CAFE

applying deal.II in cosmic-ray propagation

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Outline

1. WHAT?
2. WHY?
3. HOWTO?
4. THEN?

WHAT?

WHAT? is CAFE

CAFE

Cosmic-ray propagation Approached with Finite Element method

(current stage, focusing on CRE propagation and its feedback to GMF turbulence)

supervised by:

Piero Ullio, Luca Heltai, Alberto Sartori

WHAT? are cosmic-rays

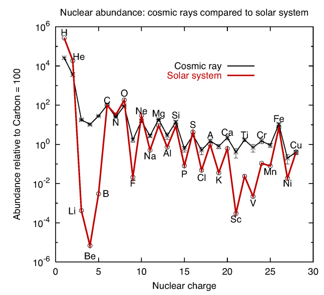
106 years ago

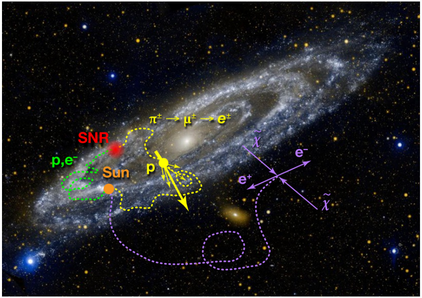
Victor Hess got a balloon ...

met the messengers from outer space

from electron to uranium, highly energetic

particle physics→astrophysics, Solar system→early Universe





CRE phase-space distribution f in $\{\mathbf{x}, p\}$
plasma turbulence power W in $\{\mathbf{x}, k\}$

$$\partial_t f + \nabla_x(D_{xx} \cdot \nabla_x f) + \nabla_p(p^2 \partial_t p \nabla_p f) + \mathbf{v} \cdot \nabla_x f = Q_{cr}$$

$$\partial_t W + \partial_k(D_{kk} W) + \Gamma_{cr} W = Q_k$$

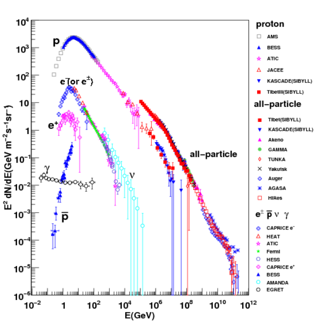
$$D_{xx} \sim 1/W(k_{res})$$

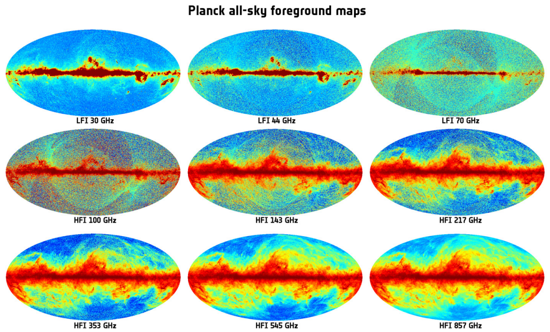
... and more

WHY?

WHY? do simulation with FEM

driven by precise observations





simulation at least in 4D

cannot be approached by lattice Boltzmann

FDM is less economic

N-body is extremely expensive

HOWTO?

HOWTO? go beyond 3D

$\otimes$ **2 triangulations**

two domains:

$$\begin{aligned} \mathcal{W} &:= span\{w_i \in H^1(R^n)\} \ , \\ \mathcal{Q} &:= span\{q_\alpha \in H^1(R^m)\} \ , \end{aligned}$$

$$\mathcal{V} := span\{v_{i\alpha} \in H^1(R^{n\times m}) | v_{i\alpha} = w_i q_\alpha\} \ .$$

solution at time t:

$$u^t = \sum_i \sum_\alpha \mathcal{U}^{t,i\alpha} w_i q_\alpha \ .$$

$\otimes$ sparsity patterns through DynamicSparsityPattern iterator

HOWTO? assemble system matrix and RHS

- loop through cells (in both domains $\Omega_x \otimes \Omega_p$)
- loop through $p_j \in \omega_p$ and $x_k \in \omega_x$
- build local-per-domain matrix or RHS
- distribute to global-per-domain cacher with constraints
- apply $\otimes$
- accumulate global-cross-domain result from each cell

working example in next slide

HOWTO? illustrate with example

solution in matrix

$$vec(\mathcal{M}_x \mathcal{U} \mathcal{M}_p^T) = (\mathcal{M}_p \otimes \mathcal{M}_x) vec(\mathcal{U})$$

weak formulation

$$(v_{j\beta}, -\hat{\mathcal{O}} u^t)_\Omega = \mathcal{U}^{t,i\alpha} (\nabla_{\mathbf{x}} v_{j\beta}, \mathcal{D} \nabla_{\mathbf{x}} v_{i\alpha})_\Omega$$

the integrand

$$\int_{\Omega_p} \int_{\Omega_x} [q_\alpha q_\beta] (\nabla_{\mathbf{x}} w_j)^T [\mathcal{D}(\mathbf{x}, p) \nabla_{\mathbf{x}} w_i] \tag{1}$$

THEN?

THEN? O what can ail thee

the expensive beauty

the Kronecker product function is executed too many times

optimisation

- multi-thread Kronecker product function?
- skip 0×0 actions in Kronecker product?

the 2nd optimisation is overkill

other expensive beauties?

THEN? verification in physical context

pure spatial diffusion problem

$$-\nabla_{\mathbf{x}} \cdot \mathcal{D}_{\mathbf{xx}} \nabla u(\mathbf{x}, \mathbf{q}) = f(\mathbf{x}, \mathbf{q})$$

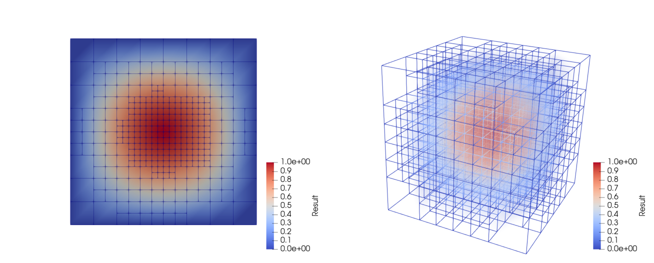
$$u(\mathbf{x}, \mathbf{q}) = 0 \ , \quad \mathbf{x} \in \partial\Omega_x$$

$$u(\mathbf{x}) = sin(\frac{(z - z_{min})\pi}{L_z})sin(\frac{(x - x_{min})\pi}{L_x})sin(\frac{(y - y_{min})\pi}{L_y})$$

$$\mathcal{D}_{\mathbf{xx}} = \begin{pmatrix} \alpha z^2 & 0 & 0 \\ 0 & \beta x^2 & \beta xy \\ 0 & \beta xy & \beta y^2 \end{pmatrix}$$

THEN? verification in physical context

solution in R^{2+1} and R^{3+1}



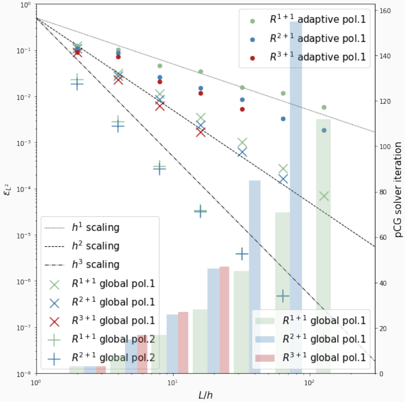
dimension settings up to R^{3+3} are allowed

THEN? performance

error estimation

$$\epsilon_{L^2} = \sqrt{\int_{\Omega_x} u(\mathbf{x}) - u_h(\mathbf{x})}$$

full solution has been interpolated from spectral domain



THANKS !