Causal structure of black holes in generalized scalar-tensor theories

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Gravitational Waves, Black Holes and Fundamental Physics

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Alternatives to general relativity

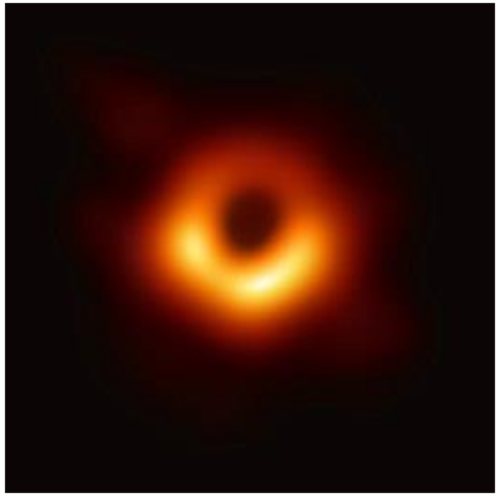
Motivations for modified gravity:

- Ignorance of the strong-field regime
- No theory of quantum-gravity
- Hope to explain the dark content of the universe

Black holes: bald or hairy?

BH in general relativity:

- Mass M
- Spin J



Black holes: bald or hairy?

BH in general relativity:

- Mass M
- Spin J

BH in modified gravity:

- Mass M
- Spin J
- Additional hair? q_s

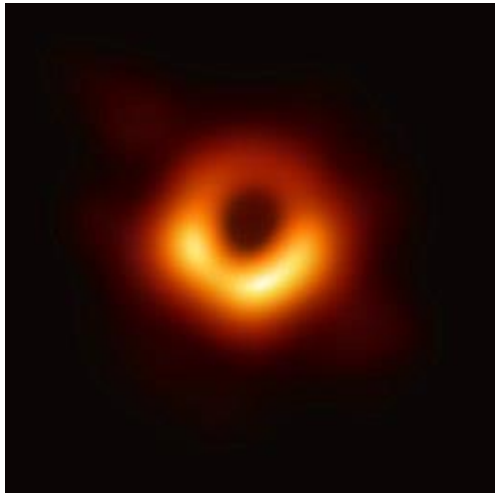


Image credit: Mario Herrero Valea

Penrose process

Extraction of angular momentum from a spinning black hole

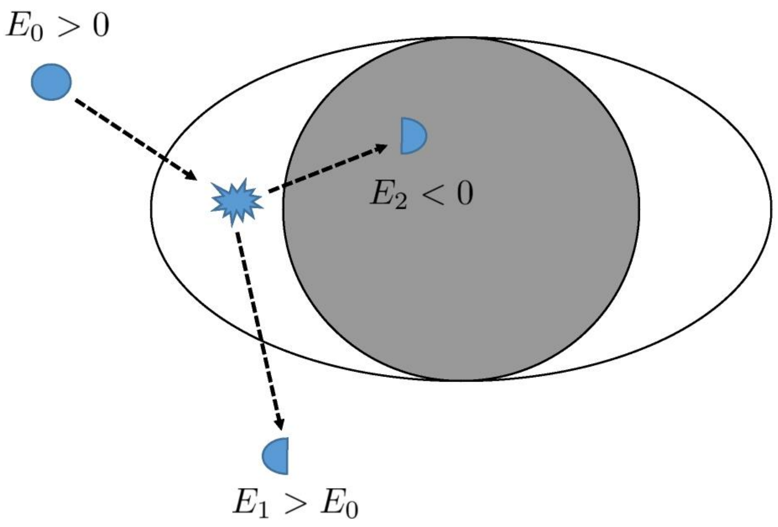


Image credit: Brito, et al. 2015

Mechanism to mine spin and energy from a BH

Evolves a Kerr BH to a Schwarzschild BH

Is there an analogue natural way to extract "hair" from a hairy BH?

Analogue Penrose process

Let's take Lorentz-violating BHs Blas, et al. 2011, Barausse et al. 2011, 2013

- Spin-0,1,2 modes propagating at speed $> c$ $\quad g^{\mu\nu}_{(s)}$
- Killing horizon for each mode $\chi^\mu \Longrightarrow g_{\mu\nu}\chi^\mu\chi^\nu = 0$
- Conjecture of analogue of Penrose process: used as "hair extraction"

Benkel, et al. 2018

Superluminality

Not only in Lorentz-violating gravity

Common feature with self-interacting scalar Babichev, et al. 2008

We analyse for the most general theory of scalar + metric

Do BHs in Horndeski gravity admit different horizons for metric and scalar perturbations?

Horndeski gravity

Horndeski 1974
Deffayet 2009
Kobayashi 2011

Lagrangian (most general yielding 2 order EoMs)

$$\mathcal{L}_H = \sum_{i=2}^{5} \mathcal{L}_i$$

$$\mathcal{L}_2 = G_2(\phi, X)$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

with

$$X = -\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi$$

Scalar effective metric

Spacetime metric $g_{\mu\nu}$

Effective metric for scalar perturbations $\phi = \varphi + \delta\phi$

$$\delta^2 \mathcal{L}_H = f^{\mu\nu} \nabla_\mu \delta\phi \nabla_\nu \delta\phi$$

Scalar effective metric

Spacetime metric $g_{\mu\nu}$

Effective metric for scalar perturbations $\phi = \varphi + \delta\phi$

$$\delta^2 \mathcal{L}_H = f^{\mu\nu} \nabla_\mu \delta\phi \nabla_\nu \delta\phi$$

Very complicated form

$$f^{\mu\nu}_{(2)} = -G_{2X} g^{\mu\nu} + G_{2XX} \nabla^\mu \varphi \nabla^\nu \varphi$$

$$f^{\mu\nu}_{(3)} = \ldots \qquad f^{\mu\nu}_{(4)} = \ldots \qquad f^{\mu\nu}_{(5)} = \ldots$$

Killing horizon of the effective metric

We assume stationarity

- For the metric: the horizon $\mathcal{H}$ is a Killing horizon $g_{\mu\nu}\chi^\mu\chi^\nu = 0$

- For the scalar field $\chi^\mu\nabla_\mu\varphi = 0$

Does $\mathcal{H}$ act as a horizon for the effective metric too?

$$f^{-1}_{\mu\nu}\chi^\mu\chi^\nu = 0$$

Killing horizon of the effective metric

$$\boxed{\begin{array}{c} g_{\mu\nu}\chi^\mu\chi^\nu = 0 \\ \chi^\mu\nabla_\mu\varphi = 0 \end{array}}$$

$f^{-1}_{\mu\nu}\chi^\mu\chi^\nu\big|_{\mathcal{H}} = 0$ is equivalent to

- $f^{\mu\nu}\chi_\mu\chi_\nu|_{\mathcal{H}} = 0$

- $f^{\mu\nu}\chi_\mu|_{\mathcal{H}} = q\chi^\nu$

We are proving that $\mathcal{H}$ is a null surface for the effective metric

Killing horizon of the effective metric

$$\boxed{\begin{array}{c} g_{\mu\nu}\chi^\mu\chi^\nu = 0 \\ \chi^\mu\nabla_\mu\varphi = 0 \end{array}}$$

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We are proving that $\mathcal{H}$ is a null surface for the effective metric

Killing horizon of the effective metric

$$\boxed{\begin{array}{c} g_{\mu\nu}\chi^\mu\chi^\nu = 0 \\ \chi^\mu\nabla_\mu\varphi = 0 \end{array}}$$

We decompose $f^{\mu\nu}\chi_\mu$ into

$$g^{\mu\nu}\chi_\mu \qquad \nabla^\mu\varphi B^\nu\chi_\mu \qquad \nabla^\mu\nabla^\nu\varphi\chi_\mu \qquad R^{\mu\nu}\chi_\mu \qquad R^{\mu\rho\nu\sigma}\chi_\mu T_{\rho\sigma}$$

Killing horizon of the effective metric

$$\boxed{\begin{gathered} g_{\mu\nu}\chi^\mu\chi^\nu = 0 \\ \chi^\mu\nabla_\mu\varphi = 0 \end{gathered}}$$

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$$\Downarrow$$

$$\chi^\nu$$

Killing horizon of the effective metric

$$\boxed{\begin{gathered} g_{\mu\nu}\chi^\mu\chi^\nu = 0 \\ \chi^\mu\nabla_\mu\varphi = 0 \end{gathered}}$$

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Killing horizon of the effective metric

$$\boxed{\begin{aligned} g_{\mu\nu}\chi^\mu\chi^\nu &= 0 \\ \chi^\mu\nabla_\mu\varphi &= 0 \end{aligned}}$$

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$$\Downarrow$$

$$\chi^\nu$$

Proportional to χ^ν on $\mathcal{H}$

Killing horizon of the effective metric

$$\boxed{\begin{gathered} g_{\mu\nu}\chi^\mu\chi^\nu = 0 \\ \chi^\mu\nabla_\mu\varphi = 0 \end{gathered}}$$

We decompose $f^{\mu\nu}\chi_\mu$ into

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$$\Downarrow$$

$$\chi^\nu$$

$$\boxed{f^{\mu\nu}\chi_\mu|_{\mathcal{H}} = q\chi^\nu}$$

Killing horizon of the effective metric

$$\boxed{f^{\mu\nu}\chi_\mu|_{\mathcal{H}} = q\chi^\nu}$$

The effective metric always shares the Killing horizon with the spacetime metric, under stationarity, for every Horndeski theory

Counterexample when stationarity assumption is violated (shown if I have time)

Conclusions

- Scalar mode Killing horizon always lies on the metric, for every Horndeski theory, assuming stationarity
- No analogue of Penrose process is found
- Superluminality does not threaten the stability of BHs in Horndeski
- Counterexample when stationarity assumption is violated

Thanks for the attention =D

Time-dependent counterexample

We take a simple lagrangian

$$\mathcal{L}_K = G_2(X) = X + \frac{1}{2}\alpha X^2$$

The effective metric reads

$$f^{\mu\nu} = G_{2X} g^{\mu\nu} - G_{2XX} \nabla^\mu \varphi \nabla^\nu \varphi$$

Time-dependent counterexample

With a time dependent scalar field ansatz

$$\varphi = qv + \psi(r)$$

$$v = t + r_*$$

Solving perturbatively in q for $\psi(r)$, one finds the perturbed effective metric

$$f^{(-1)}_{\mu\nu} dx^\mu dx^\nu \propto \left(-1 + \frac{2M'}{r}\right) dv'^2 + 2\left[1 - \alpha q^2 M' \frac{4M'^2 + 2M'r + r^2}{r^3}\right] dr dv' + r^2 d\Omega^2$$