Spontaneous scalarization in generalised scalar-tensor theory

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Contents

Introduction

- Definition of spontaneous scalarization (SP): A mechanism that endows systems of relativistic stars and black holes with a nontrivial scalar configuration
- SP is probably the most promising mechanism that can reveal strong-gravity deviation from General Relativity (GR)
- Necessary and sufficient condition for the trigger of SP through tachyonic instability: The squared effective mass that appears at the linear scalar equation becomes negative
- However, trigger of SP does not guarantee stable hairy solutions; nonlinear terms determine their end state.

Brief history of Spontaneous Scalarization

- In 1993, Damour and Esposito-Farese (Phys. Rev. Lett. 70, 2220) have non-perturbatively shown that deviation from GR can occur in systems with neutron stars in the context of a certain scalar tensor theory (DEF).
- In 2018, three independent groups (e.g. arXiv:1711.02080) have shown that in the context of scalar Gauss-Bonnet gravity (sGB) there is a similar mechanism for systems of black holes
- Therefore, we were motivated study more general theories (of which all the studied theories are subclasses) for which we are led to a tachyonic instability

Description of our work

We started from the Horndeski action,

$$S = \frac{1}{2\kappa} \sum_{i=2}^{5} \int \mathrm{d}^4 x \sqrt{-g} \mathcal{L}_i + S, \qquad (1)$$

where $\mathcal{L}_i$ contains:

- $G_i(\phi, X)$ [i=1 to 5] and their first derivatives w.r.t X [$X \equiv -\nabla_\mu \phi \nabla^\mu \phi / 2$]
- derivatives of ϕ up to second order
- R and $G_{\mu\nu}$

By varying the action w.r.t the metric $g^{\mu\nu}$ and the scalar field ϕ:

$$\sum_{i=2}^{5} \mathcal{G}^i_{\mu\nu} = \kappa T_{\mu\nu}, \qquad (2)$$

$$\sum_{i=2}^{5} \left(P^i_\phi - \nabla^\mu J^i_\nu \right) = 0, \qquad (3)$$

We then impose that the Gi functions are analytical at X = 0 to ensure that are theories are solutions of GR. Therefore,

$$G_i = g_{i0}(\phi) + g_{i1}(\phi)X + \dots . \tag{4}$$

However, we should also consider the case of the sGB Gi functions which are non-analytic but they generate an analytic representation of the action. For that reason,

$$G_i(\phi, X) = \tilde{G}_i(\phi, X) + G_i^{GB}(\phi, X), \tag{5}$$

$$\tilde{G}_i(\phi, X) = g_{i0}(\phi) + g_{i1}(\phi)X + \dots \tag{6}$$

Then we substituted the functions of Eq. (5) into the scalar field equation (3) and kept only the terms that contribute to the linearization of the equation around a constant value $\phi = \phi_0$ and we got,

$$\tilde{g}^{\mu\nu}\nabla_\mu\nabla_\nu\phi + \frac{g_{20\phi} + g_{40\phi}R + \xi^{(1)}\mathscr{G}}{A(\phi)} = 0, \tag{7}$$

where

$$A(\phi) \equiv g_{21} + g_{41}R. \tag{8}$$

and where $\xi(\phi)$ is a generic function of ϕ and $\mathscr{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ is the GB invariant.

We then imposed that $\phi = \phi_0$ at the simplified scalar equation. We conluded, that there are two distinct cases for which we retrieve GR:

$$\text{case I:} \quad {g^0}_{20\phi} + {g^0}_{40\phi}R + {\xi^{(1)}}_0\mathscr{G} = 0, \quad A_0 \text{ finite}; \tag{9}$$

$$\text{case II:} \quad g^0_{20\phi} + g^0_{40\phi}R + \xi^{(1)}_0\mathscr{G} \neq 0, \quad A_0 \to \infty, \tag{10}$$

Then by linearizing our simplified scalar field equation for small $\delta\phi = \phi - \phi_0$ we got,

$$\tilde{g}^{\mu\nu}\nabla_\mu\nabla_\nu\delta\phi - {m_I}^2\delta\phi - {m_{II}}^2\delta\phi = 0, \tag{11}$$

where

$$ {m_I}^2 = -\frac{g^0_{20\phi\phi} + g^0_{40\phi\phi}R + \xi^{(2)}_0\mathscr{G}}{A_0}, \tag{12}$$

$$ {m_{II}}^2 = \frac{g^0_{20\phi} + {g^0}_{40\phi}R + \xi^{(1)}_0\mathscr{G}}{A_0^2}\frac{\partial A}{\partial\phi}\bigg|_{\phi_0} \tag{13}$$

Theories which can exhibit a tachyonic instability around a GR background either satisfy the case I condition and have ${m_I}^2 < 0$ or they satisfy the case II condition and have ${m_{II}}^2 < 0$.

Minimal action compatible with GR solutions with only terms that contribute to the linearized equation:

$$S = \int \mathrm{d}^4x \frac{\sqrt{-g}}{2\kappa}\Big[R - 2\Lambda + (1 + a_{41} R)X + \frac{m_\phi^2\phi^2 + \alpha\phi^2 R + \beta\,\phi^2\,\mathscr{G}}{2}\Big] + S_m, \quad (14)$$

where all the new terms that where introduced for simplicity are functions of the g terms that contribute to the linearized equation.

Next step was to map the above action, with already know model that can exhibit SP and check if there are any new models.

After a certain choice of parameters ($m_\phi = \alpha = 0, \alpha_{41} = 0$) we obtain our first action,

$$S = \int \mathrm{d}^4 x \frac{\sqrt{-g}}{2\kappa} \left[R - \frac{1}{2} \nabla^\mu \phi \nabla_\mu \phi + \beta \phi^2 \mathscr{G} \right] + S_m, \qquad (15)$$

However, this model is the known scalar-Gauss-Bonnet scalarization model.

Then, if we set $m_\phi = \beta = 0$ and after some scalar field redefinitions we can get,

$$S = \int \mathrm{d}^4 x \frac{\sqrt{-g}}{2\kappa} \left[\Phi R - \frac{\omega(\Phi)}{\Phi} \nabla^\mu \Phi \nabla_\mu \Phi \right] + S_m \,, \qquad (16)$$

where Φ is a function of ϕ. After some further scalar field redefinitions, it can be shown that this model is the already known DEF model.

But what changes if we also consider the term a_{41}?

$$g_{\mu\nu} \to C(\phi)\left[g_{\mu\nu} + D(\phi)\nabla_\mu\phi\nabla_\nu\phi\right]. \tag{17}$$

$\Rightarrow$ Our minimal action remains formally invariant

We also prove that the condition a_{41} is equivalent to a specific type of disformal coupling. Therefore, when a_{41} term is nonzero, it's producing already known scalarization models which are disformally coupled to matter.

Conclusion

Theories within Hordenski theory that can exhibit spontaneous scalarization triggered by tachyonic instability:

- DEF theory with scalar field potential
- sGB theory with scalar field potential
- Any of the above with disformal coupling to matter
- Combination of DEF and sGB theory with scalar field potential and disformal coupling to matter

This study is important because it narrowed down the possible theories with interesting strong field phenomenology

Possible future research directions

In the context of the combination of DEF and sGB theory with scalar field potential and disformal coupling to matter:

- Perturbative study of the threshold of the instability of the system of neutron stars when non-linear terms are added
- Full analysis of perturbation that includes the backreaction on the metric, for quantitative estimates of the thresholds
- Stability of the scalarized system of black holes and neutron stars through numerical simulations with or without non-linear terms
- Differences of possilble stable solutions with solutions in the context of DEF or sGB with or without non-linear terms
- Lastly, same analysis as the one with did with consideration of functions of the square root of X or other non-analytic functions which might produce models we missed

The End

Thank you for your attention!