Tidal deformability of black holes immersed in matter (arXiv:1912.07616)

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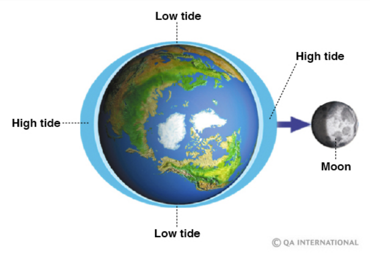
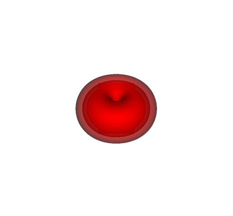
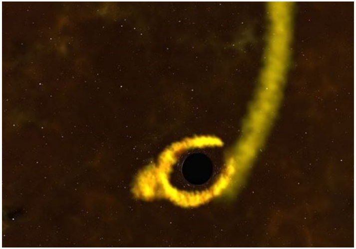
CENTRA, Instituto Superior Tecnico

Tides

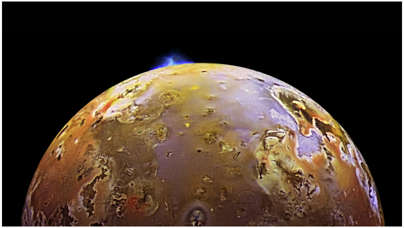




Tidal disruption of a scalar cloud around a black hole *(Image source: [Cardoso et al., 2020])*

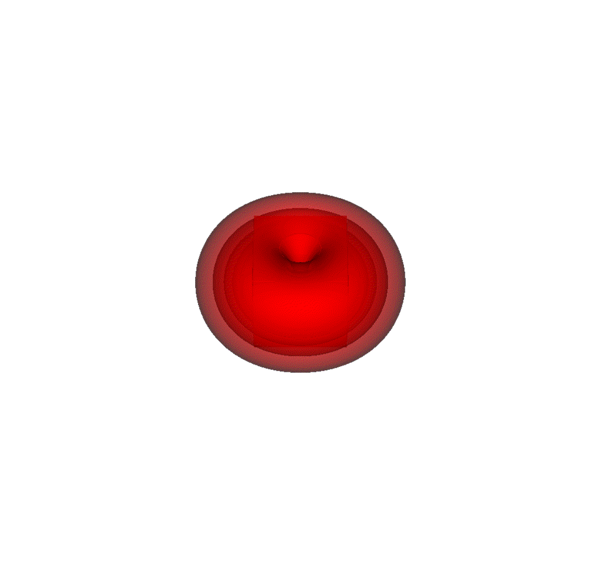


Tidal Disrupton Event ASASSN-19bt *(Image Credit: NASA's Goddard Space Flight Center)*



Volcanic Explosion in Io *(Image Credit: NASA's Goddard Space Flight Center Cover image courtesy of NASA/JPL/University of Arizona)*

Tides in Superradiance Clouds (arxiv:2001.01729)



Brief History of Tidal Love Numbers (TLNs)

Constraining neutron star tidal Love numbers with gravitational wave detectors

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Ground-based gravitational wave detectors may be able to constrain the nuclear equation of state using the early, low frequency portion of the signal of detected neutron star - neutron star inspirals. In this early adiabatic regime, the influence of a neutron star's internal structure on the phase of the waveform depends only on a single parameter λ of the star related to its tidal Love number, namely the ratio of the induced quadrupole moment to the perturbing tidal gravitational field. We analyze

Relativistic theory of tidal Love numbers

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(Dated: September 16, 2009)

In Newtonian gravitational theory, a tidal Love number relates the mass multipole moment created by tidal forces on a spherical body to the applied tidal field. The Love number is dimensionless, and it encodes information about the body's internal structure. We present a relativistic theory of Love numbers, which applies to compact bodies with strong internal gravities; the theory extends and completes a recent work by Flanagan and Hinderer, which revealed that the tidal Love number of a neutron star can be measured by Earth-based gravitational-wave detectors. We consider a spherical body deformed by an external tidal field, and provide precise and meaningful definitions for electric-type and magnetic-type Love numbers; and these are computed for polytropic equations of state. The theory applies to black holes as well, and we find that the relativistic Love numbers of a nonrotating black hole are all zero.

TLNs depend on the body's internal structure and the underlying theory of gravity.

Question: what if we measure a non-zero TLN for a body as compact as a Black Hole?

Possible Answers:

1. Correct GR

$$G_{\mu\nu} + ... = 8\pi T_{\mu\nu}$$

2. Exotic matter

(Image Credit: Shutterstock)

3. Environment effects

(Image Credit: Mark Garlick/Science photo library/Corbis)

1. and **2.** were explored in [Cardoso et al., 2017, Maselli et al., 2018, Maselli et al., 2019].

Tidal deformability in General Relativity

Natural framework: Black Hole Perturbation Theory

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$$

Focus on spherically-symmetric, static backgrounds

$$ds^2 = -F(r)\,dt^2 + G(r)\,dr^2 + r^2 d\Omega^2$$

In **Regge-Wheeler gauge**

$$h^{\text{even}}_{\mu\nu} = \begin{pmatrix} F(r)\ H_0^{lm}(r)\ Y^{lm} & H_1^{lm}(r)\ Y^{lm} & 0 & 0 \\ H_1^{lm}(r)\ Y^{lm} & G(r)\ H_2^{lm}(r)\ Y^{lm} & 0 & 0 \\ 0 & 0 & r^2\,K^{lm}(r)\ Y^{lm} & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta\,K^{lm}(r)\ Y^{lm} \end{pmatrix}$$

$$h^{\text{odd}}_{\mu\nu} = \begin{pmatrix} 0 & 0 & h_0^{lm}(r)\ S_\theta^{lm} & h_0^{lm}(r)\ S_\varphi^{lm} \\ 0 & 0 & h_1^{lm}(r)\ S_\theta^{lm} & h_1^{lm}(r)\ S_\varphi^{lm} \\ h_0^{lm}(r)\ S_\theta^{lm} & h_1^{lm}(r)\ S_\theta^{lm} & 0 & 0 \\ h_0^{lm}(r)\ S_\varphi^{lm} & h_1^{lm}(r)\ S_\varphi^{lm} & 0 & 0 \end{pmatrix}$$

Regime of **Static Tides** $\Rightarrow$ no t dependence and the system evolves **adiabatically**

Asymptotic expansions in **Multipole Moments**

$$g_{tt} = -1 + \frac{2M}{r} + \sum_{l\geq 2} \frac{2}{r^{l+1}}\sqrt{\frac{4\pi}{2l+1}}M_l\, Y^{l0} - \frac{2}{l\,(l-1)}r^l\mathcal{E}_l Y^{l0}$$

$$g_{t\varphi} = \frac{2J}{r}\sin^2\theta + \sum_{l\geq 2} \frac{2}{r^l}\sqrt{\frac{4\pi}{2l+1}}\frac{S_l}{l}\, S_\varphi^{l0} + \frac{2r^{l+1}}{3l\,(l-1)}\mathcal{B}_l S_\varphi^{l0}$$

$\mathcal{E}_l\,,\mathcal{B}_l$: External tidal field multipole moments

$M_l\,,S_l$: Induced multipole moments of the compact object

Polar TLNs

$$k_l^E = -\frac{1}{2}\frac{l\,(l-1)}{M^{2l+1}}\sqrt{\frac{4\pi}{2l+1}}\frac{M_l}{\mathcal{E}_{l0}}$$

Axial TLNs

$$k_l^B = -\frac{3}{2}\frac{l\,(l-1)}{(l+1)\,M^{2l+1}}\sqrt{\frac{4\pi}{2l+1}}\frac{S_l}{\mathcal{B}_{l0}}$$

Matter away from the horizon: Thin Shells

$$\begin{cases} F(r) = \bar{\alpha}\left(1 - \frac{2M}{r}\right) \ , \ G(r) = \frac{\bar{\alpha}}{F(r)} \ , \ r < r_0 \\ F(r) = \left(1 - \frac{2M_0}{r}\right) \ \ , \ G(r) = \frac{1}{F(r)} \ , \ r > r_0 \end{cases}$$

$$\bar{\alpha} = \frac{1 - 2M_0/r_0}{1 - 2M/r_0}$$

$$\delta M \equiv M_0 - M$$

Shell composed by a perfect fluid: $S_{ab} = (\sigma + p)\, u_a u_b + p\, \gamma_{ab}$

Intrinsic coordinates of the shell: $y^a = (T, \Theta, \Phi)$

Shell located at: $x_+^\mu = (T, r_0, \Theta, \Phi) \quad , \quad x_-^\mu = (A\,T, r_0, \Theta, \Phi)$

Darmois-Israel junction conditions

$$\begin{aligned} [[\gamma_{ab}]] &\equiv \gamma_{ab}(r_{0_+}) - \gamma_{ab}\left(r_{0_-}\right) = 0 \\ S_{ab} &= -\frac{1}{8\pi}\left([[K_{ab}]] - \gamma_{ab}\,[[K]]\right) \end{aligned}$$

where

$$\begin{aligned} e_a^\mu &\equiv \frac{\partial x^\mu}{\partial y^a} \qquad \gamma_{ab} \equiv g_{\mu\nu}\, e_a^\mu\, e_b^\nu \qquad K_{ab} \equiv e_a^\mu\, e_b^\nu\, \nabla_\mu n_\nu \\ n_\mu\, e_a^\mu &= 0 \qquad n^\mu n_\mu = 1 \qquad u_a u^a = -1 \end{aligned}$$

Polar Sector "Walkthrough"

Strategy: solve the perturbed problem inside *and* outside the shell, and then impose the junction conditions

Solutions to the linearized field equations are Schwarzschild-type ones with the corresponding mass

$$H_0^{\text{ext}} = A_1 P_l^2\left(r/M_0 - 1\right) + A_2 Q_l^2\left(r/M_0 - 1\right) \quad , \quad H_0^{\text{int}} = A_3 P_l^2\left(r/M - 1\right)$$

$$H_1^{\text{int}} = H_1^{\text{ext}} = 0 \quad , \quad H_2 = H_0 \quad , \quad K = \mathcal{F}\left(H_0, H_0', r\right)$$

In addition to $h_{\mu\nu}$, we need to perturbe

1. Determined by junction conditions

$$\delta r_\pm = \sum_{l,m} \delta r_\pm^{lm} Y^{lm}\left(\Theta, \Phi\right) \quad , \quad \left(\delta\sigma, \delta p\right) = \sum_{l,m} (\delta\sigma^{lm}, \delta p^{lm}) Y^{lm}\left(\Theta, \Phi\right)$$

2. Determined by normalization/physical arguments

$$\delta u^a \quad , \quad \delta n_{\mu_\pm}$$

Quadrupolar TLNs

Important limits $(\delta p = v_s^2\, \delta\sigma \quad , \quad v_s^2 \equiv \left(\frac{dp}{d\sigma}\right)\Big|_{\sigma_0})$

1. $r_0 \to \infty$

$$k_2^E \to \; - \; \frac{8\,\delta M}{9M + M_0}\frac{r_0^5}{M_0^5} \; + \; \mathcal{O}\left(r_0^4/M_0^4 v_s^2\right)$$

$$k_2^B \to \frac{\delta M}{5M_0}\frac{r_0^4}{M_0^4}$$

Dependence on r_0/M_0 expected from dimensional grounds

Linear dependence on δM agrees with results from other models studied

2. $M_0 \to M$ and $r_0 \to 2M$

$$k_2^E \to \frac{8\left(3 - 8v_s^2\right)}{5}\frac{\delta M}{M}\left(\frac{r_0}{M} - 2\right)$$

$$k_2^B \to \frac{8}{5}\frac{\delta M}{M}\left(\frac{r_0}{M} - 2\right)$$

Both results consistent with the vanishing of TLNs for a Black Hole.

Binaries in Astrophysical Setting

(Image Credit: SXS project)

Question: What portion of matter is relevant for the tidal effects in the dynamics and radiation emitted by the binary?

Leading order tidal effects appear in equations of motion at a Newtonian level

$$\frac{d^2 r^j}{dt^2} \quad = \quad -\frac{M_{\rm tot}}{r^2}\left(1 + \frac{3}{2r^5}\left(k_{2_1}\frac{M_2}{M_1} + k_{2_2}\frac{M_1}{M_2}\right)\right)\frac{r^j}{r}$$

To treat tidal effects perturbatively we need to guarantee

$$\frac{r_0}{r} \ll \min\left(1\,,\,\left(\frac{M_1}{\delta M}\frac{M_1}{M_2}\right)^{1/5}\,,\,\left(\frac{M_2}{\delta M}\frac{M_2}{M_1}\right)^{1/5}\right)$$

Binaries in Astrophysical Setting: the Shakura-Sunayev model

LISA detector frequency band: $f \in \left[10^{-5}, 1\right]$ Hz

Binary in circular orbit (d - binary separation)

$d \sim \left(\frac{GM_{\rm tot}}{(\pi f)^2}\right)^{1/3} \Rightarrow$ Lower bound: $r \sim 10^6 \left(M_{\rm tot}/M_\odot\right)^{1/3}$ km

Model environmental matter as a **Shakura-Sunayev thin accretion disk** (2 “free” parameters)

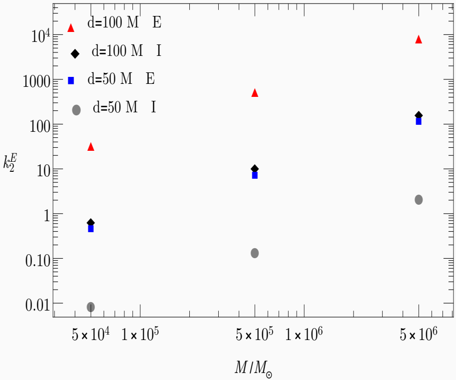
1.Mass accretion rate parameter: $\sim 10^{-2} \leq f_{\rm Edd} \leq\sim 0.2$

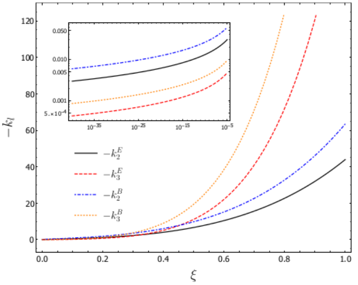
2.Viscosity parameter: $\sim 0.01 \leq \alpha \leq\sim 0.1$

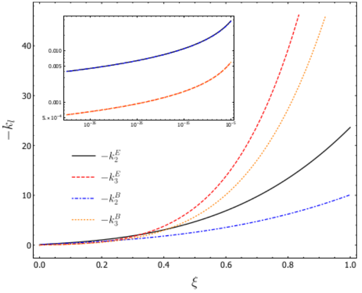
Gives r_0, δM and v_s in terms of M.

The minimum measurable TLN

E/I - most/least efficient accretion







Right upper panel: TLNs for a wormhole
Right lower panel: TLNs for a gravastar
$\xi = r_0/2M - 1$

source: [Cardoso et al., 2017]

How can we distinguish between a massive object surrounded by matter and an ultracompact object pointing towards new physics, from a measurement of Tidal Love Numbers?

$$[[H_0]] = \left[\left[\frac{\delta r\, F'}{F}\right]\right] \qquad , \qquad \frac{2}{r_0}\,[[\delta r]] = -\,[[K]] \qquad , \qquad \left[\left[\sqrt{G}\,\delta r\right]\right] = 0$$

$$\frac{2}{r_0^2}\left[\left[\frac{\delta r}{\sqrt{G}}\right]\right] + \frac{2}{r_0}\left[\left[\frac{H_0}{\sqrt{G}}\right]\right] + \frac{1}{r_0}\left[\left[\frac{H_2}{\sqrt{G}}\right]\right] - \left[\left[\frac{K'}{\sqrt{G}}\right]\right] + \frac{1}{r_0}\left[\left[\frac{\delta r\, G'}{\sqrt{G^3}}\right]\right] -$$
$$-\frac{2}{r_0}\left[\left[\frac{\delta r F'}{F\sqrt{G}}\right]\right] = 8\pi\,\delta\sigma + 8\pi\,\sigma\left(\frac{F'\delta r}{F} - H_0\right)$$

$$\frac{1}{2\,r_0^2}\left[\left[\frac{\delta r}{\sqrt{G}}\right]\right] - \frac{1}{2r_0}\left[\left[\frac{H_2}{\sqrt{G}}\right]\right] + \frac{2}{r_0}\left[\left[\frac{K}{\sqrt{G}}\right]\right] - \frac{1}{4}\left[\left[\frac{H_2 F'}{F\sqrt{G}}\right]\right] +$$
$$+\frac{1}{2}\left[\left[\frac{KF'}{F\sqrt{G}}\right]\right] + \frac{1}{2}\left[\left[\frac{K'}{\sqrt{G}}\right]\right] - \frac{1}{2}\left[\left[\frac{H_0'}{\sqrt{G}}\right]\right] - \frac{1}{2r_0}\left[\left[\frac{\delta r\, G'}{\sqrt{G^3}}\right]\right] +$$
$$+\frac{1}{r_0}\left[\left[\frac{\delta r F'}{F\sqrt{G}}\right]\right] + \frac{1}{2}\left[\left[\frac{\delta r F'}{f\sqrt{G}}\right]\right]' = 8\pi\,\delta p + 8\pi\,p\left(K + 2\frac{\delta r}{r_0}\right)$$

$$\delta p = v_s^2\,\delta\sigma\,, \qquad v_s^2 \equiv \left(\frac{dp}{d\sigma}\right)\Big|_{\sigma_0}$$

6 equations for 7 constants but 1 overall constant is irrelevant to compute the TLNs

Model environmental matter as a **Shakura-Sunayev thin accretion disk** (2 "free" parameters)

1. Mass accretion rate parameter: $\sim 10^{-2} \leq f_{\rm Edd} \leq\sim 0.2$

2. Viscosity parameter: $\sim 0.01 \leq \alpha \leq\sim 0.1$

$$\tilde{r} \equiv r/\left(GM/c^2\right)$$

$$\Sigma_{\rm disk}(r) \approx 7 \times 10^8 \frac{f_{\rm Edd}^{7/10}}{\tilde{r}^{3/4}} \left(1 - \sqrt{\frac{\tilde{r}_{\rm in}}{\tilde{r}}}\right)^{7/10} \left(\frac{0.1}{\alpha}\right)^{4/5} \left(\frac{M}{10^6 M_\odot}\right)^{1/5} \mathrm{kg}\cdot\mathrm{m}^{-2}$$

$$\frac{H}{GM/c^2}(r) \approx 3 \times 10^{-3} f_{\rm Edd}^{3/20} \left(1 - \sqrt{\frac{\tilde{r}_{\rm in}}{\tilde{r}}}\right)^{3/20} \left(\frac{0.1}{\alpha}\right)^{1/10} \left(\frac{10^6 M_\odot}{M}\right)^{1/10} \tilde{r}^{9/8}$$

$$\delta M \approx 2\pi \int_{r_{\rm in}}^{r_{\rm out}} \Sigma_{\rm disk} r\, dr \qquad r_0 = \frac{2\pi}{\delta M} \int_{r_{\rm in}}^{r_{\rm max}} \Sigma_{\rm disk} r^2\, dr \qquad H \sim \frac{v_s r}{(GM/r)^{1/2}}$$

If we start without a BH, i.e. $M = 0$, and analyse now the BH limit $r_0 \to 2M_0$ we obtain

$$k_2^E \to \frac{8}{5\left(9 + \sqrt{\frac{2}{\xi}} + 4v_s^2 + 3\log\xi\right)}$$

$$\xi \equiv 1 - \frac{2M_0}{r_0} \qquad \mathcal{C} \equiv \frac{M_0}{r_0}$$

Cardoso and Pani claimed that the TLNs of an ECO would behave as $k \sim 1/\log\xi$ in the BH limit [Cardoso and Pani, 2019] . Their proof relies on imposing Robin type boundary conditions on the Zerilli function $a\Psi + b\Psi' = c$. There are two nuances

1. True scaling goes as $k \propto 1/(b + \log\xi)$ and b might diverge faster than $\log\xi$

2. For a thin-shell the perturbations are not differentiable *at* the boundary so it is not clear how to rephrase our boundary conditions in terms of these Robin-type ones.

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