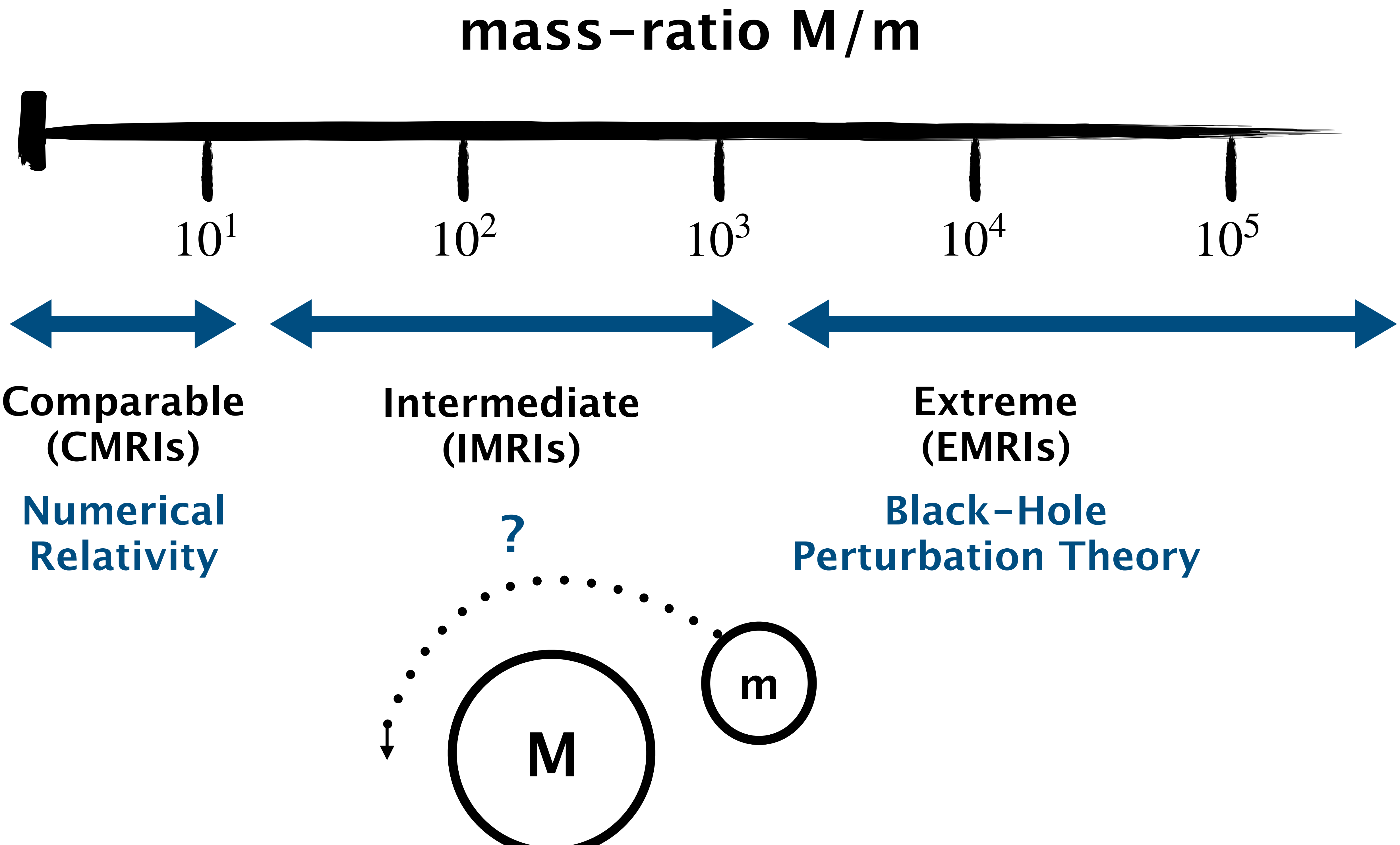
Modelling black-hole binaries in the intermediate-mass-ratio regime

Mekhi Dhesi
Supervisors: Leor Barack, Adam Pound
Collaborators (AEI): Harald Pfeiffer, Hannes Rüter

Outline

- Motivation and Challenge
- Proposed Method
- Scalar-Field Toy Model
- Next Steps

Types of BBH mergers



NR runtime with M/m

$$\left(\frac{M}{m}\right)$$ ➡ **Number of orbits (physics)**

$$\left(\frac{M}{m}\right)$$ ➡ **Time steps per orbit (numerics)**

$$\text{NR runtime} \propto \left(\frac{M}{m}\right)^2$$

If 1:10 ~ 100 days ~ 3 months
Then 1:100 ~ 10,000 days ~ 27 years!

IMRIs: 1:100–1,000

Why IMRIs

IMBH: ultra-luminous x-ray sources in globular clusters and recent LIGO trigger

#1: Stellar-mass CO + IMBH = IMRI with Advanced LIGO

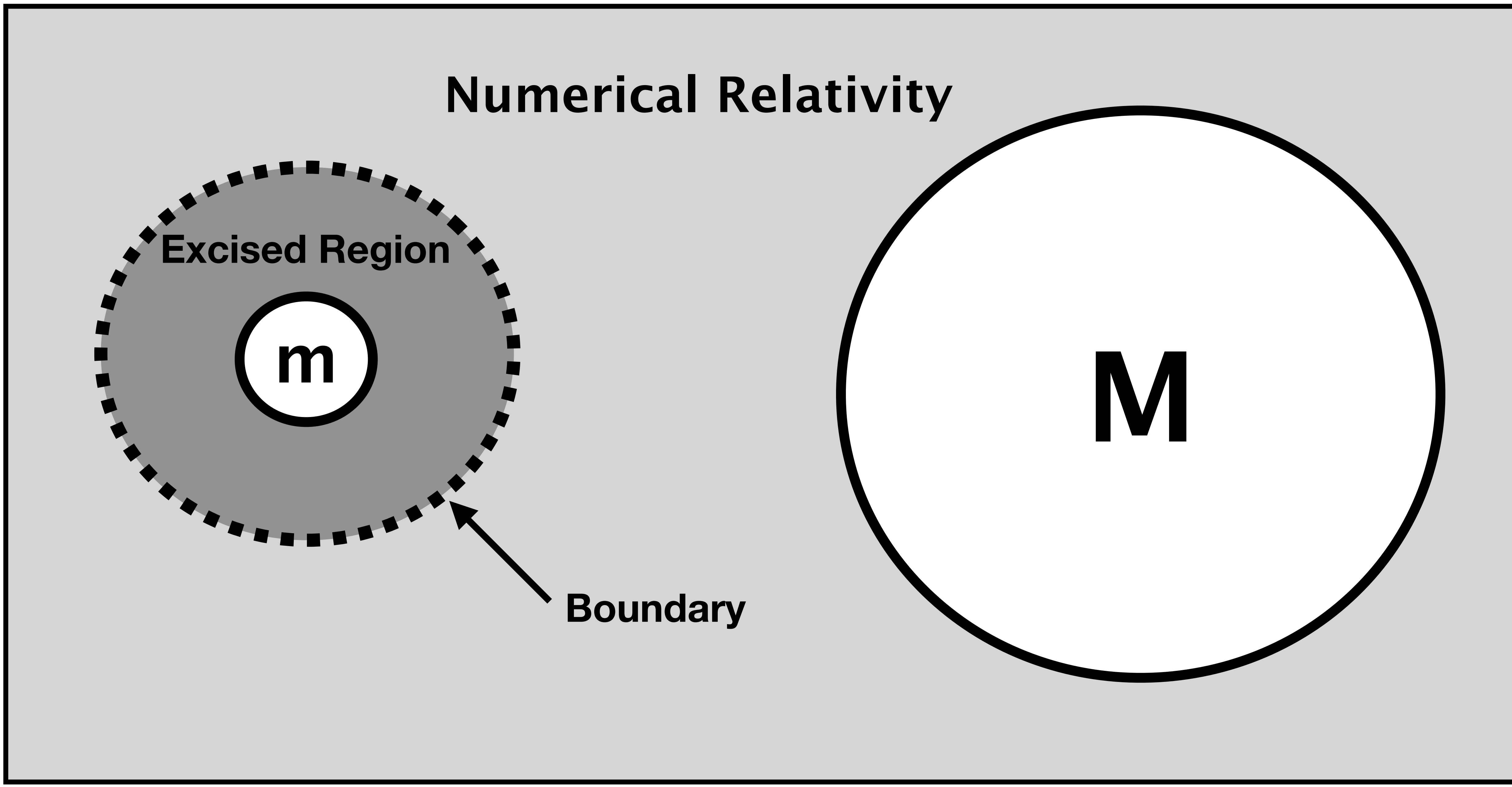
➡ **Explore the dynamics of globular clusters**

#2: IMBH + SMBH = IMRI with LISA

➡ **Explore the dynamics of galactic nuclei**

Proposed Method (3+1)

Matching of approximate analytical solution to NR elsewhere in the spacetime



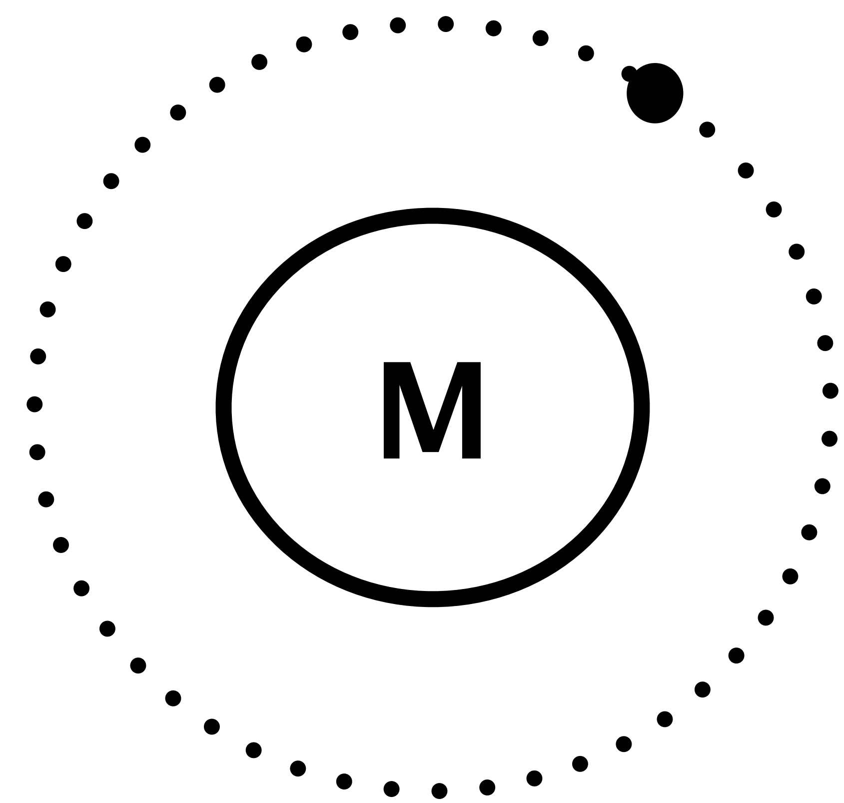
Toy Model: Linear Scalar-Field on Schwarzschild

$$g^{\alpha\beta}\nabla_\alpha\nabla_\beta\Phi(x) = -4\pi\rho(x)$$

Spherical Harmonic Mode Decomposition

$$\Phi = \frac{1}{r}\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\Psi_{lm}(r,t)Y_{lm}(\theta,\phi)$$

$$\rho = \frac{Q}{r^2(t)}\frac{F(r(t))}{E(r_p)}\sum_{lm}Y_{lm}(\theta,\phi)Y^*_{lm}(\frac{\pi}{2},\phi_p(t))\delta(r-r_p)$$

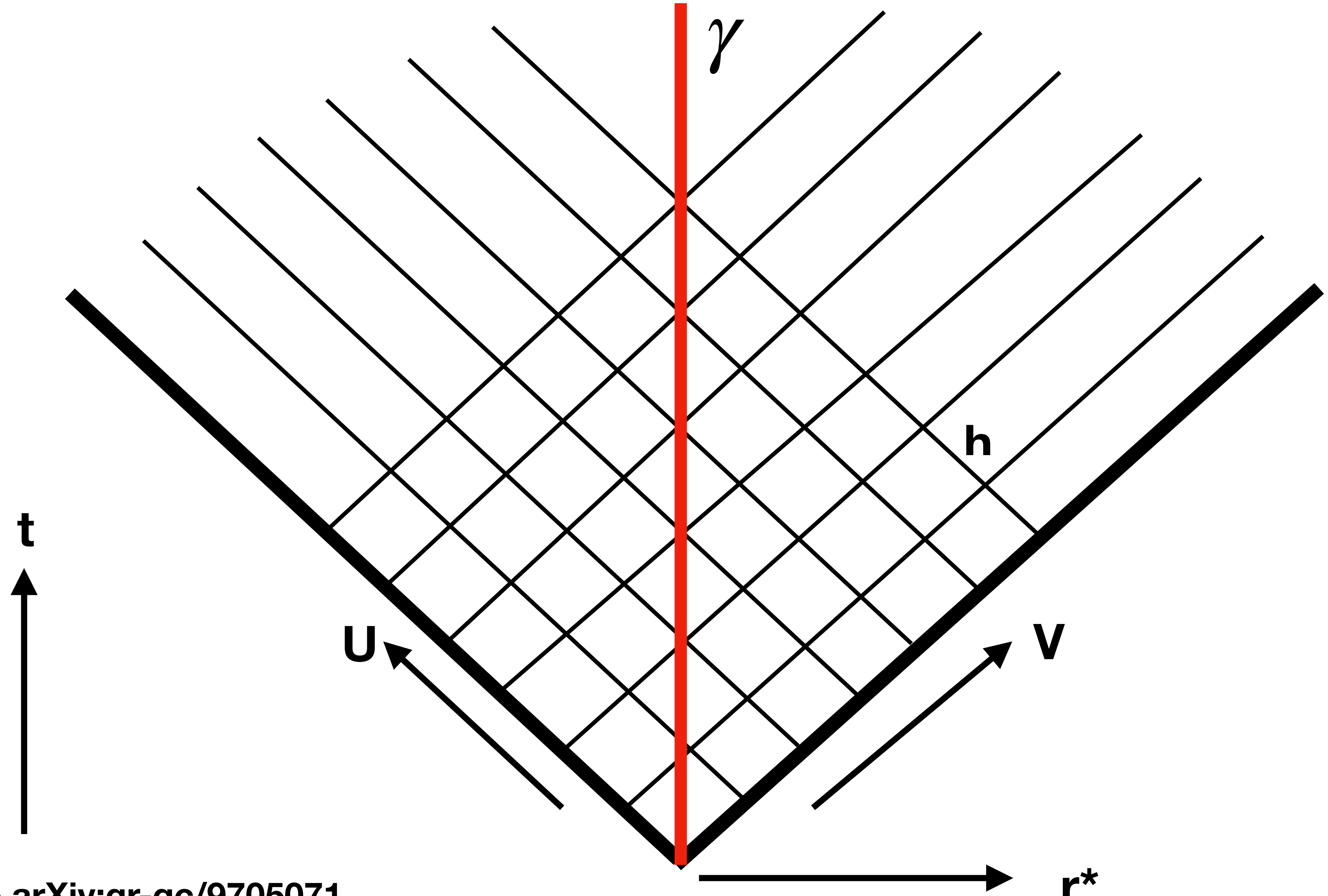


Where $F(r) = \left(1-\frac{2M}{r}\right)$ $E(r_p) = \frac{r_p-2M}{\sqrt{r_p(r_p-3M)}}$

Characteristic Grid

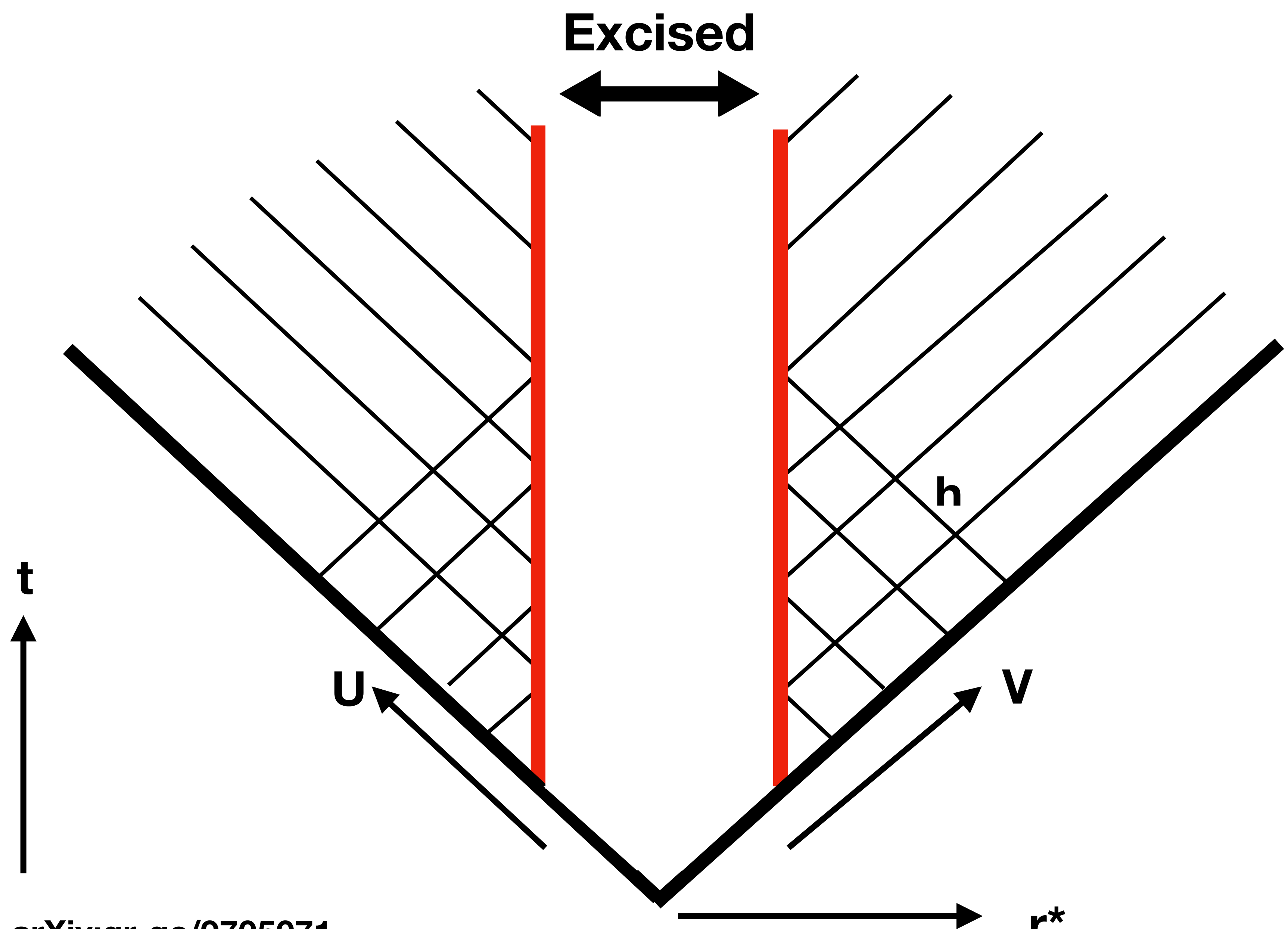
Transformation to Double-Null Coordinates

$$\partial u \partial v \Psi + \frac{V(r)}{4}\Psi = S(r(t))\delta(r - r_p)$$



Lousto and Price arXiv:gr-qc/9705071

Toy Model
Worldtube Excision on Schwarzschild



Lousto and Price arXiv:gr-qc/9705071

Approximate Analytical Solution inside Worldtube

Matching the retarded field across the worldtube boundary

$$\Phi^{ret}(x) \approx \Phi^P + \Phi^R$$

$$\Phi^P_{l,m}(t,r) = [a_0 + a_1 \Delta r + b_0 |\Delta r| + \mathcal{O}(\Delta r^2)]e^{im\Omega t}$$

a_0, a_1, b_0 = known coefficients

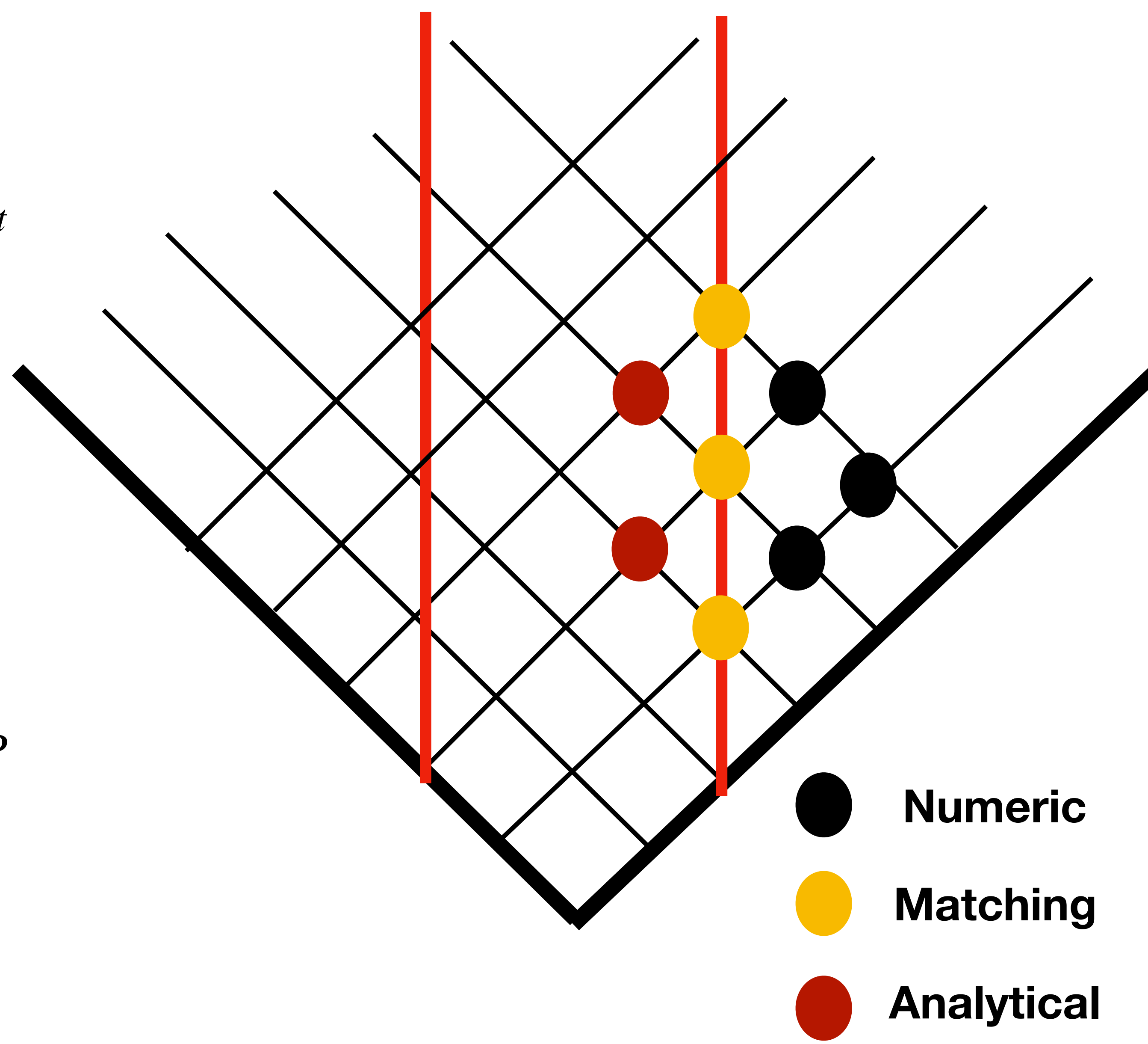
$$\Phi^R_{l,m} = \Phi^R_{l,m}(r_p, t) + \mathcal{O}(\Delta r)$$

↓

first stage = unknown constant

$$\Phi^{ret} = \Phi^N$$

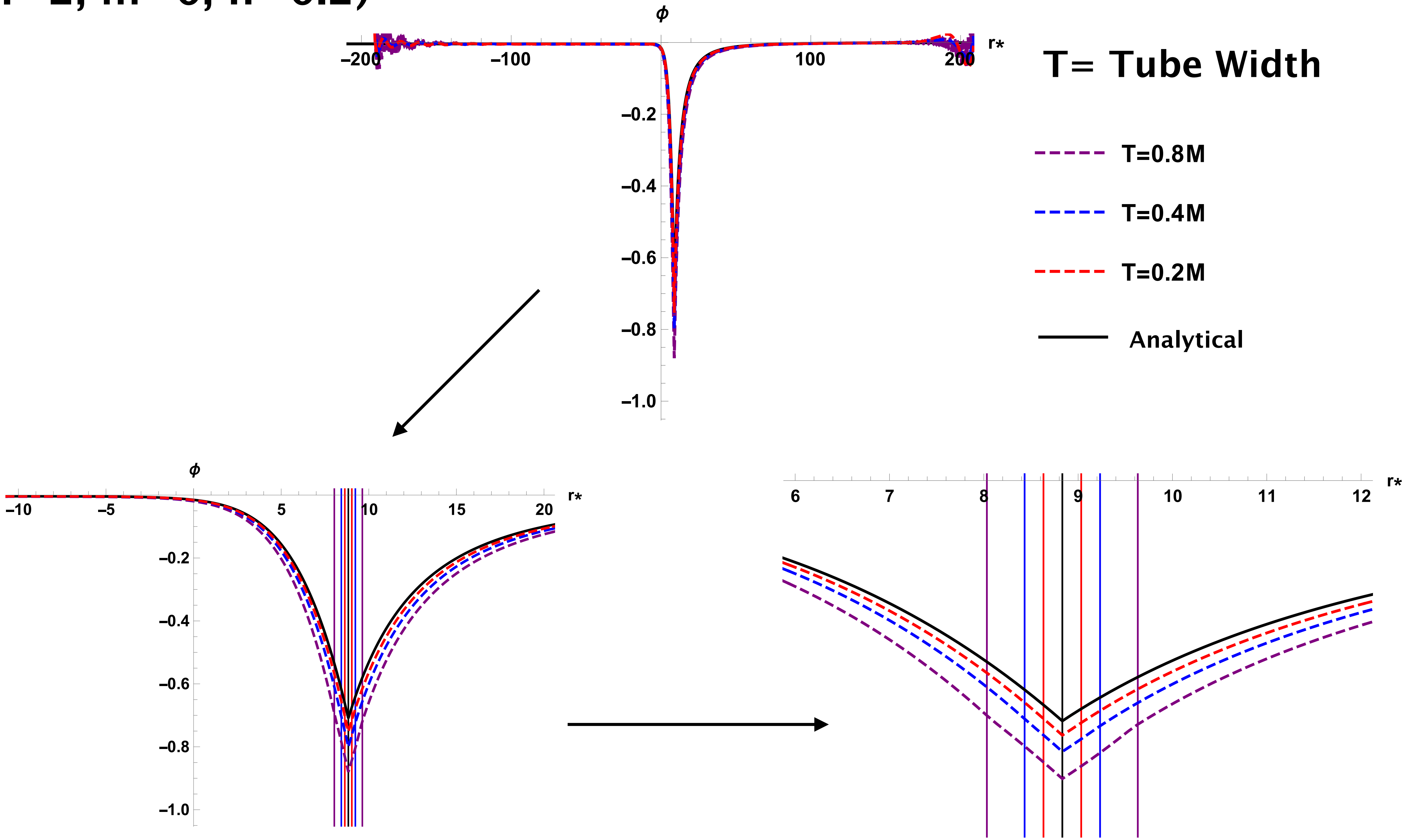
$$\Phi^R = \Phi^N - \Phi^P$$



Wardell, Warburton 2015 arXiv:1505.07841v2

Sample Results

(l=2, m=0, h=0.2)



Next Steps

- Higher orders in the approximate analytical solution

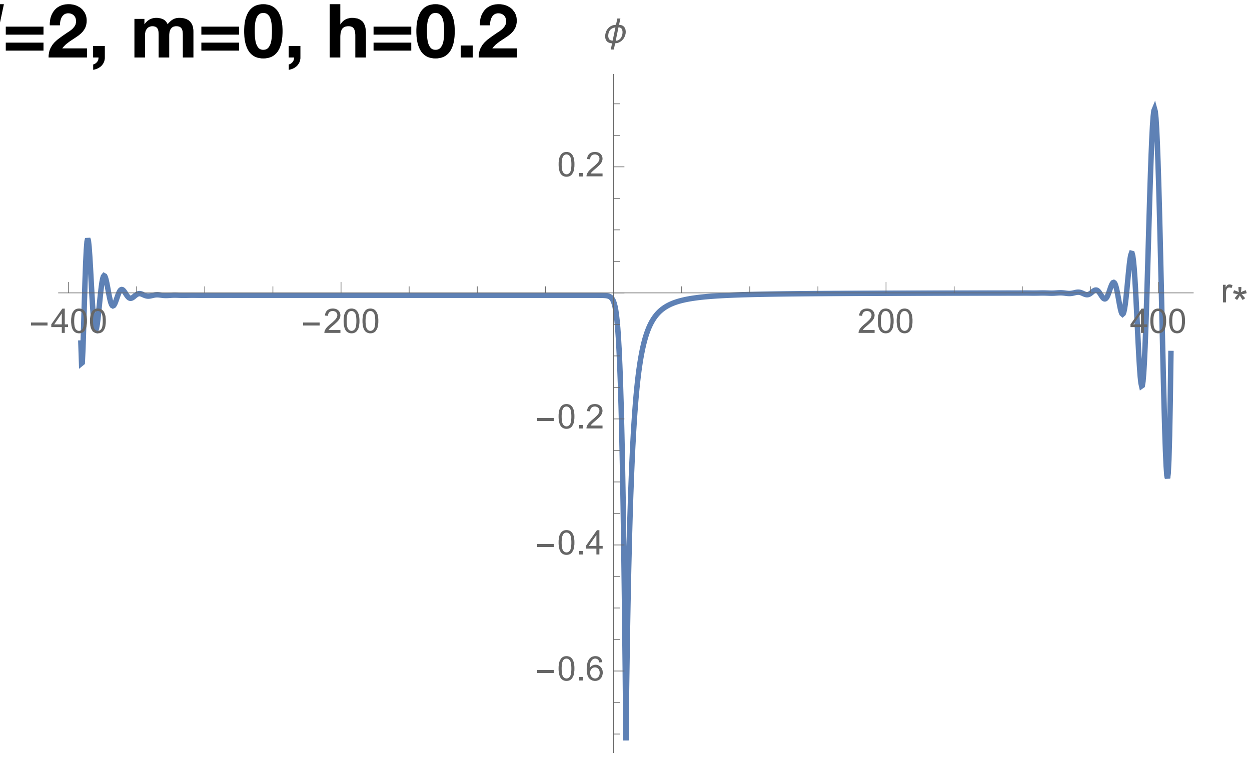
$$\Phi^{ret}(x) \approx \Phi^{P} + \Phi^{R}$$

- SpEC Implementation in full GR (3+1)
- Approximate analytical solution:
  Tidally perturbed black-hole metric
- Inclusion of small-body spin

Reserve Slides

Linear Scalar-Field: Results

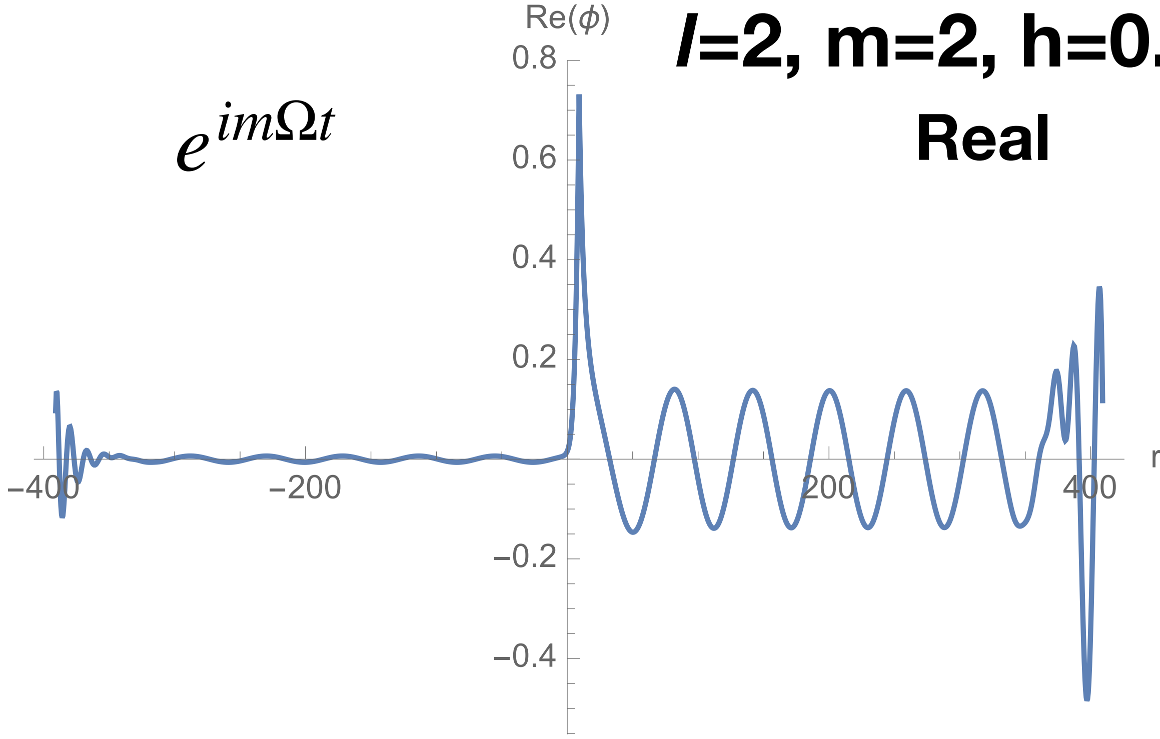
***l*=2, m=0, h=0.2**

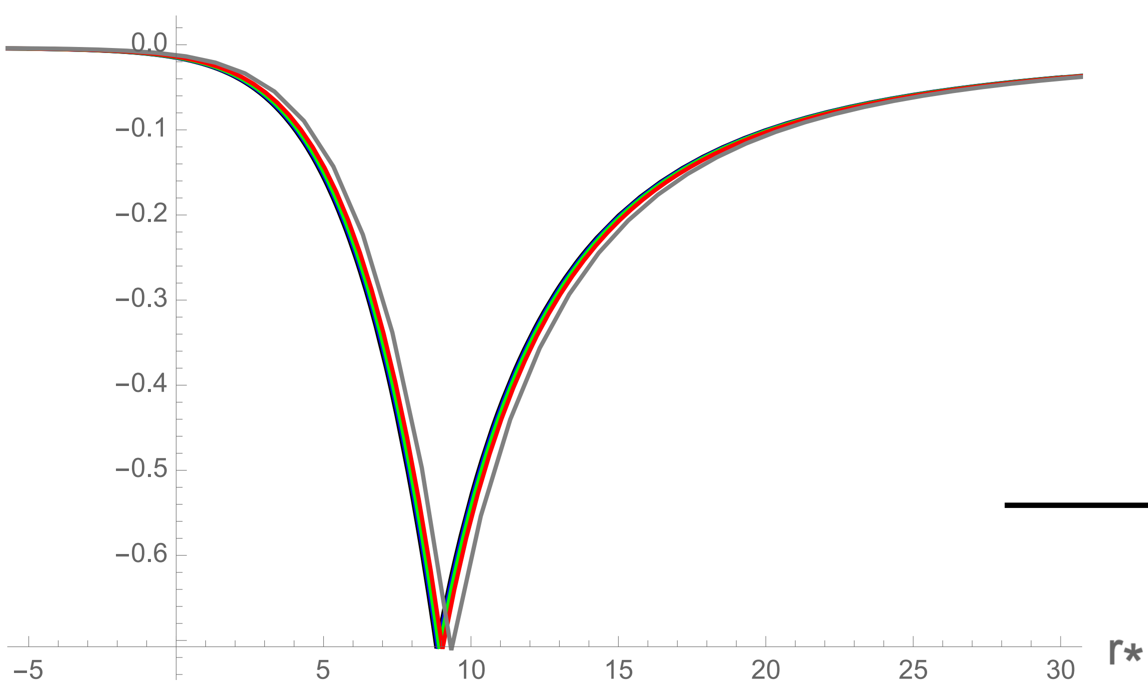


$e^{im\Omega t}$

***l*=2, m=2, h=0.2**

Real





- **Analytical**
- **h=0.1**
- **h=0.2**
- **h=0.4**
- **h=1**