BMS flux balance equations

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Based on arXiv:1912.03164 with **Geoffrey Compère** and **Roberto Oliveri**

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Introduction

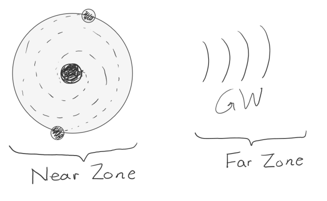
- **Goal:** Analytic study of gravitational waves from compact objects
- **Methodology:** Use the power of *symmetries*
- **Noether theorem.** Symmetry $x \to x + \xi \implies$ conserved current $\nabla \cdot J_\xi = 0 \implies$ Noether charge Q_ξ

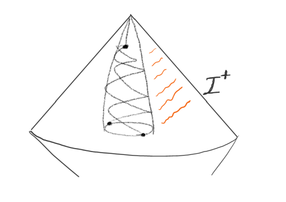
No Leakage	Conserved charges	$\dot{Q}_\xi = 0$
Leakage	Flux balance equations	$\dot{Q}_\xi = -\mathcal{F}$

- **BMS symmetries.** Infinite extension of Poincare symmetries emerging (holographically) for asymptotically flat spacetimes

Importance of Flux balance eqs

The system we have in mind





Why are flux balance equations important?

Because they constraint the evolution of the source

Known results

Einstein quadrupole formula [*Einstein '18*]

$$\dot{\mathcal{E}} = -\frac{1}{5}\frac{G}{c^5}\dddot{I}_{ij}\dddot{I}_{ij}$$

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$$\dot{\mathcal{J}}_i = -\frac{2}{5}\frac{G}{c^5}\epsilon_{ijk}\ddot{I}_{jl}\dddot{I}_{kl}$$

Linear momentum flux [*Thorne '80*]

$$\dot{\mathcal{P}}_i = -\frac{2}{63}\frac{G}{c^7}\overset{(4)}{I}_{ijk}\overset{(3)}{I}_{jk}$$

Center of mass flux [*Kozameh '16, Blanchet,Faye '18*]

$$\dot{\mathcal{G}}_i = \mathcal{P}_i - \frac{1}{21}\frac{G}{c^7}\Big(\overset{(3)}{I}_{jk}\overset{(3)}{I}_{ijk} - \overset{(2)}{I}_{jk}\overset{(4)}{I}_{ijk}\Big)$$

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We want to write exact equations and extend the list to the whole BMS.

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- **Setup:** Bondi Asymptotic expansion of metric in powers of $1/r$

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$$
\begin{aligned}
ds^2 = & -c^2\, du^2 - 2c\, dudr + r^2 \gamma_{AB}\, dx^A dx^B \\
& + \frac{2G}{c^2 r} m\, du^2 + \frac{1}{r}\frac{4G}{3c^2} N_A\, dudx^A + r\, C_{AB}\, dx^A dx^B \\
& + \dots
\end{aligned}
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- **Coloumbic data:** $m = m(u, x^A), N_A = N_A(u, x^A)$ Bondi mass, angular momentum aspect,
- **Radiative data:** $C_{AB}(u, x^A)$ Bondi shear, $\dot{C}_{AB}(u, x^A)$ Bondi news tensor

Dynamics in Bondi gauge

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- Exact equations (no approximation)
- Time variation of Coloumbic data $\propto$ fluxes of radiative data
- *Soft* (linear) and *hard* (quadratic) terms in the flux

BMS symmetries and their flux balance equations

BMS symmetries and charges

- Extended BMS symmetry generators $U(1)_{S^2} \times \mathrm{Diff}(S^2)$

$$\xi = \underbrace{T(x^A)\partial_u}_{\text{supertranslation}} + \underbrace{\epsilon^{BC}\partial_C\Phi(x^A)\partial_C}_{\text{superrotation}} + \underbrace{\gamma^{BC}\partial_B\Psi(x^C)\partial_C}_{\text{superboost}}$$

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- Corresponding BMS charges

$$\mathcal{P}_T \equiv \frac{1}{c}\oint_S T\, m\,, \qquad \textit{Supermomentum}$$

$$\mathcal{J}_\Phi \equiv \frac{1}{2}\oint_S \epsilon^{AB}\partial_B\Phi N_A, \qquad \textit{S-angular momentum}$$

$$\mathcal{K}_\Psi \equiv \frac{1}{2c}\oint_S \gamma^{AB}\partial_B\Psi N_A \qquad \textit{S-center of mass}$$

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- Charge pairs the symmetry parameter with the Bondi aspect

BMS flux balance laws

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$$\dot{P}_T = \frac{1}{c} \oint_S T \, \dot{m}$$

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$$\begin{aligned}\dot{P}_T &= \frac{1}{c}\oint_S T\,\dot{m} \\ &= \frac{1}{c}\oint_S T\left(-\frac{c^3}{8G}\dot{C}_{AB}\dot{C}^{AB} + \frac{c^4}{4G}D_A D_B \dot{C}^{AB}\right)\end{aligned}$$

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- Super-angular momentum flux balance laws
- Super-center of mass flux balance laws

Multipole expansion

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- Multipole expansion of shear tensor $\left(n_i = \frac{x_i}{r}, N_L = n_{i_1} \cdots n_{i_\ell}\right)$

$$C_{ij} = \sum_{\ell=2}^{+\infty} a_\ell \left(N_{L-2}\, \boldsymbol{U}_{ijL-2} - \frac{b_\ell}{c} N_{aL-2}\, \epsilon_{ab(i} \boldsymbol{V}_{j)bL-2} \right)^{TT}$$

in terms of mass and spin radiative multipole moments.

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- One balance equation for each harmonic of the symmetry parameters

Poincarè flux balance equations

- Energy and angular momentum $(T = c, \Phi = n_i)$

$$\dot{\mathcal{E}} = -\sum_{\ell=2}^{+\infty} \frac{G}{c^{2\ell+1}} \mu_\ell \left\{ \dot{\boldsymbol{U}}_L \dot{\boldsymbol{U}}_L + \frac{b_\ell b_\ell}{c^2} \dot{\boldsymbol{V}}_L \dot{\boldsymbol{V}}_L \right\},$$

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- Momentum and boost $(T = n_i, \Psi = n_i)$

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$$\dot{\mathcal{K}}_i = -u \, \dot{\mathcal{P}}_i - \sum_{\ell=2}^{+\infty} \frac{G}{c^{2\ell+3}} (\ell+1)^2 \mu_{\ell+1} \left\{ \dot{\boldsymbol{U}}_L \boldsymbol{U}_{iL} - \boldsymbol{U}_L \dot{\boldsymbol{U}}_{iL} + \frac{b_\ell b_\ell}{c^2} \left(\dot{\boldsymbol{V}}_L \boldsymbol{V}_{iL} - \boldsymbol{V}_L \dot{\boldsymbol{V}}_{iL} \right) \right\}$$

where

$$\mu_\ell \frac{(\ell+1)(\ell+2)}{(\ell-1)\ell\ell!(2\ell+1)!!}, \qquad \sigma_\ell \frac{8(\ell+2)}{(\ell-1)(\ell+1)!(2\ell+1)!!},$$

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- No soft terms

Poincarè flux balance equations-PN approximation

- Leading terms

$$\dot{\mathcal{E}} = -\frac{G}{c^5}\left(\frac{1}{5}I^{(3)}_{ij}I^{(3)}_{ij}\right) - \frac{G}{c^7}\left(\frac{1}{189}I^{(4)}_{ijk}I^{(4)}_{ijk} + \frac{16}{45}J^{(3)}_{ij}J^{(3)}_{ij}\right) + O(c^{-9})$$

$$\dot{\mathcal{J}}_i = -\frac{G}{c^5}\left(\frac{2}{5}\epsilon_{ijk}I^{(2)}_{jl}I^{(3)}_{kl}\right) - \frac{G}{c^7}\epsilon_{ijk}\left(\frac{1}{63}I^{(3)}_{jlm}I^{(4)}_{klm} + \frac{32}{45}J^{(2)}_{jl}J^{(3)}_{kl}\right) + O(c^{-9})$$

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$$\dot{\mathcal{G}}_i = \mathcal{P}_i - \frac{G}{c^7}\left[\frac{1}{21}\left(I^{(3)}_{jk}I^{(3)}_{ijk} - I^{(2)}_{jk}I^{(4)}_{ijk}\right)\right] + O(c^{-9})$$

- Agree with existing results [*Thorne '80, Kozameh '16, Blanchet, Faye '18*]

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- Proper BMS balance laws $\ell \geq 2$.

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- **PN analysis**

Supermomentum	
ℓ-pole	PN order
0, 2, 4	3
1, 3, 5	3.5

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- Example: Octupolar super angular momentum

$$\dot{\mathcal{J}}_{ijk} - \frac{2}{7c^2}\, u\, \dot{\boldsymbol{V}}_{ijk} = -\frac{G}{c^5}\left(\frac{6}{35}\,\epsilon_{pq\langle i}\,\dot{\boldsymbol{U}}_{j|p|}\boldsymbol{U}_{k\rangle q}\right) + \mathcal{O}(c^{-7})$$

Conclusion

Summary

- New flux balance equations
- At the same PN order as that of energy and angular momentum
- In principle relevant for radiation reaction forces

Outlook

- Wrtie BMS charges in terms of source variables
- New differential equations for source parameters

Thank you very much