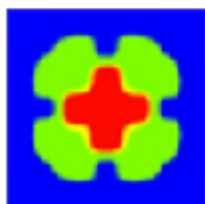




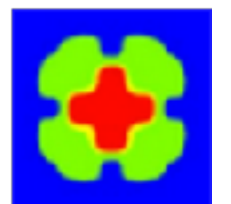
NUMERICAL (MICRO)AQUARIUM USING THE DEAL.II LIBRARY

Nicola Giuliani

(ngiuliani@sissa.it)



Sixth deal.II Users and Developers Workshop



APPLICATION: MICRO-SWIMMERS

A Micro-swimmer is a biologically propelled organism that moves through a liquid medium. In particular it exploits periodic shape changes in order to gain a net rigid displacement.



ERC Advanced Grant 340685 MicroMotility

MOTIVATION

Growing interest in artificial “bio-like” swimmer driven by spectacular biological organisms



WHY?

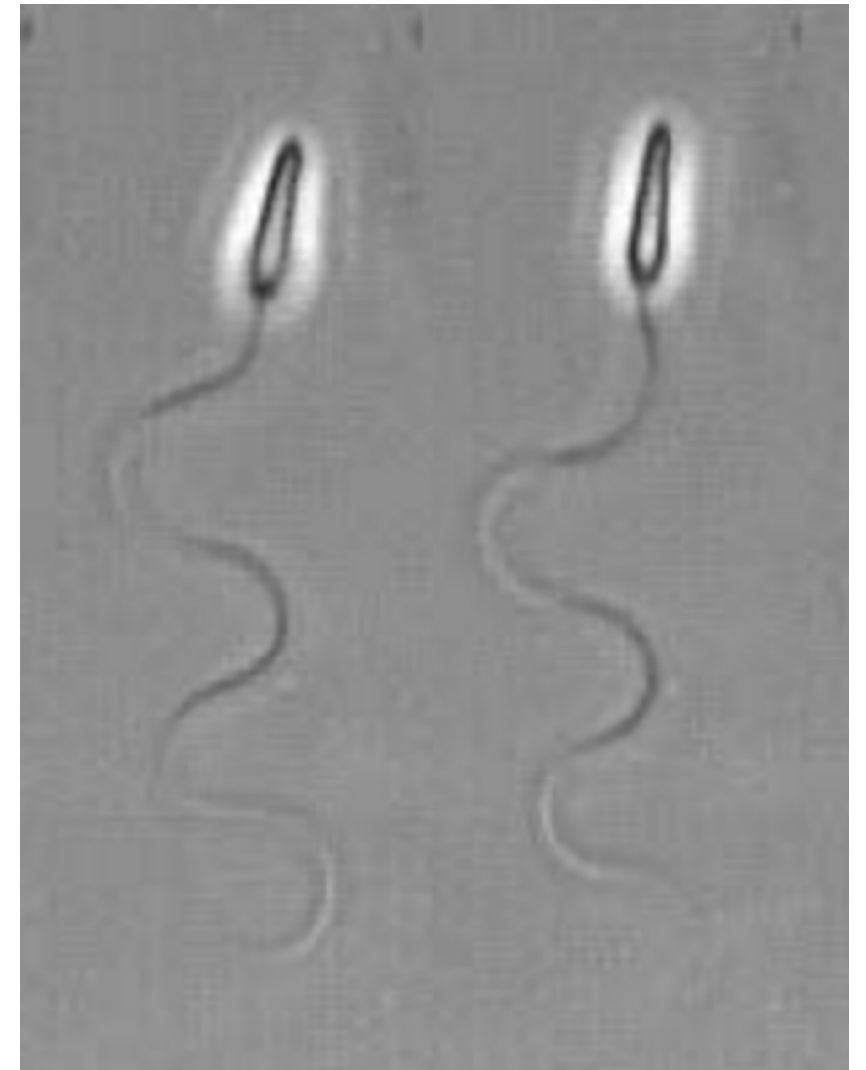
Understand nature to solve everyday complex problems

THE LIGHT SIDE

Sperm cells



Human



Sturgeon

Why so different? Internal Vs External fertilisers?

THE DARK SIDE

Extremely motile pathogens (bacteria)



Clostridium tetani



Treponema pallidum

MATHEMATICAL MODEL

$$u_s(t) = u_r(t) + v(t)$$

Kinematic model

$$f = \sigma(u, p)n = Tu_s$$

Fluid model

$$\int_{\Gamma} f(x) d\gamma(x) = 0$$

$$\int_{\Gamma} f(x) \wedge (x - x_0) d\gamma(x) = 0$$

Swimmer model

$$\begin{bmatrix} A & B \\ B^T & C \end{bmatrix} \begin{bmatrix} \dot{q} \\ \omega \end{bmatrix} + \begin{bmatrix} \int_{\Gamma} T v d\gamma(x) \\ \int_{\Gamma} M v d\gamma(x) \end{bmatrix} = 0$$

$$\mathcal{R}(X, t) \dot{p}(t) + \theta(X, t) v(X, t) = 0$$

$$\dot{p}(t) = -\mathcal{R}^{-1}(X, t) \theta(X, t) v(X, t)$$

FLUID MODEL

Newtonian incompressible material

$$\sigma = \sigma(u, p) = 2\mu \left(\frac{1}{2} \nabla u + \frac{1}{2} \nabla u^T \right) - pI$$

Incompressible Navier–Stokes equations

$$\nabla \cdot u = 0 \text{ in } \Omega$$

$$\rho \left[\frac{\partial u}{\partial t} + (\nabla \cdot u)u \right] = \nabla p + \mu \Delta u \text{ in } \Omega$$

Using the Reynolds number $Re = \frac{\rho U L}{\mu}$

$$\nabla \cdot \hat{u} = 0 \text{ in } \Omega$$

$$Re \left(\frac{\partial \hat{u}}{\partial \hat{t}} + (\nabla \cdot \hat{u})\hat{u} \right) = (\nabla \hat{p} + \Delta \hat{u}) \text{ in } \Omega$$

DIMENSIONAL ANALYSIS

$$L \sim 10^{-5}m, U \sim 10^{-5}\frac{m}{s}, \mu \sim 10^{-3}\frac{Pa}{s}, \rho \sim 10^3\frac{kg}{m^3}$$

$$Re \sim 10^{-4} \sim 0$$

$$\nabla \cdot u = 0 \text{ in } \Omega$$

$$\Delta u + \nabla p = 0 \text{ in } \Omega \quad \longrightarrow \quad u \in (H^1(\Omega))^d, p \in L_2(\Omega)$$

$$u = u_s \text{ on } \partial B_t$$

We use the fundamental solution to retrieve a unique relation between stresses and velocities on the boundary

We recast stokes as a Boundary Integral Equation

$$\alpha(x)_{it}u_t(x) - \int_{\Gamma}^{PV} W_{ijk}(x,y)n_k(y)u_j(y)d\gamma_y = \int_{\Gamma} G_{ij}(x,y)f_j(y)d\gamma_y \quad \forall x \in \Gamma$$

$$u \in V := \left\{ \phi \in (H^{\frac{1}{2}}(\Gamma))^d \right\}$$

$$f \in Q := \left\{ \gamma \in (H^{-\frac{1}{2}}(\Gamma))^d \right\}$$

BEM OPERATOR ANALYSIS

$$Vf = \int_{\Gamma} G_{ij}(x, y) f_j(y) d\gamma_y$$



NO
compact
support

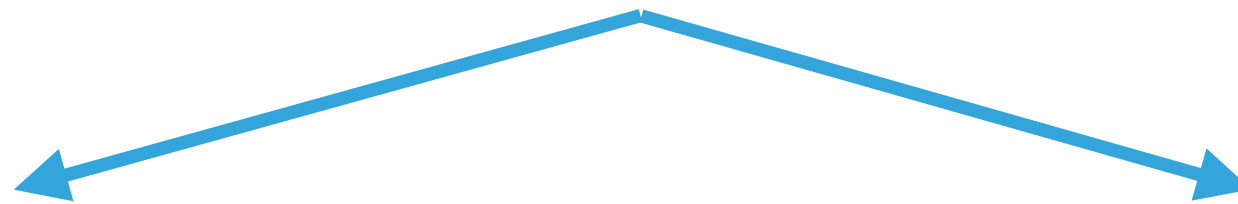


Compact support
depending on FE
basis functions

The discretisation of the BEM operators results in the assembling of dense matrices

HPC?

- ▶ Fundamental solutions not compactly supported
- ▶ Dense algebraic systems
- ▶ Quadratic costs, $O(N^2)$ with N unknowns
- ▶ Using 1000 unknowns 2 minutes (standard computations)
- ▶ Using 100000 unknowns 14 days



No HPC and very few
simulations
(finger crossed)

MHPC investment and
multiple fast simulation
(learn from errors)

BUILDING A NUMERICAL AQUARIUM

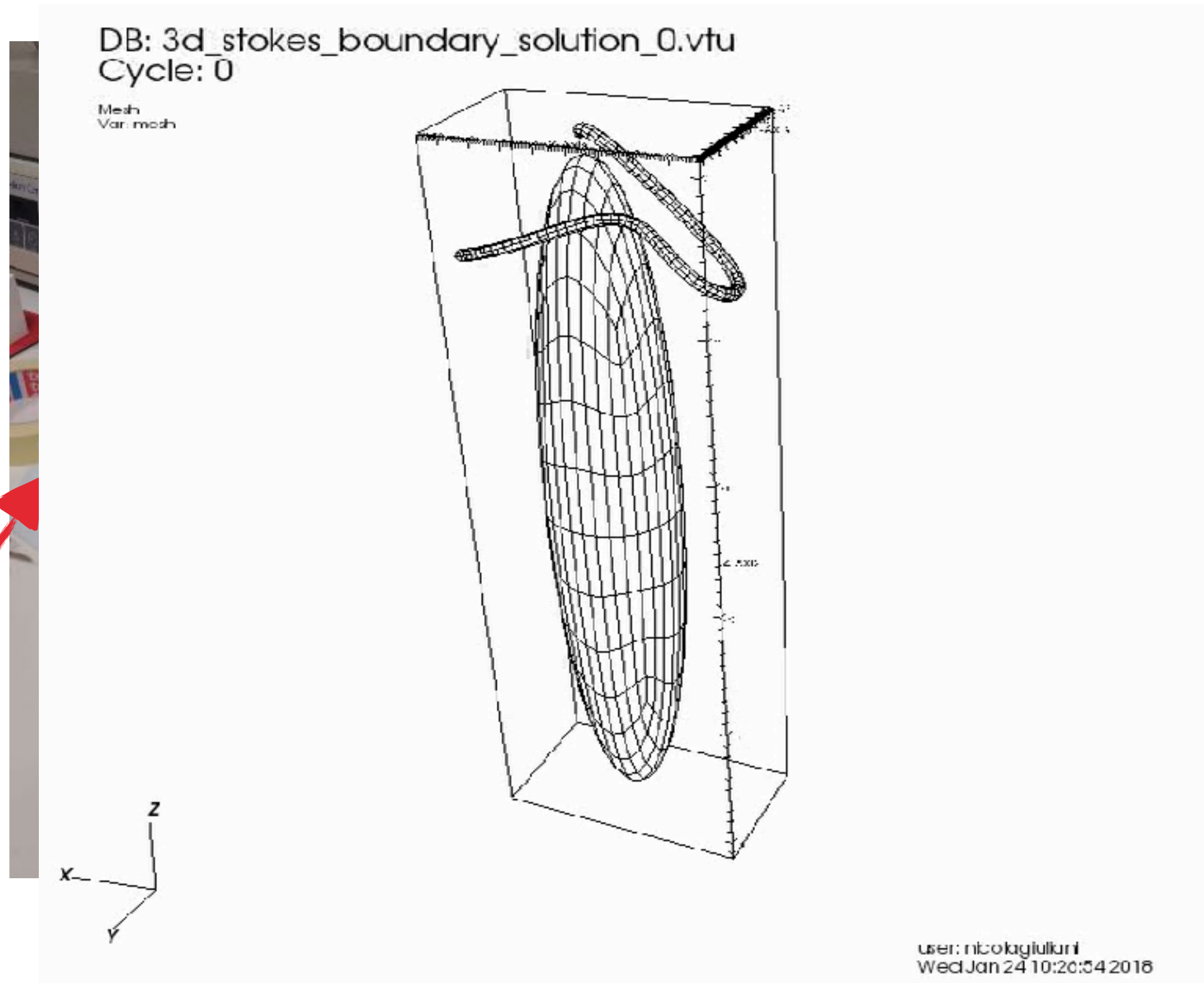
- ▶ Mesh generation starting from a video/series of images using 3D modelling tools and NURBS flagellum reconstructions
- ▶ HPC BEM Stokes simulator
 - ▶ Hybrid parallelism
 - ▶ Wall interactions
 - ▶ Continuous integration (120 tests)
- ▶ HPC post processor for velocity field analysis
- ▶ Python script to analyse the results



J.Guasto Tufts University

Everything has been strictly validated: more than 10000
parallel computations

FUTURE PERSPECTIVES



PROBLEMS

Simply a lot: datatypes, preconditioners, mesh refinements, quadratures...

REFERENCES

- N. Giuliani. Modelling Fluid Structure Interaction problems using Boundary Element Method. PhD thesis, SISSA, Scuola Internazionale Superiore di Studi Avanzati, 2017.
- N.Giuliani, A. Mola, L.Heltai. π -BEM: A flexible parallel implementation for adaptive, geometry aware, and high order boundary element methods, Advances in Engineering Software, 121 (2018) 39-58 (<https://github.com/mathLab/pi-BEM>)
- N. Giuliani, L. Heltai, A. DeSimone. Predicting and optimizing micro-swimmer performance from the hydrodynamics of its components: the relevance of interactions, Soft Robotics, 2018. (<https://gitlab.com/Giuliani/BEMStokes>, <https://gitlab.com/Giuliani/PostProcessBEMStokes>)

Questions? Curiosities? Feel free to ask anything
(ngiuliani@sissa.it)