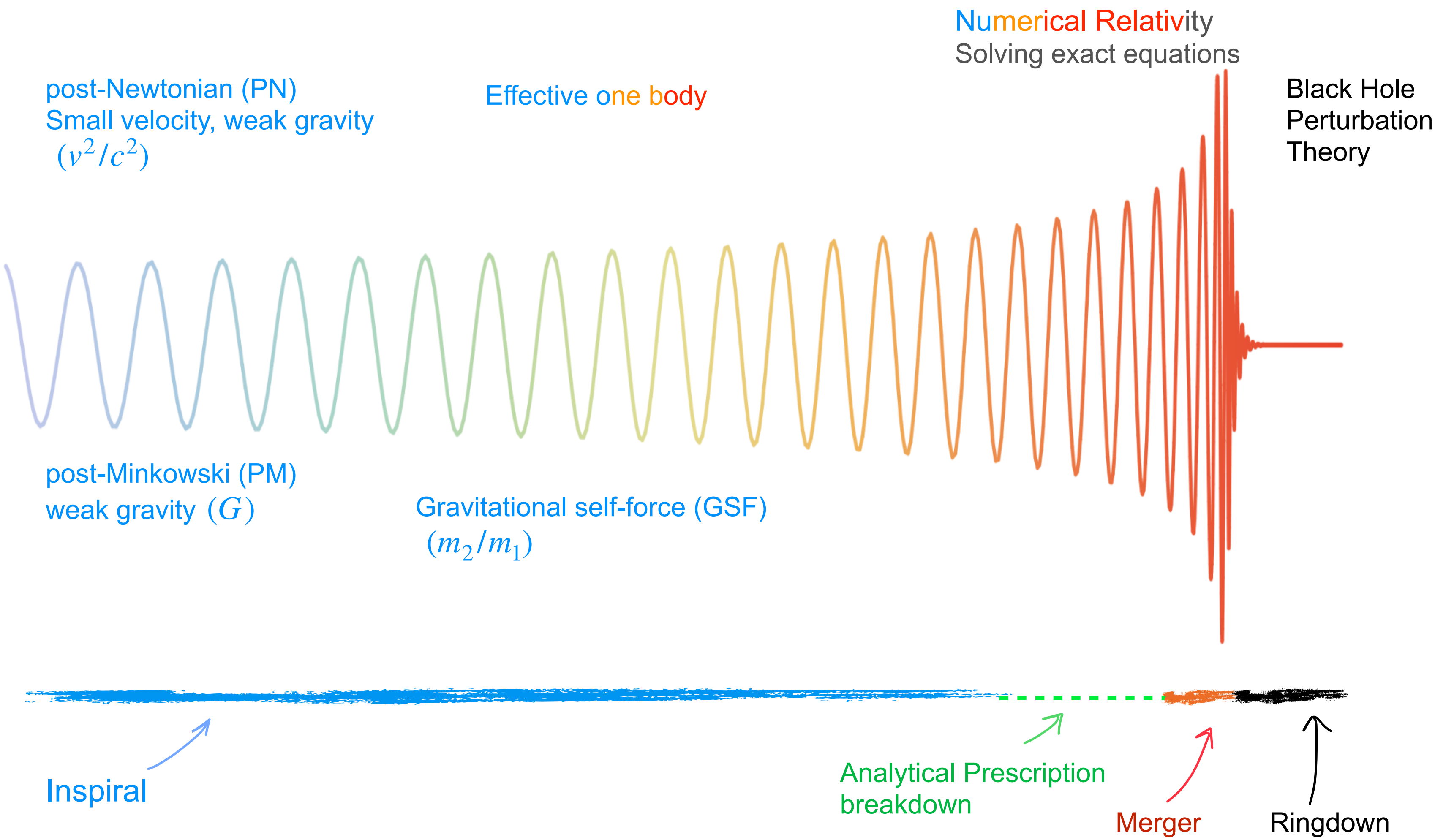
Inspiral

Analytical Prescription
breakdown

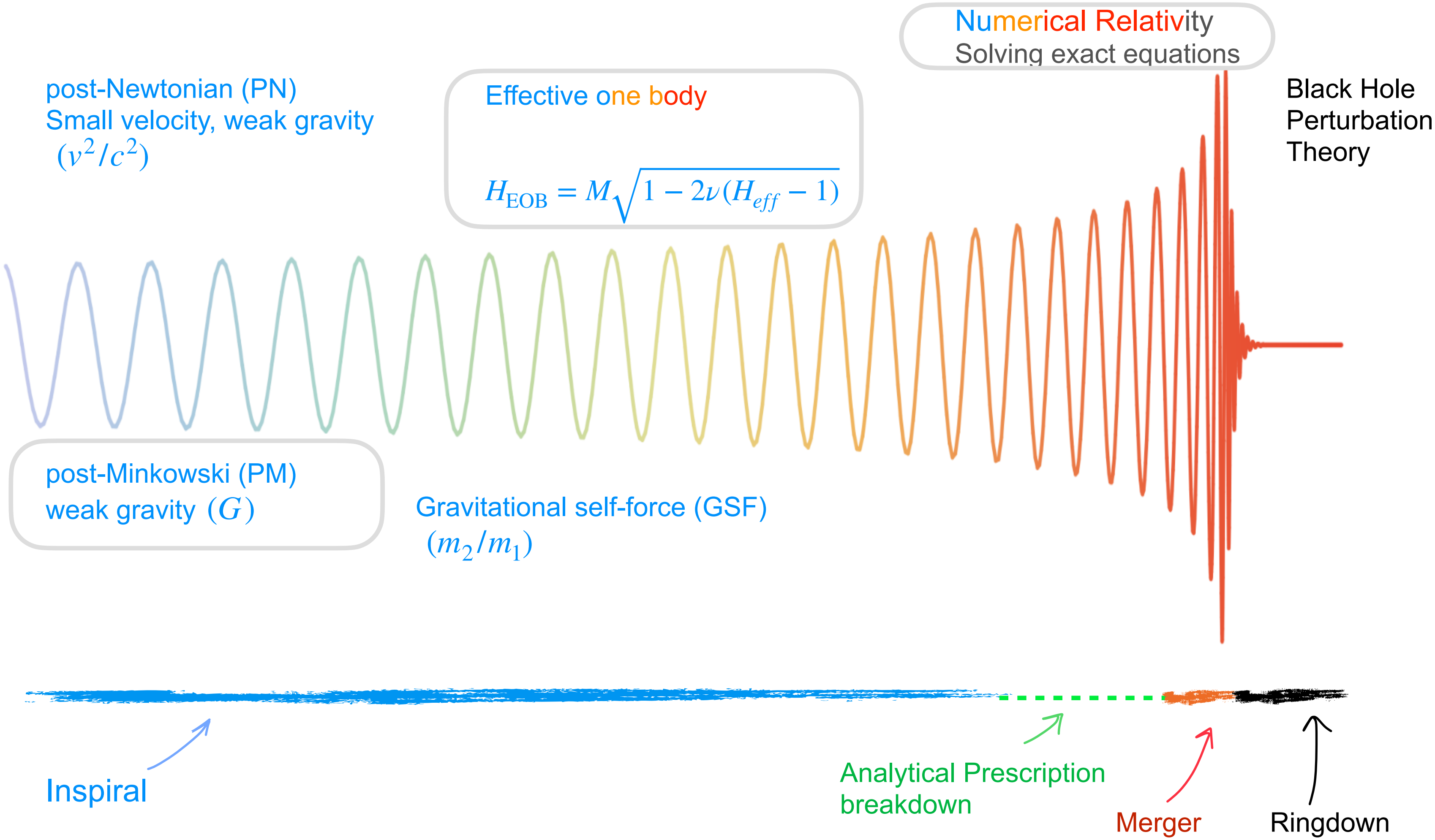
Anatomy of an inspiral



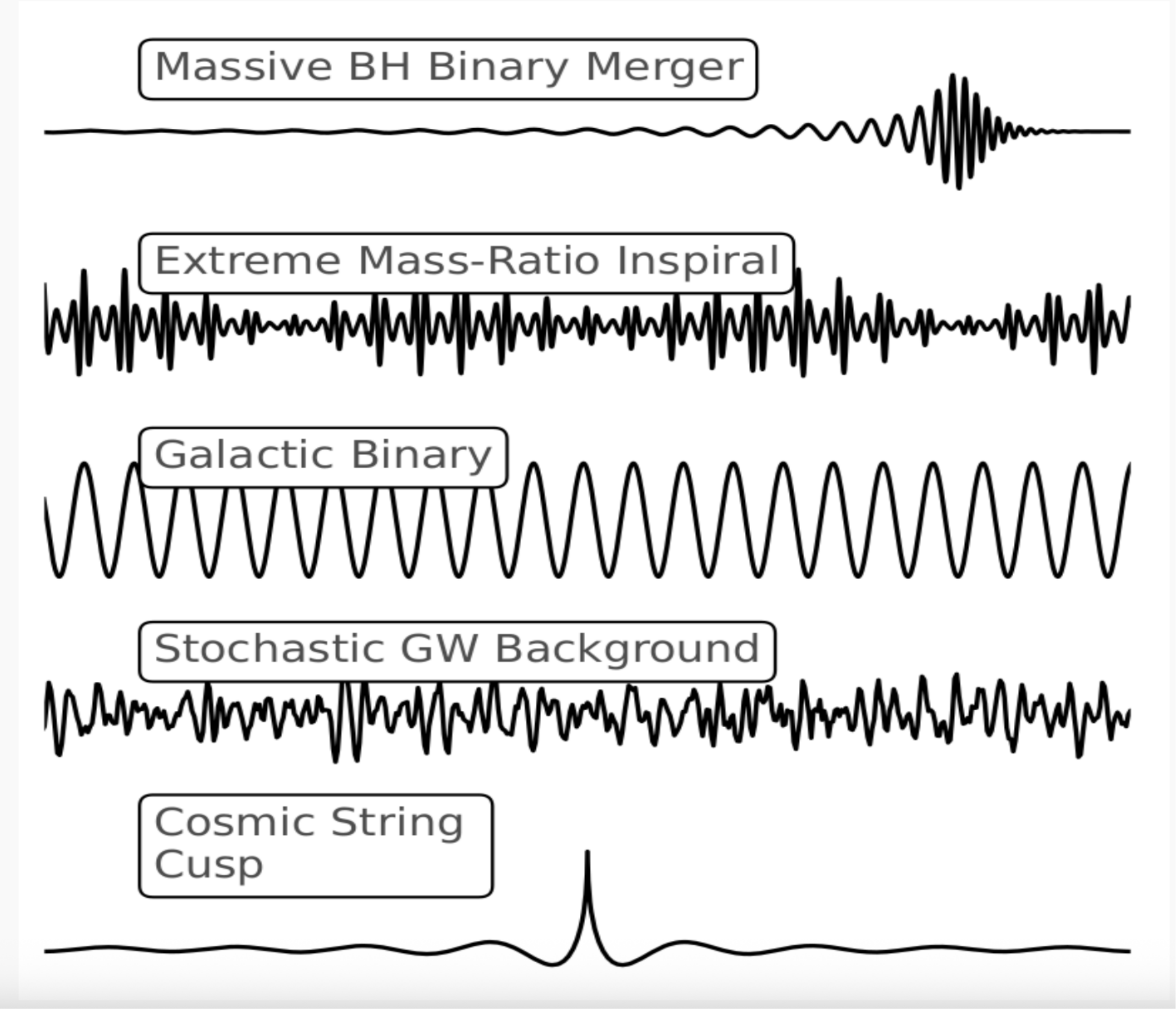
Anatomy of an inspiral



Anatomy of an inspiral



Gravitational Waveforms

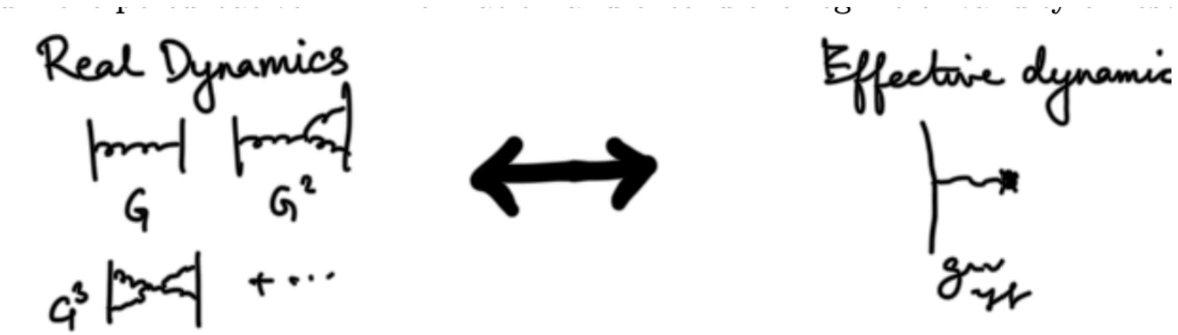


LISA definition study report 2023

Effective One Body

PN is slowly and oscillating convergent (e.g. test mass around a Schwarzschild BH) → needs resummation, such as Padé. Damour, Iyer and Sathyaprakash 1998, 2000 and 2001

Novel approach introduced by Buonanno and Damour in 1999 with the basic idea to map two-body problem onto an effective one-body problem

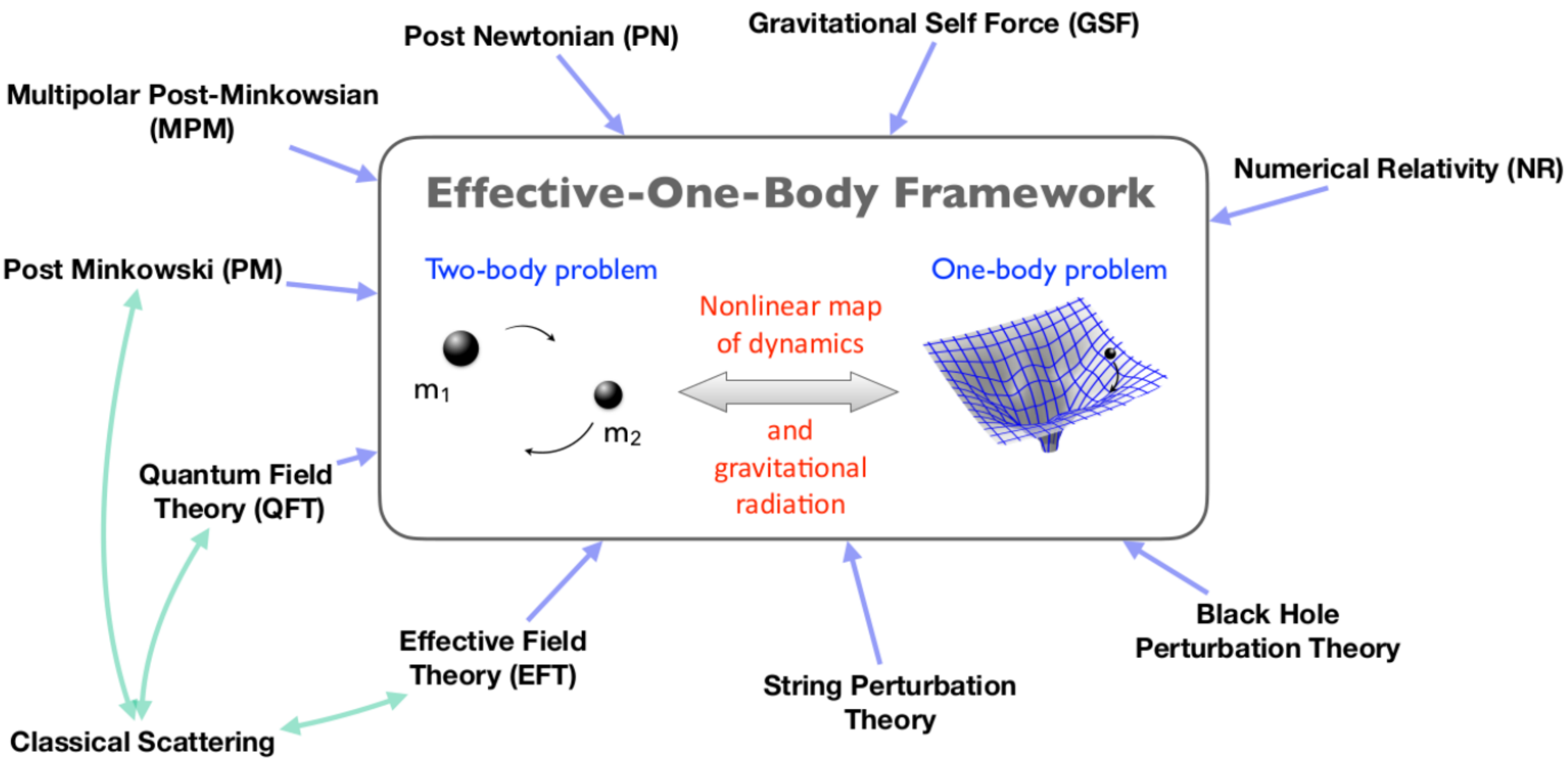


Energy map relating the two systems

$$\frac{E_{\rm eff}}{\mu} = \frac{E^2_{\rm real} - m_1^2 - m_2^2}{2 m_1 m_2}$$

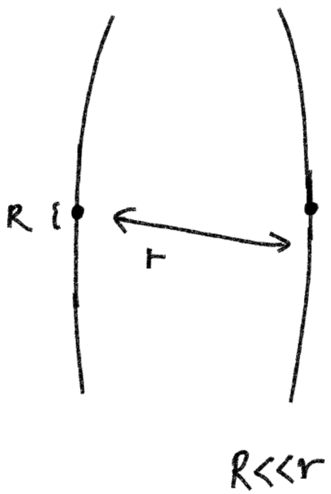
EOB resums the perturbative PN information and extend the regime of validity of results

Effective One Body



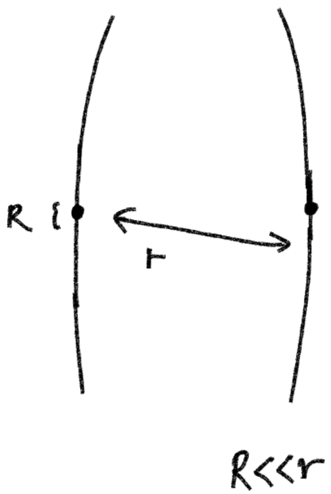
LISA waveform white paper 2023

Effective Field Theory



See Luca's and Fillipo's talk

Effective Field Theory



Post-Minkowskian expansion (G): weak gravity

Various approaches, such as: scattering amplitudes, eikonalization, *post-Minkowskian effective field theory*, worldline quantum field theory, …

For point-particle dynamics full results known up to 4PM with partial results at 5PM order
Bini+(2021), Bern+(2022,2021), Dlapa+(2022), Kalin+(2020, 2020), *etcetera*

Results also known for tidal and spinning effects

See Luca's and Fillipo's talk

From EFT to Numerical Simulations

Calibration of analytical waveforms with numerical results for *reliable* waveform templates

Hyperbolic orbits (scattering configurations) offer direct comparison of analytical and NR

Well tested (and calibrated) for point-particle, i.e. BHs in GR Damour+(2022), Rettegno+(2023), Buonanno+(2024), Long+(2025), Swain+(2025), *etcetera*

Direct map to bound orbits using EOB → generate robust analytical waveform templates (eccentric+hyperbolic encounters) Buonanno+(2024), Damour+(2026)

Can we test the effect due to *tidal interactions* using objects other than NSs? And modified theories?

An example of Boson Stars

Increased interest in beyond GR+exotic compact objects (BH-mimickers) → Boson stars

Well explored numerically (critical collapse, waveforms), simpler to evolve than neutron stars and tidally deformable →ideal system to measure tides

The action describing the boson star with potential $V(|\varphi|^2)$

$$S_{\text{bulk}} = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} - \frac{1}{2} g^{\mu\nu} \nabla_\mu \varphi \nabla_\nu \bar{\varphi} - V\left(|\varphi|^2\right) \right],$$

Ansatz for spherically symmetrical solution is $\varphi(t, r) = A(r) e^{i(\epsilon\omega t + \Phi)}$, where $A(r)$ is the (real) scalar field amplitude profile, ω is the frequency, and Φ is an initial phase

An example of Boson Stars

PM-expansion is an expansion of $1/j$ which loses accuracy is strong field systems → map to EOB.

Using EOB gravitational potential w to resum the PM information Damour+(2022)

$$\pi + \chi(\gamma, j) = 2j \int_0^{\bar{u}_{\max}(\gamma,j)} \frac{d\bar{u}}{\sqrt{p_\infty^2 + w(\bar{u},\gamma) - j^2\bar{u}^2}}$$

Decomposition of w to include tidal interactions

$$w(\gamma, \bar{r}) = w^{\mathrm{BH}}(\gamma, \bar{r}) + w^{\mathrm{tid}}(\gamma, \bar{r})$$

Inverting the relation to obtain

$$\chi_6^{\mathrm{tid}} = \frac{15\pi}{32} p_\infty^4 w_6^{\mathrm{tid}}, \quad \chi_7^{\mathrm{tid}} = 4p_\infty^3 w_1 w_6^{\mathrm{tid}} + \frac{8}{5} p_\infty^5 w_7^{\mathrm{tid}}$$

The analytic and numerical results differed by *~40 degrees* when we included only point-particle+tidal effects → suggesting *missing* physics

Analytics

Damour, Jain and Sperhake (2025), accepted in PRL

Missed scalar field effects

From faraway boson stars act as point-like particle with scalar strength $s_A \sim c_A e^{i\omega_A \tau_A}$
→ similar to scalar-tensor theories where the scalar source strength is determined by $m_A(\phi)$ and its derivatives

The point particle action is,

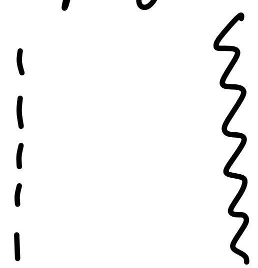
$$S_{pp} = -\sum_A \int \left\{ m_A - 2\pi \left[\varphi(\mathbf{z}_A)\bar{s}_A(\tau_A) + \bar{\varphi}(\mathbf{z}_A)s_A(\tau_A) \right] \right\} d\tau_A$$

where s_A is the scalar source

Analytics

Feynman Rules → world-line vertices, bulk vertices and propagators

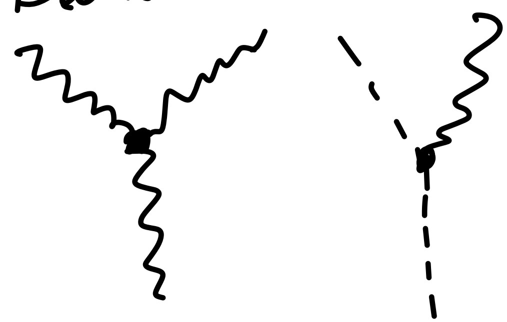
Propagators

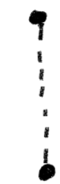


$\frac{i}{k^2-\mu^2}$ $\frac{i P_{\alpha\beta\mu\nu}}{k^2}$

World-line vertices

Bulk vertices





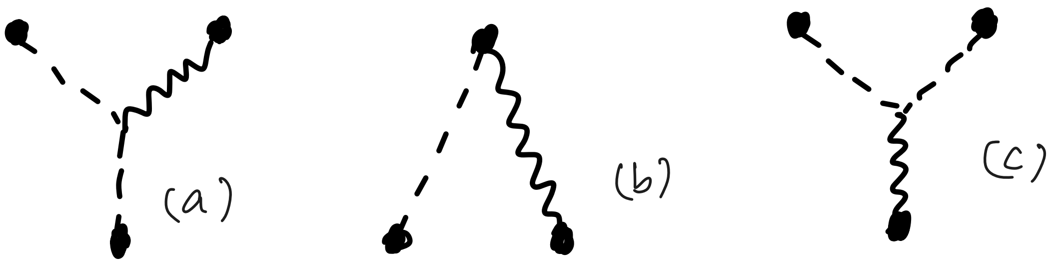
For one scalar-exchange the impulse is,

$$\Delta p_{1\,\mathrm{LO}}^{\nu\,\mathrm{scalar}} = -\frac{8\pi\tilde{\mu}c_1c_2}{\sqrt{\gamma^2-1}}K_1\big(\tilde{\mu}\,|\,b\,|\,\big)\frac{b^\nu}{|\,b\,|}\cos(\Phi_{21}) \underbrace{-\frac{8\pi c_1c_2}{\sqrt{\gamma^2-1}}(\omega_1\gamma-\omega_2)\check{u}_2^\mu\sin(\Phi_{21})K_0(\tilde{\mu}\,|\,b\,|\,)}_{\text{non-conservative piece}}$$

Where K's are Bessel functions.

Analytics

Therefore, scalar-exchange at LO is Yukawa-type → the important contributions at next order are due to gravitationally dressing of scalar exchange. The Feynman diagrams up to $\mathcal{O}(Gc_Ac_B)$ are



At $\mathcal{O}(Gc_Ac_B)$ no closed analytical solution → expansion in small-velocity to obtain scale of the system. The structure of these integrals are

$$\Delta p_1^\mu \propto \frac{\partial}{\partial b^\mu}\int \frac{d|k_\perp|}{2\pi}\, C_{a/b}\, J_0(k_\perp b)\, \arctan(|k_\perp|/\tilde{\mu})$$

(a) $\sim G m_A \left(\frac{2\omega^2-\mu^2}{\tilde{\mu}^2}\right)\log\left(2e^{\gamma_E}\tilde{\mu} b\right)$

(b) $\frac{G m_A}{b}$

(c) $\sim \frac{e^{-2\tilde{\mu} r}}{r}$

Mapping to EOB

Decomposition of w to include scalar interactions as well

$$w(\gamma,\bar{r}) = w^{\rm BH}(\gamma,\bar{r}) + w^{\rm tid}(\gamma,\bar{r}) + w^{\rm scalar}(\gamma,\bar{r})$$

By inverting χ and w relation, at LO we obtain

$$w_{\rm LO}^{\rm scalar} = \frac{8\pi c_1 c_2}{G m_1 m_2}\frac{e^{-\bar{m}\bar{r}}}{\bar{r}}\cos(\Phi_{21})$$

For high-orders, factorising w as scalar $w_{\rm HO}^{\rm scalar} = w_{\rm LO}^{\rm scalar} w_{\rm g-dressing}^{\rm scalar}$, and motivated by the solution of the scalar profile, we consider exponentiated structure of the g-dressing effects

$$w_{\rm g-dressing}^{\rm scalar} = e^{GM((2\omega^2-\mu^2)/\tilde{\mu})\log(2e^{\gamma_{\rm E}}GM\tilde{\mu}\bar{r})}$$

Solve for scattering angle using the EOB potentials to obtain the resummed angles → validate with NR simulations

Numerics

Solve 3+1 equations for the action of boson stars S_{bulk} using *LEAN* code

$$\partial_t \chi = \beta^m \partial_m \chi + \frac{2}{3}\chi(\alpha K - \partial_m \beta^m)\,, \tag{8}$$

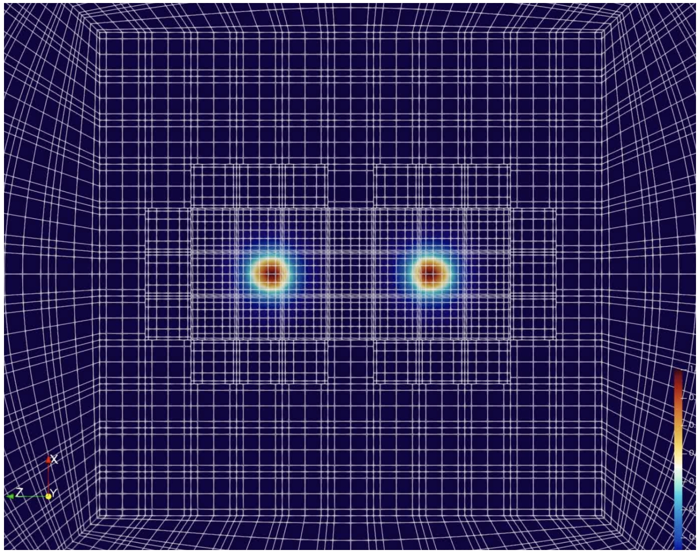
$$\partial_t \tilde{\gamma}_{ij} = \beta^m \partial_m \tilde{\gamma}_{ij} + 2\tilde{\gamma}_{m(i}\partial_{j)}\beta^m - \frac{2}{3}\tilde{\gamma}_{ij}\partial_m \beta^m - 2\alpha \tilde{A}_{ij}\,, \tag{9}$$

$$\partial_t K = \beta^m \partial_m K - \chi \tilde{\gamma}^{mn} D_m D_n \alpha + \alpha \tilde{A}^{mn}\tilde{A}_{mn} + \frac{1}{3}\alpha K^2 + 4\pi G\alpha(S+\rho)\,, \tag{10}$$

$$\begin{aligned}\partial_t \tilde{A}_{ij} &= \beta^m \partial_m \tilde{A}_{ij} + 2\tilde{A}_{m(i}\partial_{j)}\beta^m - \frac{2}{3}\tilde{A}_{ij}\partial_m \beta^m + \alpha K \tilde{A}_{ij} - 2\alpha \tilde{A}_{im}\tilde{A}^m{}_j \\ &\quad + \chi(\alpha \mathcal{R}_{ij} - D_i D_j \alpha - 8\pi G \alpha S_{ij})^{\text{TF}}\,,\end{aligned} \tag{11}$$

$$\begin{aligned}\partial_t \tilde{\Gamma}^i &= \beta^m \partial_m \tilde{\Gamma}^i + \frac{2}{3}\tilde{\Gamma}^i \partial_m \beta^m - \tilde{\Gamma}^m \partial_m \beta^i + \tilde{\gamma}^{mn}\partial_m \partial_n \beta^i + \frac{1}{3}\tilde{\gamma}^{im}\partial_m \partial_n \beta^n \\ &\quad - \tilde{A}^{im}\left(3\alpha \frac{\partial_m \chi}{\chi} + 2\partial_m \alpha\right) + 2\alpha \tilde{\Gamma}^i_{mn}\tilde{A}^{mn} - \frac{4}{3}\alpha \tilde{\gamma}^{im}\partial_m K - 16\pi G \frac{\alpha}{\chi} j^i\,,\end{aligned} \tag{12}$$

Superposed initial data $\gamma_{ij} = \gamma^A_{ij} + \gamma^B_{ij} - \gamma^B_{ij}(x^i_A)$ to take into account influence of one star onto the other Helfer+ 2021

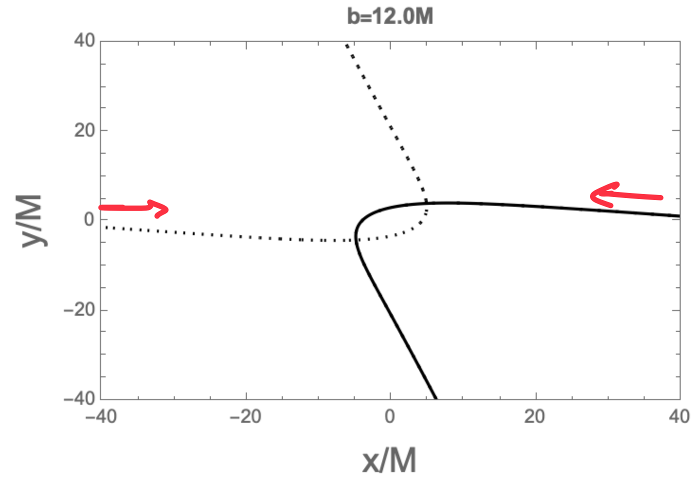


Box-in-box approach

Numerics

Considering four different sequences: 3 sequences with phase difference (Φ = 0, 90 and 180) with ϵ = 1 and one sequence with Φ = 0, $\epsilon = -1$, i.e. an anti boson star configuration.

ε	1			-1
Φ	0	90	180	0



Considered a binary boson star configuration with velocity = 0.23c, separation X = 50.49GM

Extrapolate the trajectories to infinity and extract the numerical scattering angle

Extract initial energy and angular momentum from NR simulation for input into analytical results.

Numerics

The scalar charge c_A required for analytical results is $\sqrt{G}\mu c_A = 1.608$, which is obtained using asymptotic expression for the scalar profile

$$A(r) \approx \frac{c_A}{r} \exp\left[-\bar{\mu}_A r + G m_A \frac{2\omega_A^2 - \mu^2}{\bar{\mu}_A} \log(2e^{\gamma_E}\bar{\mu}_A r)\right]$$

with $\bar{\mu}_A = \sqrt{\mu^2 - \omega_A^2}$.

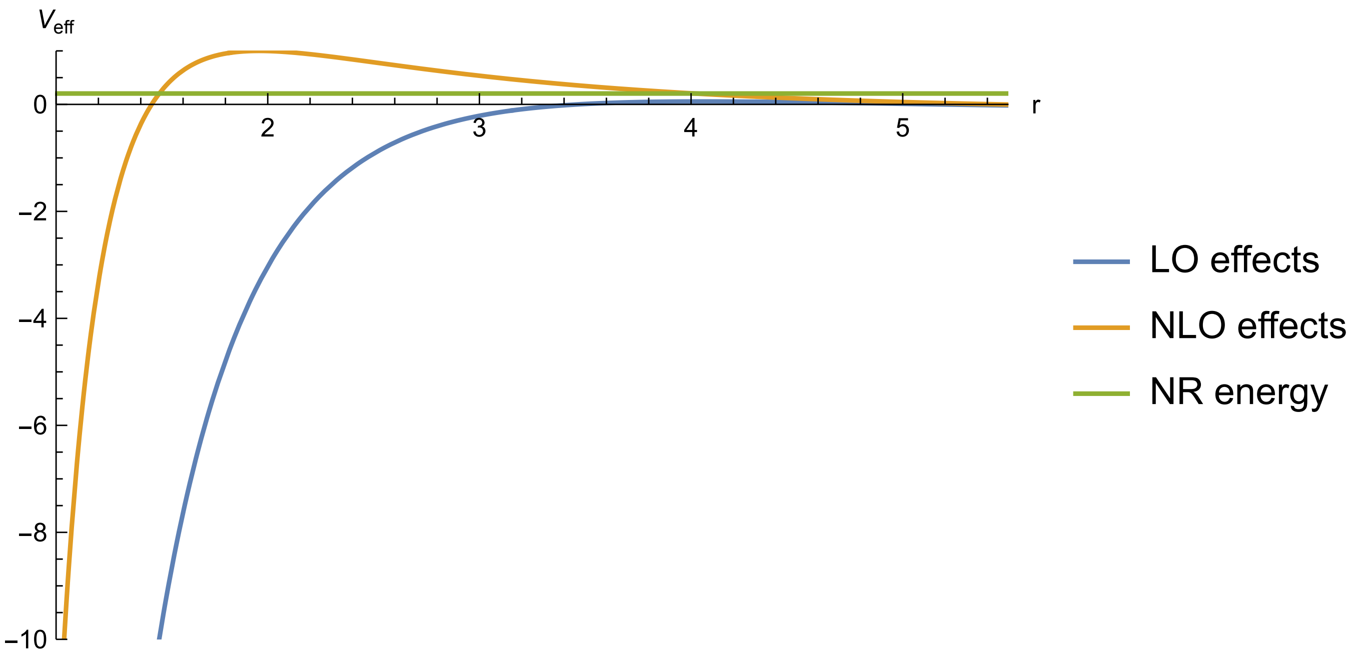
Numerical Data

TABLE I. Impact parameter b, total angular momentum $J_{\rm NR}$, and the scattering angles (together with their error bars) for the four BS binary sequences as extracted from the numerical simulations. For all configurations, the total energy is $E/M = 1.02394$.

$\frac{b}{GM}$	$\frac{J_{\rm NR}}{GM^2}$	$\chi_{\rm NR}^{\rm BSBS}$ [°]	$\chi_{\rm NR}^{\rm BS\overline{BS}}$ [°]	$\chi_{\rm NR}^{\rm BSBS^{\frac{\pi}{2}}}$ [°]	$\chi_{\rm NR}^{\rm BSBS^{\pi}}$ [°]
9.9	1.15315	258.79(95)	216.41(1.41)	214.18(1.40)	198.41(1.26)
10.5	1.22360	170.51(89)	165.42(81)	165.28(81)	161.23(75)
11.0	1.28186	143.91(62)	142.32(60)	142.28(60)	140.83(58)
12.0	1.39839	114.43(64)	114.20(63)	114.33(53)	113.96(65)
13.0	1.51492	96.89(84)	96.89(84)	96.75(60)	96.81(82)
14.1	1.64982	83.39(1.14)	83.38(1.14)	83.32(1.10)	83.32(1.11)
15.0	1.74798	75.51(1.23)	75.73(1.37)	75.62(1.30)	75.62(1.30)
16.0	1.86450	68.16(1.27)	68.44(1.44)	67.97(1.14)	68.50(1.47)

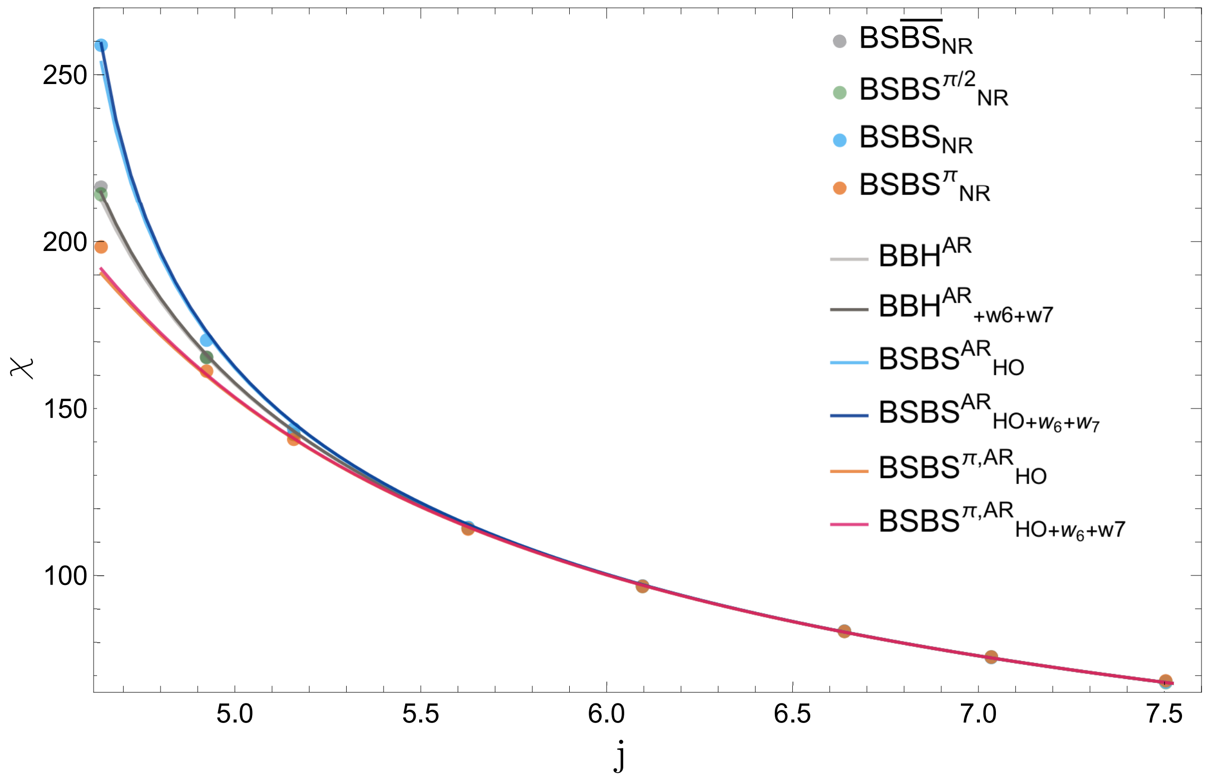
The errors due to finite differencing in NR simulations ~ 0.5 degrees in χ (much less than polynomial fitting error ~1.5 degrees)

Analytics vs Numerics



Effective potential vs radius curve for b = 10.5*M* impact parameter.

Analytics vs Numerics



- Constituent matter effects dominate in strong gravity!
- Subtle but real effects of tides!

Scattering angle comparison between the NR data (filled circles) and the EOB-resummed, PM based analytical prediction (AR; solid lines) for equal-mass, non-spinning boson star configurations for various rescaled initial angular momentum.

Analytics vs Numerics

TABLE II. Summary of the analytically predicted scattering angle along with the deviation from NR results including tidal interactions for BSB$\overline{\text{S}}$ and for BSBS$^{\frac{\pi}{2}}$ (denoted by $\frac{\pi}{2}$) configurations. For all configurations, the total energy is $E_{\text{in}}/M = 1.02394$.

$\frac{b}{GM}$	$\chi_{\text{AR}}^{\text{BBH}}$ [°]	$\delta_\chi^{\text{BSB}\overline{\text{S}}}$	$\delta_\chi^{\frac{\pi}{2}}$	$\chi_{\text{AR}}^{\text{BBH}+w6}$	$\delta_\chi^{\text{BSB}\overline{\text{S}}}$	$\delta_\chi^{\frac{\pi}{2}}$	$\chi_{\text{AR}}^{\text{BBH}+w6+w7}$	$\delta_\chi^{\text{BSB}\overline{\text{S}}}$	$\delta_\chi^{\frac{\pi}{2}}$
9.9	212.44	-2.81	-1.24	213.55	-2.03	-0.45	214.78	-1.16	0.43
10.5	165.62	0.25	0.42	165.89	0.58	0.75	166.13	0.87	1.04
11.0	143.10	1.30	1.36	143.21	1.49	1.55	143.30	1.64	1.70
12.0	114.90	0.12	1.08	114.93	1.16	1.14	114.95	1.20	1.18
13.0	97.21	0.38	0.76	97.22	0.39	0.78	97.22	0.40	0.79
14.1	83.12	-0.23	-0.19	83.12	-0.23	-0.19	83.11	-0.23	-0.19
15.0	75.39	-0.24	-0.18	75.39	-0.24	-0.18	75.39	-0.24	-0.17
16.0	68.03	-0.28	-0.05	68.03	-0.28	-0.06	68.03	-0.28	-0.06

Subtle but real effects of tides!

Broader Picture: NR vs Analytics in EsGB

Bernard, Jain and Mougiakakos (2026), accepted in JHEP

Same PMEFT machinery pushed to 3PM (two loop) computations in massless scalar tensor theories

The point particle action is

$$S_{\rm mat} = -\sum_{a=1,2} \int \mathrm{d}\tau_a \frac{m_a(\phi)}{2} \left[g_{\mu\nu}\left(x_a^\alpha(\tau_a)\right) v_a^\mu(\tau_a)\, v_a^\nu(\tau_a) + 1 \right]$$

The scattering result at 1PM is

$$\frac{\chi_b^{(1)}}{\Gamma} = \frac{1}{(\gamma^2-1)}\left(2\gamma^2 - 1 + \frac{\overbrace{\left(\tilde{d}_1^{(1)} + \tilde{f}_1^{(1)}\right)}^{\alpha_1}\overbrace{\left(\tilde{d}_1^{(2)} + \tilde{f}_1^{(2)}\right)}^{\alpha_2}}{2}\right)$$

The scalar field is massless so all the diagrams contribute (including pure scalar exchange)

The Feynman topologies at 2PM are

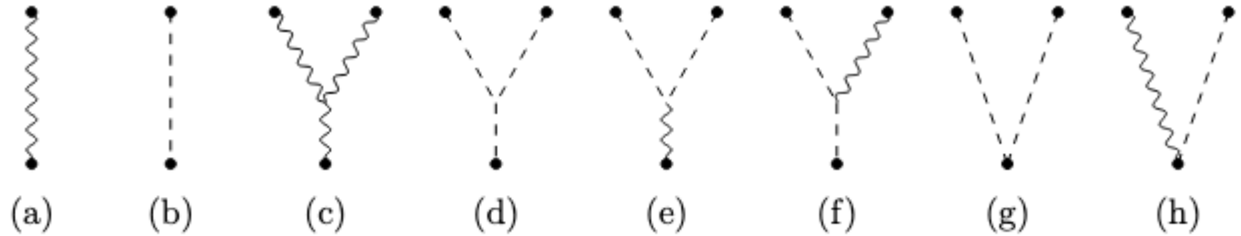


Figure 1: Feynman diagrams needed for the computation of the effective action up to 2PM order. The wavy lines represent the graviton propagator, dashed lines represents the scalar propagator and the black dots represent the external sources.

Broader Picture: NR vs Analytics in EsGB

Bernard, Jain and Mougiakakos (2026)

Integrals simplify if considering straight line (undeflected) trajectories!

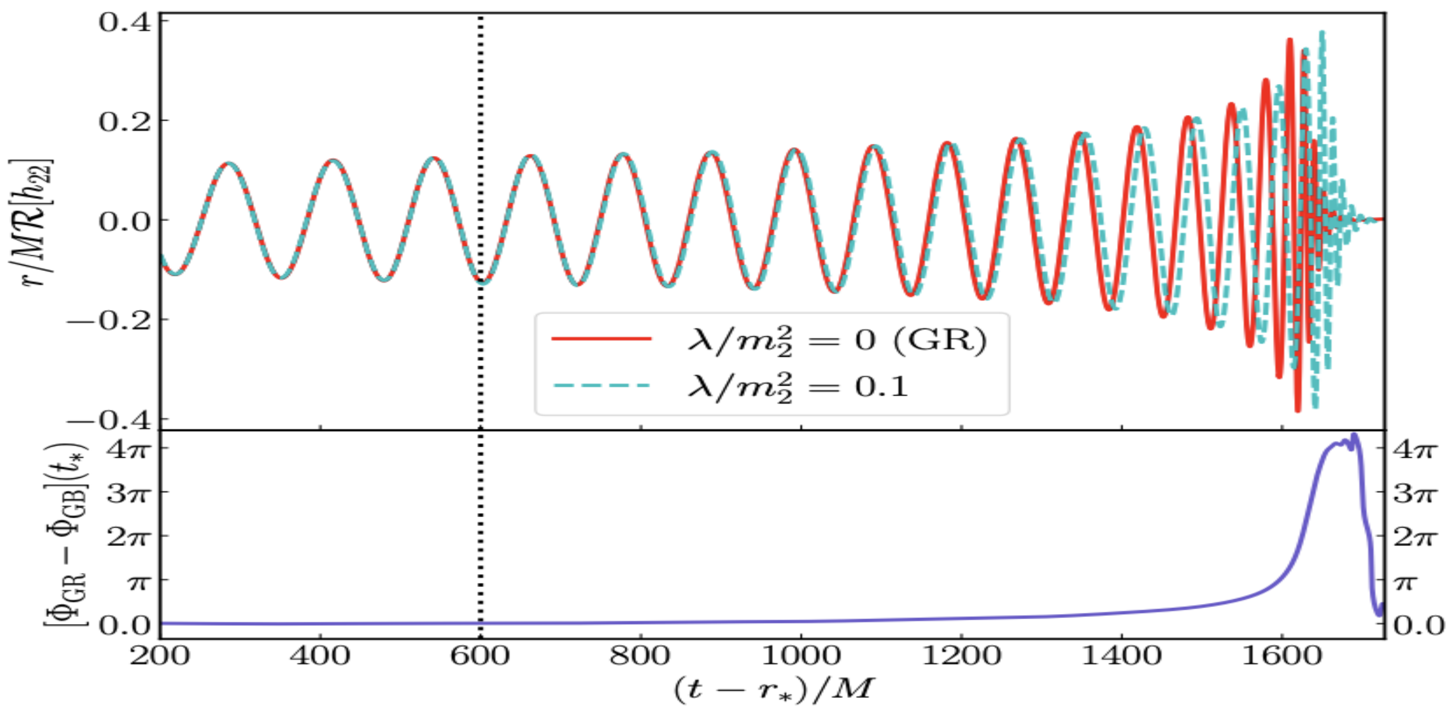
At 3PM order, we obtain ~60 Feynman Diagrams

Obtained the scattering angle using the master integrals of Jackobsen and Mogull 2022

The results verified with PM results in GR, up to 3PN order results with known PN results Jain (2024), and 3PM results of Almeida+ (2026)

Broader Picture: NR vs Analytics in EsGB

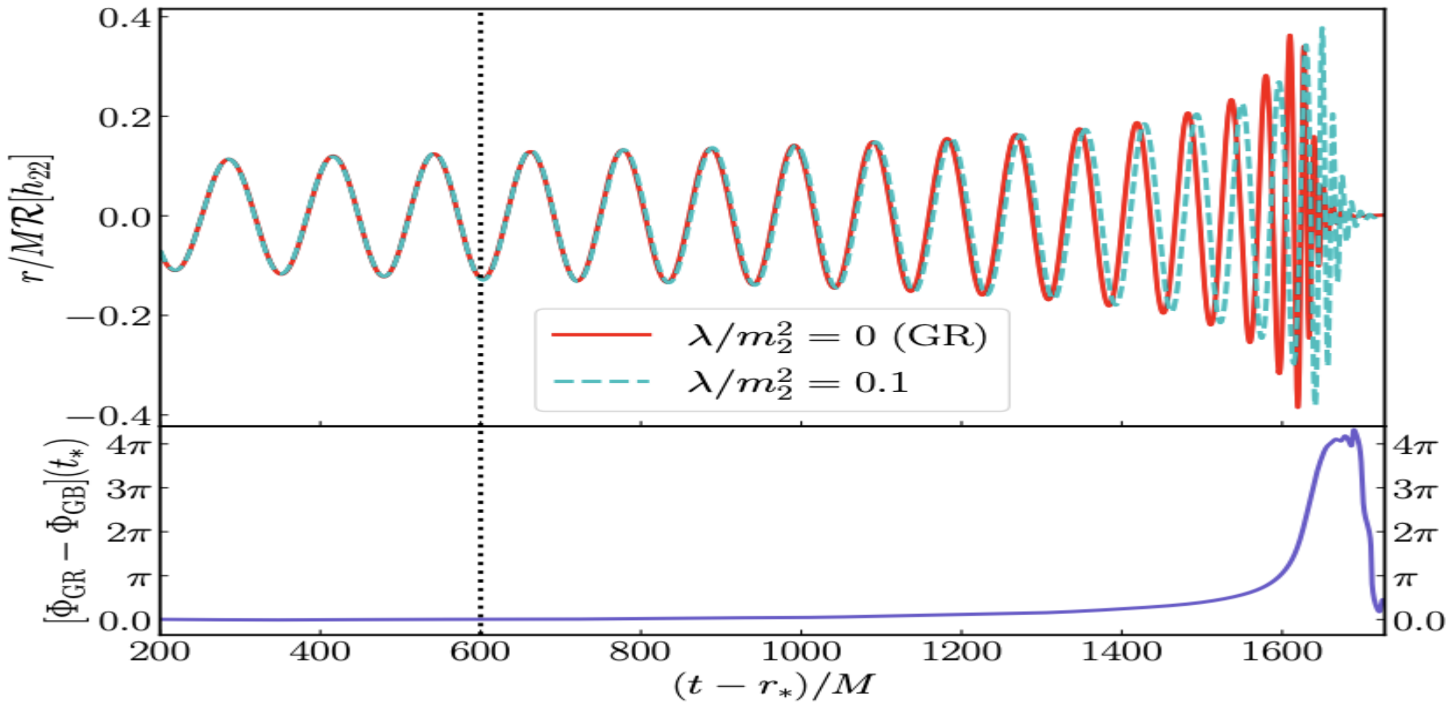
The NR results of Corman+(arXiv:2511.19073), showed that inspirals in EsGB theories are delayed compared to GR (*counterintuitive* to PN result)



Broader Picture: NR vs Analytics in EsGB

Motivated us to test using hyperbolic encounters

The NR results of Corman+(arXiv:2511.19073), showed that inspirals in EsGB theories are delayed compared to GR (*counterintuitive* to PN result)



Analytics: Derived resummed EOB w-potential derived up to 3PM (scalar + scalar-gravitational corrections) to compare to NR by decomposing the potential as

$$w_{\rm EsGB}^{\rm BH} = w_{\rm GR}^{\rm BH} + w_{\rm mod}^{\rm BH}$$

Inverting χ and w relation, we obtain

$$\begin{aligned} w_{\rm 1PM,mod}^{\rm BH} &= 2\alpha_1\alpha_2 \ , \\ w_{\rm 2PM,mod}^{\rm BH} &= \frac{1}{2h}\Big\{8\alpha_1\alpha_2 + 4(1+2\alpha_1\alpha_2)\left(\frac{m_2}{M}\beta_1 + \frac{m_1}{M}\beta_2\right) \\ &\quad + \frac{m_2}{M}\alpha_2^2\left(\gamma^2 - 1 + 4\alpha_1^2(1+\beta_1)\right) \\ &\quad + \frac{m_1}{M}\alpha_1^2\left(\gamma^2 - 1 + 4\alpha_2^2(1+\beta_2)\right)\Big\} \ , \qquad (6) \end{aligned}$$

$w^{\rm BH}_{\rm 3PM,mod}$ (given in the paper)

Broader Picture: NR vs Analytics in EsGB

Swain, Jain and Areste-Salo (2026)

__Numerics__: Hyperbolic encounters using *GRFolres* code of GRChombo for EsGB

Considered a set of *5* simulations exploring the parameter space
$\mathscr{P} \in \{\lambda/M^2, b_{\rm NR}/M\}$ (details on the next page)

Key diagnostic: $\mathscr{P} \in \{0,\, 9.7\}$ to recover GR limit *i.e*. scattering angle results using ETK given in Swain+, arXiv: 2411.09652 (2024)

A lot of fine tuning! [AMR approach]

The errors due to finite differencing in NR simulations ~ 0.1 degrees in χ (much less than polynomial fitting error ~1.2 degrees)

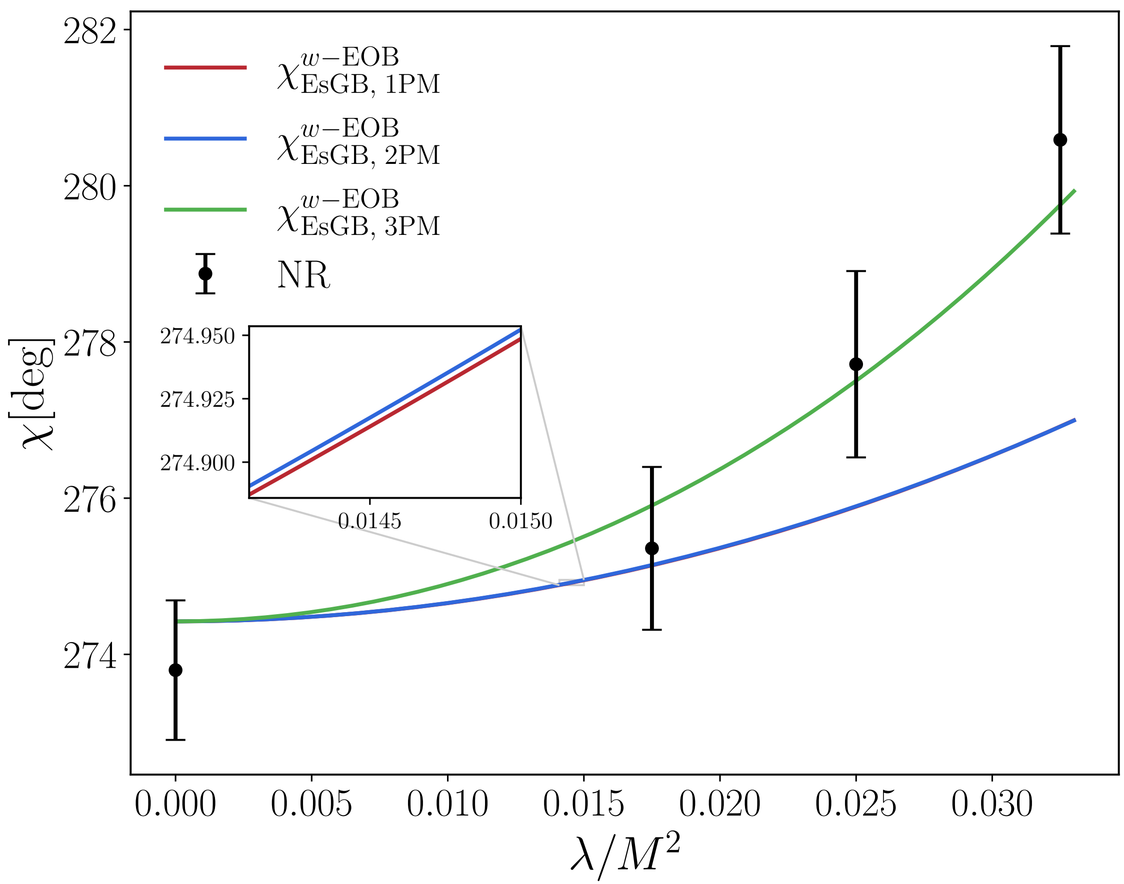
The parameter space considered in our work

$b_{\rm NR}/M$	$E_{\rm ADM}/M$	$J_{\rm ADM}/M^2$	λ/M^2	μ_i/M	$\chi_{\rm NR}\,(\Delta\chi_{\rm NR})$ [deg]
9.7	1.02264	1.11128	0.0000	0.5000	273.798 (0.893)
9.7	1.02264	1.11128	0.0175	0.5005	275.357 (1.043)
9.7	1.02264	1.11128	0.0250	0.5010	277.715 (1.193)
9.7	1.02264	1.11128	0.0325	0.5017	280.588 (1.200)
11.0	1.02264	1.26021	0.0250	0.5010	152.263 (0.649)

TABLE I. Summary of the initial data and scattering angle for all simulations considered in this work. Note that the symmetric errors quoted on the scattering angle are from the extrapolation procedure only, since resolution errors are subdominant (see supplemental material [64])

Broader Picture: NR vs Analytics in EsGB

Swain, Jain and Areste-Salo (2026)



- Analytics work for hyperbolic encounters!
- 3PM results improve the results!
- Structure of PM results

$$\underbrace{\frac{2}{r}\alpha_A^2}_{\text{1PM}} + \underbrace{\frac{0.093}{r^2}\alpha_A^2}_{\text{2PM}} + \underbrace{\frac{1.34}{r^3}\alpha^2}_{\text{3PM}} + ??$$

Probe (test-mass) limit?

Why delayed merger in bound orbits? Systematics error?

Scattering angle comparison between the NR data (filled circles) and the EOB-resummed, PM based analytical prediction (AR; solid lines) for equal-mass, non-spinning BH configurations in EsGB for various coupling strengths at *b = 9.7M*.

Key Results

First treatment of binary boson stars as an EFT

First PM-informed EOB potential for boson-star binaries

Excellent agreement between EOB-resummed analytics and NR simulations

Evidence that scalar interactions dominate over tidal effects in boson-star dynamics

Observed collapse of BSs into BHs after their close encounter and before separating to infinity → potentially due to time-dependent scalar perturbations (?)

Hybrid waveform construction and IMRCT test 2604.25582

Extension of PM-EFT methods to 3PM scalar-tensor and EsGB gravity

First validation of PM/EOB analytical results in EsGB with NR for hyperbolic black-hole encounters

Thank you!