# Plasma-driven superradiant instabilities

## Numerical investigation of the stability of accreting Kerr black holes

Alexandru Dima

SISSA, Trieste, Italy

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*Black holes, gravitational waves and fundamental physics*

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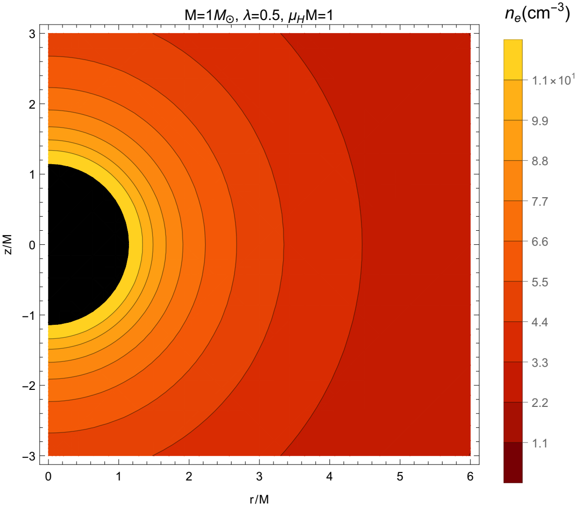
[1] *J.Conlon, C.Herdeiro*, Phys.Lett. B780 (2018) 169-173

# Density profiles

Spherically symmetric density distribution (Bondi accretion)



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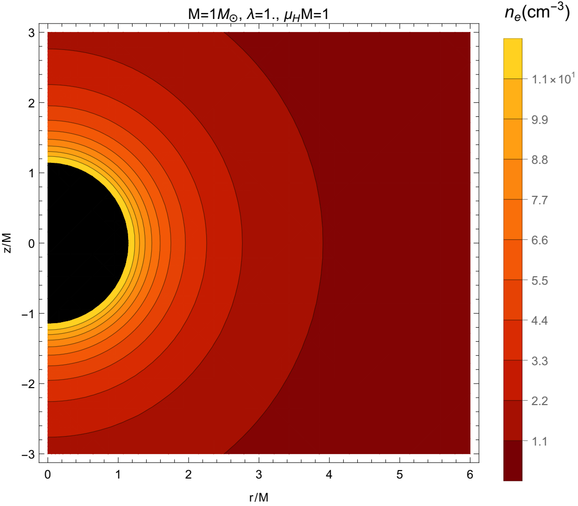

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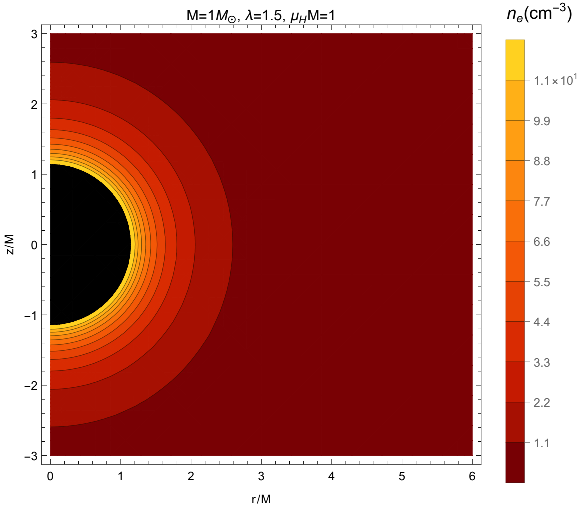

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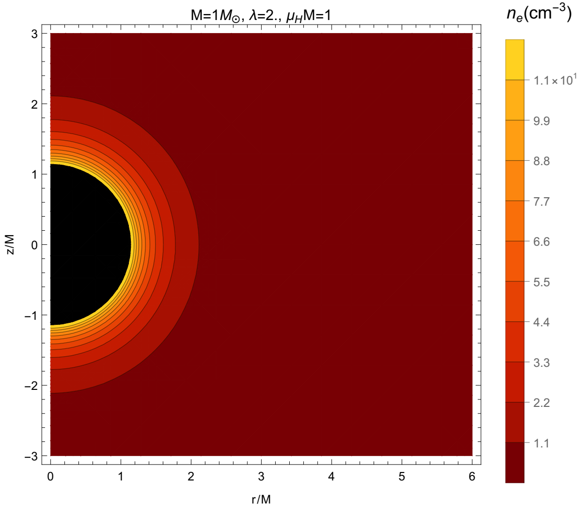

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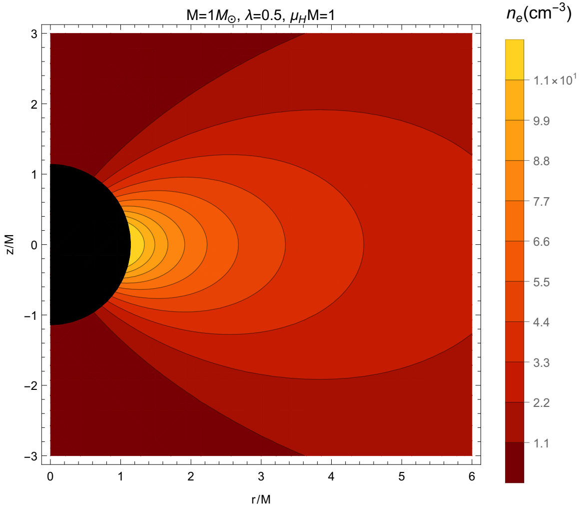

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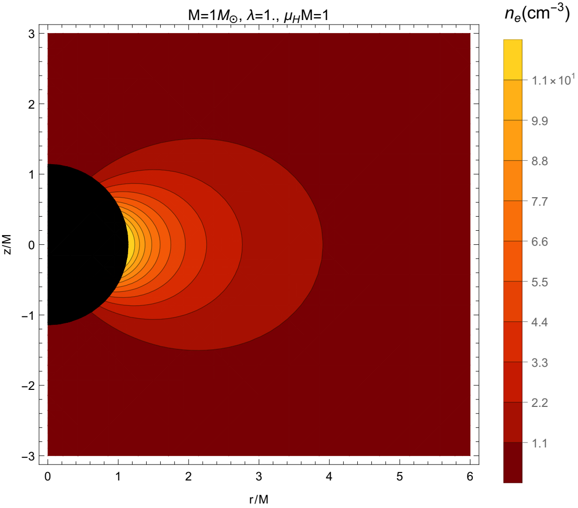

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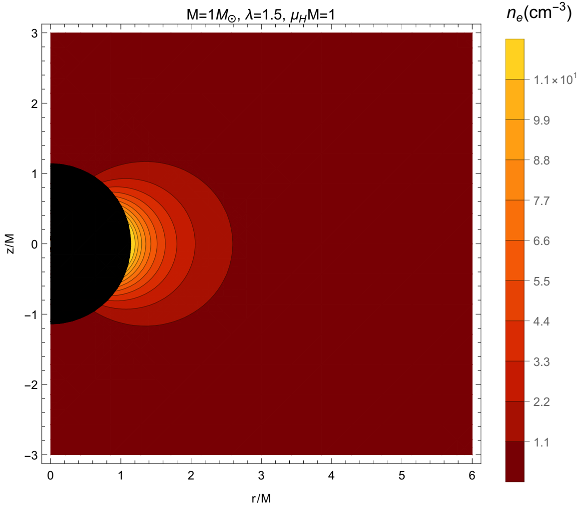

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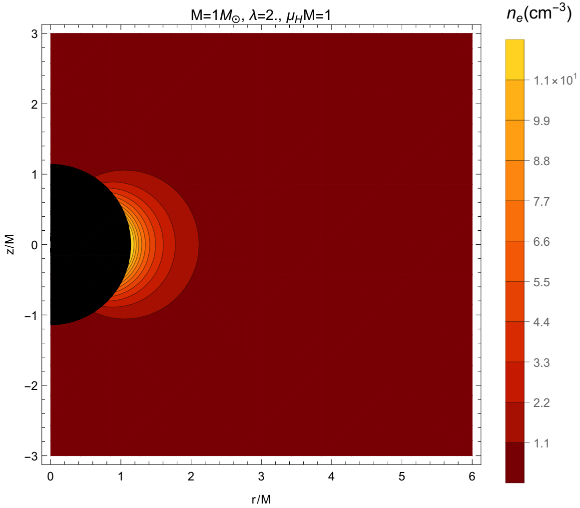

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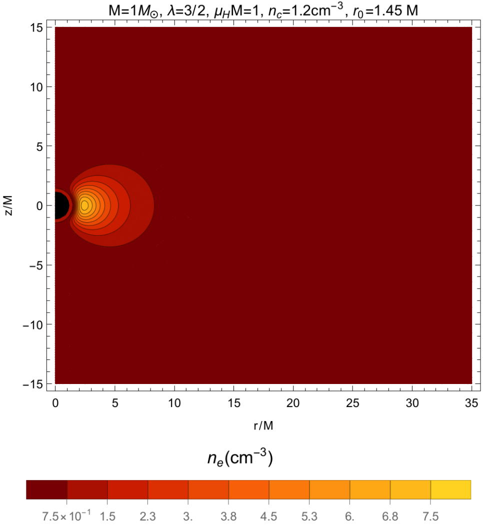

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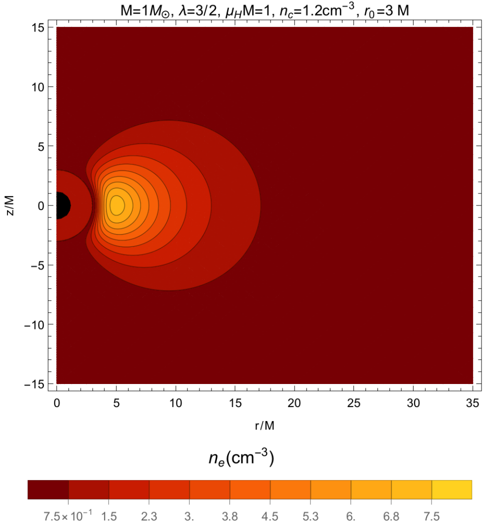

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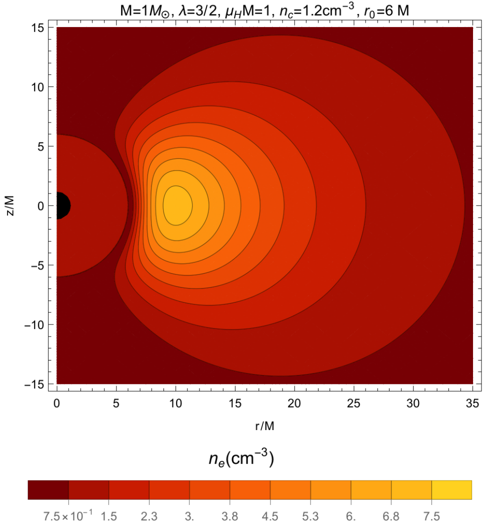

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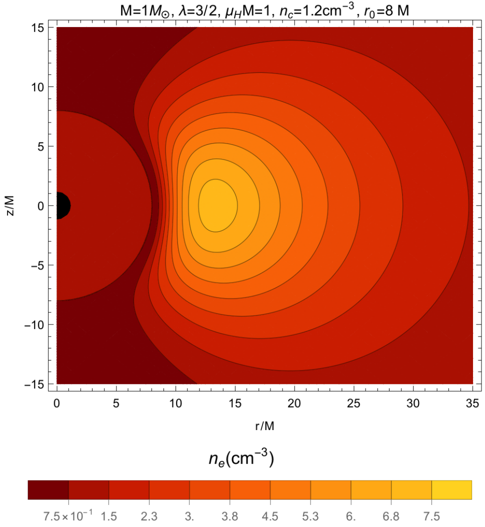


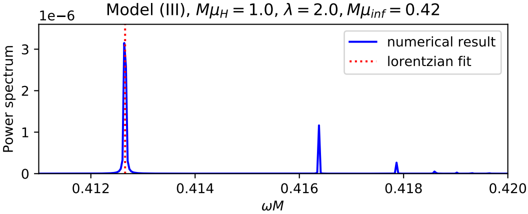
# Numerical setup

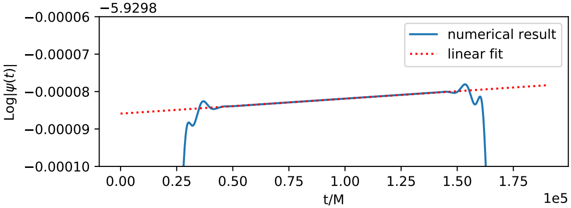
Time evolution of Klein-Gordon equation on Kerr spacetime with method of lines

- Decomposition: spherical harmonics
- Stepper algorithm: 4th order Runge-Kutta (RK4)
- Boundary Conditions:
- At infinity: outgoing BC
- Horizon: ingoing BC + artificial damping region (PML)
- Tests: Convergence, E/J Conservation, Kerr QNMs, BH-bomb instabilities, constant mass QBSs, separable mass functions[2]

[2] V. Cardoso, I. P. Carucci, P. Pani, and T. P. Sotiriou, Phys. Rev. D 88, 044056 (2013)

# Results: Bondi



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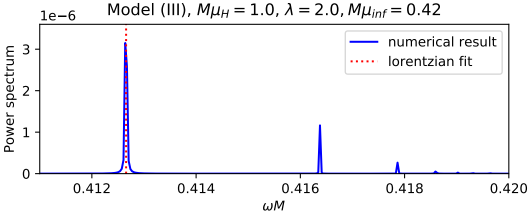


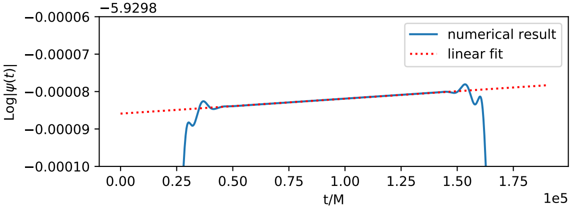


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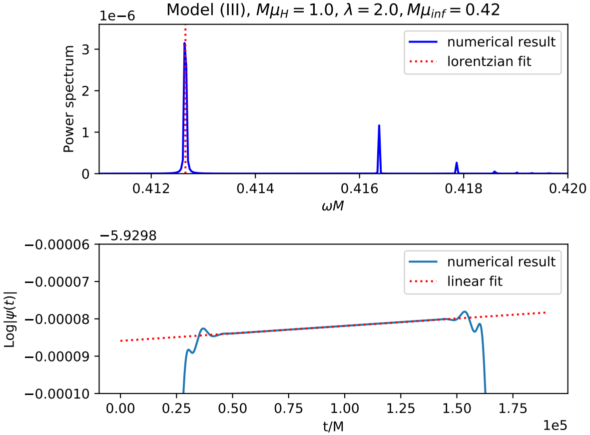


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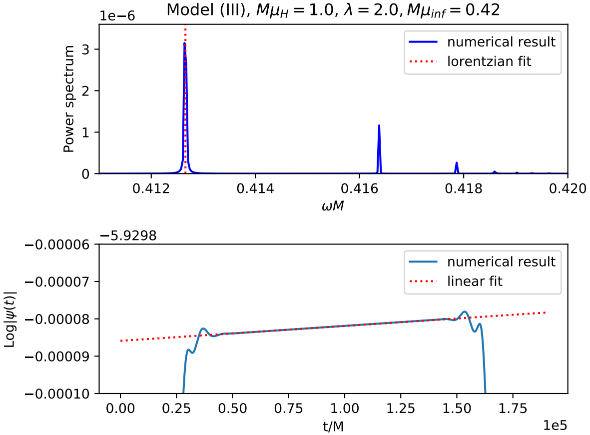


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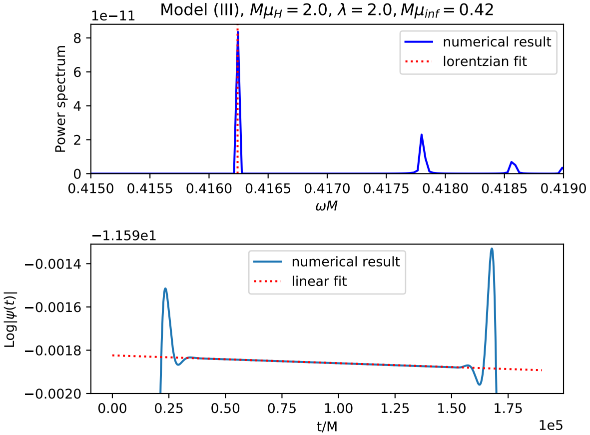

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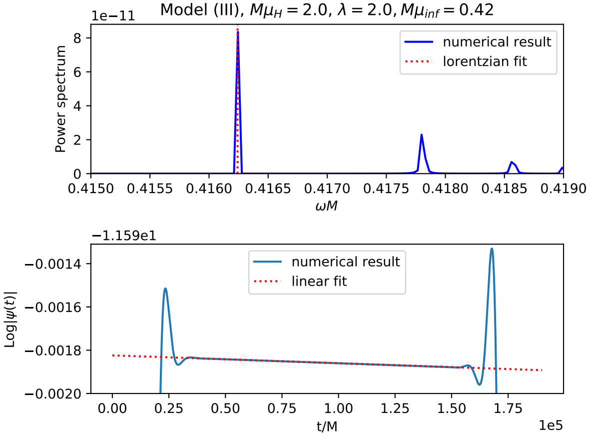


- $n_H \sim 0.5\ \text{cm}^{-3}$

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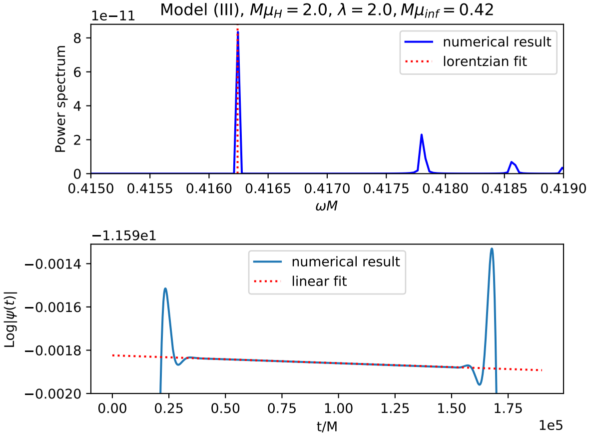


- $n_H \sim 0.5\ \mathrm{cm}^{-3}$
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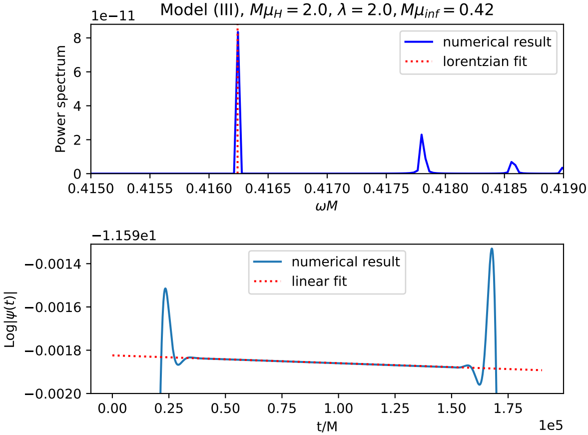


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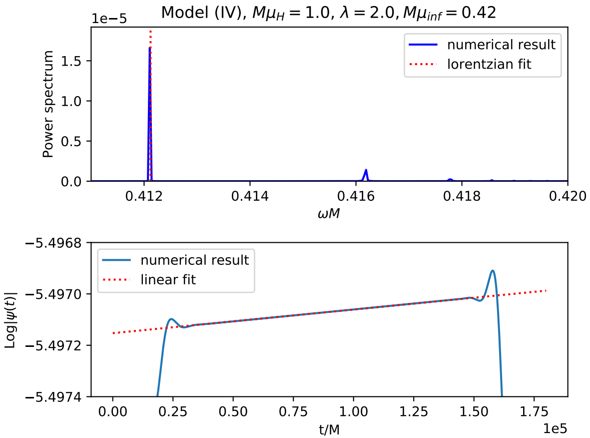

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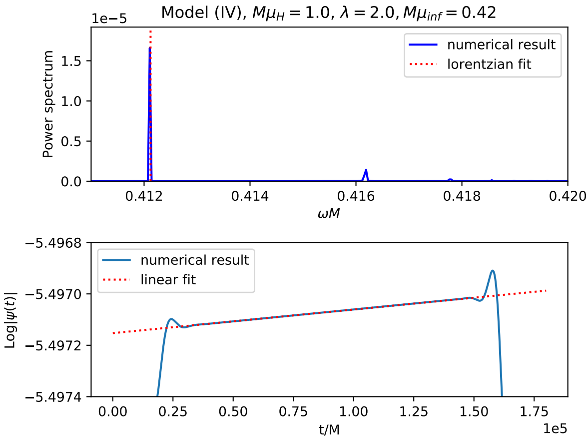


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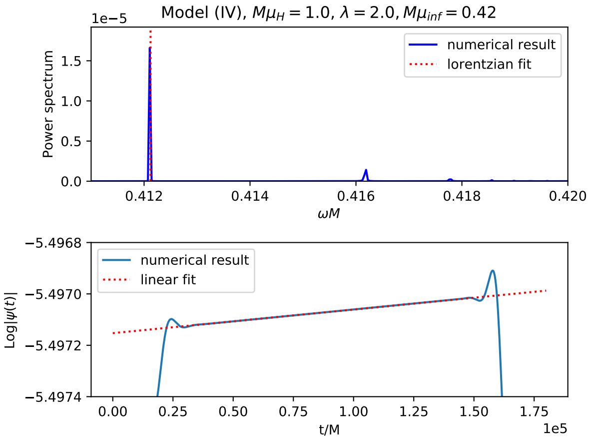


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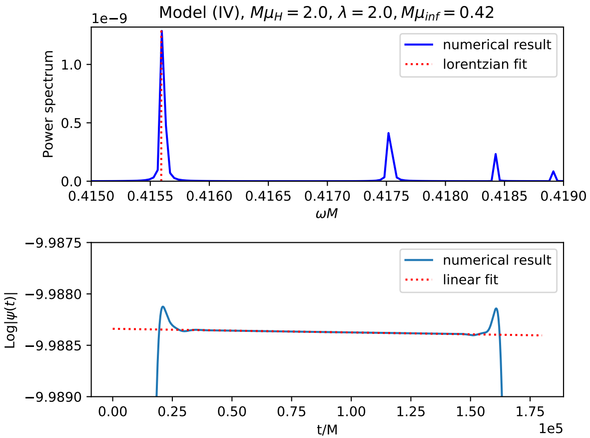

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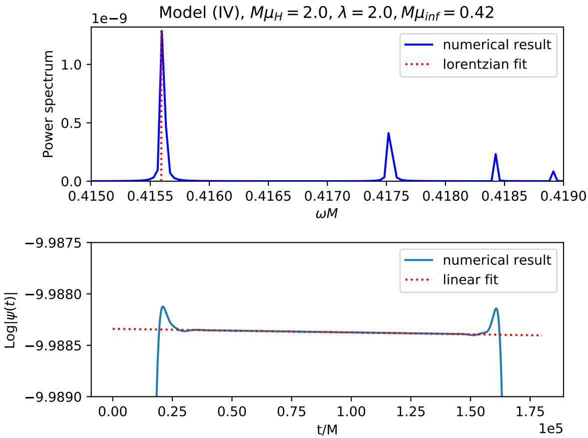


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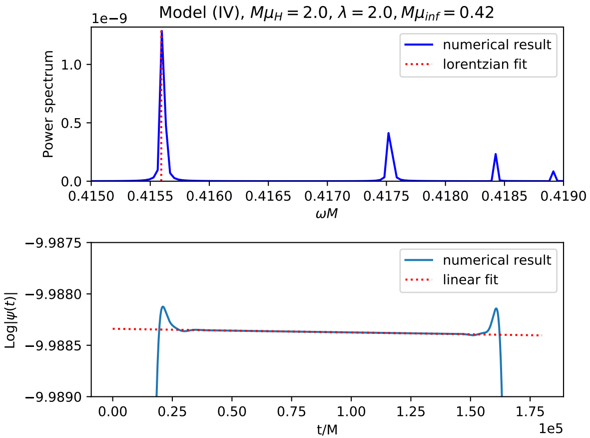


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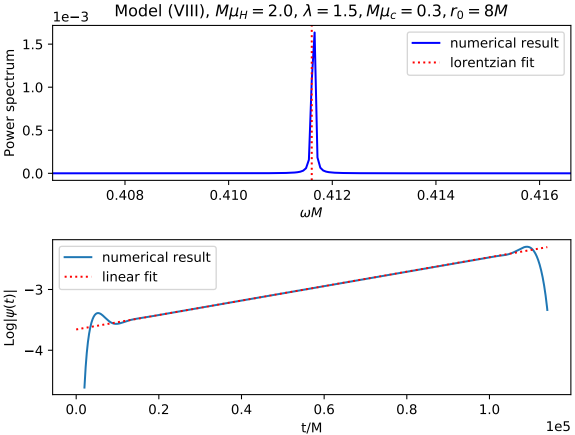

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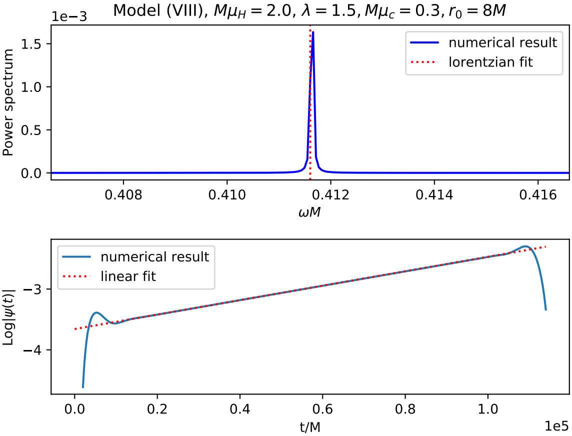


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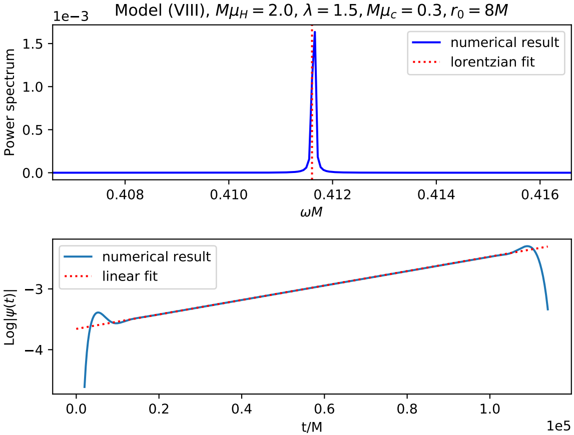


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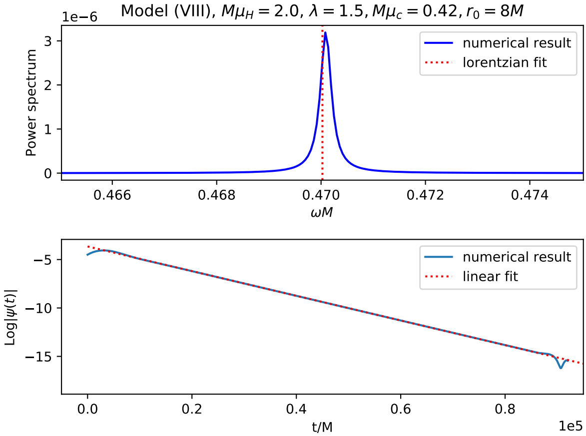

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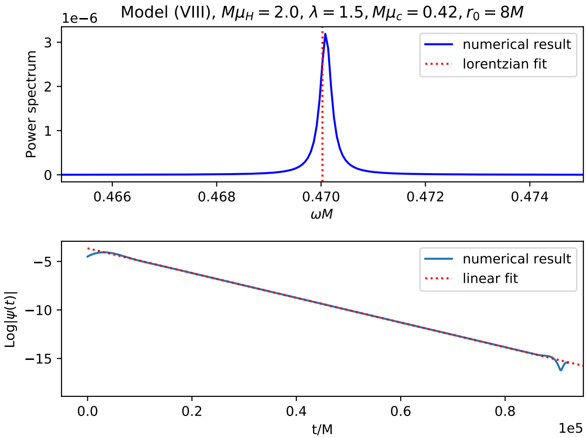


Increase to $n_c \simeq 0.02\ \mathrm{cm}^{-3}$: $\omega M = 0.471 - i\ 1.4 \cdot 10^{-4}$ !

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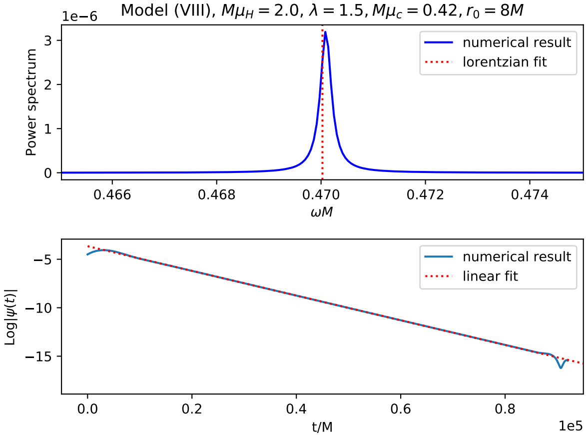


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- ...highly sensitive to effective mass in ergoregion!
- Likely to be suppressed by realistic densities in the ergoregion ($n_e \gg 1\ \mathrm{cm}^{-3}$)

# THANK YOU FOR YOUR ATTENTION!

# BONUS SLIDES

# Tests

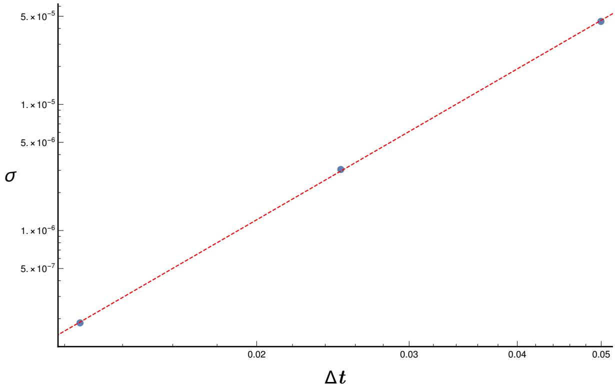
- Convergence test: x- and t-error scaling: check!
- Conserved quantities test: check!
- Comparison with original 1+1D code[3]: check!
- Comparison with approximated analytical formulas[4]: check!
- Comparison with numerical $\omega$-domain[5] results: check!

[3] S. Dolan

[4] V. Cardoso et al., PRD 70,044039 (2004)

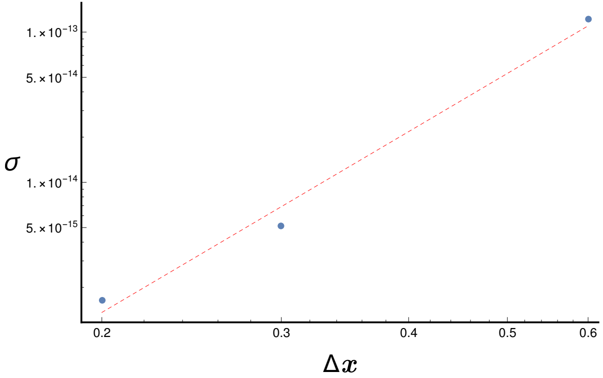
[5] E. Berti, https://pages.jh.edu/ eberti2/ringdown/

# Numerical convergence: $\Delta t$ resolution



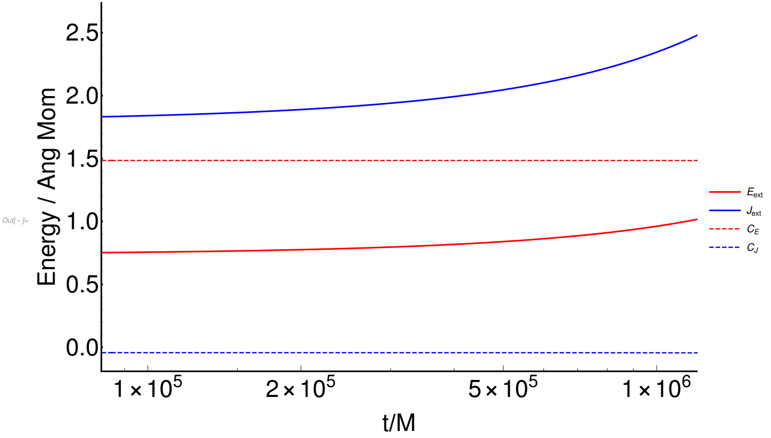

$$\log \sigma = (1.93 \pm 0.15) + (3.97 \pm 0.04) \log \Delta t$$

# Numerical convergence: $\Delta x$ resolution




$$\log \sigma = (-27.80 \pm 0.56) + (3.99 \pm 0.47) \log \Delta x$$

# E/J conservation



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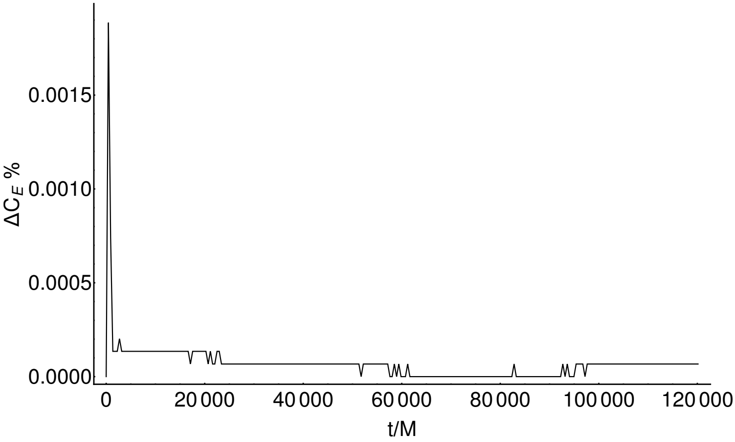

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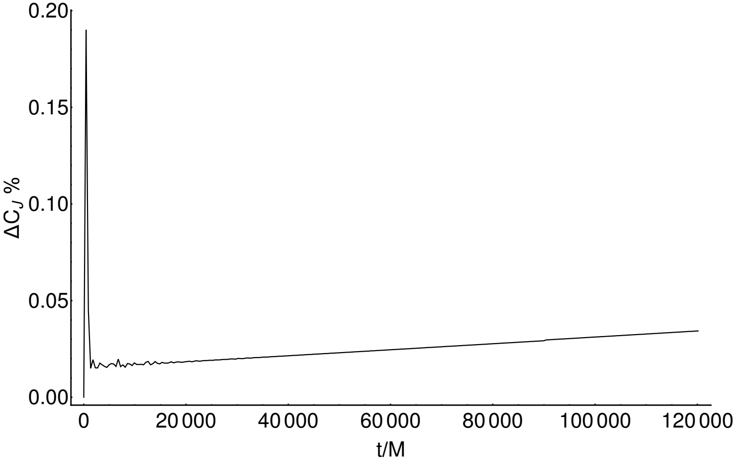
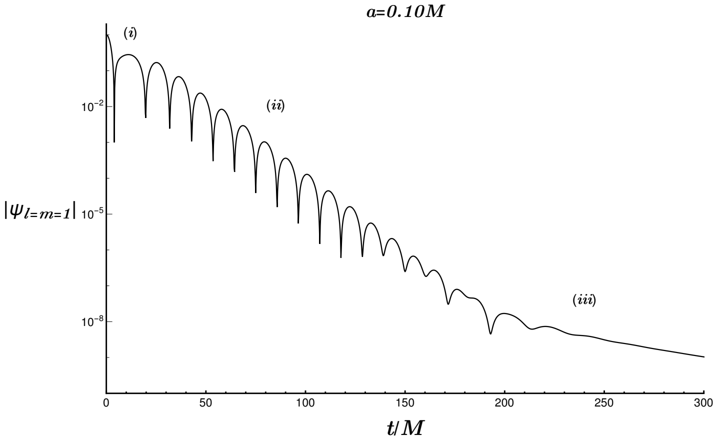
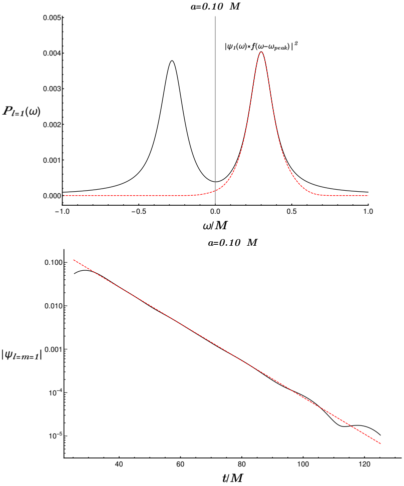


# QNM



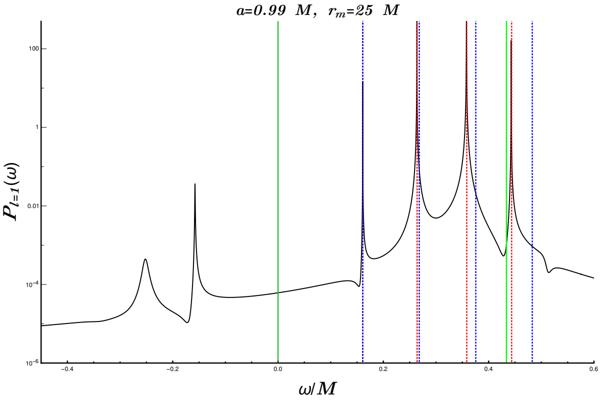

- (i) Prompt-response: depends on the specific initial data
- (ii) Ringdown: quasinormal modes
- (iii) Power-law tail

# QNM test



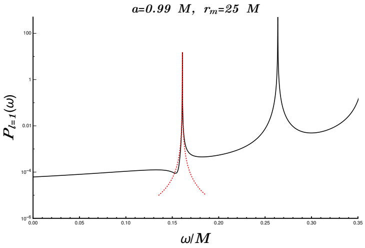

- Peak finding
- Fit with Lorentzian: $\frac{(\pi\nu)^{-1}}{(\omega-\omega_{peak})^2+\nu^2} \rightarrow \hat{\omega} = \omega_{peak}$
- Filter: $f(\omega - \hat{\omega}) = \exp(-\frac{(\omega-\hat{\omega})^4}{(nd\omega)^4})$
- linear fit: $\log|\psi| = q + \nu_{peak} \cdot t \rightarrow \nu = \nu_{peak}$

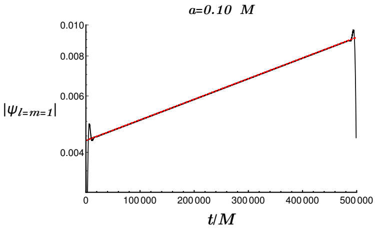
# Mirror test




- green: superradiant regime
- red: results from reference 1+1D code
- blue: approximated formula $\omega_{nlm} \simeq \frac{(n+\frac{\ell}{2}+1)}{r_{*,mirror}}$

# Extraction of superradiant modes




$$\omega_{l=m=1} = 0.160877(\pm 0.03\%) + i7.42416 \cdot 10^{-6}(\pm 0.2\%)^{6}$$

[6] relative errors w.r.t. results of Dolan(2013), in agreement with Cardoso et al.(2004) analytical and numerical results.

# Superradiance

## Superradiance with rotating BHs

Classical extraction of the BH's rotational energy in favor of bosonic degrees of freedom

## **Superradiant scattering**

Waves coming from infinity scattering off rotating BHs get amplified if

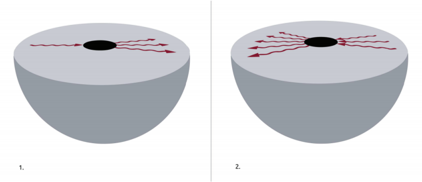
$$0 < \omega \leq m\Omega_H \implies |\mathcal{R}|^2 = |\mathcal{I}|^2 - \frac{\omega - m\Omega_H}{\omega}|\mathcal{T}|^2 \geq |\mathcal{I}|^2$$

# Superradiant instabilities

## Confinement leads to instability

Confined superradiant modes are forced to be repeatedly amplified, resulting in an exponential growth: **instability**

- Black-hole bomb: BH enclosed by a mirror




- Mass term, $\mu$: natural confinement of modes with $\omega \leq \mu$

$$\Psi(r \to \infty) \sim \frac{e^{-qr}}{r} \qquad q = \sqrt{\mu^2 - \omega^2}, Re(q) \geq 0$$

# Spectrum of the perturbations

## How to study the (in)stability?

Solve for characteristic oscillations:

$$(\Box - \mu^2)\psi = 0 \to \psi_{nlm}$$

$$(a, \mu M) \to \{\omega_{nlm}, \nu_{nlm}\}$$

## Quasi-bound states

$$\psi_{QBS}(t) \sim e^{-i(\omega + i\nu)t}$$

for $\nu > 0$, instability with associated growth rate $\tau = \frac{1}{\nu}$

# Frequency-domain vs Time-domain approach

- Analytical approximations (Detweiler, 1980; Zouros & Eardley, 1979; eikonal approximation; Pösche-Teller potential)
- Numerical $\omega$-domain techniques (WKB, shooting, continued fraction method)

### Non-separability requires time-domain approach

Space-time dependent mass term in perturbation equation ruins separability, on which these techniques rely

# Effective mass from non-linear plasma effects

In unmagnetized, cold plasma: charges ($e^-$) oscillate with characteristic frequency:

## Photon propagation in plasma

$$\omega^2 = k^2 + \omega_p^2, \qquad \omega_p \equiv \sqrt{\frac{4\pi\alpha}{m_e} n_e}$$

Non-uniform plasma distribution: effective mass is space-time dependent

$$m_{eff}^2 \equiv \omega_p^2(t, \vec{x}) \sim n_e(t, \vec{x})$$

# Can BH superradiance be induced by galactic plasmas?

J. Conlon, C. Herdeiro, PHYS LETT B 780 (2018):

- Superradiance most efficient for

$$M\mu = \left(\frac{M}{M_\odot}\right)\left(\frac{\mu}{10^{-10}\ \text{eV}}\right) \sim 1$$

- Conditions in ISM seem ideal for *stellar mass BHs*:

$$n_e \sim 10^{-3}\text{–}10\ \text{cm}^{-3} \iff \mu \sim 10^{-12}\text{–}10^{-10}\ \text{eV}$$

- PROBLEM: accretion!

# The briefest review of accretion theory

Based on the accretion rate ($\dot{M}$ in units of $\dot{M}_{Ed}$), three scenarios:

- $\dot{M} \gtrsim 1$: hot, dense, "thick" disk
- $0.01 \lesssim \dot{M} \lesssim 1$: cold, geometrically "thin" disk
- $\dot{M} \lesssim 0.01$: hot, low density, advection dominated accretion flow (ADAF)
- $\dot{M} \ll 1$: spherically symmetric Bondi accretion

# Superradiance in plasma: a toy model

Background: Kerr

Perturbations: massive scalar

Mass profile:

| | | |
|---|---|---|
| **(I)** | $\mu_0^2(r) = \mu_H^2 \left(\frac{r_+}{r}\right)^\lambda$ | Bondi (sph. sym.) |
| **(II)** | $\mu_0^2(r)\sin^2\theta$ | ADAF ("thick" disk) |
| **(III)** | $\mu_0^2(r) + \mu_\infty^2$ | Bondi + $\mu_\infty$ |
| **(IV)** | $\mu_0^2(r)\sin^2\theta + \mu_\infty^2$ | ADAF + $\mu_\infty$ |
| **(V)** | $\mu_1^2(r) = \mu_H^2 \Theta(r - r_0)\left(1 - \frac{r_0}{r}\right)\left(\frac{r_0}{r}\right)^\lambda$ | Bondi + inner edge |
| **(VI)** | $\mu_1^2(r)\sin^2\theta$ | ADAF + inner edge |
| **(VII)** | **(V)** + $\mu_\infty^2$ | Bondi + inner edge + $\mu_\infty$ |
| **(VIII)** | **(VI)** + $\mu_\infty^2$ | ADAF + inner edge + $\mu_\infty$ |

# Numerical setup

Details in *Sam Dolan*, arXiv:1212.1477:

- $1+1$D decomposition:

$$\Phi(t,r,\theta,\phi) = \sum_{\ell=|m|}^{\infty} \frac{\psi_{\ell m}(t,r)}{r} e^{im\phi} \mathbf{Y}_{\ell m}(\theta)$$

- System of coupled PDEs ($\ell$-mode couples to $\ell \pm 2$)
- O(4) finite difference scheme for spatial derivatives: system of coupled ODEs
- Evolve with Runge-Kutta O(4) time step
- Boundary Conditions:
- At infinity: outgoing BC
- Horizon: artificial damping region (perfectly-matched layers, PML) introduced close to left boundary to kill spurious reflections

# Boundary conditions: Spurious reflections

- Implementation of BC on finite grids: spurious reflections
- Artificial mirror-like instability form right boundary:

$$\nu_{\ell,mirror} \sim r_{mirror}^{-2(\ell+1)}$$

$$r_\infty = 10^3 M \rightarrow \tau_\infty \sim 10^{12} M \gg (\nu_{l=m=1})^{-1} \sim 10^6 M$$

- Left boundary: PML

# Boundary conditions: Perfectly-matched layers

Introduce artificial damping that suppresses spurious reflections from left boundary (approximated horizon)

$$\ddot{u} = u''$$

First-order rearrangement: $\dot{u} = v'$, $\dot{v} = u'$
Solution: $u \sim \exp -i\omega(t \pm x)$

Introduce x-dependent damping term:

$$\dot{u} = v' - \gamma(x)u$$
$$\dot{v} = u' - \gamma(x)v$$

Solution:

$$u \sim \exp\left(-i\omega(t \pm x) \pm \int_y^x \gamma(y)dy\right)$$

Exponential attenuation for both left and right going waves in PML!