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JÜLICH FORSCHUNGSZENTRUM

Coupling a temperature field to level set advection in a Stefan phase change problem

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Introduction

- Coupling two finite element fields sequentially

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- Physical Model: Melting ice / Freezing water

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- Physical Model: Melting ice / Freezing water
- Interface Problem: Level Set Methods

Overview

Goal: Simulate phase change through coupling a temperature field to a level set interface solver

- Stefan phase change problem statement

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- Coupling: Influence from one to the other
- Solution strategy

Phase Change Problem

Consider a block of ice that is heated from one side so that it melts. The domain, Ω, is split into Ω_l and Ω_s, separated by the interface Γ.
Energy conservation with temperature, T, is,

$$\frac{\partial T}{\partial t} - \nabla \cdot (D_l \nabla T) = 0 \tag{1}$$

and

$$\frac{\partial T}{\partial t} - \nabla \cdot (D_s \nabla T) = 0 \tag{2}$$

Phase Change Problem

With appropriate initial and boundary conditions, there is also the interface condition on Γ

$$T = T_{freezing} \tag{3}$$

$$\vec{w} \cdot \hat{n} = [(D_l \nabla T) - (D_s \nabla T)] \cdot \hat{n} \tag{4}$$

where $\vec{w}$ is the interface velocity and $\hat{n}$ is the unit normal to the interface.

Relations to tutorials and code gallery

- Advection solver, step-9

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- Diffusion solver, step-26

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- two-phase flow, code gallery

Level Set Advection

$$\frac{\partial u}{\partial t} + \vec{w} \cdot \nabla u = 0 \qquad (5)$$

where $\vec{w}$ is the velocity field for the interface motion and u is the advected quantity (level set function) .
Defining the speed function, F, to be the the magnitude of the vector field normal to the front surface, $F = \vec{w} \cdot \hat{n}$, and with the definition of the unit normal vector, $\hat{n} = \frac{\nabla u}{|\nabla u|}$, Eq. 5 can be rewritten as

$$\frac{\partial u}{\partial t} + F|\nabla u| = 0 \qquad (6)$$

Coupling

- In assembly for the temperature system, return the diffusion constant, D_i, based on the value of the level set potential funtion, u. For an intersected cell, weight the coefficient by quadrature point.
- In assembly for the advection system, compute the interface condition, based on the gradients of the temperature field, to obtain speed.

Outlook for solution strategy

- For temperature, Crank-Nicolson time stepping, AMG and CG

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- For temperature, Crank-Nicolson time stepping, AMG and CG
- For advection, SUPG, BlockJacobi and GMRES