The sound of DHOST

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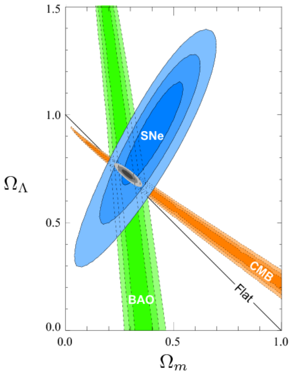
Summary of the talk

$$\Phi' = \frac{G_{\rm N} M}{r^2} + \frac{\Upsilon_1 G_{\rm N}}{4} M''$$

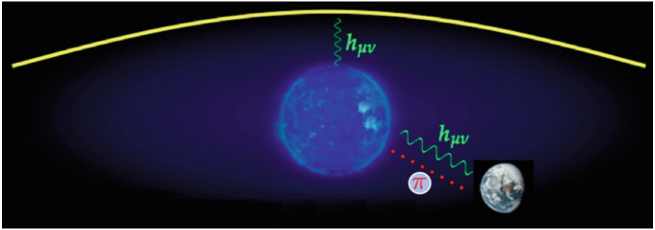
Modifying gravity: why?

Many reasons; mostly evoked to solve dark energy problems (large cosmological constant problem, coincidence problem)



Modifying gravity: why with a scalar field?

- Simplest additional degree of freedom
- Many theories related in specific regimes



Modifying gravity with a scalar field: how?

DHOST theories

Most complicated way of adding the simplest degree of freedom

Modifying gravity with a scalar field: how?

DHOST theories Langlois & Noui '15 (+ others...)

$$S_{\mathrm{DHOST}} = \int d^4x\sqrt{-g}\left[F_0(\varphi, X) + F_1(\varphi, X)\Box\varphi + F_2(\varphi, X)R + \sum_{I=1}^{5} A_I(\varphi, X)L_I^{(2)}\right]$$

$$L_1^{(2)} = \varphi^{\mu\nu}\varphi_{\mu\nu}$$
$$L_2^{(2)} = (\Box\varphi)^2$$
$$L_3^{(2)} = \Box\varphi\,\varphi^{\rho}\varphi_{\rho\sigma}\varphi^{\sigma}$$
$$L_4^{(2)} = \varphi^{\mu}\varphi_{\mu\nu}\varphi^{\nu\rho}\varphi_{\rho}$$
$$L_5^{(2)} = (\varphi^{\rho}\varphi_{\rho\sigma}\varphi^{\sigma})^2$$

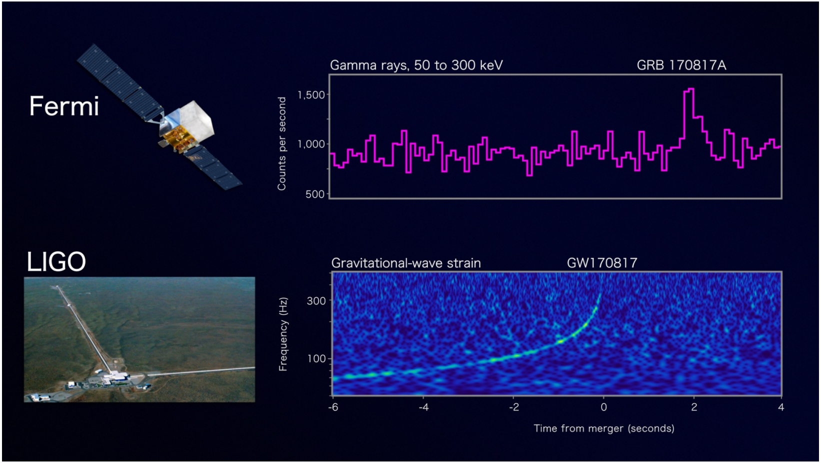
with $\varphi_{\mu\nu} = \nabla_\nu\nabla_\mu\varphi$, $\varphi_\mu = \nabla_\mu\varphi$ and $X = \varphi_\mu\varphi^\mu$

DHOST theories

From now on, $\varphi =$ dark energy

DHOST theories

From now on, φ = dark energy



DHOST theories

From now on, φ = dark energy

$c_{\rm grav} = c_{\rm light}$

$$A_1 = 0$$

$$\Downarrow$$

Degeneracy relations (no ghost)

$$A_2 = 0$$

$$A_4 = A_4[F_2, A_3]$$

$$A_5 = A_5[F_2, A_3]$$

Breaking of Vainshtein mechanism inside matter

Spherical object in expanding Universe

$$ds^2 = -[1 + 2\Phi(t, r)]dt^2 + a(t)^2[1 + 2\Psi(t, r)](dr^2 + r^2 d\Omega^2)$$

Assumptions:

- Small field limit $\Phi, \ \Psi \ll 1$, and $(\Phi'/\dot{\Phi})^2 \ll 1$
- Quasi-stationarity
- Vainshtein regime

Modified Newton law Kobayashi et al. '15

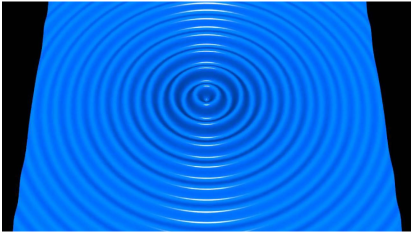
$$\Phi' = \frac{G_{\rm N}(t)M(t,r)}{r^2} + \frac{\Upsilon_1(t)G_{\rm N}(t)}{4}M''(t,r)$$

$$M = 4\pi a(t)^2 \int r^2\, dr \rho(t,r)$$

$$G_{\rm N} = G_{\rm N}[F_2, A_3]$$

$$\Upsilon_1 = \Upsilon_1[F_2, A_3]$$

Spherical sound waves



Hydrodynamics equation

$$\frac{\partial(\rho\vec{v})}{\partial t} + (\vec{v}\cdot\vec{\nabla})(\rho\vec{v}) = -\rho\vec{\nabla}\Phi - \vec{\nabla}P$$

$$\frac{\partial\rho}{\partial t} + \mathrm{div}(\rho\vec{v}) = 0$$

$$P = P_0 + \delta P, \qquad \rho = \rho_0 + \delta\rho, \qquad \frac{\delta P}{\delta\rho} \simeq \left.\frac{\partial P}{\partial\rho}\right|_S$$

Small wavelength perturbations

$$\frac{r}{\lambda} \gg 1, \quad \frac{r}{\lambda} \gg \frac{1}{\Upsilon_1}\frac{M}{\rho_0 r^3} \Rightarrow \delta\rho' \simeq \frac{\delta\rho}{\lambda} \text{ becomes dominant}$$

Wave equation

$$\ddot{\delta\rho} - \left(\left.\frac{\partial P}{\partial\rho}\right|_S + 4\pi G_{\mathrm{N}}\frac{\Upsilon_1}{4}\rho_0 r^2\right)\delta\rho'' = 0$$

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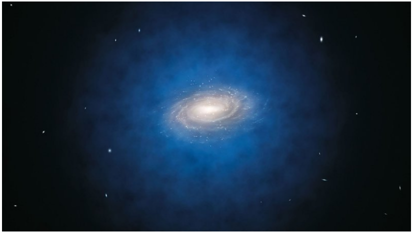
Modified dispersion relation

Babichev and AL '18

$$c^2_{\mathrm{DHOST}} = \left.\frac{\partial P}{\partial \rho}\right|_s + 4\pi G_{\mathrm{N}} \frac{\Upsilon_1}{4} \rho_0 r^2$$

$$c^2_{\mathrm{GR}} = \left.\frac{\partial P}{\partial \rho}\right|_s$$

Dust fluid (cosmology)

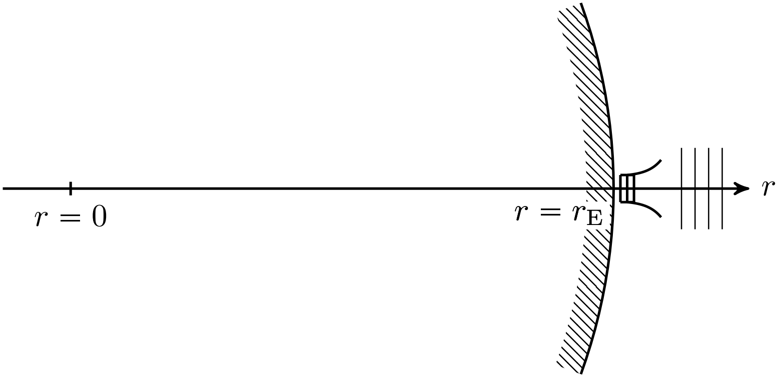


$$c^2_{\mathrm{DHOST}} = 4\pi G_{\mathrm{N}} \frac{\Upsilon_1}{4} \rho_0 r^2$$

$\Upsilon_1 < 0 \Rightarrow$ Gradient instability, way worse than Jeans (tachyonic) instability

Stability $\Rightarrow \Upsilon_1 > 0$

Sound waves on Earth



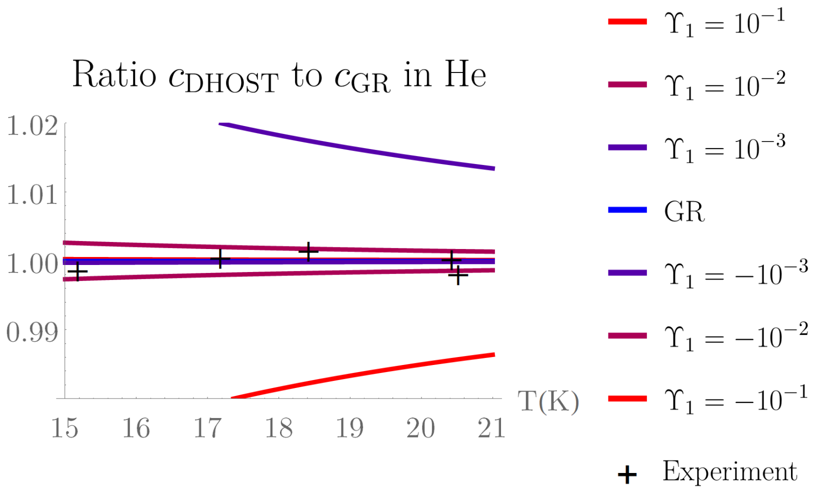
Sound waves on Earth

Perfect gas

$$c^2_{\rm DHOST} = \frac{\gamma R T}{M} + 4\pi G_{\rm N} \frac{\Upsilon_1}{4} \frac{P_0 M}{R T} r_{\rm E}^2$$

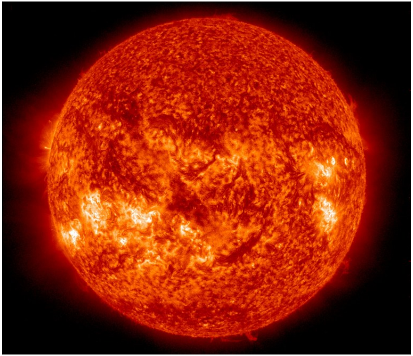
$$\gamma = \frac{7}{5}, \quad r_{\rm E} \simeq 6.4 \times 10^6 \ {\rm m}, \quad M \simeq 4.0 \times 10^{-3} {\rm kg/mol \ for \ helium}$$

Strong temperature dependence, amplified at small temperature



$$|\Upsilon_1| \lesssim 10^{-2}$$

Astrophysical constraints



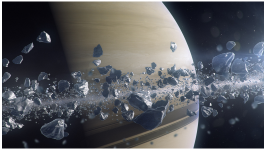
→ See Ippocratis' talk

Fundamental issue

$$F_{\rm grav} = -\phi' = -\frac{G_{\rm N} M}{r^2} - \frac{\Upsilon_1 G_{\rm N}}{4} M''$$

Gravitational force depends on matter description!

$\neq$



$M'' \neq 0$

$M'' = 0$

Conclusions

- Constraint $0 < \Upsilon_1 \lesssim 10^{-2}$, can easily be refined
- Overall, DHOST theories for dark energy have a hard time
- Caveat: some models with different modified Newton law

Thank you!